Reducing the Cost of Delay: On the Interaction of Cap-and-Trade and Subsidies for Clean Energy

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Abstract

Relying on theoretical and numerical modeling I show that subsidies for clean energy can be welfare enhancing even if a cap-and-trade (CAT) program is already in place and if there is only the carbon externality. The growth rate of the permit price in the CAT program is too high if intertemporal permit trading is unconstrained implying too low prices early on. Without affecting the permit price, optimal subsidies shift some of the resulting excessive emissions to the future and thereby postpone carbon damages. However, the optimal subsidy path is not time consistent. The Markovian subsidy has a permit price reducing effect but is still welfare enhancing compared to a CAT-only policy. Subsidies also reduce the permit price volatility and stabilize the abatement path. In this sense, subsidies can be a reasonable second-best alternative if ideal CAT programs (e.g. with permit price collars) are not available.

Keywords: cap-and-trade, renewable support, subsidies, carbon pricing

JEL codes: D62, D92, H23, Q48, Q58

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April 2020
1. Introduction

The economic textbook solution to tackle climate change is to price the carbon externality. The optimal price should reflect the present value of marginal social damages of emissions\(^1\) - the social costs of carbon (SCC). However, many countries rely on alternative policies as clean energy subsidies to decarbonize (parts of) their economies and virtually all jurisdictions that have implemented carbon pricing via cap-and-trade (CAT) programs, as the European Union (EU) and California, subsidize clean energy in the same sector as well (ICAP 2019; REN21 2019). In the absence of other market failures, such overlapping policies are often considered to be inefficient and ineffective (e.g. Böhringer and Rosendahl 2010; Fankhauser et al. 2010);\(^2\) while subsidies distort (reduce) the permit price of the CAT program they do not affect cumulative emissions due to the cap. This is also called waterbed effect because the amount of emissions is fixed and thus overlapping policies only change the distribution of emissions (Perino 2018).

In this paper, I reconsider the waterbed effect on the basis of a theoretical model and show that adding clean subsidies to a sector that is already regulated by a CAT program can indeed be welfare enhancing. This result also holds if the cap of the CAT program is chosen optimally and if there are no other market failures. The reason is that CAT with free intertemporal trading induces a too steep permit price path which implies that prices are too low early on as they are lower than the SCC. In consequence, abatement is delayed and carbon damages are inefficiently high. I show that subsidies can complement such a CAT program since they shift emissions to the future and thereby reduce the “cost of delay” (Goulder 2020).

But why is the CAT-induced price path too steep? The permit price of a CAT program with unconstrained intertemporal trading rises at the rate of interest (Cronshaw and Kruse 1996; Rubin 1996) and the optimal carbon price (i.e. the SCC) rises among

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\(^1\)For simplicity, the terms emission and carbon are used interchangeably in this paper.

\(^2\)A portfolio of several instruments can be justified if other market failures are considered, for example related to innovation and network effects (e.g. Fischer and Newell 2008; Jaffe et al. 2005). Multiple instruments can also be justified if there is a probability that the cap is not binding (Lecuyer and Quirion 2013; 2019). In order to focus attention on the main results I only include the climate externality in this analysis and assume that a cap, if implemented, always binds.
others at the growth rate of the economy (e.g. Golosov et al. 2014; Rezai and Van der Ploeg 2016; van den Bijgaart et al. 2016). Under reasonable parameter assumptions, this implies that the growth rate of the SCC is typically significantly smaller than the rate of interest and consequently the permit price path is too steep.\(^3\) Excessive interest rates applied to permit banking is also seen as a reason for inefficient low prices in real CAT markets as the EU Emission Trading System (ETS). Regulatory and market failures, as a lack of credibility of the cap (Salant 2016) and myopic market participants, increase the discount rate applied by firms which, in turn, raises the growth rate of the permit price (Fuss et al. 2018).

A potential solution to this problem would be to prohibit intertemporal permit trading. However, when regulating the climate externality under uncertainty, banking has welfare advantages compared to fixed periodical caps because the impact of shocks is reduced (Fell et al. 2012; Weitzman 2020). In fact, banking is virtually in all larger CAT programs allowed (ICAP 2019). Although borrowing is usually prohibited, intertemporal trading is practically unconstrained in many CAT programs since, as for example in the EU ETS, they exhibit large banks implying that borrowing constraints do not play a role. An alternative to prohibit banking is an intertemporal trading ratio which adjusts banked permits by a certain factor so that the growth rate of the permit price is corrected (Kling and Rubin 1997; Leiby and Rubin 2001; Yates and Cronshaw 2001). Yet, in the presence of uncertainty, intertemporal trading ratios only lead to the optimum if the cap and the trading ratio are constantly updated according to predefined policy rules (Pizer and Prest 2019) which seems challenging from a regulatory perspective. To the best of my knowledge, regular intertemporal trading ratios have indeed never been implemented in the context of climate change.\(^4\) Therefore, the case of free intertemporal trading combined

\(^3\)For example, in van der Ploeg (2018) the annual growth rate of the carbon price is 2%, while the interest rate grows at 4.4% and in the latest DICE version the carbon price rises about 3% and the interest rate is 4.25% (Nordhaus 2017). Dietz and Venmans (2019) provide an overview of recent parameter estimates, showing that the interest rate is greater than the SCC growth rate.

\(^4\)Pizer and Prest (2019) outline where intertemporal trading ratios are or were discussed. Besides the Waxman-Markey Bill in the USA, which was never implemented, they mention China and New Zealand. In both countries discount factors for permits are seen as a temporary measure to reduce overall banked permits. Interestingly, this implies a trading ratio of less than one, whereas I show below that a ratio of greater than one in combination with a lower cap would be required for optimality.
with clean subsidies analyzed in this paper is of high practical relevance.

I use a theoretical model in which a regulator implements a policy or a policy mix in order to internalize a stock pollutant (carbon emissions) from an energy sector in an infinite horizon setting. Based on analytical modeling and in a numerical application to the EU electricity sector, I find that adding a subsidy if a CAT program is already in place enhances welfare about 15 billion Euro compared to the CAT-only case. The optimal subsidy path declines over time if combined with a CAT program: in the beginning a high subsidy supports the inefficient low permit price and thereby postpones damages. Since the permit price rises faster than the SCC less support is needed over time. Moreover, the optimal subsidy does not affect the permit price because the subsidy becomes negative at the end of the transition which compensates for the permit price decreasing effect of the positive subsidy at other times. Yet, the optimal subsidy path within the CAT program creates a time consistency problem. The Markovian subsidy starts higher and never turns negative and thus the permit price is reduced by the waterbed effect. However, overall effects are similar to the commitment case and the welfare advantage is still 11 billion Euro compared to CAT-only.

Under abatement cost uncertainty subsidies have an additional advantage as they reduce the permit price volatility because subsidies prolong the transition or permit banking phase such that shocks spread over to more periods. The Markovian subsidy further stabilizes the abatement path since the subsidy and the permit price are negatively correlated. Having that said, both the commitment and the time consistent subsidy also have disadvantages under uncertainty: since the former is not adjusted when new information arrive and since the latter amplifies the permit price reducing effect, both induce some additional inefficiencies under uncertainty. Yet, in total the welfare advantage of a subsidy and CAT mix is comparable to the case under perfect information in my numerical simulation.

This paper relates to the literature on instrument choice to regulate pollutants. Hoel and Karp (2001; 2002) and Newell and Pizer (2003) are the first who extend Weitzman’s (1974) seminal work to stock pollutants and multiple periods, but without intertemporal
permit trading. Similar to me, Fell et al. (2012) integrate intertemporal permit trading in this setting and find that it improves the performance of CAT programs, but according to Weitzman (2020) banking and borrowing is either dominated by prices or fixed quantities. Pizer and Prest (2019) show that CAT can induce the first best notably because of intertemporal permit trading but only if policy updating rules are in place. However, cap-updating is not time consistent and the Markovian policy implies a welfare loss (Lintunen and Kuusela 2018; Karp 2019; Kuusela and Lintunen 2020). Instead, I focus on subsidy-updating within CAT programs for which I derive the commitment and time consistent solution.\footnote{Other theoretical papers that consider policy updating either assume time-invariant damages (Gerlagh and Heijmans 2018) or ignore explicit damages altogether (Kollenberg and Taschini 2016; Newell et al. 2005). Karp and Traeger (2018) and Karp (2019) study updating of taxes and quantities but the former ignore intertemporal trading of permits and the latter model a flow pollutant.}

A related strand of literature analyzes the welfare effects of a combination of price- and quantity-based instruments going back to Roberts and Spence (1976) and Weitzman (1978). In the context of climate change, such hybrid policies are typically analyzed as a CAT program with price collars (e.g. Pizer 2002; Fell and Morgenstern 2010; Grüll and Taschini 2011; Fell et al. 2012). I find that subsidies share some properties of a price collar as they reduce the permit price volatility and stabilize the abatement path. Nonetheless, subsidies are only a second-best alternative because they distort prices and therefore lead to an over-consumption of energy in my example.

After explaining the general model setup in section 2.1, I derive a carbon tax in section 2.2 that implements the first best and serves as optimal benchmark. Before I analyze the combination of a clean subsidy and CAT in section 2.5, I focus on both in isolation. In section 2.3 I consider the CAT program with and without intertemporal trading ratio and in section 2.4 the isolated clean subsidy is analyzed. The model is applied numerically to the EU’s electricity sector in section 3 where I differentiate between the case of perfect information in 3.2 and uncertain abatement costs in 3.3.
2. The Model

First I describe the general model setting in the next section and thereafter I analyze the respective policies.

2.1. General Setup

I consider a model that is close to the standard framework for competitive permits markets and more broadly to the price vs. quantities literature beginning with Weitzman (1974). In this setting abatement costs are usually assumed to be quadratic which I express by the following function

\[ AC_t = \frac{\psi}{2} (\varphi_t - e_t)^2 \]  

with \( \varphi_t \) as the time-dependent business-as-usual (BAU) emissions, \( \psi \) as the slope of the marginal abatement costs and \( e_t \) as the realized emissions in period \( t \). Emissions accumulate in the atmosphere and the stock of emissions in \( t \) is

\[ \Phi_t = \Phi_{t-1} + e_t. \]  

To keep the model parsimonious I ignore the decay of emissions which is sometimes explicitly considered in similar models.\(^6\) The emission stock causes societal damages equal to \( \delta_t \Phi_t \) where the parameter \( \delta_t \) is the marginal damage of emissions in period \( t \) and thus I assume marginal damages are constant which is a standard assumption in climate economics (e.g. Newell and Pizer 2003; Lintunen and Kuusela 2018; Weitzman 2020). However, I assume that the marginal damage increases over time at rate \( g \),

\[ \delta_{t+1} = \delta_t (1 + g). \]  

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\(^6\)The decay rate for carbon is low, for example Newell and Pizer (2003) assume 0.83% and Karp and Traeger (2018) 0.3%. Moreover, Dietz and Venmans (2019) argue based on results from natural sciences that the effect of the decay rate is more or less offset by other effects as the saturation of carbon sinks. One could also assume that the decay rate is part of rate \( g \) introduced below.
Below I show that $g$ is the growth rate of the social costs of carbon (SCC), that is, the discounted marginal damages of emissions.

Since I want to analyze different policy instruments that imply different production incentives, I explicitly consider the firm problem. I therefore implement the model as a two stage game: at the lower level, firms maximize (intertemporal) profits via production decisions in a competitive market. At the upper level, the regulator maximize intertemporal welfare via the levels of the respective policy instruments and given the reaction function of the firms.

I assume two representative firms $i$ that generate energy $x_{it}$: the dirty firm, $i = d$, has $\phi_d > 0$ emissions per unit of production such that its emits $e_t = \phi_d x_{d,t}$, and the clean firm, $i = c$, is equipped with a carbon free technology, $\phi_c = 0$. Production costs are assumed to be quadratic $C_{it} = \alpha_{it} x_{it} + \frac{\beta_i}{2} x_{it}^2$ and energy demand is assumed to be linear, $D_t = a_t - bw_t$, with energy price $w_t$. In the next section, I show that these assumptions lead to the quadratic abatement cost function (2.1) with $\psi$ and $\varphi_t$ defined as in equations (2.9) and (2.10) if firms maximize profits under perfect competition and given that the energy market equilibrium,

$$\sum_i x_{it} = X_t = D_t,$$

holds. In case of uncertainty, I always assume an additive shock on demand function parameter $a_t$ which is equivalent to an additive shock on BAU emissions $\varphi_t$ (see below). Uncertainty about the level of $a_t$ resolves at the beginning of each period $t$, where firms and the regulator have the same information. While I generally consider an infinite time horizon, I assume there is a single finite transition phase $t = 0, 1, ..., \hat{t}$ with $\hat{t}$ as the last period with positive emissions, $e_t > 0$. For $t > \hat{t}$ the transition to carbon neutrality is completed, demand is fully satisfied by the clean firm and thus $e_t = 0$.

2.2. Carbon Tax

I begin with deriving the optimal carbon tax. The firm problems at the lower level of the two stage model is solved first. In doing so, I derive the reaction functions of the
firms which serve as input for the regulator’s problem at the upper level. Firms have the following expected profits,

\[ E_t[\pi_{it}] = \sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} E_t \left[ w_s x_{is} - \alpha_{is} x_{is} - \frac{\beta_i}{2} x_{is}^2 - \tau_s \phi_i x_{is} \right] \]  

(2.5)

where \( \tau_t \) is the carbon tax. Since firms do not have any intertemporal decision to make, the problem breaks down to repeated static decisions. Further note that because demand uncertainty resolves at the beginning of each period, firms make decisions under certainty. Maximizing profits via production \( x_{it} \) gives the production function as

\[ x_{it} = \left( w_t - \alpha_{it} - \tau_t \phi_i \right) \beta_t^{-1}. \]

Using this in the energy market equilibrium (2.4) yields the energy price,

\[ w_t = \frac{\beta_c \beta_d a_t + \beta_c (\alpha_{dt} + \tau_t \phi_d) + \beta_d \alpha_{ct}}{\beta_c + \beta_d + \beta_c \beta_d b}. \]

(2.6)

Intuitively, the energy price increases with demand parameter \( a_t \) and the carbon tax \( \tau_t \). Inserting the energy price in the production function gives production depending only on the tax and parameters,

\[ x_{dt} = \frac{\beta_c a_t + \alpha_{ct} - (\alpha_{dt} + \tau_t \phi_d) (1 + \beta_c b)}{\beta_c + \beta_d + \beta_c \beta_d b}. \]

(2.7)

\[ x_{ct} = \frac{\beta_d a_t + \alpha_{dt} + \tau_t \phi_d - \alpha_{ct} (1 + \beta_d b)}{\beta_c + \beta_d + \beta_c \beta_d b}. \]

(2.8)

Since total emissions are \( e_t = \phi_d x_{dt} \), I get BAU emissions by multiplying expression (2.7) with \( \phi_d \) and setting \( \tau_t = 0 \),

\[ \varphi_t = \frac{\phi_d (\beta_c a_t - \alpha_{dt} (1 + \beta_c b) + \alpha_{ct})}{\beta_c + \beta_d + \beta_c \beta_d b}. \]

(2.9)

In order to derive the slope of the marginal abatement costs (the second derivative of equation (2.1)), I set marginal abatement costs equal to the tax, \( C_t' = \psi (\varphi_t - e_t) = \tau_t \), which is required for optimality. Inserting BAU emissions (2.9) and \( e_t = \phi_d x_{dt} \) in \( \psi (\varphi_t - e_t) = \tau_t \) and solving for \( \psi \) yields the slope of the marginal abatement cost curve,
\[ \psi = \frac{\beta_c + \beta_d + \beta_c \beta db}{\phi_d^2 (1 + \beta b)}. \]  

(2.10)

Hence the energy market problem with the two firm types is isomorphic to the standard quadratic abatement cost problem where BAU emission and the slope of the marginal abatement cost curve are defined as in (2.9) and (2.10), respectively. BAU emissions increase with the energy demand \((a_t)\), emission factor \((\phi_d)\) and costs of the clean technology \((\alpha_{c,t}, \beta_c)\) and they decrease with the costs of the dirty firm \((\alpha_{d,t}, \beta_d)\). The slope of the marginal abatement costs also depends on technology costs, emission factor and demand reaction to price \((b)\). Expressions (2.9) and (2.10) also hold under the CAT program which will be useful throughout the paper.

Facing reaction functions (2.7) and (2.8), the regulator decides about the carbon tax level at the beginning of the first period. Expected welfare is given by

\[
E_0 [W] = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} E_0 \left[ CS_t - \sum_i \left( \alpha_{it} x_{it} + \frac{\beta_i}{2} x_{it}^2 \right) - \delta_t \Phi_t \right],
\]

(2.11)

where \(CS_t = (a_t X_t - 0.5 X_t^2) b^{-1}\) is the consumer surplus and \(\delta_t \Phi_t\) damages due to the stock of emissions as explained in the previous section.\(^7\) By maximizing (2.11) via \(\tau_t\), I get,

\[
\tau_t = \delta_0 \sum_{s=t}^{\infty} \frac{(1+g)^s}{(1+r)^s-t} = \delta_0 (1+g)^t \frac{1+r}{r-g},
\]

(2.12)

with growth rate

\[
\frac{\tau_{t+1} - \tau_t}{\tau_t} = g
\]

(2.13)

Hence the optimal carbon tax (2.12) in each period is the sum of the discounted marginal damages of all upcoming periods and thus it is equal to the SCC. Since I assume constant marginal damages the SCC and the tax are always equalized implying that the tax also establishes the first best outcome under uncertain abatement costs. Intuitively,

\(^7\)I assume that the tax revenues are allocated to the households on a lump sum basis.
the optimal tax (and the SCC) increases with damages, $\delta_t = \delta_0 (1 + g)^t$, and decreases with the discount rate because future damages weigh less. Given the infinite horizon setting it must hold $r > g$ for the series $\sum_{s=t}^{\infty} \frac{(1+g)^s}{(1+r)^s}$ to converge. Put differently if $r \leq g$ marginal damages rise as fast or faster than the weight of future periods decreases and thus $\tau_t \to \infty$, implying zero emissions would always be optimal. In order to avoid such trivial solutions I concentrate on the case $r > g$. As explained in the introduction a higher interest rate $r$ than growth rate of the SCC $g$ is the relevant case in climate economics.

2.3. Cap-and-Trade

If the regulator implements a cap-and-trade (CAT) program, compliance requires that the dirty firm holds for each unit of emission, $e_t = \phi_d x_{d,t}$, one permit. Permits can be bought in auctions at the beginning of each period. The permit supply of the regulator is $s_t$ and purchases by the firms are $y_{it}$, such that the equilibrium condition of the permit market can be written as

$$s_t = \sum_i y_{it},$$

(2.14)

which is satisfied by the permit price $p_t$. If firms have more permits than required, they can bank them for later use. I assume that a potential permit borrowing constraint never binds before all permits are used up which simplifies the analysis without affecting the main insights. In particular, it implies that the temporal allocation of permits does not affect the results (e.g. Salant 2016).\(^8\) The banking dynamics are given by

$$b_{it} = b_{it-1} + y_{it} - x_{it} \phi_i,$$

(2.15)

with $b_t$ as the banked permits at the end of period $t$. The firm’s problem is to maximize expected profits,

\(^8\)Below I sometimes assume that all permits are issued in the first period, which also implies that the borrowing constraint never binds. Ignoring borrowing constraints can be justified by real markets as the EU ETS in which borrowing constraints do not play a role so far.
\begin{equation}
\begin{aligned}
E_t [\pi_{it}] = \sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} E_t \left[ w_s x_{is} - \alpha_{is} x_{is} - \frac{\beta_i}{2} x_{is}^2 - p_s y_{is} \right],
\end{aligned}
\end{equation}

subject to the banking dynamics (2.15). Solving this problem leads to the following permit price (I relegate all derivations for this section to Appendix A.1),

\begin{equation}
p_t = \psi (\varphi_t - s_t - B_{t-1} + B_t),
\end{equation}

with \( B_t = \sum_i b_{it} \). The permit price reflects the marginal abatement costs and the term in the brackets is the amount of abatement. From the first order conditions (see Appendix A.1) further follows that the expected permit price rises at the discount rate as long as the bank is not depleted,

\begin{equation}
\frac{E_t [p_{t+1}] - p_t}{p_t} = r \quad \forall t \leq \hat{t}
\end{equation}

with \( \hat{t} \) as the last period of the transition to emission neutrality and thus in which permits are available. This relationship reflects intertemporal arbitrage which is exploited by profit maximizing firms. After the transition is followed through, for \( t > \hat{t} \), energy production of the dirty firm must be zero and the permit price is as least as high to guarantee this.

In the following I focus on the problem under certainty because upcoming expressions under uncertainty would be complicated since they grow with the number of periods. I rely on numerical simulations below to show effects of uncertainty. By combining (2.17) and (2.18) I get the permit banking path under certainty,

\begin{equation}
B_t = \frac{\sum_{s>t}^\hat{t} ((\varphi_s - s_s) + (B_{t-1} + s_t - \varphi_t)(1+r)^{s-t})}{\sum_{s=t}^{\hat{t}} (1+r)^{s-t}}.
\end{equation}

Higher BAU emissions \( \varphi_s \) in the future increase permit banking because more permits are required for energy generation. Banking is also higher when the future supply \( s_s \) is lower because banked permits are a substitute to future supply. The bank (2.19) can be used to derive production of the dirty and clean firm depending only on parameters,
where $S = \sum_{t=0}^{\hat{t}} s_t$ is the total supply of permits or the emissions cap. The first term in (2.20) is the BAU production, i.e. the energy generation level with zero carbon price. Since I am interested in the case in which abatement is required, implying that the overall BAU emissions are larger than the cap, $\sum_{s=0}^{\hat{t}} \varphi_s - S > 0$, the second term reduces the production of the dirty sector and essentially reflects abatement that is needed for compliance. The first term for the clean firm (2.27) implies that its production increases with demand $a_t$ and decreases with its technology costs $\alpha_{c,t}$ and $\beta_c$, as well as price reaction to demand reflected by demand function parameter $b$.

Given the reaction functions of both firms (2.20) and (2.21), the regulator maximizes welfare as defined in equation (2.11) at the beginning of the first period via the total cap $S$. Taking the first order condition with respect to the cap $S$ and rearranging yields,

$$S = \sum_{t=0}^{\hat{t}} \left( \varphi_t - \frac{\tau_t}{\psi} \right),$$

where $\tau_t$ reflects the SCC but is not a policy parameter (the tax) as in the previous section. The optimal overall cap is the difference between periodic BAU emissions $\varphi_t$ and the ratio between the SCC and the slope of the marginal abatement costs, $\frac{\tau_t}{\psi}$, summed over the transition period. The ratio $\frac{\tau_t}{\psi}$ is optimal abatement in period $t$ and thus the difference $\varphi_t - \frac{\tau_t}{\psi}$ is the optimal realized emissions.\(^9\)

Higher BAU emissions and a steeper marginal abatement cost curve (larger $\psi$) lead to a larger cap since both parameters increase the abatement costs and thus imply higher optimal emissions. In contrast, the optimal cap decreases with the SCC ($\tau_t$) which reflect the benefits of abatement. Comparing the cap

\(^9\)Optimal periodic abatement can be derived from the static model with welfare $W = \frac{\psi}{2} (\varphi - e)^2 - e\tau$ where $e\tau$ are damages. Taking the derivative with respect to emission $e$ gives optimal abatement as $\frac{\tau}{\psi} = \varphi - e$. 

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to the cumulative emissions under tax regime leads to the following result (see Appendix A for all proofs).

**Lemma 1.** Cumulative emissions under the tax and CAT program are identical if and only if they induce the same transition length, \( \hat{t}_{\text{tax}} = \hat{t}_{\text{cat}} \).

As in the CAT program, under the tax regime the optimal cumulative emissions depend on the periodic abatement costs and benefits summed over the transition periods, 
\[
\sum_{t=0}^{\hat{t}_{\text{tax}}} x_{d,t} \phi_d = \sum_{t=0}^{\hat{t}_{\text{tax}}} \left( \varphi_t - \frac{\psi_t}{\psi} \right).
\]
After the transition \( t > \hat{t} \) periodic emissions are zero under both regimes. Hence if transition lengths are equal, cumulative emissions must be equal as well. However, equal transition lengths, \( \hat{t}_{\text{tax}} = \hat{t}_{\text{cat}} \), do not imply that both instruments lead to the same result.

**Proposition 1.** Even if cumulative emissions of the tax and CAT regime are equal only the tax regime is efficient.

This follows from the higher growth rate of the permit price compared to the tax, \( r > g \), implying different transition paths. Since only the tax induced transition path is optimal, the CAT path must be inefficient even if it leads to the same cumulative emissions. If cumulative emissions in both cases are the same, the permit price would be lower than the tax in the early phase of the CAT program and higher in the later phase. The initially lower permit prices lead to an earlier accumulation of emissions compared to the tax. If cumulative emissions would be equal at the end of the transition if \( \hat{t}_{\text{tax}} = \hat{t}_{\text{cat}} \), the emission stock and thus damages would be higher during the entire transition under the CAT regime. These are the cost of delay induced by free intertemporal trading in the CAT program.

However, having the same transition length and thus cumulative emissions as under the tax is also not (second best) optimal in the CAT program. The regulator has incentives to reduce the cap if \( \hat{t}_{\text{tax}} = \hat{t}_{\text{cat}} \), because a lower cap partly compensates for the too high damages of the CAT program during the transition. Put differently, by lowering the cap the permit price increases in all periods and thus cumulative emissions are always lower. Hence the failure of the too high permit price growth rate implies less than optimal
cumulative emissions. This also implies that the transition period of the CAT program is shorter than under the tax, $\hat{t}_{cat} < \hat{t}_{tax}$, because a lower cap reduces $\hat{t}_{cat}$ and because of lemma 1.

In order to correct the inefficient growth rate of the permit price, the literature suggests to implement intertemporal trading ratios (e.g. Kling and Rubin 1997). The intertemporal trading ratio $\left(1 + r_b\right)$ adjusts banked permits in the following way,

$$b_{it} = y_{it} - x_{it} \phi_i + b_{it-1} \left(1 + r_b\right). \quad (2.23)$$

Instead of an one-to-one exchange of permits over time, firms get for each banked permit $\left(1 + r_b\right)$ permits in the next period. Following the same steps as in the CAT regime without intertemporal trading ratio one can derive the following optimal total supply of permits (see Appendix A.1.3),

$$s_0 = \sum_{t=0}^{\hat{t}} \frac{1}{\left(1 + r_b\right)^t} \left(\varphi_t - \tau_t \psi\right), \quad (2.24)$$

where I assume that all permits are supplied in $t = 0$.\(^{10}\) The optimal issuance of permits is thus the discounted sum of optimal periodic emissions with intertemporal trading ratio $\left(1 + r_b\right)$ as discount factor. This restores the first best outcome if the trading ratio is set optimally as well.

**Proposition 2.** The CAT program is optimal if permits $s_0$ are issued according to (2.24) and $s_t = 0 \ \forall \ t > 0$ and the intertemporal trading ratio is set to $\left(1 + r_b\right) = (1 + r) \left(1 + g\right)^{-1}$.

First, the trading ratio of $\left(1 + r_b\right) = (1 + r) \left(1 + g\right)^{-1}$ implies that the permit price grows at the optimal rate $g$. Second, since the trading ratio is positive, $\left(1 + r_b\right) > 0$, the overall cap is increased through banking because of (2.23). Hence the initially issued permits $s_0$ must be lower than the desired cap. This is achieved by discounting the

\(^{10}\)Note that since the available permits depend on the bank level, the temporal issuance of permits matters. For simplicity I assume that all permits are issued in the first period. However, other allocations that also lead to the first best are possible.
optimal periodic emissions with \((1 + r_b)\) in (2.24). As a result, the permit price always optimally equals the SCC, \(p_t = \tau_t\). Yet, this does not hold under uncertainty and if the regulator does not adjust the trading ratio in each period as I show with the numerical simulation in Appendix B.3 (see also Pizer and Prest 2019).

2.4. Clean Subsidy

In this section I analyze subsidies for clean energy as another policy alternative. The subsidy \(\rho_t\) is paid per unit of clean production and thus \(\rho_t = 0\) for the dirty firm. Profits are given by

\[
E_t [\pi_{it}] = \sum_{s=t}^{\infty} \frac{1}{(1 + r)^{s-t}} E_t \left[ (w_s + \rho_s) x_{is} - \alpha_{is} x_{is} - \frac{\beta_i}{2} x_{is}^2 \right].
\]

(2.25)

I can obtain optimal dirty and clean energy generation by following the same steps as under the tax regime in section 2.2,

\[
x_{d,t} = \frac{\beta_c a_t + \alpha_{c,t} - \rho_t - \alpha_{d,t} (1 + \beta_c b)}{\beta_c + \beta_d + \beta_c \beta_d b}
\]

(2.26)

\[
x_{c,t} = \frac{\beta_d a_t + \alpha_{d,t} + (\rho_t - \alpha_{c,t}) (1 + \beta_d b)}{\beta_c + \beta_d + \beta_c \beta_d b}.
\]

(2.27)

Using these reactions functions, welfare (as defined in equation (2.11)) is maximized again but this time via the subsidy level \(\rho_t\). This gives the following subsidy path,\(^{11}\)

\[
\rho_t = \tau_t \frac{\phi_d}{1 + \beta_d b} \quad \forall t \leq \hat{t}.
\]

(2.28)

The subsidy equals the SCC (or optimal tax) multiplied with the term \(\phi_d (1 + \beta_d b)^{-1}\). The emission factor \(\phi_d\) reflects that the subsidy is paid for avoided damages. If energy demand would not depend on the price, \(b = 0\), clean energy would perfectly replace dirty energy and hence one unit of clean energy would reduce damages by \(\tau_t \phi_d\). Hence for \(b = 0\) the subsidy is \(\rho_t = \tau_t \phi_d\), which can be used in (2.26) and (2.27) to show that the subsidy

\(^{11}\)Abrell et al. (2019) show that alternative financing rules can significantly increase welfare. However, to streamline the analysis I assume that the subsidy is financed lump sum by consumers.
induces the first best as it leads to the same productions levels as the tax in (2.7) and (2.8). However, with price sensitive demand, $b > 0$, one unit of clean energy replaces less than one unit of dirty energy because the subsidy reduces the energy price which triggers a higher demand. To counteract the resulting over-consumption, the subsidy is reduced by multiplying it with $(1 + \beta_d b)^{-1}$.

Consequently, by distorting energy prices the subsidy leads to inefficient low abatement levels for two reasons: first, the subsidy leads to over-consumption and more emissions even if the regulator would not account for the price distorting effect and set the subsidy to $\rho_t = \tau_t \phi_t$. Second, the regulator reduces the subsidy by the factor $(1 + \beta_d b)^{-1}$ to counteract the price distorting effect. Hence the price distorting effect directly reduces abatement through over-consumption and indirectly reduces abatement through the reduction of the subsidy by the regulator. Overall, there are more (cumulative) emissions and the transition length is longer under the subsidy compared to the tax, $\hat{t}_{sub} > \hat{t}_{tax}$.

Like the tax the subsidy rises at rate $g$ over time during the transition period, $t \leq \hat{t}$. After the transition, however, the optimal subsidy should be just as high to set dirty production to zero,

$$\rho_t = \beta_c a_t + \alpha_{c,t} - \alpha_{d,t} (1 + \beta_c b) \quad \forall \ t > \hat{t},$$

which I get by setting (2.26) to zero. If the subsidy would further rise at rate $g$, it would be higher than necessary to reach emission neutrality, which would be inefficient because the subsidy triggers over-consumption and thus reduces welfare. In fact, it would be welfare enhancing to prohibit dirty energy after the transition, $t > \hat{t}$, because it would also lead to zero emissions and in addition would avoid the inefficiencies due to over-consumption since the subsidy could be set to zero. Clearly, setting the optimal production levels via a command-and-control policy would always be welfare enhancing. Yet, one could argue that determining the optimal levels is difficult during the transition, but simple after the transition since the optimal dirty production is zero.
2.5. Cap-and-Trade and Clean Subsidy Combined

While the tax and, under certainty, the CAT program with intertemporal trading ratio can reach the first best solution, a clean subsidy or a CAT program without trading ratio perform worse. Yet, in combination a CAT program (without trading ratios) and subsidies perform better than in isolation which is the focus of this section.

The problem of the dirty firm is the same as in the CAT-only case of section 2.3 as it maximizes profits (2.16) subject to the banking constraint (2.15). The clean firm maximizes again profits (2.25) as in the subsidy-only case of section 2.4. Applying the same solution steps as in the CAT-only case yields:

$$x_{d,t} = \frac{\varphi_{t}^\text{sub}}{\phi_d} - (1 + r)^t \frac{\sum_{s=0}^{t} \varphi_s^\text{sub} - S}{\sum_{s=0}^{t} (1 + r)^s \phi_d} \quad \forall \ t \leq \hat{t} \quad (2.30)$$

$$x_{c,t} = \frac{a_t - (\alpha_{c,t} - \rho_t) b - x_{d,t}^*}{1 + \beta_{c}b} \quad \forall \ t \leq \hat{t}, \quad (2.31)$$

where I have adjusted the BAU emissions with the subsidy,

$$\varphi_t^\text{sub} = \frac{\phi_d (\beta_c a_t - \alpha_{d,t} (1 + \beta_{c}b) + \alpha_{c,t} - \rho_t)}{\beta_c + \beta_d + \beta_c \beta_d b}, \quad (2.32)$$

such that $\varphi_t^\text{sub}$ are emissions with the impact of the subsidy but without the effect of the permit price. Given the reaction functions (2.30) and (2.31) welfare can be maximized, while I again focus on the case of certainty in this analytical section. The regulator now has two choice variables, the cap $S$ and the subsidy $\rho_t$. Following the same procedure as in the CAT-only scenario (but with equations (2.30) - (2.32)) gives again,

$$S = \sum_{t=0}^{\hat{t}} \left( \varphi_t - \frac{a_t}{\varphi} \right)$$

as in the CAT-only case. This, however, does not imply that the subsidy does not affect the cap since the subsidy affects the transition length $\hat{t}$ as I show in the numerical simulation.

Before I analyze the optimal subsidies there are two remarks. First, after the transition, $t > \hat{t}$, optimal subsidies are always zero because periodic emission are zero since all permits of the CAT program are exhausted after $\hat{t}$. Subsidies would therefore only distort energy prices without having any impact on emissions. Hence due to the binding cap $S$
the problem to find optimal (second best) subsidies turns the infinite into a finite horizon setting until the end of the transition $\hat{t}$. Second, the interaction between the subsidy and the CAT program introduces a time consistency problem. If the regulator sets optimal subsidies in period $t$ for future periods $s > t$, she has incentives to deviate from this plan when arriving at $s$ which I explain in more detail below. I therefore analyze two cases: Under the commitment solution the regulator sets the entire subsidy path at the beginning of the first period once and for all and under time-consistency the regulator may adjust the subsidy in each period such that I get the Markovian policy. I refer to Appendix A.2 for first order conditions and solution steps.

2.5.1. Commitment Solution

In the commitment case welfare is maximized from perspective of the first period via all subsidies $\rho_t$ given (2.30) and (2.31), which yields,

$$\rho_t = \frac{\delta_0 \phi_d}{(1 + \beta_d b) \sum_{s=0}^{t} (1 + r)^s} \left( \sum_{s=t+1}^{t} \left( (1 + r)^s \sum_{s'=0}^{t-s} (1 + g)^{t-s'} \right) - \sum_{s=1}^{t} \left( (1 + r)^s \sum_{s'=1}^{s} (1 + g)^{t-s'} \right) \right) \quad \forall t \leq \hat{t}. $$

Similar as in the subsidy-only case the term $\delta_0 \phi_d (1 + \beta_d b)^{-1}$ reflects again that more avoided damages $\delta_0 \phi_d$ and a lower price elasticity of demand (lower $b$) justify higher subsidies and vice versa. The two terms in the brackets, however, imply that the subsidy does not rise at rate $g$ as in the subsidy-only case. The first term reflects the damages of all upcoming periods until the end of the transition $\hat{t}$ from the perspective of period $t$ and the second term reflects the damages of the present and past periods. Since the first term decreases and the second term increases over time as $t$ approaches $\hat{t}$, the subsidy declines over time and even becomes negative at the end of the transition.

The subsidy path can be explained by the interplay of the subsidy and the CAT program. First, the permit price in isolation induces a too steep carbon price (cp. 2.3) which is partly corrected by a declining subsidy. Taking the time derivative of the subsidy,
\[
\rho_t - \rho_{t-1} = \rho_{t-1} g - \frac{\delta_0 \phi_d}{1 + \beta_c b} (1 + r)^t \frac{\sum_{s=0}^{t} (1 + g)^s}{\sum_{s=0}^{t} (1 + r)^s} \quad \forall t \leq \hat{t}, \tag{2.34}
\]

shows that it grows with the SCC, as in the subsidy-only case, due to the first term and decreases due to the second term. The second term is always larger than the first term such that the subsidy strictly declines as explained above. In addition, the subsidy declines faster over time to counteract the faster increasing permit price (in absolute terms). Hence the subsidy enhances the abatement paths in the early phase by a positive and in the later phase by a negative subsidy to bring the paths closer to the optimal carbon price path.

In addition, equation (2.33) shows that the subsidy interacts with the CAT program in yet another way: all transition periods are connected via permit banking and therefore setting a subsidy in one transition period affects the permit price in all transition periods, ceteris paribus. This can be shown by writing the permit price as,

\[
p_t = (1 + r)^t \left( \frac{\sum_{s=0}^{t} \rho_s}{\sum_{s=0}^{t} (1 + r)^s} - \frac{\sum_{s=0}^{t} \rho_s}{\phi_d (1 + \beta_c b) \sum_{s=0}^{t} (1 + r)^s} \right) \quad \forall t \leq \hat{t}, \tag{2.35}
\]

and thus the price is a negative function of all subsidies. A positive subsidy in any transition period reduces the price in the same way since the subsidy reduces the (anticipated) demand for permits equally, irrespective of the period when the subsidy is paid. However, the opposite is true for negative subsidies. For the optimal subsidy path (2.33) I get the following result.

**Proposition 3.** In the commitment solution, optimal second best subsidies (2.33) do not affect the permit price.

Positive and negative subsidies are perfectly balanced such that they do not crowd out the permit price. This makes sense from a welfare perspective because the permit

---

\(^{12}\)This can be derived by using emissions, \(x_{d,t} \phi_d\) (with \(x_{d,t}\) from (2.30)) in the expression for the permit price \(p_t = \psi(\varphi_t^{sub} - S_t - b_{t-1} + b_t) = \psi(\varphi_t^{sub} - x_{d,t} \phi_d)\) while considering that \(S = \sum_{t=0}^{\hat{t}} (\varphi_t - \frac{\varphi_t}{\psi})\).
price is an efficient abatement instrument (within a period) whereas subsidies distort the energy price. The purpose of the clean energy support is to adjust the abatement path by shifting emissions and damages to the future while avoiding to affect the permit price. In doing so, the subsidy enhances welfare as summarized in the following proposition for the case with two transition periods.

**Proposition 4.** Given that the transition phase consists of two periods, \( \hat{t} = 0, 1 \), the welfare advantage of subsidies within a CAT program compared to CAT-only is

\[
W_{\text{cat+sub}} - W_{\text{cat}} = \frac{\delta_0^2 \phi_d^2 (1 + 2b (\beta_d + \beta_c + \beta_b \beta_d b)) (1 + r)}{2 (\beta_d + \beta_c + \beta_d \beta_c b) (1 + \beta_c b) (1 + \beta_d b)^2 (1 + 1 + r)} > 0 \tag{2.36}
\]

The welfare advantage is strictly positive and arises because damages are postponed. Yet, the first best cannot be achieved because subsidies still distort the energy price. Therefore, the welfare effect of subsidies also depends on technology and demand function parameters. Note this result also holds for longer transition phases but the expression for the welfare differences grows rapidly with more periods and the major insights do not change, see also Appendix A.2.2.

So far I assumed that the regulator is able to commit. However, the subsidy path (2.33) is not time consistent as the regulator wants to deviate from the subsidy path after the first period has passed. For example, in the last transition period \( \hat{t} \) the subsidy is negative according to (2.33). If the regulator could reset the subsidy at the beginning of this period, she would choose \( \rho_t = 0 \) because from the perspective of \( \hat{t} \) there is no value of a non-zero subsidy. Being in \( \hat{t} \) the problem simplifies to a single period problem with given cap since the permit supply \( s_t \) and the initial bank \( b_{t-1} \) are fixed. Hence any non-zero subsidy distorts prices because it cannot affect damages. In the following section I analyze the time consistent subsidy path.

2.5.2. Time Consistent Solution

The regulator maximizes welfare again but this time only via the current subsidy \( \rho_t \) which yields,
\[
\rho_t = \frac{(1 + \beta_d b) \delta \phi d \sum_{s=t+1}^t \left( (1 + r)^{s-t} \sum_{s'=0}^{t-s} (1 + g)^{t+s'} \right) + \sum_{s>t}^t \rho_s}{(\beta_d + \beta_c + \beta d\beta_c) b \sum_{s=0}^{t-t} (1 + r)^s + \sum_{s=1}^{t-t} (1 + r)^s} \quad \forall t < \hat{t}.
\] (2.37)

In contrast to the commitment solution, the time consistent subsidy is a Markov policy and thus does not depend on the past. Put differently, subsidies in \( t \) are only set to maximize welfare in \( s \geq t \). This implies that the relevant dates for the optimal subsidy level decrease as time \( t \) approaches to the end of the transition \( \hat{t} \). Hence there is no negative term in (2.37) which would reflect damages of past periods as in the previous case (2.33). Without negative term that grows over time, the Markovian subsidy declines at a lower rate compared to the commitment solution and never becomes negative. This reflects that it never makes sense to shift emissions from the future to the present as implied by negative subsidies.

Moreover, since subsidies are always positive they partly crowd out the permit price. That is, the second term in (2.35) is positive and thereby reduces the permit price in all periods. This explains why the time consistent subsidy is affected by the technology costs and not only by the consumption increasing effect, \( (1 + \beta d b)^{-1} \), as in the commitment case. The permit price reduction has an ambiguous effect on the subsidy level compared to the commitment subsidy: on the one hand, a lower permit price implies that subsidies need to be higher than in the commitment case to compensate the gap to the optimal abatement path. On the other hand, crowding out the permit price increases the inefficiency of the subsidy implying that it should be lower than the commitment subsidy. In the numerical simulation below I find a higher subsidy in the time consistent case which, however, leads to more emissions in the early phase of the transition due to the crowding out of the permit price.

Apart from these differences the Markovian policy has the same effects as the subsidy in the commitment solution. Both subsidies are positive in the beginning and decline over time in order to shift emissions and damages to the future such that the abatement path is closer to the first best path. The commitment solution leads to higher welfare than the Markovian subsidy from the perspective of \( t = 0 \) since it allows for a stronger
adjustment of the abatement path. Moreover, shifting emissions to the future leads to a prolonged transition length \( \hat{t} \) compared to the CAT-only case. Since the optimal cap

$$S = \sum_{t=0}^{\hat{t}} \left( \varphi_t - \frac{\psi_t}{2} \right)$$

increases with \( \hat{t} \), both subsidy regimes lead to a larger cap. Recall that the reason for the low cap in the CAT-only case is to partly compensate the too low permit price and thus too high damages early on (see 2.3). Since the subsidies counteract this problem the cap can be lifted compared to CAT-only. I analyze these aspects in more detail in the following numerical simulation.

### 3. Numerical Simulation

In this section I use numerical simulations to illustrate the analytical results of the previous section and to investigate the effects of uncertainty. In section 3.1 I explain how I calibrate the model to the electricity sector of the European Union. It is an interesting case because it is part of a cap-and-trade program, the EU ETS, and in addition subsidy schemes for renewable energies are in place. However, I do not explicitly model the EU ETS nor the country-specific renewable support. These policy instruments have specific characteristics which are out of scope of this paper. Instead, I am interested in idealized (second best) instruments which are based on the SCC as derived in the previous section.

#### 3.1. Model Calibration and Scenarios

<table>
<thead>
<tr>
<th>Table 3.1: Firm data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_i ) (EUR/kWh)</td>
</tr>
<tr>
<td>clean firm</td>
</tr>
<tr>
<td>dirty firm</td>
</tr>
</tbody>
</table>

Table 3.1 shows the assumed parameters for the clean and dirty sector. The parameters are based on the detailed numerical electricity sector model LIMES-EU. LIMES-EU optimizes the electricity sectors of 29 European countries until 2050 while taking short-term variability of demand and renewable energies (wind, solar), grid connection between countries and others into account (see Osorio et al. 2018 for more details). For estimating the production cost function of the clean sector I consider 15 emission-free technologies.
(renewable energies and nuclear) and for the dirty sector 21 technologies (mainly coal and gas) of LIMES-EU.\textsuperscript{13} The emission intensity of the dirty sector $\phi_d$ is assumed to be 650 g/kWh as compromise between coal (about 900 g/kWh) and gas (about 350 g/kWh) technologies.

I consider 100 periods in three year steps starting in the year 2020. The transition (endogenously) ends at the latest after 28 periods and the remaining periods are included to mimic the infinite horizon. Notice that only the SCC are directly affected by the time horizon and including more than 100 periods hardly affects the SCC. I set demand parameter $a_t$ to 4.4 PWh in the first model year (2020) and $b$ (time independent) to 15 (kWh reduction of demand per EUR). This leads to a realized demand between 3.3 PWh and 3.6 PWh in 2020 depending on the scenario, while actual electricity generation in the EU was 3.47 PWh in 2017 according to Eurostat. I further assume that $a_t$ increases by 0.5\% per year. These parameters imply an average (long-run) price elasticity of demand\textsuperscript{14} about -0.2 to -0.3 which is in line with recent empirical estimates (Deryugina et al. 2020). Furthermore, these assumptions lead to BAU emissions $\varphi_t$ of 1.5189 Gt in 2020 and the slope of the MAC curve $\psi$ is 0.0481. In case of uncertainty, I assume that energy demand parameter $a_t$ and thus BAU emissions are random, modeled as AR1 process, $a_t = \omega a_{t-1} + \epsilon_t$, with $\omega = 0.6$ and $\epsilon_t \in \{-\theta a_0, \theta a_0\}$. Hence there is either a positive or negative shock on demand parameter $a_t$ equal to $\theta a_0$. I compute two uncertainty scenarios: one with a relative weak $8\%$ shock every three years, $\theta = 0.08$, and one with a relative strong $12\%$ shock, $\theta = 0.12$. The first shock emerges in $t = 2023$ and, to keep the model computable, the last shock is in $t = 2044$.

According to data used by Dietz and Venmans (2019) the growth rate of the social cost of carbon $g$ is about 1.5\% to 3.3\%, van der Ploeg (2018) applies 2\% and Nordhaus (2017) 3\%. I assume 2.5\% for the growth rate of the social cost of carbon and 5\% for

\textsuperscript{13}In order to derive the cost functions I perform several LIMES simulations with increasing clean production shares. Per simulation, this results in total clean and dirty costs (sum of 15 and 21 technologies, respectively) as well as total clean and dirty production levels. The cost function parameters are derived from fitting these costs and production data.

\textsuperscript{14}The price elasticity of demand in the model is $\varepsilon_t = \frac{dD_t}{dw} \frac{w_t}{D_t} = -\frac{bw_t}{a_t - bw_t}$. I take the average over the model periods since the elasticity increases with price $w_t$ and decreases with parameter $a_t$. 

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the discount rate. Initial marginal damages are assumed to be $\delta_{2020} = 0.00201$ such that according to equation (2.12) the social costs of carbon are 30 EUR/t in 2020. This is roughly in line with SCC estimates from the literature (e.g. Nordhaus 2017; van der Ploeg 2018).

I consider the six policies analyzed above: a tax, which serves as optimal benchmark, a CAT program with and without intertemporal trading ratio (ITR), a subsidy and a CAT program combined with a subsidy in the commitment (indicated by a 'C') and time consistent (indicated by a 'T') case.\(^{15}\) I first show the results for the model under perfect foresight, and thereafter analyze the effects of abatement cost uncertainty.

3.2. Transition Under Certainty

The first best transition is achieved with the tax and with the CAT program plus ITR because in both cases the policy induced carbon price is always equal to the SCC. All other instruments are imperfect and imply a welfare loss compared to the first best as depicted in Table 3.2 (a). While the highest welfare loss is induced if only a subsidy is implemented (250.91 billion EUR), the second worst alternative is the CAT-only policy with free intertemporal trading (no ITR). However, CAT-only is with a welfare loss of 24.64 billion EUR much more efficient than the subsidy. Hence the inefficient permit price growth rate of the CAT program weighs less than the energy price distorting effect of the subsidy (cp. section 2.4). Furthermore, combining the CAT program with a subsidy reduces the welfare loss to 9.81 and 13.59 billion EUR under the commitment and time consistent subsidy, respectively. That is, by shifting emissions to the future within the CAT program, subsidies significantly reduce damages early on and thus enhance welfare. Put differently, the subsidies reduce the cost of delay caused by the CAT-only policy.

The welfare effects can be explained by the different abatement and carbon price

\(^{15}\) The model is solved using the software GAMS as non-linear program (NLP) while I maximize welfare subject to the first order conditions of the firms as defined above. In CAT scenarios the cap is always exogenous and adjusted between model runs until the welfare optimum is found. However, the scenario with CAT combined with the time consistent subsidy is solved as extended mathematical programming (EMP) in which only the firm profits are maximized. In this case the subsidy is also exogenous to the model and set according to the level defined in section 2.5.2 to account for time consistency. The code is available upon request.
paths engendered by the policy instruments as shown in Figure 3.1. For reasons of comparability the subsidies as shown in part (c) of the Figure are converted into an implicit carbon price used to calculate the carbon prices in part (b). This implicit price is the carbon price that leads to the same abatement as the subsidy, which I get by setting dirty production in the tax and subsidy case, equations (2.7) and (2.26), equal and solve for the tax \( \tau_t = \rho_t (\phi_d (1 + \beta_c b))^{-1} \). In the two policy mix scenarios, this implicit carbon price is added to the permit price which yields the total carbon price. Further note that after the transition is followed through, carbon prices in all cases are just as high as to guarantee zero emissions.\(^{16}\) This implies that prices rise linearly after the transition along the increasing marginal abatement costs due to growing energy demand.

In the CAT-only scenario the first best cannot be obtained because the permit price rises at the rate of interest, \( r = 5\% \), but it should rise with the SCC, \( g = 2.5\% \). This implies that the abatement path in CAT-only case is too steep (see Figure 3.1): emissions are too high early on due to the low permit price, but decline quickly such that the entire transition only takes 36 years compared to 54 years under the first best. Adding subsidies to the CAT program adjusts the growth rate of the total carbon price (permit price + implicit carbon price of the subsidy). Under commitment the total carbon price rises at 3.2\% and thus it is significantly closer to the optimum of 2.5\%. The Markovian subsidy implies a somewhat higher growth rate of 3.5\% because time consistency is taken into account (cp. section 2.5.2). As a result, the abatement and carbon price paths of the policy mixes (‘CAT+subsidy C’ and ‘CAT+subsidy T’) lie in between the first best and the CAT-only case.

The reduction of the growth rate of the total carbon prices in the two policy mix scenarios can be explained by the falling subsidies over time as depicted by the dashed and dotted line in Figure 3.1 (c). While both subsidy paths start at a comparable level, the commitment subsidy (‘CAT+subsidy C’) declines faster and turns negative in the mid 2040s, implying that the growth rate of the total carbon price is lower than under

\(^{16}\)The carbon price that sets emissions to zero is \( \tau_t = \varphi_t \psi \) which I get from setting (2.7) to zero and solving for the tax while considering (2.9) and (2.10).

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time consistency (3.2% vs 3.5%). The negative subsidy is also the reason why there is no waterbed effect in the commitment solution. In contrast, the waterbed effect reduces the CAT price by 2.66 EUR/t in 2020 under time consistency.\textsuperscript{17}

Cumulative emissions also exhibit large differences between the policy instruments (see Table 3.2 (b)). The largest amount of cumulative emissions is caused by the subsidy-only policy (57.36 Gt) because the subsidy distorts energy prices and leads to over-consumption. In the CAT-only scenario, cumulative emissions (i.e. the cap) are much lower (27.15 Gt) and essentially also significantly lower than under the first best solution (31.84 Gt). The regulator chooses a lower cap to reduce the inefficient high damages in the early phase of the transition due to the too low permit price (cp. section 2.3). By adding subsidies to the CAT program, emissions and thus damages are shifted to the future such that, compared to CAT-only, the cap can be increased to 29.73 Gt and 28.55 Gt under the commitment and time consistent subsidy, respectively. That is, subsidies do not directly affect cumulative emissions, but they influence the regulator’s choice of the cap and hence indirectly increase cumulative emissions.

\textsuperscript{17}In order to calculate the permit price reduction of the waterbed effect I use the cap of the ‘CAT+subsidy T’ scenario but set the subsidy to zero. The difference in the permit price between this and the ‘CAT+subsidy T’ scenario is attributed to the waterbed effect of the subsidy.
Figure 3.1: Emission, carbon price and subsidy paths under certainty

(a) Emissions

(b) Carbon prices

(c) Subsidies

Note: The lines for 'CAT+subsidy C' and 'CAT+subsidy T' in part (b) reflect the sum of the permit and implicit carbon prices of the subsidy.
Table 3.2: Welfare loss, cumulative emissions and transition length

<table>
<thead>
<tr>
<th></th>
<th>tax</th>
<th>CAT</th>
<th>CAT+ITR</th>
<th>subsidy</th>
<th>CAT+sub C</th>
<th>CAT+sub T</th>
</tr>
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<tbody>
<tr>
<td><strong>(a) Welfare loss (billion EUR)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>no shock</td>
<td>-</td>
<td>24.64</td>
<td>-</td>
<td>250.91</td>
<td>9.81</td>
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<tr>
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<td>-</td>
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<td>11.27</td>
<td>249.62</td>
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<tr>
<td>12% shock</td>
<td>-</td>
<td>46.83</td>
<td>24.98</td>
<td>247.89</td>
<td>31.84</td>
<td>35.34</td>
</tr>
<tr>
<td><strong>(b) Cumulative emissions (Gt)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no shock</td>
<td>31.84</td>
<td>27.15</td>
<td>31.84</td>
<td>57.36</td>
<td>29.73</td>
<td>28.55</td>
</tr>
<tr>
<td>8% shock</td>
<td>32.53</td>
<td>26.88</td>
<td>32.20</td>
<td>57.95</td>
<td>29.76</td>
<td>29.67</td>
</tr>
<tr>
<td>12% shock</td>
<td>33.44</td>
<td>27.57</td>
<td>32.68</td>
<td>58.73</td>
<td>30.30</td>
<td>30.22</td>
</tr>
<tr>
<td><strong>(c) Transition length (years)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no shock</td>
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<td>36.00</td>
<td>54.00</td>
<td>72.00</td>
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<td>8% shock</td>
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<td>53.07</td>
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<td>12% shock</td>
<td>51.65</td>
<td>35.79</td>
<td>53.12</td>
<td>71.31</td>
<td>41.80</td>
<td>43.36</td>
</tr>
</tbody>
</table>

**Note:** Welfare is always maximized with the tax because of the assumed constant marginal damages. Welfare losses in the other cases are the differences to the tax scenario.

### 3.3. Transition Under Uncertainty

Uncertainty generally elevates the welfare advantage of price based policies because of the assumed constant marginal damages. This is why the tax is always optimal and the welfare disadvantage of the subsidy does not increase with uncertainty, whereas scenarios with CAT exhibit an increasing welfare loss with higher uncertainty (see Table 3.2 (a)). Moreover, the hybrid policies consisting of CAT and subsidy also do not perform better under uncertainty than the CAT-only scenario (welfare losses in all CAT scenarios increase about the same magnitude when a shock is added). Indeed, under uncertainty subsidies lead to welfare advantages and but also disadvantages if added to the CAT program.

An advantage is that subsidies reduce the total carbon price volatility as shown in Figure 3.2. Therefore, the deviations from the optimal carbon price due to shocks are reduced which is welfare enhancing. This even holds for the commitment subsidy (‘CAT+subsidy C’) although the entire subsidy path is determined in the first period and it is not state-contingent. That is, the subsidy is a fixed term which would not affect the carbon price.
volatility if the transition length would be given. However, the subsidy shifts emissions to the future and thereby prolongs the transition or banking phase by about six years compared to CAT-only (see 3.2 (c)). A longer banking phase reduces the impact of shocks on the permit price because shocks are spread over to more periods (Fell et al. 2012) and therefore the subsidy indirectly reduces the permit price volatility. Yet, committing to a subsidy path has the disadvantage of a lower flexibility. Put differently, the regulator does not react to new information which causes a welfare loss that offsets the advantage of more stable carbon prices. The initially set subsidy path is a compromise between a potentially shorter or longer transition phase: Compared to the certainty scenario (see Figure 3.3), this explains why the subsidy is less negative before 2060 since non-zero subsidies induce a welfare loss if emissions are already zero. It also explains why the subsidy is more negative after 2060 because from the perspective of the first period there is a positive probability that the transition is still ongoing.

![Figure 3.2: Carbon price variance](image)

Note: The figure depicts the variance from the perspective of the first period until the period in which the last shock emerges for the 12% shock case. There is no difference between the variance of the CAT price and the total carbon price in the scenario 'CAT+subsidy C' because the subsidy is fixed, see footnote 18.

---

To see why consider the simplest case with only one period. The total carbon price is equal to the permit price plus the implicit carbon price of the subsidy, \( I = p + \rho (\phi_d (1 + \beta_c b)^{-1}) \). The second term is fixed in the commitment case and thus does not affect the volatility of \( I \). The permit price is equal to the marginal abatement costs, \( p = \psi (\varphi^a - S) \) while \( \psi \) and \( S \) are fixed and \( \varphi^a \) are random BAU emissions. Since the subsidy \( \rho \) is not random and additive to the shock on energy demand parameter \( a \) (see (2.32)), the subsidy does not affect the price variance, which is \( Var[p] = \left(\frac{\psi \varphi_d}{\beta_s + \beta_d + \beta_c b}\right)^2 Var[\epsilon] \) with \( \epsilon \) as shock on the demand parameter \( a \).
In contrast, under time consistency the regulator has the opportunity to react to shocks. Therefore, she continues the subsidy program after she learned that the transition is ongoing and she sets subsidies to zero after she learned that the transition ends in the upcoming period. The subsidy is raised when new information imply that the transition takes more time and vice versa because a longer (shorter) transition allows to shift more (less) emissions to the future. This implies that the correlation between the subsidy and the permit price is negative: a positive abatement cost shock increases (expected) emissions in the current and future periods such that more permits are needed and the expected transition length is shorter than expected before the shock occurred. Hence if abatement costs and thus permit prices are high, subsidies are low because the transition phase is expected to be short and vice versa. Put differently, subsidy updating counteracts the permit price shocks such that the total carbon price volatility is much lower (see Figure 3.2) which stabilizes the abatement path. Moreover, the regulator prolongs the expected transition length if she can update the subsidy levels. On average she raises the subsidy more after abatement costs decline than she reduces it after abatement costs increase. This is welfare enhancing because under CAT the transition length is too short and thus on average higher subsidies bring it closer to the optimum (see Table 3.2 (c)).

However, although subsidy updating significantly reduces the total carbon price volatility and prolongs the transition length, it is not welfare enhancing under uncertainty com-
pared to the commitment solution. The reason is that subsidy updating significantly increases the (expected) subsidy level as shown in Figure 3.3. This reduces the permit price through the waterbed effect: the price reducing effect of the subsidy is 3.36 EUR/t in 2020 under the 12% shock vs. 2.66 EUR/t under certainty. Hence the subsidy takes on a stronger role under uncertainty relative to the permit price which is inefficient because of the energy price distorting effect. In consequence, the opportunity to react to shocks may increase or decrease welfare compared to the commitment solution, while in this simulation effects roughly cancel out, indicated by similarly increasing welfare losses due to uncertainty in Table 3.2 (c). For further simulation results on the ITR and cumulative emissions under uncertainty see Appendix B.

4. Conclusion

In this paper I analyze the interaction of subsidies for clean energy and a cap-and-trade (CAT) program with free intertemporal trading. I show that optimally set subsidies enhance welfare if added to such a CAT program, even if there is only the carbon externality. The reason is that the permit price of the CAT program never equates the social costs of carbon (SCC) because the permit price rises at a higher rate than the SCC. Hence the permit price is initially too low implying excessive emissions and damages early on. A subsidy partly corrects this flaw by shifting emissions to the future such that the costs of delayed action are reduced.

I derive the optimal subsidy path which strictly declines over time if a CAT program is in place as well. Since the subsidy even becomes negative at the end of the transition it has no effect on the permit price. Hence the ideal waterbed effect of the subsidy is permit price preserving and only shifts emissions to the future. Yet, such a subsidy path is unlikely to be implemented in reality. For one, it is not time consistent as the regulator has incentives to deviate from subsidies that were scheduled in the past. Moreover, negative subsidies for clean energy seem to be unrealistic especially from a political perspective. Therefore, I also derive the time consistent subsidy path which never turns negative. The drawback is that the time consistent subsidy partly crowds out the permit price.
However, welfare is still significantly higher compared to CAT-only.

In the numerical simulation of the EU electricity sector I show that subsidies within a CAT program cause further advantages but also disadvantages if abatement cost uncertainty is considered. On the one hand, subsidies reduce the permit price volatility because the transition or banking phase is prolonged implying that shocks spread over to more periods. Especially if the regulator can react to new information in the time consistent solution, she counteracts the permit price development with higher or lower subsidies such that the (implicit) total carbon price is relative stable. On the other hand, I find that the price reducing effect of the subsidy is exacerbated under uncertainty if the regulator can react to shocks in the time consistent case. The inability to react to shocks (commitment solution) in turn, implies that the initially set subsidy path is a compromise between a potentially longer or shorter transition phase which induces some welfare losses. Yet, in the numerical simulation subsidies lead to comparable welfare advantages as under certainty if added to the CAT program.

These findings suggest that the CAT and subsidy policy mix may be better than its reputation in the economic literature. Clearly, it is only a second-best solution which falls behind idealized hybrid instruments as CAT with price collars. However, the widespread implementation of the CAT and subsidy mix indicates its high political feasibility. Hence as long as price floors cannot be implemented at a reasonable level and permit prices are far below the SCC, subsidies can in principle be a useful addition to a CAT program even if there are no other market failures.

Having said that, implementing subsidies in an welfare enhancing way is not without problems. For one, subsidies should reflect avoided damages, besides potential other market failures, which hardly seems to be the case in practice. Abrell et al. (2019) show that this requires technology differentiated subsidies in electricity markets because technologies (e.g. wind, photovoltaics) have different production profiles. Yet, falsely set subsidies contain the risk of an excessive reduction of the permit price and thereby subsidies may undermine the relevance and credibility of CAT programs (Fankhauser et al. 2010). Future research could analyze which specific support schemes (e.g. renewable
quotas or feed-in tariffs) are best suited to complement a CAT program and avoid adverse
effects on the permit price. In addition, the analysis of this paper builds upon several
simplifying assumptions as I ignore borrowing constraints in the permit market or other
market failures as myopia or market power that may play an important role in real
markets, all of which are interesting avenues for future research in this field.

Acknowledgments

The author would like to thank Ottmar Edenhofer, Marina Friedrich, Christian Gamb-
bardella, Samuel Okullo, Sebastian Osorio and Michael Pahle for valuable comments.
This work was funded by the European Union’s Horizon 2020 research and innovation
program under grant agreement No 730403 (INNOPATHS).

Appendix A. Derivations

Appendix A.1. Cap-and-Trade

The Bellmann equation of the firm problem described in section 2.3 is

\[
V_{it}(b_{it}) = \max_{x_{it},y_{it}} \left( w_{it}x_{it} - \alpha_{it}x_{it} - \beta_{it}^2 x_{it}^2 - p_{it}y_{it} \right) + \frac{1}{1+r} E_t \left[ V_{it+1}(b_{it+1}) \right],
\]

(A.1)

subject to the banking constraint (2.15). The first order conditions are:

\[
w_{it} - \alpha_{it} - \beta_{it} x_{it} - \frac{\phi_d}{1+r} E_t \left[ \frac{\partial V_{it+1}}{\partial b_{it+1}} \right] = 0 \quad (x_{it} \geq 0)
\]

(A.2)

\[
p_{it} - \frac{1}{1+r} E_t \left[ \frac{\partial V_{it+1}}{\partial b_{it+1}} \right] = 0 \quad (y_{it})
\]

(A.3)

\[
\frac{\partial V_{it}}{\partial b_{it}} - \frac{1}{1+r} E_t \left[ \frac{\partial V_{it+1}}{\partial b_{it+1}} \right] = 0 \quad (b_{it})
\]

(A.4)

Using (A.2) and using (A.3) yields the production function \( x_{it} = (w_{it} - \alpha_{it} - p_{it}\phi_i) \beta_i^{-1} \).

Inserting the production functions of both firms into the energy market equilibrium (2.4)
yields the energy price \( w_t(p_t) \) as in (2.6) but with \( p_t \) instead of \( \tau_t \). Inserting the energy price into the production functions gives production depending on the permit price,

\[
x_{d,t} = \frac{\beta_d a_t + \alpha_{c,t} - (\alpha_{d,t} + p_t \phi_d) (1 + \beta_c b)}{\beta_c + \beta_d + \beta_c \beta_d b}, \quad (A.5)
\]

\[
x_{c,t} = \frac{\beta_d a_t + \alpha_{d,t} + p_t \phi_d - \alpha_{c,t} (1 + \beta_d b)}{\beta_c + \beta_d + \beta_c \beta_d b}. \quad (A.6)
\]

These expressions can now be inserted in the permit equilibrium, \( s_t = \sum_{i} y_{it} = \phi_d X_t + B_t - B_{t-1} \) which yields,

\[
p_t = \frac{\beta_d + \beta_c + \beta_d \beta_d b}{\phi_d^2 (1 + \beta_c b)} \left( \frac{\phi_d (\beta_c a_t - \alpha_{d,t} (1 + \beta_d b) + \alpha_{c,t})}{\beta_c + \beta_d + \beta_c \beta_d b} - s_t - B_{t-1} + B_t \right). \quad (A.7)
\]

By using the expressions for BAU emissions \( \varphi_t \) and the slope of the marginal abatement cost curve \( \psi \) from (2.9) and (2.10), the price (A.7) can be written as \( p_t = \psi (\varphi_t - s_t - B_{t-1} + B_t) \). I get the same result by minimizing (2.1) in (A.1) subject to (2.15) and hence the two problems are identical.

Combining (A.3) and (A.4) yields the intertemporal price dynamics shown in (2.18) and thus the permit price rise at rate \( r \). Inserting the permit price \( p_t = \psi (\varphi_t - s_t - B_{t-1} + B_t) \) in the expression for the intertemporal price dynamics (2.18) gives the permit bank,

\[
B_t = \frac{R (s_t + B_{t-1} - \varphi_t) + E_t [\varphi_{t+1} + B_{t+1}]}{1 + (1 + r)} \quad \forall t \leq \hat{t}. \quad (A.8)
\]

Once the budget is used up the bank is empty, that is \( B_t = 0 \) for all \( t \geq \hat{t} \). Inserting \( B_{\hat{t}} = 0 \) for \( B_{\hat{t}+1} \) in (A.8), gives the last bank before it becomes zero, \( B_{\hat{t}-1} \), which only depends on parameters. Inserting \( B_{\hat{t}-1} \) in \( B_{\hat{t}-2} \) and so forth until the first period is reached gives the whole banking path depending only on parameters as shown in (2.19) under certainty. Note that uncertainty makes this expression complicated because it implies that the transition length \( \hat{t} \) is random which in turn affects the desired bank. I therefore focus in the analytical part on the case of certainty and analyze effects of uncertainty in
the numerical simulations.

Next using (A.7) in (A.5) yields dirty energy generation as 
\[ x_{d,t} = (s_t - B_t + B_{t-1}) \phi_t^{-1}. \]
Inserting (2.19) shows that dirty production is (2.20). For the clean production I similar use (A.7) in (A.6) and insert (2.19) for the bank which yields (2.21).

The reaction functions (2.20) and (2.21) can be used in the welfare function 
\[ W = \sum_{t=0}^{\infty} (1 + r)^{-t} (CS_t - \sum_i C_{it} - \delta_t \Phi_t) \]
which is maximized via the cap \( S \) and subject to (2.2). This yields the optimal cap (2.22).

Appendix A.1.1. Proof of Lemma 1

Total emission under the CAT program are given by the total cap, 
\[ S = \sum_{t=0}^{\hat{t}_{cat}} (\varphi_t - \frac{\tau_t}{\psi}). \]
If the tax is implemented, the periodic production \( x_{d,t} \) is given by (2.7) and emissions are \( x_{d,t}\phi_d \). By using the expression for the BAU emissions (2.9) and the slope of the marginal abatement cost curve (2.10) in (2.7), multiplying with \( \phi_d \) and taking the sum over \( t \) until the transition period ends in \( \hat{t}_{tax} \), I get cumulative emissions under the tax regime, 
\[ \sum_{t=0}^{\hat{t}_{tax}} x_{d,t}\phi_d = \sum_{t=0}^{\hat{t}_{tax}} (\varphi_t - \frac{\tau_t}{\psi}). \]
Comparing both expressions for cumulative emissions implies that they differ only with respect to the transition lengths \( \hat{t} \). It follows that only if transition periods under the tax and CAT program are of equal length \( \hat{t}_{tax} = \hat{t}_{cat} \), total emissions are equal as well.

Appendix A.1.2. Proof of Proposition 1

In order to show that the CAT program is not efficient it requires to show that the permit price \( p_t \) is not equal to the tax (SCC) in all periods of the transition. By noting that emissions under the CAT regime can be written as 
\[ x_{d,t}\phi_d = s_t + B_{t-1} - B_t, \]
I can insert dirty energy generation \( x_{d,t}\phi_d \) in the permit price (2.17) while using \( x_{d,t} \) from (2.20). Further inserting the optimal cap (2.22) gives the permit price as
\[ p_t = \frac{(1 + r)^{\hat{t}} \sum_{s=0}^{\hat{t}} \tau_t}{\sum_{s=0}^{\hat{t}} (1 + r)^s}. \tag{A.9} \]

It follows that the permit price is not equal to the SCC \( \tau_t \) in all periods and therefore the CAT program is not efficient.
Appendix A.1.3. Extension by Intertemporal Trading Ratio

The only difference to the previous problem is the adaptation of the permit banking equation by the intertemporal trading ratio \((1 + r_b)\) as given by (2.23). The first order conditions for the firms remain unchanged apart from the intertemporal condition which becomes

\[
\frac{\partial V_{it}}{\partial b_{it}} - \frac{1 + r_b}{1 + r} E_t \left[ \frac{\partial V_{it+1}}{\partial b_{it+1}} \right] = 0 \quad (b_{it}), \quad (A.10)
\]

and therefore the growth rate of the permit price is

\[
\frac{r - r_b}{1 + r_b} = \frac{E_t [p_{t+1} - p_t]}{p_t}. \quad (A.11)
\]

I get the permit price again by inserting the production functions in the permit equilibrium, \(p_t = \psi (\varphi_t - s_t - b_{t-1} (1 + r_b) + b_t)\). Using this price in (A.11) yields the permit bank,

\[
B_t = \sum_{s=t}^{i} \frac{(1 + r_b)^{2t-s-t} (\varphi_s - s_s) + (B_{t-1} (1 + r_b) + s_t - \varphi_t) (1 + r_b)^{2(t-s)} (1 + r)^{s-t}}{\sum_{s=t}^{i} (1 + r_b)^{2(t-s)} (1 + r)^{s-t}}.
\]

This expression for the permit bank can again be used in the production functions (A.5) and (A.6) where \(p_t\) is replaced by \(p_t = \psi (\varphi_t - s_t - b_{t-1} (1 + r_b) + b_t)\). Based on the resulting reaction functions and assuming that all permits are issued in the first period the regulator maximizes welfare via the cap, which yields (2.24).

Appendix A.1.4. Proof of Proposition 2

In order to reach the first best the permit price must be equal to the SCC, and therefore it must rise at rate \(g\). From (A.11) follows that the intertemporal trading ratio must be equal to \((1 + r_b) = (1 + r) (1 + g)^{-1}\) such that the permit price rises at rate \(g\). Inserting the expressions for the bank (A.12), the cap (2.24) and the trading ratio \((1 + r_b) = (1 + r) (1 + g)^{-1}\) into the permit price, \(p_t = \psi (\varphi_t - s_t - b_{t-1} (1 + r_b) + b_t)\),
shows that the permit price is equal to the SCC, \( p_t = \tau_t \) and therefore the CAT program is, under certainty, optimal and leads to the same result as the tax.

**Appendix A.2. Clean Subsidy and Cap-and-Trade**

Assuming the regulator is able to commit, she maximizes welfare \( W_0 (\rho_t) \) via all subsidies \( \rho_0, ..., \rho_t, ..., \rho_i \) at the beginning of the first period. The first order condition is,

\[
\frac{dW_0}{d\rho_t} = \sum_{s=0}^{i} (1 + r)^{-s} \left( \frac{dCS_s}{d\rho_t} - \frac{dC_{d,s}}{d\rho_t} - \frac{dC_{c,s}}{d\rho_t} - \frac{d\Phi_s}{d\rho_t} \delta_s \right) = 0, \tag{A.13}
\]

where

\[
\frac{dCS_s}{d\rho_t} = -\frac{dw_t}{d\rho_t} bw_t \tag{A.14}
\]

\[
\frac{dC_{is}}{d\rho_t} = \frac{dx_{is}}{d\rho_t} (\alpha_{is} + \beta_i x_{is}) \tag{A.15}
\]

\[
\frac{d\Phi_s}{d\rho_t} = \sum_{s'=-0}^{s} \frac{dx_{d,s'}}{d\rho_t} \phi_d \tag{A.16}
\]

with \( x_{d,t} \) and \( x_{c,t} \) given by (2.30) and (2.31) and

\[
w_t = \frac{\beta_c \beta_d a_t + \beta_c (\alpha_{d,t} + p_t \phi_d) + \beta_d (\alpha_{c,t} - \rho_t)}{\beta_c + \beta_d + \beta_c \beta db}, \tag{A.17}
\]

while the permit price \( p_t \) is (2.35). Inserting all expressions in (A.13) finally yields (2.33).

In the time consistent case the regulator maximizes welfare via the subsidy \( \rho_t \) in each period. The first order condition becomes

\[
\frac{dW_t}{d\rho_t} = \sum_{s=t}^{i} (1 + r)^{-s} \left( \frac{dCS_s}{d\rho_t} - \frac{dC_{d,s}}{d\rho_t} - \frac{dC_{c,s}}{d\rho_t} - \frac{d\Phi_s}{d\rho_t} \delta_s \right) = 0 \tag{A.18}
\]

while all other expressions still apply. Hence the difference to the commitment solution is that the regulator only considers present and future impacts of the subsidy whereas under commitment subsidies in \( t \) also optimize welfare in \( s < t \).
Appendix A.2.1. Proof of Proposition 3

In the commitment solution the subsidy is given by equation (2.33). Inserting this subsidy into the permit price (2.35) shows that for any $t$ and $\hat{t}$ the second term in (2.35) is always zero,

$$\sum_{s=0}^{\hat{t}} \rho_s \phi_d (1 + \beta d b) \sum_{r=0}^{t-r} (1 + r)^r = 0,$$

and therefore the permit price is not affected by the subsidy.

Appendix A.2.2. Proof of Proposition 4

Using abatement cost function (2.1), welfare can be written as

$$W = \sum_{t=0}^{\infty} (1 + r)^{-t} \left(-\frac{\psi}{2} (\varphi_t - e_t)^2 - \delta_t \Phi_t\right),$$  \hspace{1cm} (A.19)

with cumulative emissions $\Phi_t = \Phi_{t-1} + e_t$ and periodical emissions $e_t = \phi_d x_{d,t}$. In the CAT-only scenario $x_{d,t}$ is given by equation (2.20) and in the combined CAT and subsidy scenario by (2.30). Next I insert the optimal policy instrument levels in the respective $x_{d,t}$. First, the optimal cap in both cases is

$$S = \sum_{t=0}^{1} \left(\varphi_t - \frac{\varphi_t}{\psi}\right)$$

because by assumption the transition phase consists of two periods, $\hat{t} = 0, 1$. Second, for emissions in the combined case I insert the optimal subsidies according to equation (2.33) which are

$$\rho_0 = \frac{\delta_0 \phi_d (1 + r)}{(1 + \beta d b) (1 + 1 + r)},$$ \hspace{1cm} (A.20)

$$\rho_1 = -\frac{\delta_0 \phi_d (1 + r)}{(1 + \beta d b) (1 + 1 + r)},$$ \hspace{1cm} (A.21)

if the transition takes two periods and the regulator can commit. Finally, the welfare difference between the combined policy and the CAT-only policy, $W_{\text{cat+sub}} - W_{\text{cat}}$, can be calculated which yields expression (2.36) that is positive because all parameters of the expression are positive. Note that it suffices to consider only welfare within the transition phase, $t = 0, 1$, because for $t > \hat{t}$ market outcomes in both policy cases are identical.

Doing the same steps for a three period transition phase gives
\[ W_{cat+sub} - W_{cat} = \]
\[
\frac{\delta_0^2 \phi_2^2 (1 + 2b(\beta_d + \beta_c + \beta_d \beta_c b)) \left( (1 + r)^2 + (1 + (1 + g))^2 (1 + r) + (1 + g)^2 \right)}{2 (\beta_d + \beta_c + \beta_d \beta_c b) (1 + \beta_c b) (1 + \beta_d b)^2 (1 + 1 + r + (1 + r)^2)} > 0. \]

The differences are the growing expressions on the right which reflect that damages from more periods needed to be considered. Since there are no main insights to gain and the expression grows rapidly with more periods, I do not include cases with more periods.
Appendix B. Additional Simulation Results for Uncertainty Scenarios

Appendix B.1. Emission and Carbon Price Path

Figure B.1: Emission and carbon price paths under uncertainty (12% shock)

(a) Emissions

(b) Carbon prices

Note: The lines for 'CAT+subsidy C' and 'CAT+subsidy T' in part (b) reflect the sum of the permit and implicit carbon prices of the subsidy.

Appendix B.2. Cumulative Emissions

Cumulative emissions are in all cases but one ('CAT' with 8% shock) higher due to uncertainty (see Table 3.2), which can be explained by the zero lower bound of emissions at the end of the transition. To see this, first consider a price policy (tax or subsidy) in which the bound implies that negative shocks, that would lead to less emissions, have no or a weaker effect if emissions are already zero or close to zero. In contrast, positive shocks
have the full emission increasing effect if emissions are (close to) zero, thus inducing larger overall (expected) emissions.

Under CAT there are two effects on the cumulative emissions: first, for a given cap the expected transition length is always shorter than the transition length under certainty (not shown. Note that the values in Table 3.2 (c) are for adjusted caps as shown in panel (b) of the Table). This can be traced back to the concavity of the emission paths that results from exponentially increasing permit prices, implying an asymmetric impact of shocks on the transition length. Since the optimal cap declines with a shorter transition length (cp. equation (2.22)), this first effect leads to less cumulative emissions. The second effect increases the optimal cap: if there is a positive probability that the bank is depleted in the next period, the expected permit price generally rises at a lower rate than the interest rate because of the convenience yield (e.g. Schennach 2000). Hence from the first period onward in which the transition could end (i.e. the bank is depleted), the expected growth rate of the price is lower than under certainty, i.e. lower than $r$. For a given cap this implies lower expected prices and more emissions later on. In turn, price are higher early on due to intertemporal trading. The resulting lower emissions (and damages) in the beginning allow to increase the cap similar to the case when subsidies are added to a CAT program (see previous section). The first effect dominates for the 8% shock in the CAT-only scenario and therefore cumulative emissions are lower (see Table 3.2), while for the 12% shock the second effect dominates.

Appendix B.3. Intertemporal Trading Ratio

While a CAT program with ITR restores the first best under certainty, it performs worse with increasing uncertainty and essentially, the welfare loss increases faster than in the CAT and subsidy cases. Hence subsidies may outperform ITR with increasing uncertainty, yet not given the parameter assumptions in this simulation. The reason for the increasing inefficiency of ITR is that uncertainty causes the expected permit price to grow at a rate that declines over time because of the convenience yield (see section 3.3). In response to the lower average permit price growth rate, the regulator also reduces the ITR slightly to 2.2% and 2.3% (compared to 2.5% under certainty) given the 8% and
12% shock, respectively, because a lower ITR increases the growth rate (see Appendix A.1.3). This, however, cannot restore the optimal abatement path because the permit price growth rate is time-variable. Since the fixed ITR is too low for the early phase and too high for the later phase, the permit price is too low in the beginning and too high later on (see Figure B.1).

References


