Emissions Trading with Clean-up Certificates: How Carbon Debt can Increase Climate Ambition Levels

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Abstract

Incentivizing and financing carbon dioxide removal (CDR) is a challenge for regulators. We show how introducing carbon debt – the obligation to remove carbon in the future – in an emissions trading scheme (ETS) can induce CDR and enable net-negative emission flows. For "clean-up certificates" that bundle emission permits with carbon debt, we characterize demand and pricing in an analytically tractable model. To ensure repayment of carbon debt, we derive the necessary value of collateral and discuss institutions as a lender of last resort. We find that introducing clean-up certificates does not reduce near-term carbon prices and mitigation efforts when they replace emission permits in the ETS, and that, by controlling the extent of carbon debt, clean-up certificates are more efficient than an ETS with full borrowing flexibility. In an exemplary calibration to a comprehensive EU ETS, we identify welfare-improving reforms that increase environmental ambition while simultaneously reducing compliance costs. With sufficiently rapid technological progress, the EU's remaining cumulative carbon budget could be halved compared to the current budget or even become negative.

1. Introduction

As global emissions continue to rise, carbon dioxide removal (CDR) is becoming an indispensable pillar of climate policy. Scenarios that keep global warming to below 2 degrees or even 1.5 degrees often rely on extensive upscaling of carbon removal. In some models, gross removal by a range of different CDR technologies amounts to

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10-20 Gt CO_2 per year – which is in the range of one-quarter to half of current carbon emissions – by the second half of the 21st century (IPCC, 2018). Besides compensating for residual, hard-to-abate emissions, annual carbon removal could even exceed emissions, turning net emissions negative, for three main reasons: First, unless progress in mitigation is accelerated, net-negative emissions may become mechanically necessary to return to the global temperature target after a period of temperature overshoot (see Fig. 1). In other words, by undoing past emissions, carbon removal makes achieving a climate target possible even after a temporary violation of that target. Second, for a sufficiently high social cost of carbon, the atmospheric carbon content may already be too high. Net-negative emissions would be optimal to return to the cost-benefit optimal carbon concentration. Recent studies that have updated climate damages upwards (Kotz et al., 2024; Bilal and Känzig, 2024) increase the likelihood of this case. Third, net-negative emissions could be motivated by justice and fairness considerations related to countries' historical responsibility in contributing to global warming. With carbon dioxide removal, countries that benefited from early industrialization could undo part of their past emissions.

While carbon prices are an effective policy instrument for reducing emissions (see, e.g., Döbbeling-Hildebrandt et al., 2024), incentivizing net-negative emissions via carbon pricing faces two key challenges. First, as the sign of the emission flow flips, a carbon tax becomes a removal subsidy. With an estimated global removal cost of up to two percent of global GDP, financing removal subsidies could put considerable pressure on public funds and therefore face political opposition.[1] As revenues from carbon pricing in a carbon-neutral economy will be low, other sources of finance would need to be accessed. Second, a cap-and-trade system, such as the EU Emissions Trading System (ETS), can deliver net-zero emissions by allowing the creation of certificates from removed carbon and integrating them in the carbon market. However, without further amendments to the regulation, existing cap-and-trade systems cannot incentivize or finance a later phase of net-negative emissions. This paper addresses these issues (for a broader discussion of the challenges of incentivizing CDR, we refer to Edenhofer et al. 2023, 2024).

One way to extend cap-and-trade systems to allow for net-negative emissions is the inclusion of "carbon debt". In analogy to the carbon budget that indicates the remaining cumulative emissions until mid-century, the cumulative net-negative emissions in the second half of the century represent a debt that needs to be repaid by removing carbon

[1] Assuming annual removal of 10-20 GtCO_2 in the second half of the century at costs of \$300/tCO_2 (Smith et al., 2023), global CDR expenditures could reach \$3-6 trillion annually. With a projected global GDP of \$365 trillion in 2075 (average over SSP1–SSP5 scenarios), this represents 0.8-1.6 percent of world GDP.

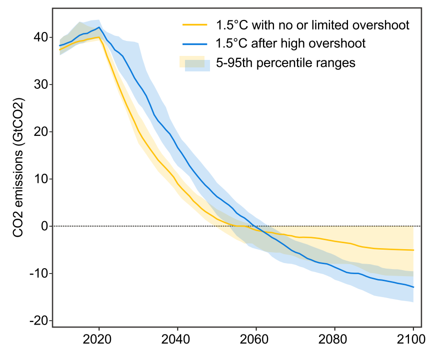

Figure 1: Global net carbon dioxide (CO_2) emissions in scenarios assessed in IPCC (2022). Figure adapted from Smith et al. (2023).

from the atmosphere. While current emission trading systems implement the carbon budget by setting rules for its allocation and the trade of emission permits, an extension to net-negative emissions requires new rules for the responsibility for carbon debt and for ensuring that the debt is paid back.

In this paper, we propose such a set of new rules and investigate how carbon debt can be integrated into an existing ETS by introducing "clean-up certificates". Clean-up certificates bundle the permission to emit CO_2 with an obligation for its removal, thereby shifting the responsibility to clean up emissions via carbon removal to the emitter in the spirit of the "polluter pays principle". To ensure that carbon debt is repaid, we suggest a collateral requirement to address limited liability of debtors, and an institutional setting with a carbon central bank as a lender of last resort.

Our analysis contributes to an emerging literature on carbon debt. Bednar et al. (2021) introduce the mechanism of carbon removal obligations, which links current emissions to carbon debt. The carbon debt enters the balance sheets of the emitters as a liability – similar to financial debt – to banks and central banks. The authors argue that the default risk of carbon debt is addressed by a mark-up on the interest rate paid for carbon debt, thereby relying on the financial sector rather than carbon markets for the implementation. Their numerical simulations show that the interest on carbon debt reduces the reliance on carbon dioxide removal.[2] The idea of linking CO_2 emissions to their removal is also fundamental for carbon takeback obligations (Jenkins et al., 2021, 2023), which require fossil fuel companies to demonstrate removal of a fraction of the associated emissions. To guide the world economy towards net-zero emissions, the fraction converges to 100 percent on a given trajectory. As removal is required before

[2]See Bednar et al. (2023a) for details on the pricing of the premium on carbon debt and Bednar et al. (2023b) for details on the implementation.

extraction (or import), the takeback obligations do not need the concept of carbon debt. In contrast, Lyngfelt et al. (2024) focus on financing CDR in the future, as tight fiscal budgets and lack of international cooperation may otherwise render future CDR infeasible. They suggest requiring emitters to deposit a fee for each unit of emissions that can only be redeemed upon its removal from the atmosphere. Lyngfelt et al. do not determine a necessary minimum value of the fee, arguing instead that when the deposit is invested as part of a public fund, its value eventually increases to a point where it becomes profitable to carry out the removal and redeem the deposit. To incentivize further negative emissions, for example to offset historical emissions, a ratio greater than unity of required removal to emissions could be required from the emitters.

The idea of ensuring removal by demanding a deposit that is redeemable and tradable is similar to carbon shares in Lemoine (2020, 2024). Lemoine proposes a carbon stock tax that emitters pay as a rental charge for storing carbon in the atmosphere until the time when they remove the CO_2 from the atmosphere. The incentive to remove emissions is impaired when firms can forego the rental charge in case of bankruptcy. To address this moral hazard, Lemoine introduces up-front payment via a bond that is redeemable upon removal, and tradable carbon shares that financialize the liability. Rickels et al. (2021) analyze the integration of negative emissions into a cap-and-trade system like the EU ETS, highlighting both legal and economic challenges. They point out that full integration of carbon removal increases efficiency, whereas imposing quantity constraints on removals reduces the efficiency of carbon markets. However, constraints may be necessary, as large-scale substitution of abatement by removal could reduce political feasibility and public acceptance.[3] Consequently, Rickels et al. (2021) propose separation of the markets for emission reductions and carbon removal. This separation allows for incentivizing carbon dioxide removal without interfering with the existing EU ETS. An intermediary could connect the two markets by buying removal credits and selling them into the ETS (at a lower price). Furthermore, Rickels et al. (2022) suggest that the intermediary could build a strategic reserve by banking removal credits, which could be used to cushion price spikes in the ETS. In sum, the incentives for technological learning in both technologies are maintained by managing the prices for removal and abatement separately.

In contrast to previous studies, we focus on the integration of carbon dioxide removal

[3]As surveyed by Burke and Gambhir (2022), the underlying worry is that integrating carbon dioxide removal in compliance markets deters mitigation efforts. This mitigation deterrence may occur for multiple reasons. Uncertainty about removal capacities and cost makes future removal an imperfect substitute for near-term mitigation. Similarly, lack of additionality, non-permanence of storage or moral hazard may cause planned net-negative emissions never to materialize. Moreover, integrating carbon removal may reduce the carbon price in the cap-and-trade system, thereby impairing the price signal that triggers mitigation and low-carbon investments.

in an existing ETS that allows for a temporary overshoot of the carbon budget and a phase of net-negative emissions. And we show how the trade-off between "mitigation deterrence" (environmental ambition) and economic flexibility can directly be fine-tuned in the design of the ETS extension. These two features allow to improve the dynamic efficiency of emissions trading schemes while simultaneously safeguarding against the environmental concerns of crowding out mitigation. Using an analytically tractable, dynamic model of an emissions trading scheme, we model the introduction of clean-up certificates, carbon debt, and collateral requirements. We derive closed-form solutions and fully characterize the equilibrium paths of net emissions and the prices of emission permits. We calculate the prices of clean-up certificates and the collateral required to ensure the repayment of carbon debt, and identify the length of different characteristic phases (transition to net zero, net zero, and net negative). We find that the demand for clean-up certificates is driven by the anticipation of technological progress in abatement and removal, the discount rate and the length of the compliance period. In contrast, without (sufficient) cost-savings in CDR, there is no market for clean-up certificates.

We show that clean-up certificates can be introduced without deterring mitigation efforts by retiring an equal amount of regular emission permits. As this leaves the number of emission permits the same, neither net emissions nor carbon prices are affected in the near-term. Simultaneously, associating carbon debt with existing emission permits reduces the cumulative emissions of the ETS below its initial carbon budget, implying a strengthening of the ambition of the ETS. For still more environmental ambition, more than one unit of carbon debt can be associated with each emission permits, such that the ETS delivers cumulative net-negative emissions, i.e. overall the ETS does not add any CO_2 to the atmospheric carbon stock but removes some CO_2 from it. While clean-up certificates assign the responsibility for CDR to emitters, they do not necessarily imply a financial burden for the private sector because, implicitly, the net-negative emissions are financed by forgone revenues from permit auctioning, as the clean-up certificates sell at a lower price than regular emission permits. In contrast, introducing clean-up certificates in addition to the carbon budget of an existing ETS exerts downward pressure on the carbon price, and mitigation efforts are deterred. The carbon budget is temporarily overshot, and cumulative emissions only return to the original target at the end of the compliance period. In this case, the net-negative emissions induced by clean-up certificates are not additional to the carbon budget. However, the regulator may also choose the extent to which this "clean-up" is additional: a share of carbon debt could be attached to existing emission permits to create additional net-negative emissions, while the remainder could be issued as newly created clean-up certificates that will crowd-out mitigation and reduce abatement cost. In this way, by choosing the degree of additionality, the regulator can choose the trade-off between dynamic efficiency and environmental integrity.

We numerically illustrate these results by calibrating our model to a comprehensive EU ETS and show the implications for emission pricing, the economic efficiency of the carbon market, for its environmental effectiveness, and for the political feasibility of including clean-up certificates. Clean-up certificates allow the carbon market to tap the intertemporal flexibility created by CDR and thus to improve its efficiency relative to a traditonal ETS. Clean-up certificates are furthermore superior to an ETS with unlimited borrowing as they allow the regulator to limit the extent of carbon debt (and, equivalently, borrowing), and, therefore, grant some control over the environmental consequences of the emissions overshoot associated with borrowing. They fall short of the full efficiency that could be implemented with intertemporal trading ratios following Leiby and Rubin (2001). We identify a trade-off for choosing the share of clean-up certificates that replace permits in the ETS: a higher share brings down climate change damages, but increases compliance costs and reduces the revenues from auctioning emission permits and clean-up certificates. However, we find that given sufficient technological progress, there exists a set of clean-up policies that reduce cumulative climate damages while simultaneously increasing fiscal revenues and lowering carbon prices and mitigation costs. The range for such win-win policies decreases with the size of carbon debt relative to the remaining carbon budget and, hence, requires to introduce clean-up certificates sooner rather than later. The (second-)best policies from the set of "Pareto" improving policies are characterized by a high additionality of removal and high levels of carbon debt (in excess of the current budget of the EU ETS). If technological progress in removal technologies is sufficiently strong, we estimate a sizable welfare gain in the order of magnitude of the EU-Canada trade agreement.

Finally, we discuss institutional aspects that are relevant to ensure a functioning market for clean-up certificates. When financial intermediaries cannot overcome liability problems and default risks due to long time horizons and large uncertainties, a public institution might step in. Such an institution could charge the carbon debt to buyers of clean-up certificates and take the risk to finance the future carbon removals. This latter property makes it a lender of last resort for carbon debt – a 'carbon central bank'. Similarly, time inconsistency problems that are inherent to the management of carbon budgets can be reduced by delegation to an independent institution. In the context of the EU ETS, a European Carbon Central Bank could address both issues.

The remainder of the paper is structured as follows: Section 2 introduces the model and Section 3 presents the characterization of net emissions and carbon prices in closed form. In Section 4, we present the model calibration to the case of the EU ETS and discuss the results. In Section 6, we discuss institutional challenges for the governance of clean-up certificates. We conclude in Section 7.

2. The model

We consider an economy where net-emissions $Q(t)$ accumulate without decay in the atmospheric carbon stock $X(t)$ with $X(0) = 0$. At any time t, net-emissions can be controlled via abatement and removal at the cost of $f(Q)$. That is, $f(Q)$ is the cost of limiting net-emissions to Q, such that net-zero emissions can be achieved at $f(0)$, and the cost of achieving q net-negative emissions is $f(-q)$.[4] The cost function $f(Q)$ captures the economic reality that achieving lower net emissions is more costly. This reflects both the direct costs of abatement and removal technologies as well as the opportunity costs of foregone economic activity that would have generated higher emissions. Costs $f(Q) = A(t)a(Q)$ are reduced by autonomous technological progress $A(t)$ and are otherwise convex with $a' < 0$ and $a'' \geq 0$.

As a starting point, we consider a conventional emissions regulation over a finite time period $[0,T] \ni t$ that constrains emissions by a cap B on cumulative emissions $X(t)$ at any time t. By assuming that the actors of a market economy comply with the regulation at the lowest possible cost, the equilibrium of the market economy is, in absence of any market distortions, equivalent to a minimization of the net present value of the cost of limiting net emissions in accordance with the regulation by a social planner:

$$\max_{Q} \int_{0}^{T} -\left[A(t)a\left(Q\right)\right] e^{-rt} \mathrm{d}t \tag{1}$$

$$\text{such that} \quad \dot{X} = Q \qquad \perp \mu \tag{2}$$

$$X(t) \leq B \quad \text{(per-period constraint)} \qquad \perp \gamma \tag{3}$$

$$\text{and} \quad X(T) = B \quad \text{(transversality condition)} \tag{4}$$

Variables $Q(t)$, $X(t)$, the co-state variable $\mu(t)$ and the Lagrangian multiplier $\gamma(t)$ are all time-dependent. Where possible, we do not indicate time-dependence to keep the equations short and simple. One easily verifies that if the constraint B was implemented by via tradable emission permits, the allowance budget would be B and their price

[4]Considering net emissions instead of separating abatement and removal implies an efficient balance of the two with equal marginal costs at every point in time where both technologies are used. Consider, for example, removal $R(t)$ and mitigation $M(t)$ as perfect substitutes such that net emissions Q for given baseline emissions $Q_0(t)$ are $Q(t) = Q_0(t) - R(t) - M(t)$. For cost functions $r(R)$ and $m(M)$, cost efficiency at t is then given for $r'(R) = m'(M)$. Without loss of generality, we can therefore proceed with $f(Q)$ as the abatement cost of reducing baseline emissions to the amount of $Q(t)$ net emissions under an efficient use of removal and mitigation.

would be the negative of the shadow price of emissions, $-\mu(t)$, see Appendix A.

We now extend the standard emission cap problem (1-4) by the concept of carbon debt D, i.e. the obligation to remove D carbon emissions from the atmosphere. In terms of permit trading, carbon debt is introduced as a new type of emission permit, *clean-up certificates*, which bundles one unit of carbon debt with one emission allowance (we relax the assumption of a one-to-one exchange rate in Section 3.4). We model the effects of introducing clean-up certificates by adjusting the carbon regulation in (4) and (3) to reflect carbon debt.

We capture two ways in which the newly introduced carbon debt D affects the carbon regulation. First, when carbon debt is required to be repaid in addition to the emission reduction for the original carbon budget B, carbon debt creates additional net-negative emissions that reduce the cumulative net-emissions of the ETS below B, i.e. the new regulation reduces cumulative emissions at time T to $X(T) = B - D$. When introducing D clean-up certificates into the permit trading system, this "additionality" of removal is implemented by retiring an equal amount of conventional allowances (thus reducing their supply B).

Second, carbon debt can ensure that emissions that temporarily exceed the carbon budget B will be undone by the end of the regulation period. That is, cumulative emissions can be allowed to overshoot the carbon budget by the amount of carbon debt, $X(t) \leq B + D$, because repayment of D will ensure that the carbon budget holds by time period T, $X(T) = B$. In a permit trading system, this is accomplished by introducing D clean-up certificates on top of the budget of existing emission allowances (i.e. without retiring permits).

In the model, we allow for both kinds of carbon debt by including the additionality parameter λ when we introduce carbon debt in equations (3) and (4) such that $\lambda = 1$ introduces additional net-negative emissions, whereas $\lambda = 0$ only increases intertemporal flexibility without effect on the ultimate cumulative emissions (i.e. carbon debt introduces limited borrowing of allowances as in Leiby and Rubin 2001).

$$X(t) \leq B + (1-\lambda)D \quad \text{(per-period constraint)} \qquad \perp \gamma \qquad (3')$$
$$X(T) = B - \lambda D \quad \text{(transversality condition)} \qquad (4')$$

Mixed policies are possible by choosing an additionality $0 < \lambda < 1$. In terms of clean-up certificates, when introducing D clean-up certificates with an additionality of λ, then D clean-up certificates are introduced, and λD conventional emission permits of the carbon budget are retired (see Figure 2). The effect is an increased environmental ambition in the final time period T by λD relative to the original carbon budget, and an increased intertemporal flexibility by temporarily expanding the carbon budget by

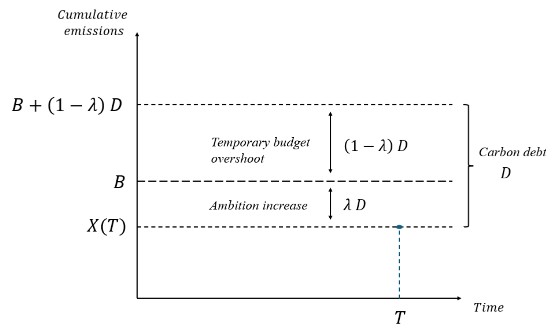

Figure 2: Relation of the budget B, the long-term target X_T, the overshoot cap $\hat{B}$ and the carbon debt D. The fraction λ of carbon debt is issued to increase the ambition of the long-term target.

$(1-\lambda)D$ at any time before T.

3. Model dynamics

The optimal solution is characterized by the following first-order conditions:

$$\mu = A(t)a'(Q) \tag{5}$$

$$\dot{\mu} = r\mu + \gamma \quad \text{or} \quad \hat{\mu} = r + \frac{\gamma}{\mu} \tag{6}$$

The co-state variable μ is the shadow price of keeping net emissions at Q. The sign of μ will be negative, as an increment in the carbon stock X tightens the remaining budget and therefore has a negative impact on Q in the objective function.[5] The Lagrangian multiplier γ of the per-period constraint is always non-negative and characterizes the stringency of the upper bound on cumulative emissions.

Therefore, according to (5), the shadow price μ balances with marginal abatement costs. Its growth, according to (6), is driven by discount rate r and the stringency of the per-period constraint. Due to the no-arbitrage condition, the price of the clean-up certificate must equal the price of the conventional emission permit minus the discounted removal costs. We will give the exact expression for this price below in Section 3.3.

[5] To see this, rewrite the transversality condition for a point in time t^* along the optimal path as $X(T) = X(t^*) + \int_{t^*}^{T} Q(\tau)d\tau + X_0 \leq B$ and rearrange to $\int_{t^*}^{T} Q(\tau)d\tau \leq \bar{X} - X_0 - X(t^*)$ to see that an increase in $X(t^*)$ negatively affects Q for $t > t^*$.

3.1. Net emission pathways

Taking the time derivative of (5) and dividing by the original equation yields

$$\hat{\mu} = g + \frac{a''}{a'}\dot{Q} \tag{7}$$

where $\hat{\mu} := \frac{\dot{\mu}}{\mu}$ is the rate of change of the shadow price and $g := \frac{\dot{A}}{A}$ is the autonomous rate of change of marginal abatement cost,[6] which, combined with (6), can be solved for $\dot{Q}$:

$$\dot{Q} = \left((r-g) + \frac{\gamma}{\mu}\right)\frac{a'}{a''} \tag{8}$$

At this point we make specific assumptions about the cost function $f(Q)$. For the variable cost we assume an exponential function $a(Q) = e^{-\alpha Q}$, that is, we assume that the costs of reducing emissions increase at a constant relative rate, α. The choice of the exponential function simultaneously implies that marginal costs (of mitigation and removal) increase a constant relative rate, $a''/a' = \alpha$. Furthermore, for the autonomous technological efficiency $A(t)$, we assume a constant rate of cost-saving technological change $-g$. With these assumptions, equation (8) can be rewritten as

$$\dot{Q} = -\left((r-g) + \frac{\gamma}{\mu}\right)\frac{1}{\alpha} \tag{9}$$

Together, the differential equations (6) and (9) characterize the dynamics of any efficient solution.

3.2. Reference case: No carbon debt

As a reference scenario, we consider the case of no carbon debt, that is, $D = 0$. The per-period constraint ($3'$) and the transversality condition ($4'$) simplify to $X(t) \leq B$ for all t. The assumptions of no carbon debt and no permit borrowing are in line with the current implementation of the EU ETS. When the remaining carbon budget is still positive, $B > 0$, the economy transitions to net-zero emissions in a first phase, followed by a second phase with continued net-zero emissions.

Phase 1: Transition to net-zero emissions. Starting at $B > X(0) = 0$ the per-period constraint ($4'$) is not binding, and we have $\gamma = 0$ such that (9) simplifies to

[6] We refer to $-g$ as the rate of technological progress, but it also encompasses broader structural changes, for example, demand changes.

$$\dot{Q} = -\frac{(r-g)}{\alpha} \tag{10}$$

That is, net-emissions decrease at a constant rate at a pace that increases with the rates of discounting (r) and cost-reducing technological progress ($-g$) as long as $r > g$, which we assume to be the case.[7] Hence, we can express net-emissions at time t as

$$Q(t) = Q_0 - \frac{(r-g)}{\alpha}t \tag{11}$$

The time T_1^{NCD} at which the budget is exhausted (in the no carbon debt (NCD) case) is found by integrating over (11) such that $B = X(T_1^{\text{NCD}}) = \int_0^{T_1^{\text{NCD}}} Q(t)dt$. With $Q(T_1^{\text{NCD}}) = 0$ in (11), we can then solve for Q_0

$$T_1^{\text{NCD}} = \sqrt{\frac{2\alpha B}{r-g}} \tag{12}$$

$$Q_0 = \sqrt{\frac{2(r-g)B}{\alpha}} \tag{13}$$

Finally, substituting (13) into (5) yields an expression for the initial shadow price μ_0, which grows at the constant rate r due to (6), together with (11), (12) and (13), this describes the transition to net-zero emissions.

$$\mu_0 = -\alpha A_0 e^{-\sqrt{2\alpha(r-g)B}} \tag{14}$$

$$\mu(t) = \mu_0 e^{rt},\ t \leq T_1^{\text{NCD}} \tag{15}$$

The exponential increase with the discount rate in time in (15) is standard Hotelling dynamics. Furthermore, we observe in (14) that the initial carbon price μ_0 is decreasing in the level of the cap B and in the rate of technological progress $-g$.

Phase 2: Net-zero emissions. Once the budget is exhausted and the per-period constraint (3′) binds, we have ($\gamma > 0$). As net-emissions must remain zero until the end of the time horizon, we know from equation (9) that

$$\frac{\gamma}{\mu} = g - r \tag{16}$$

[7]If $r > g$, the costs of abatement grow at a lower rate than the discount rate. This seems to be a plausible case, as many abatement and carbon removal technologies still have significant potential for cost reductions, implying even $g < 0$.

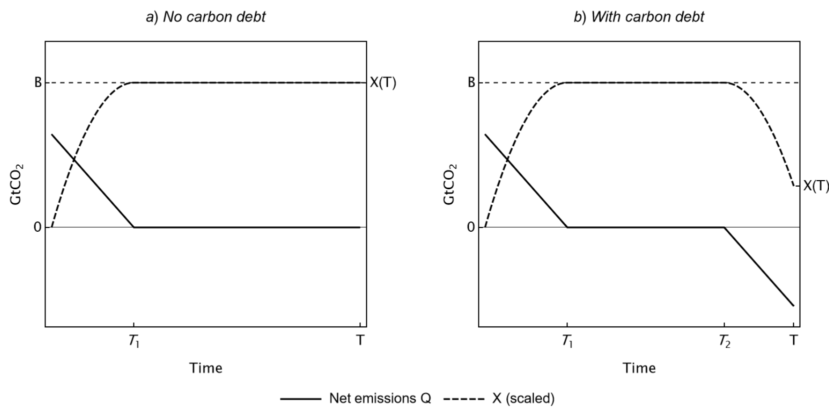

Figure 3: Illustration of the carbon dynamics: the carbon stock (dashed lines) and flow (solid lines) for the cases of no carbon debt (a), carbon debt with increased ambition (b).

Together with equation (6) this implies that the shadow price for emissions decreases at the rate of technological progress. For (16) to hold, this must also be true for γ.

$$\hat{\mu} = \hat{\gamma} = g \tag{17}$$

As $\mu(T_1^{\text{NCD}})$ is known and g and r are exogenous, equation (16) also pins down the level of γ.

$$\mu(t) = \mu(T_1)e^{g(t-T_1^{\text{NCD}})} = -\alpha A_0 e^{gt} \tag{18}$$

$$\gamma(t) = (g-r)\mu(t) = (r-g)\alpha A_0 e^{gt} \tag{19}$$

In summary, net emissions and carbon prices in the case of no carbon debt are given by

$$Q(t) = \begin{cases} Q_0 - \frac{(r-g)}{\alpha}t, & \text{if } t \leq T_1^{\text{NCD}} \\ 0, & \text{if } T_1^{\text{NCD}} < t \leq T \end{cases} \tag{20}$$

$$\mu(t) = \begin{cases} \left(-\alpha A_0 e^{-\sqrt{2\alpha(r-g)B}}\right) e^{rt}, & \text{if } t \leq T_1^{\text{NCD}} \\ -\alpha A_0 e^{gt}, & \text{if } T_1^{\text{NCD}} < t \leq T \end{cases} \tag{21}$$

The left panel of Fig. 3 illustrates the emission dynamics. The case of a carbon budget without carbon debt is the well-known case of standard ETS with partial temporal flexibility, that is, with free banking of permits but no borrowing. As in the seminal resource extraction model in Hotelling (1931), the permit price grows with the discount rate until the emission permits are exhausted. Once the carbon budget is reached, the permit price remains at the marginal costs of maintaining net-zero emissions by abating

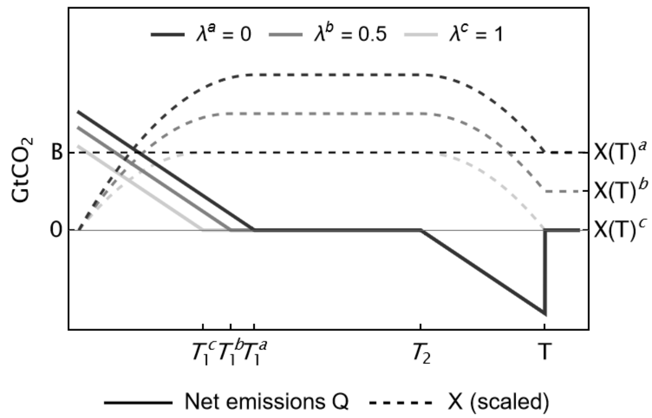

Figure 4: Net-emissions and cumulative emissions over time for a given amount of carbon debt ($D = B$). Depending on the policy parameter λ, carbon debt can lead to *a)* clean-up of a temporary overshoot of the budget ($\lambda = 0$), *c)* lower long-term cumulative emissions due to additional clean-up ($\lambda = 1$), or *b)* a mix of the two ($0 < \lambda < 1$).

or removing emissions. As net emissions are constant (and zero), marginal costs in this phase change only with technological progress $-g$.

To implement this solution in a permit trade system with banking but without borrowing, the regulator could, for example, issue all B emission permits at time $t = 0$. The firm would bank permits and use them until T_1^{NCD}. The permits would be traded at the price $p_X(t) = -\mu(t)$, although net permit trade of the representative firm would be zero (cf. model of the firm in Appendix A).

3.3. *Carbon debt*

Whether carbon debt $D > 0$, introduced by the regulator, affects the per-period constraint ($3'$) or the transversality condition ($4'$) depends on the choice of the policy parameter λ, which governs the extent to which clean-up is additional to the budget B. The share of carbon debt that is attached to existing certificates (λD) increases the long-term ambition of the long-term target as reflected by the transversality condition $X(T) = B - \lambda D$. In contrast, the share of carbon debt that is issued by creating new emission permits ($\lambda < 1$) relaxes the per-period constraint and the original budget will be overshot up to $\hat{B} = B + (1-\lambda)D > B$ (cf. Figure 2).

In the following, we first derive the net-emissions and carbon price pathways for the general case of $\lambda \in [0,1]$ and then discuss the implications of different degrees of additionality.

Phase 1: Transition to net-zero emissions. As before, we have $\gamma = 0$ until the per-period constraint is reached, and net-emissions are

$$Q(t) = Q_0 + \frac{(g-r)}{\alpha}t \tag{22}$$

Following the same steps as in Section 3.2 we find T_1^{CD} where X hits the upper bound $\hat{B}$ in the case with carbon debt (superscript CD). Using β, the relative size of carbon debt with respect to the original target B, that is $D = \beta B$ with $\beta > 0$, we can rewrite T_1^{CD} in terms of T_1^{NCD} of the no carbon debt case.

$$T_1^{\text{CD}} = \sqrt{\frac{2\alpha\hat{B}}{r-g}} = T_1^{\text{NCD}}\sqrt{1-\lambda\beta+\beta} \tag{23}$$

Hence, introducing carbon debt ($\beta > 0$) that is not fully additional ($\lambda < 1$) moves the point of net-zero emissions further into the future. Plugging (23) into (22) shows that Q_0 is increased by the same factor.

$$Q_0 = \sqrt{\frac{2(r-g)\hat{B}}{\alpha}} = Q_0^{\text{NCD}}\sqrt{1-\lambda\beta+\beta} \tag{24}$$

Finally, we combine (24) with (5) to derive the initial level of the shadow price μ_0, and by virtue of (6) we obtain $\mu(t)$.

$$\mu_0 = -\alpha A_0 e^{-\sqrt{2\alpha(r-g)\hat{B}}} = \mu_0^{\text{NCD}} e^{\sqrt{2\alpha(r-g)B}\left(1-\sqrt{1-\lambda\beta+\beta}\right)} \tag{25}$$

$$\mu(t) = \mu_0 e^{rt},\ t \leq T_1^{\text{CD}} \tag{26}$$

Together with (22), (23) and (24), this describes the transition to net-zero emissions as a function of the exogenous variables in the model. Figure 4 illustrates how a lower degree of additionality of clean-up λ implies higher initial net-emissions Q_0 and a later time T_1^{CD} of reaching net-zero.

Moreover, equation (25) shows that the initial carbon price falls as the level of carbon debt β increases or the degree of additionality λ decreases. Consequently, introducing carbon debt without full additionality of clean-up leads to mitigation deterrence. Contrary, in case of full additionality ($\lambda = 1$), the initial net-emissions Q_0, the time of net-zero T_1^{CD} and the initial carbon price μ_0 are exactly equal to the case of no carbon debt. Hence, the introduction of carbon debt with $\lambda = 1$ translates into an increase in ambition via net-negative emissions without mitigation deterrence or stranded assets in mitigation.

Phase 2: Net-zero emissions. Once the per-period constraint is reached, we have $\gamma > 0$ and, more specifically, the dynamics of $\gamma(t)$ and $\mu(t)$ from equation (6) and (9) during the net-zero emissions phase are

$$\frac{\gamma}{\mu} = g - r \tag{27}$$

$$\hat{\mu} = \hat{\gamma} = g \tag{28}$$

Thus, γ and μ converge to zero at the rate of technological progress.

$$\mu(t) = \mu(T_1^{\text{CD}})e^{g(t-T_1^{\text{CD}})} = -\alpha A_0 e^{gt}, \qquad T_1^{\text{CD}} \leq t \leq T_2^{\text{CD}} \tag{29}$$

Emissions remain at net-zero from T_1^{CD} to the end of the net-zero phase, T_2^{CD}. The latter is determined by the optimal duration of the clean-up phase, which we consider next.

Phase 3: Net-negative emissions. During the clean-up phase, the carbon debt D needs to be settled by net-negative emissions. With the beginning of the clean-up phase, the per-period constraint is no longer binding and the associated multiplier γ falls back to zero. The change in net-emissions $\dot{Q}$ and their level $Q(t)$ thus evolve as in the first phase, that is, Q is falling linearly, starting at $Q(T_2^{\text{CD}}) = 0$. For $t \geq T_2^{\text{CD}}$ net emissions are given by

$$Q(t) = -\frac{(r-g)}{\alpha}(t - T_2^{\text{CD}}) \tag{30}$$

To find the time at which the clean-up phase begins, we set the cumulative net-emissions ΔQ from time T_2^{CD} to T equal to the carbon debt D.

$$\Delta Q = \int_{T_2^{\text{CD}}}^{T} Q(t)dt = \frac{(g-r)(T - T_2^{\text{CD}})^2}{2\alpha} = -D \tag{31}$$

Solving for T_2^{CD} yields

$$T_2^{\text{CD}} = T - \sqrt{\frac{2\alpha\beta B}{r-g}} = T - \sqrt{\beta}T_1^{\text{NCD}} \tag{32}$$

Unsurprisingly, the higher the relative carbon debt β, the longer the clean-up phase. Similarly, we observe that the longer the time horizon the later the beginning of the clean-up phase, because discounting and technological progress reduce the cost when removal is delayed. This finding suggests that clean-up policies may suffer from time inconsistency, and we discuss institutional implications for ensuring commitment in

Section 6. Furthermore, Figure 4 illustrates that for a given level of carbon debt, the timing and duration of the clean-up phase depends on the amount of carbon debt β but not on its allocation λ.

Note that the per-period constraint (3′) will only bind when there is sufficient time to pay back the carbon debt. The minimum time horizon $\tilde{T}$ such that a given carbon debt budget is exhausted is therefore:

$$\tilde{T} = \left(\sqrt{1-\lambda\beta+\beta} + \sqrt{\beta}\right) T_1^{\text{NCD}} \tag{33}$$

For shorter time horizons ($T < \tilde{T}$) not all of the available clean-up certificates would be sold. For the remainder of this section we assume that the time horizon is sufficiently long, that is, $T \geq \tilde{T}$. Finally, note that during the clean-up phase, the shadow price of the per-period constraint γ is zero, and the shadow price of the emissions cap μ grows with the discount rate.

$$\mu(t) = \mu(T_2)e^{r(t-T_2)} = -\alpha A_0 e^{(g-r)T_2+rt}, \qquad t \geq T_2^{\text{CD}} \tag{34}$$

In summary, net emissions and carbon prices during the three phases of the case of carbon debt are given by

$$Q(t) = \begin{cases} Q_0^{\text{NCD}}\sqrt{1-\lambda\beta+\beta} - \frac{(r-g)}{\alpha}t, & \text{if } t < T_1^{\text{CD}} \\ 0, & \text{if } T_1^{\text{CD}} \leq t < T_2^{\text{CD}} \\ -\frac{(r-g)}{\alpha}\left(t + \sqrt{\beta}T_1^{\text{CD}} - T\right), & \text{if } T_2^{\text{CD}} \leq t \leq T \end{cases} \tag{35}$$

$$\mu(t) = \begin{cases} \mu_0^{\text{NCD}} e^{\sqrt{2\alpha(r-g)B}\left(1-\sqrt{1-\lambda\beta+\beta}\right)+rt}, & \text{if } t < T_1^{\text{CD}} \\ -\alpha A_0 e^{gt}, & \text{if } T_1^{\text{CD}} \leq t < T_2^{\text{CD}} \\ -\alpha A_0 e^{gT_2^{\text{CD}}+r(t-T_2^{\text{CD}})}, & \text{if } T_2^{\text{CD}} \leq t \leq T \end{cases} \tag{36}$$

Figure 5 illustrates the development of the carbon price for a given level of carbon debt and various degrees of additionality. Once again, we observe that carbon prices decrease along the transition to net zero if carbon debt is not fully additional ($\lambda < 1$). However, if carbon debt is solely associated with existing certificates ($\lambda = 1$), the carbon price trajectory is initially identical to the case of no carbon debt. Thus, an increase in ambition and a sustained phase of net-negative emissions can be achieved without increasing short- to medium-term mitigation cost. Only as the economy enters into the clean-up phase the long-term carbon prices exceed the carbon price in the case of no carbon debt.

We can use the permit price equation (36) to determine the price of the carbon debt, i.e. the price of the obligation to remove the carbon later. In an emissions trading

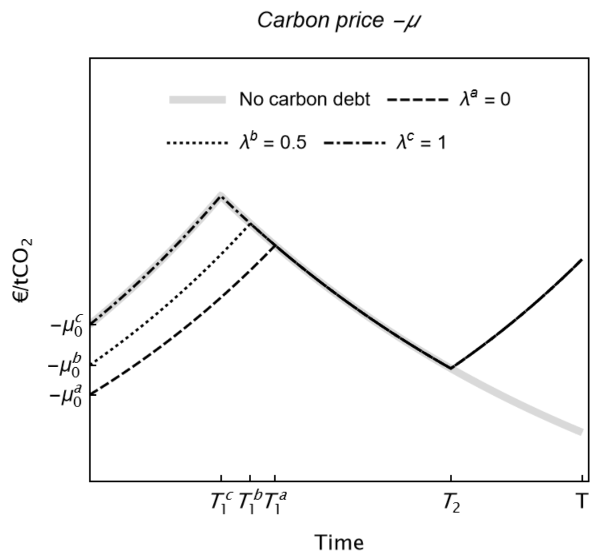

Figure 5: Development of the carbon price in the case of *i)* no carbon debt (solid line), and *ii)* relative carbon debt of $\beta = 1$ with various degrees of additionality (dashed lines). In the case of $\lambda^c = 1$, carbon prices are identical up to the time step T_2 where the clean-up phase begins.

scheme, a firm that wants to emit a ton of CO_2 at time $t \leq T_2^{\text{CD}}$ can choose between a conventional permit or a clean-up certificate. As the latter comes with the obligation for later removal in the phase with net-negative emissions, it implies additional costs. If removal occurs at $t' \geq T_2^{CD}$, the discounted costs at time t are $-\mu(t')e^{-r(t'-t)}$ which is equivalent to $-\mu(T_2^{\text{CD}})e^{-r(T_2^{\text{CD}}-t)}$, because the shadow price grows at r after T_2^{CD}. In particular, the present value of the carbon debt at the initial time period, p_0, is then

$$p_0 = -\mu(T_2^{CD})e^{-rT_2^{CD}} \tag{37}$$

Are clean-up certificates an attractive option for firms? The following equation compares their value as an emission permit with the obligation of the carbon debt in present value terms. Ruling out arbitrage between clean-up and regular certificates, this expression is also equal to the price of clean-up certificates:

$$\begin{aligned} p^{CUC} &= -\mu_0 - p_0 \\ &= \alpha A_0 \left(e^{-(r-g)T + \sqrt{2\alpha(r-g)\beta B}} - e^{-\sqrt{2\alpha(r-g)(1-\lambda\beta+\beta)B}} \right) \end{aligned} \tag{38}$$

Equation (38) is positive for any $T \geq \tilde{T}$ given by (33). Intuitively, a minimum length of the time horizon is needed to reduce the present value of future marginal abatement costs sufficiently by cost-saving technological progress and discounting at the combined rate $r-g$. For shorter time horizons $T<\tilde{T}$, the willingness-to-pay expressed by (38) falls below zero and not all clean-up certificates can be auctioned. Conversely, for $T>\tilde{T}$ the net price for clean-up certificates during the transition to net-zero $t\in[0,T_1^{\text{CD}}]$ is strictly positive, reflecting the scarcity of clean-up certificates. Moreover, by setting carbon debt equal to zero and solving for $(r-g)$, we can identify at which rate of technological progress and at which minimum time horizon the introduction of clean-up certificates meets a positive demand.

$$r-g>\frac{2\alpha B}{T^2} \tag{39}$$

If this condition holds, the wedge between current and future mitigation cost is sufficient to finance net-negative emissions via clean-up certificates. We observe that for longer time horizons, less technological progress is required, as there is more time to reduce cost. In contrast, if the remaining budget B is large, faster technological progress is needed to make the clean-up commitment financially viable. Note that the condition in (39) can be reformulated in a condition for the length of the time horizon, that is $T>\sqrt{2\alpha B/(r-g)}=T_1^{NDC}$. Hence, for a given technological progress and budget, the time horizon must be long enough to allow for the cost-efficient transition to net-zero emissions without carbon debt, which we assume to be the case. For a longer time horizon, clean-up certificates could be introduced, while the number of clean-up certificates is constrained by the length of the time horizon given by (33).

Note that in the special case of full additionality ($\lambda=1$), the level of the relative carbon debt cannot exceed $\beta=1$ (i.e. $D=B$) if we assume a one-to-one exchange rate between emission permits and carbon debt. The introduction of the maximum amount of clean-up certificates with ($\lambda=1,\beta=1$) is thus equivalent to a net-zero long-term target $X(T)$ in the ETS. In this case, all emission permits would be replaced by clean-up certificates. However, whether there would be positive demand for this amount of clean-up certificates depends on the level of technological progress. The price of clean-up certificates is declining in β and reaches zero (i.e. demand vanishes) for

$$\tilde{\beta}=1-\frac{T\left(2\sqrt{2B(r-g)\alpha}-(r-g)T\right)}{2B\alpha} \tag{40}$$

Hence, if technological progress is slow, no more clean-up certificates than $\tilde{\beta}<1$ could be sold. On the other hand, note that $\tilde{\beta}$ exceeds one for $(r-g)>\frac{8\alpha B}{T^2}$. Therefore, the maximum amount of carbon debt with full additionality is given by

$$\beta^{\max}=\begin{cases}0, & \text{if } r-g<\frac{2\alpha B}{T^2}\\ 1-\frac{T\left(2\sqrt{2B(r-g)\alpha}-(r-g)T\right)}{2B\alpha}, & \text{if } \frac{2\alpha B}{T^2}\leq r-g<\frac{8\alpha B}{T^2}\\ 1, & \text{if } r-g\geq\frac{8\alpha B}{T^2}\end{cases} \tag{41}$$

That is, the regulator could use clean-up certificates to implement an ETS with a net-zero long-term target. Its feasibility, again, depends crucially on the level of technological progress and the assumed discount rate, relative to the size of the remaining budget and the time horizon of the regulator. If technological progress is fast, even net-negative long-term targets seem possible. We explore this option in more detail in the following section.

3.4. Net-negative long-term targets

Our analysis so far has assumed that clean-up certificates link emissions permits and carbon debt by a fixed exchange rate of one-to-one. In this section, we relax this assumption by introducing an exchange rate ϕ greater than one, which enables the regulator to set net-negative long-term targets. A negative long-term net-emission target $X(T) < 0$ is achieved for $X(T) = (1-\lambda\beta)B < 0$ (cf. (4′)). When $\lambda\beta > 1$, the long-term target becomes negative and each clean-up certificate is associated with a carbon debt greater than one. Specifically, the amount of carbon removal that has to be delivered by the ETS is $D = \beta B$. The amount of certificates in the system is equal to $\hat{B} = (1+(1-\lambda)\beta)B$ (cf. (3′)). Hence, an exchange rate ϕ between emission allowances and carbon debt ($D = \phi\hat{B}$) is required to yield the desired amount of carbon removal:

$$\begin{aligned} \beta B &= \phi(1+(1-\lambda)\beta)B \\ \Leftrightarrow \quad \phi &= \frac{\beta}{1+(1-\lambda)\beta} \end{aligned} \tag{42}$$

As certificate holders now have to remove more than one tonne of carbon per tonne of carbon allowance, their willingness-to-pay for clean-up certificates decreases accordingly.

$$\begin{aligned} p^{CUC} &= -\mu_0 - \phi p_0 \\ &= \alpha A_0 \left(e^{-\sqrt{2\alpha(r-g)(1-\lambda\beta+\beta)B}} - \phi e^{-(r-g)T + \sqrt{2\alpha(r-g)\beta B}} \right) \end{aligned} \tag{43}$$

As before, the regulator sets the long-term target by choosing the level of relative carbon debt β and the additionality share λ.[8] A net-negative long-term target can be achieved via clean-up certificates as long as the price of clean-up certificates is greater than or equal to zero. The regulator can thus increase β and λ until the price of the clean-up certificates becomes zero. Note that this would be equivalent to earmarking all of the ETS revenues for financing future carbon removal. If the regulator was willing

[8] Note that choosing β and λ also determines the necessary exchange rate ϕ.

to forgo these revenues, what is the lowest possible long-term target the regulator could implement via clean-up certificates? To derive this, we calculate the maximum relative carbon debt in the case of full additionality ($\lambda = 1$). Setting the price of clean-up certificates equal to zero and solving for β yields

$$\beta^{\max,\phi} = \frac{2}{\alpha B(r-g)} W\left(\sqrt{\frac{1}{2}\alpha B(r-g)e^{(r-g)\left(T-\sqrt{2}\sqrt{\frac{\alpha B}{r-g}}\right)}}\right)^2 \tag{44}$$

where $W(y)$ is the inverse of $y = x\exp\{x\}$ (the Lambert W function), which (on the positive real axis) resembles a log function that passes through the origin. We explore $\beta^{\max,\phi}$ in more detail in an numerical illustration in Section 5.3, where we show that $\beta^{\max,\phi}$ is increasing in $r-g$ on the relevant range of $r-g$.

Some questions cannot be addressed analytically as their answers are specific to parameter choices, or involve specific trade-offs in the model, including: How would clean-up certificates be priced, if integrated into the EU ETS? What would the efficiency gains from introducing clean-up certificates be? How much removal, how much net-negative emissions could be financed via a carbon market like the EU ETS? And what are the implications of increased environmental ambition versus increased economic flexibility for different societal groups, and ultimately, for the political feasibility of introducing removal via clean-up certificates? To address these questions, we calibrate our model to the case of the EU ETS.

4. Calibration

To illustrate the model dynamics and quantitative effects, we calibrate the model to a comprehensive emissions trading scheme for the economies of the European Union covering all sectors from 2030 onward. This set-up is in line with the EU's ambition to include buildings and transport emissions in the EU ETS2 and eventually expanding regulation to cover all emissions.

We calibrate our model to fit projections of the carbon price and emissions in a comprehensive EU ETS in the year 2030 as in Rickels et al. (2023). Parameter values are summarized in Table 1. We set the duration of the ETS, T, such that the ETS ends by 2100. For the remaining budget of emissions in the EU, we set $B = 14$ $GtCO_2$ based on the recommendation in ESABCC 2023. We choose a discount rate of 2% (in line with, e.g., WH.GOV, 2024). For the cost of the mitigation technology (which represents both the abatement and the removal sector), we calibrate the model to three scenarios, as the cost assumptions are at the same time decisive and very uncertain. For our reference scenario, we calibrate g to the "middle-of-the-road" scenario of the Shared Socioeconomic Pathways (SSP2, see O'Neill et al. 2014), and consider a "fast progress" scenario with double the speed of cost-saving technological progress as well as a "no progress" scenario with constant marginal costs. Cost parameters α and A_0 are calibrated such that initial emissions Q_0 and the initial permit price μ_0 match values from Rickels et al. (2023), see Appendix B for details.

Table 1: Calibration parameters

Symbol	Description	Value	Source
Reference scenario			
T	length of the time horizon (years)	70	assumption
B	1.5°-target GHG budget for the EU ($GtCO_2$)	14	ESABCC (2023)
r	discount rate	0.02	WH.GOV (2024)
α	abatement cost parameter	0.453	calibrated to SSP2
g	rate of change in the abatement efficiency	−0.0165	calibrated to SSP2
Q_0	initial emissions in 2030 ($GtCO_2$)	1.501	Rickels et al. (2023)
μ_0	initial carbon price in 2030 (€/tCO_2)	155	Rickels et al. (2023)
A_0	initial value of the abatement efficiency	675.063	calibrated to SSP2
$\hat{\beta}_0$	social cost of carbon in 2030 (€/tCO_2)	216	U.S. EPA (2023)
$\hat{\beta}_1$	linear increase in social cost of carbon (€/tCO_2 per year)	3.65	U.S. EPA (2023)
γ	incomplete internalization of global SCC	0.76	assumption
Variations*			
g^{FP}	"Fast progress" scenario for g	−0.033	assumption: $2 \times g$
g^{NP}	"No progress" scenario for g	0	assumption

* In the two variation scenarios, Q_0 and μ_0 are updated with the different value of g.

To discuss the environmental benefits of the ETS in a consistent way, we introduce a climate change damage function. Since the EU's contribution to global warming is rather small, a linear damage function using the social cost of carbon in U.S. EPA (2023) is a good approximation. With these damages, the global cost-benefit optimal carbon budget for the EU falls below the budget recommended in ESABCC (2023). For consistency, we assume that the EU regulators find it optimal for the EU to internalize the global social cost of carbon only partially and, therefore, we scale the EPA's social cost of carbon by a factor $\gamma = 0.76$ such that an initial budget of 14 $GtCO_2$ minimizes the sum of compliance cost and climate damages. Thus, we consider social cost of carbon

$$SCC(t) = \gamma(\hat{\beta}_0 + \hat{\beta}_1 t) \tag{45}$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are estimated to approximate the time-path of the SCC in U.S. EPA (2023), see Table 1. In the initial year 2030, we then have $SSC(0) = \gamma\hat{\beta}_0 = 163$ €/tCO_2.

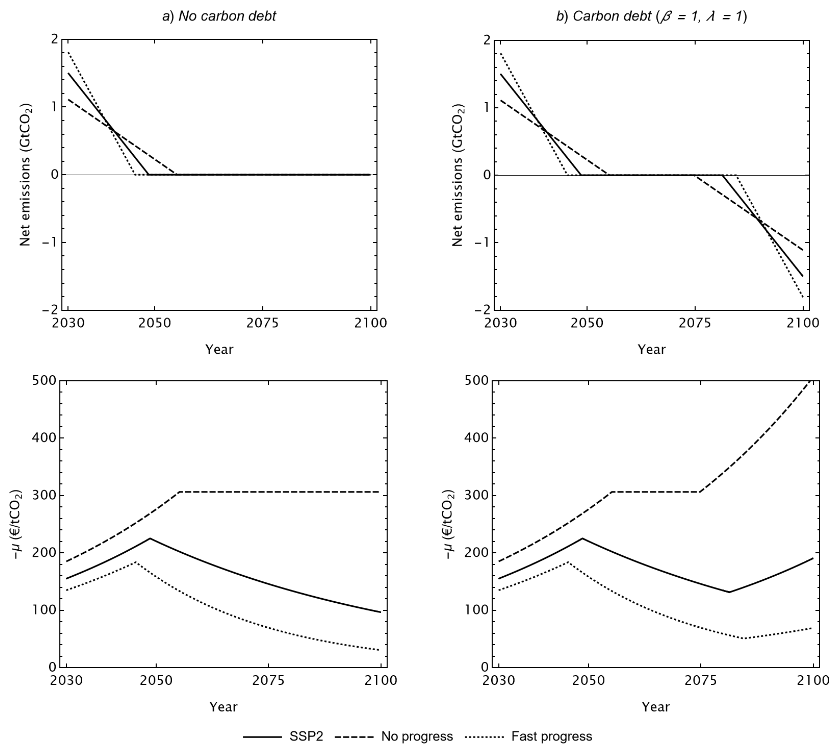

Figure 6: Net-emissions and the carbon price over time in a calibrated comprehensive EU ETS with and without clean-up certificates that fully replace conventional certificates ($\lambda = 1$). See Table 1 for calibration parameters.

5. Numerical analysis

Integrating the option to remove emissions from the atmosphere in an emission permits system creates additional flexibility. Clean-up certificates grant the regulator control over this new flexibility and its implications for economy and environment. In the following numerical analysis, we explore the implications of clean-up certificates for carbon pricing, for economic efficiency, for the environmental effectiveness, and for the political feasibility of clean-up certificates.

5.1. Pricing clean-up certificates

Using the closed-form solutions derived in Section 2, we present the net emission pathways and the evolution of the carbon price in Figure 6. Figure 6a shows that without carbon debt and clean-up certificates, net emissions in a comprehensive EU ETS decrease linearly over time and reach zero around the year 2050. This development is driven by an exponentially increasing carbon price, rising from 155 €/tCO_2 in 2030 to

Table 2: Price of clean-up certificates as a fraction of the price for regular emission allowances. Degree of additionality λ equal to 1. See Table 1 for calibration parameters.

	Technological progress $-g$		
Clean-up certificates issued	No progress	SSP2	Fast progress
2 $GtCO_2$	0.51	0.80	0.92
7 $GtCO_2$	0.42	0.75	0.90
14 $GtCO_2$	0.32	0.70	0.87

approximately 225 €/tCO_2 by 2050. In the scenario with fast technological progress, more abatement is shifted into the future, such that net-zero is reached about 3 years earlier. Without technological progress, net-zero is achieved about 6 years later, as it is optimal to distribute the abatement burden over a more extended period.

For the case of carbon debt, we start by setting β and λ equal to 1 and assume that 100 percent of the remaining budget can be converted into clean-up certificates. Figure 6b shows that the introduction of clean-up certificates leads to the same net-emission and carbon price trajectories as in Figure 6a in the short- and medium run. Since in this example all regular emission permits are converted into clean-up certificates and no certificates are issued in excess of the budget, the total amount of emission allowances in the market is unchanged. Thus, we do not observe mitigation deterrence or stranded assets in mitigation.

As the economy enters the net-zero phase, the carbon price decreases due to technological progress in the abatement technology. Approximately by the year 2081, which is 19 years prior to the end of the time horizon, the economy enters the clean-up phase. Net emissions begin to decrease as the carbon price rises. By the end of the time horizon, all carbon debt has been repaid through net-negative emissions. As before, we observe that a change in the rate of technological progress leads to a shift of the burden of mitigation over time. The higher the technological progress, the shorter the transition to net zero and the later the start of the clean-up phase.

Table 2 shows the price of clean-up certificates as a fraction of the price for regular emission allowances for a set of exemplary parameter values. For a given amount of clean-up certificates, the price increases in technological progress. Intuitively, a stronger cost reduction over time decreases the net present value of removal cost and firms would be willing to pay more for clean-up certificates. Similarly, for a given level of technological progress, the price of clean-up certificates decreases with the amount of clean-up certificates in the market. The intuition is that higher aggregate carbon debt implies higher future removal demand, resulting in a higher net present value of future removal cost. In the example considered above ($\beta = 1$, i.e., carbon debt of 14 $GtCO_2$),

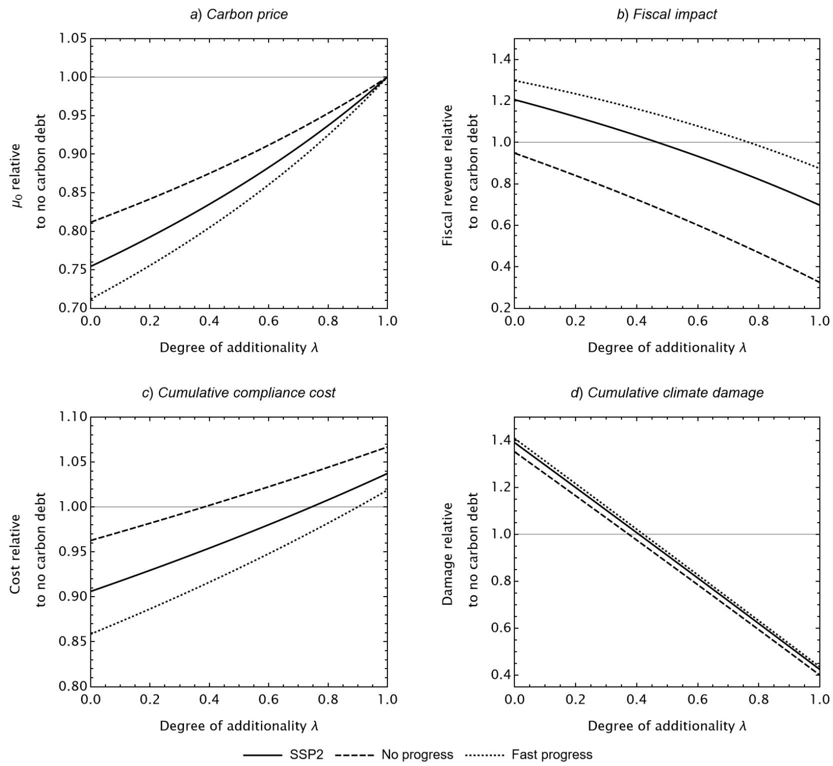

Figure 7: Impact of the introduction of clean-up certificates on the initial carbon price *(a)*, fiscal revenue from permit auctioning *(b)*, cumulative compliance cost *(c)* and cumulative climate damages *(d)* in a calibrated comprehensive EU ETS, for a given relative carbon debt of $\beta = 1$ (i.e. equal to the initial carbon budget). Impacts are shown relative to the case of no carbon debt and as a function of the degree of additionality λ. If $\lambda = 1$, clean-up certificates replace conventional certificates by 100 percent. See Table 1 for calibration parameters.

the value of the collateral would be between 13 and 68 percent of the carbon price, depending on the rate of technological progress.

Carbon pricing is substantially effected by the degree to which removal is additional (parameter λ). Figure 7 illustrates how additionality (parameter λ, x-axis) and technological progress (three scenarios) affect the initial carbon price and subsequent model results. When clean-up certificates are issued in excess of the budget ($\lambda < 1$), the expanded supply of permits reduces the initial carbon price. This effect is reinforced by technological progress (Figure 7a).

The reduced permit price and the additionality of removal have a twofold effect on the fiscal revenue from the auctioning of emission allowances and clean-up certificates, which we compute as

$$\textit{Fiscal impact} = \frac{\mu_0^{\mathrm{CD}}\,(B-\lambda D) + (\mu_0^{\mathrm{CD}} - p_0^{\mathrm{CD}})D}{\mu_0^{\mathrm{NCD}} B} \tag{46}$$

While the auctioning of D clean-up certificates (even at the reduced price $\mu_0^{\mathrm{CD}} - p_0^{\mathrm{CD}}$) increase revenues, the more permits are substituted by clean-up certificates (λD in the first term) the less permits can be sold at μ_0^{CD}, which reduces revenues, and, hence, revenues are declining in λ. Furthermore, as the introduction of clean-up certificates decreases the equilibrium price for both regular allowances and for clean-up certificates, the fiscal impact can also be negative in the case of low technological progress.

The upward-sloping curves of the carbon price in Figure 7a directly translate to upward-sloping cumulative compliance costs (relative to compliance costs without carbon debt) in Figure 7c, computed as

$$\textit{Compliance cost impact} = \frac{\int_0^T \left[A(t)a\left(Q(t)^{\mathrm{CD}}\right)\right] e^{-rt}\mathrm{d}t}{\int_0^T \left[A(t)a\left(Q(t)^{\mathrm{NCD}}\right)\right] e^{-rt}\mathrm{d}t} \tag{47}$$

We observe that the introduction of clean-up certificates can lower the cumulative compliance cost if the degree of additionality is not too high. Hence, if the regulator allows a temporary overshoot of the budget, the resulting near-term cost savings can be sufficient to achieve a lower long-term target without increasing cumulative compliance cost. However, if clean-up certificates are created mostly by attaching carbon debt to existing certificates (high λ), cumulative compliance cost increase due to the increase in ambition. This result is aggravated in the case of no technological progress.

Finally, Figure 7d shows the environmental impact of clean-up certificates in terms of cumulative climate damages, calculated as

$$\textit{Climate damage impact} = \frac{\int_0^T \left[Q(t)^{\mathrm{CD}} SCC(t)\right] e^{-rt} dt}{\int_0^T \left[Q(t)^{\mathrm{NCD}} SCC(t)\right] e^{-rt} dt} \tag{48}$$

where $SCC(t)$ is an exogenously given function of annual social cost of carbon. $SCC(t)$ is based on U.S. EPA (2023), and scaled such that the assumed remaining budget of 14 $\mathrm{GtCO_2}$ is cost-benefit optimal for the EU.[9, 10] In Figure 7d the effect of clean-up certificates on cumulative damages and be aggravating or dampening, depending on

[9]Note that the assumption of exogenously given *SCC* ignores the potential impact of the choice of $\lambda > 0$ on global emissions. For small economies the impact is negligible and can be omitted.

[10]Scaling the social cost of carbon such that the assumed budget is optimal ensures that no incentive to increase or decrease the budget due to the *SCC* enters the welfare analysis in Section 5.4. Note furthermore, that we use the estimated social cost of carbon for a discount rate of 2 percent. Refer to Appendix B for more details.

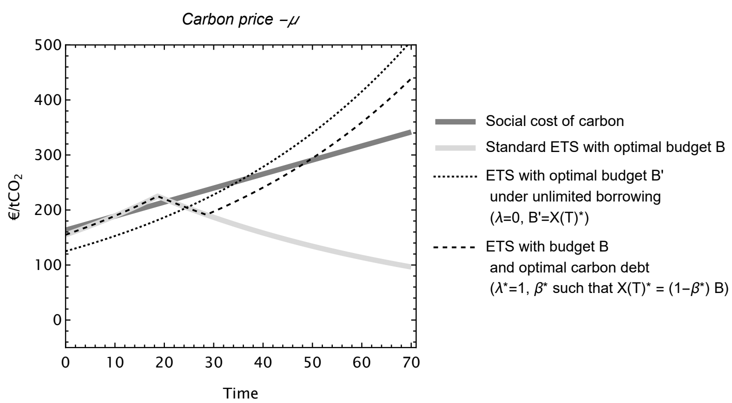

Figure 8: Economic efficiency of ETS designs

the degree of additionality. If clean-up certificates allow extensive overshoot (low λ), cumulative climate damages are higher. If, in contrast, clean-up is mostly additional (high λ), the cumulative climate damage of the EU ETS is lower.

5.2. *Economic efficiency*

Clean-up certificates introduce carbon debt, i.e. intertemporal borrowing, into the emission permit system. Should we expect greater efficiency and economic gains from this additional flexibility? Indeed, the availability of emissions removal makes a difference for the efficiency of permit trade. Without the option to remove, borrowing is not essential for emission permit trade to be (cost) efficient: emission permits can be front-loaded (i.e. issued in the first period) such that emitters can bank permits and are not restricted in their cost-efficient utilization of permits. However, when removal is introduced, the efficient emissions profile includes overshoot, which necessitates borrowing of permits. With CDR technologies, an emission permit system without borrowing is inefficient.

The carbon prices in the ETS in Figure 8 visualize the efficiency improvement from introducing clean-up certificates. We compare the outcome of three ETS designs to the social cost of carbon: (i) without borrowing, (ii) with borrowing, and (iii) with clean-up certificates; for each design the overall budget is chosen optimally.[11] A (cost-benefit) efficient carbon price would coincide with the social cost of carbon (SCC), thus any deviation from the SCC path implies inefficiency.

While the ETS without borrowing (solid light gray) is closer to the SCC in the first

[11] Socially optimal emission budgets are found by minimizing the sum of compliance costs as in (47) and climate change damages as in 48.

periods, it deviates strongly in later periods. The ETS with borrowing (dotted) appears to approximate the SCC much better on average. As it allows for borrowing, this ETS can make use of net-negative emissions and can allow more gross emissions early on, which shifts the price path (otherwise following a Hotelling dynamics) downward. This results in an 11.4 % welfare improvement relative to the ETS without borrowing.

With clean-up certificates, both the overall budget as well as the carbon debt were chosen optimally. That is, in contrast to the ETS with borrowing, clean-up certificates put a (per-period) constraint on the carbon in the atmosphere and, thus provide additional control over climate change impacts. In the carbon price profile, limited borrowing introduces corner points, which indicate the beginning and end of the zero-emissions phase, and allow a closer approximation of the SCC. The carbon price of the optimal ETS is the same with and without carbon debt until the end of the zero-emissions phase (as we show in Appendix D). The resulting welfare improvement is 12.9 % relative to the ETS without borrowing.

A forth possible ETS design is omitted in Figure 8: Leiby and Rubin (2001) show that intertemporal trading ratios (ITRs), which put an "interest rate" on permit trade, can be used to align the case of unlimited borrowing with the social optimum, i.e. with ITRs the carbon price in Figure 8 would be first-best and track the social cost of carbon perfectly. Note though, that the creation of additional permits by ITRs greater unity (or their destruction by ITRs less than unity) affects the carbon budget. Thus, to match the optimum carbon budget in an ETS with ITRs, the regulator needs to take an extra step, e.g. adjusting the carbon budget to a gross budget such that the net carbon budget – net of the created/destructed permits – matches the social optimum.[12] The informational requirement of anticipating all creation and destruction of permits by permit trade is large if not infeasible. In the context of these four ETS designs, clean-up certificates are a second-best instrument with low informational requirements that improve upon unlimited borrowing by improving the intertemporal profile of the carbon price but fall short of the complex adjustment by means of ITRs.

The optimal carbon budgets of the ETS with removal (with unconstrained borrowing as well as with the clean-up certificates) are very ambitious, arriving at long-term net-negative targets of -59 $GtCO_2$ and -55 $GtCO_2$, respectively. If implemented with clean-up certificates, high removal costs would put the price of the collateral p_0 well above the permit price, such that the price of clean-up certificates would be negative. The associated efficiency gains are, thus, hypothetical. We take this observation into account in the following sections when we derive the maximum removal that can be

[12] Leiby and Rubin (2001) do not discuss adjusting of the permit allocation in anticipation of created or deleted permits but observe that "there is generally no way, using non-unitary ITRs, to assure an unchanged level of cumulative emissions".

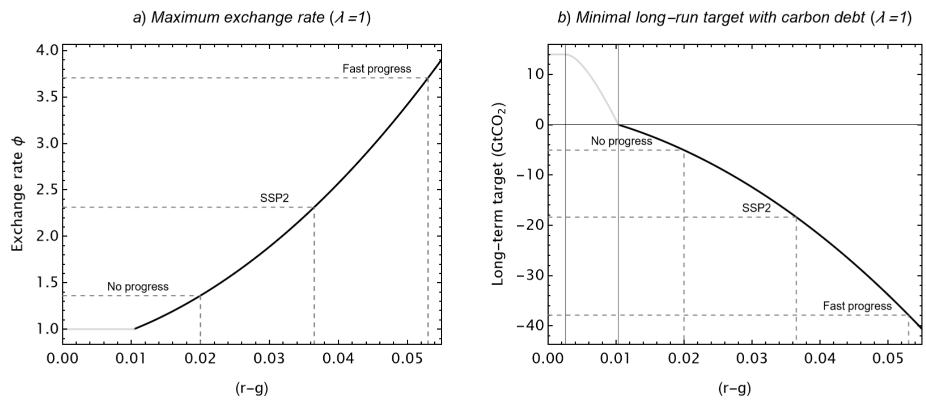

Figure 9: Maximum exchange rate $\phi^{\max}$ *(a)* and minimal long-run target $X(T)$ *(b)* depending on the sum of discount rate r and rate of cost-saving technological progress $-g$.

financed by the carbon markets, and the political economy of clean-up certificates.

5.3. Environmental effectiveness

Up to this point we assumed a one-to-one exchange rate between the emission allowance and the carbon debt, thus limiting removal by the amount of remaining emission permits. In this section, we explore the full potential of the carbon market to finance emissions removal by introducing a flexible exchange ratio, ϕ. This relaxation further expands the scope for financing removal (Section 3.4). Figure 9 displays the maximum exchange rate that could be implemented via clean-up certificates while maintaining a positive price for certificates and the corresponding lowest achievable long-term target in the case of full additionality ($\lambda = 2$). The maximum exchange rate $\phi^{\max}$ is shown in panel *(a)* and given by the piecewise function

$$\phi^{\max} = \begin{cases} 1, & \text{if } (r-g) \leq \frac{8B\alpha}{T^2} \\ \frac{2}{\alpha B(r-g)} W\left(\sqrt{\frac{1}{2}\alpha B(r-g) e^{(r-g)\left(T-\sqrt{2}\sqrt{\frac{\alpha B}{r-g}}\right)}}\right)^2, & \text{if } (r-g) > \frac{8B\alpha}{T^2} \end{cases} \tag{49}$$

Note that, in line with the previous analysis, we assume an exchange rate of one-to-one if the level of carbon debt that can be financed via clean-up certificates is insufficient to yield a net-negative long-term target. The threshold for an exchange rate larger than one is determined by the sum of the rates of technological progress $(-g)$ and discounting (r) according to (40). As panel *a* illustrates, our calibration suggests a threshold of approximately 1%.

When $(r-g)$ exceeds this threshold, the maximum exchange rate that can be required from firms exceeds one and the ETS can achieve net-negative long-term targets. In our "SSP2" reference scenario, we assume that $r-g=3.65\%$ and find a maximum exchange rate of 2.3. Thus, firms would be willing to commit to a carbon debt of up to 2.3 times the emission allowance bundled in the clean-up certificate.

The maximum exchange rate translates to a lowest achievable long-term target (Panel *(b)*). The lowest achievable long-term target $X(T)^{\min}$ is given by

$$X(T)^{\min} = \begin{cases} B, & \text{if } (r-g) \leq \frac{2B\alpha}{T^2} \\ \frac{T\left(2\sqrt{2B(r-g)\alpha}-(r-g)T\right)}{2\alpha}, & \text{if } \frac{2B\alpha}{T^2} < (r-g) \leq \frac{8B\alpha}{T^2} \\ \left(1-\frac{2}{\alpha B(r-g)} W\left(\sqrt{\frac{1}{2}\alpha B(r-g)e^{(r-g)\left(T-\sqrt{2}\sqrt{\frac{\alpha B}{r-g}}\right)}}\right)^2\right)B, & \text{if } (r-g) > \frac{8B\alpha}{T^2} \end{cases} \tag{50}$$

For values between 0.0026 and 0.01, the minimum long-term target gradually decreases (by an increasing maximum β) but remains positive (the range between the two vertical lines). [13] In this case, not all regular allowances are converted into clean-up certificates. For $(r-g)>0.01$, the long-term target can be negative and decreases in $(r-g)$ by an increasing $\phi^{\max}$. While the *"No progress"* scenario supports a long-term target of -5 $GtCO_2$, a long-term target of -18 $GtCO_2$ can be achieved in our reference scenario. In the optimistic *"Fast progress"* scenario, the long-term target could even drop to -38 $GtCO_2$ below the initial carbon stock.

5.4. *Political economics of clean-up certificates*

The preceding sections showed the implications of taking clean-up certificates to the extremes – exploring their optimum implementation from a social planner perspective (Section 5.2) and the maximum environmental impact (Section 5.3). Here, we turn to the question how removal can be integrated into an existing ETS identifying, first, the reform space where no losers are created by clean-up certificates and, second, the optimal policy design for given political economy constraints.

For this analysis, we assume that a policy reform is feasible, if it receives support from politically relevant groups that focus on specific outcome variables. Firms and consumers prefer outcomes with lower carbon prices and lower compliance costs. Environmental groups, but also voters or governments, may want to reduce climate damages with a reform. Finally, governments may care about the fiscal implications related to

[13] Note that according to (39) no clean-up certificates are economically feasible for values of $(r-g)$ below $\frac{2B\alpha}{T^2}=0.0026$. In this case, the minimum long-term target would remain equal to the initial budget (see the range to the left of the first vertical line). This case is rather unlikely as it requires either a very low discount rate or substantial cost-increasing technological progress.

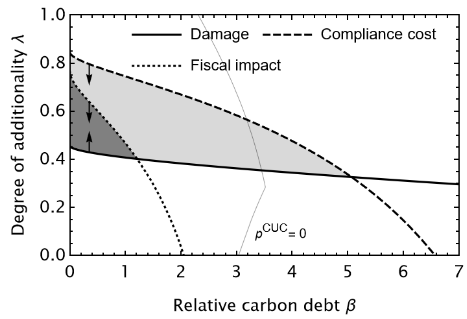

Figure 10: Indifference curves indicating (β, λ) combinations with the same outcome as without carbon debt. Arrows indicate the direction of improvement with respect to "no carbon debt". The enclosed areas are sets of strong "Pareto" improvements along all dimensions (dark) and weaker "Pareto" improvements (light). Certificate prices are only positive left of the $p^{CUC} = 0$ line, which assumes a one-to-one exchange rate for $\lambda\beta \leq 1$ and a flexible exchange rate otherwise.

the revenues from carbon pricing and may prefer outcomes with non-decreasing revenues. Note that in contrast to compliance costs and climate damages, the fiscal impact does not reflect real costs for society but rather the political economy consideration (or constraints) of the government. Therefore, we distinguish a "weak" Pareto set of policies that improve compliance costs and climate damages from a "strong" Pareto set that includes the political economy constraint of maintaining fiscal revenues.

Figure 10 shows which policy combinations of relative carbon debt β and additionality λ result in an improvement across the dimensions of cumulative compliance cost and cumulative climate damage – the "weak Pareto set", shown in light gray – and policies that, in addition maintain or improve the fiscal revenues – shown as the dark gray "strong Pareto set".[14] Neither the weak nor the strong Pareto set are empty, i.e. there are policies that reconcile these often opposed motives such that introducing clean-up certificates ought to be a politically feasible option. The light gray $p^{CUC} = 0$ frontier shows that all of the strong Pareto set and much of the weak Pareto set can be implemented by selling the corresponding clean-up certificates into the ETS (at a positive price).

Dedicating about half of the clean-up certificates to additional removal ($\lambda \gtrsim 0.45$) is necessary to prevent that clean-up certificates water-down the environmental effect of the ETS. Conversely, to reduce compliance costs and increase fiscal revenue, the degree of additionality cannot be set too high.

However, Figure 11 illustrates that the size of the Pareto sets varies significantly with

[14] As the initial carbon price always decreases (as $\lambda < 1, \beta > 0$) or remains unchanged ($\lambda = 1$ or $\beta = 0$), we disregard this dimension.

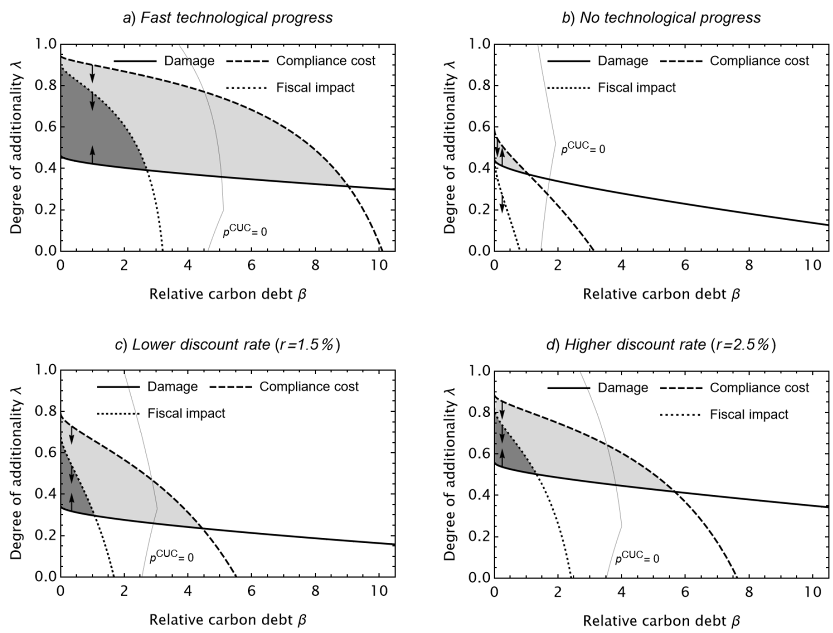

Figure 11: Indifference curves for alternative assumptions about the rate of technological progress and discount rates. Arrows point into the direction of improvement with respect to "no carbon debt". The dark shaded area indicates the set of possible "Pareto" improvements along all dimensions. The light shaded area indicates the weaker "Pareto"-set without consideration of fiscal revenue impacts. Certificate prices are only positive left of the $p^{CUC} = 0$ line, which assumes a one-to-one exchange rate for $\lambda\beta \leq 1$ and a flexible exchange rate otherwise.

the assumptions about the future technological progress and the discount rate. As shown in Figure 11a, the sets expand for faster technological progress, which (i) reduces compliance costs and (ii) reduces the collateral value and therefore improves fiscal revenues. In the case without technological progress the strong Pareto set is empty, i.e. there exists no combination of parameters that achieves an improvement across all three dimensions (Figure 11b). However, even without technological progress there exists a weak Pareto set where both cumulative abatement cost and climate damages are reduced. Consequently, the model hints at possible improvements for industry, consumers and environmental groups and, hence, suggests scope for agreement if governments can be convinced to consider the real effects rather than the fiscal revenues. Finally, a change in the discount rate leads to a vertical shift of the option space. As shown in Figure 11c, a lower discount rate with higher estimates of the social cost of carbon leads to a downward shift of the set. Conversely, a higher discount rate with lower estimates of the social cost of carbon shifts the set upwards (Figure 11d).

Welfare optimal policy reform

While the preceding section demonstrated the substantial range of clean-up certificates specifications that should be politically feasible and ought to receive broad support, this section focuses on the question what kind of welfare improvements are possible with clean-up certificates, and it identifies the (second-best) policy designs within the different feasibility constraints, as well as the first-best policy.

We measure the welfare implications of integrating clean-up certificates (with specific λ and β) by calculating the net present value of the sum of compliance cost and climate damages. In Table 3, we show the welfare gain ΔW relative to the optimal ETS without clean-up certificates. For this exercise, we adjust the budget to its optimal size for the "no progress" and "fast progress" scenarios (as shown in the first row of the table) to preclude the incentive to correct a suboptimal budget and focus on the incremental gains from clean-up certificates relative to a conventional ETS. We visualize the location of the optimal policies for the default calibration in Figure B.2 in the appendix.

We first restrict the use of clean-up certificates to the introduction of extra certificates, i.e. we rule out any retiring of emission permits ($\lambda = 0$) and, therefore, we commit an unchanged environmental ambition of the ETS. With this constraint, a substantial amount of carbon debt ($\beta = 1.42$) is optimal. As we exclude additional removal, this is a scenario of pure overshoot. The corresponding welfare gain of 3.15% reflects the availability of net-negative emissions and the improved intertemporal efficiency. While this welfare gain is substantial, the following scenarios that relax this restriction show that focusing on pure overshoot severely limits the welfare gains that can be realized.

Consensus policy options that respect not only benefits and costs but also fiscal con-

Table 3: Welfare-maximizing (λ, β)-combinations for different scenarios

	No progress	**SSP2**	**Fast progress**
Optimized budget ($GtCO_2$)	31.2	14.0	9.1
Constrained to intertemporal efficiency ($\lambda = 0$)			
λ	0.00	0.00	0.00
β	0.03	1.42	3.20
ΔW (%)	0.01	3.15	7.36
Financed CDR ($GtCO_2$)	1.0	19.9	29.1
Overshoot ($GtCO_2$)	1.0	19.9	29.1
Long-term target ($GtCO_2$)	31.2	14.0	9.1
Feasible with clean-up certificates ($p^{CUC} \geq 0$)	yes	yes	yes
Constrained to strong Pareto set			
λ	0.40	0.40	0.62
β	0.04	1.21	5.63
ΔW (%)	0.01	4.45	25.25
Financed CDR ($GtCO_2$)	0.9	17.0	51.2
Overshoot ($GtCO_2$)	0.6	10.2	19.3
Long-term target ($GtCO_2$)	30.8	7.2	−22.7
Feasible with clean-up certificates ($p^{CUC} \geq 0$)	yes	yes	yes
Constrained to weak Pareto set			
λ	0.42	0.59	0.71
β	0.04	2.92	9.73
ΔW (%)	0.02	8.36	39.48
Financed CDR ($GtCO_2$)	1.2	40.9	88.4
Overshoot ($GtCO_2$)	0.7	16.7	25.8
Long-term target ($GtCO_2$)	30.6	−10.1	−53.6
Feasible with clean-up certificates ($p^{CUC} \geq 0$)	yes	yes	no
Constrained by willingness to pay ($p^{CUC} \geq 0$)			
λ	1.00	0.82	0.78
β	0.05	2.72	6.51
ΔW (%)	0.02	9.66	32.33
Financed CDR ($GtCO_2$)	1.4	38.1	59.2
Overshoot ($GtCO_2$)	0.0	6.9	12.7
Long-term target ($GtCO_2$)	29.7	−17.2	−37.4
Feasible with clean-up certificates ($p^{CUC} \geq 0$)	yes	yes	yes
Unconstrained welfare maximum			
λ	1.00	1.00	1.00
β	0.05	4.95	16.98
ΔW (%)	0.02	12.88	62.46
Long-term target ($GtCO_2$)	29.7	−55.3	−145.2
Feasible with clean-up certificates ($p^{CUC} \geq 0$)	yes	no	no

sideration (the strong Pareto set) necessitate less carbon debt but, by requiring an additionality of 40% ($\lambda = 0.4$), deliver a larger welfare gain. The constraint of the strong Pareto set to maintain the same fiscal revenues as in the initial ETS turns out to be a severe limitation: When this constraint is relaxed (the weak Pareto set) the welfare gain almost doubles to 8.36%. In our calibration, this corresponds to an annualized welfare gain of €28 billion, which arise from the cost reductions due to a generous overshoot as well as from the climate benefits of a net-negative long-term target of -10.1 $GtCO_2$. The total welfare gain in this scenario corresponds to roughly 64 percent of the fiscal revenues in the baseline ETS without carbon debt. It is of similar magnitude of estimated benefits of the EU-Canada trade agreement (Government of Canada and European Commission, 2008), The carbon debt and the amount of removal that is financed to repay it are substantial. In the "fast progress" case, the carbon debt is too extensive to be financed by the willingness-to-pay in the permit market, indicated by a negative certificate price $p^{CUC} < 0$.

Slightly higher welfare gains of 9.66% can be achieved with a policy that is not bound by the consideration of Pareto improvement but exploits the full willingness-to-pay of the carbon market. In this case, compliance costs exceed their previous level, as the additionality is significantly increased from 0.59 to 0.82, while the level of carbon debt is slightly dialed back.

The first-best, unconstrained policy focuses exclusively on financing additional removal ($\lambda = 1$), reflecting the value of avoiding climate damages by use of CDR once technological progress has reduced the cost of these technologies. This scenario could not be sustained by the willingness-to-pay in the carbon market. The government, however, could implement this scenario by subsidizing removal, e.g. by subsidizing the purchase of clean-up certificates.

Here, for the case of fast progress we observe large volumes of carbon debt (more than three times the default) but also very high welfare gains. These numbers need to be taken with caution as, for example, the benefits are computed with *SCC* that are constant in the removal quantity. Nevertheless, these number suggest that if removal costs do turn out to be low in the future, it would be worthwhile for a government to finance large scale utilization of CDR technologies.

6. Institutional aspects: time inconsistency and liability

In the formal model, we implicitly made two strong assumptions: (i) the regulator can credibly commit to its policy, and (ii) there exist a functioning intertemporal permit market where firms do not fail to meet their removal obligations. We consider these aspects and potential institutional amendments related to introducing clean-up certificates in this section.

6.1. Commitment problem of the regulator

The assumption of an exogenously given and fixed time horizon T requires that the regulator can credibly commit to the end date of the policy. The regulator, however, has an incentive to reconsider and postpone T because the obligation of settling the "carbon debt" can be met at lower costs in the future. In our setting, the net-present value of costs are reduced by technological progress and discounting and therefore decline with time (if $g < 0$ and $r > 0$). Consequently, it is always beneficial to move the end of the policy further into the future and the regulator's policy thus suffers from time inconsistency.

The fundamental reason for this inconsistency is the disregard of climate damages in the optimization problem of the social planner. If the benefits of reducing carbon emissions are unaccounted for, the social planner values any relaxation of the carbon budget, the ambition level, or the compliance period as beneficial. This problem is not unique to the introduction of clean-up certificates but present in any analysis of emissions trading schemes that ignores the benefits. The first-best approach to address this problem would be to adjust the emission trading scheme such that it yields a welfare-maximizing outcome. In principle, intertemporal efficiency could be achieved by introducing intertemporal trading ratios to correct for the sub-optimal time path of the carbon price arising due to unaccounted climate damages (Kling and Rubin, 1997; Leiby and Rubin, 2001). However, this requires a rather complex ETS reform and an institution that determines the trading ratios according to a pre-defined rule.

The lack of credible commitment can alternatively be addressed by delegation to an independent authority with a clear and narrow mandate that can only be changed by a qualified political majority. After establishing the desired final period T, the regulator could delegate control of the clean-up certificates to an independent agency, for example, a European Carbon Central Bank (ECCB) for the EU ETS as proposed in Edenhofer et al. (2023). The carbon central bank receives the mandate to manage emission permits (regular allowances and clean-up certificates) with the aim to guarantee a cost-efficient achievement of the climate target in period T. As the carbon central bank is created rather than elected, it would be more resilient to lobbying and influence from interest groups than regulators that depend on re-election.

6.2. Limited liability problem of the private sector

Firms that acquire clean-up certificates commit themselves to removing the associated emissions before the final period T. As the clean-up certificates have a debt component, there will be strong incentives for firms to close down business or to go bankrupt to refrain from paying the debt (i.e. from removing the carbon later). The removal obligation could even be transferred to another firm that declares bankruptcy on purpose. The incentive problem due to the limited liability of firms is often referred to as the "judgment proof problem" (Shavell, 1986).

There are two basic approaches to overcome this problem: (i) further financialization of the carbon debt or (ii) charging a collateral by a public institution. The first approach aims to transfer the carbon debt to financial intermediaries that are large enough – compared to the carbon debt – so that they can always be held liable. This approach corresponds to the carbon removal obligation proposal in Bednar et al. (2021).[15] Following this proposal, a firm that uses a clean-up certificate needs to find a bank or insurance company that guarantees the removal to the regulator. Such a guarantee involves additional costs that will reduce the demand for clean-up certificates. Some of these costs are real (e.g., risk premiums associated to unexpected changes in future removal costs), while others are related to market failures (e.g., premiums associated with the default of the respective firm). There might also be economies of scale effects that increase costs for small-to-medium-size financial intermediaries that have to invest in new skills for assessing carbon markets and carbon removal technologies.

The second approach links the use of clean-up certificates to a collateral that is charged by a public institution. Recall from Section 2 that if there were no uncertainties on future removal costs, the collateral at time t would equal the discounted removal costs, i.e. $-\mu(T_2^{CD})e^{r(t-T_2^{CD})}$. If the firm goes bankrupt, this collateral could be used by the public institution to finance the necessary removal. Alternatively, firms could transfer the removal obligation directly to the public institution by paying p_0 as a fee. In both cases, the liability for the removal shifts from private firms to the public institution. Hence, the public institution would need to determine the height of the collateral and ensure that the removal occurs even when removal costs are higher than expected. For this case, the institution would also need to be equipped with a credible financing mechanism. As with the time inconsistency problem, it would also be necessary to ensure sufficient independence for the institution to safeguard against political influence on the calculation of the level of the collateral. In the case of the EU, the European Carbon Central Bank could fulfill this function by determining the level of the collateral and ensuring that all removal obligations associated to clean-up certificates are carried out.

[15] This approach is also similar to the carbon shares proposed by Lemoine (2020) that rely, however, on a carbon tax rather than an emissions trading scheme.

6.3. *Publicly financed removal*

The institutional challenges related to the liability problem raise the fundamental question: is it easier to let the regulator pay directly for the net removal in the last phase of the emissions trading scheme? In principle, it would be possible to adjust the conventional carbon budget such that cumulative emissions until the beginning of the net-negative phase, T_2^{CD}, are identical to the ETS with clean-up certificates. The regulator would need to earmark a certain fraction of the revenues from carbon pricing in the first phase for the financing of the net-negative phase. The regulator would also need to credibly commit to implementing the optimal net removal path $Q(t) < 0$ for $t > T_2^{CD}$, for example, through reversed auctions on removals. If the introduction of the clean-up certificates leads to an increase in fiscal revenue as illustrated in Section 4, the financing of the net-negative phase would also be possible without additional revenues from alternative sources. In this case, an equivalent outcome with respect to fiscal cost, climate damages, compliance costs, and carbon prices could be achieved.

However, this approach, too, requires a strong institutional set-up. First, the net-removal flows must be credibly announced early on to ensure investments in upscaling removals (the inertia of which we have neglected in our model). Within the augmented emissions trading scheme, the credibility would be built-in due to the creation of the clean-up certificates that signal future removal demand to investors. Second, the time-inconsistency problem sketched in Section 6.1 for the regulator is exacerbated as she has to (i) pay for the removals in the last phase while (ii) withstanding opposition of firms and consumers against the high carbon price. In the augmented ETS, the carbon price can only be reduced by a deliberate intervention in the ETS (e.g. by postponing the compliance period). Third, the regulator controls time and quantity of removals in the reverse auctions. Creating the optimal demand for removal, therefore, requires that the regulator has information on the optimal path of removal, i.e. when the net-removal phase begins, as well as the cumulative amount of net removals at each point in time. Hence, any solution to directly finance net-removals is also institutionally and informationally demanding and hardly effective without delegation to a strong independent institution.

7. Conclusion

Augmenting emissions trading schemes to allow for a later phase of net-negative emissions constitutes a challenge for regulators. With conventional emission permits, a regulator can limit the flow of emissions – to zero emissions in the extreme – but cannot turn their flow negative. Standard financial incentives for CDR, for example, via a subsidy on removal or programs to purchase removal credits, suffer from two key problems: (i) They require additional revenues in the future, and (ii) planned future removals might be subject to political rollbacks (in particular, with looming fiscal costs for removals that might become substantial).

The clean-up certificates proposed in this study operate through an extension of ETS systems, committing emitters to undo their emissions by bundling emission permits with the obligation for later removal (carbon debt). By allocating carbon debt to emitters, emitters become responsible for financing CDR, thereby relieving future public finances of this burden. Though the private sector will pay for the removal, part of the removal costs may also accrue to the current fiscal budget: since clean-up certificates sell at a lower price than emission permits, removal may in part be financed by forgone carbon pricing revenues. Additionally, overall compliance costs can be kept at a minimum by exploiting the efficiency of the intertemporal carbon market of the ETS.

If carbon debt is introduced with the sole purpose of enabling a net-negative phase after the current ETS, the regulator should issue clean-up certificates by substituting emission permits from the carbon budget such that the subsequent CDR is fully additional. In this case, the environmental ambition of the ETS is increased by the extent of carbon debt issued. The remaining carbon budget of the EU ETS, for example, would be sufficient to finance the complete removal of all emissions in its anticipated carbon budget until 2100. If exploited to the fullest, we estimate that clean-up certificates could even enable a net-negative long-term targets for the ETS and finance up to 18 GtCO2 of net-negative emissions until the end of the century. When the ambition of the ETS is not increased to the full extent of the carbon debt, clean-up certificates create a temporary overshoot of the original carbon budget that is undone when emitters carry out their removal obligation. Such an overshoot reduces compliance costs, albeit at the cost of deterring mitigation and inducing additional climate change damages due to higher emissions in the near term.

The degree to which clean-up certificates translate either to additional emission reductions or to a limited temporary overshoot of the carbon budget creates room for compromise between important players in policy-making. Importantly, these players may focus on different dimensions of the policy, such as the environmental concerns that motivate climate policy, the economic burden of compliance costs, or the fiscal implications for budget-constrained finance ministers. We find that clean-up certificates can be introduced in a way that improves all of these dimensions. This finding suggests a window of political support for increasing ambition levels. In our numerical analysis, we find that to reduce climate damages, at least half of the carbon debt should be additional, i.e., replacing conventional emission permits. Otherwise, there is the risk that mitigation costs fall at the expense of the higher cumulative climate damages.

Since clean-up certificates constitute a long-term policy that extends far into the future, care must be taken to safeguard against moral hazards of emitters to leave carbon debt unpaid, and of regulators to revise their policy. While the former risk can be hedged by collateral requirements, it remains unclear whether financial markets can price such collateral efficiently due to large uncertainties and long time horizons. To solve for both problems – the time inconsistency of the regulator and the pricing of the collateral – an independent institution like a European Carbon Central Bank could be established. While such an institution has been suggested for a number of tasks (e.g. valuing and issuing certificates, certification of removals, linking of carbon markets, providing technology support or stabilizing carbon prices, see Rickels et al. (2022); Edenhofer et al. (2023)) this paper emphasizes two new functions that follow from integrating net-negative emissions in an ETS.

Many open questions remain for future research. The regulation of an ETS with clean-up certificates needs to cover long time horizons. Due to these long time horizons, some key determinants in our analysis, such as the rate of technological progress, are only known with large uncertainties. While we considered some parameter variations, a stochastic analysis could provide more insights on the integration of clean-up certificates in the case of uncertainties. Furthermore, technological progress is exogenous in our analysis. Therefore, differences in technological learning for abatement and removal or the implications of learning-by-doing cannot be accounted for in our model. The question of how to complement clean-up certificates with technology support policies therefore remains open. Finally, our analysis takes a marginal perspective by assuming that climate change damages from a budget overshoot of the modeled economy can be captured by the social cost of carbon and does not trigger any tipping points. However, if a prominent player like the EU engaged in emissions overshoot, others might follow suit – triggering a substantial global overshoot and significantly increasing tipping point risks. Therefore, future research is needed on the governance of global overshoot and the role of carbon debt in an international context. The model presented in this paper could serve as the starting point for this important avenue for future research.

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Appendix

A. Permit trade model

The solution of the social planner model of Section 2 can be implemented by a market equilibrium of a competitive economy with emission permit trade. Consider a representative firm that buys permits $z(t)$ at price $p_X(t)$ and is required to hold a permit for each unit of emissions. The permit debt account $\mathsf{D}(t)$ tracks the net deficit of permits of the representative firm, that is, if $\mathsf{D} > 0$ the firm has "carbon debt", and if $\mathsf{D} < 0$ the firm has excess permits for additional emissions.

A carbon budget B of the social planner economy maps to an initial allocation of permits $\mathsf{D}_0 = -B$, and the case of "no carbon debt" is reflected by a per-period constraint on carbon debt, $\mathsf{D}(t) \leq 0$.

Then, an introduction of carbon debt that increases the ambition of the permit trade system is subtracted from the carbon budget. In contrast, carbon debt that only facilitates inter-temporal borrowing implies a relaxation of the per-period constraint on carbon debt. We have

$$\mathsf{D}_0 = -B + \lambda D \tag{A.1}$$

$$\mathsf{D}(t) \leq (1-\lambda) D \tag{A.2}$$

Note that the model does not need to track carbon in the atmosphere (stock X), it suffices to know the permit debt account of the firm. The problem of the firm reads:

$$\max_{Q} \int_0^T -\left[A(t)a\left(Q(t)\right) + p_X z(t)\right] e^{-rt} dt$$

$$\text{such that} \quad \dot{\mathsf{D}} = Q - z \qquad \perp \mu \tag{A.3}$$

$$\text{with } \mathsf{D}(0) = D_0 = -B + \lambda D \tag{A.4}$$

$$\mathsf{D}(t) \leq (1-\lambda) D \qquad \perp \gamma \tag{A.5}$$

$$\mathsf{D}(T) = 0 \tag{A.6}$$

$$\mathscr{H} = -\left[A(t)a\left(Q\right) + p_X z\right] + \mu \left[Q(t) - z(t)\right] + \gamma\left[(1-\lambda) D - \mathsf{D}\right]$$

First-order conditions.

$$\frac{\partial \mathscr{H}}{\partial Q} = -A_t a'(Q) + \mu = 0 \quad \Leftrightarrow \quad \mu = A_t a'(Q) \tag{A.7}$$

$$\frac{\partial \mathscr{H}}{\partial z} = -p_X - \mu \quad \Leftrightarrow \quad p_X = -\mu \tag{A.8}$$

$$\dot{\mu} = r\mu - \frac{\partial \mathscr{H}}{\partial \mathsf{D}} = r\mu - [-\gamma] \quad \Leftrightarrow \quad \dot{\mu} = r\mu + \gamma \tag{A.9}$$

Equation (A.8) shows that $(-\mu)$, which is the same in the planner model, is the permit price $p_X = -\mu$. First-order conditions (A.7) and (A.9) are the same as (5) and (6), the market equilibrium therefore implements the solution of the social planner economy. Note that the levels of X and D differ (cf. X_0 versus D_0).

B. Model calibration

Calibration of g

To calibrate the constant rate of change g of the scaling parameter $A(t)$, we decompose g according to the Kaya identity, assuming that the rate of change of the scaling parameter is proportional to the autonomous rate of change of emissions as a first-order approximation:

$$g = \Delta_{GDP} - \Delta_{EI} - \Delta_{CI} \tag{B.1}$$

where Δ_{GDP} is the average annual GDP growth rate in the EU from 2025 to 2100, Δ_{EI} is the average annual reduction in energy intensity from 2010 to 2100, and Δ_{CI} is the average annual reduction in carbon intensity from 2010 to 2100. We take data from the "middle-of-the-road" scenario (SSP2) of the shared socio-economic pathways (Fricko et al., 2017) and set $\Delta_{GDP} = 0.95\%$, $\Delta_{EI} = 1.5\%$ and $\Delta_{CI} = 1.1\%$.

Calibration of α and A_0

To calibrate the abatement cost parameter α, we solve equation (13) for α and plug in the parameter values given in Table 1. In particular, Q_0 is equal to the amount of emissions in a comprehensive EU ETS in 2030 as modelled in Rickels et al. (2023). Similarly, we calibrate the initial value of the scaling parameter A_0 by solving (5) for A_0 and plugging in the parameter values given in Table 1. In particular, μ_0 is equal to the carbon price in a comprehensive EU ETS in 2030 as modelled in Rickels et al. (2023).

Table B.4: Coefficients of the linear *SCC* function.

r (%)	$\hat{\beta}_0$	$\hat{\beta}_1$
1.5	362	4.15
2.0	216	3.36
2.5	135	2.65

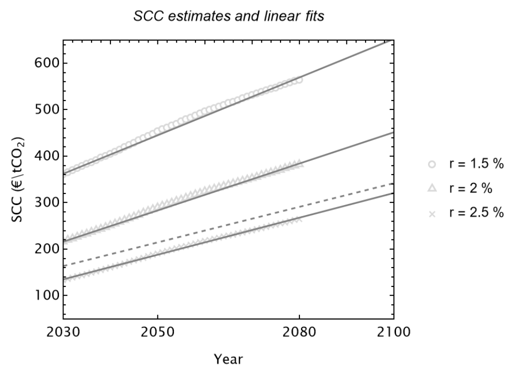

Figure B.1: Annual estimates of the social cost of carbon for different discount rates from U.S. EPA (2023) and linear regression lines used for constructing the linear $SCC(t)$ function. The dashed line represents the downscaled social cost of carbon at a discount rate of 2 %. Linear extrapolation starts in the year 2080.

Social cost of carbon

We convert the annual estimates of the social cost of carbon provided in U.S. EPA (2023) from Dollar to Euro using a USD-EUR exchange rate of 0.94 and fit a linear regression model $y(t) = \beta_0 + \beta_1 t + \varepsilon$ to the data (cf. Figure B.1). Table B.4 reports the estimated coefficients for the reference scenario with $r = 2\%$, and for higher and lower discount rates for the variation of r in Section 5.4. Note that since estimates are only provided until the year 2080, we use linear extrapolation to calculate the social cost of carbon in the remaining years. The cost-benefit optimal carbon budget for the EU with these social cost of carbon falls below the budget recommended in ESABCC (2023). We assume that regulators find it optimal for the EU to internalize the social cost of carbon only partially and scale the obtained social cost of carbon function by the parameter $\gamma = 0.76$ such that an initial budget of 14 $GtCO_2$ is welfare-maximizing in the sense of maximizing the benefit of climate damages that are avoided by the ETS net of its compliance costs. That is, we consider social cost of carbon

$$SCC(t) = \gamma \hat{y}(t)$$

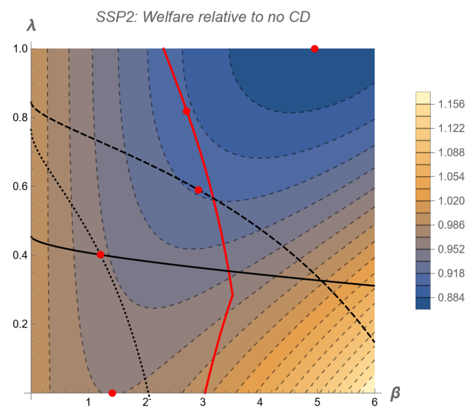

Figure B.2: Contour plot of welfare for different (λ,β)-combinations relative to the case without carbon debt, overlaid with indifference curves for damages (solid line), cumulative cost (dark dashed line) and fiscal impact (light dashed line). The area below and up to the red line is implementable with clean-up certificates, i.e. with non-negative prices. In ascending order, the red dots indicate the optimal (λ,β)-combinations in the case of *a)* efficiency only, *b)* strong Pareto set, *c)* weak Pareto set, *d)* optimal feasible ETS, *e)* welfare optimum.

C. Time inconsistency problem

Figure C.4 demonstrates the time inconsistency problem regarding the choice of the length of the time horizon. Extending the time horizon can reduce cumulative compliance cost due to discounting and prolonged technological progress. Consequently, policymakers at time $t = 0$ cannot credibly commit to the end date of the clean-up policy. Similarly, future policymakers would face similar incentives to extend the time horizon. This underscores the fundamental challenge of time inconsistency of the introduction of clean-up certificates.

D. Proof: Coinciding carbon prices with and without carbon debt

Figure 8 shows that the carbon prices in the optimal ETS with and without carbon debt coincide during the phases of carbon accumulation ($t < T_1$) and of zero emissions ($T_1 < t < T_2$), i.e. the optimal per-period limit on aggregate emissions in the atmosphere $X(t) \leq B$ is the same regardless whether a clean-up phase of net-negative emissions ($t > T_2$) follows or not.

If there was an intertemporal link between the phases, one would expect that a divergence of two carbon price paths with and without carbon debt for $t > T_2$, e.g. a phase of net-negative emissions instead of a continuation of zero emissions, would also trigger differences in earlier phases. Intuitively, the reason why nothing can be gained by additional emission in phase 1 that are cleaned-up in phase 3 is the assumption of

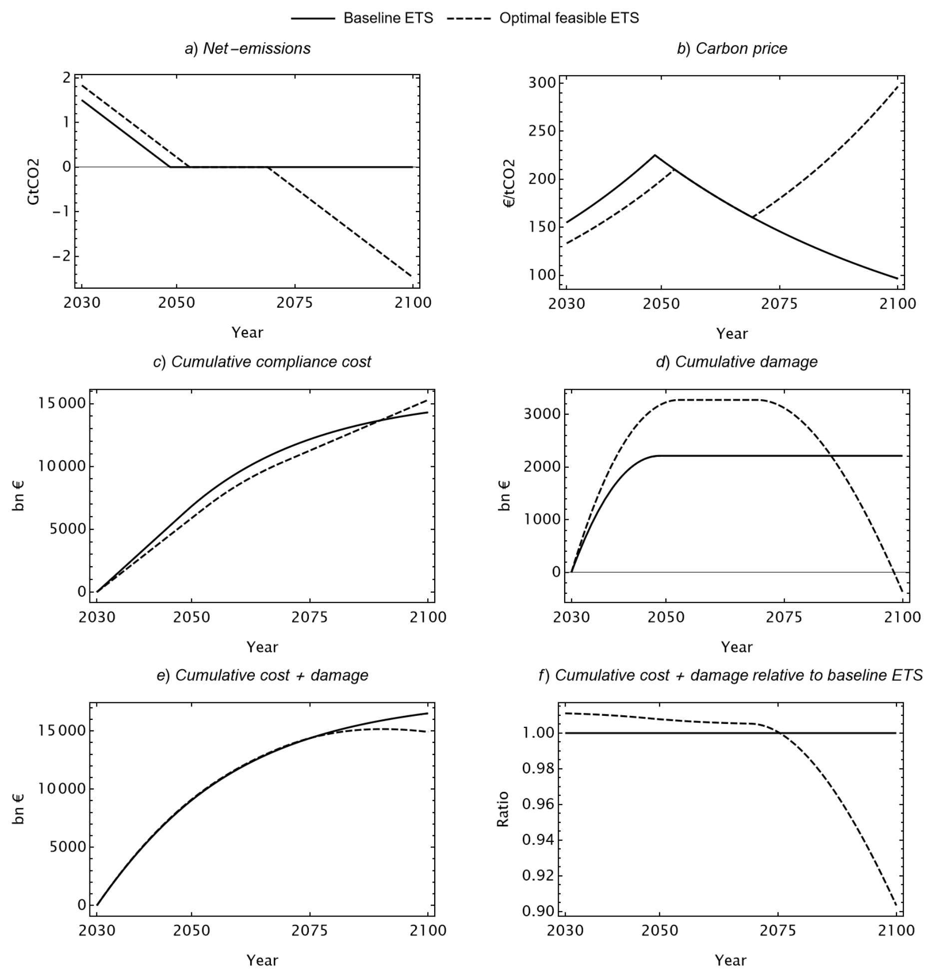

Figure B.3: Illustration of the model dynamics and welfare components in the baseline ETS without carbon debt and in the optimal feasible ETS ($\lambda = 0.82$, $\beta = 2.72$).

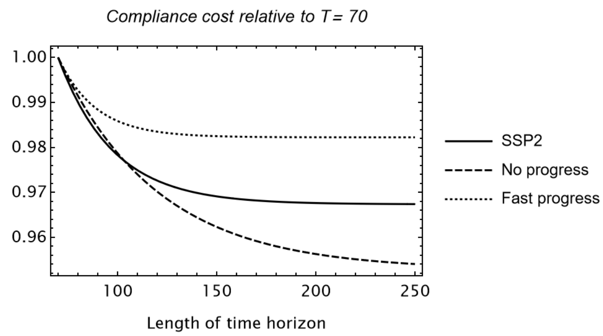

Figure C.4: Time inconsistency of the choice of the time horizon for a given relative carbon debt of $\beta = 1$ and full additionality of clean-up ($\lambda = 1$). Cumulative compliance cost relative to a time horizon of 70 years decrease by shifting the end of the policy into the future. See Table 1 for calibration parameters.

cost-benefit optimal budgets X_T for the ETS in this analysis. When the budgets of the ETS are adjusted to their optimal values the constraint $X(T) \leq X_T$ becomes nonbinding because conforming with it is optimal, thus removing the intertemporal links created by this "Hotelling constraint".

To see this, we consider the family of function of emission paths $Q(t)$ for the cases of an ETS with carbon debt $\mathscr{Q}^{\text{CD}}$ and without carbon debt $\mathscr{Q}^{\text{NCD}}$.

$$\mathscr{Q}^{\text{NCD}} = \left\{ Q^{\text{NCD}} : Q^{\text{NCD}}(t) = Q(t) \text{ as in (20), with } B \in \mathbb{R}^+ \right\}$$
$$\mathscr{Q}^{\text{CD}} = \left\{ Q^{\text{CD}} : Q^{\text{CD}}(t) = Q(t) \text{ as in (35), with } B \in \mathbb{R}^+, \beta \in \mathbb{R}^+, \lambda \in [0,1] \right\}$$

Functions in Q^{NCD} have only one corner point at T_1^{NCD} where the zero emissions phase begins, functions Q^{CD} have an additional second corner point T_2^{CD} point where the net-negative phase begins. The cost-benefit optimal budget B^{NCD} that we assume in Figure 8 is the budget that minimizes an objective function W,which is the sum of compliance costs (47) and climate damages (48), on the set $\mathscr{Q}^{\text{NCD}}$. Likewise, the cost-benefit optimal triple $(B^{\text{CD}}, \beta^{\text{CD}}, \lambda^{\text{CD}})$ minimizes W on the set $\mathscr{Q}^{\text{CD}}$.

Now, for the case of cost-benefit optimal ETS Q^{CD} consider the set S of all emission paths that enter the net-negative phase at the optimal time $T_2 = T_2^{\text{CD}}$, truncated to the interval $[0, T_2]$:

$$S = \left\{ Q|_{[0,T_2]} : Q(t) \in \mathscr{Q}^{\text{CD}}, T_2^{\text{CD}} = T_2 \right\}$$

Similarly, let S^* denote the set of all emission paths without carbon debt that reach the net-zero phase no later than $t = T_2$, also truncated to the interval $[0, T_2]$:

$$S^* = \left\{ Q|_{[0,T_2]} : Q(t) \in \mathscr{Q}^{\text{NCD}}, T_1^{\text{NCD}} < T_2 \right\}$$

For all elements in S we truncated the same net-negative emissions phase $Q^{\text{CD}}(t)$, $t > T_2$. For all elements in S^* we have truncated part of the zero-emissions phase $Q(t) = 0, t > T_2$, which is also the same for all $Q \in S^*$.

Note that every element Q' in S, defined by the triple (B', λ', β') can be mapped to an element Q'' in S^*, defined by B'', that describes the same emission path by choosing $B'' = B'(1+(1-\lambda')\beta')$. With this substitution, the first corner points of Q' and Q'' (as given by (12) and (23)) coincide and, thus, $Q'(t) = Q''(t)$ for $t < T_2$. Conversely, any Q'' in S^* can be mapped to a Q' in S by choosing $B' = B''$, $\lambda' = 0$, which aligns the first corner points, and $\beta' = \frac{r-g}{2\alpha B''}\left(T - T_2^{\text{CD}}\right)^2$, which aligns the second corner point with T_2^{CD} (as given by (32)), such that $Q''(t) = Q'(t)$ for $t < T_2$ and $T_2 = T_2^{\text{CD}}$.

Observe that $Q(t)$ is the only endogenous component of W (as technological progress, social cost of carbon, and discounting all follow exogenous paths), such that W is additive-separable in $Q(t)$ in the sense that we can split the integrals at any point in time T^* and evaluate W in two parts: up to this time T^* and beyond T^*, $W(Q(t)) = W_0^{T^*}(Q(t)) + W_{T^*}^{T}(Q(t))$. We use this to evaluate emissions paths up to T_2 and beyond T_2.

As all elements in S had the same continuation before we truncated, we know that for all $Q \in S$ the values $W_{T_2}^{T}(Q)$ are the same, and therefore the optimality of Q^{CD} on $\mathbb{R}^+$ implies for $t < T_2$:

$$Q^{\text{CD}} = \text{argmin}_{Q \in S} W_0^{T_2}(Q)$$

Likewise, all elements in S^* had the same continuation before we truncated and, thus, for all $Q \in S^*$the values $W_{T_2}^{T}(Q)$ are the same, and therefore for $t < T_2$:

$$\begin{aligned} Q^{\text{NCD}} &= \text{argmin}_{Q \in S^*} W_0^{T_2}(Q) \\ &= \text{argmin}_{Q \in S} W_0^{T_2}(Q) = Q^{\text{CD}} \end{aligned}$$

The latter follows because the feasible set S^* and S are the same, i.e. since both Q^{NCD} and Q^{CD} minimize W on S we can conclude that they coincide on the interval up to T_2, which in turn implies that the carbon prices of optimized ETS with and without carbon debt coincide for $t \in [0, T_2^{\text{CD}}]$.