POTSDAM INSTITUTE FOR
CLIMATE IMPACT RESEARCH

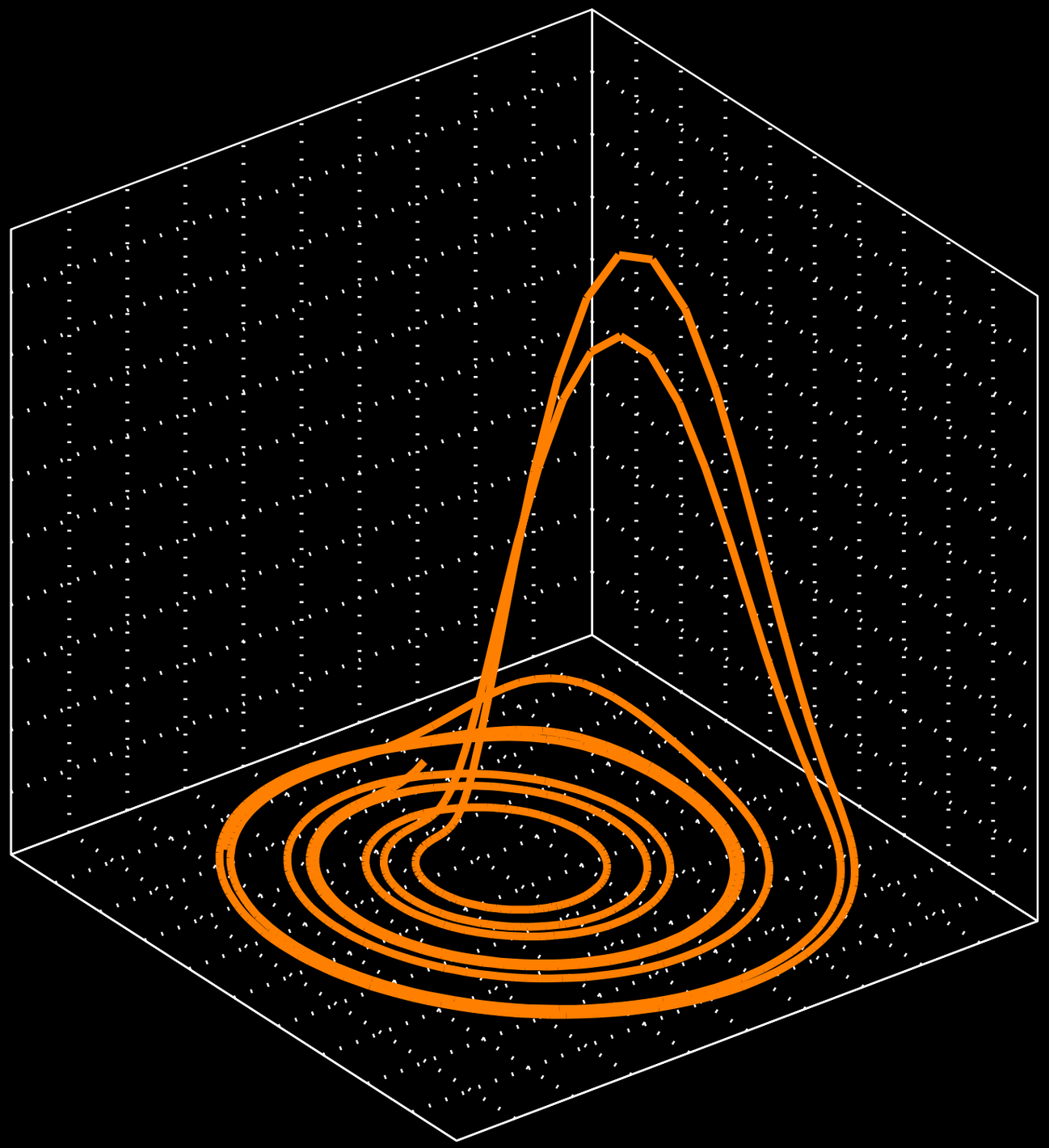
Norbert Marwan, Niels Wessel, Jürgen Kurths

Complex Network Approach

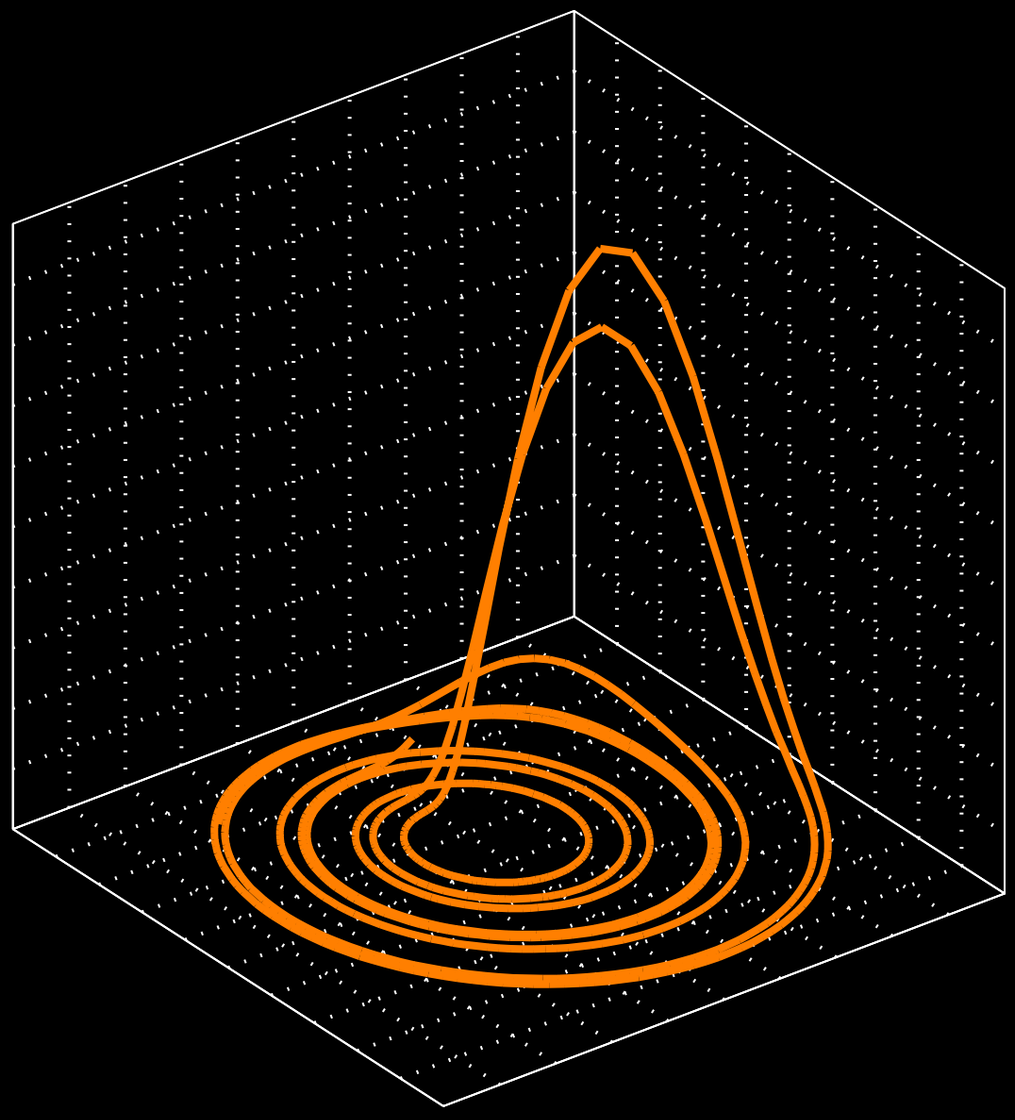
for Recurrence Analysis of Cardiovascular Oscillations

Dynamical System

- States represented in phase space
- Trajectory represents the system‘s dynamics
- Recurrences



Recurrence Plot



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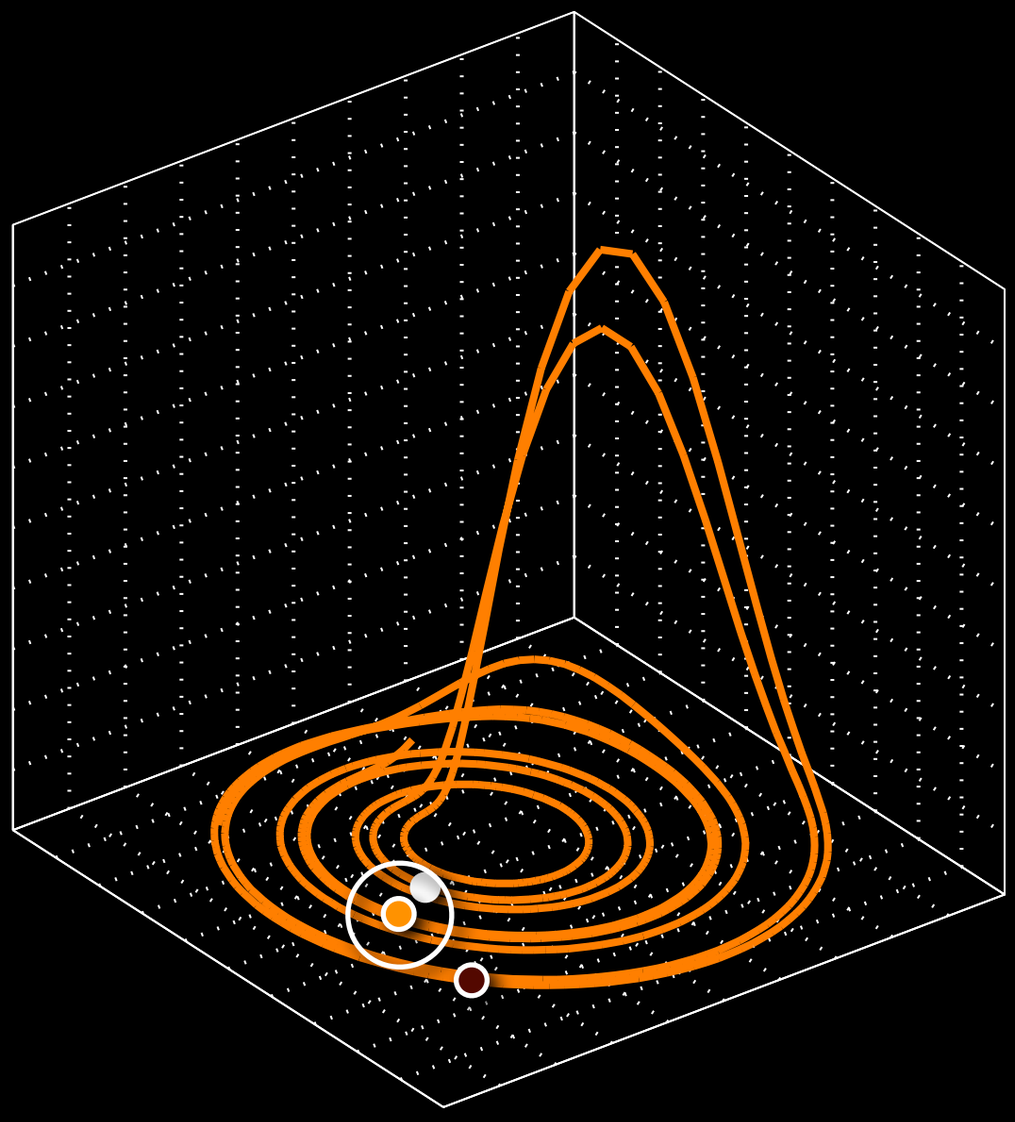

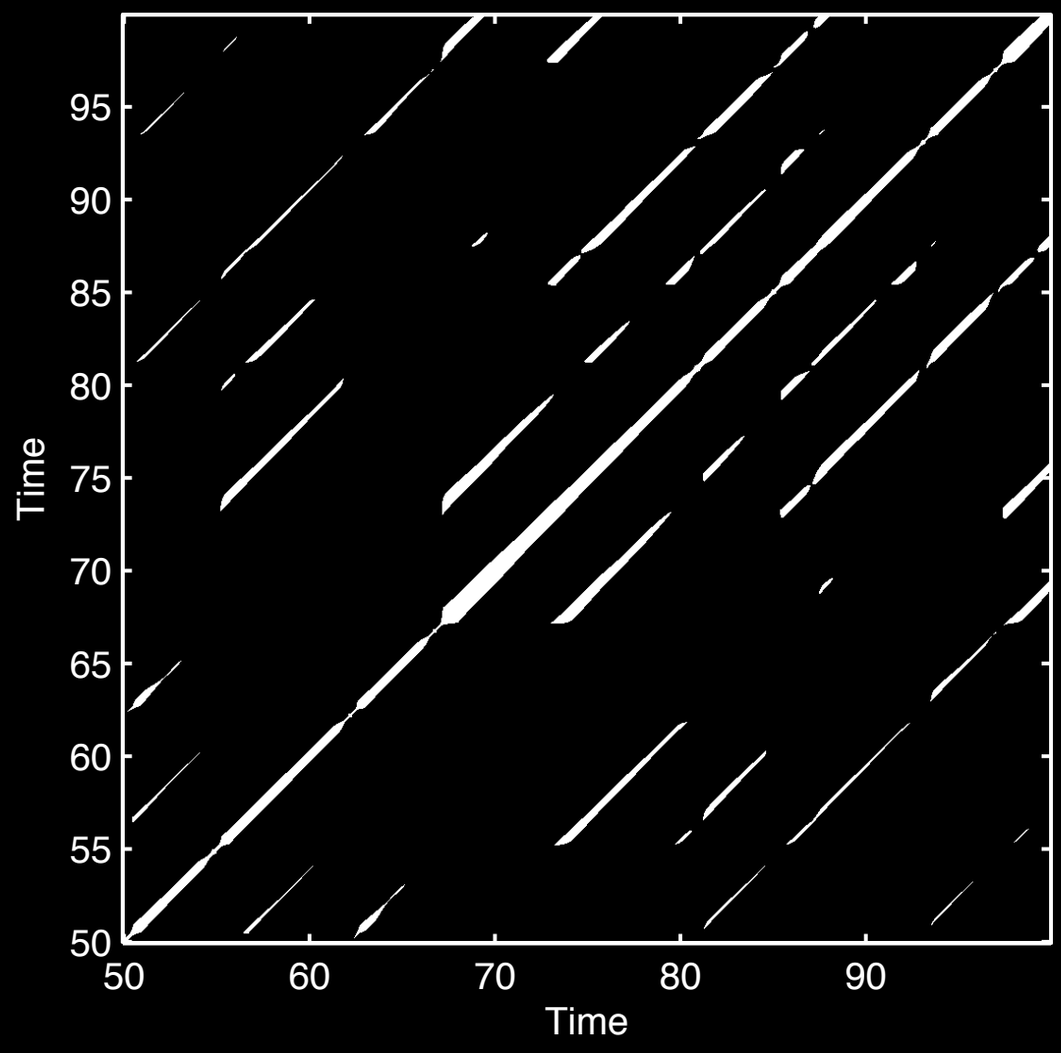

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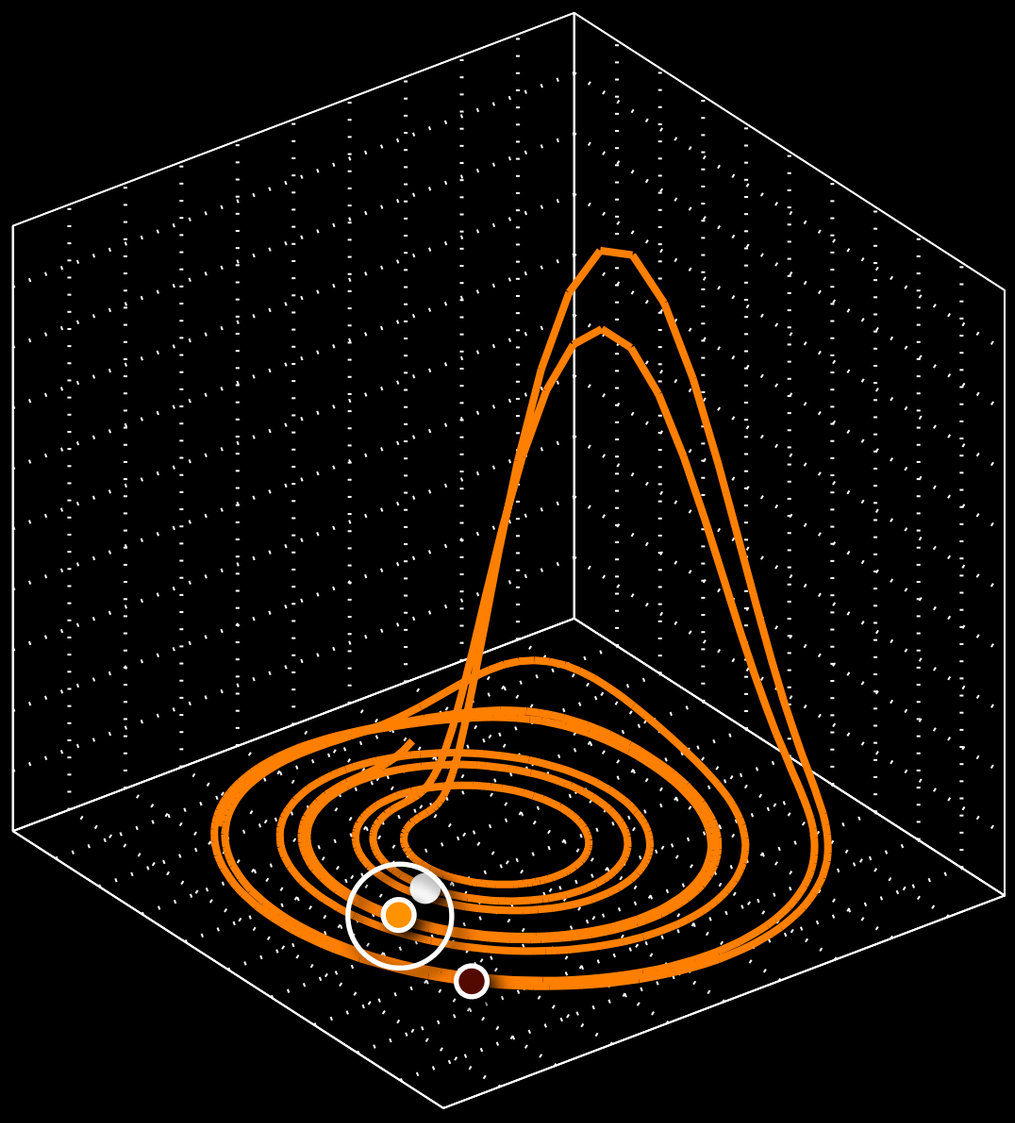
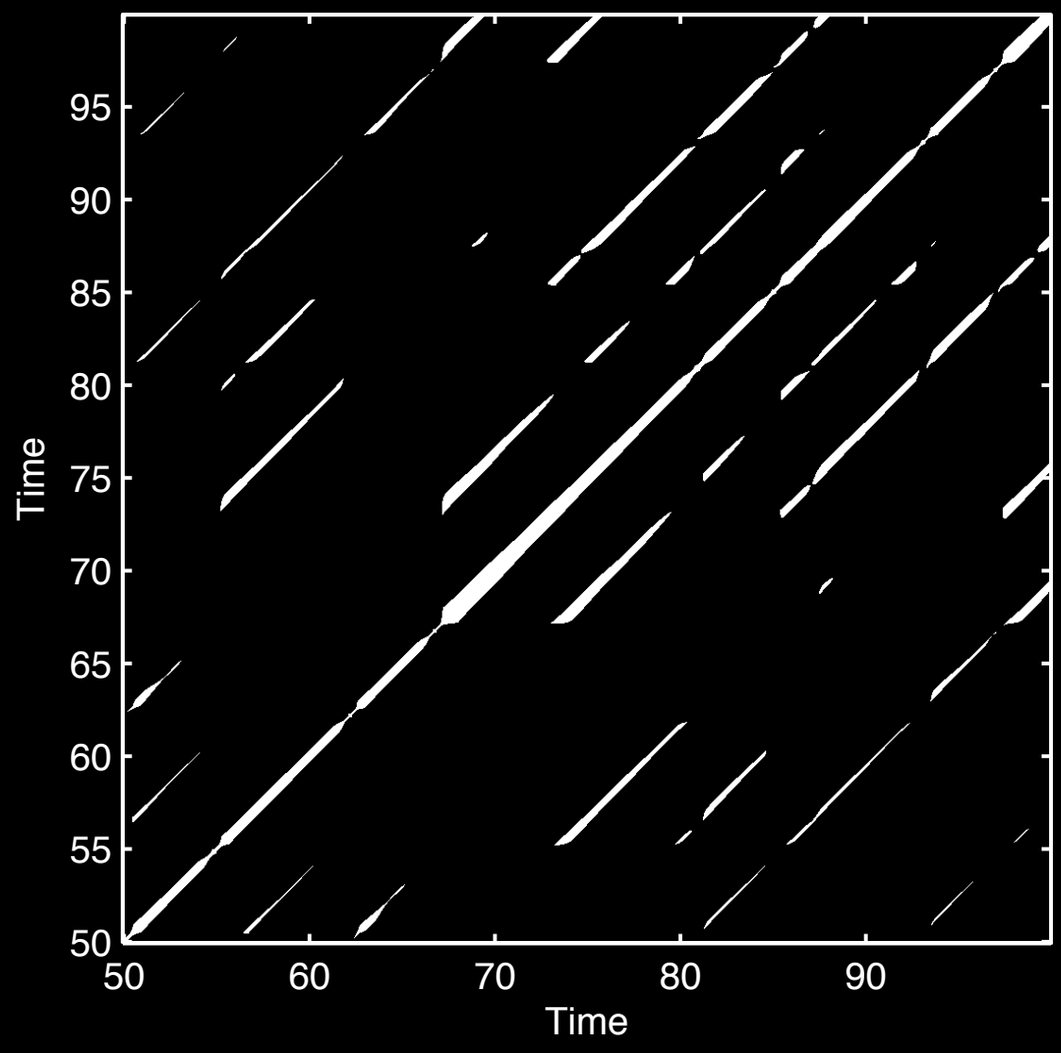
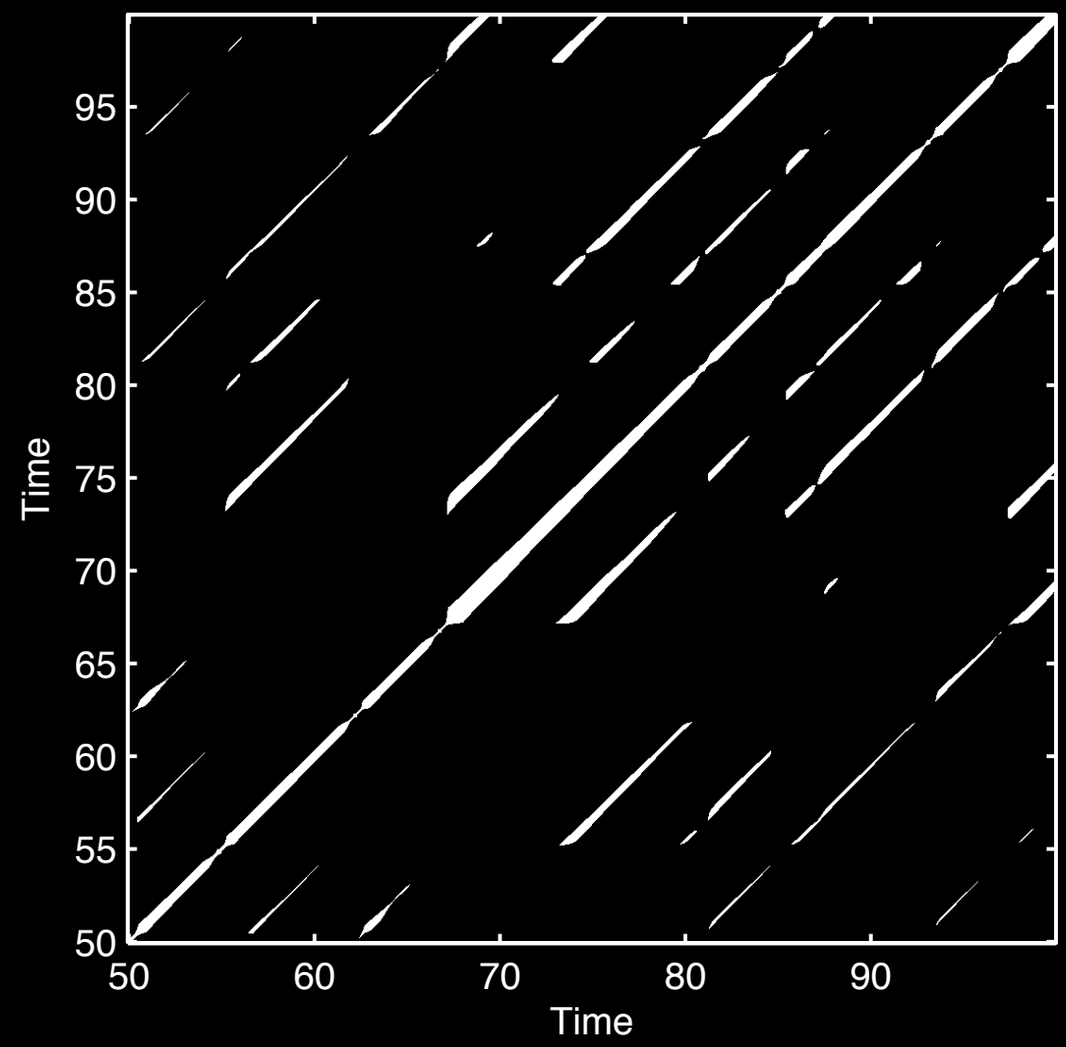




- recurrence plot: symmetric and binary matrix

Recurrence Plot

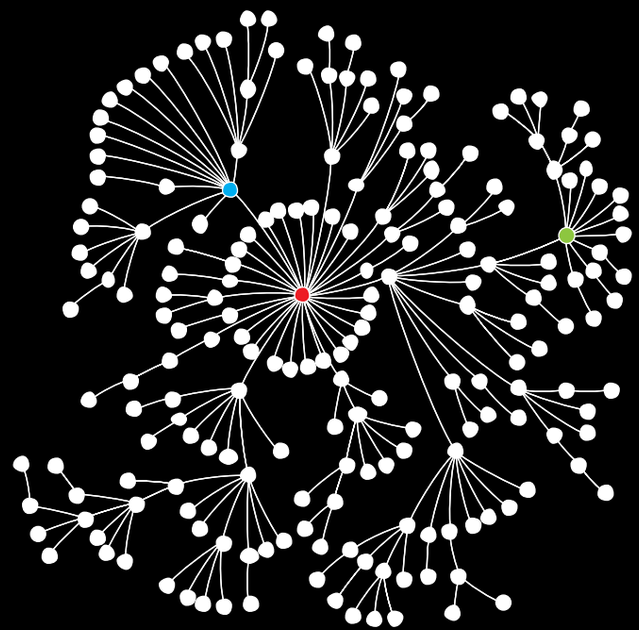
- Transition detection
- Differentiate dynamics
- Finding time scales
- Synchronisation analysis
- etc.

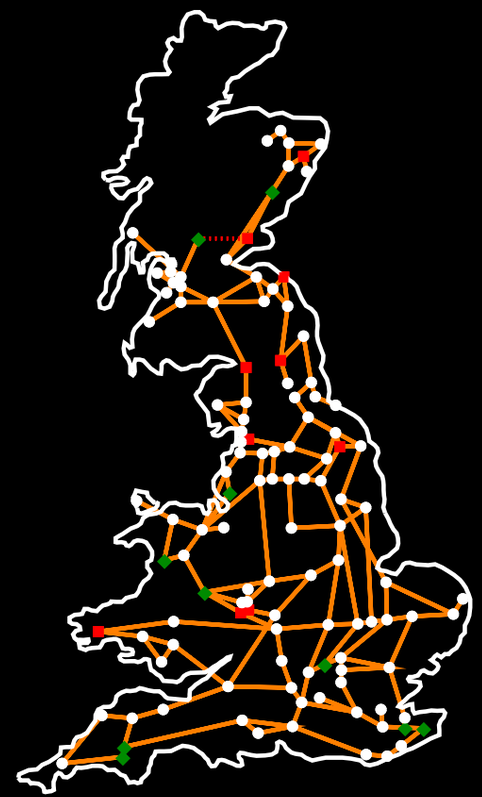


- recurrence plot: symmetric and binary matrix

Complex Networks

- Successful applications in many fields
  - social networks
  - brain dynamics
  - power grids
  - metabolic networks etc.





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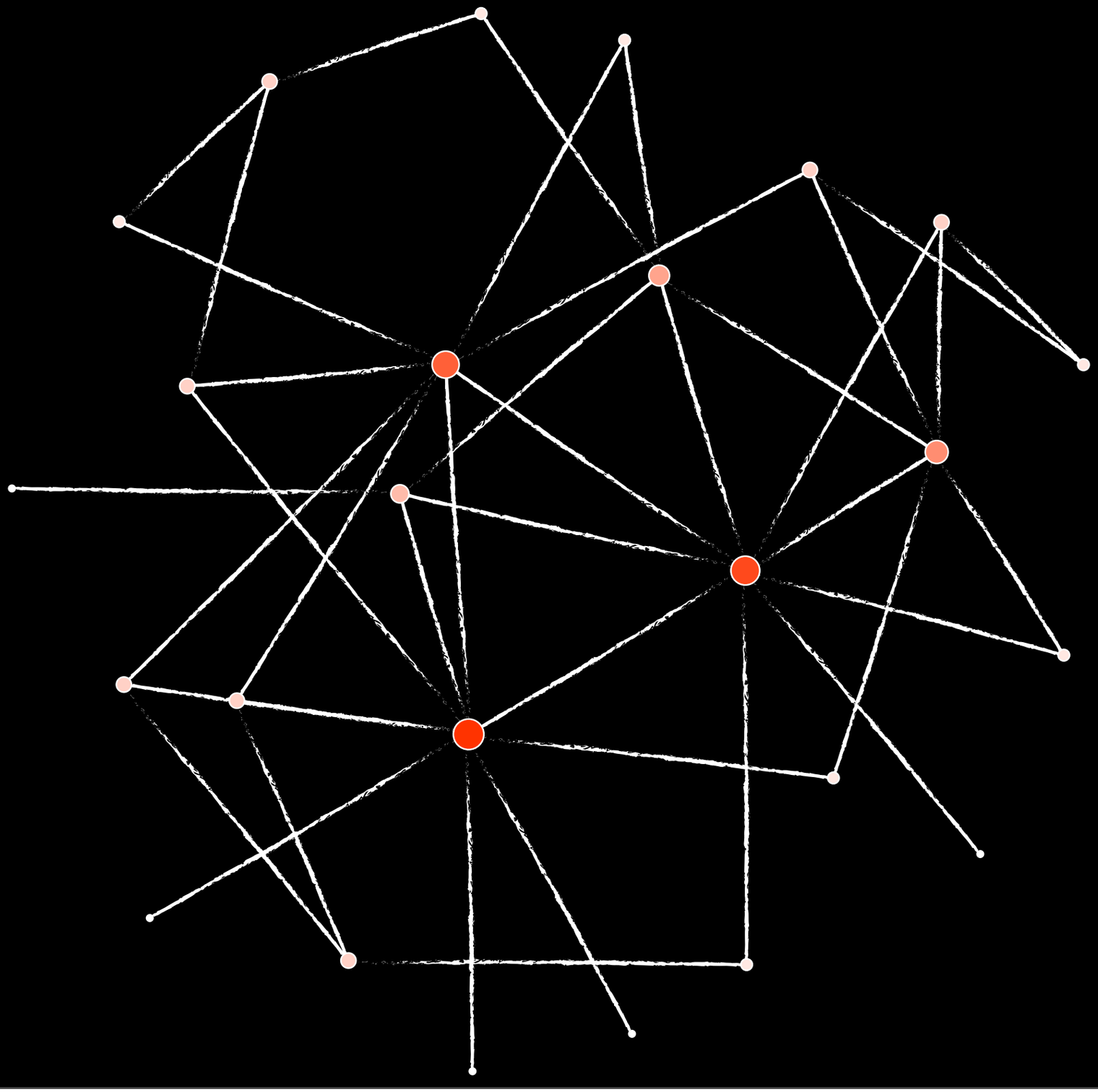

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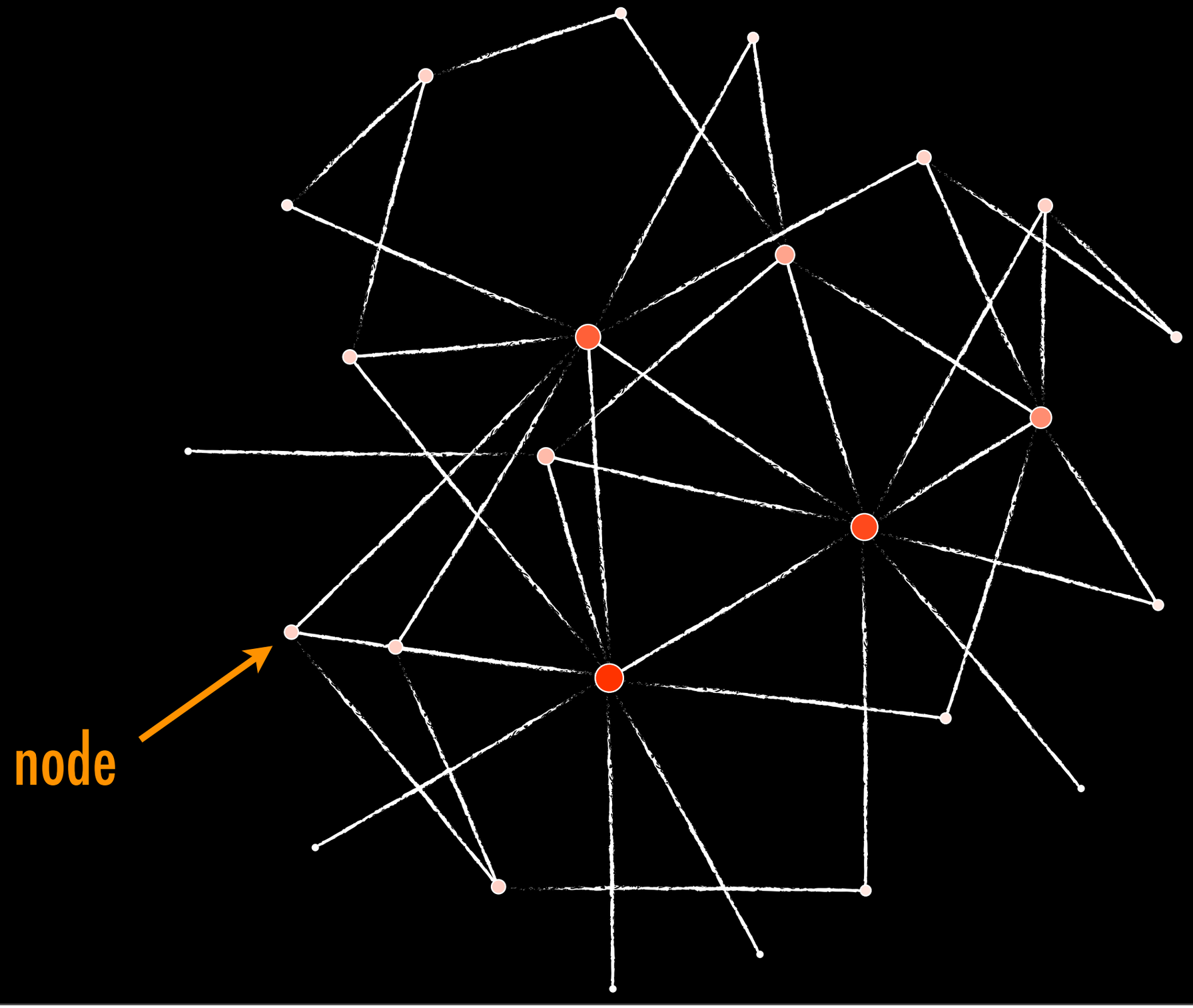

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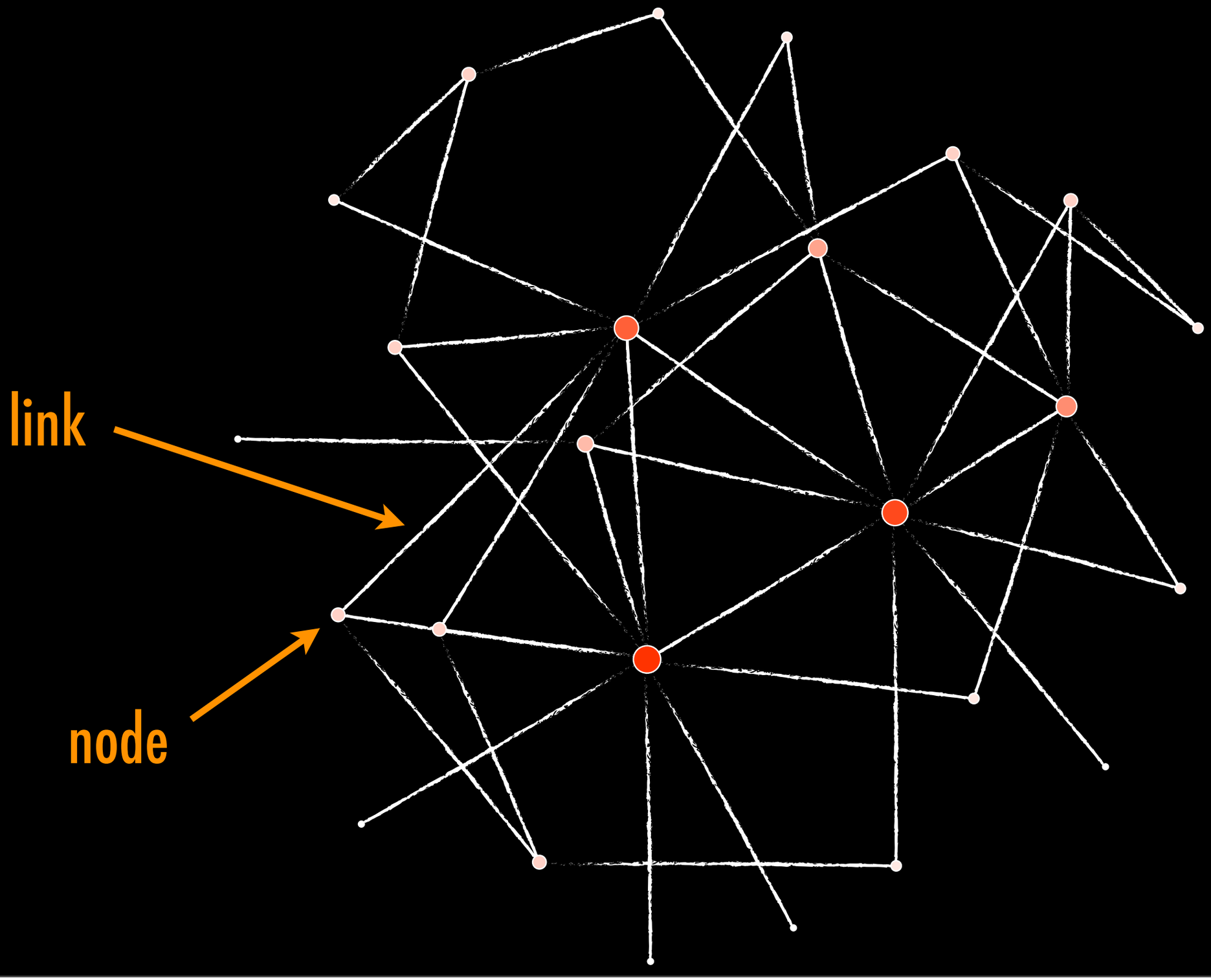

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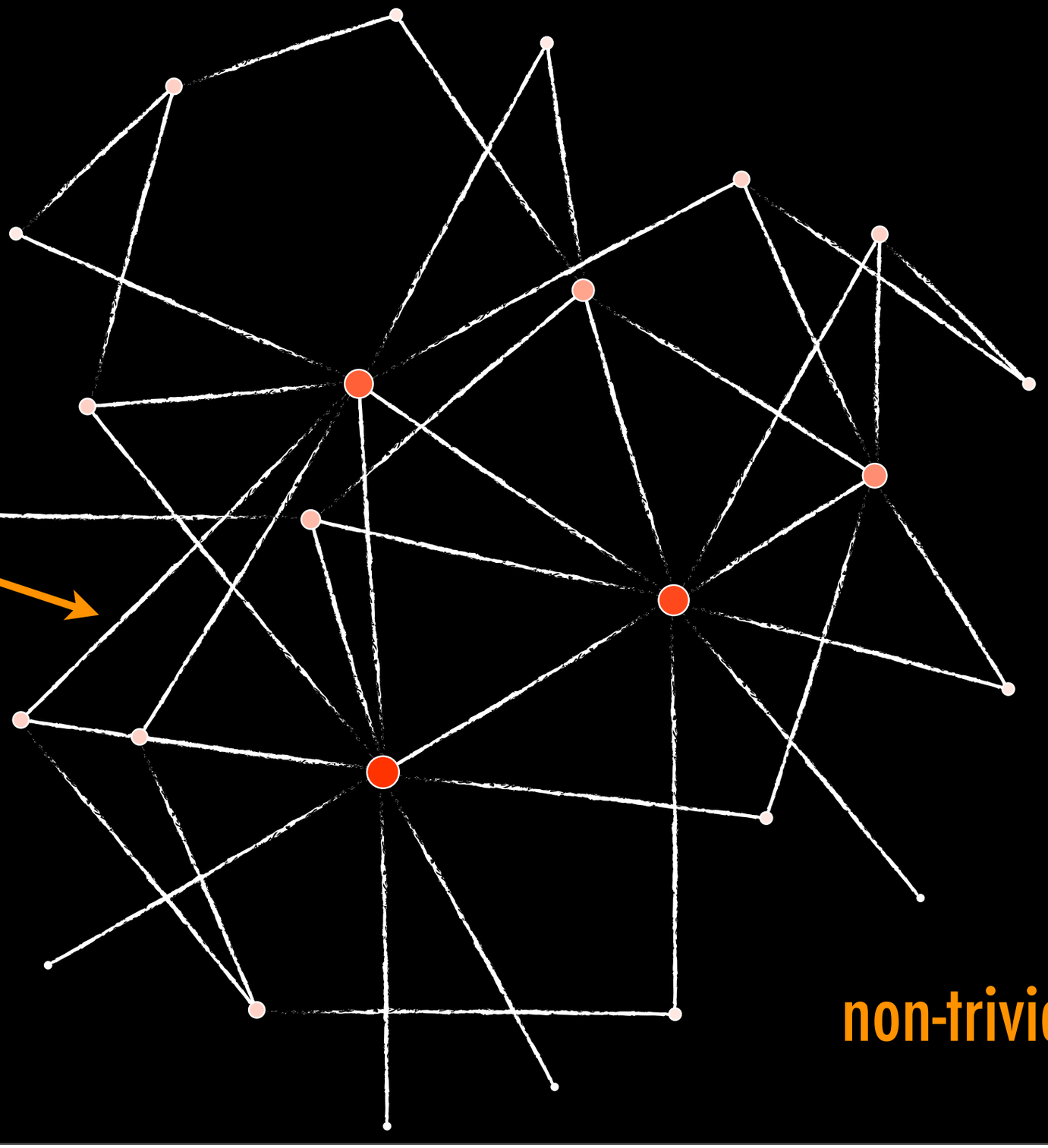
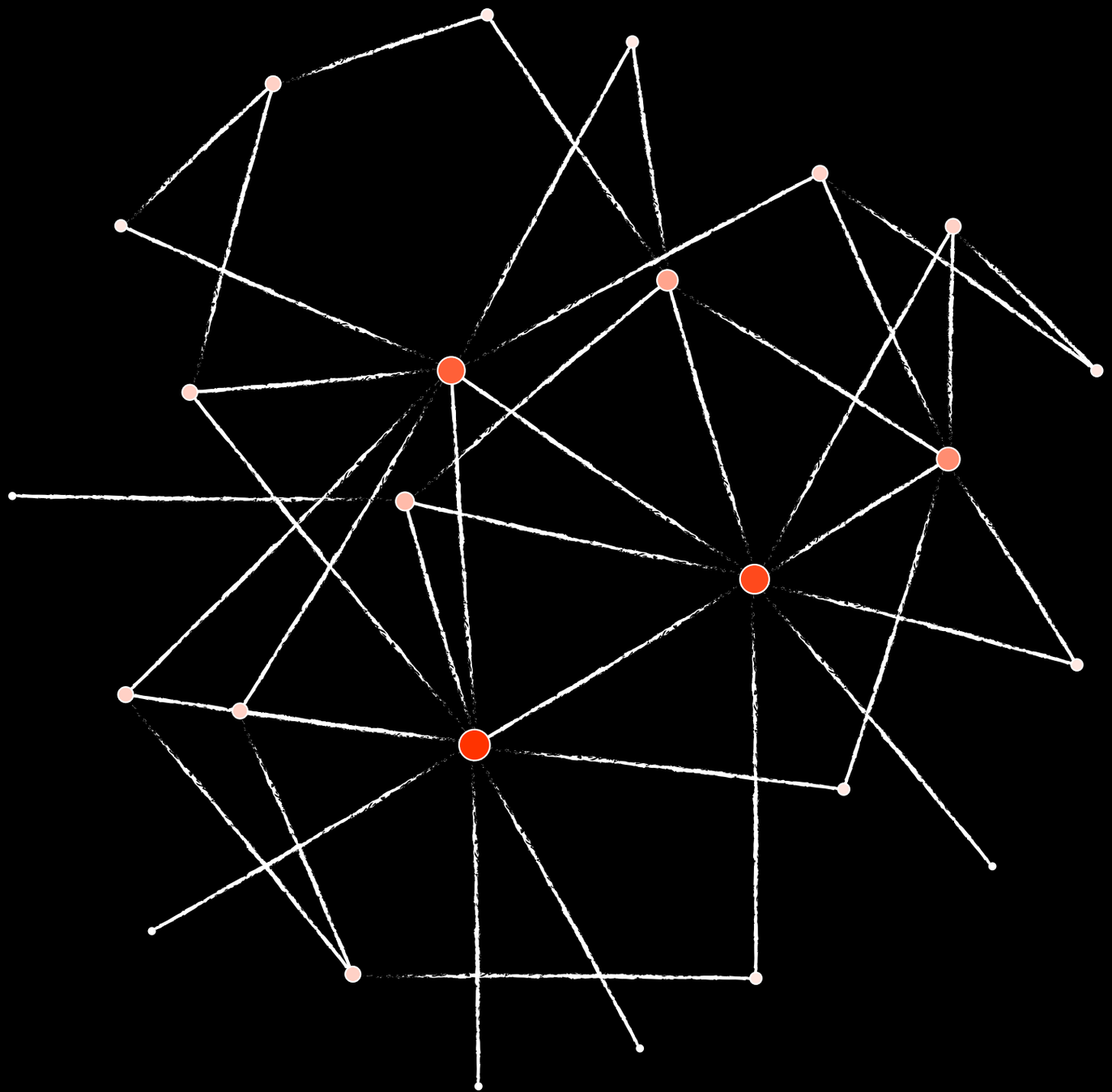


link

node

non-trivial topology

Complex Networks



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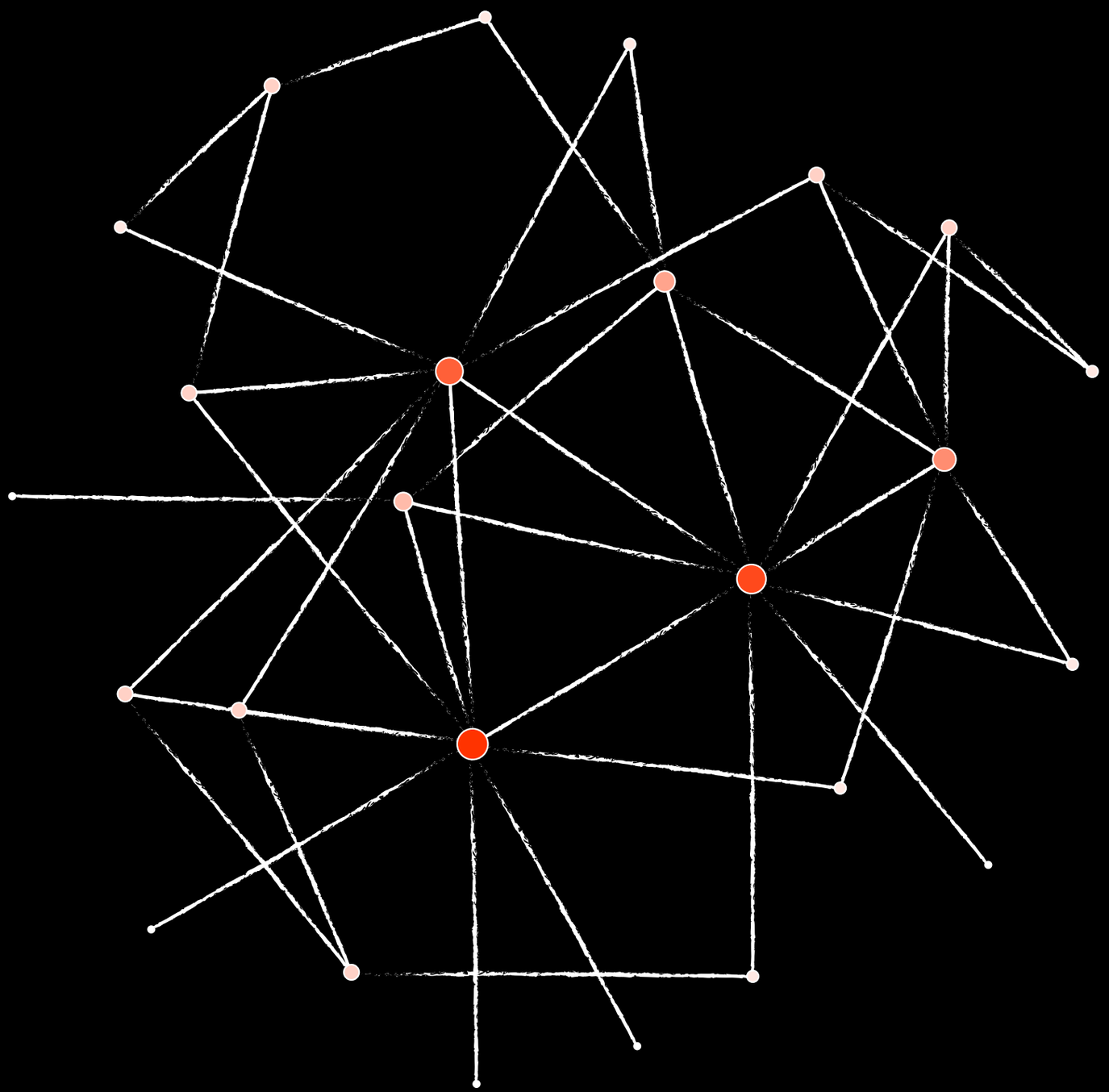

- link matrix (undirected, unweighted network):
  - binary
  - symmetric

$A_{i,j} =$

0	1	0	0	1
1	0	1	0	1
0	1	0	0	0
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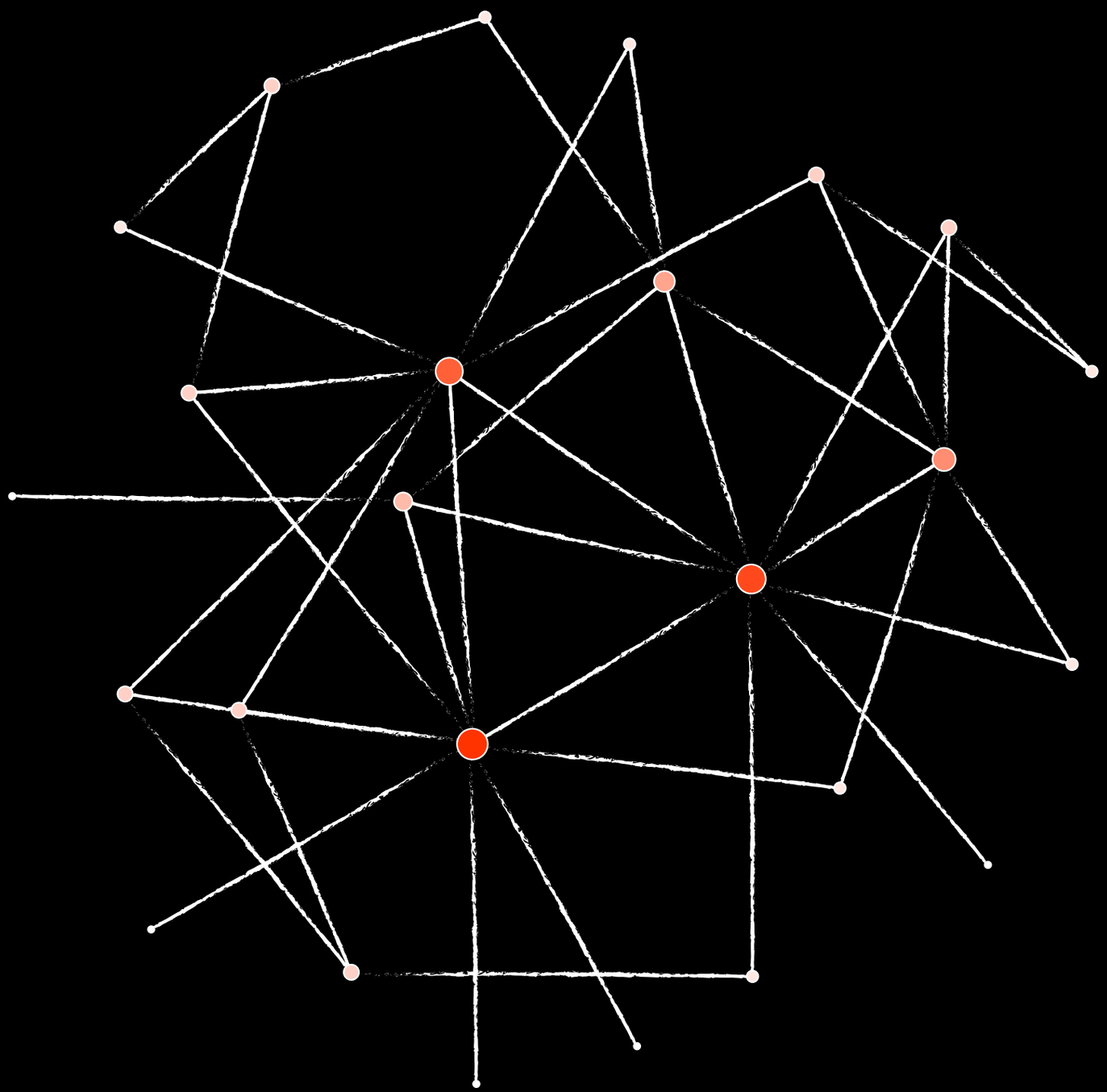
Complex Networks



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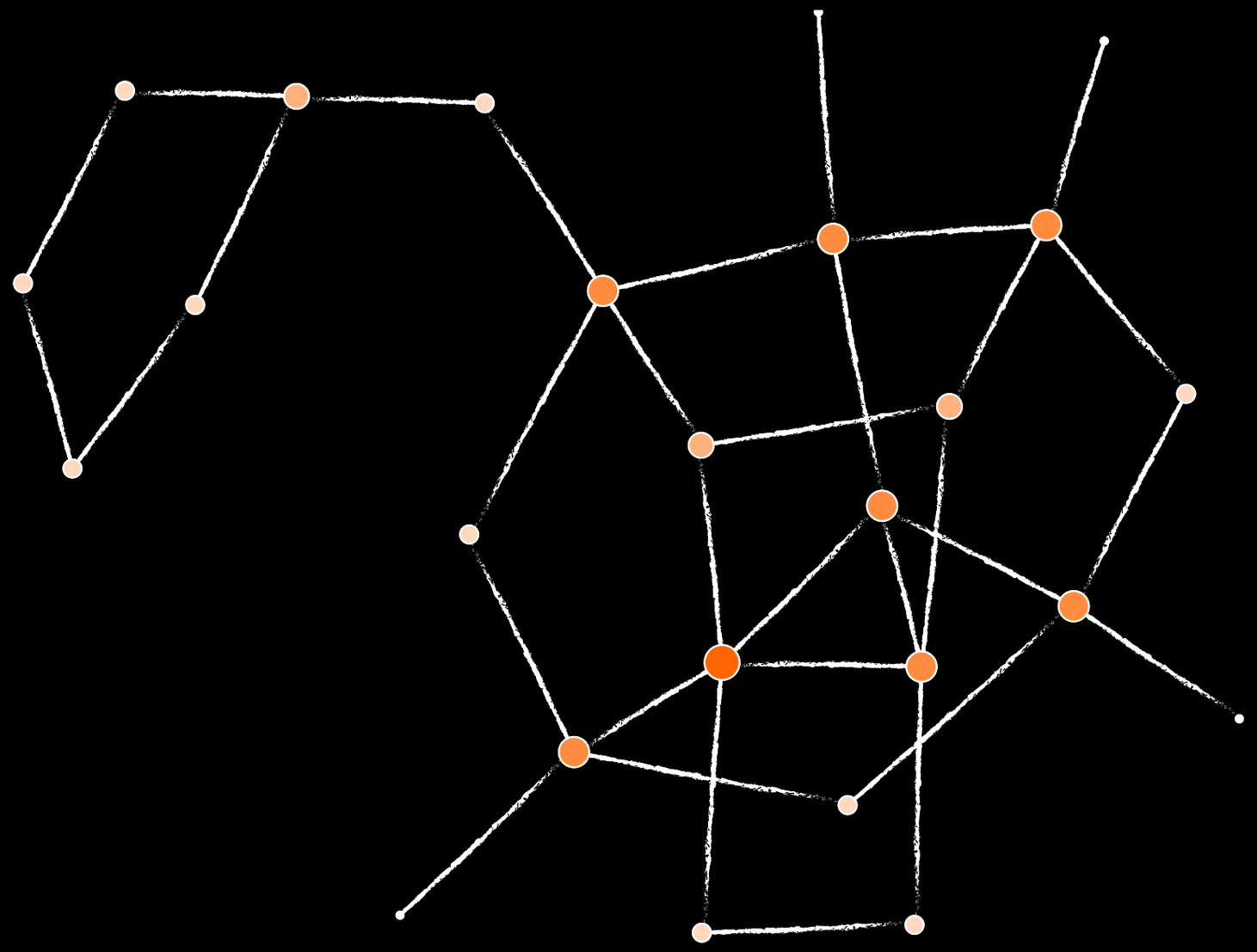
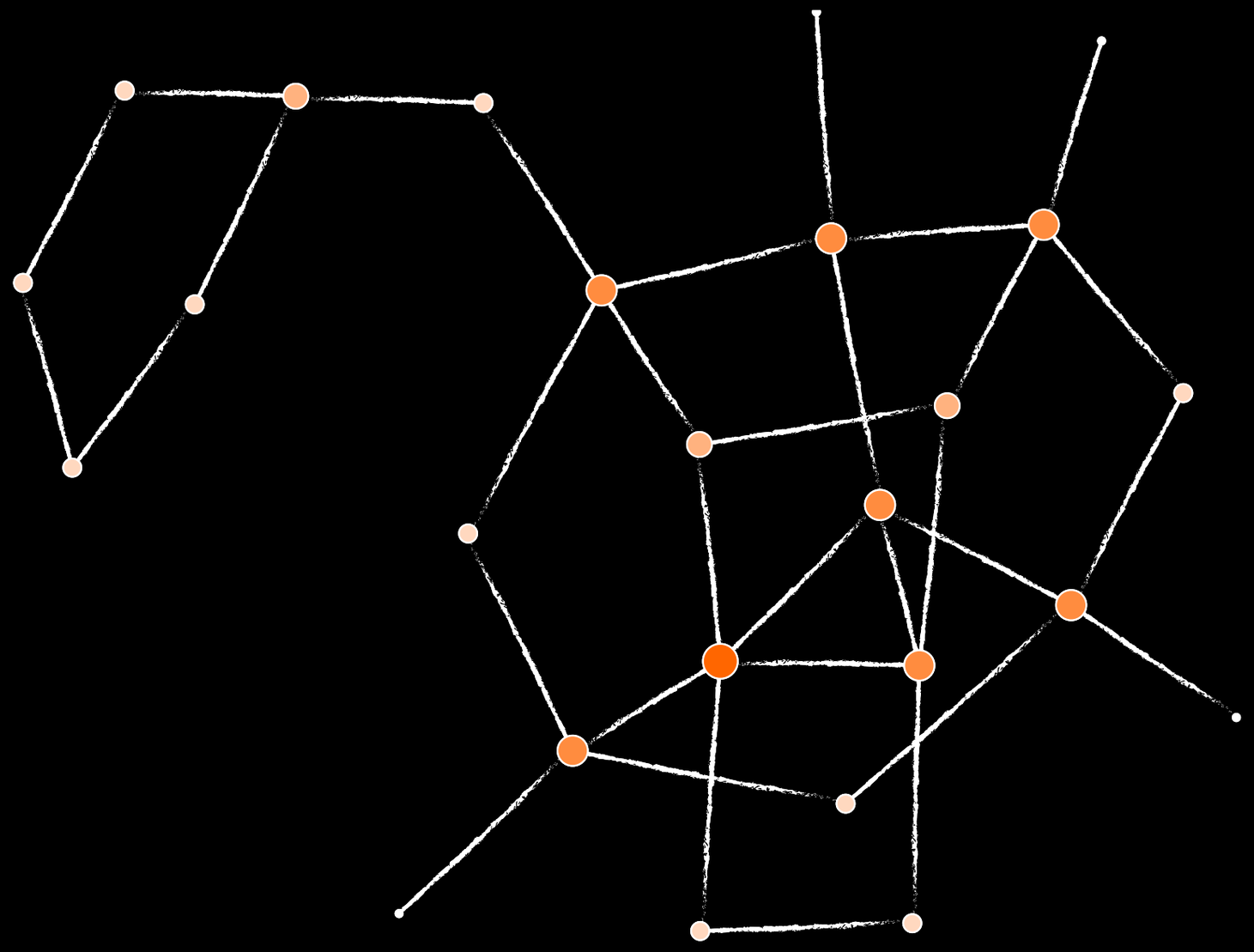
- link matrix: similar to recurrence plot

Complex Networks



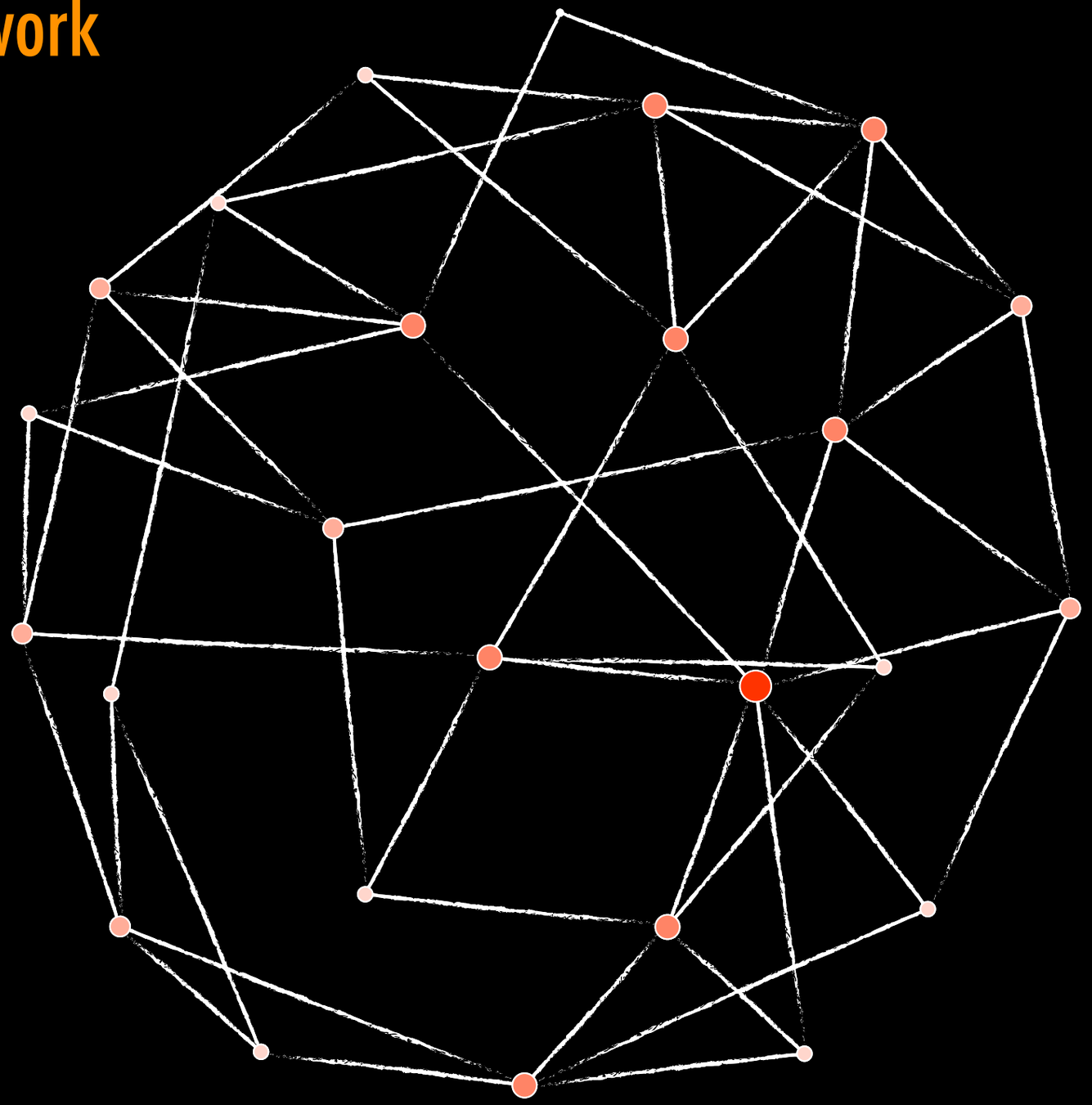
Complex Networks

random network



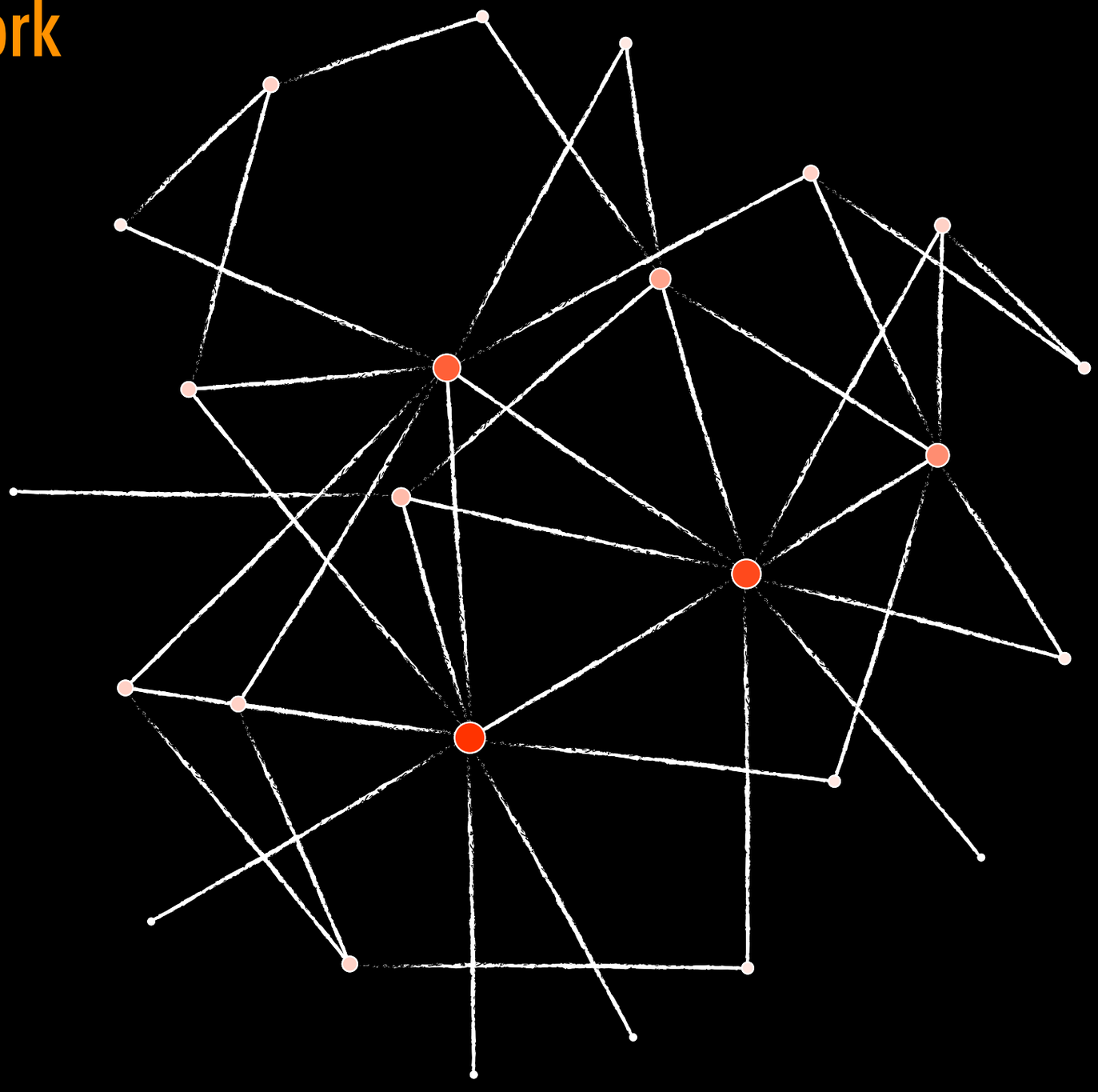
Complex Networks

small-world network



Complex Networks

scale-free network



Complex Networks

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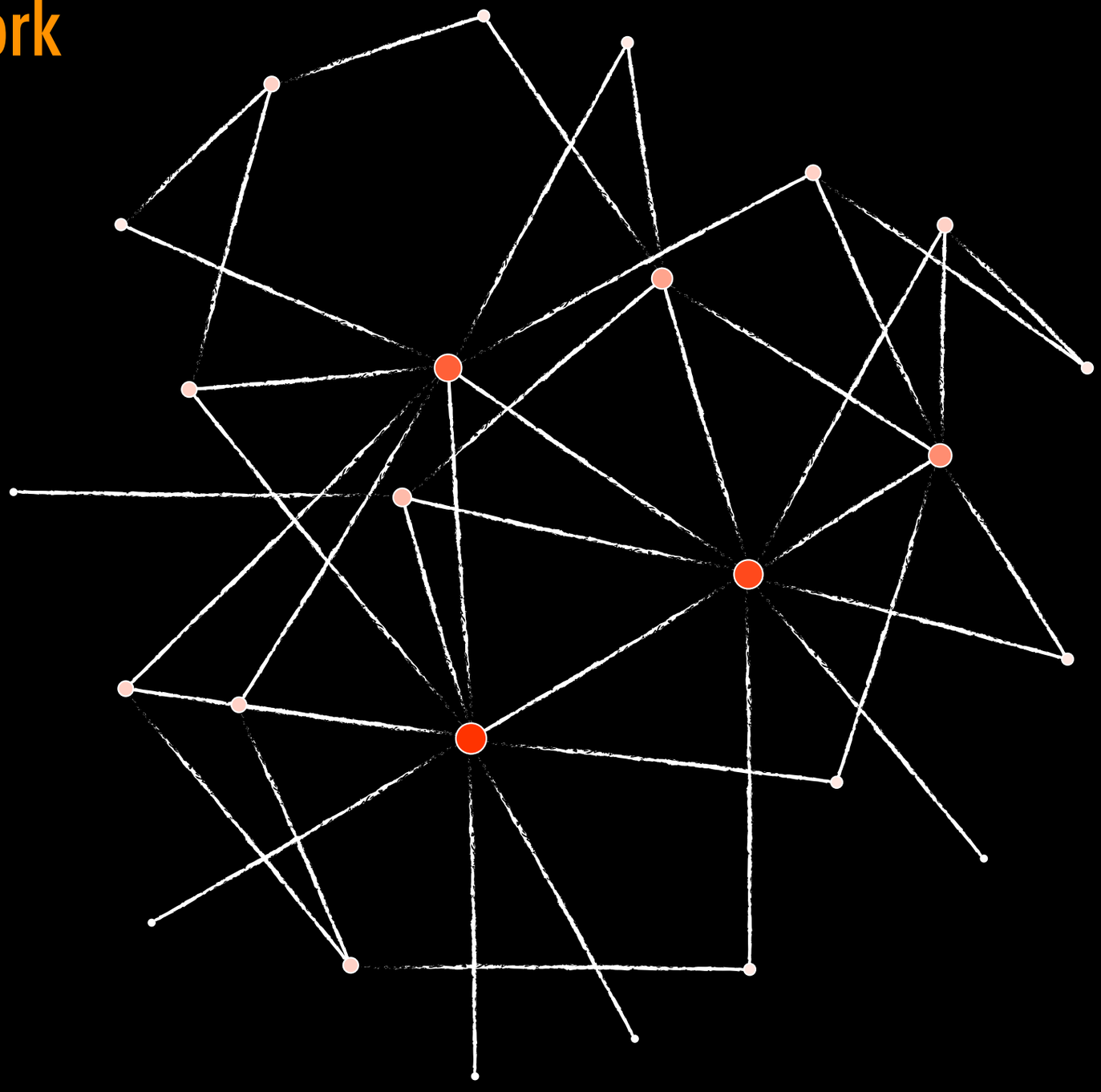

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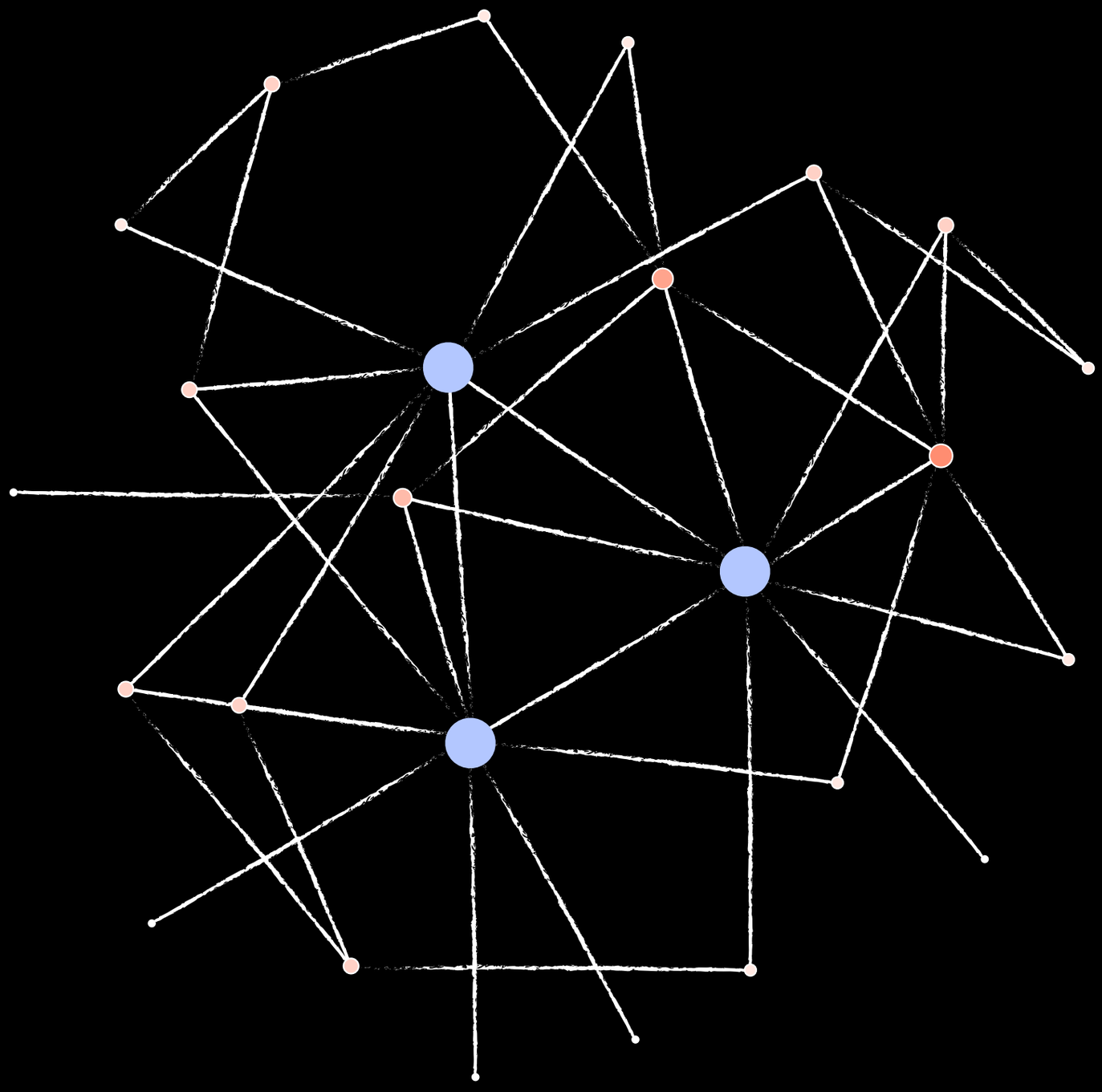

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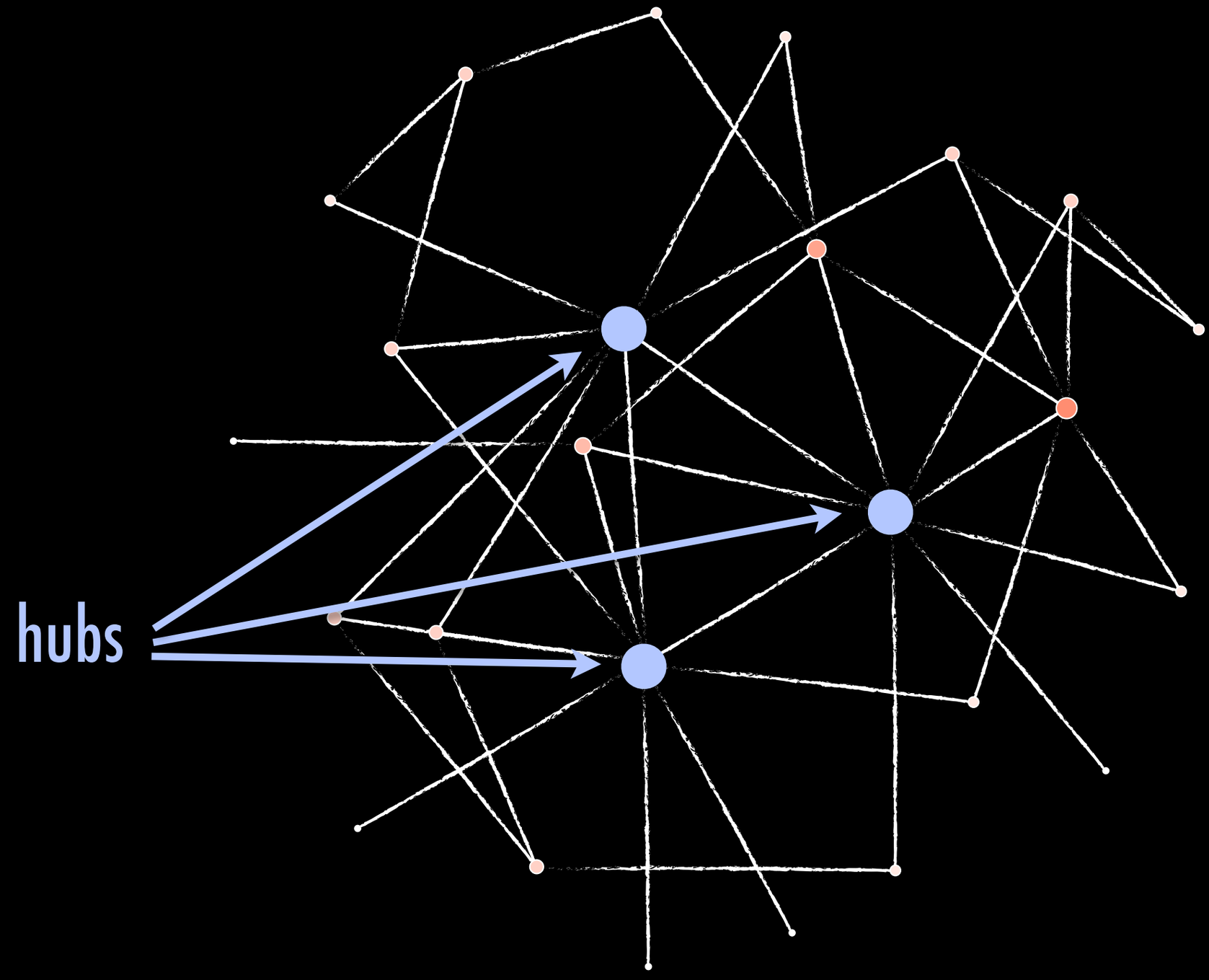

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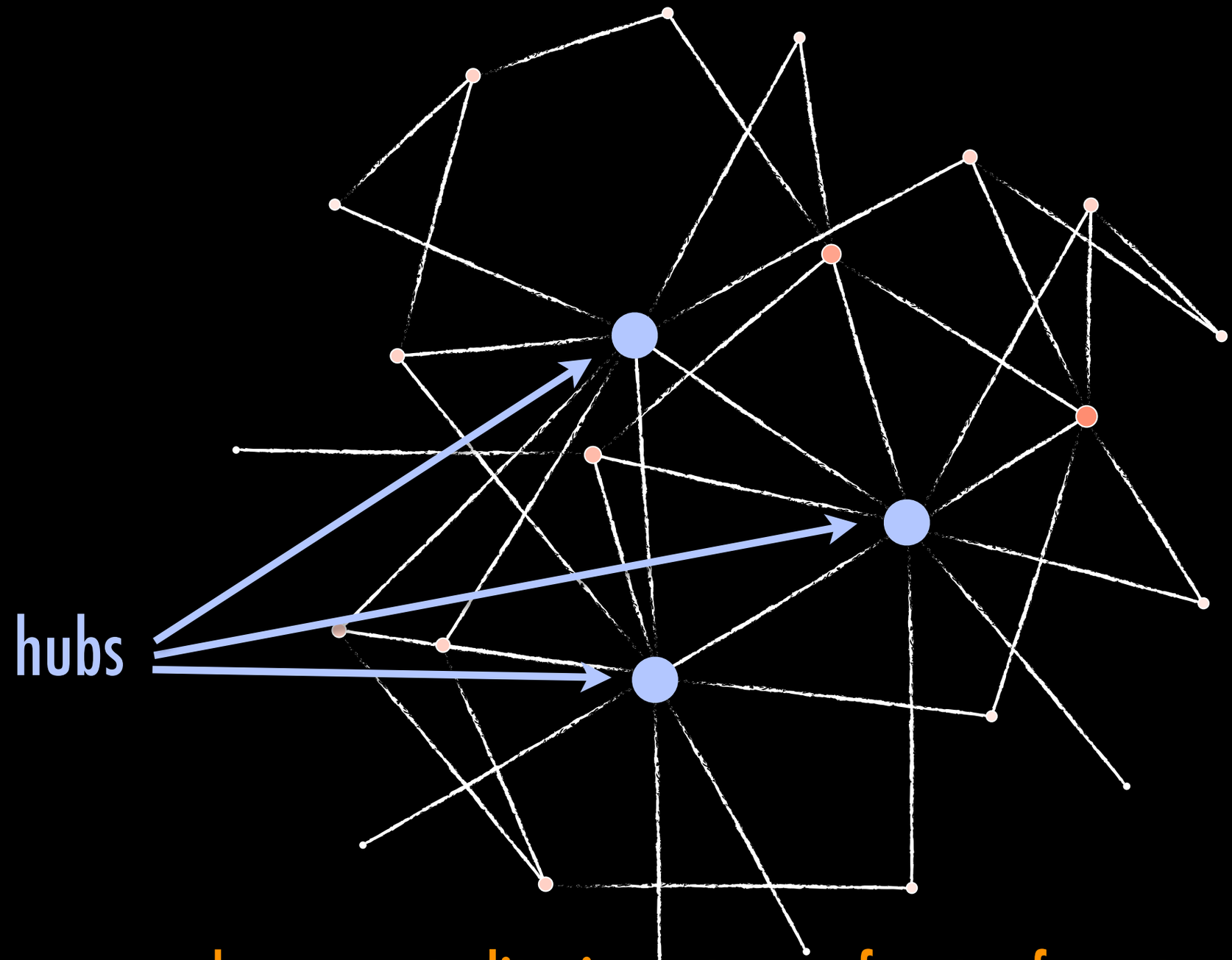
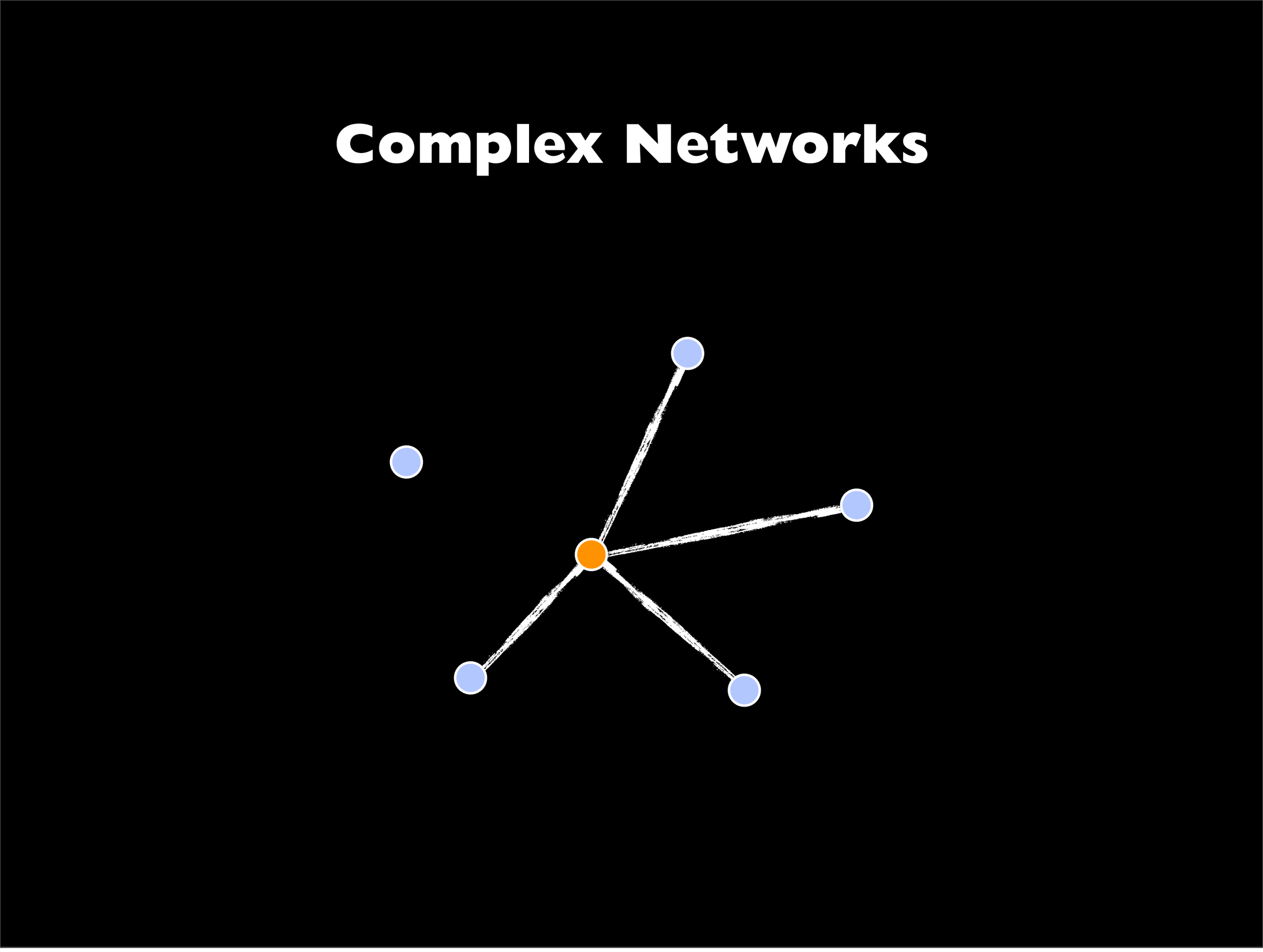


- degree centrality: importance of vertex for network

Complex Networks



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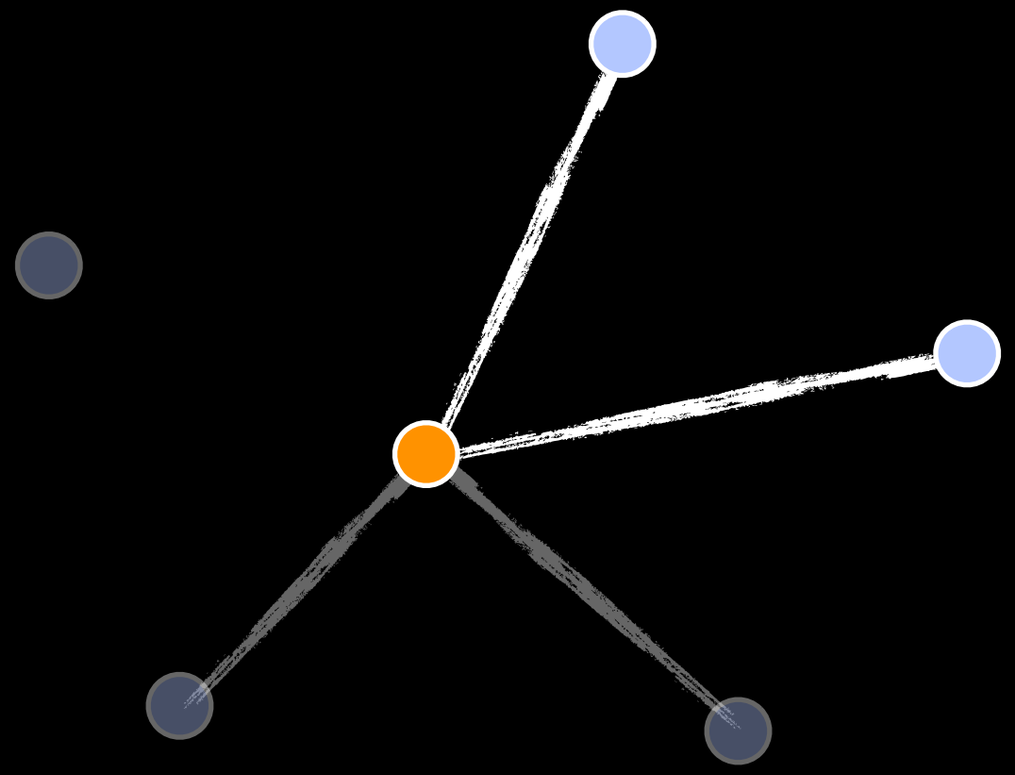

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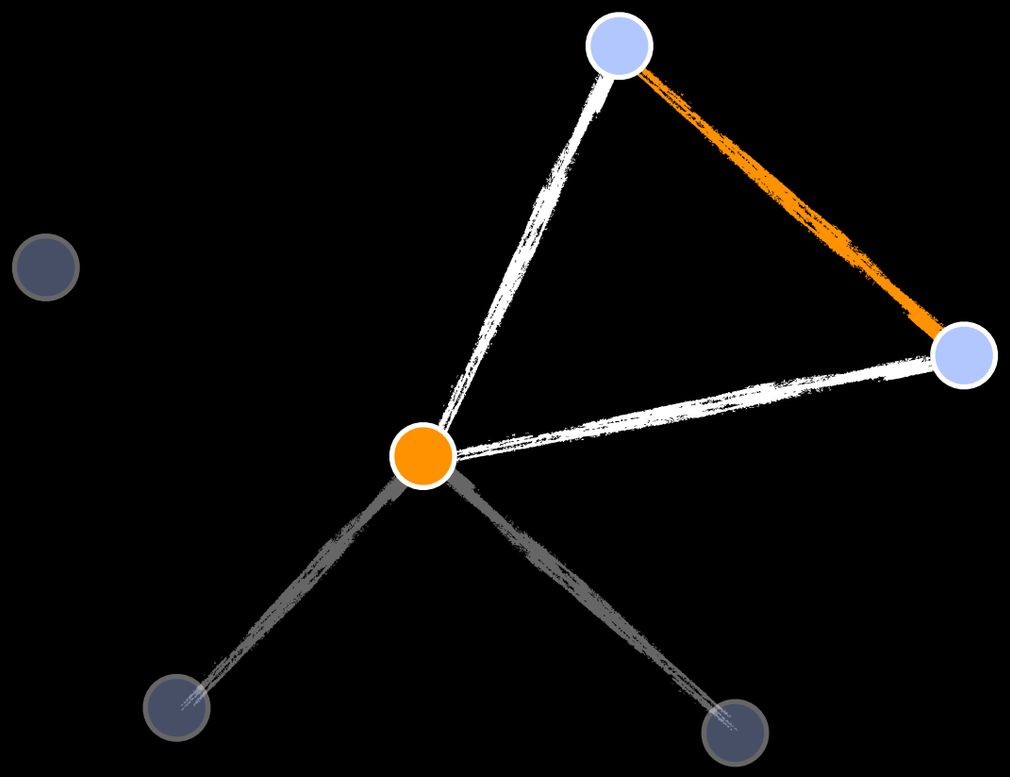

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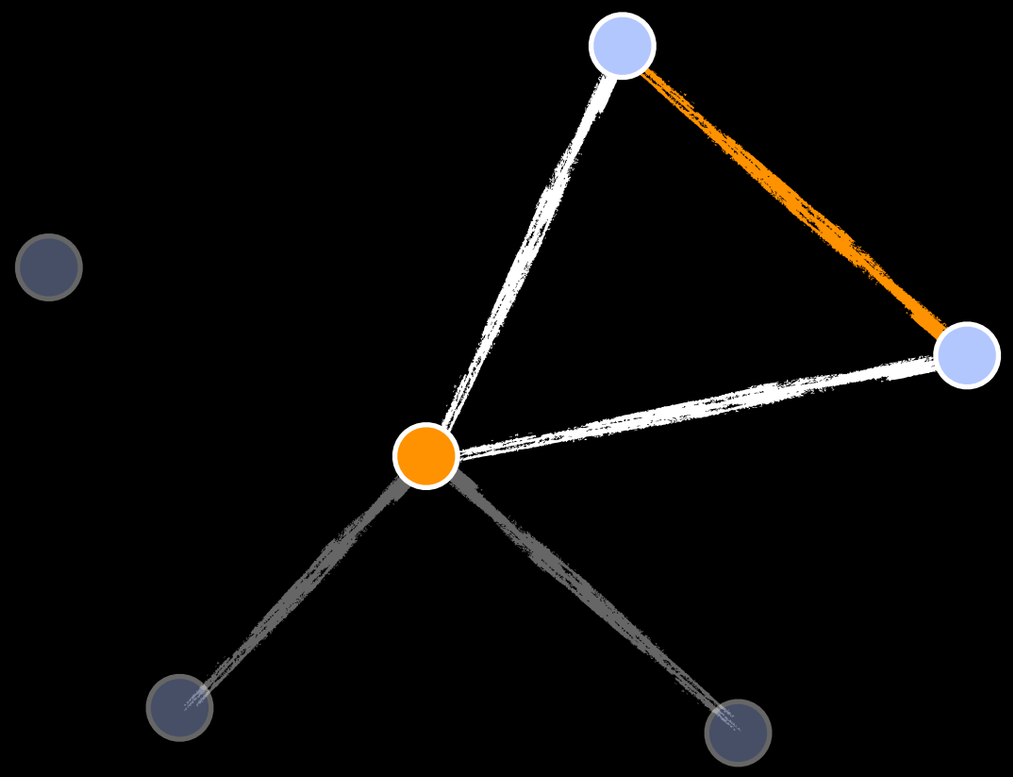

- clustering coefficient: clustering of nodes

Time Series Analysis using Complex Networks

- Link matrix = recurrence matrix of time series

- Nodes: states in phase space
- Links: local neighbours of states

Marwan et al., Phys. Lett. A 373, 4246-4254 (2009)

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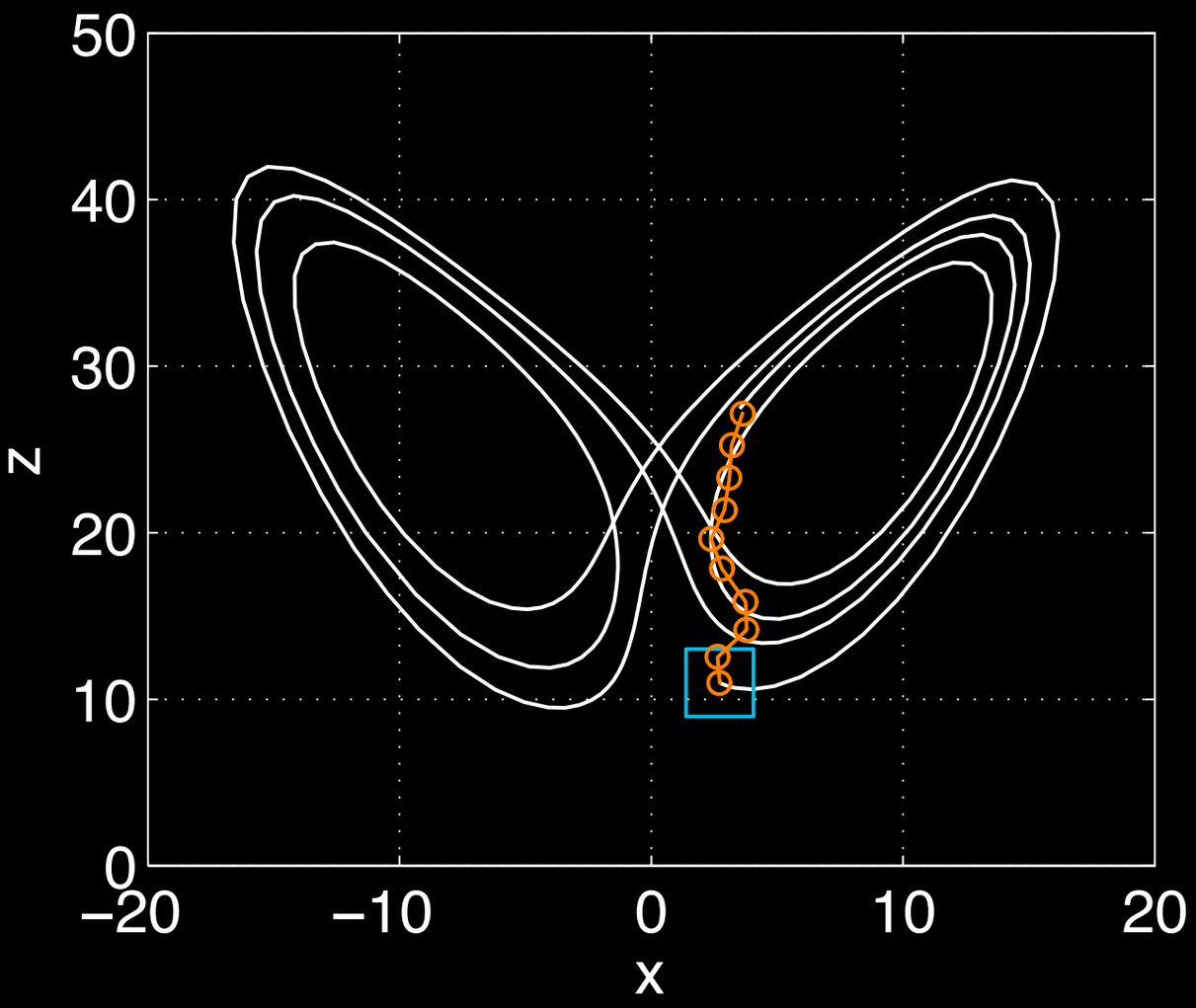

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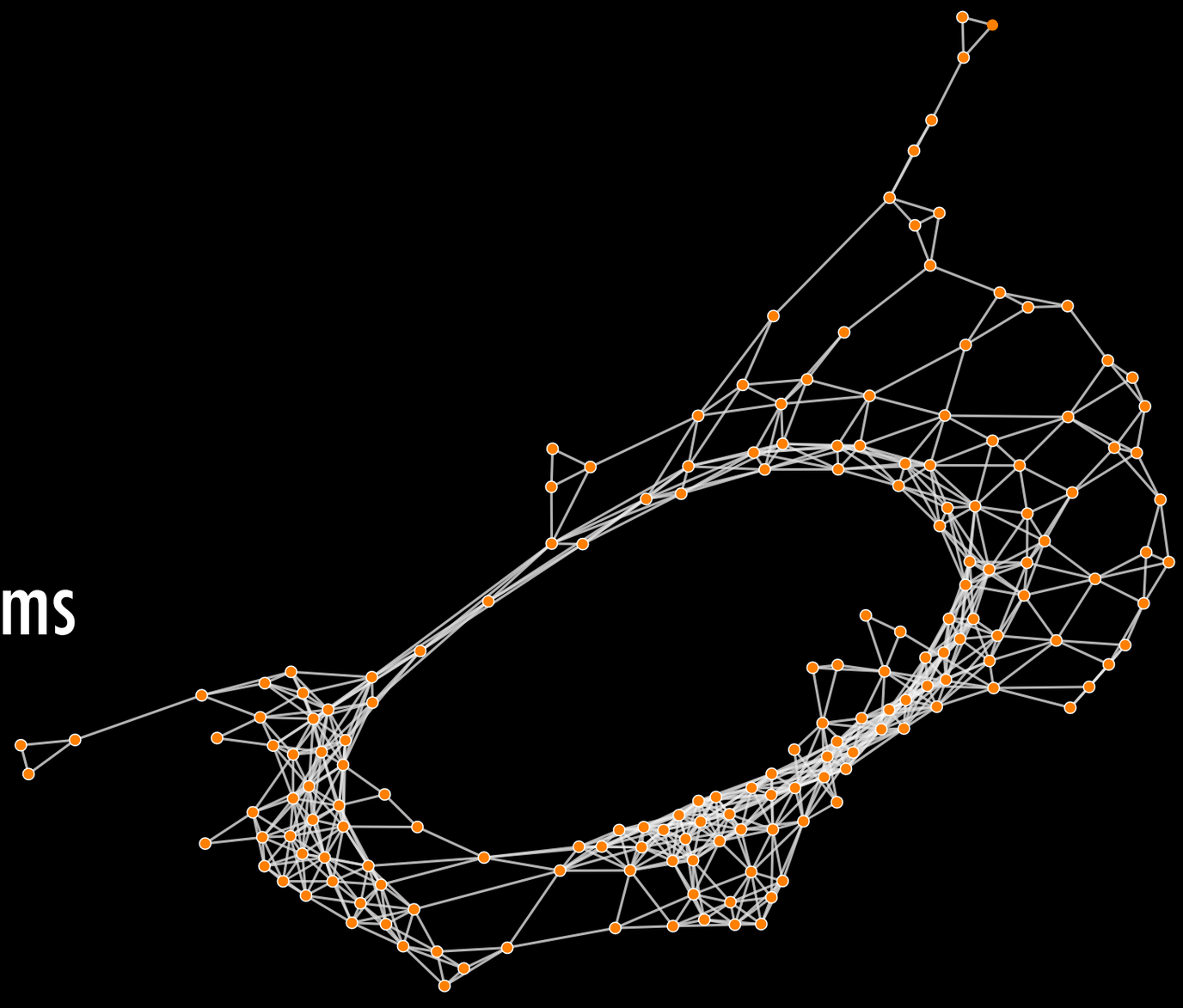
Time Series Analysis using Complex Networks

- Complex network measures applied to recurrence plot
  - measures of complexity explaining dynamical properties complex systems



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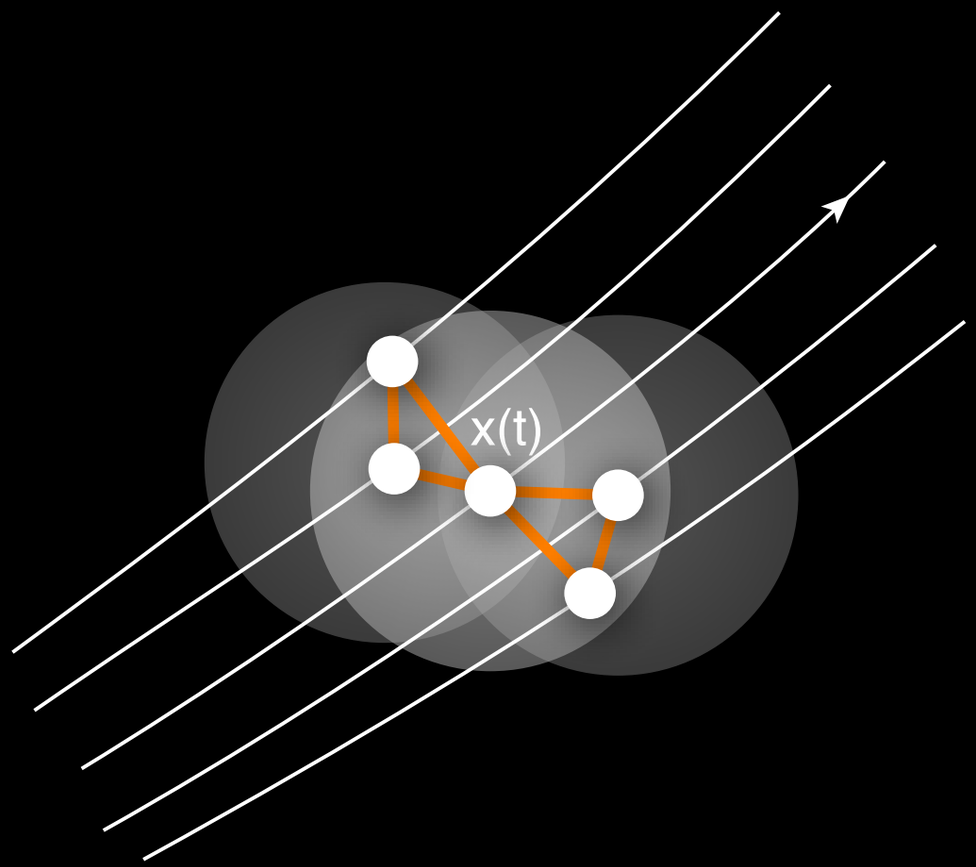

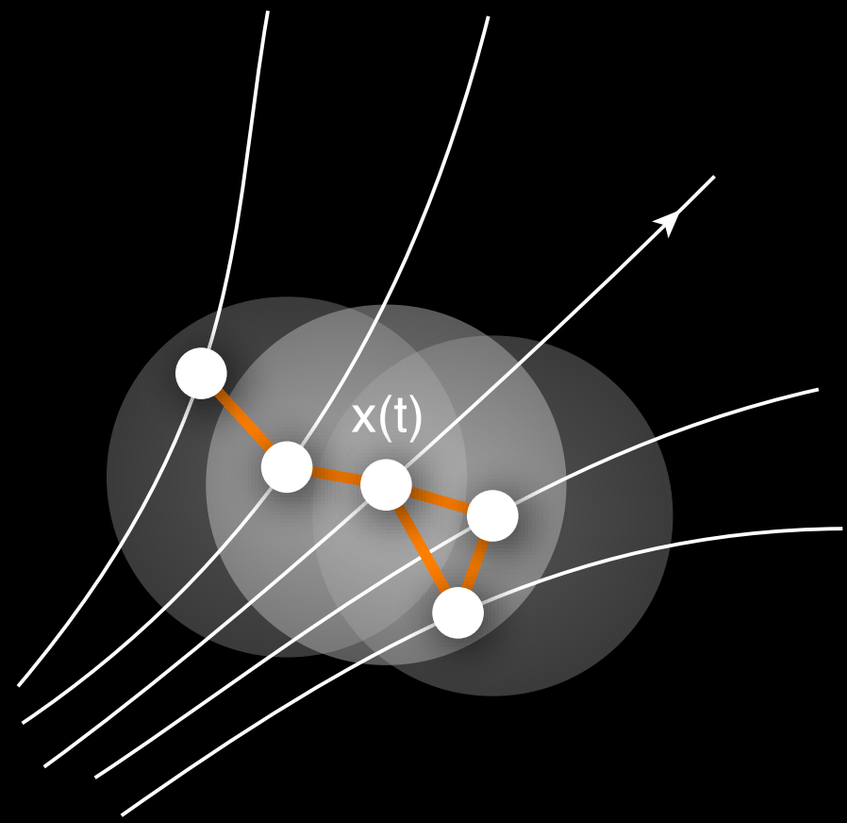

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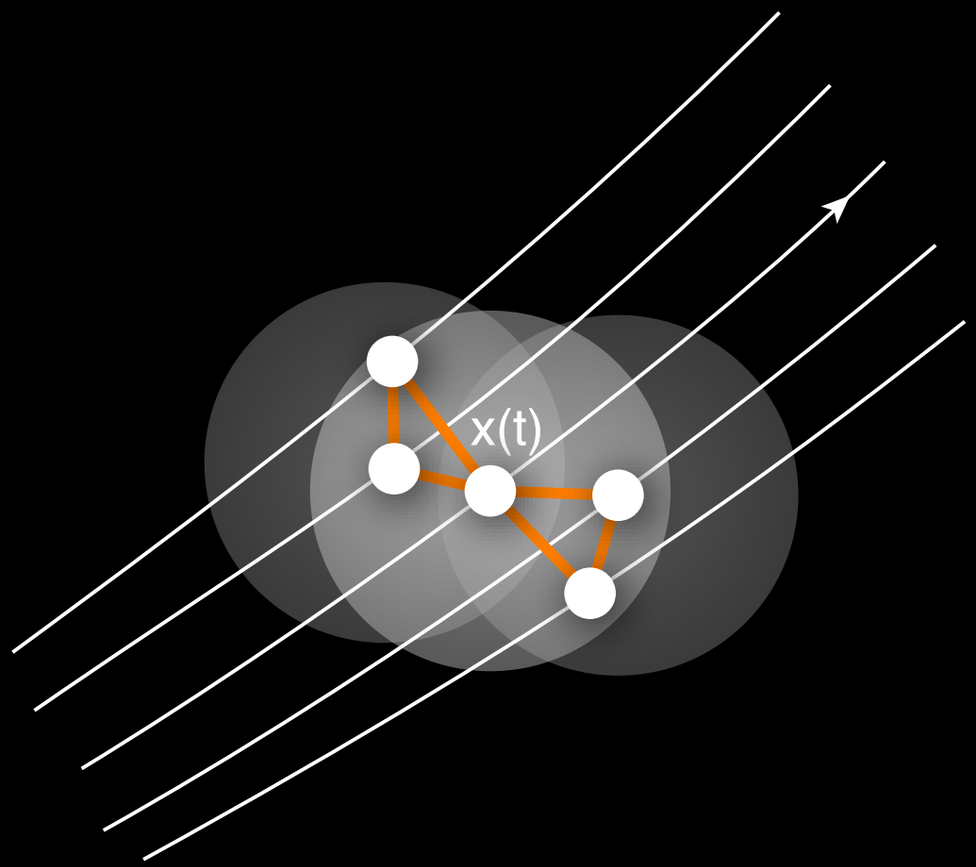

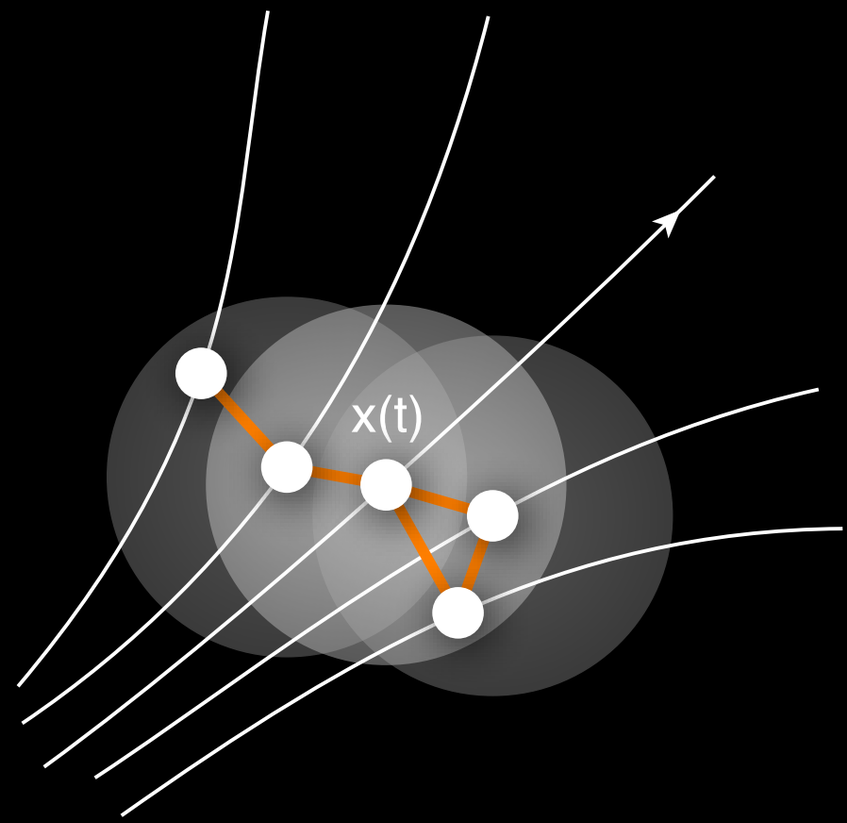

- clustering coefficient: regularity of dynamics

Early Detection of Preeclampsia in Pregnancy

- Life-threatening cramps for mother and fetus
- Under-supply of the fetus
- Growth retardation

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- Under-supply of the fetus
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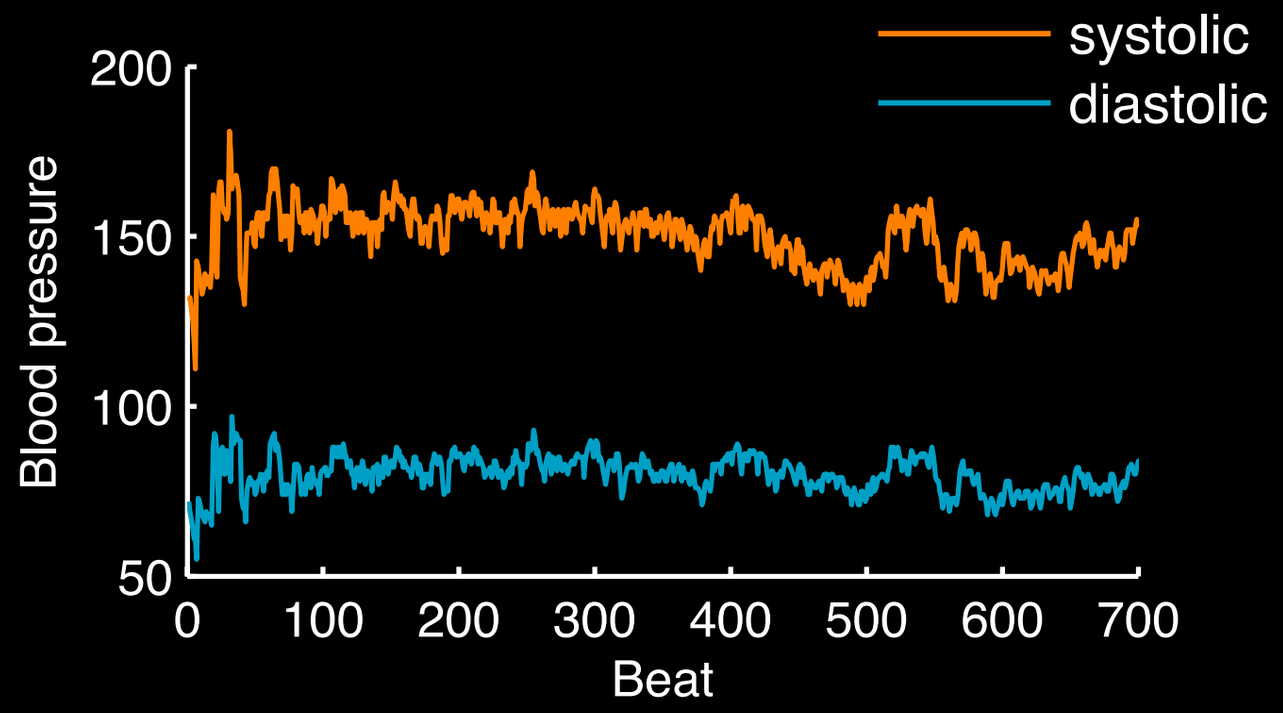
- positive predictive value appr. 20-30%

Early Detection of Preeclampsia in Pregnancy

- 20th week of gestation
- Systolic and diastolic blood pressure (S, D)



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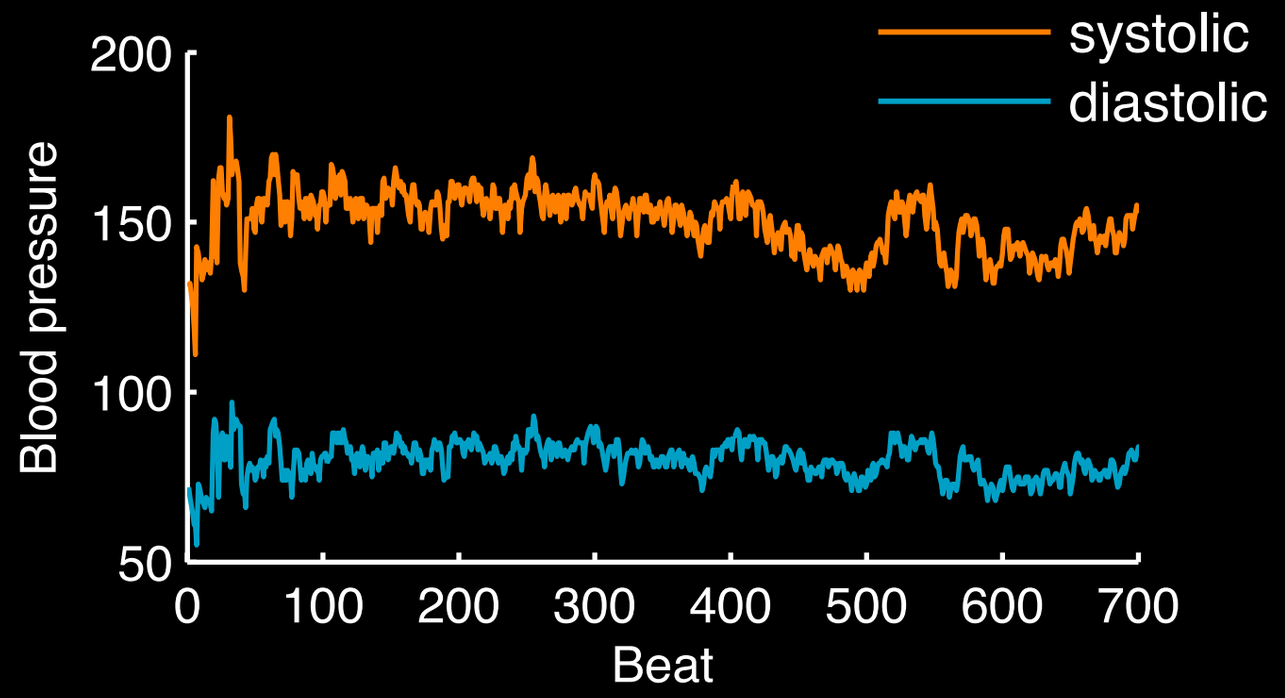

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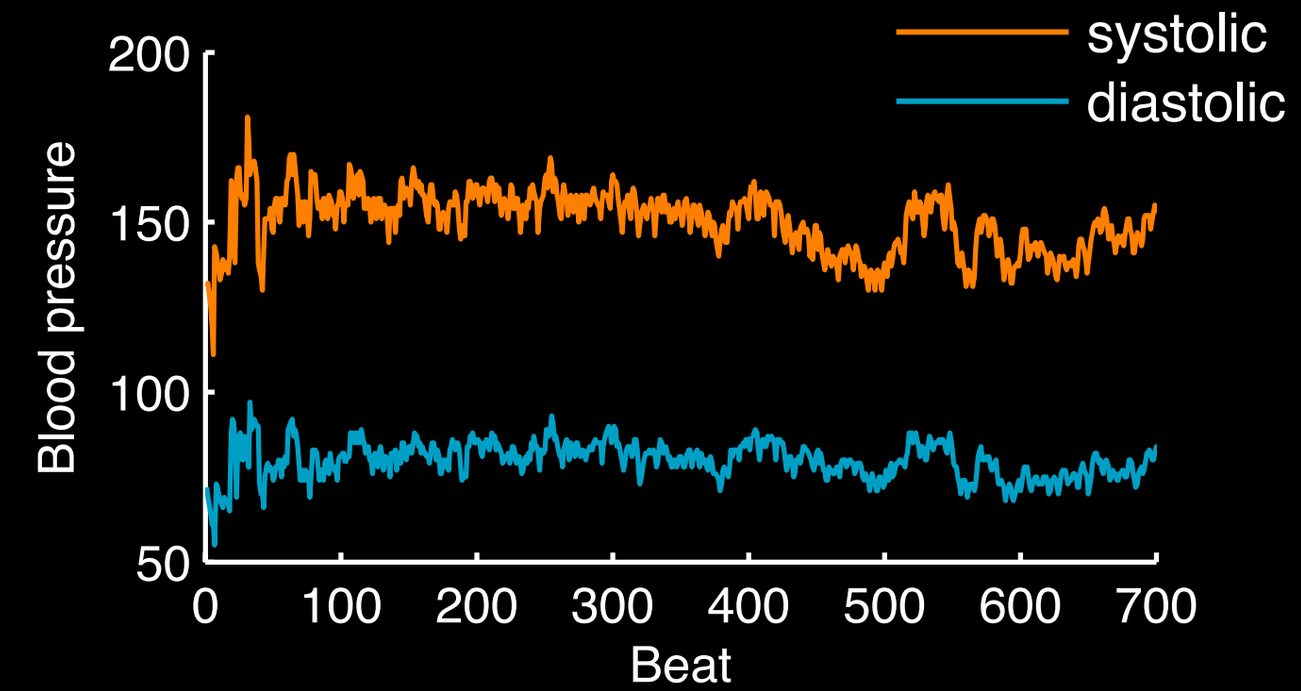

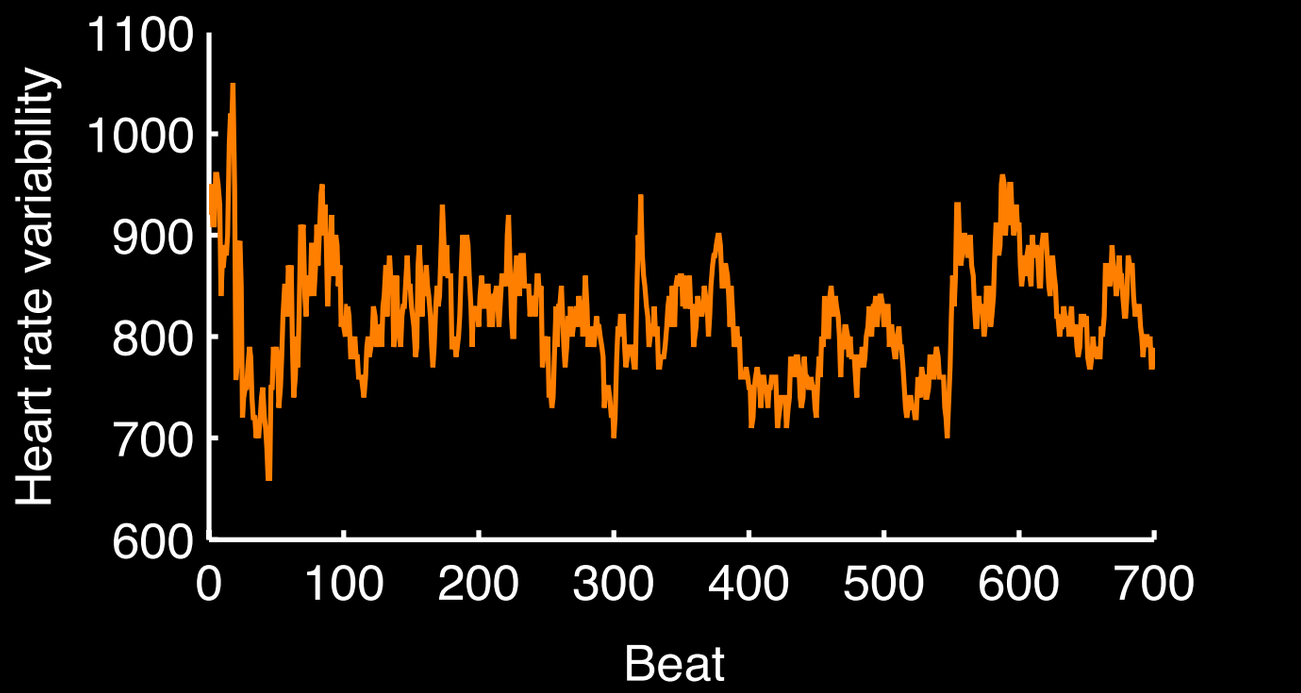

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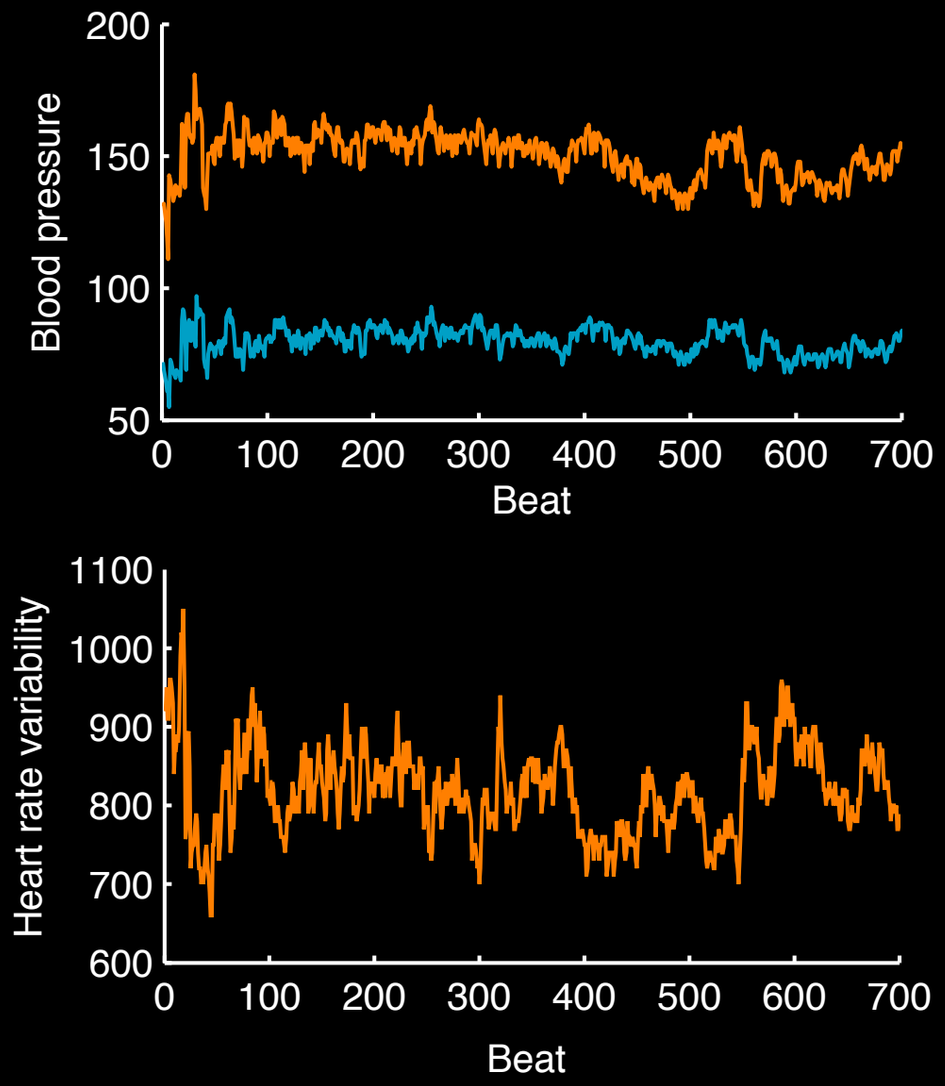
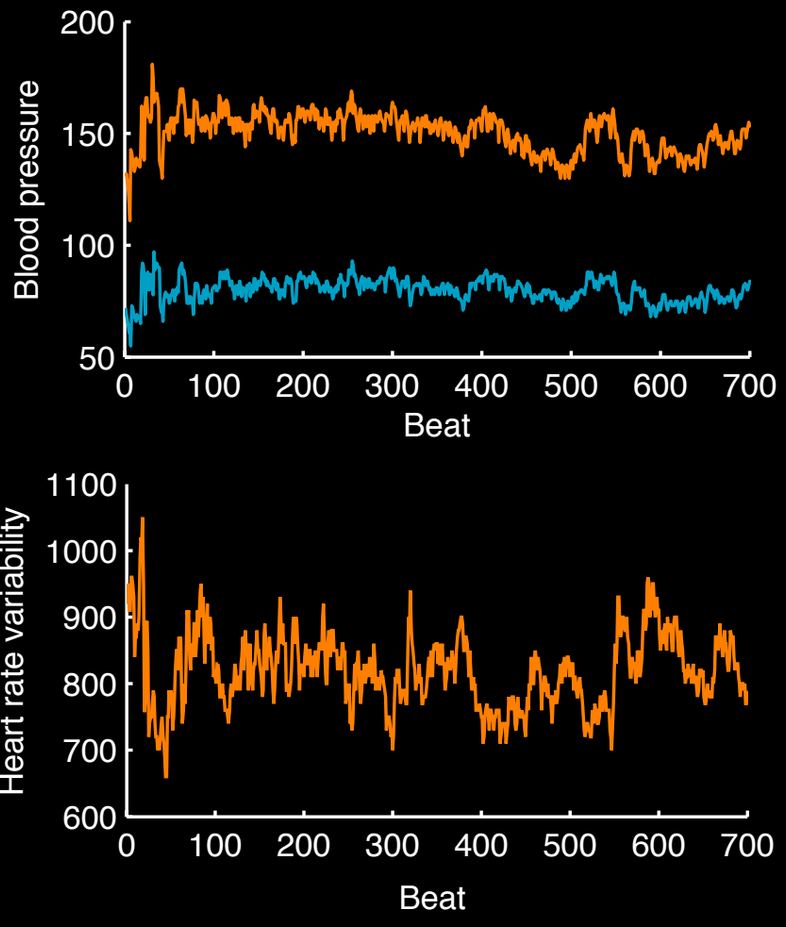


construct phase space

$$\vec{x}(t) = \begin{pmatrix} H(t) \\ D(t+1) \\ S(t+2) \end{pmatrix}$$

Early Detection of Preeclampsia in Pregnancy



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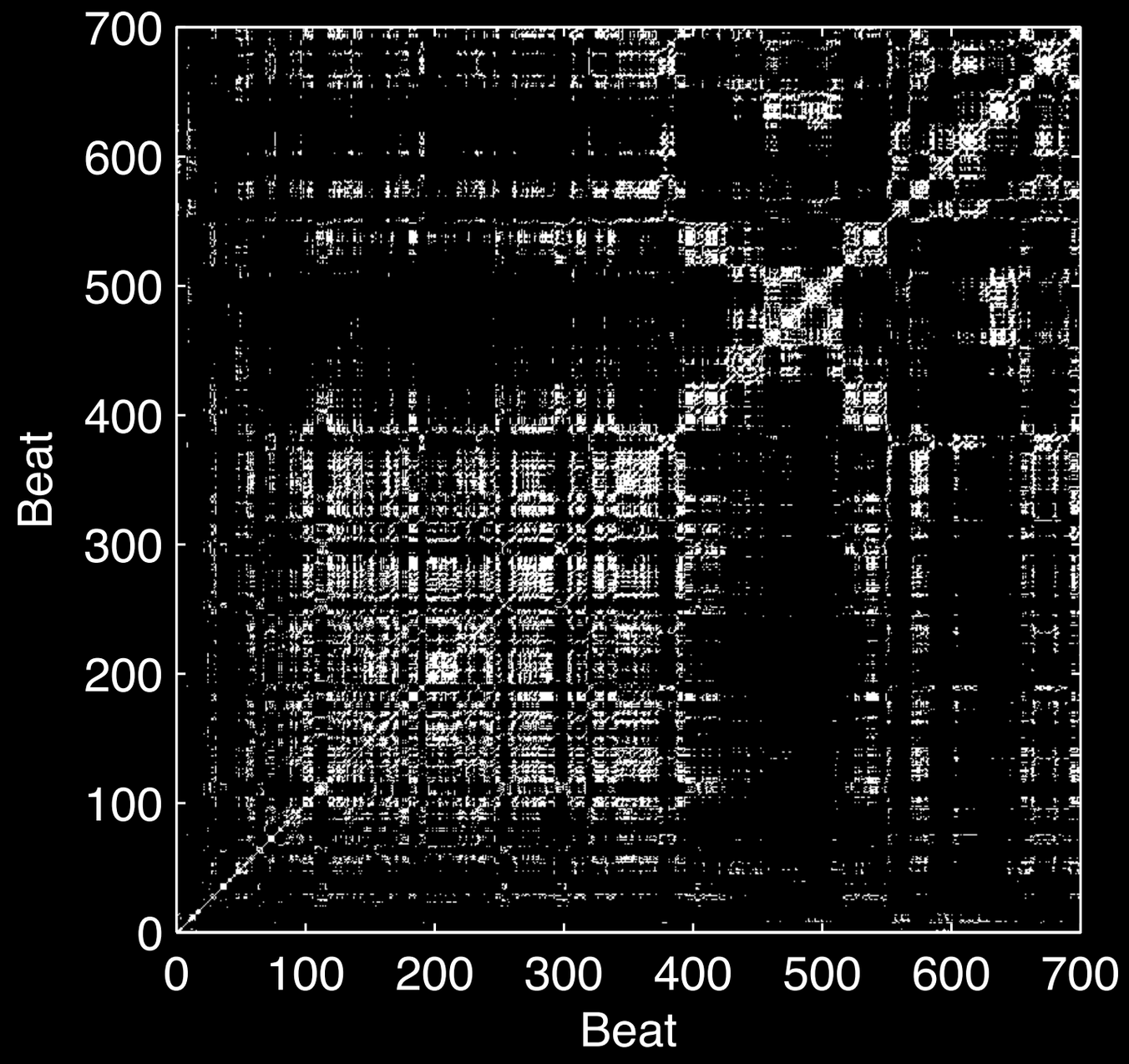

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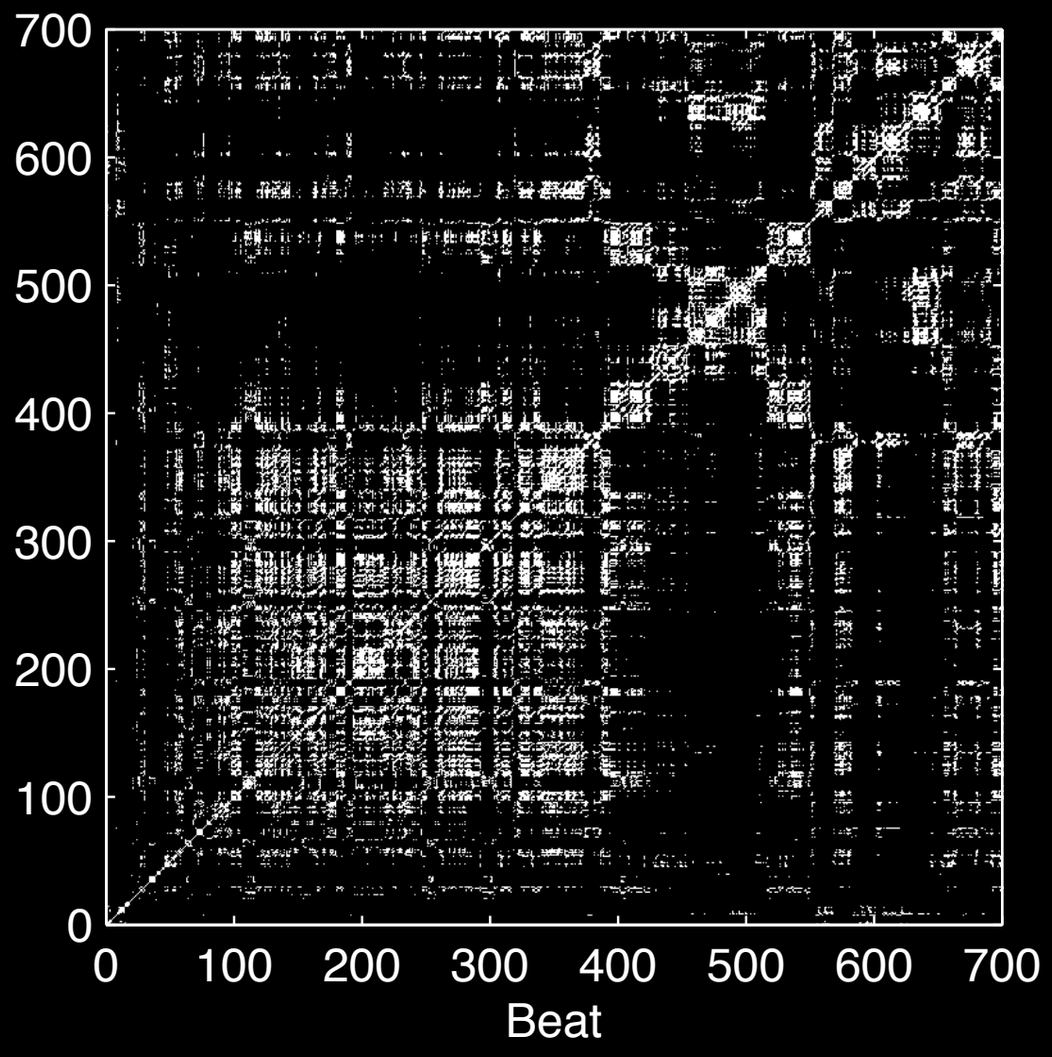

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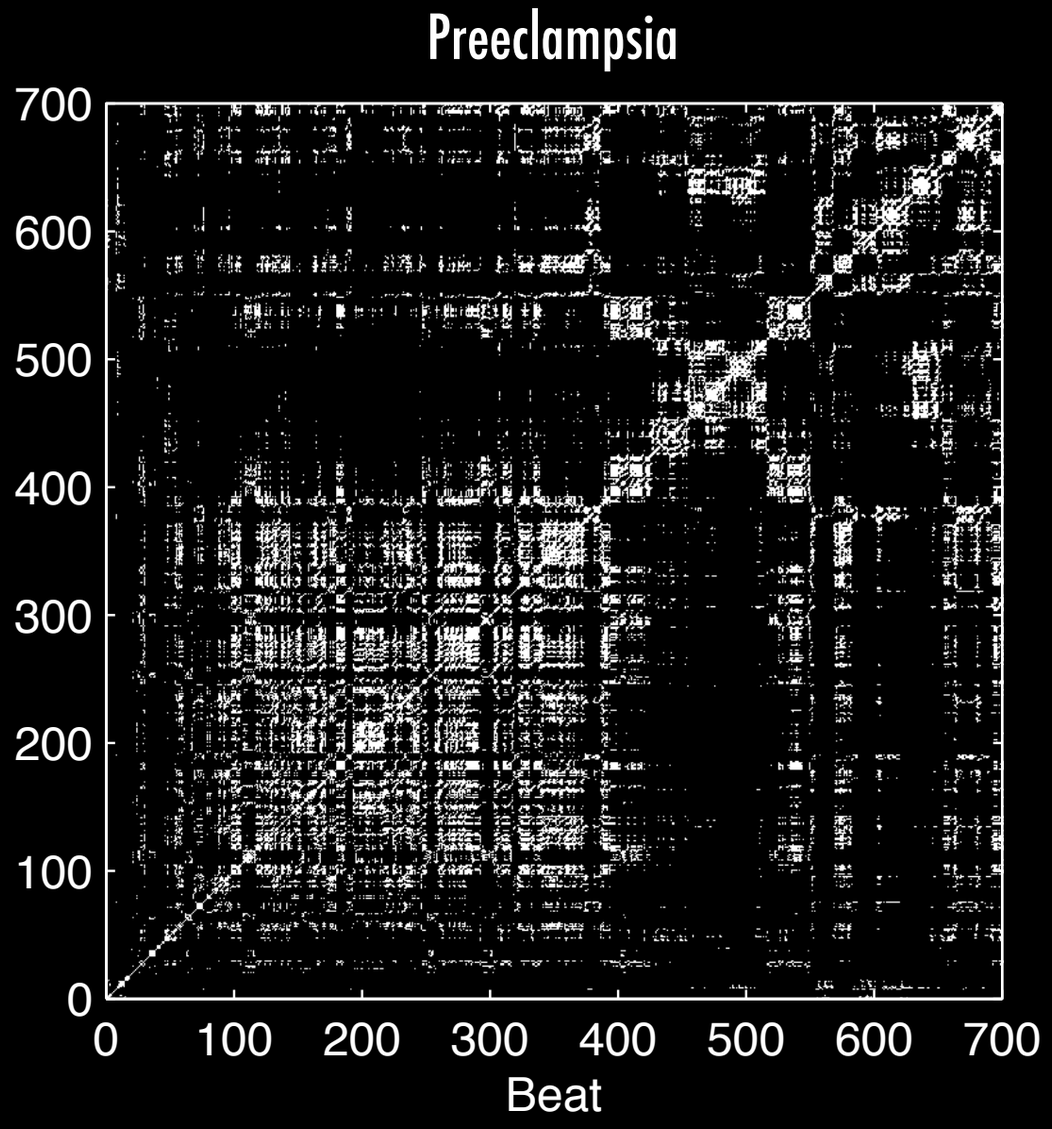

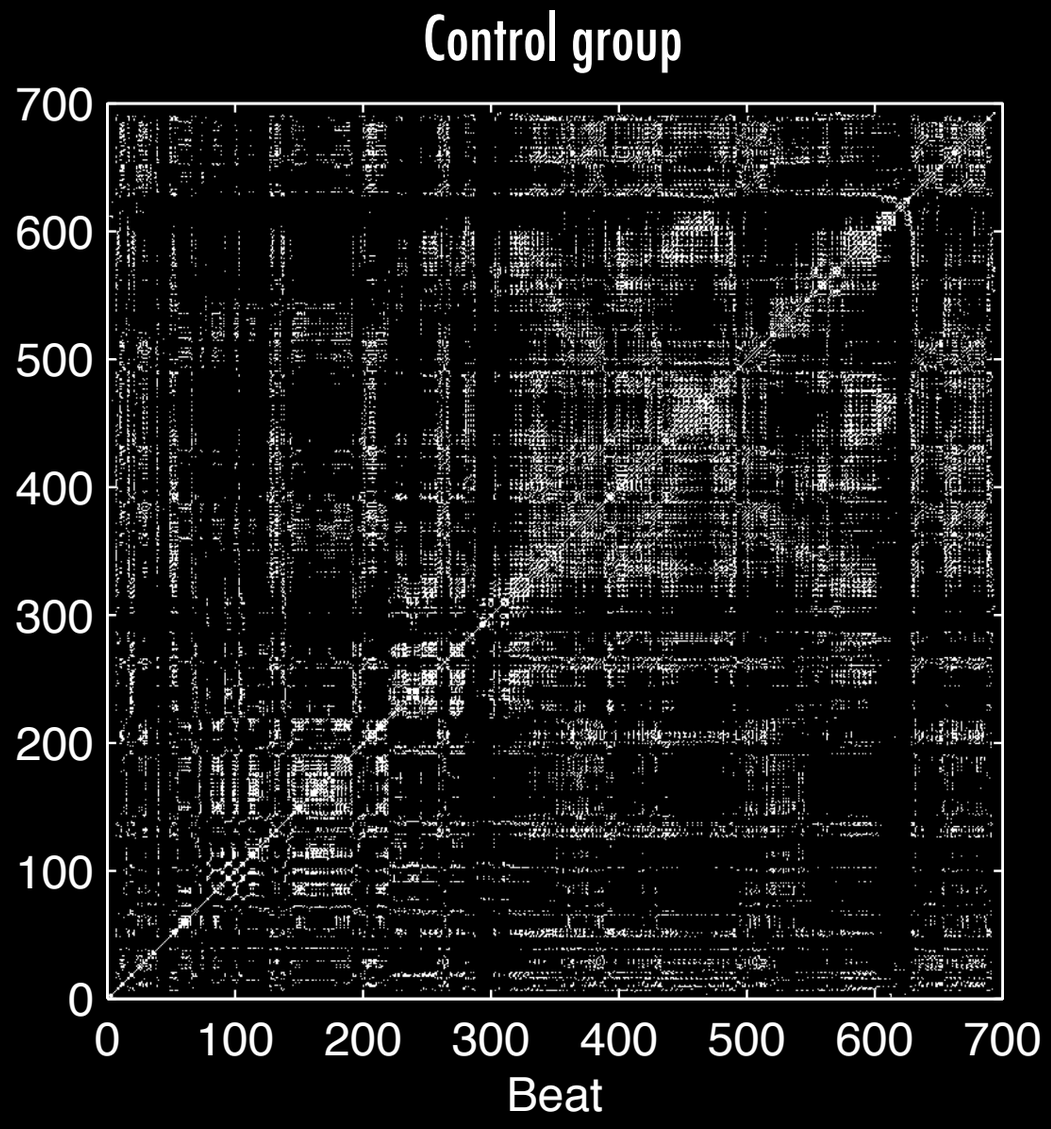

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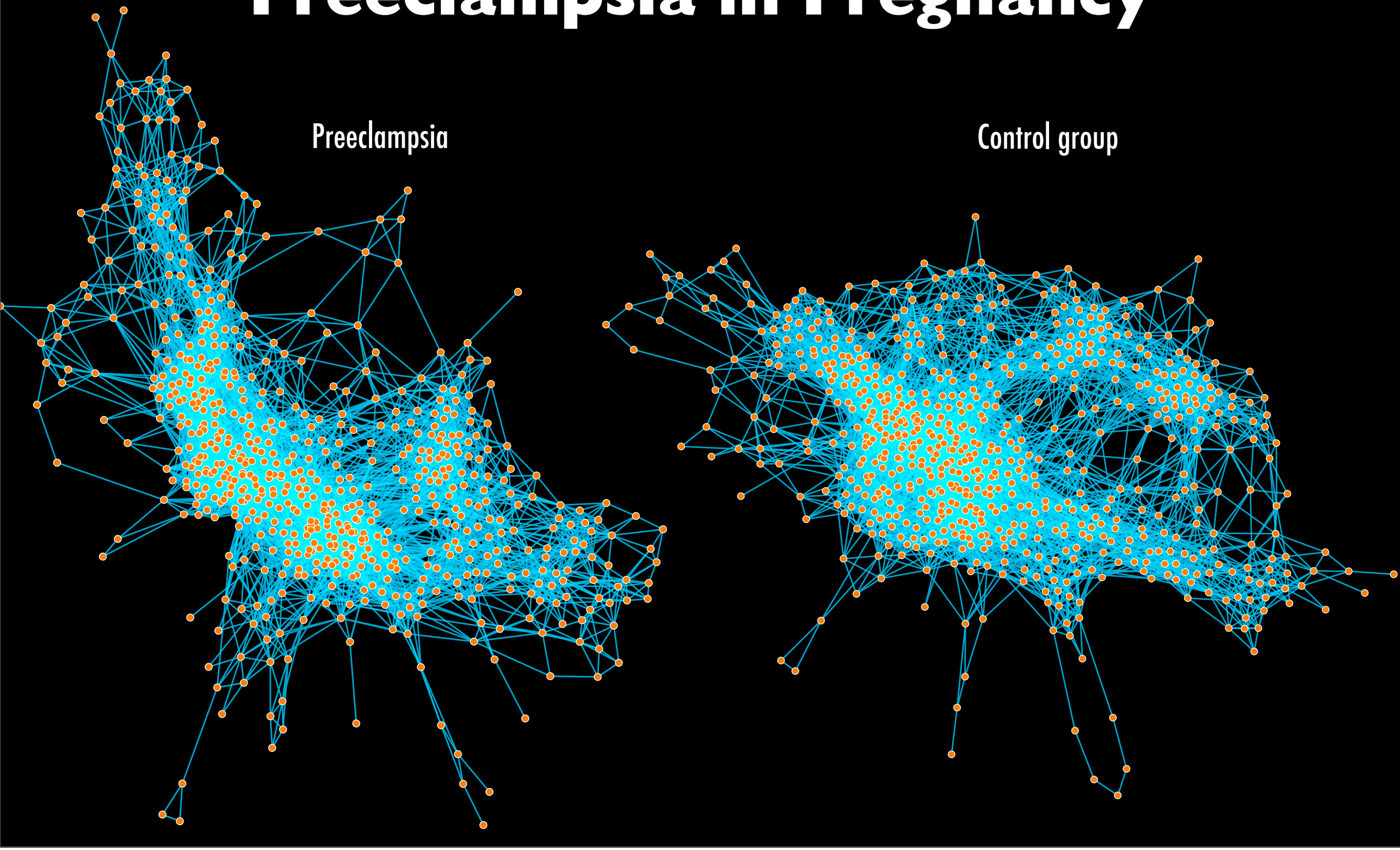

Early Detection of Preeclampsia in Pregnancy

	Preeclampsia	Control	p
H (ms)	734.5 (±110.8)	760.5 (±111.7)	n.s.
S (mmHg)	123.0 (±15.4)	123.5 (±20.0)	n.s.
D (mmHg)	75.5 (±10.4)	66.6 (±13.9)	n.s.
recurrence rate	0.14 (±0.04)	0.16 (±0.05)	0.0024
laminarity	0.80 (±0.10)	0.83 (±0.08)	n.s.
clustering	0.60 (±0.03)	0.62 (±0.04)	0.0015

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positive accuracy value: 60% negative accuracy value: 80%

Conclusions

- Complex network analysis of time series
- Complementary insights
- Applicable to non-stationary and noisy data
- Preeclampsia: first promising results

Publications

- N. Marwan, M. C. Romano, M. Thiel, J. Kurths: Recurrence Plots for the Analysis of Complex Systems, Physics Reports, 438(5–6), 237–329 (2007)
- N. Marwan, J. Donges, Y. Zou, R. Donner, J. Kurths: Complex network approach for recurrence analysis of time series, Phys. Lett. A 373, 4246–4254 (2009)
- R. V. Donner, Y. Zou, J. F. Donges, N. Marwan, J. Kurths: Recurrence networks – A novel paradigm for nonlinear time series analysis, New Journal of Physics, 12(3), 033025 (2010)