DeMo: a simple generic model for making decisions influenced by friends

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Abstract

Decisions are rarely made in solitude. Here we present a very simple model that balances the inner drive towards an individual optimum with the influence of the neighbors in a social network. As a third element the model incorporates inertia towards change. The model generates structure from an unstructured initial state by clustering individuals through their social interaction. This clustering is associated with a cognitive dissonance that arises when individuals’ states differ from their own optimum due to the influence of their neighbors. We find this cognitive dissonance to

I. INTRODUCTION

A. Social dynamics literature

• pioneering work on the modelling of social behaviour was conducted by e.g. Sakoda [1], Schelling [2] and Granovetter [3]

• one of the first models on opinion formation was proposed by De Groot [4], studying the convergence to consensus in a group of individuals

• we refer to reviews on the benefits of agent-based modelling [5] and for a general summary of the developments in social dynamics modelling [6] [7]

• In the last years, several specialized models where used to show specific aspects of social dynamics. Research topics include coalition formation on networks and network evolution [8] [9], social tipping [10], bounded confidence opinion dynamics extended by a measure of cognitive dissonance [11]

• applied work focused on social dynamics of the COVID-19 pandemic [12] [13] and begins to evolve to include a coupled modelling of social and natural systems with respect to the Climate Crisis [14]

• A recent example of theoretical work on polarization and segregation of opinions [15] shows the challenge and interest to model observed opinion dynamics of the real world in a minimalist approach.

• Our model takes into account the competition between two of the most fundamental human social desires: Belonging to a group and sticking out of the crowd.
Brewer’s optimal distinctiveness theory (ODT) [16] posits the need to balance assimilation and distinction from others.

Deci’s and Ryans’s self determination theory [17] also differentiates between individual and collective needs, labelled as ”autonomy” and ”relatedness”

more references in chapter 28, pp. 395-410 [18], empirical study on ODT e.g. [19],

some studies on social distinctiveness theory already used agent-based modelling, e.g. [20] and called for more agent-based research into socio-psychological effects [21]

Our model integrates the strive to be individual and integrated into a collective at the same time in an opinion dynamic simulation, thus bridging the gap between theories of self-expression and models of opinion formation. In the following we present our generic **Decision Model** with social dynamics based on as simple as possible assumptions.

**II. MODEL SETUP, DESCRIPTION AND ANALYSIS**

**A. Model setup**

This agent-based approach models the decision dynamics on a grid as a competition between personal belief, social influence and a personal need for independence. The model setup and dynamics are visualized in fig. 1

$G$ agents are placed on a regular, periodic grid. Each agent $i$ has two time-invariant attributes: An *avantgarde factor* $a_i$ and a *belief* $b_i$. The avantgarde factor $a_i \in [0, 1]$ expresses the agent’s need for individualism with $a_i = 1$ corresponding to independent decisions (“pioneer”) and $a_i = 0$ for decisions strongly adapted to the social surroundings (“follower”). The belief $b_i \in [0, 1]$ expresses the agent’s intrinsic, personal opinion on the matter to be decided. For each agent $i$ both $a_i$ and $b_i$ are drawn from independent random distributions. This can easily be adapted to reflect individualist behaviour and beliefs in the context of a particular application.

Each agent $i$ in the grid interacts with a set of neighbors $N_{i,R}$ where $R$ is the radius of the neighborhood. All agents $j$ with Chebyshev Distance $\leq R$ to agent $i$ are part of $N_{i,R}$.
In our basic model we set $R = 1$ resulting in $|N_{i,1}| = 8$ on a grid with periodic boundary conditions. Each agent $i$ is influenced by all agents $j$ in their neighborhood. The strength of the influence of $j$ on $i$ is determined by the weight $w_{j,i}$. In the basic model, we set $w_{j,i} = \frac{1}{|N_{i,R}|}$ $\forall$ $i, j$. This is again adaptable to contextually model different influences.

B. Model dynamics

At each timestep $t$, each agent adjusts a continuous choice parameter $c_i \in [0, 1]$ based on the following equation:

$$\frac{\Delta c_i}{\Delta t} = \lambda ((1 - a_i)W_i + a_i (b_i - c_i) |W_i|)$$  \hspace{1cm} (1)

At timestep $t = 0$, for each agent $i$, $c_i$ is drawn from independent random distributions. $W_i$ describes the overall influence of the neighborhood $N_{R,i}$ and is defined as

$$W_i = \sum_{j \in N_{R,i}} w_{j,i} (c_j - c_i)$$  \hspace{1cm} (2)

with $\sum_{j \in N_{R,i}} w_{j,i} \equiv 1$ and $w_{j,i} \in [0, 1]$ $\forall j \in N_{R,i}$ \hspace{1cm} (3)

Since $(c_j - c_i) \in [-1, 1]$, it follows that $W_i$ is also bounded:

$$W_i \in [-1, 1] \quad \forall i \in \{1, \ldots, G\}$$  \hspace{1cm} (4)

The model equilibrium is given by... DAMIAN ADD ADAPTED EQUILIBRIUM EQUATIONS AND SOME EXPLANATION ON THEM.

simple implications / equilibrium:

$$\Delta o_i \ll 1$$

$$c_i - o_i = \frac{a_i}{a_i} - 1 \frac{W_i}{|W_i|} + \frac{\Delta o_i}{a_i|W_i|}$$  \hspace{1cm} (5)
Step 1: Initialize Agents

Agent $i$, $i \in [1, G]$:
- $a_i$ - avantgarde $\in [0, 1]$ indicates strength of dependence on others
- $b_i$ - belief $\in [0, 1]$ indicates the intrinsic opinion of the agent

Step 2: Grid placement

Regular, periodic grid holding G agents

Neighborhood $N_{i,R}$ of agent $i$ with radius $R$

Strength of the influence of $j$ on $i$ ($W_{j,i}$)

Step 3: Dynamics

At each timestep $t$ each agent $i$ makes a choice $c_i$:

$$\frac{\Delta c_i}{\Delta t} = (1 - a_i)W_i + a_i(b_i - c_i)|W_i|$$

Directed neighbor influence weighted by (in)dependence
Influence of own belief in relation to neighbors

$W_i$ measures the weighted difference to the neighbor’s choices:

$$W_i = \sum_{j \in N_R} w_{j,i}(c_j - c_i) \in [?, ?]$$

Step 4: Interpretation and further analysis

After reaching an equilibrium, each agent is assigned a decision $d_i$:

$$d_i = \begin{cases} 1 & \text{if } c_i > 0.5 \\ 0 & \text{else} \end{cases}$$

Cluster analysis
Does my decision align with others?
Cognitive dissonance
How far is my choice from my belief?
Satisfaction analysis
Does my choice generally align with my belief?

FIG. 1. Model setup and dynamics. Agents are initialized with an avantgarde factor $a_i$, expression individualism, and a personal belief $b_i$. They are placed on a regular periodic grid and assigned neighborhoods based on a Chebyshev distance. At each timestep, agents make a choice with the dynamics factoring in the need for individualism, the influence of the neighborhood and the striving towards the personal belief. Once the equilibrium is reached, each agent is assigned a final decision and the final parameters are then processed in downstream analysis.
\[ |c_i - o_i||W_i| = |W_i + a_i^{-1} (\Delta o_i - W_i)| \]  

(6)

\[ |W| \leq \frac{\epsilon}{|1 - a| |\text{sign}(W)| + a} \]  

(7)

Once the model has reached its equilibrium, each agent is assigned a final decision \(d_i\) based on their final choice \(c_i\):

\[
d_i = \begin{cases} 
1, & \text{if } c_i > 0.5 \\
0, & \text{else} 
\end{cases}
\]  

(8)

Based on the final decision and choice we then conduct downstream analysis.

C. Model analysis

We conduct analysis on the individual agents level as well as summarizing clustering analysis based on these.

For individual analysis we consider: First, the cognitive dissonance \(CD_i\) of agent \(i\), i.e. the distance between belief \(b_i\) and choice \(c_i\).

\[
CD_i := |b_i - c_i|
\]  

(9)

This metric summarizes the effects of the neighbourhood on agent \(i\), since deviations from the belief are driven by the choices of neighbours modulated by an agents avantgarde \(a_i\).

Second, we define an agents \(i\) satisfaction \(S_i\) as a binary measure based on belief \(b_i\) and final decision \(d_i\):

Since beliefs are constant, we define the endogenous decision \(\tilde{d}_i\) of agent \(i\) as

\[
\tilde{d}_i := \begin{cases} 
1, & \text{if } b_i > 0.5 \\
0, & \text{else.} 
\end{cases}
\]  

(10)

If the endogenous decision concurs with the decision finally taken, an agent is satisfied. Vice-versa, if belief and final decision are opposed, an agent is not satisfied:

\[
S_i = \begin{cases} 
1, & \text{if } d_i = \tilde{d}_i \\
0, & \text{otherwise.} 
\end{cases}
\]  

(11)
Thus the satisfaction measures the distortion of the final decision compared to an agent's inherent decision.

We define *neighbourhood uniformity* $U_{N_{i,r}}$ for agent $i$ with respect to a neighbourhood $N$ with extent $r$ as

$$
U_{N_{i,r}} := \frac{\sum_{j \in N_{r}} \mathbf{1}(\tilde{d}_i = \tilde{d}_j) - \sum_{j \in N_{r}} \mathbf{1}(\tilde{d}_i \neq \tilde{d}_j)}{|N_{r}|}.
$$

Thus positive uniformity indicates a majority of neighbours with the same decision, while negative uniformity indicates that the majority of the neighbouring agents has contradicting beliefs.

For population level analysis, we consider summary statistics of $CD_j$ and $S_j$ for (sub)groups of the $G$ agents, i.e. for $j \in g \subseteq \{1, \ldots, G\}$.

**D. ideas for further analysis**

We conduct cluster analysis ...

- connectivity of same decision clusters
- (K-means) clustering by decisions
- similarity of choice in a cluster?! (just an idea, not sure how to measure and if interesting)
- are there clusters of high / low CD resp. satisfaction?

**III. RESULTS**

**Simulation ensemble** We run 100 simulations on a 100x100 lattice. Random initialisation draws the avantgarde factor, belief and the initial choice parameter from an uniform distribution $\mathcal{U}([0, 1])$. The time-scaling factor is chosen as $\lambda^{-1} = 10$.

We analyze the state after 200 simulation steps, since the model reaches a semi-stable state of small changes of choice parameter. [fig. S2]
FIG. 2. **Polarization of decisions emerging from random initial configurations.** Sample realization from a single simulation. (a,b) show uniform distributed initial choice and the initial decisions. (c) shows emergence of clustered choice parameter in the final state. The resulting decisions in (d) thus show the emerging polarization of final decisions.
FIG. 3. Accumulation of agents with choice parameter around 0.5. A majority of agents (approx. 58%) make a final choice according to their inherent belief.
FIG. 4. Uniformity of neighbourhood leads to higher satisfaction. High avantgarde agents are faster satisfied regardless of their neighbourhood. (a) shows results for neighbourhood radius $r = 1$, (b) for $r = 2$. Satisfaction is averaged over neighbourhood uniformity bins.
FIG. 5. Cognitive Dissonance (CD) of agents declines for high avantgarde factors. For avantgarde factors $a_i \leq 0.6$ we found the average $CD$ to be approximately constant. (a) shows the full span of the CD distributions per avantgarde interval including the 25% and 75% ranges. (b) shows the mean CD (black) and satisfaction (blue) for the initial and final states and the respective expected value (dashed) for the initial configurations. While the average CD declines with increasing avantgarde factors the average satisfaction lags behind and only changes substantially for $a \gtrsim 0.5$. 
FIG. 6. Large clusters with uniform conviction lead to higher satisfaction. Multiple simulations on a 100x100 lattice with isolated positive (negative) conviction clusters of different size and density were used to calculate the respective average satisfaction inside the clusters. Only agents inside the cluster with an avantgarde \( a > x \) \( (x < 1.0) \) were used. For initializing a positive cluster a position was chosen to be the cluster center and \( p \) percent of all neighbours within radius \( r \) have been assigned a random (uniform distribution) conviction \( c > 0.5 \) \( (< 0.5 \) for the others). All agents outside of the cluster have a conviction \( 0 < c < 1.0 \) (uniform distribution).


Appendix A: Supplementary information

FIG. S1. Initial avantgarde factors and conviction sampled from uniform distributions.

FIG. S2. Convergence of model dynamics. The average change in choice parameters between subsequent model steps declines exponentially indicating the convergenced into a equilibrium state. Here, for better resolution, only every second model step was considered.