

REMIND: The equations

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ReMIND is a modeling environment that is developed for the implementation of energy-economic models in a multi-regional framework. The current framework provides a number of features that allows the representation of energy carriers and conversion technologies with various techno-economic characteristics. Moreover, the macroeconomic part contains a nested CES function that can have any structure. The regional models are solved as optimal growth models with equilibrium at the energy and capital markets. The regional models are linked by trade in energy carriers, tradeable permits and the generic goods, thus, the markets for traded goods are in equilibrium as well. The present documentation introduces the *GAMS* implementation of the model code. It gives an introduction to the abstract structure of the model and the modeling possibilities. The present documentation does not introduce the particular realization of a model version. Hence, the documentation opens up the possibility to implement individual realizations of energy-economy models.

Contents

1	Preliminary remarks	4
1.1	Model Versions	4
1.2	Notation convention	4
1.3	Sets: The 'lattice' of the equations	4
1.4	Mappings: combining set elements	5
1.5	Equations and symbols used in the equations	5
2	Economy module	6
2.1	The Intertemporal Social Welfare Function (<code>welffun</code>)	6
2.2	Budget equation (<code>budget</code>)	6
2.3	The Production Function (<code>production</code>)	7
2.4	Capital stocks (<code>kapmo, kapmo0</code>)	8
2.5	Labor (<code>labbal</code>)	9
2.6	Final Energy balance (<code>balfinen</code>)	9
2.7	Trade balances and restrictions	9
2.8	Emissions permit allocation (<code>perm_alloc</code>)	10
3	Energy System Module	12
3.1	Energy system costs	12
3.1.1	Fuel costs (<code>ccostfu</code>)	12
3.1.2	Investment Costs (<code>ccostin</code>)	12
3.1.3	Operation and Maintenance Costs (<code>ccostom</code>)	13
3.2	Energy Balance Equations	14
3.2.1	Primary Energy Balance (<code>pebal</code>)	14
3.2.2	Secondary Energy Balance (<code>sebal</code>)	15
3.2.3	Share of technology <code>te</code> in total electricity production (<code>eq_shareseel</code>)	16
3.2.4	Required storage production (<code>eq_reqstorprod</code>)	18
3.2.5	Required long distance grid production (<code>eq_reqgridprod</code>)	19
3.2.6	Competition for geographical potential for renewable energies (<code>eq_geopot</code>)	20
3.3	Energy Transformation Equations	21
3.3.1	Primary Energy to Secondary Energy (<code>pe2setrans</code>)	21
3.3.2	Secondary Energy to Secondary/Final Energy (<code>se2fetrans, se2setrans</code>)	22
3.4	Stock equations (<code>stockenty, stockconst</code>)	23
3.5	Capacities	23
3.5.1	Capacity constraints for energy transformations	23
3.5.2	Capacity constraints for CCS technologies (<code>capconstccs</code>)	25
3.5.3	Capacity Depreciation (<code>ccap</code>)	25

3.5.4	Cumulated Capacities (<code>capcummo</code>)	26
3.6	Learning equation (<code>llearn</code>)	26
3.7	Resource and Potential Constraints	27
3.7.1	Fuel extraction (<code>fuelconst2</code>)	27
3.7.2	Constraints on energy production from renewable sources	28
3.8	The Emission Equations	29
3.8.1	Production and Capture of Emissions (<code>emissions</code>) . .	29
3.8.2	The CO_2 emission constraint (<code>emiconst</code>)	30
3.8.3	Total emissions (<code>emissions2</code>)	31
3.8.4	SO_2 and Aerosol emissions (<code>so2emi</code> , <code>cscouple</code> , <code>cscouple0</code>)	31
3.9	The CCS Equations (<code>ccsbal</code> , <code>ccstrans</code> , <code>ccsconst</code>)	32
4	Climate Module	34
4.1	ACC2 (Interface and further reference)	34
4.2	Simple box model	34
5	Negishi procedure	36
6	Overviews	39
6.1	Variables	39
6.2	Parameters	40
6.3	Sets and Subsets	42
6.4	Mappings	43
7	Literature	44

1 Preliminary remarks

1.1 Model Versions

The REMIND model comprises different model versions. The versions differ with respect to the number of regions and the climate module used to run policy scenarios.

The REMIND model is designed in a multi-regional structure. We maintain two model versions: **REMIND-R** is a multi-regional model which includes inter-regional interactions. **REMIND-G** represents the whole world as the only region. In this paper, we document both model versions. We point out explicitly if an equation shows varieties between the versions or is just included in one versions.

Both versions are coupled to a climate module. We use either a simple box-model or the more sophisticated **ACC2** model.

1.2 Notation convention

We use the following convention on notation:

- **Variables** are written as capital Latin letters. Variables which occur only in the Negishi procedure and in the climate module are written in fraktur, e.g. \mathfrak{T} . (Please note an exception: \mathfrak{M} is used for mappings.)
- **Parameters** are written as Greek letters. Exception: initial and boundary conditions on variables are denoted as the associated variable plus index, although they are parameters. (E.g.: K_0 is the initial value associated with the variable "capital" K .)
- **Sets and Subsets** are written as small Latin letters.
- **Mappings** are written as M with index and \mathfrak{M} with index. Mappings are used in GAMS to identify certain combinations of members of more than one set. The concept of mappings is explained in sec. 1.4.

Indices are used for additional distinctions, e.g. of subsets.

Additional symbols denote some special cases which may occur in any of the four types defined just above:

- Temporal changes of an items are symbolized by " Δ ". (E.g.: ΔS is the change in the amount of a stockable quantity.) The time step length is symbolized by Δt .
- A hat denotes cumulative values. (E.g.: \hat{Z} is the cumulated capacity of a technology.)

1.3 Sets: The 'lattice' of the equations

Sets and subsets form the 'lattice' on which the equations are defined.

- t is the set of time steps from the initial point t_0 to the end point t_{end} .

- r is the set of regions.
 - In REMIND-G, r contains only one element representing the whole world.
 - In REMIND-R, r contains more than one element, representing disjunctive parts of the world.
- v is the set of economic factors (production factors and capital types as well as the macroeconomic output).
- **Energy types e :** Various energy types like coal, electricity, natural gas for household use are defined and grouped into subsets according to their characteristics (for example: primary, secondary, and final energy types e_p, e_s, e_f).
- **Technologies c :** This group covers all transformation technologies in the energy transformation or CCS chain. Again, there are subsets according to different characteristics).
- **Grade levels g :** Some items are characterized by different levels of quality.

1.4 Mappings: combining set elements

Mappings are used in GAMS to define combinations of set elements in order to avoid redundancy in the code. Consider the following example:

In the secondary to final energy transformation equation (cf. sec. 3.3.2, eq. 29), the variables "demand for secondary energy" (D_S) and "production of secondary energy" (P_S) are indexed by time step, region, secondary energy type (e_s), final energy type (e_f), and transformation technology (c).

The equation is evaluated for all time steps and all regions ($\forall t, r$) and all defined combinations of secondary energy type, final energy type and technology ($\forall M_{s \rightarrow f}$). The definition of the mapping contains the desired combinations $e_s \times e_f \times c$. This reduces the number of single equations generated in the compilation process, as "meaningless" combinations can be avoided.

Mappings can also be used in a summation index.

1.5 Equations and symbols used in the equations

The model equations are documented in the following chapters. The variables, parameters, sets/subsets and mappings are explained in tables at the end of each section sorted by these four groups. GAMS code notations are marked by a **special font**. The basic sets and subsets named above in sec. 1.3 are not included in the tables again due to their high frequency of occurrence.

For an overview of symbols used in the equations, see sec. 6.

2 Economy module

2.1 The Intertemporal Social Welfare Function (welffun)

The objective of the optimization is to maximize the total discounted intertemporal welfare U . It is calculated from the time dependent regional utility $\tilde{U}(t, r)$ by summing about all regions r weighted by their Negishi weight $W(r)$ and summing about all time steps taking into account the pure time preference rate $\zeta(r)$. Δt is the time step length. In REMIND-G, Negishi weights $W(r)$ do not appear in the code.

$$U = \sum_r \left(W(r) \sum_{t=t_0}^{t_{end}} \left(\Delta t \cdot e^{-\zeta(r)(t-t_0)} \tilde{U}(t, r) \right) \right) \quad (1)$$

The region- and time dependent annual welfare $\tilde{U}(t, r)$ is calculated from consumption $C(t, r)$ and labor (equivalent to population) $L(t, r)$, assuming an intertemporal elasticity of substitution of 1:

$$\tilde{U}(t, r) = L(t, r) \cdot \ln \left(\frac{C(t, r)}{L(t, r)} \right) \quad \forall t, r \quad (2)$$

In the code, equations (1) and (2) are combined:

$$U = \sum_r \left(W(r) \sum_{t=t_0}^{t_{end}} \left(\Delta t \cdot e^{-\zeta(r)(t-t_0)} L(t, r) \cdot \ln \left(\frac{C(t, r)}{L(t, r)} \right) \right) \right) \quad (3)$$

C	consumption	<code>cons</code>
L	labour (equivalent to population)	<code>vari("lab")</code>
U	total discounted intertemporal welfare	<code>welf</code>
\tilde{U}	region- and time dependent annual utility	
W	Negishi weight	<code>w</code>
Δt	time step length	<code>ts</code>
ζ	time preference rate	<code>disrate</code>

2.2 Budget equation (budget)

Exports of the final good (X_G) are deduced from macroeconomic output $Y(t, r)$, imports of the final good (M_G) are added, taking specific trade costs τ_T into account, which are assigned to the importer. The resulting output is used for consumption, $C(t, r)$, for investments into the capital stock, $I(t, r)$,¹ and for the energy system cost components fuel costs $G_F(t, r)$, investments $G_I(t, r)$ and operation & maintenance $G_O(t, r)$.

¹Please note that the capital stock dynamics of the energy sector is treated separately in the energy system module. Associated investments enter the macroeconomic budget as investment costs $G_I(t, r)$.

$$\begin{aligned}
Y(t, r) - X_G(t, r) + (1 - \tau_G) M_G(t, r) &\geq C(t, r) + I(t, r) \\
&+ G_F(t, r) + G_I(t, r) + G_O(t, r) + G_{AC}(t, r) \\
&+ \sum_p (\tau_{itr}(r, p) \cdot M_E(t, r, p)) \\
&+ \sum_p (\tau_{dom}(r, p) \cdot X_E(t, r, p)) \quad \forall t, r \quad (4)
\end{aligned}$$

C	consumption	<code>cons</code>
G_{AC}	(non-energy) abatement costs	<code>costabat</code>
G_F	fuel costs	<code>costfu</code>
G_I	investment costs	<code>costin</code>
G_O	operation & maintenance costs	<code>costom</code>
I	investments into individual stocks of capital	<code>invest</code>
M_E	imports of primary energy	<code>MpRes</code>
X_E	exports of primary energy	<code>XpRes</code>
M_G	imports of the final good	<code>Mpgood</code>
X_G	exports of the final good	<code>Xpgood</code>
Y	macroeconomic output	<code>vari("inco")</code>
τ_{dom}	additional domestic trade costs of primary energy export	<code>tradecost3</code>
τ_{itr}	inter-regional trade costs of primary energy	<code>tradecost2</code>
τ_G	specific trade costs	<code>tradecost</code>

2.3 The Production Function (production)

The production function is a nested 'CES' (constant elasticity of substitution) production function. The macroeconomic output Y is generated by the inputs capital K , labor L , and total energy E . The generation of total energy is described by a CES production function also, whose input factors are CES function outputs again. Sector-specific final energy types represent the bottom end of the 'CES-tree'.

In the code, you will find only the general form of the production function. It calculates the amount of factor output in a time-step and region, $V(t, r, v_{out})$, from the associated factor input amounts $V(t, r, v_{in})$ according to the following quantities:

- parameter $\phi(r, v_{out})$: total factor productivity
- parameter $\rho(r, v_{out})$. ρ is calculated from the elasticity of substitution, σ , according to the relation

$$\sigma = \frac{1}{1 - \rho}$$

- $\theta(t, r, v_{in})$: efficiency. It is calculated as the product of an initial value and a time-dependent scaling factor.

All outputs (intermediate outputs and GDP) in the CES-tree represent monetary values.

$$V(t, r, v_{out}) = \phi(r, v_{out}) \cdot \left(\sum_{M_{CES}} (\theta(t, r, v_{in}) \cdot V(t, r, v_{in}))^{\rho(r, v_{out})} \right)^{1/\rho(r, v_{out})} \quad \forall t, r, v_{out} \quad (5)$$

$$M_{CES} = (v_{in} \times v_{out}) \in \mathfrak{M}_{CES}$$

The mapping M_{CES} assigns the correct input types v_{in} to each output v_{out} .

On top of the CES-tree, macroeconomic output/GDP is calculated from capital, labor, and total energy:² If ϕ' and ρ' denote the total factor productivity and substitution elasticity, resp., associated with GDP, we thus have³

$$Y(t, r) = \phi'(r) \cdot \left((\theta_K \cdot K)^{\rho'(r)} + (\theta_L \cdot L)^{\rho'(r)} + (\theta_E \cdot E)^{\rho'(r)} \right)^{1/\rho'(r)} \quad \forall t, r \quad (6)$$

E	total final energy (as a production factor)	<code>vari("en"</code>
K	capital	<code>vari("kap"</code>
L	labor (equivalent to population)	<code>vari("lab"</code>
V	amount of production factor output	<code>vari(in)</code>
Y	macroeconomic output	<code>vari("inco")</code>
θ	efficiency	<code>eff('2005'), effscal</code>
ρ	parameter, calculated from substitution elasticity σ	<code>cesdataout("rho")</code>
ϕ	total factor productivity	<code>cesdataout("phi")</code>
\mathfrak{M}_{CES}	combination of input types and associated output	<code>cescomp</code>

2.4 Capital stocks (`kapmo, kapmo0`)

To calculate the capital stock K , its amount in the previous time step is devaluated by an annual depreciation factor δ_k and enlarged by investments

²Set v_{out} contains the element 'GDP', mapping M_{CES} assigns the v_{in} -elements 'capital' $K(t, r)$, 'labor' $L(t, r)$, and 'total energy' $E(t, r)$.

³For clarity, the arguments of K , L , E and the associated efficiency parameters have been dropped here.

I . Both depreciation and investments are expressed as annual values, so the time step length Δt is taken into account.

$$K(t+1, r) = K(t, r) \cdot (1 - \Delta t \cdot \delta_k(r)) + \Delta t \cdot I(t, r) \quad \forall t, r \quad (7)$$

Initial values are assigned from exogenous data K_0 :

$$K(t_0, r) = K_0(r) \quad \forall r \quad (8)$$

I	investments	invest
K	capital stock	vari("kap")
K_0	Initial values for V	cesdataout("inp")
δ_k	annual depreciation factor	cesdataout("delta")
Δt	time step length	ts

2.5 Labor (labbal)

The labor available in every time step and every region, $L(t, r)$, comes from exogenous data $L_t(t, r)$:

$$L(t, r) = L_t(t, r) \quad \forall t, r \quad (9)$$

L	labor available in every time step and every region	vari("lab")
L_t	exogenous data for available labor	datapop

2.6 Final Energy balance (balfinen)

The final energy balance equals the production of final energy P of type e_f in time-step t and region r to its demand as an input factor of the production function $V(t, r, e_f)$.

$$V(t, r, e_f) = P_f(t, r, e_f) \quad \forall t, r, e_f \quad (10)$$

P_f	final energy production	feprod
V	production factor	vari

2.7 Trade balances and restrictions

This chapter applies to **REMIND-R** only!

Trade balances of energy, final good, and permits (tradebal1, tradebal2, tradebal3)

In each time step, exports X_i and imports M_i of each tradeable entity are globally balanced. This applies for exports and imports of each energy type e_T (X_E, M_E), final good (X_G, M_G), and emission permits (X_Q, M_Q):

$$\sum_r (X_E(t, r, e_T) - M_E(t, r, e_T)) = 0 \quad \forall t, e_T \quad (11)$$

$$\sum_r (X_G(t, r) - M_G(t, r)) = 0 \quad \forall t \quad (12)$$

$$\sum_r (X_Q(t, r) - M_Q(t, r)) = 0 \quad \forall t \quad (13)$$

Emission permit trade restriction (perm_restr)

To avoid fictitious permits generated from negative emissions, permit exports X_Q must be lower than the initial allocation of permits Q_{init} by region and time step:

$$X_Q(t, r) < Q_{init}(t, r) \quad \forall r, t > t_0 \quad (14)$$

M_E	energy imports	MpRes
M_G	import of final goods	MpGood
M_Q	permit imports	MpPerm
Q_{init}	initial allocation of permits	emicap
X_E	energy exports	XpRes
X_G	export of final goods	XpGood
X_Q	permit exports	XpPerm
e_T	tradable energy type	entra

2.8 Emissions permit allocation (perm_alloc)

This chapter applies to REMIND-R only!

To calculate the initial allocation of emission permits Q_{init} (emicap), three different scenarios are possible:

Contraction and Convergence

$$Q_{init}(t, r) = \left(\lambda(t) \cdot \frac{L(t, r)}{\sum_r L(t, r)} + (1 - \lambda(t)) \cdot \frac{Q_0(r)}{\sum_r Q_0(r)} \right) \cdot Q_{CO_2}^{ES}(t) \quad \forall t, r \quad (15)$$

The convergence parameter λ increases linearly from zero at the beginning of the time horizon (2005) to 1 at the convergence time (2050).

Intensity: Proportional to BAU-GDP

$$Q_{init}(t, r) = \frac{Y_{BAU}(t, r)}{\sum_r Y_{BAU}(t, r)} \cdot Q_{CO_2}^e(t) \quad \forall t, r \quad (16)$$

Multistage

For each time step, every region is assigned to one subset (r_1 to r_4) with different calculation of the initial allocation Q_{init} . The assignment is based on the per capita income levels of the BAU scenario.

- r_1 : Q_{init} follows emissions in the business as usual.
- r_2 : Q_{init} is proportional to GDP in the business as usual (Y_{BAU}).
- r_3 : Q_{init} is fixed to a value proportional to GDP in the business as usual in t' . t' is the last time step where the region was grouped into r_2 .
- r_4 : Remaining permits are distributed following a contraction and convergence procedure.

$$Q_{init}(t, r) = Q_{BAU}(t, r) \quad \forall t, r \in r_1 \quad (17)$$

$$Q_{init}(t, r) = 0.15 \cdot Y_{BAU}(t, r) \quad \forall t, r \in r_2$$

$$Q_{init}(t, r) = 0.15 \cdot Y_{BAU}(t', r) \quad \forall t, r \in r_3$$

$$Q_{init}(t, r) =$$

$$\left(\lambda(t) \cdot \frac{L(t, r)}{\sum_{r' \in r_4} L(t, r')} + (1 - \lambda(t)) \cdot \frac{Q_0(r)}{\sum_{r' \in r_4} Q_0(r')} \right) \cdot \left(Q_{CO_2}^e(t) - \sum_{r' \notin r_4} Q_{init}(t, r') \right) \quad \forall t, r \in r_4$$

$Q_{CO_2}^e$	global energy-related CO_2 emissions	en_emi - lucemi
$Q_{init}(t, r)$	initial permit allocation	emicap
Q_0	emissions in the year 2000 (data)	dataes
Q_{BAU}	energy-related CO_2 emissions in the business-as-usual (data)	emi_bau
Y_{BAU}	GDP in the business-as-usual (data)	gdp_bau
λ	convergence parameter	lambda

3 Energy System Module

3.1 Energy system costs

3.1.1 Fuel costs (ccostfu)

Fuel costs are associated with the use of exhaustible primary energy (fossils, uranium) and biomass. In the latter case, resources are divided into several grades, and each grade has fixed specific costs. In the former case, specific fuel costs are a function of previous cumulative extraction ("**Rogner-curve**").

$$G_F(t, r) = \sum_{M_{e_p \leftrightarrow g}} (\tau_F(r, e_p, g, t) \cdot P_p(t, r, e_p, g)) + \sum_{e_r} \left(\chi_1(r, e) + \chi_2(r, e) \left(\frac{\sum_t \Delta t P_p(t, r, e_p, g)}{\chi_3(r, e)} \right)^{\chi_4(r, e)} F(t, r, e_r) \right) \quad \forall t, r \quad (18)$$

$$M_{e_p \leftrightarrow g} = (e_p \times g) \in \mathfrak{M}_{e_p \leftrightarrow g}$$

In REMIND-G, fuel extraction F is replaced by primary energy production P_p .

F	fuel extraction of primary energy e_p or e_r	fuel <code>ex</code>
G_F	overall fuel costs	cost <code>fu</code>
P_p	primary energy production	pe <code>prod</code>
τ_F	cost per unit of fuel e_q with grade level g	dataperen("cost")
χ_i	parameters to characterize the exhaustible fuel cost curve (i=1,2,3,4)	datarog
e_r	exhaustible primary energy types	pe <code>tyrog</code>
$\mathfrak{M}_{e_p \leftrightarrow g}$	combinations of primary energy types and grade levels (covers only biomass)	peren2r <code>lf</code>

3.1.2 Investment Costs (ccostin)

Specific investment costs of learning technologies are a model-endogenous variable; those of non-learning technologies are exogenous parameters. Total investment costs G_I are the product of specific costs and capacity additions ΔZ :

$$G_I(t, r) = \sum_{M_{e \rightarrow e}} \left(\sum_{c_{NL}} \left(\tau_{INL}(r, c_{NL}) \sum_{M_{c \leftrightarrow g}} \Delta Z(t, r, c_{NL}, g) \right) \right. \\ \left. + \sum_{c_L} J(t, r, c_L) \sum_{M_{c \leftrightarrow g}} \Delta Z(t, r, c_L, g) \right) \quad \forall t, r \quad (19)$$

$$M_{e \rightarrow e} = (e_{in} \times e_{out} \times c) \in \mathfrak{M}_{e \rightarrow e}, \quad M_{c \leftrightarrow g} = (c \times g) \in \mathfrak{M}_{c \leftrightarrow g}$$

In equation 19, $\mathfrak{M}_{c \leftrightarrow g}$ is restricted to c_{NL} or c_L , resp. through the second step summation.

G_I	investment costs	<code>costin</code>
J	specific investment costs per unit of capacity addition of a learning technology c_L	<code>investcost</code>
ΔZ	addition to the capacity of technology c of grade level g	<code>deltacap</code>
τ_{INL}	specific investment costs per unit of capacity addition of a non-learning technology c_{NL}	<code>data("inco0")</code>
c_{NL}	non-learning energy transformation technology	<code>nolearnte(te)</code>
c_L	learning energy transformation technology	<code>learnte(te)</code>
$\mathfrak{M}_{e \rightarrow e}$	definition of general energy transformation	<code>temapall</code>
$\mathfrak{M}_{c \leftrightarrow g}$	combination of technologies and grade levels	<code>teall2rlf</code>

3.1.3 Operation and Maintenance Costs (`ccostom`)

O & M costs result from

- maintenance of existing facilities according to their capacity (**fixed O & M costs**) and
- operation of energy transformations according to the amount of produced secondary and final energy (**variable O & M costs**).

Addition of both contributions yields total O & M costs C_O :

$$G_O(t, r) = \sum_{M_{e \rightarrow e}} \left(\tau_{fix}(r, c) \sum_{M_{c \leftrightarrow g}} \left((\tau_{INL}(r, c_{NL}) + J(t, r, c_L)) \cdot Z(t, r, c, g) \right) \right. \\ \left. + \tau_{var}(r, c) \cdot \left(P_s(t, r, e_p, e_s, c) + P_f(t, r, e_s, e_f, c) \right) \right) \quad \forall t, r \quad (20)$$

$$M_{e \rightarrow e} = (e_{in} \times e_{out} \times c) \in \mathfrak{M}_{e \rightarrow e}, \quad M_{c \leftrightarrow g} = (c \times g) \in \mathfrak{M}_{c \leftrightarrow g}$$

G_O	operation & maintenance costs	costom
J	specific investment costs for adding capacity of a learning technology c_L	investcost
P_s	production of secondary energy	seprod
P_f	production of final energy	feprod
Z	capacity of technology c	cap
P_s	production of secondary energy	seprod
P_f	production of final energy	feprod
τ_{fix}	fixed specific O&M costs	data("omf")
τ_{INL}	specific investment costs per unit of capacity addition of a non-learning technology c_{NL}	data("inco0")
τ_{var}	variable specific O&M costs	data("omv")
c_{NL}	non-learning energy transformation technology	nolearnte(te)
c_L	learning energy transformation technology	learnte(te)
$\mathfrak{M}_{e \rightarrow e}$	definition of general energy transformation	temapall
$\mathfrak{M}_{c \leftrightarrow g}$	combination of technologies and grade levels	teall2rlf

3.2 Energy Balance Equations

Energy balance equations equate the production P of and demand D for each primary, secondary and final energy; so the general structure is:

$$\sum_{all} P_j = \sum_{all} D_j \quad \forall t, r \quad j \in \{p, s, f\}$$

where "all" means all possible ways of energy transformation relevant for the respective transformation stage (primary, secondary, final).

3.2.1 Primary Energy Balance (pebal)

$$\sum_{M_{e_p, g}} \sum_{M_{p \rightarrow s}} P_p(t, r, e_p, e_s, c, g) = \sum_{M_{p \rightarrow s}} D_p(t, r, e_p, e_s, c) \quad \forall t, r \quad \forall e_p \quad (21)$$

$$M_{p \rightarrow s} = (e_p \times e_s \times c) \in \mathfrak{M}_{p \rightarrow s}$$

D_p	demand for primary energy	pedem
P_p	production of primary energy	peprod
$\mathfrak{M}_{e_p, g}$	combination of primary energy types and grade levels	enty2clf
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se

3.2.2 Secondary Energy Balance (sebal)

The secondary energy balance comprises the following terms:

- Secondary energy can be produced (P_s) from primary or (another type of) secondary energy ($e_{in} \rightarrow e_p, e_s$).
- Secondary energy can be demanded (D_s) to produce final or (another type of) secondary energy ($e_{out} \rightarrow e_{s'}, e_f$).
- Own consumption of secondary energy occurs from the production of secondary and final energy, and from CCS technologies. Own consumption is calculated as the product of the respective production (P_s, P_f , or R as the amount of CO_2 in the respective CCS chain step) and a coefficient ξ . The 2nd, 3rd and 4th argument of ξ define the underlying transformation process and the 5th argument specifies the consumed energy type. Mapping M_{own} defines possible combinations.
- Couple production is modeled as own consumption, but with a negative ξ .
- Stockable secondary energy (e_s) can be transferred to storage (ΔS).

$$\begin{aligned}
& \sum_{M_{p \rightarrow s}} P_s(t, r, e_p, e_s, c) + \sum_{M_{s \rightarrow s}} P_s(t, r, e_{s'}, e_s, c) \\
& + \sum_{M_{own}} (\xi(r, e_p, e_{s'}, c, e_s) \cdot P_s(t, r, e_p, e_{s'}, c)) \\
& + \sum_{M_{own}} (\xi(r, e_{s'}, e_f, c, e_s) \cdot P_f(t, r, e_{s'}, e_f, c)) \\
& + \sum_{M_{own}} \sum_{M_{c \rightarrow CCS}} (\xi(r, q_i^{CCS}, q_{i+1}^{CCS}, c, e_s) \cdot R(t, r, q_i^{CCS}, q_{i+1}^{CCS}, c, g)) \\
& = \sum_{M_{s \rightarrow f}} D_s(t, r, e_s, e_f, c) + \sum_{M_{s \rightarrow s}} D_s(t, r, e_s, e_{s'}, c) \\
& + \Delta S(t, r, e_s) \quad \forall t, r \quad \forall e_s \quad (22)
\end{aligned}$$

$$M_{p \rightarrow s} = (e_p \times e_s \times c) \in \mathfrak{M}_{p \rightarrow s}$$

$$M_{s \rightarrow s'} = (e_s \times e_{s'} \times c) \in \mathfrak{M}_{s \rightarrow s'}$$

where e_s and $e_{s'}$ denote two not necessarily different secondary energy types.

$$M_{s \rightarrow f} = (e_s \times e_f \times c) \in \mathfrak{M}_{s \rightarrow f}$$

$$M_{own} = (e_{in} \times e_{out} \times c \times e_{own}) \in \mathfrak{M}_{own}$$

$$M_{c \rightarrow CCS} = (c \times g) \in \mathfrak{M}_{c \rightarrow CCS}$$

D_s	demand for secondary energy	sedem
P_f	production of final energy	feprod
P_s	production of secondary energy	seprod
R	amount of CO_2 in the i th step of the CCS chain to be transformed to the next one using technology c with grade level g	ccs
ΔS	change per time in stock of e_s if e_s is a stockable quantity	deltaenty
ξ	own consumption coefficient	dataoc
q_i^{ccs}	CCS (captured CO_2) of stage i , $i=1, \dots, 4$	ccsco2(enty)
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se
$\mathfrak{M}_{s \rightarrow s'}$	definition of secondary to secondary energy transformation	se2se
$\mathfrak{M}_{s \rightarrow f}$	definition of secondary to final energy transformation	se2fe
\mathfrak{M}_{own}	definition of own consumption	oc2te
$\mathfrak{M}_{c \rightarrow CCS}$	combination of technology and grade levels for CCS	teccs2rlf

3.2.3 Share of technology te in total electricity production (eq_shareseel)

The share of electricity generation of one technology is calculated by dividing all secondary energy electricity (seel) production of one technology by the combined seel production. As storage losses are simply losses and not used for anything, they are discounted (else the share of a technology needing storage might be larger than 100 percent). The secondary energy balance comprises the following terms:

- Seel can be produced (P_s) from primary or secondary energy, including storage technologies.
- Seel can be the byproduct of pe2se and se2fe production (only positive terms (couple production, $\xi > 0$) are counted, not the own consumption terms).
- All seel that goes into storage has to be deducted.

$$\begin{aligned}
& A_{seel}^{c'}(t, r) / 100 * \\
& \left[\sum_{M_{p \rightarrow seel}} P_s(t, r, e_p, seel, c) + \sum_{M_{s \rightarrow seel}} P_s(t, r, e_s, seel, c) \right. \\
& + \sum_{M_{own_prod}^{p, s \rightarrow s'}} (\xi(r, e_{p,s}, e_{s'}, c, seel) \cdot P_s(t, r, e_{p,s}, e_{s'}, c)) \\
& + \sum_{M_{own_prod}^{s \rightarrow f}} (\xi(r, e_s, e_f, c, seel) \cdot P_s(t, r, e_s, e_f, c)) \\
& \left. \sum_{M_{stor}^{c \rightarrow c''}} \frac{1}{\eta(r, c'')} P_s(t, r, seel, seel, c'') \right] \\
& = \\
& \left[\sum_{M_{p \rightarrow seel}} P_s(t, r, e_p, seel, c') + \sum_{M_{s \rightarrow seel}} P_s(t, r, e_s, seel, c') \right. \\
& + \sum_{M_{own_prod}^{p, s \rightarrow s'}} (\xi(r, e_{p,s}, e_{s'}, c', seel) \cdot P_s(t, r, e_{p,s}, e_{s'}, c')) \\
& + \sum_{M_{own_prod}^{s \rightarrow f}} (\xi(r, e_s, e_f, c', seel) \cdot P_s(t, r, e_s, e_f, c')) \\
& \left. \sum_{M_{stor}^{c' \rightarrow c''}} \frac{1}{\eta(r, c'')} P_s(t, r, seel, seel, c'') \right] \\
& \qquad \qquad \qquad \forall t, r \quad \forall c' \in tenostor \quad (23)
\end{aligned}$$

$$M_{p \rightarrow seel} = (e_p \times seel \times c) \in \mathfrak{M}_{p \rightarrow s}$$

$$M_{s \rightarrow seel} = (e_s \times seel \times c) \in \mathfrak{M}_{s \rightarrow s'}$$

$$M_{own_prod}^{p, s \rightarrow s'} = (e_{p,s} \times e_s \times c \times e_{own}) \in \mathfrak{M}_{own}$$

where ‘_prod’ denotes couple production ($\xi > 0$).

$$M_{own_prod}^{s \rightarrow f} = (e_s \times e_f \times c \times e_{own}) \in \mathfrak{M}_{own}$$

where ‘_prod’ denotes couple production ($\xi > 0$).

$$M_{stor} = (c_{tns} \times c_{ts}) \in \mathfrak{M}_{c_{tns} \rightarrow c_{ts}}$$

$A_{seel}^{c'}$	share of electricity generation by technology c'	v_shareseel
P_f	production of final energy	feprod
P_s	production of secondary energy	seprod
ξ	own consumption coefficient	dataoc
η	efficiency of technology c , can depend on time or not	data("eta"), dataeta
c_{tns}	technologies that require storage	teneedstor
c_{ts}	technologies that store electricity	testor
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se
$\mathfrak{M}_{s \rightarrow s'}$	definition of secondary to secondary energy transformation	se2se
$\mathfrak{M}_{s \rightarrow f}$	definition of secondary to final energy transformation	se2fe
\mathfrak{M}_{own}	definition of own consumption	oc2te
$\mathfrak{M}_{c_{tns} \rightarrow c_{ts}}$	definition of storage requirements	te2stor

3.2.4 Required storage production (eq_reqstorprod)

- Each fluctuating renewable technology ($teneedstor, c_{tns}$) requires a storage technology ($testor, c_{ts}$) ($M_{c_{tns} \rightarrow c'_{ts}}$).
- The storage requirements of a renewable technology c_{tns} are increased through the use of other c_{tns} with similar fluctuations ($M_{c_{tns} \rightarrow c''_{tns}}$).
- Total storage requirement is given by the product of the share of electricity production ($A_{seel}^{c_{tns}}$) of a c_{tns} multiplied with the total usable seel production of this c_{tns}
- Total usable seel production of c_{tns} is equal to the total seel production of c_{tns} minus the losses of the associated storage technology.

$$\begin{aligned}
P_{seel}^{c'_{ts}}(t, r) &> \\
&\sum_{M_{c_{tns} \rightarrow c'_{ts}}} \left\{ \left(A_{seel}^{c_{tns}} + \sum_{M_{c_{tns} \rightarrow c''_{tns}}} \frac{A_{seel}^{c''_{tns}}}{4} \right) \right. \\
&\quad \left. * \sum_{M_{p \rightarrow seel}^{c_{tns}}} \left(P_s(t, r, e_p, seel, c_{tns}) - P_s(t, r, seel, seel, c'_{ts}) * \frac{1 - \eta_{c_{ts}}}{\eta_{c_{ts}}} \right) \right\} \\
&\quad \forall t, r \quad \forall c' \in teneedstor \quad (24)
\end{aligned}$$

$$M_{c_{tns} \rightarrow c'_{ts}} = (c_{tns} \times c_{ts}) \in \mathfrak{M}_{c_{tns} \rightarrow c_{ts}}$$

$$M_{c_{tns} \rightarrow c'_{tns}} = (c_{tns} \times c''_{tns}) \in \mathfrak{M}_{c_{tns} \rightarrow c_{tns}}$$

$$M_{p \rightarrow seel}^{c_{tns}} = (e_p \times seel \times c_{tns}) \in \mathfrak{M}_{p \rightarrow s}$$

A_{seel}^c	share of electricity generation by technology c	<code>v_shareseel</code>
P_s	production of secondary energy s	<code>seprod</code>
η_c	efficiency of technology c , can depend on time or not	<code>data("eta"), dataeta</code>
c_{tns}	technologies that require storage	<code>teneedstor</code>
c_{ts}	technologies that store electricity	<code>testor</code>
$\mathfrak{M}_{c_{tns} \rightarrow c_{ts}}$	definition of storage requirements	<code>te2stor</code>
$\mathfrak{M}_{c_{tns} \rightarrow c_{tns}}$	definition of similar fluctuation patterns	<code>te2teneedstorlinked</code>
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	<code>pe2se</code>

3.2.5 Required long distance grid production (eq_reqgridprod)

- The renewable technologies wind, wind offshore, spv and csp (teneedgrid, c_{tnng}), for which the potential is usually concentrated in one region of a continent, require a grid technology (tegrid, c_{tg}) ($M_{c_{tnng} \rightarrow c'_{tg}}$).
- The grid requirements of a renewable technology c_{tnng} are increased through the use of other c_{tnng} with similar locations for their potentials ($M_{c_{tnng} \rightarrow c'_{tnng}}$).
- Total grid requirement is influenced by the the share of electricity production ($A_{seel}^{c_{tnng}}$) of a c_{tnng} and the total usable seel production of this c_{tnng} .
- Logic: Assuming an equal distribution of demand centers, the length of a power line from the production center increases with the square root of demand centers, which is proportional to the square root of the share of electricity generation (`shareseel`, A_{seel}). Integration over this results in $\frac{1}{1.5} * A_{seel}^{1.5}$, which is approximated for numerical reasons by $0.5 * A_{seel} + 0.5 * A_{seel}^2$.
- Total usable seel production of c_{tnng} is equal to the total seel production of c_{tnng} minus the losses of the associated storage technology, as c_{tnng} usually also require storage.
- The prefactor 0.85 counts as a capacity factor for grid infrastructure.

$$\begin{aligned}
& Z_{c'_{tg}}(t, r) * 0.85 * 1.5 > \\
& \sum_{M_{c_{tns} \rightarrow c'_{tg}}} \left\{ \left[0.5 + 0.5 * \left(A_{seel}^{c_{tns}} + \sum_{M_{c_{tns} \rightarrow c''_{tns}}} \frac{A_{seel}^{c''_{tns}}}{4} \right) \right] \right. \\
& \quad \left. * \sum_{M_{p \rightarrow seel}^{c_{tns}}} \left[P_s(t, r, e_p, seel, c_{tns}) - \left(P_s(t, r, seel, seel, c'_{ts}) * \frac{1 - \eta_{c_{ts}}}{\eta_{c_{ts}}} \right) \right] \right\} \\
& \quad \forall t, r \quad \forall c' \in teneedgrid \quad (25)
\end{aligned}$$

$$M_{c_{tns} \rightarrow c'_{tg}} = (c_{tns} \times c_{tg}) \in \mathfrak{M}_{c_{tns} \rightarrow c_{tg}}$$

$$M_{c_{tns} \rightarrow c''_{tns}} = (c_{tns} \times c''_{tns}) \in \mathfrak{M}_{c_{tns} \rightarrow c_{tns}}$$

$$M_{p \rightarrow seel}^{c_{tns}} = (e_p \times seel \times c_{tns}) \in \mathfrak{M}_{p \rightarrow s}$$

$$M_{c_{tns} \rightarrow c'_{ts}} = (c_{tns} \times c_{ts}) \in \mathfrak{M}_{c_{tns} \rightarrow c_{ts}}$$

A_{seel}^c	share of electricity generation by technology c	v_shareseel
Z_c	capacity of technology c	cap
η_c	efficiency of technology c, can depend on time or not	data("eta"), dataeta
c_{tns}	technologies that require long-distance electricity transport	teneedgrid
c_{tns}	technologies that transport electricity	tegrid
c_{tns}	technologies that require storage	teneedstor
c_{ts}	technologies that store electricity	testor
$\mathfrak{M}_{c_{tns} \rightarrow c_{tg}}$	definition of long-distance grid requirements	te2grid
$\mathfrak{M}_{c_{tns} \rightarrow c_{tns}}$	definition of similar production locations	te2teneedgridlinked
$\mathfrak{M}_{c_{tns} \rightarrow c_{ts}}$	definition of storage requirements	te2stor
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se

3.2.6 Competition for geographical potential for renewable energies (eq_geopot)

- Several renewable technologies c_{ren} use the same geographical potential, e.g., CSP and PV use the primary energy pesol.

$$GP_{e_{pc}}(r, g) > \sum_{M_{pc \rightarrow c_{ren}}} \frac{Z_{c_{ren}}(t, r, g)}{\lambda \mu_{c_{ren}}(r) * 1000} \quad \forall t, r \quad \forall e_{pc} \in petycomp \quad (26)$$

$$M_{pc \rightarrow c} = (e_{pc} \times c) \in \mathfrak{M}_{e_{pc} \rightarrow c}$$

Z_c	capacity of technology c	cap
$\lambda \mu_{c_{ren}}$	land use parameter of technology c_{ren}	data("luse")
c_{ren}	all renewable technologies	ter
e_{pc}	all renewable PE that experience potential competition from different technologies	petycomp
$\mathfrak{M}_{e_{pc} \rightarrow c}$	definition of PE potential competition	petyren2ter

Final Energy Balance

The final energy balance is placed in the economy module. See section 2.6.

3.3 Energy Transformation Equations

Taking the technology-specific transformation efficiency η into account, the equations describe the transformation of an energy type to another type; note that energy entering a transformation is *demanded* (D_j), the resulting energy is *produced* (P_k):

$$\eta(t, r, c) \cdot D_j(t, r, c) = P_k(t, r, c) \quad \forall t, r \quad \forall c \quad j, k \in p, s, f \quad (27)$$

and the allowed combinations of j and k are primary to secondary, secondary to secondary, and secondary to final energy.

3.3.1 Primary Energy to Secondary Energy (pe2setrans)

The transformation technology's efficiency η can be constant in time (c_{η_c}) or time dependent ($c_{\eta(t)}$); in the latter case, production P_s is replaced by the equivalent product of capacity addition ΔZ and load factor ν to assign the η value valid at the time step of the capacity addition (compare with sections 3.5.3 and 3.5.1):

$$\begin{aligned}
D_p(t, r, e_p, e_s, c) &= \frac{1}{\eta(r, c_{\eta_c})} P_s(t, r, e_p, e_s, c_{\eta_c}) \\
&+ \sum_{M_{c_s \leftrightarrow g}} \nu(r, c_{\eta(t)}) \Delta t \sum_{M_{c \leftrightarrow t_l}} \frac{\omega(r, t_l, c_{\eta(t)}) \Delta Z(t - t_l, r, c_{\eta(t)}, g)}{\eta(t - t_l, r, c_{\eta(t)})} \\
&\qquad \qquad \qquad \forall t, r \forall M_{p \rightarrow s} \quad (28)
\end{aligned}$$

$$M_{p \rightarrow s} = (e_p \times e_s \times c) \in \mathfrak{M}_{p \rightarrow s}$$

$$M_{c_s \leftrightarrow g} = (c_s \times g) \in \mathfrak{M}_{c_s \leftrightarrow g}$$

$$M_{c \leftrightarrow t_l} = (c_{vin} \times t_l) \in \mathfrak{M}_{c \leftrightarrow t_l}$$

D_p	demand for primary energy	pedem
P_s	production of secondary energy	seprod
ΔZ	addition to the capacity of technology c of grade level g	deltacap
η	efficiency of technology c , can depend on time or not	data("eta"), dataeta
ν	load factor of technology c	data("nu")
ω	weight factor of addition to technology c 's capacity prior to initial time	datacap("omeg")
t_l	life time	tlt
c_{η_c}	technology with constant η	teneta(te)
$c_{\eta(t)}$	technology with η variable over time	teeta(te)
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se
$\mathfrak{M}_{c_s \leftrightarrow g}$	combination of secondary energy technologies and grade levels	tese2rlf
$\mathfrak{M}_{c \leftrightarrow t_l}$	set of possible combinations of vintage technologies and life time indices	tlt2te

3.3.2 Secondary Energy to Secondary/Final Energy (se2fetrans, se2setrans)

Secondary Energy to Final Energy (se2fetrans)

$$\eta(r, c) \cdot D_s(t, r, e_s, e_f, c) = P_f(t, r, e_s, e_f, c) \quad \forall t, r \quad \forall M_{s \rightarrow f} \quad (29)$$

$$M_{s \rightarrow f} = (e_s \times e_f \times c) \in \mathfrak{M}_{s \rightarrow f}$$

Between Secondary Energy types (se2setrans)

$$\eta(r, c) \cdot D_s(t, r, e_s, e_{s'}, c) = P_s(t, r, e_s, e_{s'}, c) \quad \forall t, r \quad \forall M_{s \rightarrow s'} \quad (30)$$

$$M_{s \rightarrow s'} = (e_s \times e_{s'} \times c) \in \mathfrak{M}_{s \rightarrow s}$$

D_s	demand for secondary energy	sedem
P_f	production of final energy	feprod
P_s	production of secondary energy	seprod
η	efficiency of technology c	data("eta")
$\mathfrak{M}_{s \rightarrow s'}$	definition of secondary to secondary energy transformation	se2se
$\mathfrak{M}_{s \rightarrow f}$	definition of secondary to final energy transformation	se2fe

3.4 Stock equations (stockenty, stockconst)

Stock change (stockenty)

$$S(t+1, r, s) = \Delta t \cdot \Delta S(t, r, s) + S(t, r, s) \quad \forall t, r \quad \forall s \quad (31)$$

Initial values of stocks (stockenty0)

$$S(t_0, r, s) = 0 \quad \forall r \quad \forall s \quad (32)$$

Constraint on stock quantities (stockconst)

$$S(t, r, s) \leq \psi_S(r, s) \quad \forall t, r \quad \forall s \quad (33)$$

S	amount in stock of quantity s	stock
ΔS	change in stockable quantity s per time	deltaenty
ψ_S	capacity of stock of quantity s	stockmax
s	stockable quantity	stockty(enty)

3.5 Capacities

3.5.1 Capacity constraints for energy transformations

Capacity constraints for primary to secondary energy transformation (capconstse)

$$P_s(t, r, e_p, e_s, c) = \sum_{M_{c_s \leftrightarrow g}} \nu(r, c) \cdot \nu_g(r, c, g) \cdot Z(t, r, c, g) \quad \forall t, r \quad \forall M_{p \rightarrow s} \quad (34)$$

$$M_{p \rightarrow s} = (e_p \times e_s \times c) \in \mathfrak{M}_{p \rightarrow s}, \quad M_{c_s \leftrightarrow g} = (c_s \times g) \in \mathfrak{M}_{c_s \leftrightarrow g}$$

**Capacity constraints for secondary to secondary energy transformation
(capconstse2se)**

$$P_s(t, r, e_s, e_{s'}, c) = \sum_{M_{c_s \leftrightarrow g}} \nu(r, c) \cdot \nu_g(r, c, g) \cdot Z(t, r, c, g) \quad \forall t, r \quad \forall M_{s \rightarrow s'} \quad (35)$$

$$M_{s \rightarrow s'} = (e_s \times e_{s'} \times c) \in \mathfrak{M}_{s \rightarrow s'}, \quad M_{c_s \leftrightarrow g} = (c_s \times g) \in \mathfrak{M}_{c_s \leftrightarrow g}$$

**Capacity constraints for secondary to final energy transformation
(capconstfe)**

$$P_f(t, r, e_s, e_f, c) = \sum_{M_{c_f \leftrightarrow g}} \nu(r, c) \cdot Z(t, r, c, g) \quad \forall t, r \quad \forall M_{s \rightarrow f} \quad (36)$$

$$M_{s \rightarrow f} = (e_s \times e_f \times c) \in \mathfrak{M}_{s \rightarrow f}, \quad M_{c_f \leftrightarrow g} = (c \times g) \in \mathfrak{M}_{c_f \leftrightarrow g}$$

Constraint on the Share of Electricity from CHP (capconstchp)

$$\sum_{M_{p \rightarrow s_{CHP}}} P_s(t, r, e_p, e_s, c) = \gamma(r) \cdot \sum_{M_{p \rightarrow s_{Elec}}} P_s(t, r, e_p, e_s, c) \quad \forall t, r \quad (37)$$

$$M_{p \rightarrow s_{CHP}} = (e_p \times e_s \times c) \in \mathfrak{M}_{p \rightarrow s_{CHP}}, \quad M_{p \rightarrow s_{Elec}} = (e_p \times e_s \times c) \in \mathfrak{M}_{p \rightarrow s_{Elec}}$$

P_f	production of final energy	feprod
P_s	production of secondary energy	seprod
Z	capacity of technology c	cap
ν	load factor associated with technology c	data("nu")
ν_g	scaling of the load factor ν dependent on grade level g	dataren("nur")
γ	share of electricity from CHP on overall electricity generation	dataschp("bscu")
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se
$\mathfrak{M}_{s \rightarrow s'}$	definition of secondary to secondary energy transformation	se2se
$\mathfrak{M}_{s \rightarrow f}$	definition of secondary to final energy transformation	se2fe
$\mathfrak{M}_{c_s \leftrightarrow g}$	combination of secondary energy technologies and grade levels	tese2rlf
$\mathfrak{M}_{c_f \leftrightarrow g}$	combination of final energy technologies and grade levels	tefe2rlf
$\mathfrak{M}_{p \rightarrow s_{CHP}}$	definition of primary to secondary energy transformation using CHP technologies	techp
$\mathfrak{M}_{p \rightarrow s_{Elec}}$	definition of primary to secondary energy transformation for electricity generation	teel_dir_nostor

3.5.2 Capacity constraints for CCS technologies (capconstccs)

$$R(t, r, q_i^{ccs}, q_{i+1}^{ccs}, c, g) = Z(t, r, c, g) \quad \forall t, r \quad \forall M_{c \rightarrow CCS} \quad \forall M_{CCS} \quad (38)$$

$$M_{CCS} = (q_i^{ccs} \times q_{i+1}^{ccs} \times c), i = 1, \dots, 4, \in \mathfrak{M}_{CCS}, \quad M_{c \rightarrow CCS} = (c \times g) \in \mathfrak{M}_{c \rightarrow CCS}$$

R	amount of CO_2 in step i of the CCS chain to be transformed to the next one using technology c with grade level g	ccs
Z	capacity of CCS transformation technology with grade level g through technology c	cap
q_i^{ccs}	CO_2 emissions in step i of the CCS process chain	
\mathfrak{M}_{CCS}	definition of CCS steps and associated technologies	ccs2te
$\mathfrak{M}_{c \rightarrow CCS}$	combination of technology and grade levels for CCS	teccs2r1f

3.5.3 Capacity Depreciation (ccap)

Capacities depreciate either according to a vintage depreciation scheme (c_{vin}) or exponentially (c_{exp}):

$$Z(t, r, c, g) = \left(\sum_{M_{c \leftrightarrow t_l}} \Delta t \cdot \omega(r, t_l, c_{vin}) \cdot \Delta Z(t - t_l, r, c_{vin}, g) \right) + \Delta t \cdot \Delta Z(t - 1, r, c_{exp}, g) + (1 - \Delta t \cdot \delta_c(r, c)) \cdot Z(t - 1, r, c_{exp}, g) \quad \forall t, r \quad \forall M_{c \leftrightarrow g} \quad (39)$$

$$M_{c \leftrightarrow t_l} = (c_{vin} \times t_l) \in \mathfrak{M}_{c \leftrightarrow t_l}, \quad M_{c \leftrightarrow g} = (c \times g) \in \mathfrak{M}_{c \leftrightarrow g}$$

Initial capacities (ccap0)

Initial capacities of technologies with exponential depreciation are assigned from data; they are assumed to have a grade level $g = 1$:

$$Z(t_0, r, c_{exp}, g = 1) = Z_0(r, c_{exp}) \quad \forall r \quad \forall c_{exp} \quad (40)$$

Z	capacity of technology c	cap
ΔZ	addition of capacity	deltacap
δ_c	depreciation of technology c	data("delta")
ω	weight factor of addition to technology c 's capacity prior to initial time	datacap("omeg")
Z_0	initial capacity of technology c_{exp}	data("cap0")
t_l	life time	tlt
c_{exp}	technologies with exponential capacity depreciation	expte(te)
c_{vin}	technologies with vintage capacity depreciation	vinte(te)
$\mathfrak{M}_{c \leftrightarrow g}$	combination of technologies and grade levels	teall2r1f
$\mathfrak{M}_{c \leftrightarrow t_l}$	set of possible combinations of vintage technologies and life time indices	tlt2te

3.5.4 Cumulated Capacities (capcummo)

$$\hat{Z}(t+1, r, c_L) = \sum_{M_{c \leftrightarrow g}} \Delta t \Delta Z(t, r, c_L, g) + \hat{Z}(t, r, c_L) \quad \forall t, r \quad \forall c_L \quad (41)$$

$$M_{c \leftrightarrow g} = (c \times g) \in \mathfrak{M}_{c \leftrightarrow g}$$

Initial cumulated capacities (capcum0)

$$\hat{Z}(t_0, r, c_L) = \hat{Z}_0(r, c_L) \quad \forall r \quad \forall c_L \quad (42)$$

\hat{Z}	cumulated capacity of a learning technology c_L	capcum
ΔZ	addition to capacity of a learning technology c_L of grade level g	deltacap
\hat{Z}_0	initial cumulated capacity of a learning technology c_L	data("ccap0")
c_L	learning technologies	learnte(te)
$\mathfrak{M}_{c \leftrightarrow g}$	combination of technologies and grade levels	teall2r1f

3.6 Learning equation (llearn)

$$J(t, r, c_L) = \alpha(r, c_L) \cdot \hat{Z}(t, r, c_L)^{\beta(r, c_L)} + \tau_{IL}(r, c_L) \quad \forall t, r \quad \forall c_L \quad (43)$$

This is equivalent to the common formulation of learning curves in the literature

$$J(t, r, c_L) = \tilde{\alpha}(r, c_L) \cdot \left(\frac{\hat{Z}(t, r, c_L)}{\hat{Z}(t_0, r, c_L)} \right)^{\beta(r, c_L)} + \tau_{IL}(r, c_L) \quad \forall t, r \quad \forall c_L$$

with

$$\alpha(r, c_L) = \frac{\tilde{\alpha}(r, c_L)}{\hat{Z}(t_0, r, c_L)^{\beta(r, c_L)}} \quad \forall r \quad \forall c_L$$

where $\tilde{\alpha}$ represents the difference between initial costs and floor costs. β is calculated from the learning rate (relative cost decrease when cumulated capacities double), $\tilde{\beta}$:

$$\beta(r, c_L) = \frac{\ln(1 - \tilde{\beta}(r, c_L))}{\ln 2} \quad \forall r \quad \forall c_L$$

J	specific investment costs for adding capacity of a learning technology c_L	<code>investcost</code>
\hat{Z}	cumulated capacity of technology c	<code>capcum</code>
τ_{IL}	floor costs of a learning technology c_L	<code>data("inco0")</code>
α, β	learning parameters of a learning technology c_L	<code>data("learna"),</code> <code>data("learnb")</code>
$\tilde{\alpha}$	difference between initial costs and floor costs	
$\tilde{\beta}$	learning rate	
c_L	technology which develops through learning (learning technologies)	<code>learnte(te)</code>

3.7 Resource and Potential Constraints

3.7.1 Fuel extraction (fuelconst2)

For exhaustible energy types (fossils, uranium) and biomass, fuel extraction equals primary energy production. In REMIND-R, import M_E and export X_E of tradable primary energy types $E_{T,p}$ is added, taking specific trade costs τ_T into account.⁴ This contribution is not included in the REMIND-G code.

$$\sum_{M_{p \rightarrow s}} P_p(t, r, e_x, e_s, c, g) = F(t, r, e_x, g) - (X_E(t, r, e_{T,p}) - (1 - \tau_T)M_E(t, r, e_{T,p})) \quad \forall t, r \quad \forall M_{e_x, g} \quad (44)$$

⁴ τ_T represents energetic losses here, whereas in case of final good import, τ_T represents monetary costs (see sec. 2.2).

$$M_{p \rightarrow s} = (e_p \times e_s \times c) \in \mathfrak{M}_{p \rightarrow s}, \quad M_{e_x, g} = (e_x \times g) \in \mathfrak{M}_{e_x, g}$$

F	fuel extraction of an exhaustible resource e_x of grade level g	fuelex
M_E	energy import	MpRes
P_p	production of primary energy	peprod
X_E	energy export	XpRes
$e_{T,p}$	tradable primary energy type	entrape
e_x	exhaustible resource	petyric(enty)
$\mathfrak{M}_{e_x, g}$	combination of exhaustible energy types and grade levels	enty2clf
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se

3.7.2 Constraints on energy production from renewable sources

Constraint on secondary energy production from renewable sources (renconst)

This equation assigns upper limits Π_c on the *technical potential* of secondary energy production technologies from renewable sources (c_{ren}).

$$\pi_c(r, g, c_{ren}) \geq \nu(r, c_{ren}) \cdot \nu_g(r, c_{ren}, g) \cdot Z(t, r, c_{ren}, g) \quad \forall t, r \quad \forall M_{c_{ren} \leftrightarrow g} \quad (45)$$

$$M_{c_{ren} \leftrightarrow g} = (c_{ren} \times g) \in \mathfrak{M}_{c_{ren} \leftrightarrow g}$$

Constraint on renewable primary energy (biomass) production (renconst2)

$$\pi_e(r, e_p, g) \geq F(t, r, e_p, g) \quad \forall t, r \quad \forall M_{e_p \leftrightarrow g} \quad (46)$$

$$M_{e_p \leftrightarrow g} = (e_p \times g) \in \mathfrak{M}_{e_p \leftrightarrow g}$$

F	extraction of primary resource e_p of grade level g	fuel _{ex}
Z	capacity of technology c	cap
ν	load factor of technology c	data("nu")
ν_g	scaling of the load factor ν dependent on grade level g	dataren("nur")
π_c	maximal production (according to technology c_{ren}) of secondary energy from non-exhaustible resource via c_{ren}, g	dataren("maxprod")
π_e	maximal production of primary energy from primary resource e_p of grade level g	datapere("maxprod")
c_{ren}	renewable energy transformation technologies	ter(te)
$\mathfrak{M}_{c_{ren} \leftrightarrow g}$	combination of renewable technologies and grade levels (mapping $\mathfrak{M}_{c \leftrightarrow g}$ is restricted on subset c_{ren})	teall2rlf
$\mathfrak{M}_{e_p \leftrightarrow g}$	combinations of primary renewable energy types and grade levels	peren2rlf

3.8 The Emission Equations

3.8.1 Production and Capture of Emissions (emissions)

Emissions of type q result from primary to secondary energy transformation or transformations within the chain of CCS steps (Leakage).⁵

The equation describes CO_2 released into the atmosphere and CO_2 capture for storage as two different emission types. In primary to secondary energy transformation processes, both types can be generated.⁶

$$\begin{aligned}
Q(t, r, e_{in}, e_{out}, c, q) &= \sum_{M_{p \rightarrow s}} \gamma(r, e_p, e_s, c, q) \cdot D_p(t, r, e_p, e_s, c) \\
&+ \sum_{M_{CCS \rightarrow Q}} \sum_{M_{c \rightarrow CCS}} \gamma(r, q_i^{CCS}, q_{i+1}^{CCS}, c, q) \cdot R(t, r, q_i^{CCS}, q_{i+1}^{CCS}, c, g) \\
&\quad \forall t, r \quad \forall M_{c \rightarrow Q} \quad (47)
\end{aligned}$$

$$M_{c \rightarrow Q} = (e_{in} \times e_{out} \times c \times q) \in \mathfrak{M}_{c \rightarrow Q}, \quad M_{p \rightarrow s} = (e_p \times e_s \times c) \in \mathfrak{M}_{p \rightarrow s}$$

⁵Emissions associated with secondary to final transformation or the usage of final energy, e.g. combustion of transport fuels, are transferred to the underlying secondary energy production.

⁶Further emission types could easily be added into this structure.

D_p	demand of primary energy	pedem
P_s	production of secondary energy	seprod
Q	amount of emissions from type q produced by conversions explained in $\mathfrak{M}_{c \rightarrow Q}$	emi
R	transformation in the CCS chain from step q_i^{ccs} to q_{i+1}^{ccs} using technology c with grade level g	ccs
γ	emission of type q per energy flow in the transformation e_{in} into e_{out} using c	dataemi
q	emission type (CO_2 , captured CO_2)	enty
q_i^{ccs}	CCS (captured CO_2) of stage i , $i=1, \dots, 4$	ccsco2(enty)
$\mathfrak{M}_{CCS \rightarrow Q}$	definition of leakage from CCS transformations	ccs2tele
$\mathfrak{M}_{c \rightarrow CCS}$	combination of technology and grade levels for CCS	teccs2rlf
$\mathfrak{M}_{c \rightarrow Q}$	definition of emissions from a transformation	emi2te
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se

3.8.2 The CO_2 emission constraint (emiconst)

REMIN-D-R:

The initial allocation of permits to a region (Q_{init}) must cover the sum of emissions Q over all domestic emitting processes plus its permit exports X_Q minus its permit imports M_Q .

$$\sum_{M_{c \rightarrow Q}} Q(t, r, e_{in}, e_{out}, c, CO_2) + X_Q(t, r) - M_Q(t, r) \leq Q_{init}(t, r) \quad \forall M_{emicon} \quad (48)$$

REMIN-D-G:

The equation is used for experiments with an exogenous emission time path Q_{exog} .

$$\sum_{M_{c \rightarrow Q}} Q(t, r, e_{in}, e_{out}, c, CO_2) \leq Q_{exog}(t, r) \quad \forall M_{emicon} \quad (49)$$

$$M_{c \rightarrow Q} = (e_{in} \times e_{out} \times c \times q) \in \mathfrak{M}_{c \rightarrow Q}$$

Mapping \mathfrak{M}_{emicon} contains combinations of regions, time steps and emissions types to be considered in the respective emission constraint time path. In other words: It governs which region caps which emission type in which time step:

$$M_{emicon} = (t \times r \times CO_2) \in \mathfrak{M}_{emicon}$$

M_E	emissions permit import	MpPerm
Q	amount of emissions from type $q = CO_2$ produced by conversions specified in $\mathfrak{M}_{c \rightarrow Q}$	emi
Q_{init}	initial permit allocation	emicap
X_E	emissions permit export	XpPerm
Q_{exog}	exogenous time path for energy-related total CO_2 emissions	dataemiconst
$\mathfrak{M}_{c \rightarrow Q}$	definition of emissions from a transformation	emi2te
\mathfrak{M}_{emicon}	combination of regions, time steps and emission types to be considered in the respective emission constraint	emicon

3.8.3 Total emissions (emissions2)

REMIND-R:

The equation calculates total global CO_2 emissions per time step, $Q_{CO_2}^{tot}$ for the calculation of radiative forcing in the climate module. Exogenous land use-change emission data are added.

$$\sum_r \sum_{M_{c \rightarrow Q}} Q(t, r, e_{in}, e_{out}, c, CO_2) + Q_{CO_2}^{lu} = Q_{CO_2}^{tot}(t) \quad \forall t \quad (50)$$

REMIND-G:

The equation calculates energy-related global CO_2 emissions, exogenous land use-change emission data enter the climate module separately.

$$\sum_r \sum_{M_{c \rightarrow Q}} Q(t, r, e_{in}, e_{out}, c, CO_2) = Q_{CO_2}^e(t) \quad \forall t \quad (51)$$

$$M_{c \rightarrow Q} = (e_{in} \times e_{out} \times c \times q) \in \mathfrak{M}_{c \rightarrow Q}$$

Q	amount of emissions from type $q = CO_2$ produced by conversions specified in $\mathfrak{M}_{c \rightarrow Q}$	emi
$Q_{CO_2}^e$	energy-related global CO_2 emissions	emifos
$Q_{CO_2}^{tot}$	total global CO_2 emissions	en_emi
$Q_{CO_2}^{lu}$	land use-change global CO_2 emissions (data)	lucemi
$\mathfrak{M}_{c \rightarrow Q}$	definition of emissions from a transformation	emi2te

3.8.4 SO_2 and Aerosol emissions (so2emi, cscouple, cscouple0)

Both SO_2 and Aerosol emissions from the energy system are not modeled explicitly but parameterized according to the following equations, shown here

for SO_2 (for Aerosols, they work exactly the same way):

Coupling SO_2 to CO_2 (so2emi)

$$Q_{SO_2}^e(t) = H(t) \cdot Q_{CO_2}^e(t) \quad \forall t \quad (52)$$

SO_2 conversion factor (cscouple)

The conversion factor $H(t)$ can change over time with a constant rate ΔH :

$$H(t+1) = H(t) \cdot (1 - \Delta H) \quad (53)$$

SO_2 conversion factor initial value (cscouple0)

$$H(t_0) = H_0 \quad (54)$$

H	conversion factor	cso2
$Q_{CO_2}^e$	energy-related global CO_2 emissions	REMIND-G: emifos, REMIND-R: en_emi
$Q_{SO_2}^e$	energy-related global SO_2 emissions	eso2
H_0	SO_2 conversion factor initial value	
ΔH	change rate of conversion factor	desulphur

3.9 The CCS Equations (ccsbal, ccstrans, ccsconst)

CCS Balance (ccsbal)

The right hand side of the equation calculates the total amount of CO_2 captured (Q with argument q_1^{ccs}) from all relevant emitting processes.⁷ This amount enters the CCS process chain, R , (left hand side).

$$\sum_{M_{c \rightarrow CCS}} R(t, r, q_1^{ccs}, q_2^{ccs} g) = \sum_{M_{c \rightarrow Q}} Q(t, r, e_{in}, e_{out}, c, q_1^{ccs}) \quad \forall t, r \quad (55)$$

$$M_{c \rightarrow CCS} = (c \times g) \in \mathfrak{M}_{c \rightarrow CCS}, \quad M_{c \rightarrow Q} = (e_{in} \times e_{out} \times c \times q) \in \mathfrak{M}_{c \rightarrow Q}$$

⁷Note that "CO₂ captured" is treated as an emission type distinct from CO₂ released into the atmosphere (see sec. 3.8.1 also).

Transformation in the CCS chain (ccstrans)

Processes in the CCS chain are subject to leakage. The amount R of CO_2 in step $i + 1$ is thus the amount R of CO_2 in the previous step i times 1 minus specific emission coefficient γ .⁸

$$\begin{aligned} & (1 - \gamma(q_i^{ccs}, q_{i+1}^{ccs}, c, CO_2)) \cdot R(t, q_i^{ccs}, q_{i+1}^{ccs}, c, g) \\ & = R(t, q_{i+1}^{ccs}, q_{i+2}^{ccs}, c, g) \quad \forall t \quad \forall M_{CCS} \quad \forall M_{c \rightarrow CCS} \end{aligned} \quad (56)$$

Constraint on CCS injection (ccsconst)

$$\sum_{t'=t_0}^t \Delta t \cdot R(t', r, q_3^{ccs}, q_4^{ccs}, c, g) \leq \psi_{CCS}(r, g) \quad \forall t \quad \forall M_{c \rightarrow CCS} \quad (57)$$

$$M_{CCS} = (q_i^{ccs} \times q_{i+1}^{ccs} \times c), i = 1, \dots, 4, \in \mathfrak{M}_{CCS}, \quad M_{c \rightarrow CCS} = (c \times g) \in \mathfrak{M}_{c \rightarrow CCS}$$

Q	amount of CO_2 emissions produced by conversions explained in $\mathfrak{M}_{c \rightarrow Q}$	<code>emi</code>
R	amount of CO_2 in step i of the CCS chain to be transformed to the next one using technology c with grade level g	<code>ccs</code>
γ	specific CO_2 emissions in the transformation q_i^{ccs} into q_{i+1}^{ccs} using c	<code>dataemi</code>
ψ_{CCS}	maximal cumulative injection for CCS of grade level g	<code>dataccs("quan")</code>
q_i^{ccs}	CCS (captured CO_2) of stage i , $i=1, \dots, 4$	<code>ccsco2(enty)</code>
$\mathfrak{M}_{c \rightarrow CCS}$	combination of technology and grade levels for CCS	<code>teccs2rlf</code>
$\mathfrak{M}_{c \rightarrow Q}$	definition of emissions from a transformation	<code>emi2te</code>
\mathfrak{M}_{CCS}	definition of CCS steps and associated technologies	<code>ccs2te</code>

⁸Note the different index numbers in the second and third arguments of R on the left hand and right hand side of the equation.

4 Climate Module

4.1 ACC2 (Interface and further reference)

The climate and carbon chemistry module *ACC2* calculates the climate system dynamics in response to various radiative forcings given in annual resolution.

For a model description, please refer to Kanaka and Kriegler (2007).

ACC2 is coupled to REMIND via emissions of *CO2*, *SO2*, and aerosols from the energy system. By linear interpolation, REMIND values are converted to annual resolution. Other emissions are taken from exogenous data.

In policy experiments, a guardrail on global mean temperature increase is assigned as an extra constraint.⁹

4.2 Simple box model

The implementation of the simple box model follows the elaboration by Kriegler and Bruckner (2004).

Carbon cycle

$$\hat{Q}_{CO_2}(t) = \sum_{t'} Q_{CO_2}^{tot}(t') \quad \forall t' = 1, \dots, t \quad (58)$$

$$\begin{aligned} \mathfrak{C}(t+1) = \mathfrak{C}(t) &+ \frac{\Delta t}{2} \left(\sigma_{source} \cdot \hat{Q}_{CO_2}(t) + \epsilon \cdot Q_{CO_2}^{tot}(t) - \sigma_{sink} \cdot (\mathfrak{C}(t) - \mathfrak{C}_H) \right. \\ &\quad \left. + \sigma_{source} \cdot \hat{Q}_{CO_2}(t+1) \right. \\ &\quad \left. + \epsilon \cdot Q_{CO_2}^{tot}(t+1) - \sigma_{sink} \cdot (\mathfrak{C}(t+1) - \mathfrak{C}_H) \right) \quad \forall t \quad (59) \end{aligned}$$

$$\mathfrak{C}(t_0) = \mathfrak{C}_0 \quad (60)$$

Radiative forcing

$$\mathfrak{R}_{CO_2}(t) = \kappa \cdot \frac{\ln(\mathfrak{C}(t)/\mathfrak{C}_H)}{\ln 2} \quad \forall t \quad (61)$$

$$\mathfrak{R}_{SO_2}(t) = \frac{\iota_D \cdot Q_{SO_2}^e(t)}{Q_{SO_2,1990}^e} + \frac{\iota_I \cdot \ln(1 + Q_{SO_2}^e(t)/Q_{SO_2}^{nat})}{\ln(1 + Q_{SO_2,1990}^e/Q_{SO_2}^{nat})} \quad \forall t \quad (62)$$

$$\mathfrak{R}(t) = \mathfrak{R}_{CO_2}(t) + \mathfrak{R}_{SO_2}(t) + \mathfrak{R}_{OGHG}(t) \quad \forall t \quad (63)$$

⁹*ACC2* can handle other guardrails as well, e.g. an ocean acidification limit. The coupling to the energy system does not yet support these alternative guardrails.

Temperature equations

$$T(t+1) = T(t) + \Delta t \frac{1}{2} \left(\mu_{drive} \frac{\ln 2}{\kappa} \mathfrak{R}(t) - \mu_{damp} T(t) + \mu_{drive} \frac{\ln 2}{\kappa} \mathfrak{R}(t+1) - \mu_{damp} T(t+1) \right) \quad \forall t \quad (64)$$

$$T(t_0) = T_0 \quad (65)$$

The estimation of the parameters μ_{drive} and μ_{damp} takes climate system factors like climate sensitivity, ocean heat capacity and vertical diffusivity into account.

\hat{Q}_{CO_2}	cumulative CO_2 emissions	cume
$Q_{SO_2}^e$	SO_2 emissions in tgs per a	eso2
$Q_{CO_2}^{tot}$	total global CO_2 emissions	en_emi
T	global mean temperature relative to preindustrial level	temp
\mathfrak{C}	atmospheric CO_2 concentrations	conc
\mathfrak{R}	total radiative forcing	ftot
\mathfrak{R}_{CO_2}	CO_2 radiative forcing	fco2
\mathfrak{R}_{SO_2}	SO_2 direct and indirect radiative forcing	fso2
ϵ	conversion factor gtc into ppmv	cconvi
ι_D	direct aerosol forcing in 1990 (W per m^2)	dso1990
ι_I	indirect aerosol forcing in 1990 (W per m^2)	iso1990
κ	radiative forcing for a doubling of CO_2 (W per m^2)	fcodb
μ_{damp}	temperature damping factor	alpha
μ_{drive}	temperature driving factor	mu
σ_{sink}	ocean biosphere as CO_2 sink	sigma
σ_{source}	ocean biosphere as CO_2 source	b
\mathfrak{C}_0	initial atmospheric CO_2 concentration	c1995
\mathfrak{C}_H	preindustrial atmospheric CO_2 concentration	c0
\mathfrak{R}_{OGHG}	radiative forcing of other GHGs - external input	foghg
$Q_{SO_2,1990}^e$	SO_2 emissions in 1990 in tgs per a	so1990
$Q_{SO_2}^{nat}$	natural SO_2 emissions in tgs per a	enatso2
T_0	global mean temperature in 2005 relative to preindustrial level	

5 Negishi procedure

This chapter applies to REMIND-R only!

The Negishi procedure adjusts the Negishi weights W in an iterative process around the model optimization. The implementation follows Leimbach and Toth (2003). In each iteration step, new Negishi weights are determined as follows:

- Trade deficits \mathfrak{T} of each tradable entity (specified by indices: tradable energy E , final good G , emission permits Q) are calculated from imports M and exports X :

$$\mathfrak{T}_E(t, r, e_T) = X_E(t, r, e_T) - M_E(t, r, e_T) \quad \forall t, r, e_T \quad (66)$$

$$\mathfrak{T}_G(t, r) = X_G(t, r) - M_G(t, r) \quad \forall t, r$$

$$\mathfrak{T}_Q(t, r) = X_Q(t, r) - M_Q(t, r) \quad \forall t, r$$

- Shadow prices \mathfrak{P} of each tradable entity are determined from the marginal values of the associated trade balance (see sec. 2.7). In case of permits, the maximum of trade balance marginal and emission summation (see sec. 3.8.3) is considered:¹⁰

$$\mathfrak{P}_E(t, e_T) = \left| \frac{\partial U}{\partial (\sum_r (X_E - M_E))} \right| \quad \forall t, e_T \quad (67)$$

$$\mathfrak{P}_G(t) = \left| \frac{\partial U}{\partial (\sum_r (X_G - M_G))} \right| \quad \forall t$$

$$\mathfrak{P}_Q(t) = \max \left(\left| \frac{\partial U}{\partial (\sum_r (X_Q - M_Q))} \right|, \left| \frac{\partial U}{\partial (\sum_r \sum_{M_c \rightarrow Q} Q + Q_{CO_2}^{tu} - Q_{CO_2}^{tot})} \right| \right) \quad \forall t$$

- The **intertemporal trade balance** \mathfrak{B} of each region is determined as the sum of trade volumes of all tradable entities. Trade volumes are products of trade deficits and associated shadow prices. i is the iteration step:

$$\mathfrak{B}_i(r) = \sum_t \left(\sum_{e_T} \mathfrak{P}_E(t, e_T) \mathfrak{T}_E(t, r, e_T) + \mathfrak{P}_G(t) \mathfrak{T}_G(t, r) + \mathfrak{P}_Q(t) \mathfrak{T}_Q(t, r) \right) \quad \forall r, i \quad (68)$$

¹⁰ Arguments of model variables are omitted here for clarity.

- Weighting factors \mathfrak{W} express regional economic power by adding the product of final good shadow price times consumption to the intertemporal trade balance:

$$\mathfrak{W}(r) = \sum_t \left(\mathfrak{P}_G(t)C(t, r) + \sum_{e_T} \mathfrak{P}_E(t, e_T)\mathfrak{T}_E(t, r, e_T) + \mathfrak{P}_G(t)\mathfrak{T}_G(t, r) + \mathfrak{P}_Q(t)\mathfrak{T}_Q(t, r) \right) \forall r \quad (69)$$

- Non-normalized Negishi weights \tilde{W} are calculated from intertemporal trade balances, weighting factors, and the non-normalized Negishi weights from the previous iteration step:

$$\tilde{W}_{i+1}(r) = \tilde{W}_i(r) \cdot \left(1 + \mathfrak{B}_i(r) \frac{\Delta i \ln(i) + 2\Delta i}{\sum_r \mathfrak{W}(r) + \mathfrak{W}(r)} \right) \quad \forall r, i \quad (70)$$

Δi is a parameter used to control the iteration step size. If \mathfrak{B} approaches zero, the correction of the weights \tilde{W} gets smaller, and the iteration converges.

- Finally, normalization yields the Negishi weights W :

$$W(r) = \frac{\tilde{W}_i(r)}{\sum_r \tilde{W}_i(r)} \quad (71)$$

$Q_{CO_2}^{tot}$	total global CO_2 emissions	en_emi
W	Negishi weights	w
\tilde{W}	Non-normalized Negishi weights	NW
\mathfrak{B}	intertemporal trade balance	defic
\mathfrak{P}	Shadow prices	PVP1,2,3
\mathfrak{T}	Trade deficits	trade1,2,3
\mathfrak{W}	Weighting factors	weight
$Q_{CO_2}^{lu}$	land use-change global CO_2 emissions	lucemi
Δi	parameter	parm
e_T	tradable energy type	entra

6 Overviews

6.1 Variables

Variables in Latin letters

C	consumption	cons
D_p	demand of primary energy	pedem
D_s	demand of secondary energy	sedem
E	total final energy (as production factor)	vari("en")
F	fuel extraction of primary energy e_p or e_r	fuelex
G_F	fuel costs	costfu
G_I	investment costs (energy system)	costin
G_O	operation & maintenance costs	costom
H	conversion factor SO_2 per CO_2 emissions	cso2
I	investments into capital stocks	invest
J	specific investment costs per unit of capacity addition of a learning technology c_L	investcost
K	capital (macroeconomy)	vari("kap")
L	labor	vari("lab")
M_E	energy import	MpRes
M_G	final good import	MpGood
M_Q	emissions permit import	MpPerm
P_f	production of final energy	feprod
P_p	production of primary energy	peprod
P_s	production of secondary energy	seprod
Q	amount of energy related emissions from a single contribution	emi
Q_{init}	initial allocation of permits	emicap
$Q_{CO_2}^e$	global energy-related CO_2 emissions	REMIND-R: en_emi-lucemi REMIND-G:emifos
$Q_{CO_2}^{tot}$	total global CO_2 emissions	en_emi
\hat{Q}_{CO_2}	cumulative CO_2 emissions	cume
$Q_{SO_2}^e$	SO_2 emissions in tgs per a	eso2
R	amount of CO_2 in the i th step of CCS chain	ccs
S	amount in stock of e_s	stock
ΔS	change per time in stock of e_s	deltaenty
T	global mean temperature relative to preindustrial level	temp
U	total discounted intertemporal welfare	welf
V	production factor (monetary amount)	vari
W	Negishi weight	w
X_E	energy export	XpRes
X_G	final good export	XpGood
X_Q	emissions permit export	XpPerm
Y	macroeconomic output	vari("inco")
Z	capacity of technology c	cap
ΔZ	addition to the capacity of technology c of grade level g	deltacap
\hat{Z}	cumulated capacity of technology c	capcum

Variables in Fraktur letters

\mathfrak{C}	atmospheric CO_2 concentrations	conc
\mathfrak{T}	Trade deficits	trade1,2,3
\mathfrak{P}	Shadow prices	PVP1,2,3
\mathfrak{R}	total radiative forcing	ftot
\mathfrak{R}_{CO_2}	CO_2 radiative forcing	fco2
\mathfrak{R}_{SO_2}	SO_2 direct and indirect radiative forcing	fso2
\mathfrak{B}	intertemporal trade balance	defic
\mathfrak{W}	Weighting factors	weight
\tilde{W}	Un-normalized Negishi weights	NW

6.2 Parameters

Parameters in Latin letters (initial and boundary values of variables)

H_0	SO_2 conversion factor initial value	
K_0	Initial values for V	cesdataout("inp")
L_t	exogenous data for available labour	datapop
Q_0	emissions in the year 2000 (data)	dataes
Q_{BAU}	energy-related CO_2 emissions in the business-as-usual	emi_bau
$Q_{CO_2}^{lu}$	land use-change global CO_2 emissions (data)	lucemi
Q_{exog}	exogenous time path for energy-related total CO_2 emissions	dataemiconst
$Q_{SO_2}^{nat}$	natural SO_2 emissions in tgs per a	enatso2
$Q_{SO_2,1990}^e$	SO_2 emissions in 1990 in tgs per a	so1990
T_0	global mean temperature in 2005 relative to preindustrial level	
Y_{BAU}	GDP in the business-as-usual (data)	gdp_bau
Z_0	initial capacity of technology c_{exp}	data("cap0")
\hat{Z}_0	initial cumulated capacity of a learning technology c_L	data("ccap0")

Parameters in Fraktur letters (initial and boundary values of variables)

\mathfrak{C}_0	initial atmospheric CO_2 concentration	c1995
\mathfrak{C}_H	preindustrial atmospheric CO_2 concentration	c0
\mathfrak{R}_{OGHG}	radiative forcing of other GHGs - external input	foghg

Parameters in Greek letters

α	learning parameter of a learning technology	<code>data("learna")</code>
	c_L	
$\tilde{\alpha}$	difference between initial costs and floor costs	
β	learning parameter of a learning technology	<code>data("learnb")</code>
	c_L	
$\tilde{\beta}$	learning rate	
γ	emission of type q per energy flow in the transformation e_{in} into e_{out} using c	<code>dataemi</code>
δ_c	depreciation of technology c	<code>data("delta")</code>
ΔH	change rate of conversion factor	<code>desulphur</code>
δ_k	annual depreciation factor	<code>cesdataout("delta")</code>
Δi	Negishi iteration step size parameter	<code>parm</code>
Δt	time step length	<code>ts</code>
ϵ	conversion factor gtc into ppmv	<code>cconvi</code>
ζ	time preference rate	<code>disrate</code>
η	efficiency of technology c , can depend on time or not	<code>data("eta"),</code> <code>dataeta</code>
θ	efficiency	<code>eff('2005'),effscal</code>
ι_D	direct aerosol forcing in 1990 (W per m^2)	<code>dso1990</code>
ι_I	indirect aerosol forcing in 1990 (W per m^2)	<code>iso1990</code>
κ	radiative forcing for a doubling of CO_2 (W per m^2)	<code>fcodb</code>
λ	convergence parameter (permit allocation)	<code>lambda</code>
μ_{damp}	temperature damping factor	<code>alpha</code>
μ_{drive}	temperature driving factor	<code>mu</code>
ν	load factor of technology c	<code>data("nu")</code>
ν_g	scaling of the load factor ν dependent on grade level g	<code>dataren("nur")</code>
ξ	own consumption coefficient	<code>dataoc</code>
π_c	maximal production (according to technology c_{ren}) of secondary energy from non-exhaustible resource via $c_{ren,g}$	<code>dataren("maxprod")</code>
π_e	maximal production of primary energy from primary resource e_p of grade level g	<code>dataperen("maxprod")</code>
ρ	parameter, calculated from substitution elasticity σ	<code>cesdataout("rho")</code>
σ_{sink}	ocean biosphere as CO_2 sink	<code>sigma</code>
σ_{source}	ocean biosphere as CO_2 source	<code>b</code>
τ_F	cost per unit of fuel e_q with grade level g	<code>dataperen("cost")</code>
τ_{fix}	fixed specific O&M costs	<code>data("omf")</code>
τ_{IL}	floor costs of a learning technology c_L	<code>data("inco0")</code>
τ_{INL}	specific investment costs per unit of capacity addition of a non-learning technology	<code>data("inco0")</code>
	c_{NL}	
τ_T	specific trade costs	<code>tradecost</code>

τ_{var}	variable specific O&M costs	<code>data("omv")</code>
ϕ	total factor productivity	<code>cesdataout("phi")</code>
χ_i	parameters to characterize the exhaustible fuel cost curve (i=1,2,3,4)	<code>datarog</code>
ψ_{CCS}	maximal cumulative injection for CCS of grade level g	<code>dataccs("quan")</code>
ψ_S	capacity of stock of quantity s	<code>stockmax</code>
ω	weight factor of addition to technology c 's capacity prior to initial time	<code>datacap("omeg")</code>

6.3 Sets and Subsets

c	transformation technology	<code>te</code>
c_{exp}	technologies with exponential capacity depreciation	<code>expte(te)</code>
c_{η_c}	technology with constant η	<code>teneta(te)</code>
$c_{\eta(t)}$	technology with η variable over time	<code>teeta(te)</code>
c_L	learning energy transformation technology	<code>learnte(te)</code>
c_{NL}	non-learning energy transformation technology	<code>nolearnte(te)</code>
c_{ren}	renewable energy transformation technologies	<code>ter(te)</code>
c_{vin}	technologies with vintage capacity depreciation	<code>vinte(te)</code>
e	physical quantity (energy types, emission types)	<code>enty</code>
e_f	final energy type	<code>fety</code>
e_p	final energy type	<code>pety</code>
e_r	exhaustible primary energy types	<code>petyrog</code>
e_s	final energy type	<code>sety</code>
e_T	tradable energy type	<code>entra</code>
$e_{T,p}$	tradable primary energy type	<code>entrape</code>
e_x	exhaustible resource	<code>petyric(enty)</code>
g	grade	<code>clf, rlf</code>
q	emission type (CO_2 , captured CO_2)	<code>emis(enty)</code>
q_i^{ccs}	CCS (captured CO_2) of stage i , $i=1,\dots,4$	<code>ccsco2(enty)</code>
r	region	<code>regi</code>
s	stockable quantity	<code>stockty(enty)</code>
t	time (model time steps)	<code>t</code>
t_l	life time	<code>tlt</code>
v	macroeconomic factors	<code>in</code>

6.4 Mappings

$\mathfrak{M}_{c \rightarrow CCS}$	combination of technology and grade levels for CCS	teccs2rlf
$\mathfrak{M}_{c \rightarrow Q}$	definition of emissions from a transformation	emi2te
$\mathfrak{M}_{c \leftrightarrow g}$	combination of technologies and grade levels	teall2rlf
$\mathfrak{M}_{c \leftrightarrow tl}$	set of possible combinations of vintage technologies and life time indices	tlt2te
\mathfrak{M}_{CCS}	definition of CCS steps and associated technologies	ccs2te
$\mathfrak{M}_{CCS \rightarrow Q}$	definition of leakage from CCS transformations	ccs2tele
\mathfrak{M}_{CES}	combination of input types and associated output	cescomp
$\mathfrak{M}_{c_f \leftrightarrow g}$	combination of final energy technologies and grade levels	tefe2rlf
$\mathfrak{M}_{c_{ren} \leftrightarrow g}$	combination of renewable technologies and grade levels (mapping $\mathfrak{M}_{c \leftrightarrow g}$ is restricted on subset c_{ren})	teall2rlf
$\mathfrak{M}_{c_s \leftrightarrow g}$	combination of secondary energy technologies and grade levels	tese2rlf
$\mathfrak{M}_{e \rightarrow e}$	definition of general energy transformation	temapall
$\mathfrak{M}_{e_p \leftrightarrow g}$	combinations of primary energy types and grade levels (covers only biomass)	peren2rlf
$\mathfrak{M}_{e_p, g}$	combination of primary energy types and grade levels	enty2clf
$\mathfrak{M}_{e_x, g}$	combination of exhaustible energy types and grade levels (mapping ... is restricted to subset ...)	enty2clf
\mathfrak{M}_{emicon}	combination of regions, time steps and emission types to be considered in the respective emission constraint	emicon
\mathfrak{M}_{own}	definition of own consumption	oc2te
$\mathfrak{M}_{p \rightarrow s}$	definition of primary to secondary energy transformation	pe2se
$\mathfrak{M}_{s \rightarrow f}$	definition of secondary to final energy transformation	se2fe
$\mathfrak{M}_{s \rightarrow s'}$	definition of secondary to secondary energy transformation	se2se

7 Literature

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