Adaptive bridge control strategy for opinion evolution on social networks

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In this paper, we present an efficient opinion control strategy for complex networks, in particular, for social networks. The proposed adaptive bridge control (ABC) strategy calls for controlling a special kind of nodes named bridge and requires no knowledge of the node degrees or any other global or local knowledge, which are necessary for some other immunization strategies including targeted immunization and acquaintance immunization. We study the efficiency of the proposed ABC strategy on random networks, small-world networks, scale-free networks, and the random networks adjusted by the edge exchanging method. Our results show that the proposed ABC strategy is efficient for all of these four kinds of networks. Through an adjusting clustering coefficient by the edge exchanging method, it is found out that the efficiency of our ABC strategy is closely related with the clustering coefficient. The main contributions of this paper can be listed as follows: (1) A new high-order social network is proposed to describe opinion dynamic. (2) An algorithm, which does not require the knowledge of the nodes' degree and other global/local network structure information, is proposed to control the "bridges" more accurately and further control the opinion dynamics of the social networks. The efficiency of our ABC strategy is illustrated by numerical examples. (3) The numerical results indicate that our ABC strategy is more efficient for networks with higher clustering coefficient. © *2011 American Institute of Physics*. [doi:10.1063/1.3602220]

The social network refers to the relatively stable relation system grounded upon the interactions between social individual members. This realm is concerning with the interactions and relations among social individuals as well as its impact exerted upon human social behaviors. In the actual social network, some insignificant gossip or thoughts, if not properly controlled, would eventually erupt on a large scale or even headline the whole network. If some control strategies against the gossip spread on the internet could be carried out based upon further understanding of its internal mechanism, its potential application value would be enormous. In this paper, we bring forward the adaptive bridge control (ABC) strategy that could control the opinion evolution without the overall or even partial information and perform very well in the numerical experiment. It is believed that this strategy is feasible, economic, and highly effective in real-world applications, especially for the real social networks bearing high clustering coefficient.

I. INTRODUCTION

The use of simple mathematical models to describe social phenomena in human populations has a long history in social sciences. At present, the spread of rumors and ideas in networks has gradually become one of the most hot topics in the study of complex networks.[1–10] Many models are proposed to describe this complex process. Recently, Kitsak *et al.*[11] discussed this topic based on SIR (susceptible-infectious-recovered) and SIS (susceptible-infectious-susceptible) models, and we have just discussed some evolution rules of opinion spreading in another way[12] based on social influence theory.[13]

With gradually deepening the study on idea spread, another question comes out. It is known that in some real social networks, if some rumors or ideas, which seem nonsignificant at first, have not been controlled, they may eventually break out in a large scope or even occupy the entire network. In recent years, in order to overcome this kind of problem, many researchers have proposed lots of immunization strategies,[14] including random immunization, targeted immunization,[15,16] and acquaintance immunization.[17,18] One of the authors has also theoretically studied the pinning control of dynamics in complex networks.[19]

The methods mentioned above take the degree of nodes as the main measuring criterion to select the immune nodes. However, does the size of degree indeed determines the importance of nodes and do some "insignificant" nodes (in the sense of degree) play very important role in the spread of rumors and ideas? We notice one kind of nodes, called as bridge, describing some nodes with small degree, but connecting different groups within the network. This kind of nodes sometimes play a decisive role in the opinion evolution. For many networks, nodes are joined together in tightly

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knit groups, among which there are only looser connections. Moreover, the traditional control strategies choose the controlled nodes before evolution. Although some researchers tried to propose some active-network-based targeted control strategies, it should be pointed out that in most cases the change of real network structure is unpredictable and it would be much harder to obtain global information when the network is time-varying.

Furthermore, for many real social networks, the communication between each node and its neighbors is radically different. We assume most of the nodes, called first-order connections, communicate only once with each other, while some of the nodes may communicate twice or even more. Motivated by these network phenomena, we propose a control strategy to restrain the spread of rumors and ideas. The opinions of all the neighbors around each node are classified, and high-order connections[20–23] are considered in the evolution rule, which extends traditional first-order social evolution to high-order social evolution, and hence make the simulated networks closer to the real social networks. The control action is conducted on the nodes (i.e., people, carrier of opinions, and abstracted as node in the paper) with vacillating opinions (such nodes are usually the nodes with large connectivity or the nodes called as bridges).

In this paper, we highlight the bridges found in many realistic networks. Erdös-Rényi (ER) random networks, Barabási-Albert (BA) scale-free networks, and Watts-Strogatz (WS) small-world networks are, respectively, studied, and numerical results are given to present the influence of adaptive bridge control (ABC) strategy on the evolution of the entire network. We find out that the control strategy detects this known structure with high efficiency. Our results in small-world networks are much better than those in other types of network topologies, and the visualization-based results show that most of the nodes selected to be controlled in our ABC strategy are indeed the bridges. We also apply our ABC strategy to random networks adjusted by the edge exchanging method, and the results show that there is a significant relationship between the efficiency of our ABC strategy and the clustering coefficient of the network. It indicates that our bridge-based control strategy is more efficient for networks with a high clustering coefficient.

II. ABC STRATEGY FOR OPINION EVOLUTION

In this section, we first review some existing control strategies for restraining the spread of rumors and ideas and discuss the cases in which these approaches may fail. Then, we propose our new control strategy to avoid some of the shortcomings of the traditional techniques.

A. Traditional control strategies

In order to restrain the spread of rumors and ideas, some researchers utilized traditional methods to select the immune nodes according to the degree of nodes. Random immunization is used to immunize a part of the network nodes in a completely random manner, and obviously, it does not own high efficiency and economy, especially for the scale-free networks. It has been shown that in order to make a successful use of this method in random networks, almost all of the nodes have to be immunized. Targeted immunization[15,16] is used to immunize a few nodes with larger connectivity in turn, and it shows to be high efficiency. However, for the targeted immunization, the global information of the network structure is required in advance, and this requirement is indeed difficult to be satisfied in realistic cases. In order to avoid the requirement of global information in targeted immunization, Cohen *et al.*[17,18] proposed a strategy known as acquaintance immunization: select "pop" neighbors from randomly selected nodes further, and there is a greater probability to select the nodes with larger connectivity, so that it can obtain much better effects than a random immunization strategy, but with lower cost.

B. Our proposed control strategies

Concerning the control strategy of rumors spread in this paper, the global information or even local information of network structure is not required, instead only the opinion changes of a node itself shall be considered. That is, the object of controlling the spread of rumors in the entire network is realized by controlling the nodes with vacillating opinions, and these are usually the nodes with large connectivity or the nodes called as bridges. Here, we highlight the "bridge"—a property of community structure, found in many networks, in which nodes are joined together in tightly knit groups, and between them there are only looser connections. As shown in Fig. 1, the six star points in the network connect two tight sub-networks with larger intensity. Although the average degree of these six points is not large, their opinion changes in opinion evolution process are more frequent. Hence, if the six points are controlled with a certain probability, the evolution of the entire network is expected to be controlled with high possibility. The two big red nodes in Fig. 1 denote the nodes with the largest degree, but they may not as important as the six star points in the opinion evolution process.

In our ABC strategy, when the opinion of any node σ_i changes from –1 to 1, it will be controlled with certain probability p. Then the nodes with vacillating opinions will be controlled at a larger probability. By controlling the vacillating node, the probability of its opinion to be changed will be strongly reduced. That is, if the opinion of node i changes from $\sigma_i(t) = -1$ to $\sigma_i(t) = 1$, then the node will be controlled with certain probability. The variable $\sigma_i(t+2), \sigma_i(t+3), \ldots$ will be determined by Eq. (2) until its opinion σ_i changes

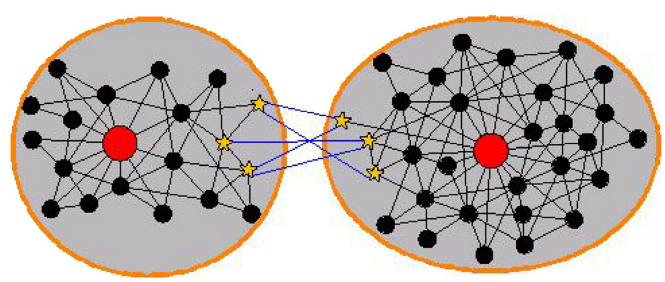

FIG. 1. (Color online) A schematic representation of a network with bridge structure. Six star points connect two tight networks with larger intensity, and the two big red nodes denote the nodes with high degree.

back to –1. It means that the nodes with opposite opinion –1 (our objective opinion is 1) will not be controlled, and our strategy only control the vacillating nodes currently with opinion 1. Our strategy is a kind of conservative control strategy which is motivated by the following reality: in a social network, it is much easier and also the cost is much lower to influence the people with same opinion than influencing the people with opposite opinion.

Consider an undirected and un-weighted network with a total number of N nodes and an average degree K. The detailed embodiment is given by the following recursion equations. When node i is not controlled, its opinion dynamics is described as follows:

$$\sigma_i(t+1) = sign\left[\sum_{j=1}^{N} a_{ij}\sigma_j(t)\xi_{ji}(t)\right]. \tag{1}$$

When node i is controlled, its opinion dynamics is described as follows:

$$\sigma_i(t+1) = sign\left[r_i(t) + \sum_{j=1}^{N} a_{ij}\sigma_j(t)\xi_{ji}(t)\right]. \tag{2}$$

Following this rule, the evolution of all nodes in the network is synchronized essentially. In the equations, $\sigma_i(t)$ denotes the opinion of node i at time t, the value of which can be 1 or –1. Different from some traditional models, the weight a_{ij} is equal to the order of the connection between nodes i and j (e.g., if the edge $i \rightarrow j$ belongs to the 2nd order connections, then $a_{ij} = 2$), and a_{ij} equals to zero if the nodes i and j are not connected. Moreover, it is reasonable to assume that each node has a self-effect, which means that every node's opinion will influence its own opinion in the next period, i.e., $a_{ii} = 1$. The parameter $\xi_{ji}(t)$ means the strength of influence of node j on node i, and it is a random variable of time t subject to a uniform distribution on [0,1]. The value $r_i(t)$ expresses the control strength of the evolution process of opinion, and it is a random variable of time t subject to the uniform distribution of [0,r], where r is the boundary of the control intensity r_i. It follows from Eq. (2) that when $r = 0$, i.e., the network is not controlled and each node is inclined to accept most opinions of its neighbors; when $r > 0$, this tendency is not obvious. The evolution rules are developed from a social influence theory.[13] The difference is that, in this paper, we assume that the influence of each node is time-varying instead of static.

Remark 1: It should be noted that, according to our strategy, the selected (controlled) nodes are bridges with high probability in our numerical examples. The reason behind this phenomenon is that, due to the special topological structure involving bridge, the bridge node typically changes its opinion frequently during the evolution process.

Remark 2: To the best of our knowledge, it is the first time that a higher-order network is introduced to model the opinion dynamics. The proposed social network model extends traditional first-order social evolution networks to higher-order social evolution networks and makes the model much closer to realistic social networks. Moreover, as the order reflect the cohesion between the neighbors, one can observe that the number of connections decreases along with the increasing of the order, i.e., $M_1 \gg M_2 \gg M_3 \gg, \cdots, M_n$ denotes the number of *nth* order connections. In many cases, the number of higher-order connections in the network is so small that they have little effect on the ultimate simulation of the entire network. If only the nodes in the first order description are considered, then the social network model will become one of the most widely used opinion dynamic models.

Remark 3: The proposed ABC strategy calls for a small probability to control an individual when its opinion changes, and our simulation results show that only probability $p = 0.01$ to $p = 0.1$ (depending on the structure of the network) is enough to get a good control result. One of the important advantages in our control strategy is that our strategy requires only the observation of the opinion of each node and not the knowledge of the node degrees or any other global or local information on the network structure, which is indeed required in targeted immunization or acquaintance immunization. Furthermore, our strategy is devoted to controlling the nodes during the evolution process, but not before the evolution. Even when the structure of the network changes over time, our method can still pick out the bridges with high probability for efficient opinion control. It means that our ABC strategy is more robust against a time-varying network structure.

III. COMPARISON AMONG RANDOM NETWORK, SCALE-FREE NETWORK AND SMALL-WORLD NETWORK

ER random network,[24] BA scale-free network[25] (generation based on a total connectivity network of $K+1$ nodes and the parameter $m = K/2$), and WS small-world network[26] (with rewiring probability 0.01 and each node has $K/2$ connections to each side of its neighbors) are three of the most typical models for complex networks, and they embody the typical topological characteristics of real networks from three different perspectives. These three types of networks are selected to study the opinion evolution in this paper. To obtain comparability, the same size and the same average degree are adopted for these kinds of networks. All the simulated networks include 1000 nodes with average degree $K = 6$ in all the networks and 5% second-order connections (the order of each connections are determined randomly), no third or higher connection is considered and the parameter r is fixed at 5. The proportion of nodes with initial opinion 1 is only 35% and opinion –1 is 65%. All simulation programs will run until one of the opinions completely occupies the network. If the network topology is connected, one of the two opinions will definitely occupy the whole network. Even if our control strategy is utilized in the opinion evolution system to maintain the opinion of vacillating nodes in 1, there is still possibility for opinion –1 to occupy the whole network. Thus, here we define the "ultimate winning rate" to be the rate that opinion 1 finally occupies the whole network among the experiments. All results are obtained by averaging 1000 independent repeated simulations. Our results show that our

ABC control strategy will efficiently make small groups win eventually.

For these three kinds of topologies, we study what control strength shall be adopted for the nodes with changing opinions to achieve better control effects? Can the node behavior-based control strategy achieve a good control effect? Fig. 2 displays the ultimate winning rate of opinion 1 for different topologies when the control intensity changes from 0 (i.e., there is no control) to 0.2. It can be observed from Fig. 2 that the results obtained for different topologies are of significant difference. The best result can be obtained for small-world network, in which control probability 0.01 has already resulted in more than 65% winning rate of opinion 1. It can be observed that when the control probability exceeds 0.l, the winning rates of opinion 1 in all three kinds of network topologies are close to 100%.

Furthermore, the cost problem shall be considered. The control strategy proposed in the paper aims that the ultimate winning of opinion 1 in the network can be achieved without the knowledge of the global information or even the local information of the network structure, and simultaneously efficient control results should be achieved with lower cost. However, if the number of nodes requiring control is too large, it is still a large cost control. Fig. 3 depicts how the proportion of controlled nodes in the network changes when the control probability p changes. It can be seen from Fig. 3 that the number of nodes to be controlled is at an acceptable ratio under our ABC strategy.

Fig. 4 presents the visualization based simulation result of our proposed ABC strategy. In Fig. 4, the opinion 1 has occupied the whole network after the evolution, and the green nodes denote the controlled nodes. It can be observed that almost all of the green nodes possess the characteristic of bridge as we expect, and the nodes within the community are seldom controlled. Moreover, the number of the controlled nodes is quite small comparing to the whole network, which indicates that our strategy is efficient and economic.

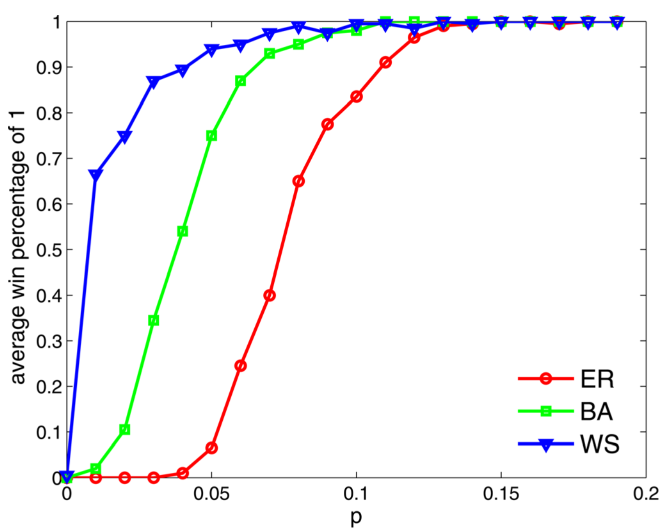

FIG. 2. (Color online) Comparison of the controlling effect. The changes of average win percentage of opinion 1, as one increases the probability p that *a* vacillating node is controlled. The three curves are, respectively, for ER random networks, BA scale free networks, and WS small world networks. The network size is 1000, the average degree of all the networks is 6, and all the data are the average of 1000 independent experiments.

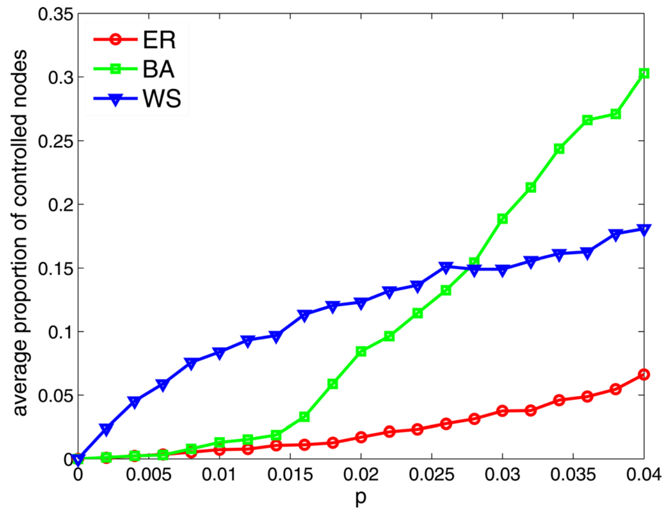

FIG. 3. (Color online) Comparison of the number of controlled nodes. In x-axis, p is the control probability, y-axis denotes the controlled proportion of all nodes. The network size is 1000, the average degree of all the networks is 6, and all the data are the average of 1000 independent experiments.

IV. APPLICATION SCOPE OF OUR STRATEGY

In order to further explore the application of our control strategy, in this section, we study the underlying causes which may influence the control effect.

Fig. 5 describes the performance-price ratio of ABC strategy proposed here for different topologies, namely, the number of nodes to be controlled for achieving the corresponding control effect. Interestingly, we can observe an interesting phenomenon in Fig. 5 that the corresponding curve for BA scale-free networks is a straight line. The three curves can be fitted very well by the equation $y=ax^b$, respectively, with $b<1$, $b=1$, and $b>1$. It can be observed from Fig. 5 that the advantage of our control strategy is most

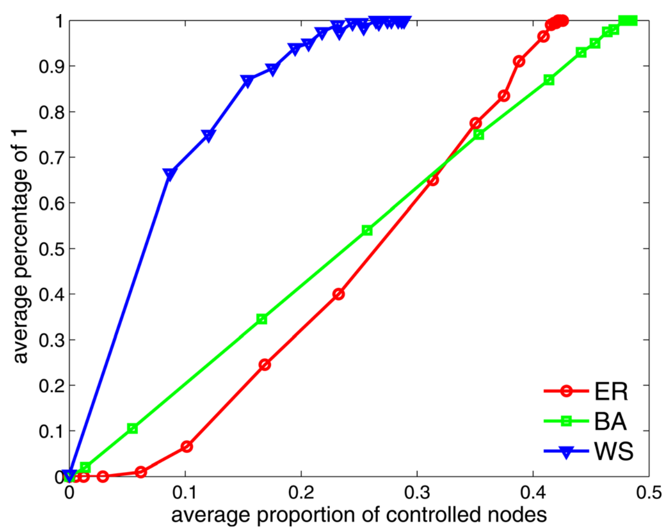

FIG. 4. (Color online) Visualization-based simulation results. The network has already been occupied by opinion 1, and the green nodes are the nodes which have been controlled during the evolution. We have emphasized them by blue circles.

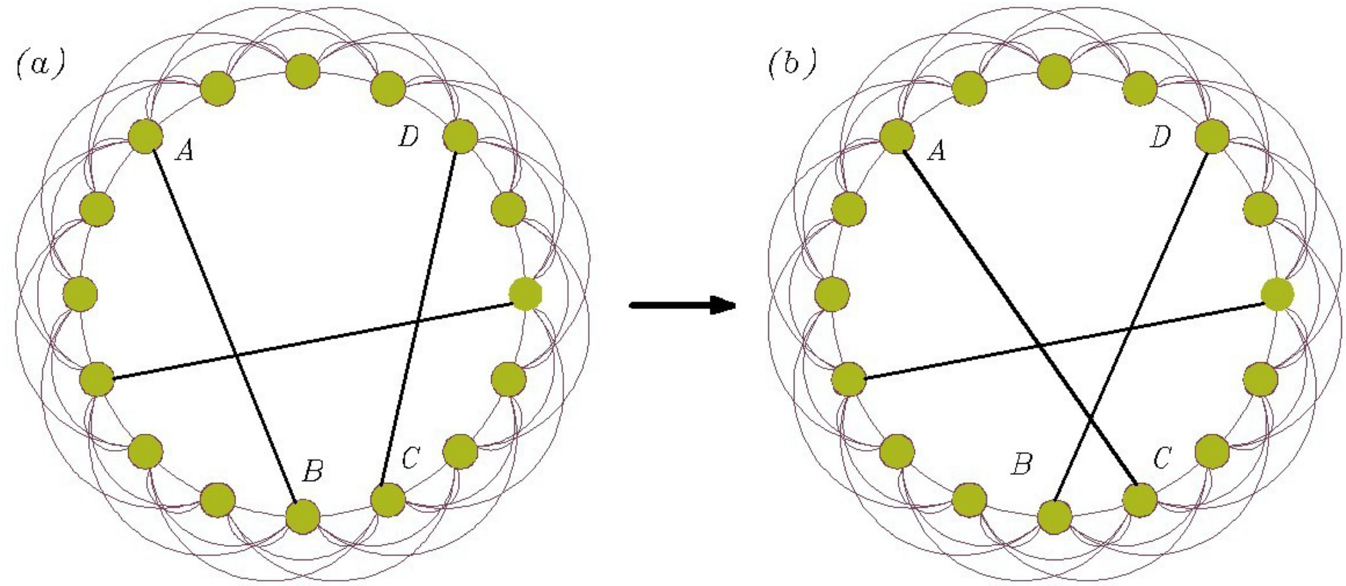

FIG. 5. (Color online) Comparison of cost and effect. The figure reflects the performance price ratio of our ABC strategy in different kinds of networks. The network size is 1000, the average degree of all networks is 6, and all the data are the average of 1000 independent experiments.

efficient for small-world networks. The total controlling effect of WS small-world networks is there much stronger than that of BA scale-free network, and the total controlling effect of BA scale-free networks is slightly stronger than that of ER random networks.

Table I shows the clustering coefficients of the three types of network topologies simulated in this paper. As we can see, the clustering coefficient of WS small-world networks is much larger than that of BA scale-free networks, and the clustering coefficient of BA scale-free networks is slightly larger than that of random networks. Considering the above-mentioned results about the controlling efficiency, we observe that there is a strong relationship between the strength of clustering coefficients of the networks and the controlling efficiency. A further question is: what kind of relationship is there between the control effects and the clustering coefficient? The following experiment is designed for a preliminary exploration of their relationship.

We study the control results by applying our control strategy to the network whose clustering coefficient is adjusted by using the edge exchanging method.[27] Kim has already successfully used this edge exchanging method to test the performance of the Hopfield neural network.[28] Note the fact that by using the edge exchanging method, the clustering coefficient of the network can be increased significantly without changing the degree of any node. Thus, we can get a set of networks with the same degree of each node, but with a different clustering coefficient. Using this method, the effect of clustering coefficient on control efficiency can be further studied by simulating this set of networks as follows.

TABLE I. Comparison of clustering coefficient.

Network	Clustering coefficient
WS small-world network	0.5645
BA scale-free network	0.0493
ER random network	0.0061

Here, we would like to give a brief review of the edge exchanging method.[27] As shown in Fig. 6, two edges AB and CD are randomly chosen from the network. Then, we introduce this method by the following steps: First, judge whether the two edges have repeated vertex, if not, each node changes its partner. Second, compare the clustering coefficient before and after edge exchanging, if the clustering coefficient increases, keep this change; if not, cancel it. Repeat these two steps until the clustering coefficient satisfies the expectation.

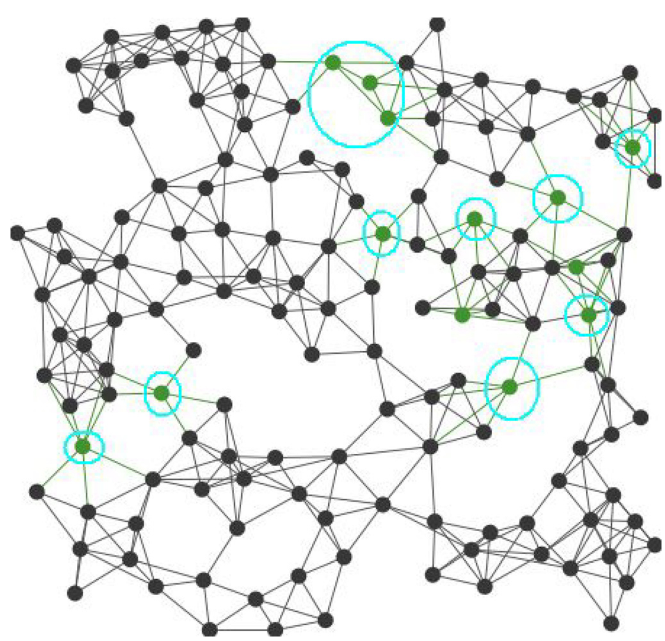

FIG. 6. (Color online) Edge exchanging method: for any network, randomly pick a pair of edge (AB and CD in graph (a), for example) then rewire to have different end nodes (AC and BD as in (b) and AD and BC is also ok). This edge exchanging method can keep each nodes unchanged.

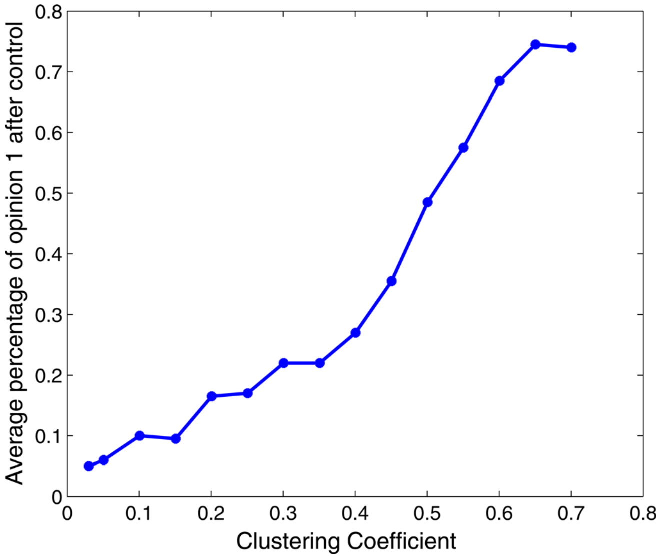

FIG. 7. (Color online) The relationship between the clustering coefficient and the controlling efficiency. The clustering coefficient here is adjusted by the edge exchanging method, so that the degree of each node will remain the same. The network size is 200, the average degree of all networks is 6, and all the data are the average of 250 independent numerical simulations.

In Fig. 7, the y-axis and x-axis, respectively, stand for the winning rate of opinion 1 and the corresponding clustering coefficient of the ER network after using the edge exchanging method and adding the adaptive control strategy. The ultimate winning rate of opinion 1 in the ER network is presented in Fig. 7, in which the initial proportion of nodes with opinion 1 is selected as 35% and control probability $p = 0.05$. It can be observed from Fig. 7 that the clustering coefficient of the network increases obviously after adding the edge exchanging method, which additionally coincides with the increasing trend of the ultimate winning rate of opinion 1 in the network. Moreover, the larger the clustering coefficient of the network is, the faster the ultimate winning rate of opinion 1 increases in the network, which verifies our statement that the larger the clustering coefficient is, the better the control acts.

Further analysis shows that the networks with high clustering coefficient are more likely to have many community structures, which means that there are more bridges in the network. Our ABC strategy achieves its controlling effect by fully using the structure of bridge. Thus, our ABC strategy performs better for networks with more bridges.

V. CONCLUSION

Effective method should be adopted to control the rumor spread in real networks. For large-scale complex networks, if the majority of nodes should be controlled, then a high cost needs to be paid. Hence, this paper is devoted to proposing an efficient control strategy concerning how to ultimately control the wide spread of rumor in the entire network by only controlling a small fraction of nodes. The key idea in our opinion control strategy is to control the nodes with vacillating opinions, which are finally illustrated to be bridges in great change by numerical examples. We have applied our ABC strategy in three different network topologies including WS small-world networks, BA scale-free networks, and ER random networks, and analyzed and discussed the simulation results. It has been observed that our ABC control strategy has achieved good control results for all three types of networks, especially for the WS small-world networks. The number of the nodes which should be controlled in WS small-world networks is much smaller than that of the other two kinds of networks.

By analyzing the network characteristics of the three topologies, we observe that the clustering coefficient is reduced in the order in WS small-world networks, BA scale-free networks, and ER random networks, and a preliminary exploration on the relationship between clustering coefficient and control results has been given. For the ER random network adjusted by the edge exchanging method, the networks' control performance becomes better along with the increasing of the network's clustering coefficient. Therefore, strengthening the tight junction of a network can improve the control efficiency of the network.

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