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Finite-time Consensus of Leader-following Multi-agent Systems with Multiple Time Delays over Time-varying Topology

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Abstract: This paper studies the finite-time consensus of leader-following multi-agent systems with multiple time delays over time-varying topology. The finite-time consensus protocol based on the agents' states and the communication topology is designed. By adopting the algebraic graph theory, Lyapunov stability theory and pinning control method, some sufficient conditions for the finite-time consensus are given. It is proved that the system can reach consensus in a finite time both over the connected and disconnected topology. Moreover, the upper bound of the convergence time is given. Several simulations are presented to verify the effectiveness of the adopted method.

Keywords: Finite-time consensus, multiple time delays, pinning control, time-varying topology.

1. INTRODUCTION

Coordinate control of multi-agent system (MAS) has attracted much attention due to its wild application, such as autonomous underwater vehicles, mobile robot networks, attitude alignment of satellites, and so on [1–10]. Consensus of the leader-following multi-agent system (LFMAS), which refers to the state of all followers ultimately reach the same as that of the leader, has become one of the important coordinate control problem because of its extensive application in various fields, such as multi-robot systems, military control, wireless sensors [5–8, 10–12]. Based on some control methods, the operating efficiency of systems is improved and the robustness of systems is achieved [7, 8, 13–15]. To avoid the continuous communication between agents, Tan *et al*. [7] studied the consensus for LFMAS through Gronwall's inequality and impulsive control method. Zhang *et al*. [8] discussed the consensus of the heterogeneous LFMAS over time-varying directed topologies. However, practically, many systems are expected to achieve the consensus in a finite time [16–21]. Besides the requirement of shorter convergence time, the finite-time consensus (FTC) can also better suppress the disturbance of uncertain factor and there is stronger robustness. In [16], FTC of the second-order LFMAS was discussed and the feedback control was designed to estimate the velocity information and the external disturbances. Du *et al*. [17] studied the FTC for the high-order MAS. However, in practice, the communication network between agents is not always connected [19, 22, 23]. The method of pinning control, which through control certain selected agents to control all agents, can be used to analyze the problem of disconnected network. Guan *et al*. [22] studied the FTC for the second-order LFMAS over undirected disconnected topologies by the pinning control method. Song *et al*. [23] used the pinning control method to analyze the consensus of the directed disconnected network, and the m-matrix is introduced for a better solution. In addition, the practical environment that the system located is usually characterized by some uncertainty [24–33], in which time delay is the most common and inevitable factor [26–32]. Wang *et al*. [26] studied the consensus for MAS with active leader and time delay. Olfati-Saber *et al*. studied the consensus problem of time-delay systems over switching topologies [27, 28]. Time delay may destroy the relative stability of a system. For example, for the vehicle-road cooperative system, vehicle has a reaction time between receiving information from neighbour vehicles and starting to perform tasks. If the reaction time, that is time delay, is too long, the cooperative

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control will not be achieved, and even worse, it may lead to the vehicle-road cooperative system paralyzed [34]. In the traffic system, time delay may lead to errors in safe driving positioning between vehicles, which will lead to traffic accidents [35]. Therefore, the study of FTC problem for the time-delay system is very significant. Due to the complexity of the environment, usually, the communication network between agents is variable, not always connected, and the time delay between different agents is different. Because of all these problems, this paper studies the FTC of the LFMAS with multiple delays over time-varying topology. The main innovations of this paper are as follows: (i) FTC for the LFMAS with multiple time delays is studied. (ii) The consider network is time-varying. (iii) Both the connected and disconnected networks are considered.

The rest of this paper is as follows: Section 2 gives some preliminaries and the problem formation. The main results are given in Section 3. Several simulations are presented in Section 4. And the conclusions are summarized in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Preliminaries

Throughout this paper, the weighted undirected graph of n nodes is denoted as $\mathcal{M} = (\mathcal{S}, \mathcal{V})$, where $\mathcal{S} = \{s_1, s_2, \cdots, s_n\}$ is the set of nodes and $\mathcal{V} \in \mathcal{S} \times \mathcal{S}$ is the set of edges. $I_n = \{1, 2, \cdots, n\}$ is the vertex set of graph $\mathcal{M}$, $W_i = \{j | (s_i, s_j) \in \mathcal{V}\}$ represents that the nodes s_i and s_j can transmit information to each other, and $\mathcal{F}(t) = [\alpha_{ij}(t)] \in R^{n\times n}$ is the weighted adjacency matrix, where $\alpha_{ij}(t) = \alpha_{ji}(t) > 0$, if $i \neq j, (s_i, s_j) \in \mathcal{V}$, otherwise, $\alpha_{ij}(t) = 0$ and $\alpha_{ii}(t) = 0$, $\forall\, i, j \in I_n$. The Laplacian matrix of graph $\mathcal{M}$ is defined as $\mathcal{L}(\mathcal{F}) = \mathcal{G} - \mathcal{F}$, where $\mathcal{G} = diag(g_1, g_2, \cdots, g_n)$, and for all $i \in I_n$, $g_i = \sum_{j\neq i}^{n} \alpha_{ij}(t)$. Let $W(t) = [l_1(t), \cdots, l_n(t)]^T$, where $l_i(t)$ is the connection weight between follower i and the leader. If follower i can receive the leader's information directly, $l_i(t) > 0$, otherwise $l_i(t) = 0$. Denote $P(t) = (\varepsilon_1(t), \cdots, \varepsilon_n(t))^T$, where $\varepsilon_i(t)$ is the pinning control gain, $\varepsilon_i(t) > 0$ if agent i is pinned and otherwise $\varepsilon_i(t) = 0$. Let $\hat{P}(t) = P(t) + W(t) = (\hat{\gamma}_1(t), \cdots, \hat{\gamma}_n(t))^T$ with $\hat{\gamma}_i(t) = \varepsilon_i(t) + l_i(t), i \in I_n$.

For further discussion, some lemmas are presented as follow.

Lemma 1 [36]**:** Assume that there is a function $f(t)$ and constants $\gamma > 0$, $0 < \zeta < 1$, such that
(i) $f(t)$ is a continuous positive-definite function;
(ii) $f(t) > 0$ and $\dot{f}(t) \leq -\gamma f^{\zeta}(t)$, $\forall\, t > 0$.
Then for any $t \geq 0$, there is

$$f^{1-\zeta}(t) \leq f^{1-\zeta}(0) - \gamma(1-\zeta)t,$$

and $f(t) \equiv 0$, $\forall\, t \geq T_0$, where $T_0 = \frac{f^{1-\zeta}(0)}{\gamma(1-\zeta)}$.

Lemma 2 [37]**:** For any $r_1, r_2, \cdots, r_n \in R^m$ and $\rho \in (0, 1]$, there is

$$\Big(\sum_{i=1}^{n} |r_i|\Big)^{\rho} \leq \sum_{i=1}^{n} |r_i|^{\rho} \leq n^{1-\rho}\Big(\sum_{i=1}^{n} |r_i|\Big)^{\rho}.$$

Lemma 3 [38]**:** Consider the following system

$$\dot{Q}_i(t) = HQ_i(t) + \sum_{k=1}^{n} B_k S_i(t-\mu_k), \ \ t \geq 0, \tag{1}$$

where $Q_i(t) \in R$ and $S_i(t) \in R$ are position states and feedback control of agent i, $i \in I_n$, respectively, parameters H, $\mu_k > 0$ and $B_k > 0$, $k \in I_n$, are constants. If

$$X_i(t) = Q_i(t) + \sum_{k=1}^{n} L^{\mu_k}_{(H,C_k)} S_i, \tag{2}$$

with $L^{\mu_k}_{(H,C_k)} S_i = \int_{-\mu_k}^{0} e^{H(-\mu_k - s)} C_k S_i(s) ds$, and $C_k = B_k e^{-H\mu_k}$, then

$$\dot{X}_i(t) = HX_i(t) + BS_i(t), \tag{3}$$

where $B = \sum_{k=1}^{n} C_k$, $S_i(t) = \phi(t) w(X_i(t))$. That is

$$S_i(t) = \phi(t) w\Big(Q_i(t) + \sum_{k=1}^{n} L^{\mu_k}_{(H,C_k)} S_i\Big),$$

with the function $\phi(t) \in R$ is bounded and function $w(\cdot): R \to R$ is continuous. If system (3) is finite-time stability, then system (1) is finite-time stability and the settling time $T(Q)$ of system (1) satisfies $T(Q) \leq T(X) + \sum_{k=1}^{n} C_k \mu_k$, where $T(X)$ is the settling time of system (3), $X(t) = (X_1(t), \cdots, X_n(t))^T$, and $Q(t) = (Q_1(t), \cdots, Q_n(t))^T$.

Lemma 4 [39]**:** For $\xi = (\xi_1, \cdots, \xi_n)^T, \rho = (\rho_1, \cdots, \rho_n)^T \in R^n$ and a symmetric matrix $D = [d_{ij}] \in R^{n\times n}$, there is an odd function $\psi: R \to R$, such that

$$\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}\xi_i \psi(\rho_i - \rho_j) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}(\xi_i - \xi_j)\psi(\rho_i - \rho_j).$$

2.2. Problem formulation

Consider the following LFMAS

$$\begin{cases} \dot{q}_i(t) = hq_i(t) + \sum_{k=1}^{n} b_k p_i(t-\tau_k), \ \ i \in I_n, \\ \dot{q}_0(t) = 0, \end{cases} \tag{4}$$

where constant $h > 0$, $b_k > 0$, time delay $\tau_k > 0$, $k = 1, 2, \cdots, n$, $q_i(t) \in R$, $p_i(t) \in R$ are the position states and the feedback control of agent i, $i \in I_n$, respectively, and $q_0(t) \in R$ is the position state of the leader.

Denote $q(t) = (q_1(t), \cdots, q_n(t))^T$ the position state vector, and $z(t) = (z_1(t), \cdots, z_n(t))^T \in R^n$.

Definition 1: If there exists a positive constant T_0, such that for any initial states, there is

$$\lim_{t\to T_0}\|q_i(t)-q_0(t)\|=0,$$
$$q_i(t)=q_0(t),\ \ \forall\, t>T_0,\ \ i\in I_n,$$

then system (4) is said to achieve the FTC.

For system (4), design the following control protocol

$$p_i(t)=-\frac{1}{b}\Bigg(-K\sum_{j\in W_i}\alpha_{ij}(t)\mathrm{sig}|z_j(t)-z_i(t)|^{\varphi}+\frac{1}{2}l_i(t)\mathrm{sig}|z_i(t)-z_0(t)|^{\varphi}+hz_i(t)\Bigg), \tag{5}$$

where $K>0$ is the control gain, $0<\varphi<1$ is a constant, $\mathrm{sig}|z_i|^{\varphi}=|z_i|^{\varphi}\mathrm{sign}(z_i)$, $\mathrm{sign}(\cdot)$ is the sign function and $|z_i|$ the absolute value of z_i, $i\in I$, constant $h>0$ is the same as that in system (4), and

$$\begin{cases} z_i(t)=q_i(t)+\sum\limits_{k=1}^{n}L^{\tau_k}_{(h,c_k)}p_i(t),\ \ i\in I_n,\\ z_0(t)=q_0(t)+\sum\limits_{k=1}^{n}L^{\tau_k}_{(h,c_k)}p_0, \end{cases}$$

where $L^{\tau_k}_{(h,c_k)}p_i=\int_{-\tau_k}^{0}e^{h(-\tau_k-s)}c_kp_i(s)ds$, $c_k=b_ke^{-h\tau_k}$, $b=\sum\limits_{k=1}^{n}c_k$, $p_0(t)=-\frac{h}{b}z_0(t)$.

3. MAIN RESULTS

This section gives the main results.

Theorem 1: If there is a spanning tree with the leader as the root, then system (4) can reach the FTC under the control protocol (5) with $K>0$, $0<\varphi<1$, and the settling time $T(q)$ satisfies

$$T(q)\le T(z)+\sum_{k=1}^{n}c_k\tau_k,$$

where $T(z)=\frac{2(V(0))^{\frac{1-\varphi}{2}}}{\tilde{\lambda}(1-\varphi)}$, $\tilde{\lambda}=\lambda_{\min}^{\frac{\varphi+1}{2}}(2\mathcal{L}(\mathcal{C})+\mathcal{H})$, $\mathcal{C}=\left((K\alpha_{ij}(t))^{\frac{2}{\varphi+1}}\right)_{n\times n}$, $\mathcal{H}=diag\left(l_1^{\frac{2}{\varphi+1}},\ \cdots,\ l_n^{\frac{2}{\varphi+1}}\right)$, and $\lambda_{\min}(2\mathcal{L}(\mathcal{C})+\mathcal{H})$ is the minimum eigenvalue of the matrix $2\mathcal{L}(\mathcal{C})+\mathcal{H}$.

Proof: Consider the following system

$$\begin{cases} \dot{z}_i(t)=hz_i(t)+bp_i(t),\ \ i\in I_n,\\ \dot{z}_0(t)=0. \end{cases} \tag{6}$$

Let $\delta_i(t)=z_i(t)-z_0(t)$, $i\in I_n$, and $\delta(t)=(\delta_1(t),\ \cdots,\ \delta_n(t))^T$. Then $\dot{\delta}_i(t)=\dot{z}_i(t)$, $i\in I_n$.

Choose the following Lyapunov function

$$V(t)=\sum_{i=1}^{n}\delta_i^{\,2}(t). \tag{7}$$

Then under the control protocol (5), the derivative of (7) along (6) is

$$\begin{aligned}\dot{V}(t)=&2\sum_{i=1}^{n}\delta_i(t)\dot{\delta}_i(t)=2\sum_{i=1}^{n}\delta_i(t)(hz_i(t)+bp_i(t))\\ =&2\sum_{i=1}^{n}\delta_i(t)\Big(hz_i(t)+b\big(-\frac{1}{b}(hz_i(t)\\ &-K\sum_{j\in W_i}\alpha_{ij}(t)\mathrm{sig}|z_j(t)-z_i(t)|^{\varphi}\\ &+\frac{1}{2}l_i(t)\mathrm{sig}|z_i(t)-z_0(t)|^{\varphi})\big)\Big)\\ =&2\sum_{i=1}^{n}\delta_i(t)\Big(K\sum_{j\in W_i}\alpha_{ij}(t)\mathrm{sig}|z_j(t)-z_i(t)|^{\varphi}\\ &-\frac{1}{2}l_i(t)\mathrm{sig}|z_i(t)-z_0(t)|^{\varphi}\Big)\\ =&2K\sum_{i=1}^{n}\sum_{j\in W_i}\alpha_{ij}(t)\delta_i(t)\mathrm{sig}|\delta_j(t)-\delta_i(t)|^{\varphi}\\ &-\sum_{i=1}^{n}l_i(t)\delta_i(t)\mathrm{sig}|\delta_i(t)|^{\varphi}.\end{aligned}$$

From Lemma 4 there is

$$\begin{aligned}\dot{V}(t)=&-K\sum_{i=1}^{n}\sum_{j\in W_i}\alpha_{ij}(t)(\delta_j(t)-\delta_i(t))\mathrm{sig}|\delta_j(t)-\delta_i(t)|^{\varphi}\\ &-\sum_{i=1}^{n}l_i(t)\delta_i(t)\mathrm{sig}|\delta_i(t)|^{\varphi}\\ =&-K\sum_{i=1}^{n}\sum_{j\in W_i}\alpha_{ij}(t)|\delta_j(t)-\delta_i(t)|^{\varphi+1}\\ &-\sum_{i=1}^{n}l_i(t)|\delta_i(t)|^{\varphi+1}\\ =&-\Bigg(\sum_{i=1}^{n}\sum_{j\in W_i}\Big((K\alpha_{ij}(t))^{\frac{2}{\varphi+1}}(\delta_j(t)-\delta_i(t))^2\Big)^{\frac{\varphi+1}{2}}\\ &+\sum_{i=1}^{n}\Big(l_i^{\frac{2}{\varphi+1}}(t)\delta_i^2(t)\Big)^{\frac{\varphi+1}{2}}\Bigg).\end{aligned}$$

According to Lemma 2, one can get

$$\begin{aligned}\dot{V}(t)\le&-\Bigg(\sum_{i=1}^{n}\sum_{j\in W_i}(K\alpha_{ij}(t))^{\frac{2}{\varphi+1}}(\delta_j(t)-\delta_i(t))^2\\ &+\sum_{i=1}^{n}l_i^{\frac{2}{\varphi+1}}(t)\delta_i^2(t)\Bigg)^{\frac{\varphi+1}{2}}\\ \le&-\Big(2\delta^T(t)\mathcal{L}(\mathcal{C})\delta(t)+\delta^T(t)\mathcal{H}\delta(t)\Big)^{\frac{\varphi+1}{2}}\\ =&-\Big(\delta^T(t)(2\mathcal{L}(\mathcal{C})+\mathcal{H})\delta(t)\Big)^{\frac{\varphi+1}{2}},\end{aligned}$$

where $\mathcal{C}=\left((K\alpha_{ij}(t))^{\frac{2}{\varphi+1}}\right)_{n\times n}$, $\mathcal{H}=diag(l_1^{\frac{2}{\varphi+1}},\cdots,l_n^{\frac{2}{\varphi+1}})$. Note that matrix $2\mathcal{L}(\mathcal{C})+\mathcal{H}$ is symmetric and positive definite. Then

$$\dot{V}(t) \le -\Big(\lambda_{\min}\big(2\mathcal{L}(\mathcal{C})+\mathcal{H}\big)\delta^T(t)\delta(t)\Big)^{\frac{\varphi+1}{2}}$$
$$\le -\lambda_{\min}^{\frac{\varphi+1}{2}}\big(2\mathcal{L}(\mathcal{C})+\mathcal{H}\big)V^{\frac{\varphi+1}{2}}(t).$$

According to Lemma 1, there is $T(z)=\frac{2(V(0))^{\frac{1-\varphi}{2}}}{\tilde{\lambda}(1-\varphi)}$, where $\tilde{\lambda}=\lambda_{\min}^{\frac{\varphi+1}{2}}(2\mathcal{L}(\mathcal{C})+\mathcal{H})$. Hence system (6) is globally finite-time stability and $T(z)$ is the settling time. Then from Lemma 3, system (4) is globally finite-time stability and the settling time $T(q)$ satisfies $T(q)\le T(z)+\sum_{k=1}^{n}c_k\tau_k$. The proof is completed. □

Assumption 1: There is at least one $\hat{\gamma}_i(t)>0$, $i\in I_n$.

Theorem 2: Suppose that Assumption 1 holds. Then whether there is a spanning tree with the leader as the root or not, system (4) can reach the FTC under the following control protocol

$$p_i(t)=-\frac{1}{b}\Bigg(-K\sum_{j\in W_i}\alpha_{ij}(t)\mathrm{sig}|z_j(t)-z_i(t)|^{\varphi}$$
$$+\frac{1}{2}\hat{\gamma}_i(t)\mathrm{sig}|z_i(t)-z_0(t)|^{\varphi}+hz_i(t)\Bigg), \tag{8}$$

with $0<\varphi<1$, $\hat{\gamma}_i(t)=\varepsilon_i(t)+l_i(t)$, and the settling time $T(q)$ satisfies

$$T(q)\le T(z)+\sum_{k=1}^{n}c_k\tau_k,$$

where

$$T(z)=\frac{2(V(0))^{\frac{1-\varphi}{2}}}{\hat{\lambda}(1-\varphi)},\quad \hat{\lambda}=\lambda_{\min}^{\frac{\varphi+1}{2}}(2\mathcal{L}(\mathcal{C})+\mathcal{O}),$$
$$\mathcal{C}=\Big((K\alpha_{ij}(t))^{\frac{2}{\varphi+1}}\Big)_{n\times n},$$
$$\mathcal{O}=diag(\hat{\gamma}_1^{\frac{2}{\varphi+1}}(t),\cdots,\hat{\gamma}_n^{\frac{2}{\varphi+1}}(t)),$$

and $\lambda_{\min}(2\mathcal{L}(\mathcal{C})+\mathcal{O})$ is the minimum eigenvalue of the matrix $2\mathcal{L}(\mathcal{C})+\mathcal{O}$.

Proof: For system (6), denote $\delta_i(t)=z_i(t)-z_0(t), i\in I_n$. Then $\dot{\delta}_i(t)=\dot{z}_i(t), i\in I_n$. Next similar to the argument in Theorem 1, choose the candidate Lyapunov function (7). Under the control protocol (8), the derivative of (7) along (6) is

$$\dot{V}(t)\le -\Big(\sum_{i=1}^{n}\sum_{j\in W_i}(K\alpha_{ij}(t))^{\frac{2}{\varphi+1}}(\delta_j(t)-\delta_i(t))^2$$
$$+\sum_{i=1}^{n}\hat{\gamma}_i^{\frac{2}{\varphi+1}}(t)\delta_i^2(t)\Big)^{\frac{\varphi+1}{2}}.$$

Denote $\delta(t)=(\delta_1(t),\cdots,\delta_n(t))^T$. Then

$$\dot{V}(t)\le -\big(2\delta^T(t)\mathcal{L}(\mathcal{C})\delta(t)+\delta^T(t)\mathcal{O}\delta(t)\big)^{\frac{\varphi+1}{2}}$$
$$=-\big(\delta^T(t)(2\mathcal{L}(\mathcal{C})+\mathcal{O})\delta(t)\big)^{\frac{\varphi+1}{2}}.$$

Since $2\mathcal{L}(\mathcal{C})+\mathcal{O}$ is a symmetric positive-definite matrix, then

$$\dot{V}(t)\le -\big(\lambda_{\min}(2\mathcal{L}(\mathcal{C})+\mathcal{O})\delta^T(t)\delta(t)\big)^{\frac{\varphi+1}{2}}$$
$$\le -\lambda_{\min}^{\frac{\varphi+1}{2}}(2\mathcal{L}(\mathcal{C})+\mathcal{O})V^{\frac{\varphi+1}{2}}(t).$$

By Lemma 1, system (6) is globally finite-time stability, and $T(z)$ is the settling time. Then according to Lemma 3, system (4) is globally finite-time stability under the pinning protocol (8), and the settling time $T(q)$ satisfies $T(q)\le T(z)+\sum_{k=1}^{n}c_k\tau_k$. That is system (4) can reach the FTC. □

Remark 1: Although there are many results on FTC of the MAS, as far as we know, there is few result on FTC for the leader-following multi-time-delay system over time-varying topology. That is to say, this work considers the consensus problem of the system with a leader, multiple time delays and time-varying networks synthetically.

Remark 2: Due to the complexity of the system with multiple time delays over time-varying topology, we adopt the algebraic theory, especially the algebraic transformation method, to transform the multi-time-delay system into the relatively simple system. For the disconnected network, we adopt the pinning control technique, which by controlling certain selected agents to control all the agents, to make the FTC.

Remark 3: Time delay is inevitable in the practical. For the MAS, due to the different structure and properties of agents, communication delay between different agents is usually different, therefore the study of systems with multiple time delays is important. There are many related applications, such as the cooperation of robotic system, the vehicle-road cooperative system, unmanned air vehicles and so on.

4. NUMERICAL EXAMPLES

This section gives several simulations to illustrate the correctness of the main results.

Consider system (4) with 10 followers, which are denoted by $i=1,\cdots,10$ and a leader $i=0$. Note that there is a spanning tree in graph $\mathcal{M}_1$ and $\mathcal{M}_2$ with the leader node being the root.

Firstly, consider system (4) over $\mathcal{M}_1$ as in Fig. 1. For simplicity, denote $\tau=[\tau_1,\tau_2,\cdots,\tau_{10}]^T$. The initial states of agents are $q_0(0)=10.1773$, $q_1(0)=4.8393$, $q_2(0)=10.3504$, $q_3(0)=2.8980$, $q_4(0)=6.2387$, $q_5(0)=1.8777$, $q_6(0)=8.3155$, $q_7(0)=1.8872$, $q_8(0)=5.0999$, $q_9(0)=7.1241$, $q_{10}(0)=9.2806$. Under the protocol (5), choose $K=1$, $h=1$, $b_k=1$, $(k=1,\cdots,10)$, $\varphi=0.5$, $\alpha_{ij}(t)=t+2$, $l_i(t)=2t+1$ $(\forall i,\ j=1,\cdots,10)$, and $\tau=[0.01,$

0.01, 0.01, 0.01, 0.01, 0.02, 0.02, 0.03, 0.03, 0.03]^T. The state trajectories are given in Fig. 2, which shows that the states of all followers achieve that of the leader in a finite time. That is under the protocol (5), system (4) over $\mathcal{M}_1$ achieves the FTC.

Secondly, consider system (4) over $\mathcal{M}_2$ as in Fig. 3. The initial states of agents are $q_0(0) = 4.8801$, $q_1(0) = 13.0861$, $q_2(0) = 13.4367$, $q_3(0) = 9.2940$, $q_4(0) = 10.4851$, $q_5(0) = 10.9491$, $q_6(0) = 4.3335$, $q_7(0) = 9.8978$, $q_8(0) = -0.7998$, $q_9(0) = 10.3240$, $q_{10}(0) = -0.1580$. Under the protocol (5), choose $K = 2$, $h = 1$, $b_k = 1$ $(k = 1, \cdots, 10)$, $\varphi = 0.8$, $\alpha_{ij}(t) = 3t+1$, $l_i(t) = t+2$ $(\forall\, i, j = 1, \cdots, 10)$, and $\tau = [0.5, 0.5, 0.5, 0.2, 0.2, 0.2, 0.6, 0.6, 0.6, 0.1]^T$. The agents' state trajectories are given in Fig. 4, which shows that the states of all followers achieve that of the leader in a finite time. That is under protocol (5), system (4) over $\mathcal{M}_2$ achieves the FTC.

Next, consider system (4) over networks $\mathcal{M}_3$ in Fig. 5. Note that there is no spanning tree with the leader being the root in $\mathcal{M}_3$. For system (4) with 3 followers, which are denoted by $i = 1, 2, 3$ and a leader $i = 0$, denote $\tilde{\tau} = [\tau_1, \tau_2, \tau_3]^T$. The initial states of agents are $q_0(0) = 1.1731$, $q(0) = [-4.3805,\ 8.9402,\ 2.3462]^T$. Under the pinning control protocol (8), choose $K = 1$, $h = 1$, $b_k = 10$ $(k = 1, 2, 3)$, $\varphi = 0.5$, $\alpha_{ij}(t) = t+2$ $(\forall i, j = 1, 2, 3)$, $P(t) = [2t+1, 0, 0]^T$, $W(t) = [0, 0, 0]^T$, and $\tilde{\tau} = [0.1, 0.2, 0.3]^T$. Then $\hat{P}(t) = [2t+1, 0, 0]^T$. The state trajectories $q_i(t)$ $(i = 0,$

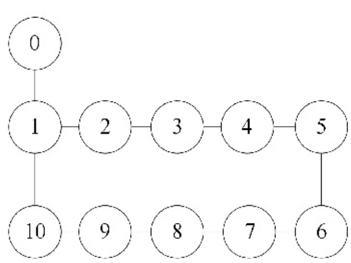

Fig. 1. Network graph $\mathcal{M}_1$.

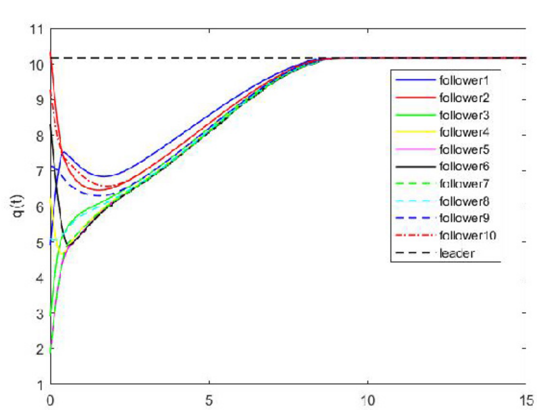

Fig. 2. State trajectories of the agents over $\mathcal{M}_1$.

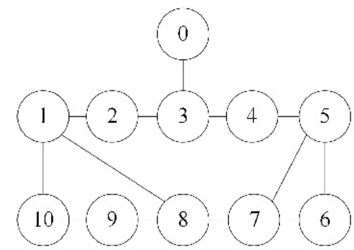

Fig. 3. Network graph $\mathcal{M}_2$.

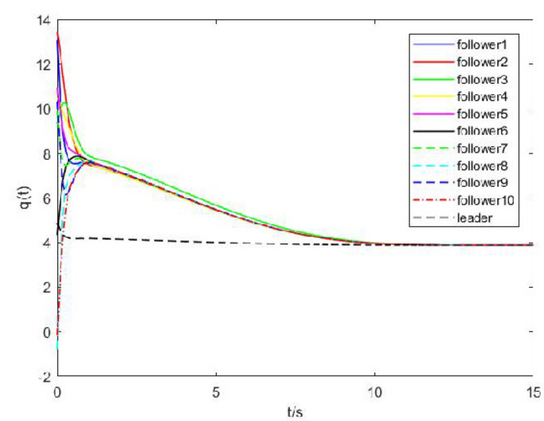

Fig. 4. State trajectories of the agents over $\mathcal{M}_2$.

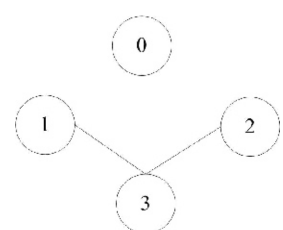

Fig. 5. Network graph $\mathcal{M}_3$.

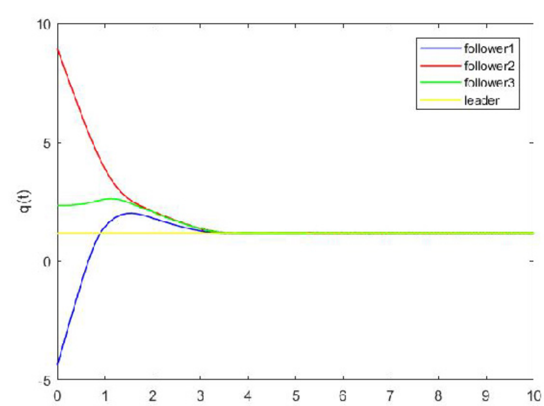

Fig. 6. State trajectories of the agents over $\mathcal{M}_3$ by pinning control.

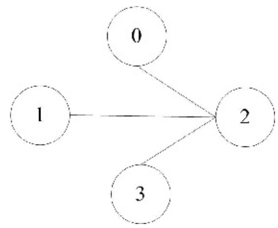

Fig. 7. Network graph $\mathcal{M}_4$.

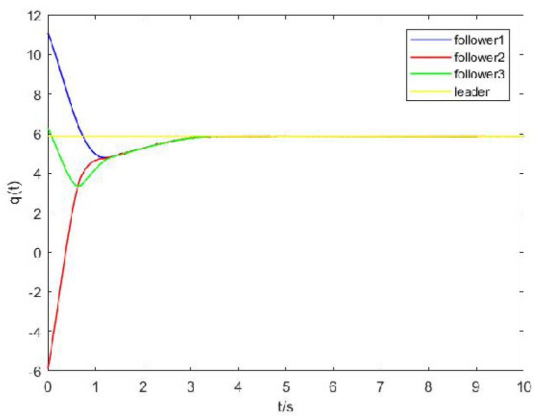

Fig. 8. State trajectories of the agents over $\mathcal{M}_4$.

1, 2, 3) are given in Fig. 6, which shows that the states of all followers achieve that of the leader in a finite time over the disconnected topology $\mathcal{M}_3$.

For system (4) over $\mathcal{M}_4$ as in Fig. 7, the initial states are $q_0(0) = 5.8654$, $q(0) = [11.0808, -5.8827, 6.2849]^T$. Choose $K = 1$, $h = 1$, $b_k = 10$ $(k = 1,2,3)$, $\varphi = 0.6$, $\alpha_{ij}(t) = 3t+1$ $(\forall i, j = 1, 2, 3)$, $P(t) = [0, 0, 0]^T$, $W(t) = [0, 2t+1, 0]^T$, $\tilde{\tau} = [0.1, 0.2, 0.3]^T$. Then $\hat{P}(t) = [0, 2t+1, 0]^T$. The state trajectories of all the agents are given in Fig. 8, which shows that the states of all followers achieve that of the leader in a finite time. That is system (4) can achieve the FTC.

For system (4) over $\mathcal{M}_3$, if there is no pinning control, that is $P(t) = [0,0,0]^T$ in protocol (8), choose the initial states $q_0(0) = 1.1731$, $q(0) = [-4.3805, 8.9402, 2.3462]^T$, and $K = 1$, $h = 1$, $b_k = 10$ $(k = 1, 2, 3)$, $\varphi = 0.5$, $\alpha_{ij}(t) = t+2$ $(i, j = 1, 2, 3)$, $\tilde{\tau} = [0.1, 0.2, 0.3]^T$. The state trajectories of all the agents are given in Fig. 9, which shows that the states of the followers can not achieve that of the leader in a finite time. That is without the pinning control, system (4) can not achieve the FTC over the disconnected network $\mathcal{M}_3$. Fig. 6 and Fig. 9 show that the method used in this work is more effective when the network is disconnected.

5. CONCLUSIONS

This paper studies the FTC of multi-time-delay LF-

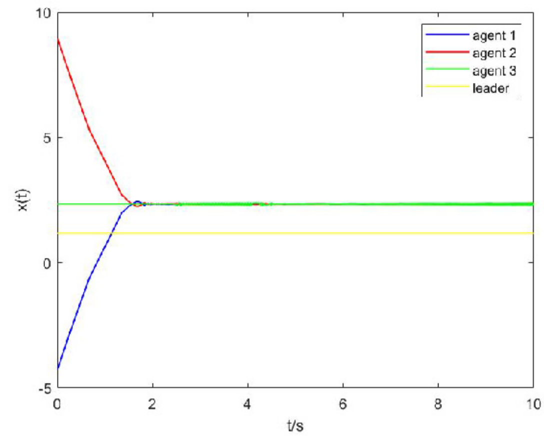

Fig. 9. State trajectories of the agents over $\mathcal{M}_3$ without pinning control.

MAS over time-varying topologies through the pinning control. Based on the Lyapunov function theory, graph theory and algebraic theory, sufficient conditions for the leader-following FTC are obtained. Several presented simulations verify the effectiveness of the adopted method. However, the related problem, such as swarming, flocking, or under the saturation control case, has not been considered. It is the future work to be done.

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