Hot Topics

in the Recurrence Plot Field

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P I K

Recurrence

- fundamental characteristic of many dynamical systems
- Poincaré, 1890:

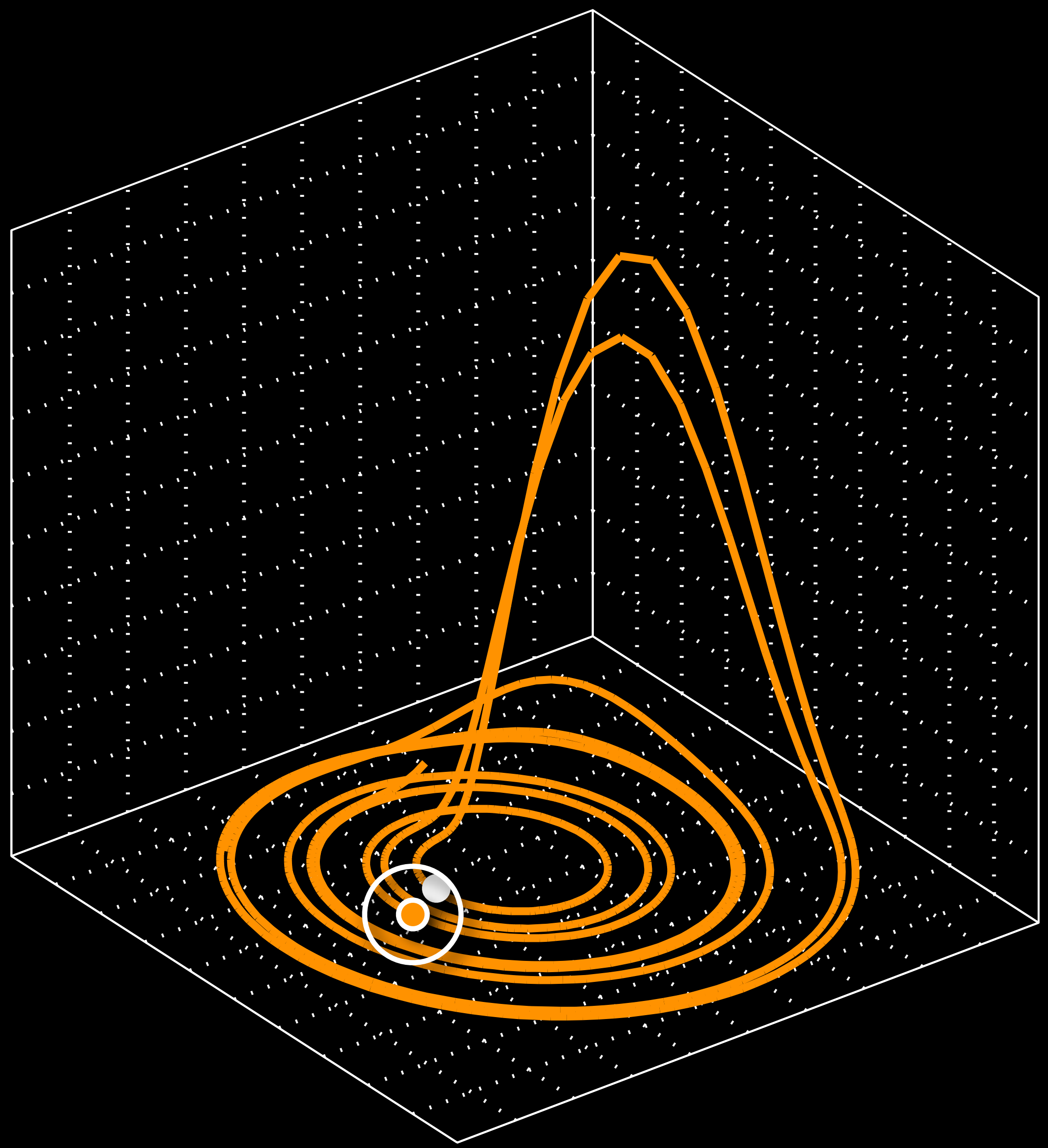
  "a system recurs infinitely many times as close as one wishes to its initial state"
- recurrences in real life:

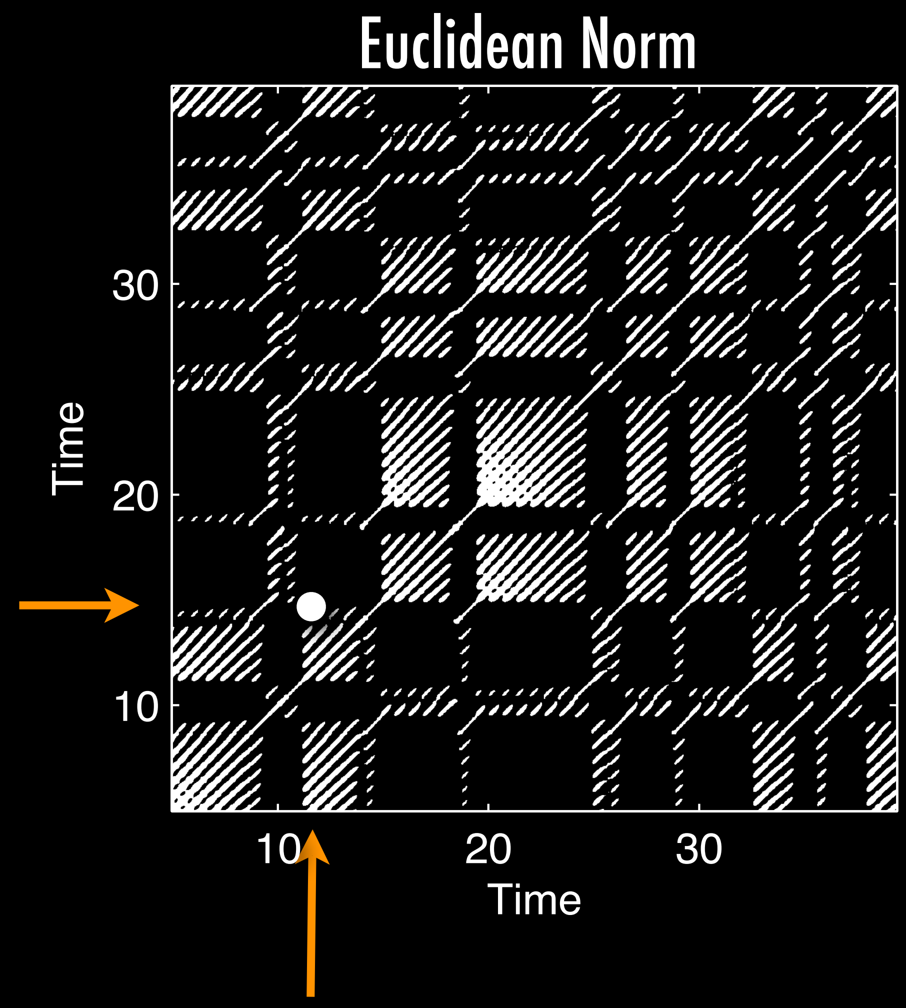
  Milankovich cycles, weather after storm, El Niño phenomenon, heart beat after exertion, Maya calendar etc.

Historical Review

Recurrence Plot

$$\mathbf{R}_{t_1,t_2} = \Theta\left(\varepsilon - \|\vec{x}(t_1) - \vec{x}(t_2)\|\right)$$





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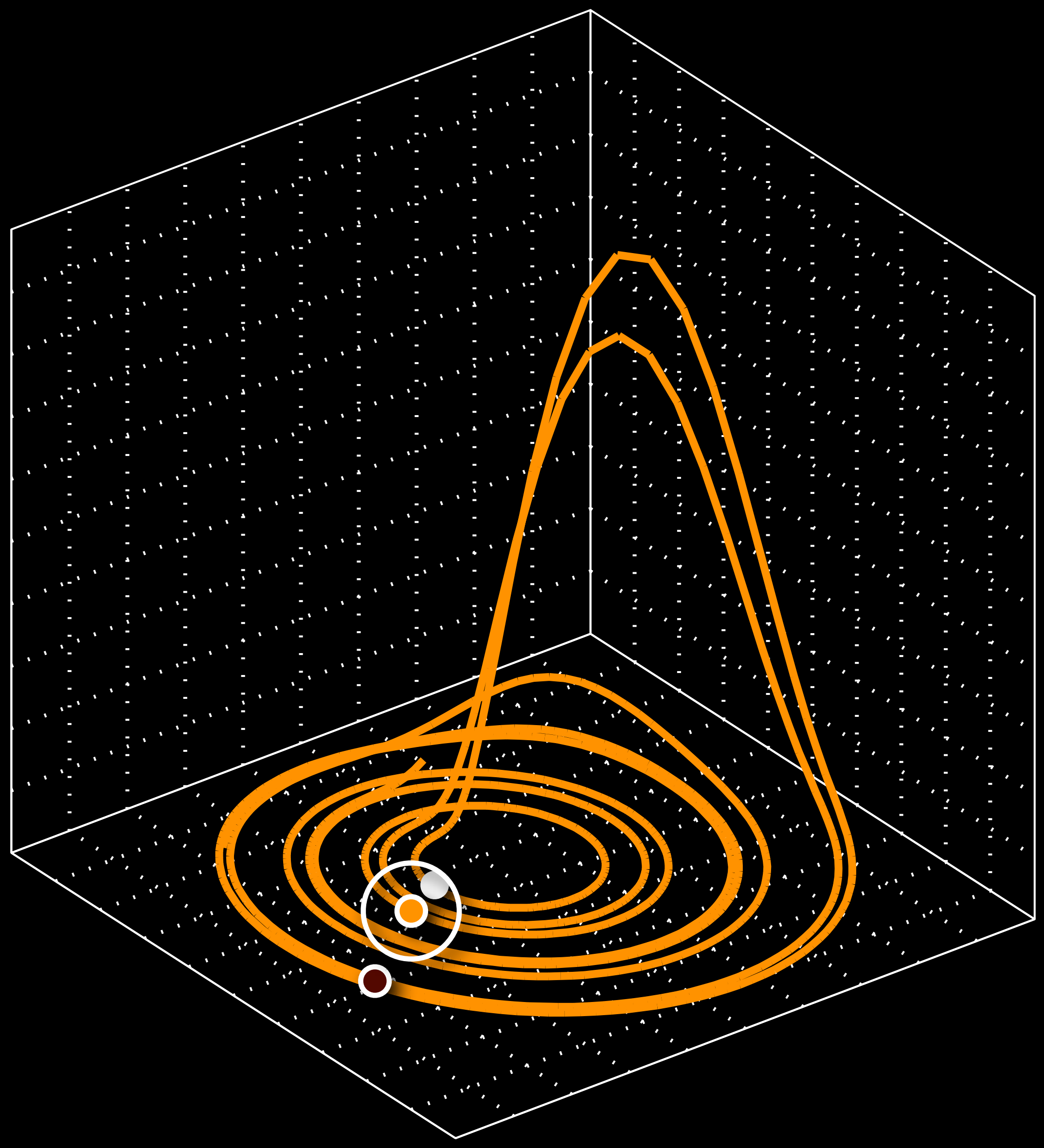
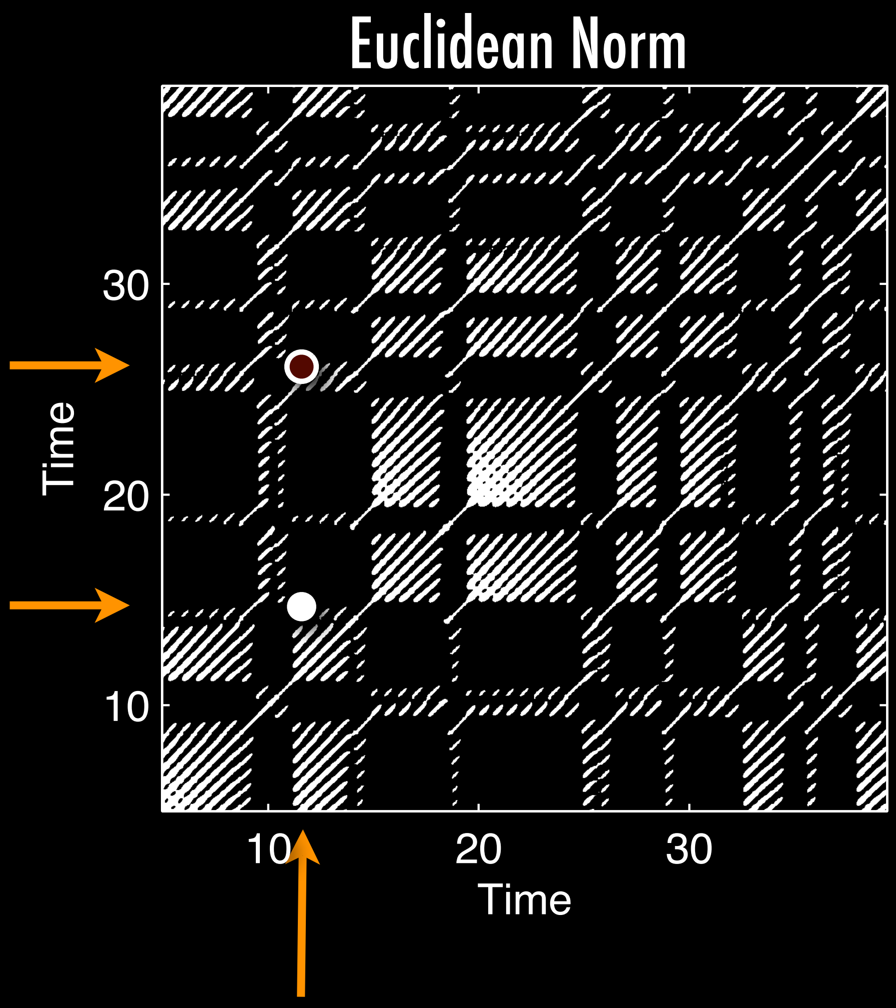
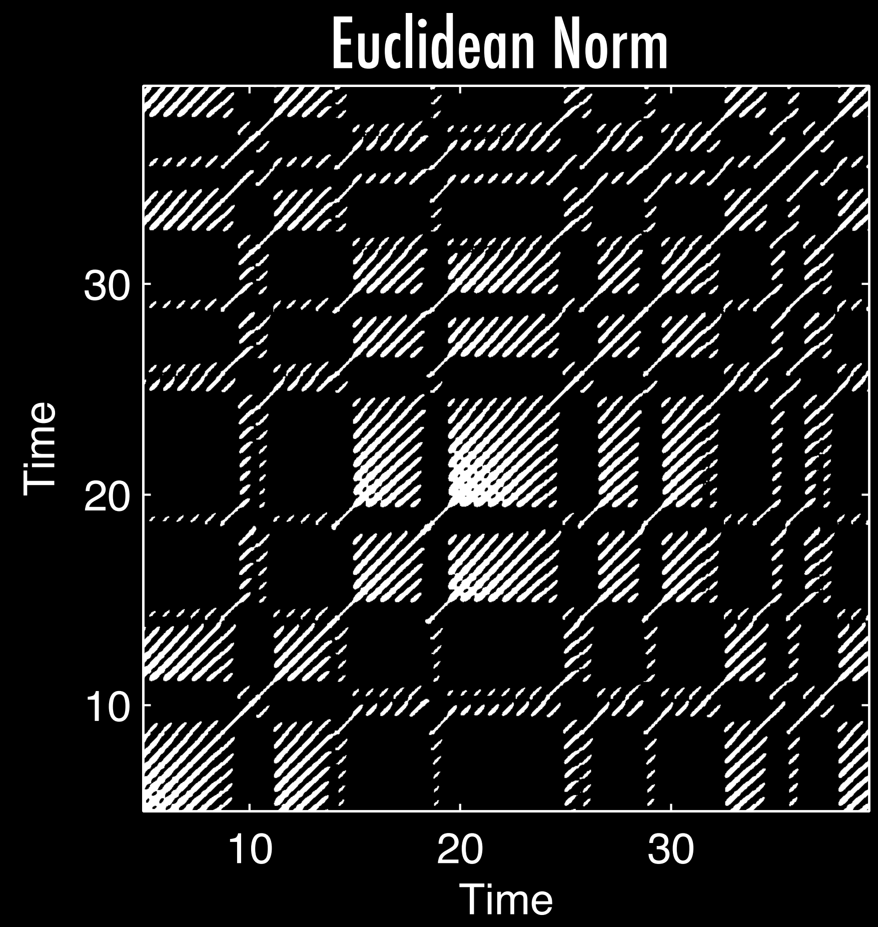


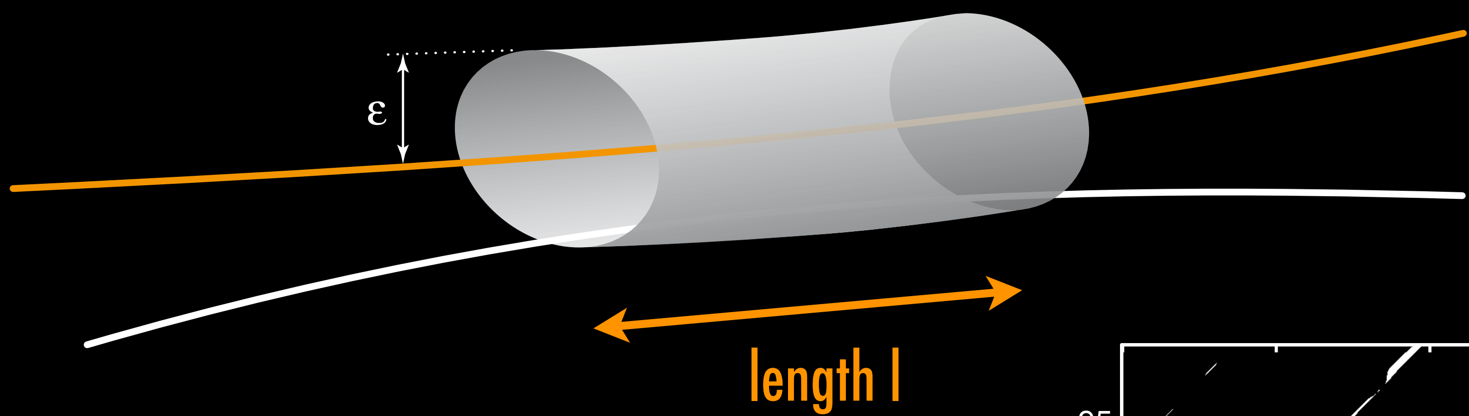


Recurrence Plot

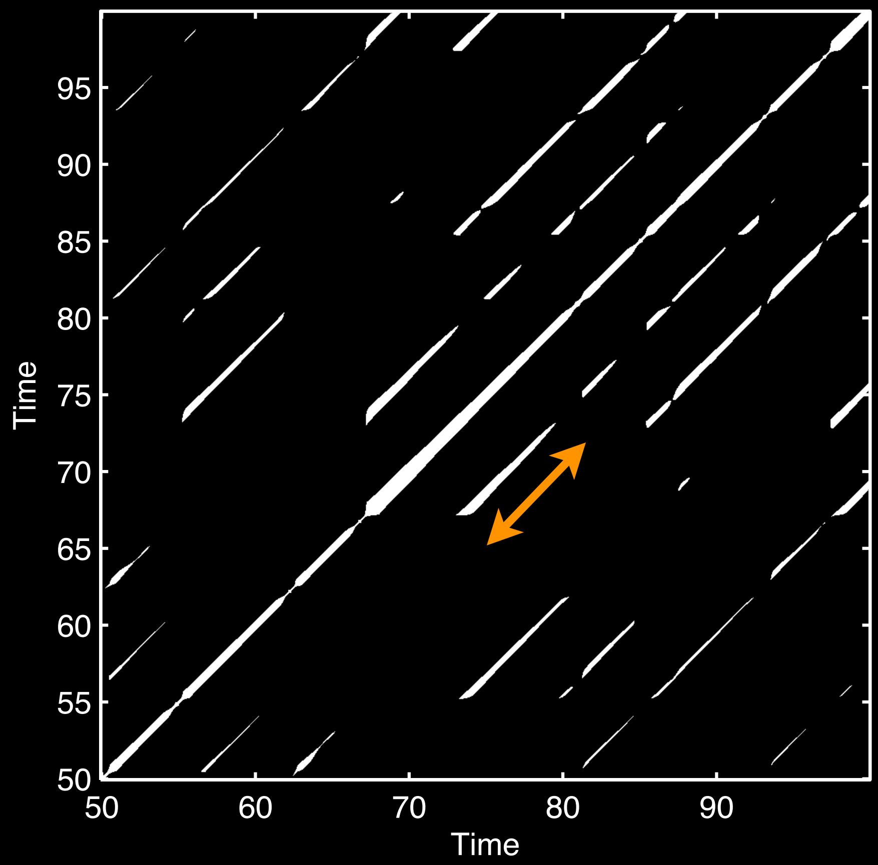
- Transition detection
- Differentiate dynamics
- Finding time scales
- Interrelation detection
- Synchronisation analysis
- Surrogates
- Recurrence time statistics
- etc.



Recurrence Quantification



- number of lines of exactly length l
  - histogram P(l)



Line Based Measures

- Determinism DET

$$DET = \frac{\sum_{l=l_{\min}}^{N} l\,P(l)}{\sum_{l=1}^{N} l\,P(l)}$$

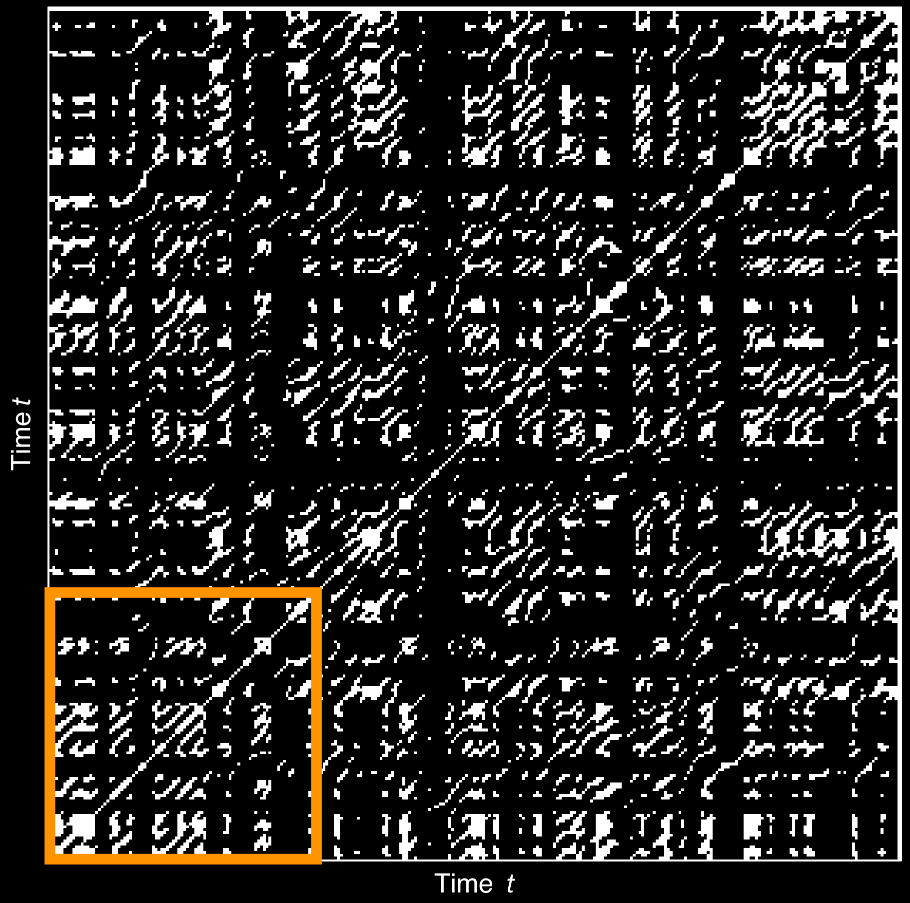
Fraction of points forming diagonal lines

- Mean diagonal line length L

$$L = \frac{\sum_{l=l_{\min}}^{N} l\,P(l)}{\sum_{l=l_{\min}}^{N} P(l)}$$

Recurrence Quantification

- Time dependent analysis:
  - sliding windows over RP
- Detection of transitions



Recurrence Quantification

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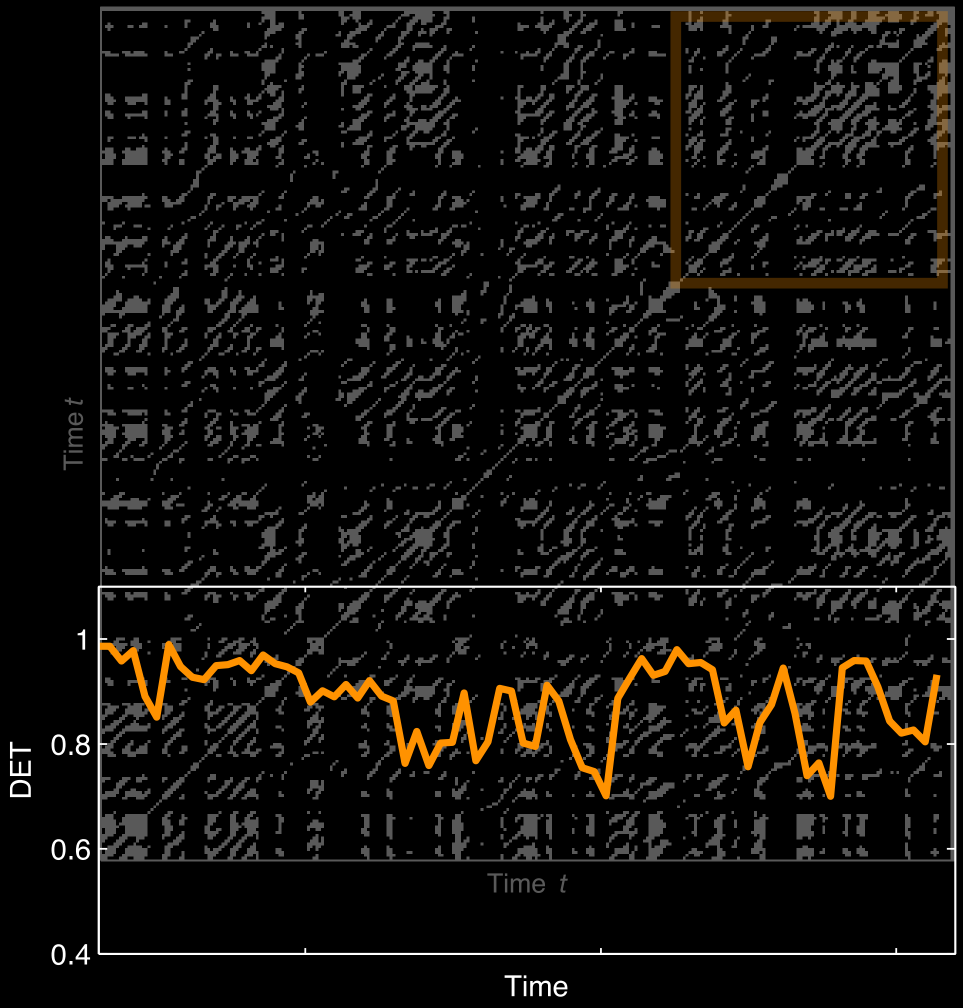
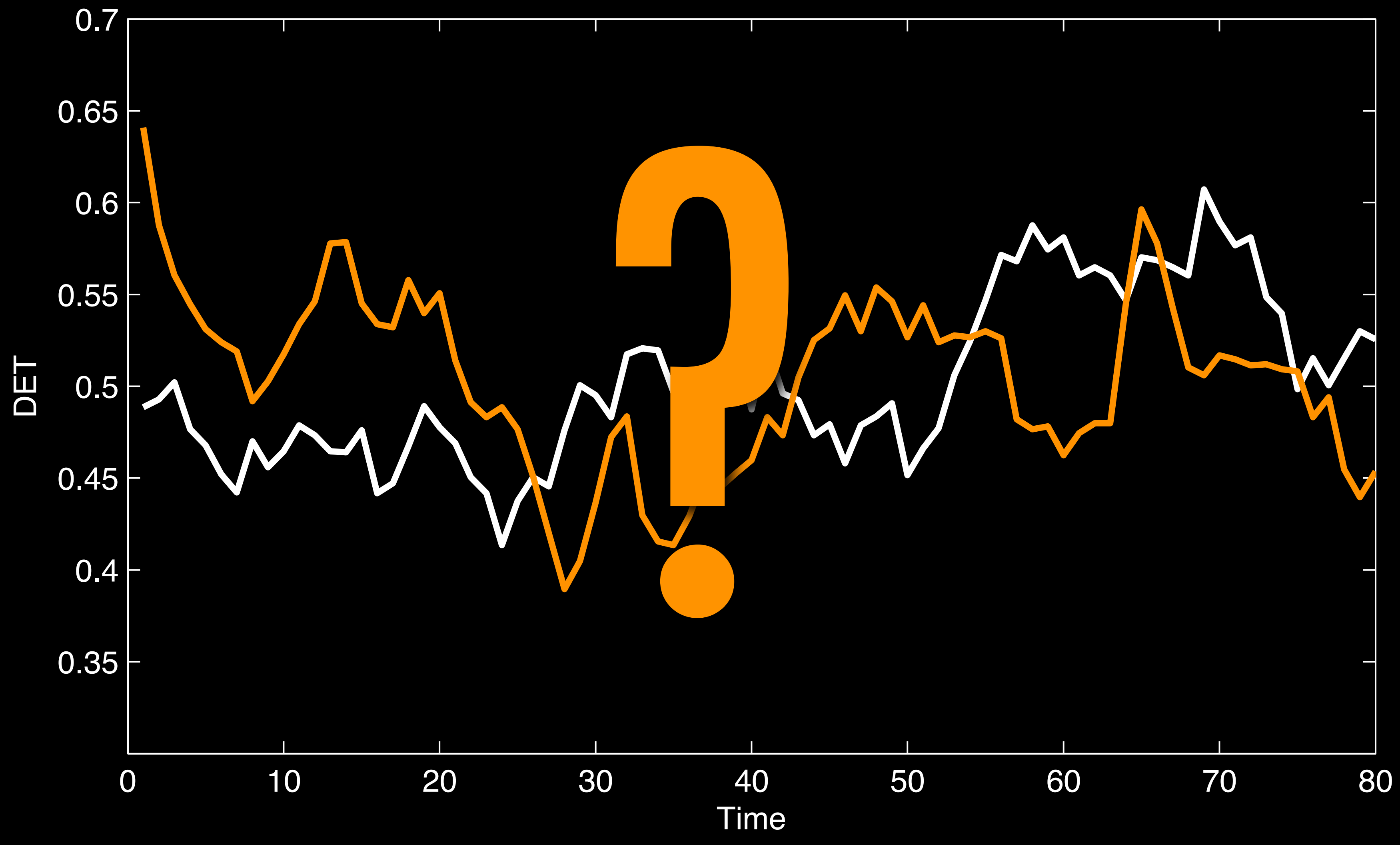
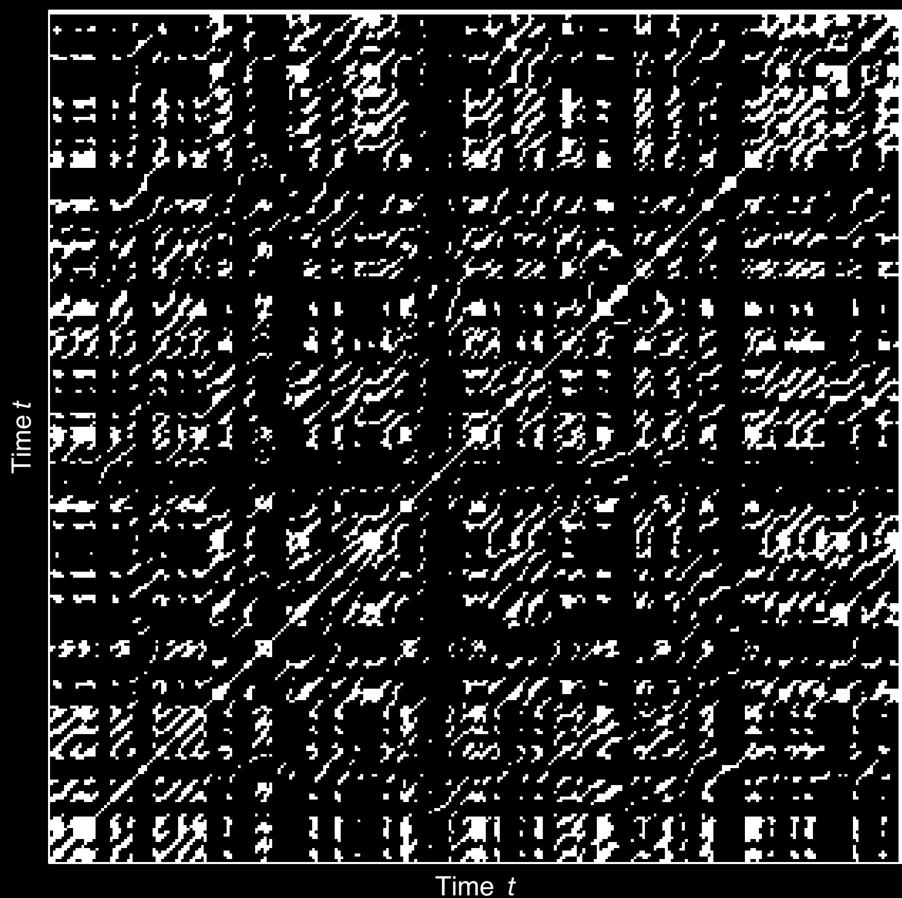


Confidence Intervals

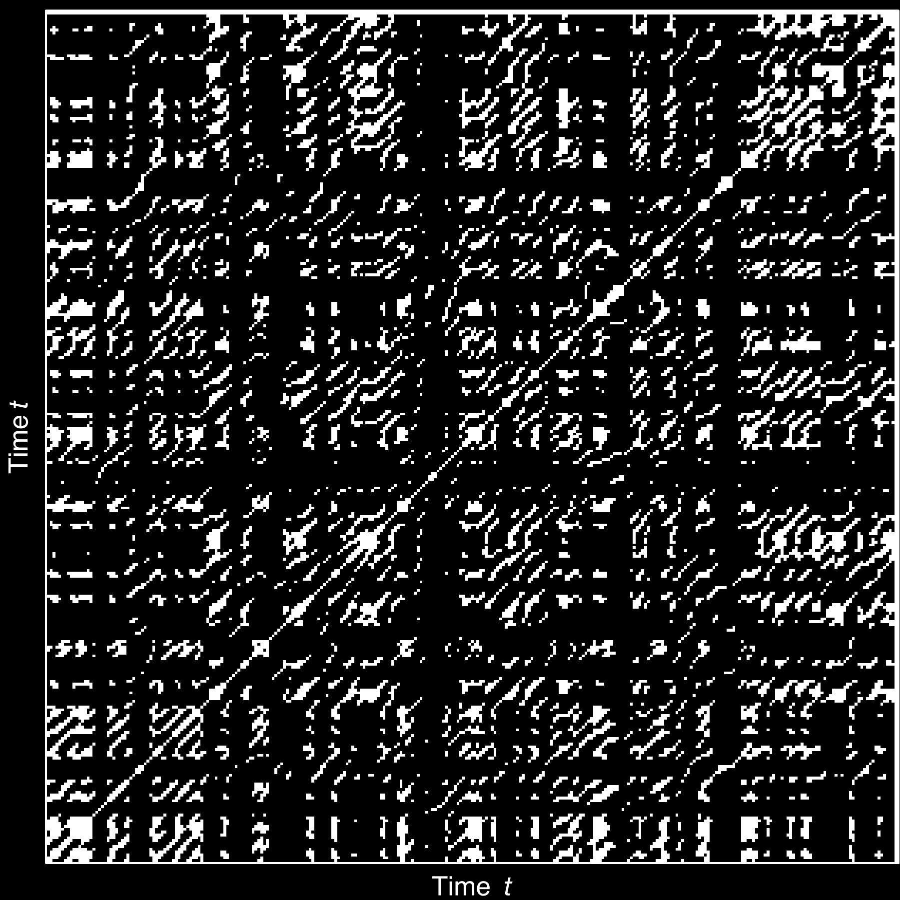


Bootstrapping Recurrence Structures



➡ Empirical distribution of RQA measures

Bootstrapping Recurrence Structures



$P_1(l) \rightarrow DET_1$

$P_2(l) \rightarrow DET_2$

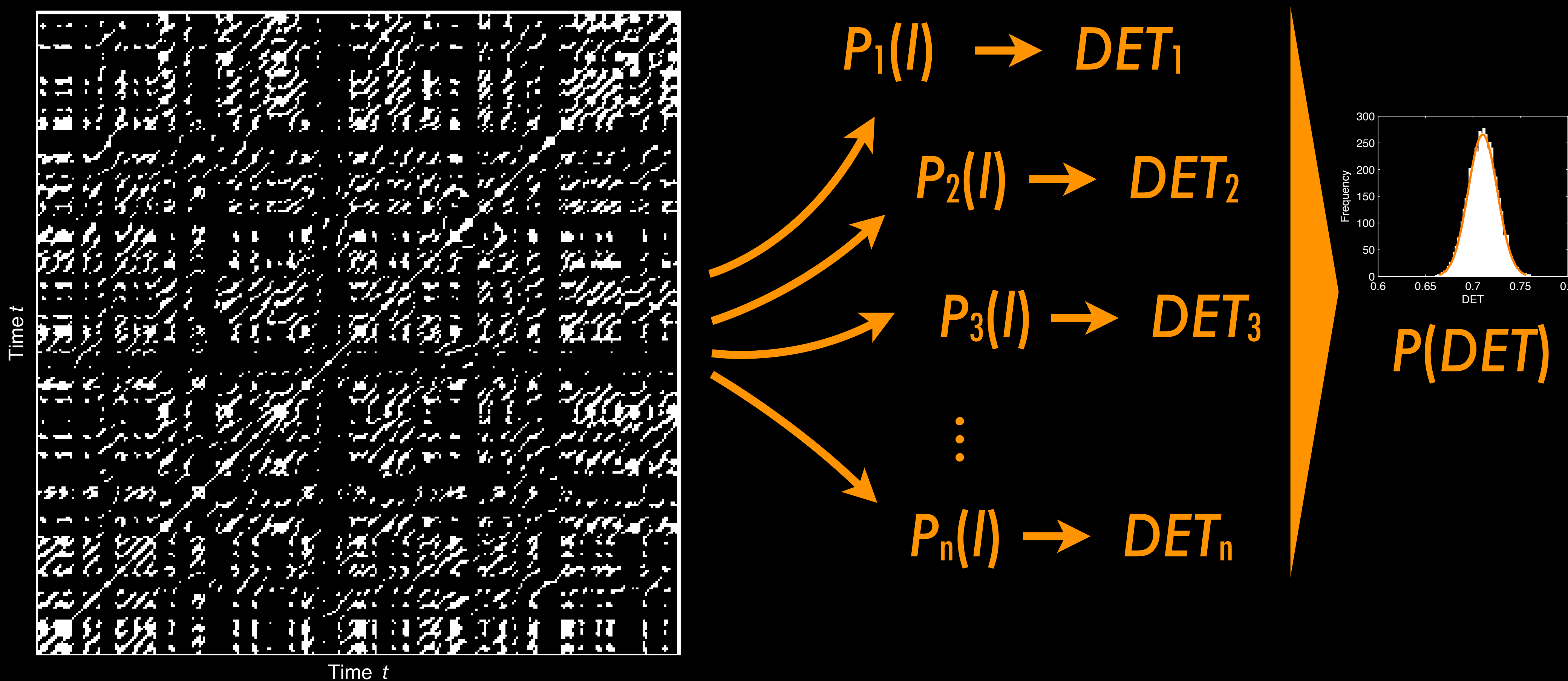
$P_3(l) \rightarrow DET_3$

$\vdots$

$P_n(l) \rightarrow DET_n$

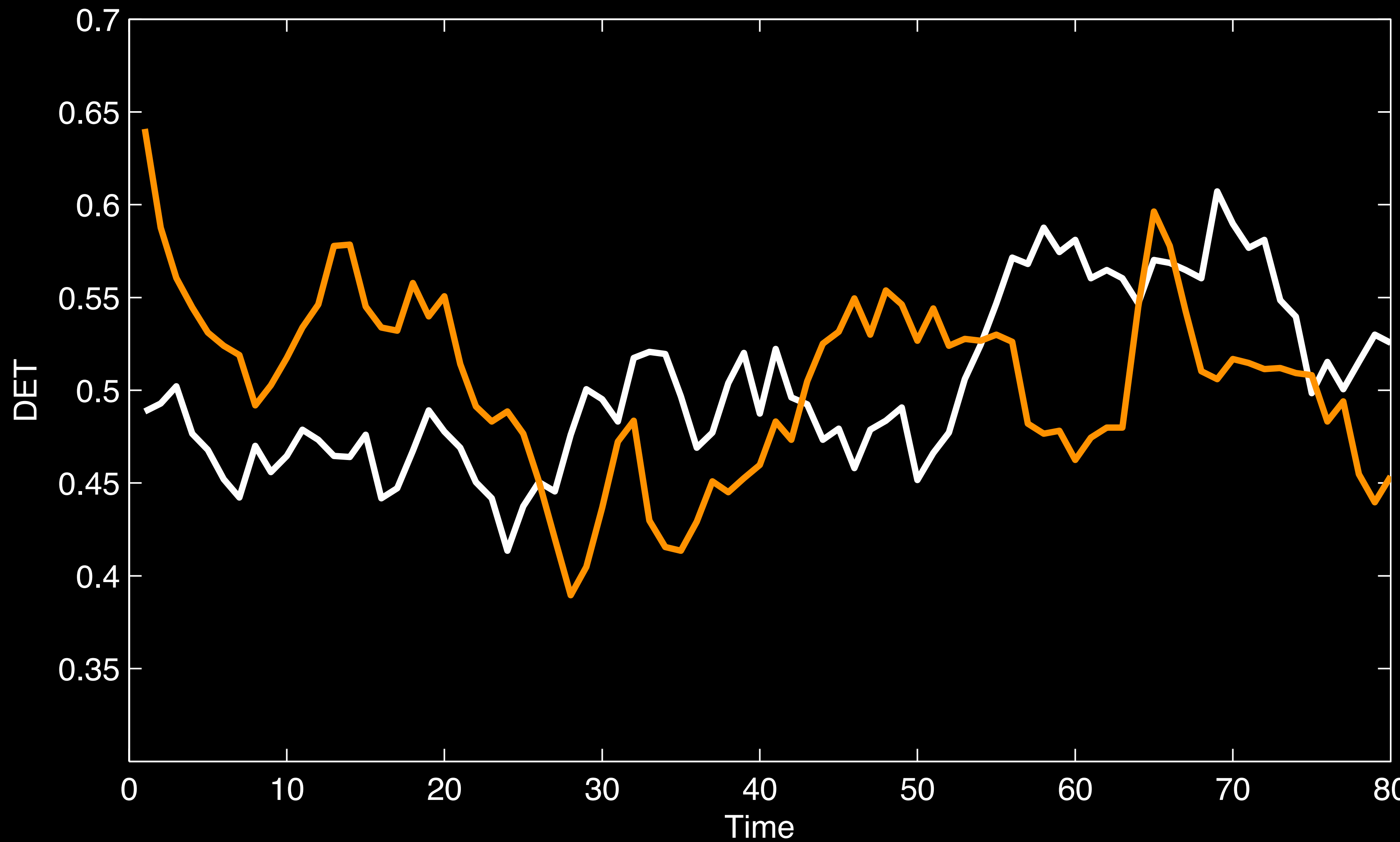
➡ Empirical distribution of RQA measures

Bootstrapping Recurrence Structures

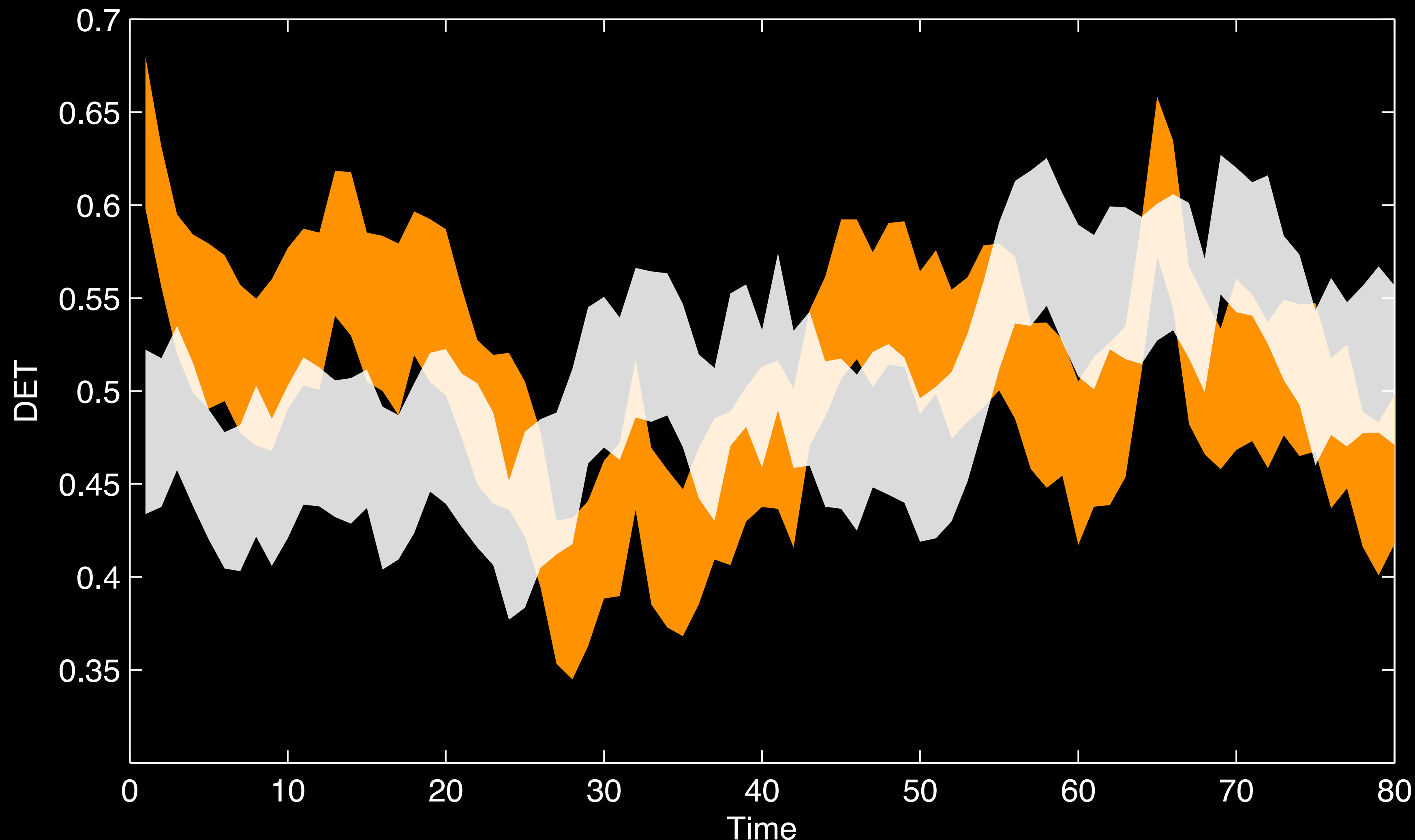


➡ Empirical distribution of RQA measures

Confidence Intervals



Confidence Intervals



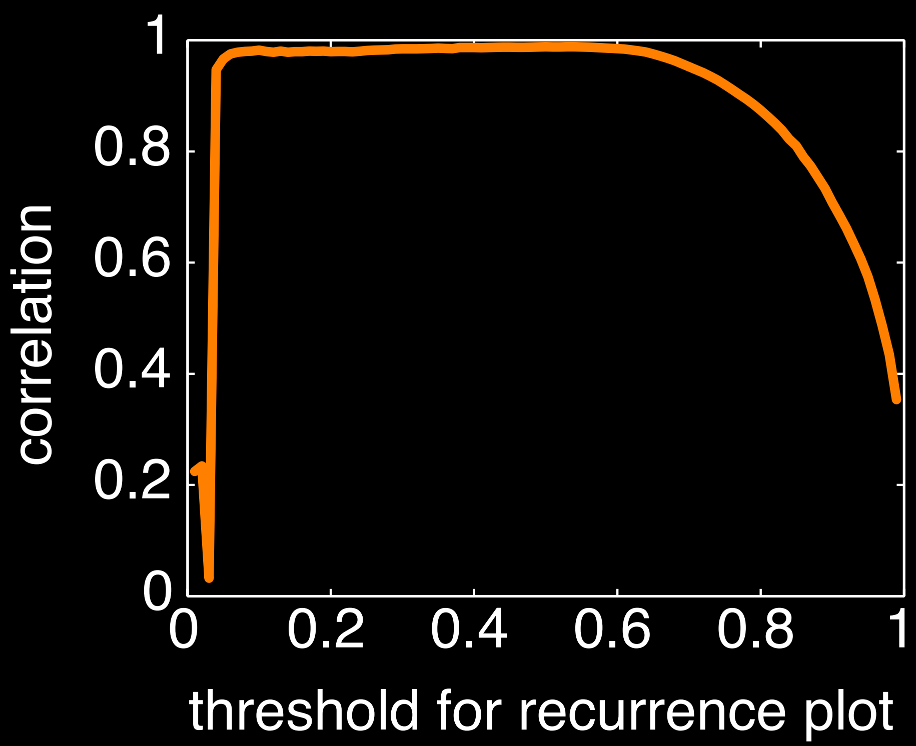
Schinkel et al., Physics Letters A, 373, 2009

Talk: Schinkel et al., Confidence bounds of recurrence-based complexity measures

Recurrence Threshold

- most crucial parameter in the RP analysis?
- only rules of thumb or some theoretical foundation?
- different threshold values for different kinds of analysis (dynamical invariants, transitions tests, signal detection, reconstruction, etc.)

Recurrence Threshold

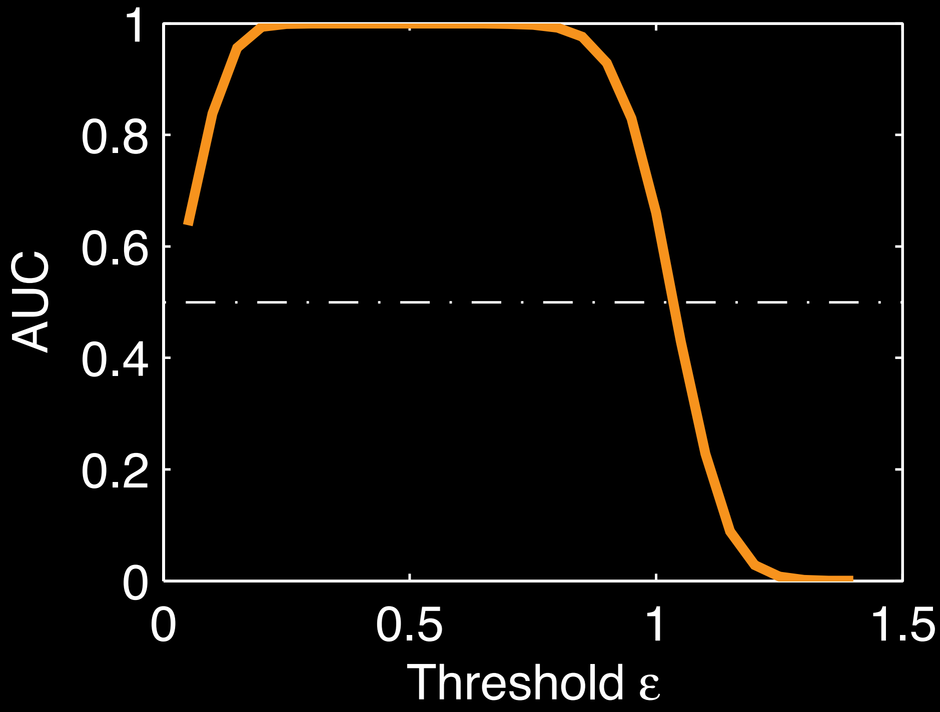


- reconstruction of distance matrix from binary RP
- correlation between original and reconstructed distance matrix

➡ range of optimal thresholds (in terms of recurrence rate)

Hirata et al., European Physical Journal ST, 164, 2008

Recurrence Threshold

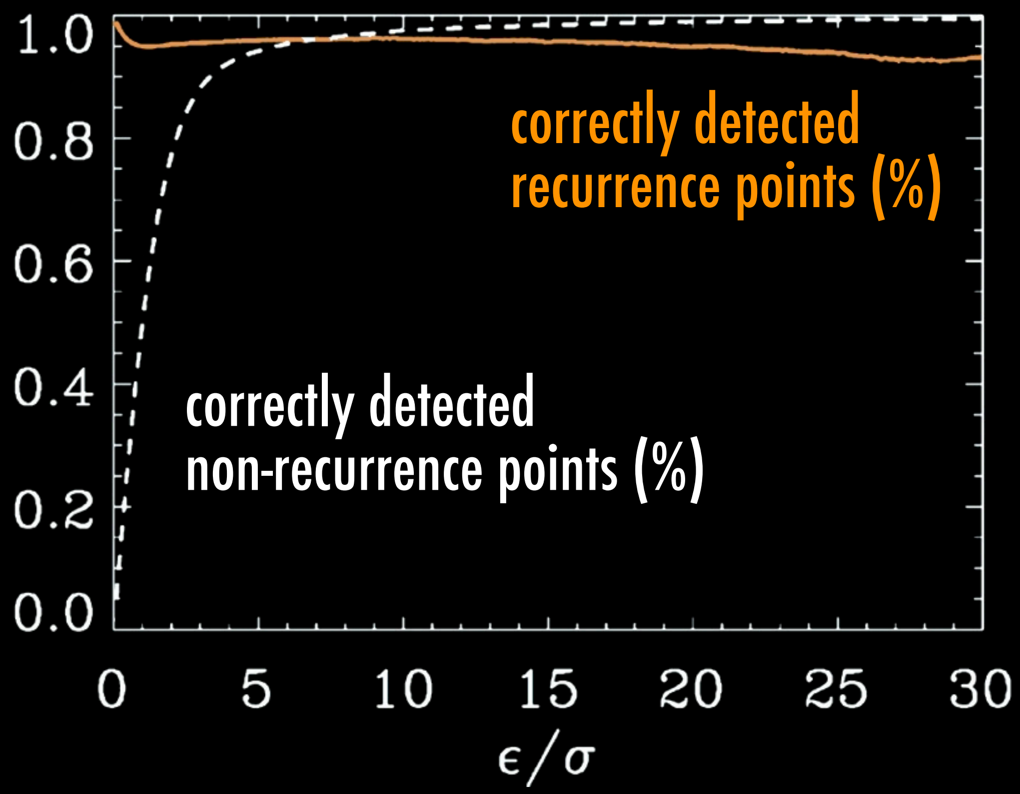


- signal detection from noise
- receiver operator characteristics (ROC) and area under curve (AUC)

➡ range of optimal thresholds (in terms of standard deviation)

Schinkel et al., European Physical Journal ST, 164, 2008

Recurrence Threshold



- observational noise

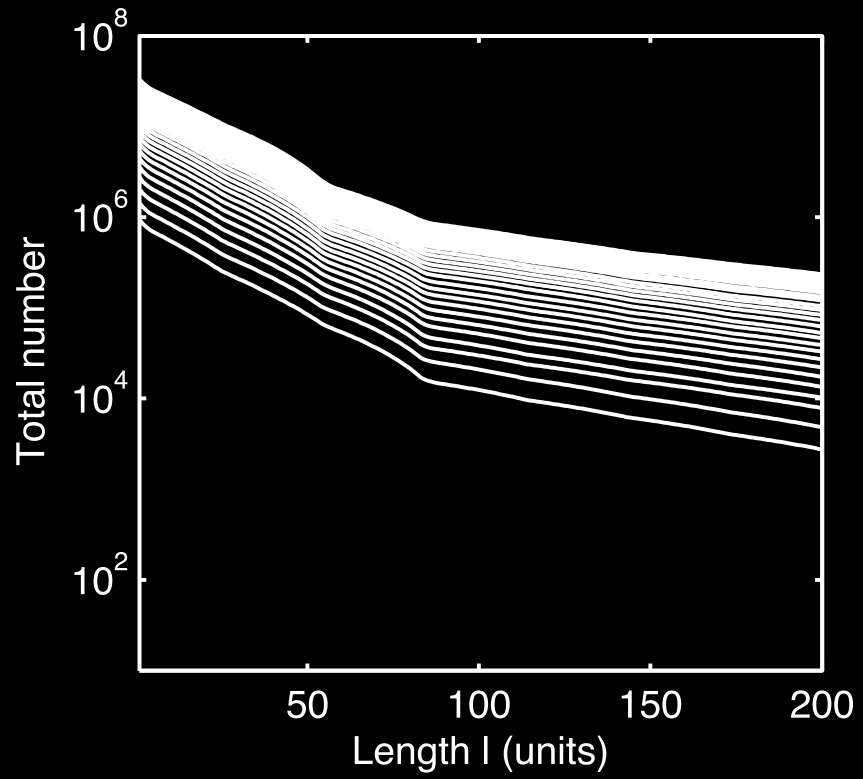
➡ threshold

$$\varepsilon > 5\sigma$$

(in terms of standard deviation)

Thiel et al., Physica D, 171, 2002

Recurrence Threshold



- dynamical invariants
  - ➡ threshold as small as possible
  - ➡ scaling with threshold

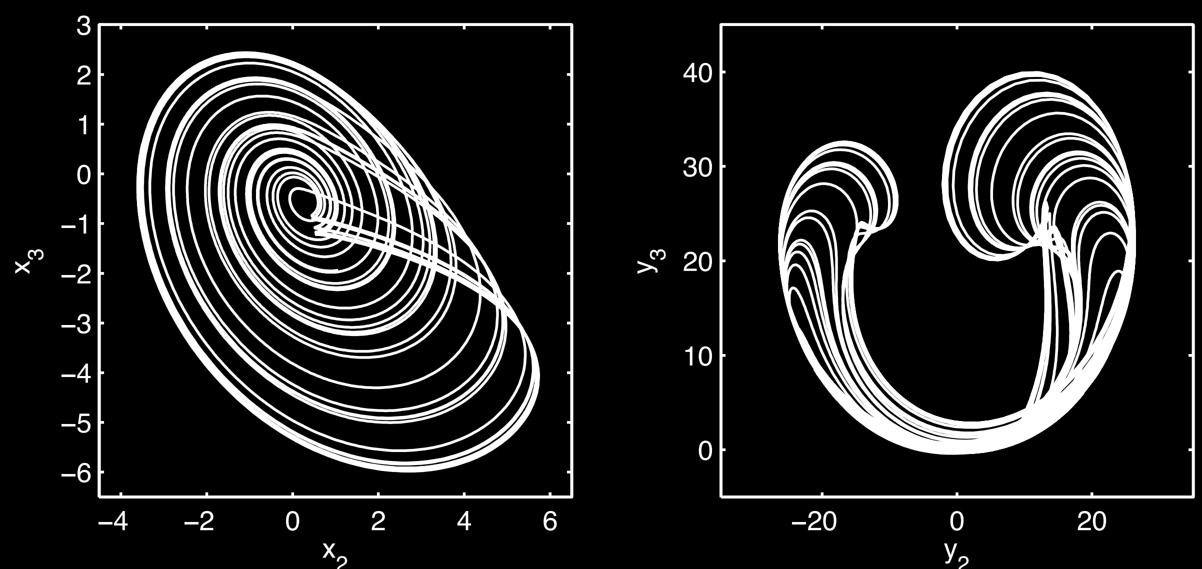
Thiel et al, Chaos, 14, 2004
Marwan et al, Physics Reports, 438, 2007

Coupling Direction

Rössler oscillator drives Lorenz oscillator

$$\begin{aligned}\dot{x}_1 &= b + x_1(x_2 - c) \\ \dot{x}_2 &= -x_1 - x_3 \\ \dot{x}_3 &= x_2 + a x_3\end{aligned} \qquad \begin{aligned}\dot{y}_1 &= -\sigma(y_1 - y_2) \\ \dot{y}_2 &= r\,u - y_2 - u\,y_3 \\ \dot{y}_3 &= u\,y_2 - b\,y_3\end{aligned}$$

$$\text{where} \quad u = x_1 + x_2 + x_3$$

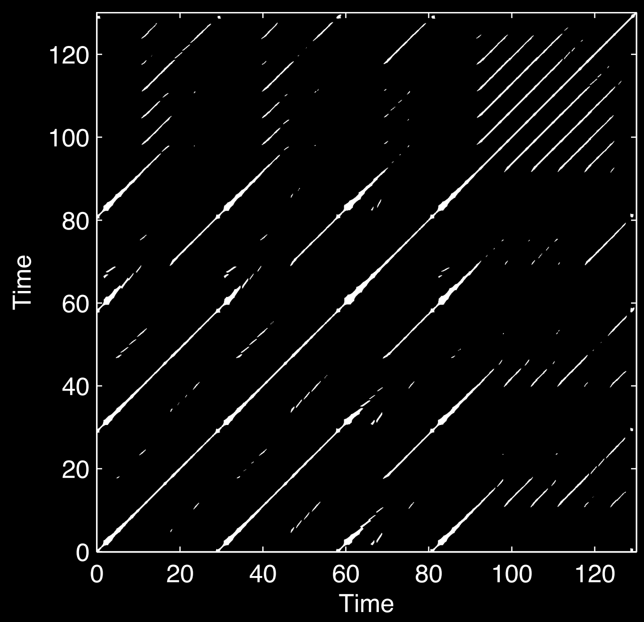


Coupling Direction

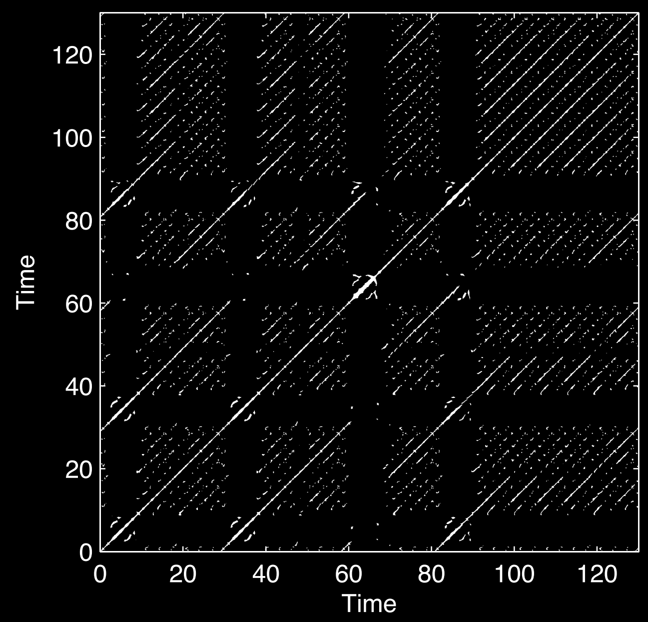
- Joint recurrence plot:

$$\mathbf{JR}_{i,j}(x,y) = \mathbf{R}_{i,j}(x) \cdot \mathbf{R}_{i,j}(y)$$

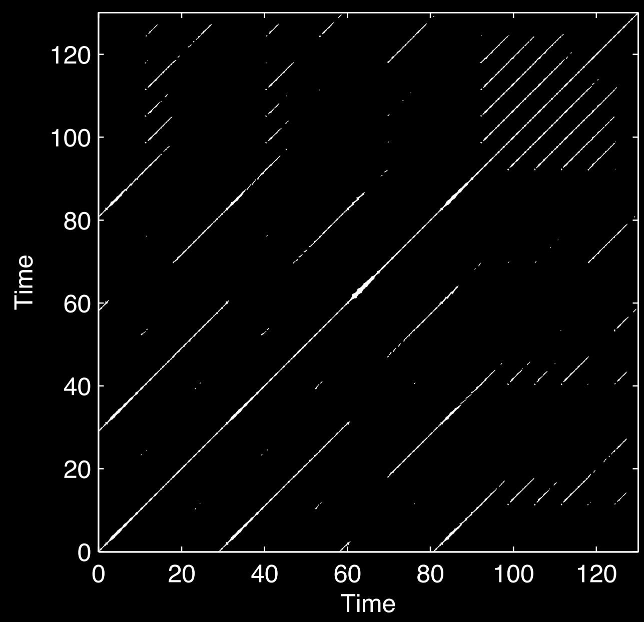
RP Rössler



RP Lorenz



JRP



Coupling Direction

prob. of recurrence of state x_i

$$p(x_i) = \sum_{j=1}^{N} \mathbf{R}_{i,j}(x)$$

joint prob. of recurrence of state x_i and y_i

$$p(x_i, y_i) = \sum_{j=1}^{N} \mathbf{JR}_{i,j}(x, y)$$

mean conditional prob. of recurrence between x and y

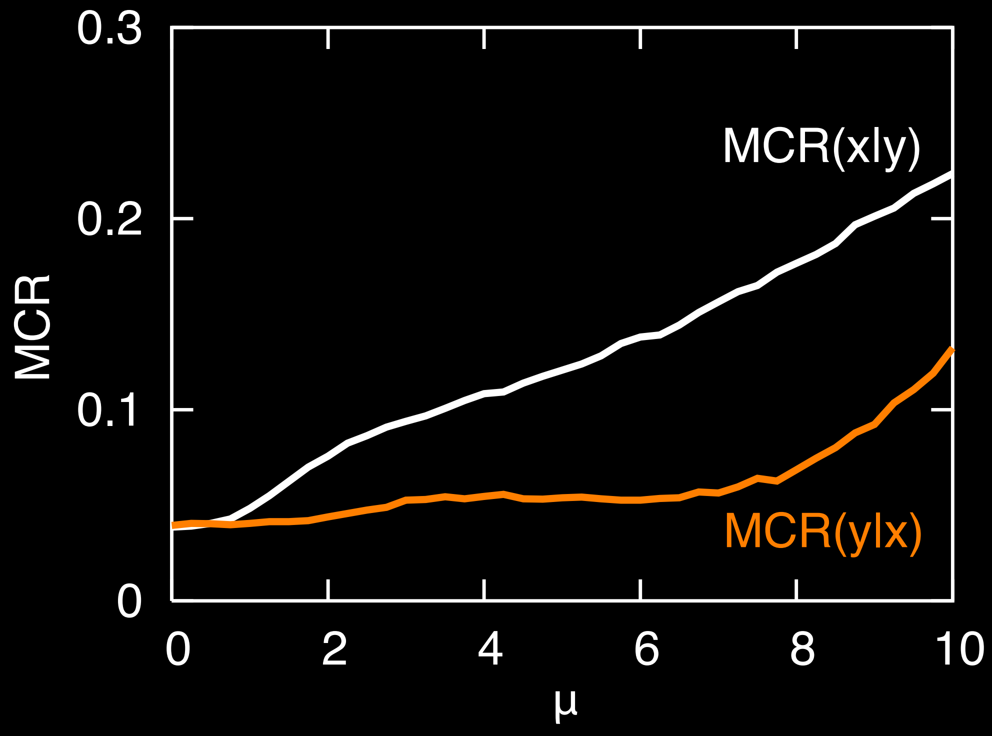
$$MCR(y|x) = \frac{1}{N} \sum_{i=1}^{N} p(y_i|x_i) = \frac{1}{N} \sum_{i=1}^{N} \frac{\sum_{j=1}^{N} \mathbf{JR}_{i,j}(x, y)}{\sum_{j=1}^{N} \mathbf{R}_{i,j}(x)}$$

Romano et al, Physical Review E, 76, 2007

Coupling Direction

- MCR(y | x) < MCR(x | y)
  - ➡ x drives y
- MCR(x | y) < MCR(y | x)
  - ➡ y drives x

weakly coupled, non-identical Lorenz oscillators



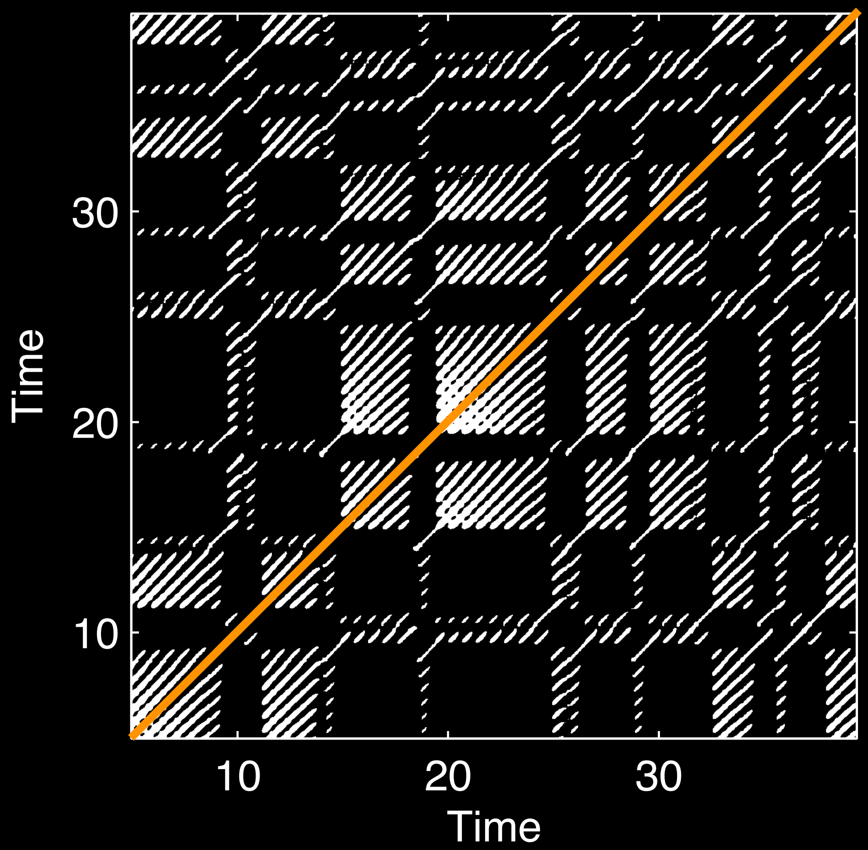
Romano et al, Physical Review E, 76, 2007

Talk: Yong Zou, Extracting indirect coupling by means of probabilities of recurrence: revisited

Recurrence Spectrum

- probability that system recurs after time τ (τ-recurrence rate)

$$RR_\tau = \frac{1}{N-\tau} \sum_{i=1}^{N-\tau} \mathbf{R}_{i,i+\tau}$$



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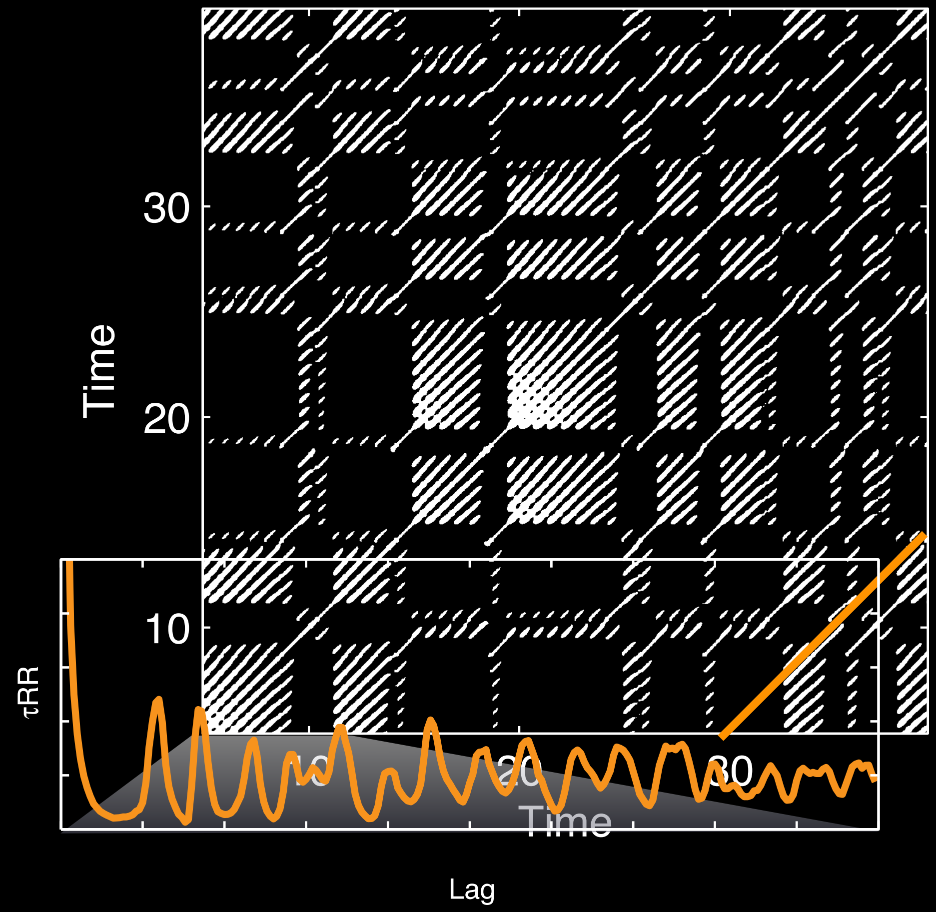
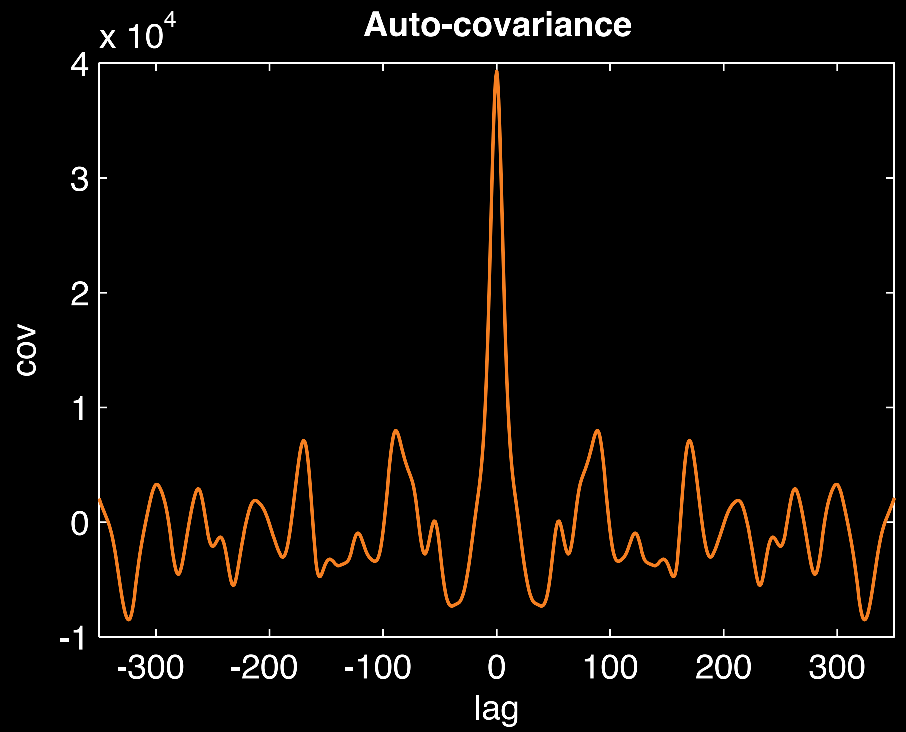
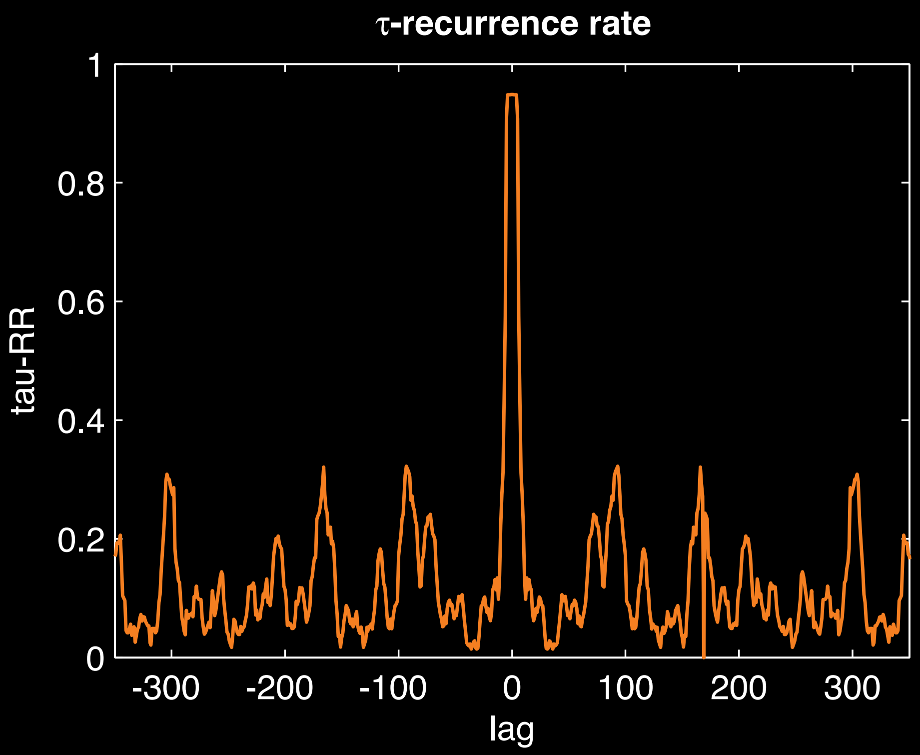


Recurrence Spectrum

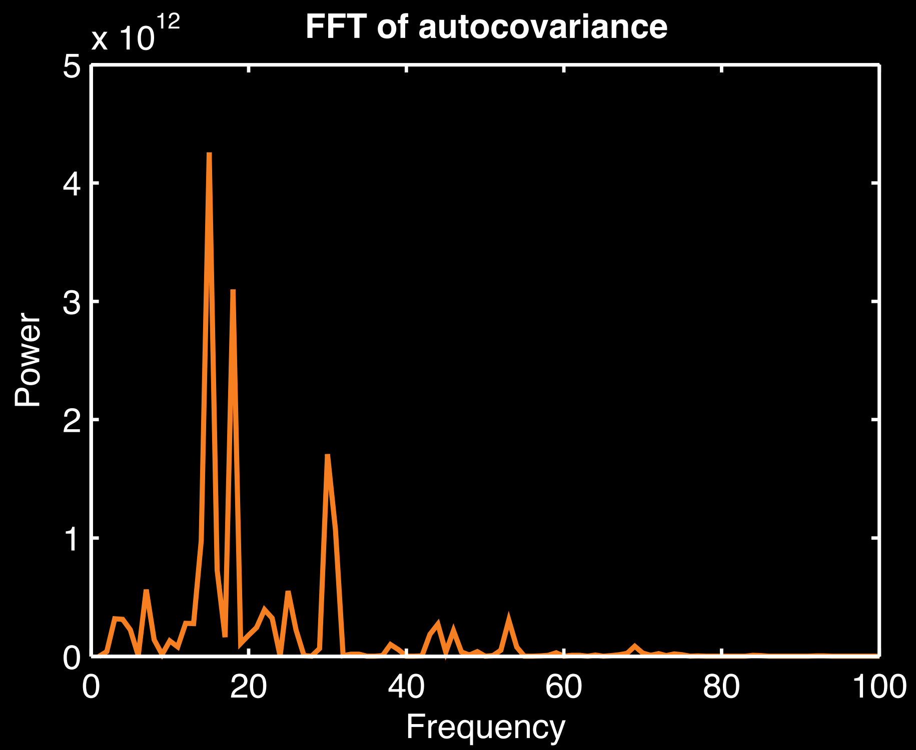
- τ-recurrence rate similar to auto-covariance

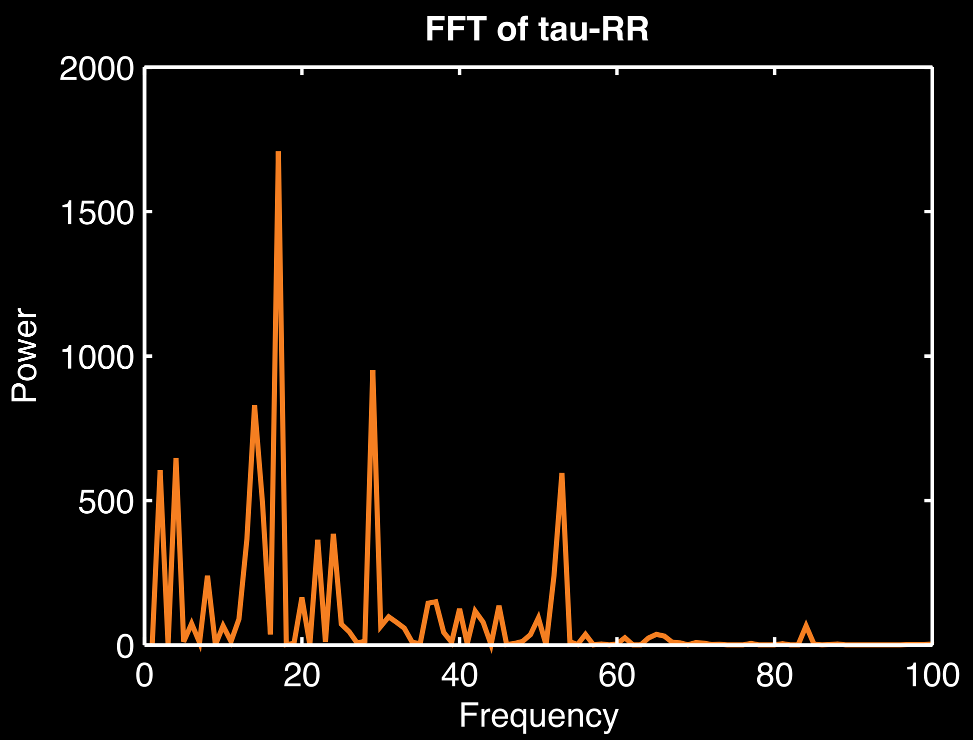




Recurrence Spectrum

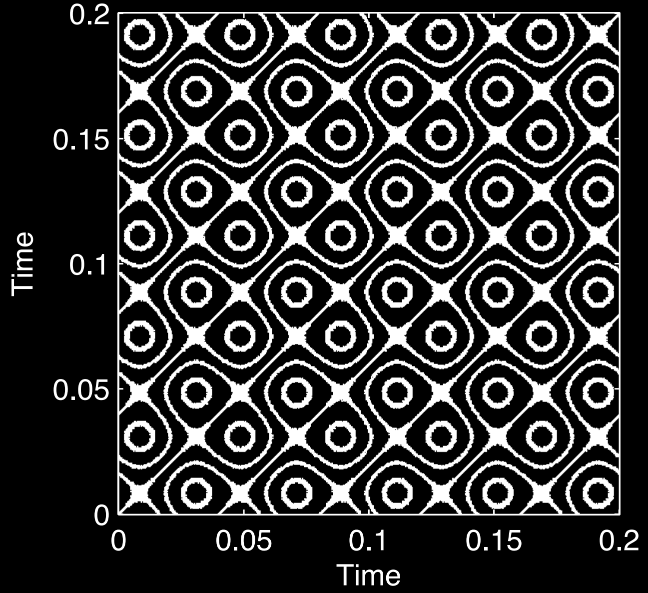
- power spectrum by FFT of τ-recurrence rate (Wiener-Khinchin theorem)





Macro Structures

- interference effect of sampling frequency and signal frequency



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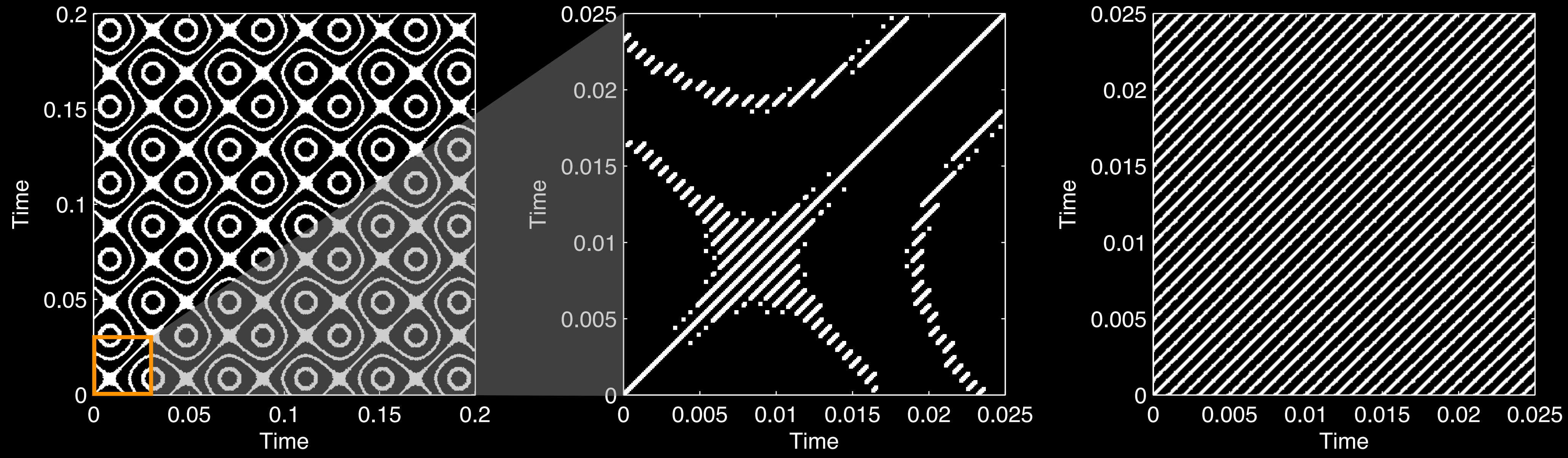
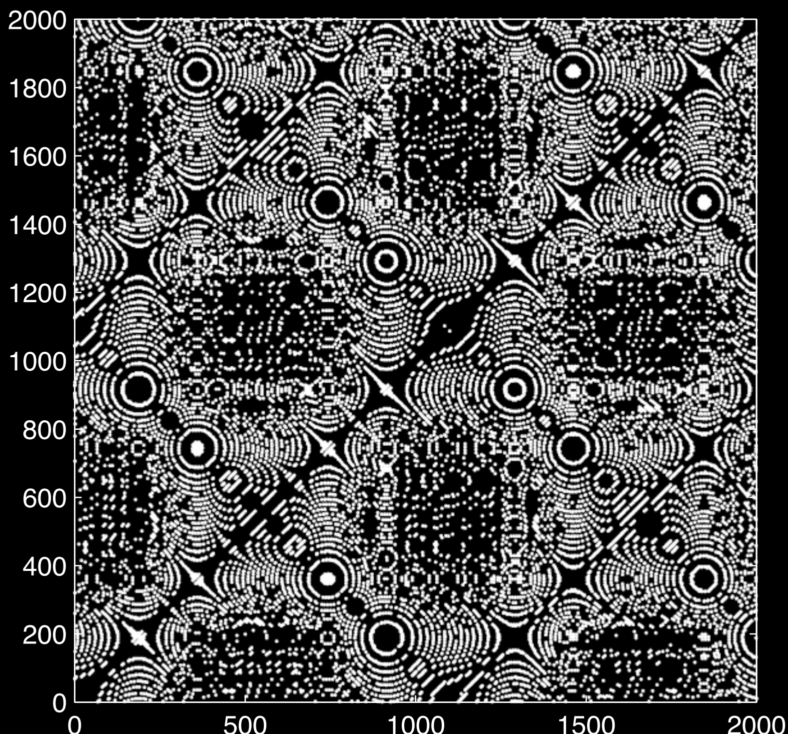


Macro Structures

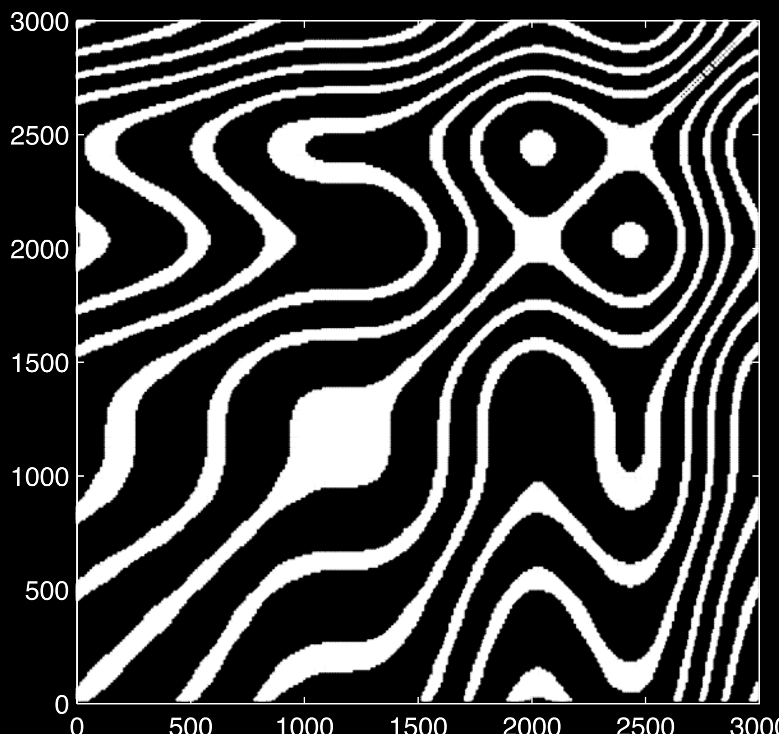
- very sensitive to slight frequency modulations

➡ magnification lens to detect tiny frequency modulations

$$\sin(2\pi f_c t + 2\pi \sin(2\pi f_m t))$$

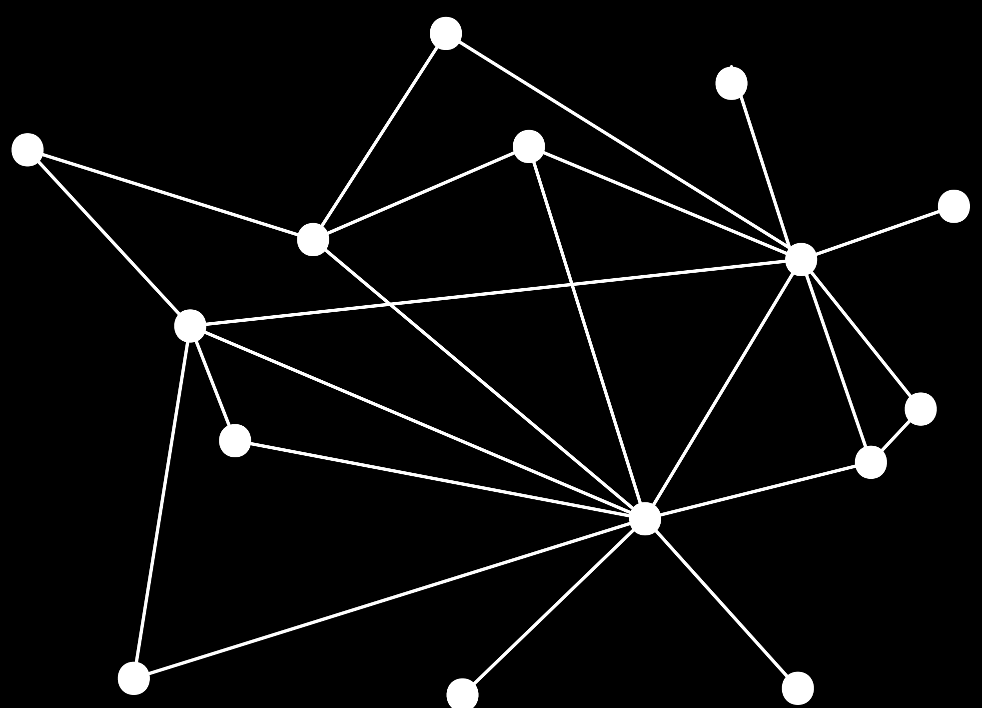


$$\sin(2\pi f_c t + 2\pi \sin(2\pi f_m t) t^{\frac{3}{2}})$$



Facchini et al, Physical Review E, 75, 2007

Complex Networks



- link matrix (undirected, unweighted network):
  - ➡ binary
  - ➡ symmetric

$A_{i,j}$ =

0	1	0	0	1
1	0	1	0	1
0	1	0	0	0
0	0	0	0	1
1	1	0	1	0

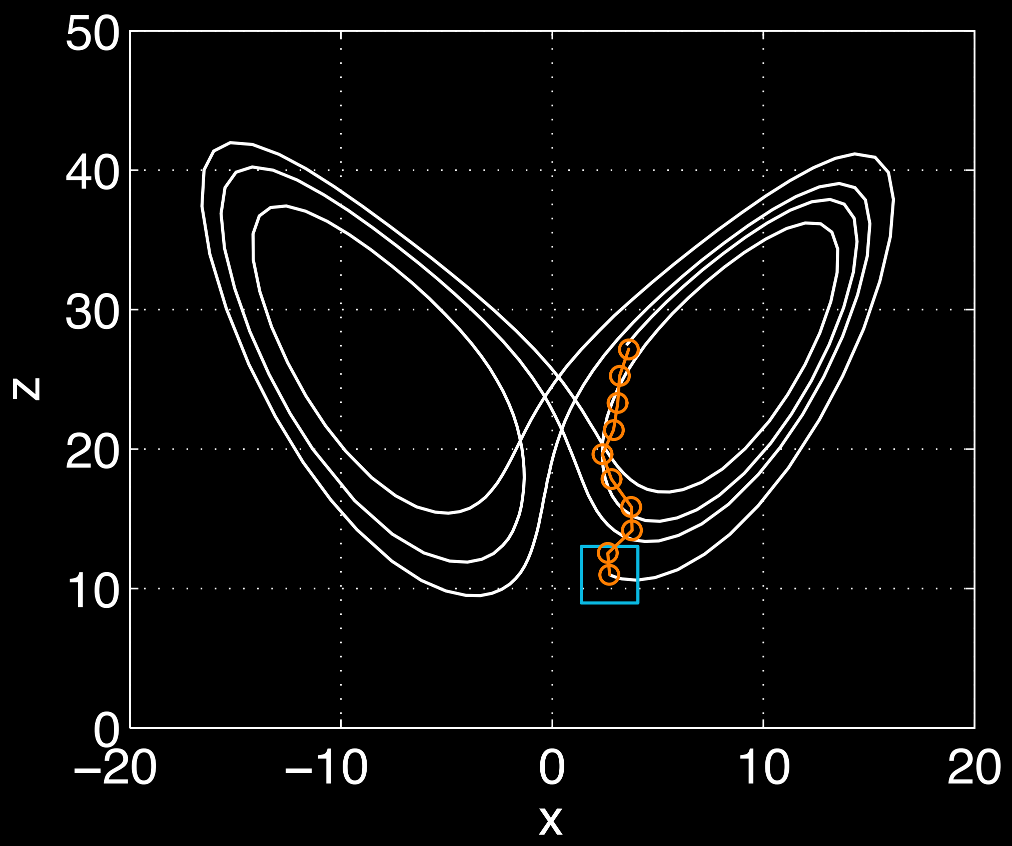
Complex Networks

- complex network analysis for time series analysis

➡ dynamical properties of time series of complex system

- E.g. mean link density:

$$\langle k \rangle = \sum_{i,j} A_{i,j}$$



Small, Complex Networks from Recurrence Plots and Time Delay Embeddings
Zaldivar et al., From complex networks to time series analysis and viceversa
Donner et al, The Complex Network Approach and Recurrence Quantification Analysis

Summary

confidence intervals	coupling direction	recurrence time spectrum
recurrence threshold	recurrence networks	macro structures