Hindawi
Complexity
Volume 2020, Article ID 6859042, 8 pages
https://doi.org/10.1155/2020/6859042

Research Article

Fixed-Time Connectivity Preserving Tracking Consensus of Multiagent Systems with Disturbances

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Received 2 September 2019; Revised 11 January 2020; Accepted 21 January 2020; Published 21 February 2020

Academic Editor: José Manuel Galán

This text studies the fixed-time tracking consensus for nonlinear multiagent systems with disturbances. To make the fixed-time tracking consensus, the distributed control protocol based on the integral sliding mode control is proposed; meanwhile, the adjacent followers can be maintained in a limited sensing range. By using the nonsmooth analysis method, sufficient conditions for the fixed-time consensus together with the upper and lower bounds of convergence time are obtained. An example is given to illustrate the potential correctness of the main results.

1. Introduction

Consensus of multiagent systems (MASs) is a very active research topic in control field [1–15]. The key to consensus problem is through designing an appropriate control protocol, such that all the agents can achieve a certain value. The convergence velocity is an important performance index of a control protocol, and the fast convergence velocity is very important for improving the performance and robustness of a system [4, 7–15]. Olfati-Saber and Murray [9] proved that the algebraic connectivity of the interaction graph determines the convergence velocity. The larger the second small eigenvalue of the Laplace matrix, the faster the convergence velocity. Zhou and Wang [4] characterized the convergence velocity for the distributed discrete-time consensus algorithm over a variety of random networks with arbitrary weights. They proposed the asymptotic and per-step convergence factors as measures of the convergence velocity and derived the exact value for the per-step convergence fact. Draief and Vojnović [11] derived an exact relation between the expected convergence time and the voting margin for some of these graphs, which revealed how the expected convergence time tends to infinity as the voting margin approaches zero. By maximizing the algebraic connectivity of the interaction graph, one can increase the convergence velocity with respect to the linear protocol, but the consensus can never be reached in finite time [4, 9–15]. That is, the convergence time is infinite or inestimable. However, in practice, it is often required the consensus to be reached in finite time. Comparing with the asymptotic consensus, the finite-time consensus has strong antiinterference, anti-uncertainty, robustness, and fast convergence velocity [16–19]. Though the finite-time consensus can ensure the system converge in finite time, the convergence time depends on the initial states. However, due to the complexity of environment, the initial states are usually not available in advance [20–28]. In this case, the convergence time cannot be estimated. Correspondingly, the practical application of the multiagents systems will be restricted. To overcome this shortcoming, the fixed-time stability is proposed and analyzed in [20–23]. Based on the fixed-time stability concept, Parsegov et al. [22] first proposed the fixed-time consensus

(FdTC), which means that the convergence time is limited and the upper bound of convergence time is independent of the initial states. Zuo and Tie [24] studied the FdTC of first-order MASs over undirected topology. It is shown that the convergence time of the proposed general framework is upper bounded for any initial states. Fu and Wang [25] studied the fixed-time tracking problem for second-order systems with bounded input uncertainties. Based on a fixed-time distributed observer and sliding mode control method, the fixed-time tracking consensus (FdTTC) for high-order MASs with unknown bounded external disturbances is studied in [26]. A new cascade control structure is developed to achieve the FdTTC. Wang et al. [27] studied the FdTC of the second-order nonlinear MASs with time delay over undirected topology. Shang and Ye studied the leader-follower fixed-time group consensus control of MASs over directed topology [28]. A display estimation of convergence time is given. Though there are many results on FdTC, the preserving connectivity between agents communicating with each other is seldom taken into account. However, in practice, the communication ability of multiple agents is usually limited, so it is very important to ensure that the agents communicate with each other within a limited communication radius [29–32]. In addition, for some complex systems, the Lipschitz condition is hard to satisfy. Hence, it is very important to study the system with the nonlinear term regardless of the Lipschitz conditions [29, 30]. By the nonsmooth analysis method, Cao et al. [29] studied the finite-time consensus of MASs with unknown Lipschitz terms and ensured that the communication between agents are always within a limited communication radius. Sun et al. [30] studied the finite-time connectivity-preserving consensus of second-order MASs with nonlinear terms which did not satisfy the Lipschitz conditions. In [30], by using the energy function idea and the technique of arc method, it is proved that the designed control protocol can ensure that the neighbor agents are always in a limited communication radius. Hong et al. [31] studied the finite-time consensus problem for second-order nonlinear MASs under communication constraints, and the interaction patterns can be preserved for the system with the objection of disturbance rejection, but the display estimation of convergence time is not given. Hong et al. [32] studied fixed-time connectivity-preserving distributed average tracking for first-order MASs without considering the interference and the nonlinear terms. Zuo et al. [33] studied the FdTC for MASs over directed and switching interaction topology. They addressed that the explicit bounds of the convergence time for both protocols are independent of the initial condition. However, as far as we know, there are few results related to the FdTTC of MASs with the disturbance and general nonlinearity under the limited communication radius between neighbor agents. Hence, inspired by [27–34], this work studies the FdTTC of the first-order leader following MASs with disturbances and general nonlinearities. Based on the integral sliding surface, a new control protocol is designed. The main contributions of this paper are as follows: (a) the FdTTC of MASs with disturbances and nonlinear term is studied; (b) the agents communicating with each other are always maintaining in a limited communication radius; (c) the upper and lower bounds of convergence time that are independent of initial conditions are given; (d) the unknown disturbances and nonlinear terms are not required to satisfy the Lipschitz condition.

The remainder of this paper is as follows. Notations and preliminaries are described in Section 2. Problem statement and some necessary lemmas are given in Section 3. Main results are given in Section 4. A simulation example is presented in Section 5 to illustrate the correctness of the obtained theoretical results. And the conclusion is concluded in Section 6.

2. Notations and Preliminaries

In this paper, R is the real set, R^n is the n-dimensional real space, $E \otimes F$ is the Kronecker product of matrices E and F, $\|\bullet\|$ is the 2-norm of a matrix, $\|\cdot\|_1$ is the 1-norm of a matrix, $1_n = (1, \ldots, n)^T$ is the n-dimensional column vector with all elements being 1, and $(\cdot)^T$ is the transposition of a matrix or a vector. Let $\operatorname{sign}(X) = [\operatorname{sign}(x_1), \ldots, \operatorname{sign}(x_n)]^T$, where $X = [x_1, \ldots, x_n]^T$, and $\operatorname{sign}(\cdot)$ is the sign function, and$\operatorname{sig}(\cdot)^a = \|\cdot\|^a \operatorname{sign}(\cdot)$.

Graph $G = \{\varepsilon, \varphi\}$ with the edge set $\varepsilon \subseteq \varphi \times \varphi$ describes the communication connection between followers, and $\varphi = \{\varphi_i \mid i = 1, 2, \ldots, N\}$ is the node set. The neighbor set of agent φ_i is denoted as $N_i = \{\varphi_j \mid (\varphi_j, \varphi_i) \in \varepsilon\}$. The adjacency matrix between followers is $\mathscr{A} = [a_{ij}]_{N\times N} \in R^{N\times N}$, where $a_{ij} = 0$ if node φ_j does not belong to N_i or the relative distance between nodes φ_i and φ_j is larger than r, and $r > 0$ is the communication radius of the agent; otherwise $a_{ij} > 0$. In the context, assume that $a_{ii} = 0$, $\forall i = 1, 2, \ldots, N$. If for any $k \in \{2, \ldots, s\}$, there is a sequence of edges (i_1, i_2), $(i_2, i_3), \ldots, (i_{s-1}, i_s)$ with $(i_{k-1}, i_k) \in \varepsilon$, it is called there is a path from agent i_1 to agent i_s. If there is a path between any two different nodes of G, graph G is a connected graph. The degree matrix of G is $\mathscr{D} = \operatorname{diag}\{d_1, d_2, \ldots, d_N\}$, where $d_i = \sum_{j=1}^{N} a_{ij}$ for $i = 1, 2, \ldots, N$. Let $\mathscr{L} = [l_{ij}] = \mathscr{D} - \mathscr{A}$ represent the Laplacian matrix of G. Denote $H = \mathscr{L} + \mathscr{B}$, where $\mathscr{B} = \operatorname{diag}\{b_1, \ldots, b_N\}$, and $b_i > 0$ if the ith follower can get information from the leader, otherwise $b_i = 0$.

3. Problem Formulation

This section studies the FdTTC for MASs with N agents. The dynamics of the i-th follower is

$$\dot{p}_i(t) = u_i(t) + f_i(p_i(t), t) + d_i(p_i(t), t), \quad i = 1, \ldots, N, \tag{1}$$

where $p_i(t) \in R$ is the position, $u_i(t) \in R$ is the control input, $f_i(p_i(t), t) \in R$ is the nonlinear item, and $d_i(p_i(t), t) \in R$ is the interference item of the ith agent, respectively.

The dynamics of the leader, that is, agent 0, is described as

$$\dot{p}_0(t) = f_0(p_0(t), t), \tag{2}$$

where $p_0(t) \in R$ is the position state and $f_0(p_0(t), t) \in R$ is the nonlinear item.

Lemma 1 (see [21]). *If there is a continuous unbounded positive definite function $V(x)$, such that for some $d_1 > 0, d_2 > 0, d_3 > 1$, and $0 \le d_4 < 1$, there is*

$$\dot{V}(x) \le -d_1 V^{d_3}(x) - d_2 V^{d_4}(x). \tag{3}$$

Then, the origin of systems (1) and (2) is globally fixed-time stable, and the convergence time T satisfies

$$\begin{aligned} \frac{1}{(d_1+d_2)(1-d_4)} &=: T_{\min} \le T \le T_{\max} \\ &:= \frac{1}{d_1(d_3-1)} + \frac{1}{d_2(1-d_4)}. \end{aligned} \tag{4}$$

Lemma 2

(i) *(see [33]) Let $\sigma_1, \sigma_2, \ldots, \sigma_N \ge 0$ and $w > 1$, then there is*

$$(\sigma_1 + \cdots + \sigma_N)^w \ge \sigma_1^w + \cdots + \sigma_N^w \ge N^{1-w}(\sigma_1 + \cdots + \sigma_N)^w. \tag{5}$$

(ii) *(see [7]) Let $\sigma_1, \sigma_2, \ldots, \sigma_N \ge 0$ and $0 < w \le 1$, then there is*

$$N^{1-w}(\sigma_1 + \cdots + \sigma_N)^w \ge \sigma_1^w + \cdots + \sigma_N^w \ge (\sigma_1 + \cdots + \sigma_N)^w. \tag{6}$$

(iii) *Sequence inequality*

Let $\varepsilon_1 \le \varepsilon_2 \le \cdots \le \varepsilon_n$ and $\mu_1 \le \mu_2 \le \cdots \le \mu_n$ be two sequences of the real number. Then, there is

$$\begin{aligned} &\varepsilon_1\mu_n + \varepsilon_2\mu_{n-1} + \cdots + \varepsilon_n\mu_1 \le \varepsilon_1\rho_1 + \varepsilon_2\rho_2 + \cdots + \varepsilon_n\rho_n \le \varepsilon_1\mu_1 \\ &\quad + \varepsilon_2\mu_2 + \cdots + \varepsilon_n\mu_n, \end{aligned} \tag{7}$$

where $\rho_1, \ldots, \rho_n$ is an arrangement of $\mu_1, \ldots, \mu_n$.

(iv) *Let $m_1, m_2, \ldots, m_n, l$ be the positive real numbers. Then, according to (iii) there is*

$$\begin{aligned} &(m_1 + m_2 + \cdots + m_n)\left(m_1^l + m_2^l + \cdots + m_n^l\right) \\ &\quad \le n\left(m_1^{l+1} + m_2^{l+1} + \cdots + m_n^{l+1}\right). \end{aligned} \tag{8}$$

Lemma 3 (see [18]). *Assume that function $\varphi: R^2 \longrightarrow R^+$ satisfies $\varphi(y_i, y_j) = -\varphi(y_j, y_i), i, j = 1, \ldots, N, i \ne j$. Then, for any symmetric matrix $\mathbb{B} = [b_{ij}]_{N\times N}$ and real numbers $\varsigma_1, \varsigma_2, \ldots, \varsigma_N$, the following equality holds:*

$$\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}\varsigma_i\varphi(y_j, y_i) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}(\varsigma_j - \varsigma_i)\varphi(y_j, y_i). \tag{9}$$

Lemma 4 (see [9]). *Laplace matrix $\mathscr{L}$ of the undirected connected graph G satisfies the following:*

(i) *For any N-dimensional column vectors $x = [x_1, \ldots, x_N]^T \in R^N$, there is*

$$x^T \mathscr{L} x = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(x_i - x_j\right)^2. \tag{10}$$

(ii) *The eigenvalues of matrix $\mathscr{L}$ are recorded as $0 < \lambda_2 \le \cdots \le \lambda_{N-1}$, and $x^T\mathscr{L}x \ge \lambda_2 x^T x$.*

Assumption 1. Graph G is connected, and there is at least one agent that can access the leader's information, and at the initial time, the distance between any follower and its neighbors is less than r.

Assumption 2. There is a constant $c > 0$, such that

$$\left|f_i(p_i(t), t) + d_i(p_i(t), t) - f_0(p_0(t), t)\right| \le c, \quad i = 1, \ldots, N. \tag{11}$$

Definition 1. For any initial state, there is

$$\begin{aligned} &\lim_{t\longrightarrow T}\left|p_i(t) - p_0(t)\right| = 0, \\ &\left|p_i(t) - p_0(t)\right| = 0, \\ &\qquad \forall t \ge T, \ i = 1, 2, \ldots, N, \end{aligned} \tag{12}$$

where T is the settling time and the bound of T is independent of the initial states; then, systems (1) and (2) are said to achieve the FdTTC.

Definition 2. If for any $t > 0$, graph G is connected, and the distance between any agent and its neighbor agents is less than the communication radius r and the states of systems (1) and (2) satisfy Definition 1; then, systems (1) and (2) are said to achieve the fixed-time connectivity-preserving tracking consensus.

For systems (1) and (2), design the following sliding mode control protocol:

$$u_i(t) = u_{i1}(t) + u_{i2}(t), \tag{13}$$

with

$$\begin{aligned} u_{i1} = &-k_1\sum_{j=1}^{N} a_{ij} sig^{\alpha_2}\left(p_i(t) - p_j(t)\right) \\ &- k_2\sum_{j=1}^{N} a_{ij}\mathrm{sig}^{\alpha_3}\left(p_i(t) - p_j(t)\right), \end{aligned} \tag{14}$$

and

$$u_{i2}(t) = -k_5\mathrm{sign}(s_i) - k_6\mathrm{sig}^{\alpha_1}(s_i), \tag{15}$$

where k_1, k_2,k_5, and k_6 are the positive constants; $\alpha_1 > 1$, $0 < \alpha_2 < 1$, $\alpha_3 > 1$, and the sliding surface $s_i(t) \in R$, $i = 1, \ldots, N$, is designed as

$$s_i = \sum_{j=1}^{N} a_{ij}\left(\overline{p}_i - \overline{p}_j\right) + b_i\left(\overline{p}_i - \overline{p}_0\right) - \int_0^t \left\{ \sum_{j=1}^{N} a_{ij}\left(u_{i1}(\tau) - u_{j1}(\tau)\right) + b_i u_{i1}(\tau) \right\} \mathrm{d}\tau, \tag{16}$$

where $\overline{p}_i = p_i(t) - p_i(0)$ with $p_i(0)$ being the initial position state of agent $i, i = 1, \ldots, N$. Throughout this paper, we denote $\widehat{a}_{\min} = \min\left\{a_{ij} | a_{ij} \neq 0, | \quad \forall i, j \in \{1, \ldots, N\}\right\}, c_0 = \max\left\{\widehat{N}_1, \widehat{N}_2, \ldots, \widehat{N}_N\right\}$, where $\widehat{N}_i$ is the number of elements in the set $N_i, \overline{H} = H \otimes I_n$, $s(t) = [s_1(t), \ldots, s_N(t)]^T$, $\overline{p} = [\overline{p}_1, \ldots, \overline{p}_N]^T$, $u_1 = [u_{11}, \ldots, u_{N1}]^T$, $u_2 = [u_{12}, \ldots, u_{N2}]^T$, $f = [f_1, \ldots, f_N]^T$ and $\mathrm{d}(t) = [\mathrm{d}_1(t), \ldots, \mathrm{d}_N(t)]^T$. Then, (16) can be rewritten as

$$s = \overline{H}\left(\overline{p} - 1_n \otimes \overline{p}_0\right) - \int_0^t \overline{H} u_1(\tau) \mathrm{d}\tau. \tag{17}$$

4. Main Results

Theorem 1. *Suppose Assumptions 1 and 2 hold, and for any $t \in [0, t_1)$, the distance between followers and their neighbors is less than r. Then, under the control protocol (13), each agent of systems (1) and (2) can maintain its dynamic behavior on the sliding surface in a limited time. That is, $s_i(t) = 0$ and $t \in [0, t_1)$, $\forall i = 1, \ldots, N$, provided that the control gain $\alpha_1 > 1$ and $k_5 > c$ and the convergence time T_1 satisfies*

$$\frac{\sqrt{2}\lambda_{\max}^{(1/2)}\left(\overline{H}^{-1}\right)}{k_5 + k_6 - c} =: T_{\min} \leq T_1 \leq T_{\max} := \sqrt{2}\lambda_{\max}^{(1/2)}\left(\overline{H}^{-1}\right)\left(\frac{1}{(k_5 - c)(\alpha_1 - 1)} + \frac{1}{k_6}\right). \tag{18}$$

Proof. Let $V_1(s, t) = (1/2)s^T\overline{H}^{-1}s$. Then, the derivative of $V_1(s, t)$ along system (17) is

$$\begin{aligned} \dot{V}_1 &= s^T\overline{H}^{-1}\dot{s} = s^T\left(-k_5 \operatorname{sign}(s) + f + d(t) - 1_n \otimes f_0\left(p_0(t), t\right) - k_6 \operatorname{sig}^{\alpha_1}(s)\right) \\ &\leq -(k_5 - c)\left\|s_1\right\| - k_6\|s\|_2^{1+\alpha_1} \\ &\leq -(k_5 - c)\sqrt{\frac{2}{\lambda_{\max}\left(\overline{H}^{-1}\right)}} V_1^{(1/2)} - k_6\sqrt{\frac{2}{\lambda_{\max}\left(\overline{H}^{-1}\right)}} V_1^{(1+\alpha_1)/2}. \end{aligned} \tag{19}$$

From Lemma 1, one can get the convergence time which satisfies

$$\frac{2}{\sqrt{2/\left(\lambda_{\max}\left(\overline{H}^{-1}\right)\right)}\left(k_5 + k_6 - c\right)} =: T_{\min} \leq T_1 \leq T_{\max} := \frac{1}{\sqrt{2/\left(\lambda_{\max}\left(\overline{H}^{-1}\right)\right)}}\left(\frac{1}{(k_5 - c)\left((\alpha_1 - 1)/2\right)} + \frac{2}{k_6}\right), \tag{20}$$

which implies that (18) holds. The proof is completed. □

Theorem 2. *For systems (1) and (2) under the control protocol (13), if $k_1 > 0$, $k_2 > 0$, $k_5 > c$, $k_6 > 0$, $m \geq 2c_1c_0$, and $E(0) < c_2 r^2 \widehat{a}_{min}$, where $E(0)$ is the initial value of an energy function, then the distance between all followers and their neighbor agents is less than r.*

Proof. Similar to the argument in [30], assume that there exists a time $t_0 > 0$ and $\varphi_j \in N_i, i = 1, \ldots, N$, such that $|p_i(t_0^-) - p_j(t_0^-)| < r$ and $|p_i(t_0) - p_j(t_0)| \geq r$. According to Theorem 1, $s_i(t) = 0$ for $t \in [0, t_0) i = 1, \ldots, N$. Denoting $e_{i1} = p_i(t) - p_0(t)$, then one can get

$$\dot{e}_{i1} = -k_1 \sum_{j=1}^{N} a_{ij} \operatorname{sig}^{\alpha_2}\left(e_{i1} - e_{j1}\right) - k_2 \sum_{j=1}^{N} a_{ij} \operatorname{sig}^{\alpha_3}\left(e_{i1} - e_{j1}\right). \tag{21}$$

Define an energy function $E(e)$ on $[0, t_0)$ as

$$E(e) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} \int_0^{|e_{i1} - e_{j1}|} h(s)\mathrm{d}s + m\sum_{i=1}^{N} e_{i1}^2, \tag{22}$$

where $h(|p_i - p_j|)$ satisfies the following conditions:

(i) $h(|p_i - p_j|)$ is continuous for $|p_i - p_j|$;

(ii) $c_2|p_i - p_j| \leq h(|p_i - p_j|) \leq c_1|p_i - p_j|, c_1 > c_2 > 0$.

The derivative of $E(e)$ along (21) is

$$\begin{aligned} \dot{E} &= 2\sum_{i=1}^{N}\left(\sum_{j=1}^{N} a_{ij} h\left(\left|e_{i1} - e_{j1}\right|\right)\right)\left(-\sum_{j=1}^{N} k_1 a_{ij} \operatorname{sig}^{\alpha_2}\left(e_{i1} - e_{j1}\right) - k_2\sum_{j=1}^{N} a_{ij} \operatorname{sig}^{\alpha_3}\left(e_{i1} - e_{j1}\right)\right) \\ &\quad + 2m\sum_{i=1}^{N} e_{i1}\left(-k_1\sum_{j=1}^{N} a_{ij} \operatorname{sig}^{\alpha_2}\left(e_{i1} - e_{j1}\right) - k_2\sum_{j=1}^{N} a_{ij} \operatorname{sig}^{\alpha_3}\left(e_{i1} - e_{j1}\right)\right), \end{aligned} \tag{23}$$

that is,

$$\begin{aligned}\dot{E} \le\ & 2c_1 \sum_{i=1}^{N}\left(\sum_{j=1}^{N} a_{ij}\left|e_{i1}-e_{j1}\right|\right)\left(\sum_{j=1}^{N} k_1 a_{ij}\left|e_{i1}-e_{j1}\right|^{\alpha_2} + k_2\sum_{j=1}^{N} a_{ij}\left|e_{i1}-e_{j1}\right|^{\alpha_3}\right)\\ & - 2m\sum_{i=1}^{N} e_{i1}\left(k_1\sum_{j=1}^{N} a_{ij}\,\mathrm{sig}^{\alpha_2}\left(e_{i1}-e_{j1}\right) + k_2\sum_{j=1}^{N} a_{ij}\,\mathrm{sig}^{\alpha_3}\left(e_{i1}-e_{j1}\right)\right).\end{aligned} \tag{24}$$

From (iv) in Lemma 2, one can get

$$\begin{aligned}\dot{E} &\le 2c_1c_0\sum_{i=1}^{N}\sum_{j=1}^{N} k_1 a_{ij}\left|e_{i1}-e_{j1}\right|^{\alpha_2+1} + k_2 a_{ij}\left|e_{i1}-e_{j1}\right|^{\alpha_3+1}\\ &\quad - 2m\sum_{i=1}^{N} e_{i1}\left(k_1\sum_{j=1}^{N} a_{ij}\,\mathrm{sig}^{\alpha_2}\left(e_{i1}-e_{j1}\right)\right.\\ &\quad \left. + k_2\sum_{j=1}^{N} a_{ij}\,sig^{\alpha_3}\left(e_{i1}-e_{j1}\right)\right)\\ &\le \left(2c_1c_0 - m\right)\sum_{i=1}^{N}\sum_{j=1}^{N}\left(k_1 a_{ij}\left|e_{i1}-e_{j1}\right|^{\alpha_2+1}\right.\\ &\quad \left. + k_2 a_{ij}\left|e_{i1}-e_{j1}\right|^{\alpha_3+1}\right)\\ &\le 0,\end{aligned} \tag{25}$$

which implies that $E(t) \le E(0) < c_1 r^2 \widehat{a}_{\min}$. Then, there is $\lim_{t\longrightarrow t_0^-}|e_{i1}-e_{j1}| = r$. Since $\int_0^r h(s)ds > (c_1r^2/2)$ and $E(t_0^-) > c_2 r^2 \widehat{a}_{\min}$, which contradicts $E(t) \le E(0) < c_1 r^2 \widehat{a}_{\min}$. Hence, the assumption is wrong. The proof is completed. □

Denote $k_3 = (k_1/2)2^{(1+\alpha_2)/2}\lambda_2^{(1+\alpha_2)/2}(\mathscr{L}_1)$ and $k_4 = (k_2/2)N^{1-\alpha_3}2^{(1+\alpha_3)/2}\lambda_2^{(1+\alpha_3)/2}(\mathscr{L}_2)$, where $\mathscr{L}_1$ and $\mathscr{L}_2$ are the Laplacian matrix of matrix $A_1 = [(a_{ij})^{2/(1+\alpha_2)}]_{N\times N}$ and $A_2 = [(a_{ij})^{2/(1+\alpha_3)}]_{N\times N}$ and $\lambda_2(\mathscr{L}_1)$ and $\lambda_2(\mathscr{L}_2)$ are the second small eigenvalue of matrix $\mathscr{L}_1$ and $\mathscr{L}_2$, respectively. □

Theorem 3. *Let Assumptions 1 and 2 hold. For systems (1) and (2) under the control protocol (13) with (14)–(16), if* $k_1 > 0, k_2 > 0,$ *and* $0 < \alpha_2 < 1, \alpha_3 > 1,$ *then for any* $i \in \{1,\ldots,N\}$,e_{i1} *could converge to zero with the convergence time* T_2 *satisfying*

$$\frac{2}{(k_3+k_4)(1-\alpha_2)} \le T_2 \le \frac{2}{k_4(\alpha_3-1)} + \frac{2}{k_3(1-\alpha_2)}. \tag{26}$$

Proof. According to Theorem 1, $s_i(t) = 0,\ i = 1,\ldots,N$. Then, based on the sliding manifold (17), there is

$$\dot{e}_{i1} = -k_1\sum_{j=1}^{N} a_{ij}\,\mathrm{sig}^{\alpha_2}\left(e_{i1}-e_{j1}\right) - k_2\sum_{j=1}^{N} a_{ij}\,\mathrm{sig}^{\alpha_3}\left(e_{i1}-e_{j1}\right). \tag{27}$$

Choose the candidate Lyapunov function

$$V_2 = \frac{1}{2}\sum_{i=1}^{N} e_{i1}^2. \tag{28}$$

The derivative of V_2 along (27) is

$$\begin{aligned}\dot{V}_2 &= \sum_{i=1}^{N} e_{i1}\dot{e}_{i1} = -k_1\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{i1}\,\mathrm{sig}^{\alpha_2}\left(e_{i1}-e_{j1}\right)\\ &\quad - k_2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{i1}^T\,\mathrm{sig}^{\alpha_3}\left(e_{i1}-e_{j1}\right).\end{aligned} \tag{29}$$

From Lemma 3, one can obtain

$$\begin{aligned}\dot{V}_2 &\le -\frac{k_1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(\left|e_{i1}-e_{j1}\right|^{1+\alpha_2}\right)\\ &\quad -\frac{k_2}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(\left|e_{i1}-e_{j1}\right|^{1+\alpha_3}\right)\\ &\le -\frac{k_1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^{2/(1+\alpha_2)}\left|e_{i1}-e_{j1}\right|^2\right)^{(1+\alpha_2)/2}\\ &\quad -\frac{k_2}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^{2/(1+\alpha_3)}\left|e_{i1}-e_{j1}\right|^2\right)^{(1+\alpha_3)/2}.\end{aligned} \tag{30}$$

According to (i) and (ii) in Lemma 2 and 4, one can get

$$\begin{aligned}\dot{V}_2 &\le -\frac{k_1}{2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}^{2/(1+\alpha_2)}\left|e_{i1}-e_{j1}\right|^2\right)^{(1+\alpha_2)/2}\\ &\quad - N^{1-\alpha_3}\frac{k_2}{2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}^{2/(1+\alpha_3)}\left|e_{i1}-e_{j1}\right|^2\right)^{(1+\alpha_3)/2}\\ &\le -\frac{k_1}{2}2^{(1+\alpha_2)/2}\lambda_2^{(1+\alpha_2)/2}(\mathscr{L}_1)V_2^{(1+\alpha_2)/2}\\ &\quad -\frac{k_2}{2}N^{1-\alpha_3}2^{(1+\alpha_3)/2}\lambda_2^{(1+\alpha_3)/2}(\mathscr{L}_2)V_2^{(1+\alpha_3)/2}\\ &\le -k_3V_2^{(1+\alpha_2)/2} - k_4V_2^{(1+\alpha_3)/2}.\end{aligned} \tag{31}$$

From Lemma 1, one can get for any $i \in \{1,\ldots,N\}$,e_{i1} converges to zero in a finite time and the settling time T_2 satisfies

$$\frac{2}{(k_3+k_4)(1-\alpha_2)} =: T_{\min} \le T_2 \le T_{\max} \\ := \frac{2}{k_4(\alpha_3-1)} + \frac{2}{k_3}(1-\alpha_2). \tag{32}$$

The proof is completed. □

From Theorems 1–3, one can find that systems (1) and (2) can achieve the fixed-time connectivity-preserving tracking consensus and the convergence time $T = T_1 + T_2$ satisfies

$$\frac{\sqrt{2}\lambda_{\max}^{(1/2)}(\overline{H}^{-1})}{k_5+k_6-c} + \frac{2}{(k_3+k_4)(1-\alpha_2)} \\ \le T \le \sqrt{2}\lambda_{\max}^{(1/2)}(\overline{H}^{-1})\left(\frac{1}{(k_5-c)(\alpha_1-1)} + \frac{1}{k_6}\right) \\ + \frac{2}{k_4(\alpha_3-1)} + \frac{2}{k_3(1-\alpha_2)}. \tag{33}$$

5. Examples

Example 1. Consider MASs (1) and (2) consisting of four followers $i = 1, 2, 3$, and 4 and one leader $i = 5$. Under the consensus protocol (13) over the communication network as shown in Figure 1, choose $\alpha_1 = 2$, $\alpha_2 = 0.3$, $\alpha_3 = 3$, $r = 20$, $k_1 = k_2 = 1$, $k_5 = 15$, $k_6 = 1$, $f_i(p_i(t), t) = -p_i(t)\cos t$, $d_i(p_i(t), t) = 2\sin(0.5it + (i\pi/6))$, $f_0(p_0(t), t) = 2\sin t$. Let $d_{12} = |p_1 - p_2|$, $d_{13} = |p_1 - p_3|$, and $d_{34} = |p_3 - p_4|$. The initial state is $e_1 = [e_{11}, e_{21}, e_{31}, e_{41}]^T = [6, 2, 3, 5]^T$, and the adjacency matrix

$$\mathscr{A} = (a_{ij})_{4\times 4} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \\ \mathscr{B} = \mathrm{diag}\{0, 0, 1, 1\}. \tag{34}$$

By calculation, one can get $T_{\max} = 28.465$, $T_{\min} = 4.785$, and $T = 7.690$, where T is the convergence time and $T_{\min}$ and$T_{\max}$ are the lower and upper bound of T. Hence, the convergence time is limited. Trajectories of the position error between followers and the leader are given in Figure 2, which shows that the position error between followers and the leader tends to zero. The distances between neighbor agents are shown in Figure 3. The maximum distance ($d_{\max}$) and minimum distance ($d_{\min}$) are given in Table 1. Figure 3 and Table 1 show that the distance between followers and their neighbors is always within a given limited range. The simulations illustrate that under the consensus protocol (13), systems (1) and (2) can achieve the FdTTC, which verifies the correctness of the main results in the work.

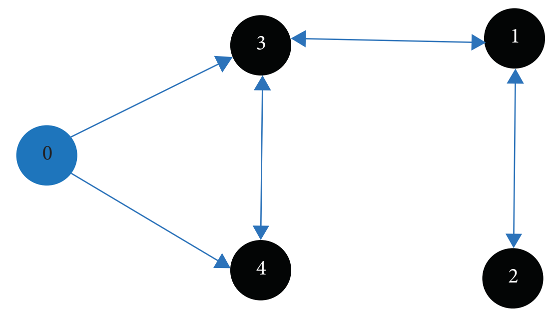

Figure 1: Network graph.

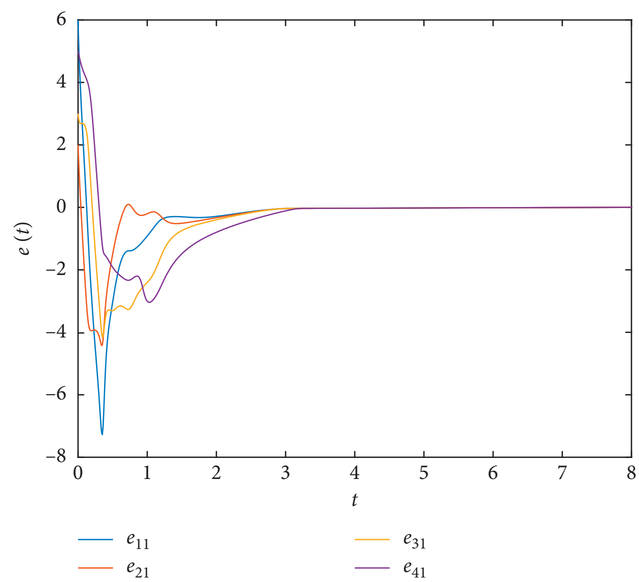

Figure 2: Trajectories of the position error between followers and the leader.

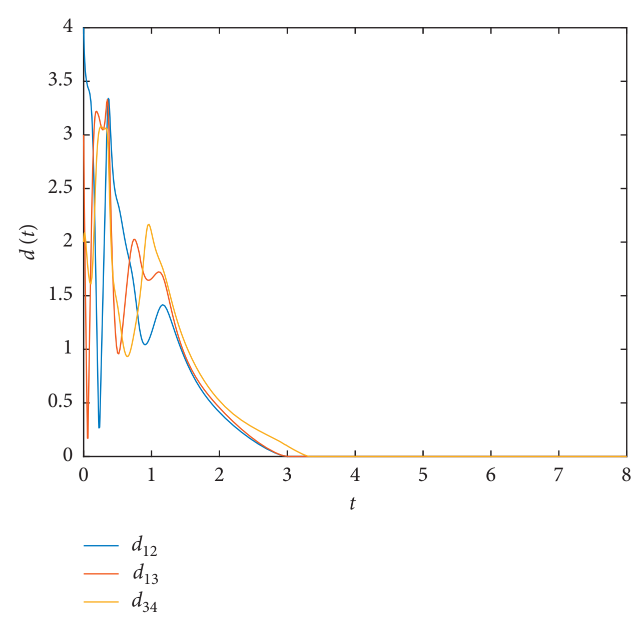

Figure 3: Distances between neighbor agents of systems (1) and (2).

Table 1: The maximum and minimum distance between neighbor agents.

	d_{12}	d_{13}	d_{34}
$d_{\max}$	4.000	3.000	2.000
$d_{\min}$	0	0	0

6. Conclusion

The FdTTC of first-order MASs with general nonlinearity and disturbances is studied in this paper. Firstly, the integral sliding surface is designed to solve the nonlinear term and disturbance in the system; then, the Lyapunov function theory and principle of fixed-time stability are adopted to prove that the integral sliding surface can switch to the surface in a finite time, and the upper and lower bounds independent of the initial conditions of the time to the sliding surface are obtained. Secondly, it is proved that the agents communicating with each other are always keeping within a limited communication radius by using the idea of arc method and designing the energy function. Thirdly, it is proved that the system can achieve FdTTC and the upper and lower bounds of convergence time that are independent of the initial conditions are given. Finally, a simulation example is presented, which shows that all the states of the followers can converge to that of the leader, and the convergence time is between the given lower and upper bounds. Table 1 gives that the agents communicating with each other are always within the given limited communication radius. Hence, the presented example verifies the correctness of theoretical results and the validity of the adopting methods. The FdTTC for the nonlinear MASs with time delay or noise over the switching network is the future work to be done.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation (NNSF) of China (Grants nos. 61503053, 61673080, and 61773082), the State Scholarship Foundation (Grant no. 201808500022), the Natural Science Funds of Chongqing CSTC (Grant no. cstc2019jcyj-msxmX0102), and the Key Project of Crossing and Emerging Area of CQUPT (Grant no. A2018-02).

References

[1] Z. Liu, G. Wen, X. Yu, Z. Guan, and T. Huang, "Delayed impulsive control for consensus of multiagent systems with switching communication graphs," *IEEE Transactions on Cybernetics*, pp. 1–11, 2019.

[2] Y. Li, C. Tang, K. Li, X. He, S. Peeta, and Y. Wang, "Consensus-based cooperative control for multi-platoon under the connected vehicles environment," *IEEE Transactions on Intelligent Transportation Systems*, vol. 20, no. 6, pp. 2220–2229, 2019.

[3] W. Yu, G. Chen, and M. Cao, "Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems," *Automatica*, vol. 46, no. 6, pp. 1089–1095, 2010.

[4] J. Zhou and Q. Wang, "Convergence speed in distributed consensus over dynamically switching random networks," *Automatica*, vol. 45, no. 6, pp. 1455–1461, 2009.

[5] Z.-W. Liu, X. Yu, Z.-H. Guan, B. Hu, and C. Li, "Pulse-modulated intermittent control in consensus of multiagent systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 5, pp. 783–793, 2017.

[6] T. H. Lee, J. H. Park, D. H. Ji, and H. Y. Jung, "Leader-following consensus problem of heterogeneous multi-agent systems with nonlinear dynamics using fuzzy disturbance observer," *Complexity*, vol. 19, no. 4, pp. 20–31, 2014.

[7] L. Wang and F. Xiao, "Finite-time consensus problems for network of dynamic agents," *IEEE Transactions on Automatic Control*, vol. 55, no. 4, pp. 950–955, 2010.

[8] F. Sun, J. Chen, Z.-H. Guan, L. Ding, and T. Li, "Leader-following finite-time consensus for multi-agent systems with jointly-reachable leader," *Nonlinear Analysis: Real World Applications*, vol. 13, no. 5, pp. 2271–2284, 2012.

[9] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[10] F. Sun, X. Liao, Y. Li, and F. Liu, "Consensus for mixed-order multiagent systems over jointly connected topologies via impulse control," *Complexity*, vol. 2019, Article ID 2720531, 7 pages, 2019.

[11] M. Draief and M. Vojnović, "Convergence speed of binary interval consensus," *SIAM Journal on Control and Optimization*, vol. 50, no. 3, pp. 1087–1109, 2012.

[12] G. Chen, F. L. Lewis, and L. Xie, "Finite-time distributed consensus via binary control protocols," *Automatica*, vol. 47, no. 9, pp. 1962–1968, 2011.

[13] H. Qing, W. Haddad, and S. Bhat, "Finite-time semistability, Filippov systems, and consensus protocols for nonlinear dynamical networks with switching topologies," *Nonlinear Analysis: Hybrid Systems*, vol. 4, no. 3, pp. 557–573, 2010.

[14] W. Ren and R. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[15] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multiagent systems and synchronization of complex networks: a unified viewpoint," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 57, no. 1, pp. 213–224, 2009.

[16] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM Journal on Control and Optimization*, vol. 38, no. 3, pp. 751–766, 2000.

[17] H. Du, S. Li, and C. Qian, "Finite-time attitude tracking control of spacecraft with application to attitude synchronization," *IEEE Transactions on Automatic Control*, vol. 56, no. 11, pp. 2711–2717, 2011.

[18] F. Jiang and L. Wang, "Finite-time information consensus for multi-agent systems with fixed and switching topologies," *Physica D: Nonlinear Phenomena*, vol. 238, no. 16, pp. 1550–1560, 2009.

[19] F.-L. Sun and W. Zhu, "Finite-time consensus for leader-following multi-agent systems over switching network topologies," *Chinese Physics B*, vol. 22, no. 11, p. 110204, 2013.

[20] V. Andrieu, L. Praly, and A. Astolfi, "Homogeneous approximation, recursive observer design, and output feedback," *SIAM Journal on Control and Optimization*, vol. 47, no. 4, pp. 1814–1850, 2008.
[21] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 8, pp. 2016–2110, 2012.
[22] S. Parsegov, A. Polyakov, and P. Shcherbakov, "Fixed-time consensus algorithm for multi-agent systems with integtor dynamics," in *Proceedings of the 4th IFAC Workshop on Distributed Estimation and Control in Networked Systems*, pp. 110–115, Koblenz, Germany, September 2013.
[23] G. Mei, X. Wu, D. Ning, and J.-A. Lu, "Finite-time stabilization of complex dynamical networks via optimal control," *Complexity*, vol. 21, no. S1, pp. 417–425, 2016.
[24] Z. Zuo and L. Tie, "Distributed robust finite-time nonlinear consensus protocols for multi-agent systems," *International Journal of Systems Science*, vol. 47, no. 6, pp. 1366–1375, 2016.
[25] J. Fu and J. Wang, "Fixed-time coordinated tracking for second-order multi-agent systems with bounded input uncertainties," *Systems & Control Letters*, vol. 93, pp. 1–12, 2016.
[26] Z. Zuo, B. Tian, M. Defoort, and Z. Ding, "Fixed-time consensus tracking for multiagent systems with high-order integrator dynamics," *IEEE Transactions on Automatic Control*, vol. 63, no. 2, pp. 563–570, 2018.
[27] C. Wang, G. Wen, Z. Peng, and X. Zhang, "Integral sliding-mode fixed-time consensus tracking for second-order nonlinear and time delay multi-agent systems," *Journal of the Franklin Institute*, vol. 356, no. 6, pp. 3692–3710, 2019.
[28] Y. Shang and Y. Ye, "Leader-follower fixed-time group consensus control of multiagent systems under directed topology," *Complexity*, vol. 2017, Article ID 3465076, 9 pages, 2017.
[29] Y. Cao, W. Ren, D. W. Casbeer, and C. Schumacher, "Finite-time connectivity-preserving consensus of networked nonlinear agents with unknown lipschitz terms," *IEEE Transactions on Automatic Control*, vol. 61, no. 6, pp. 1700–1705, 2016.
[30] C. Sun, G. Hu, L. Xie, and M. Egerstedt, "Robust finite-time connectivity preserving coordination of second-order multi-agent systems," *Automatica*, vol. 89, pp. 21–27, 2018.
[31] H. Hong, W. Yu, J. Fu, and X. Yu, "Finite-time connectivity-preserving consensus for second-order nonlinear multiagent systems," *IEEE Transactions on Control of Network Systems*, vol. 6, no. 1, pp. 236–248, 2018.
[32] H. Hong, W. Yu, X. Yu, G. Wen, and A. Alsaedi, "Fixed-time connectivity-preserving distributed average tracking for multiagent systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 64, no. 10, pp. 1192–1196, 2017.
[33] Y. Zuo, W. Yang, L. Tie, and D. Mend, "Fixed-time consensus for multi-agent systems under directed and switching interaction topology," in *Proceedings of the American Control Conference*, pp. 5133–5138, Portland, OR, USA, June 2014.
[34] J. P. Mishra, C. Li, M. Jalili, and X. Yu, "Robust second-order consensus using a fixed-time convergent sliding surface in multiagent systems," *IEEE Transactions on Cybernetics*, vol. 50, no. 2, pp. 846–855, 2020.