Klimageschichte Übung 4
1-D EBM - Snowball Earth

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1 Introduction - the model

So far we have seen various versions of a 0-D energy balance model (EBM), although some form of spatial variability was introduced implicitly into the Daisyworld model. Here we consider an extension of this EBM to 1-D by including explicitly a latitudinal dependence (Fig. 1).

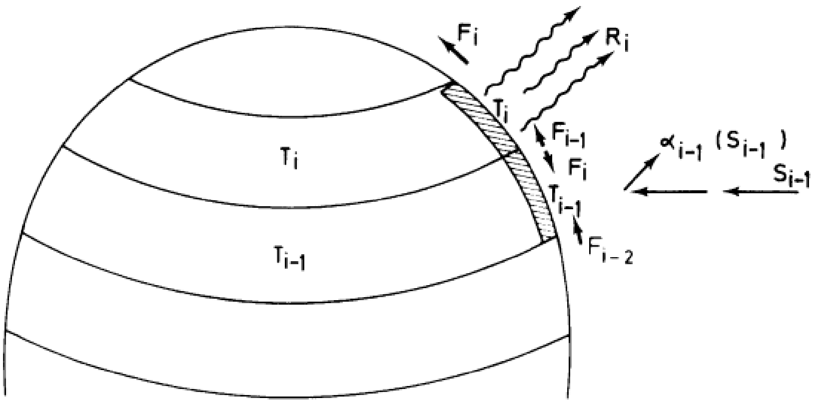

Figure 1: 1-D EBM (McGuffie and Henderson-Sellers, 2005).

Each latitudinal band will have its own temperature, determined by solving the energy balance equation orginally devised by both Sellers (1969) and BUDYKO (1969):

$$\{Shortwave\ in\} = \{Longwave\ out\} + \{Transport\ out\}, \quad (1)$$

or, mathematically:

$$S(\phi)\{1 - \alpha(\phi)\} = \{A + BT(\phi)\} + k_t\{T(\phi) - \overline{T}\} \quad (2)$$

where:
$S(\phi)$ is the mean annual radiation incident at latitude ϕ,
$\alpha(\phi)$ is the albedo at latidue ϕ,
A and B are constants governing the longwave radiation loss (here taking values $A = 204.0\ Wm^{-2}$ and $B = 2.17\ Wm^{-2\circ}C^{-1}$),
$T(\phi)$ is the temperature at latitude ϕ in °C,
k_t is the transport coefficient (here set equal to 3.81 $Wm^{-2\circ}C^{-1}$),
and $\overline{T}$ is the global mean temperature.
The outgoing longwave radiation described by the Stefan-Boltzmann law ($\epsilon\sigma T^4$) in previous exercises, is here approximated by the linear relation $A + BT$ with parameters A and B determined empirically.

1. Under which condition does a linear relation give a reasonable first approximation to the Stefan-Boltzmann law? Recall the linearization performed in Übung 2, $T^4 = (T_0+T')^4 = T_0^4(1+T'/T_0)^4 \sim T_0^4(1+4T'/T_0) = T_0^4+4T_0^3T'$ with the temperatures in K.

2. What physical processes are parameterized by the transport term?

3. Observations indicate that $S(\phi) = \overline{S}\{1 - 0.477P_2(sin\phi)\}$ where $P_2(x) = (3x^2 - 1)/2$ is the second Legendre polinomial. Compute the mean annual insolation at the equator and at the pole. Assume a global mean insolation $\overline{S} = 340\ Wm^{-2}$ as in previous exercises.

4. Assuming that the albedo of the planet would be uniform (e.g. $\alpha = 0.3$) and that there was no meridional heat transfer, what would the temperature difference between pole and equator be? Use the S values found in point 3.

5. Which do you think are the (two) major factors introducing latitudinal variations in planetary albedo? Can these processes be parameterized as a function of temperature (as it is assumed in the simple model above, $\alpha(\phi)$) in a sensible way?

2 Snowball Earth

To a first approximation surface albedo can be assumed to be affected mostly by the presence of snow or ice. Since the presence of snow or ice can be related to the temperature of the zonal band the albedo can be taken as a function of temperature only. If we are interested in the albedo at the top of the atmosphere (planetary albedo), the effect of clouds is also important. However, clouds can not be easily correlated with temperature. We will assume that the planetary albedo is mostly influenced by the surface albedo and can hence be described by a simple temperature dependence. For simplicity the snow-free land surface albedo is set to 0.3 and the albedo of snow/ice to 0.6. If temperature in a latitudinal band drops below a critical temperature of $T_{cr} = -10°C$, the surface is assumed to be covered by snow/ice. The model gives a simple representation of the ice-albedo feedback mechanism. For instance, assume that the amount of incoming solar radiation is reduced for some reason, then the temperature will drop everywhere. At some latitude temperature will drop below T_{cr} and ice will grow. This will cause an additional drop in temperature because less energy is reflected back to space due to the increased albedo, closing the (positive) feedback loop.

Now the model can be solved numerically to get the latitudinal profile of temperature, albedo and ice cover for different values of the incoming solar radiation. The model supports two stable equilibrium solutions for a range of solar constant values (Fig. 2).

To answer the following questions you can use different versions of the 1-D EBM:
- use the iPython notebook provided (available at `http://www.pik-potsdam.de/members/willeit/klimageschichte-ubungen`
- download the model from `http://www.climatemodellingprimer.net/downloads`, available for Windows (executable or Microsoft Excel spreadsheet), Mac and Linux.

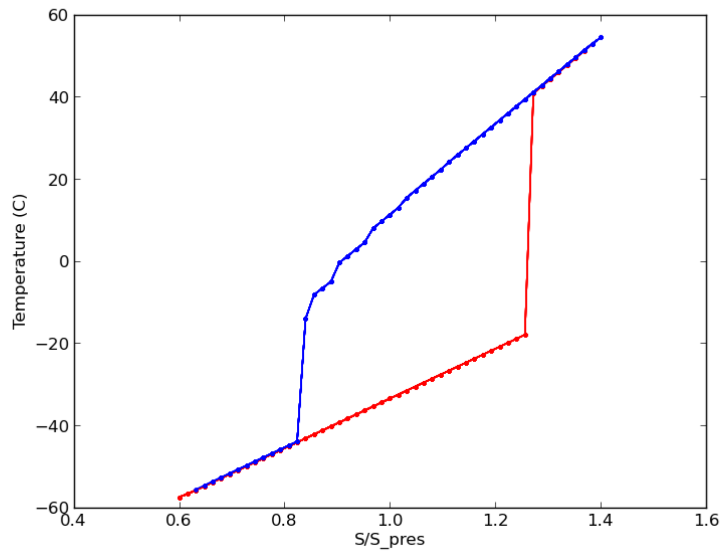

Figure 2: 1-D EBM hysteresis.

1. Using the default values of albedo, k_t, A and B, determine what decrease in the solar constant is required just to glaciate the Earth completely (ice edge at 0°C) if you start the experiment from a warm present day climate with a present day value of the solar constant (Hint: look at the hysteresis loop when solar constant is decreasing.).

2. In the above experiment, how far south does the ice cover have to expand in order to cause a runaway feedback that glaciates the Earth completely?

3. Observations show that land will be totally snow-covered during winter for an annual mean surface temperature of 0°C, and oceans totally ice-covered all year for a temperature of about -13°C.The model specifies a change from land/sea to snow/ice at $T_{cr} = -10°C$. Alter this critical temperature and investigate how the threshold in the solar constant decrease leading to a Snowball Earth is affected.

4. Once the Earth is in a glaciated state, what increase in the solar constant is required to deglaciate the Snowball in this simple model?

5. Recent modeling results have raised doubts about the ability to deglaciate from a global glaciation at atmospheric carbon dioxide levels that are realistic for a Neoproterozoic Snowball Earth. Abbot and Pierrehumbert (2010) argue that over the lifetime of a Snowball event, ice dynamics could lead to the development of a layer of continental and volcanic dust at the ice surface in the tropics that would significantly lower the tropical surface albedo. How does a reduction of the ice albedo to 0.5 in the model affect the solar constant increase required to deglaciate the Earth? (For simplicity we assume that changes in the solar constant can be used to represent changes in atmospheric CO_2.)

Supporting material:
McGuffie and Henderson-Sellers (2005) is an excellent introductory book to climate modelling, including descriptions and applications of simple EBMs.

References

Abbot, D. S. and Pierrehumbert, R. T. (2010). Mudball: Surface dust and Snowball Earth deglaciation. *Journal of Geophysical Research*, 115(D3):D03104.

BUDYKO, M. I. (1969). The effect of solar radiation variations on the climate of the Earth. *Tellus*, 21(5):611–619.

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Sellers, W. D. (1969). A Global Climatic Model Based on the Energy Balance of the Earth-Atmosphere System. *Journal of Applied Meteorology*, 8(3):392–400.