Synchronization route to weak chimera in four candle-flame oscillators

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Synchronization and chimera are examples of collective behavior observed in an ensemble of coupled nonlinear oscillators. Recent studies have focused on their discovery in systems with least possible number of oscillators. Here we present an experimental study revealing the synchronization route to weak chimera via quenching, clustering, and chimera states in a single system of four coupled candle-flame oscillators. We further report the discovery of multiphase weak chimera along with experimental evidence of the theoretically predicted states of in-phase chimera and antiphase chimera.

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I. INTRODUCTION

Synchronization leading to self-organization within a population of oscillators due to their mutual coupling has awed the human mind since time immemorial [1]. Following the observation of mutual adjustment of rhythms of pendulum clocks, Huygens in the 17th century named this behavior sympathy, later called synchronization [2], wherein all oscillators adjust their timescales to a common value upon coupling [1]. Subsequently both experimental and theoretical studies [3–5] have shown the existence of various types of coupled behavior apart from synchronization and oscillation quenching [4]. Such coupled behavior includes weak chimeras [6,7] and chimeras [8,9] in large populations (number of oscillators, N, ranging from 20 to 100) of oscillators.

Ashwin and Burylko [6] recently discovered the state of weak chimera, a symmetry-breaking phenomena found in minimal (three to six oscillators) networks of coupled phase oscillators. Weak chimera is defined as the state where two or more frequency-synchronized oscillators coexist with one or more oscillators having different frequencies with respect to the synchronized group. The pioneering experimental evidence of the weak chimera state was reported by Wojewoda *et al.* [7] in a system of three coupled pendula, followed by experimental and theoretical studies of pendulum-like nodes [10], electrochemical oscillators [11], and Stuart-Landau oscillators [9]. Chimera is one of the theoretically [3,12–19] and experimentally [5,8,13,20–25] well-studied cases of weak chimera. Discovered by Kuramoto and Battogtokh [3] in complex Ginzburg-Landau oscillators and named by Abrams and Strogatz [12], chimera is a symmetry-breaking phenomenon where oscillators having identical individual properties and coupling structure separate into phase-locked (synchronized) and phase-drifting (desynchronized) group of oscillators. Within the last decade, different types of chimera came to light such as clustered chimera [13,15,23,26,27], breathing chimera [14,15], symmetric and asymmetric chimera [16], metastable chimera [17], type I chimera and type II chimera [18], and virtual chimeras [19], characterized on the basis of instantaneous phase and amplitude relations between oscillators.

Apart from the theoretical studies discussed so far, experimental investigations of chimera have also been reported in literature. The pioneering experimental studies showing the existence of chimera in a natural system used coupled chemical oscillators [20] and coupled map lattices [21]. Further experimental studies observed the existence of chimera in mechanical oscillators, such as coupled pendula [22], metronomes [13], electrochemical oscillators [5,23], electronic oscillators [24], optical oscillators [8] and in thermoacoustic oscillators [25]. Chimera states have also been predicted for different global [28], weak nonlocal [3], and local couplings [29]. The pioneering study by Hart *et al.* [8] experimentally witnessed the existence of chimera in four globally coupled chaotic optoelectronic oscillators. which is the minimal network of oscillators required to support the state.

Although the phenomena of chimera, synchronization, and oscillation quenching have been widely studied, recent theoretical and experimental studies find interest in identifying their existence in systems with a minimum number (N ranging from three to six) of oscillators [8–11]. The insights obtained from such minimal systems can be equally applied to the dynamics of large, spatially extended systems. Further, the occurrence of these states has been identified in various governable systems, systems whose behavior can be externally controlled, such as optoelectronic oscillators [8], pendulum-like nodes [10], and mechanical oscillators [7] to name a

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few. However, the exhibition of all such states in a single self-governing system remains unreported. Therefore, the present study is a quest for the experimental evidence of such phenomena in an autonomous system with a minimum number of coupled candle-flame oscillators [30,31] by varying a single control parameter.

The experiments involving candle-flame oscillators, while being unconventional, are simple to conduct and can be used to simulate synchronization phenomena of coupled flames in more complicated systems such as can-annular combustors and the afterburner of a gas turbine engine [32]. The oscillatory nature of such coupled flames after interaction with the acoustic modes of the combustor results in the generation of large-amplitude self-sustained oscillations, known as thermoacoustic instability [33,34]. The presence of such oscillations is undesirable for safe operation of the engine; hence, the systems are always operated away from the stability margin of such instabilities [33].

Previous studies on coupled candle-flame oscillators, with two [30,31] and three [35] oscillators, show the existence of rich dynamical behaviors such as synchronization, amplitude death, and rotation mode. Subsequent studies [30,36,37] reported various possible reasons behind the exhibition of these coupled dynamics in candle-flame oscillators. Here we investigate the presence of several nonlinear phenomena such as clustering and chimeras (including weak chimera and bare minimum chimera) in a system of four coupled candle-flame oscillators, positioned in a rectangular topology.

II. EXPERIMENTAL SETUP

The setup consists of two platforms, one stationary and the other movable, each consisting of two candle-flame oscillators [Figs. 1(a) and 1(b)]. The distances between the centers of the oscillators, d_x and d_y, are varied as control parameters from 2 to 7 cm in steps of 0.5 cm. All experiments are conducted in a dark room with a quiescent environment. The candles used in the experiments are 10 cm long and have a diameter of 1 cm. Four such candles are bundled together and lit in order to form a single oscillating flame; such an oscillator is referred to as a candle-flame oscillator. As the distances between the oscillators are varied, they exhibit different dynamical states. These dynamics are captured using the high-speed imaging feature of an iPhone 5S outfitted with a CH* chemiluminescence filter (wavelength centered at 435 nm with a bandwidth of 10 nm). With the employment of the filter, the luminous intensity of the flame would correspond to the local heat release rate of the flame in each frame [38]. Images for each combination of d_x and d_y are captured at a frame rate of 119 fps for 10 s. The global heat release rate value for each candle-flame oscillator at a given instant (or frame) is obtained after isolating the oscillator and summing up the luminous intensity values (local heat release rate) of the oscillator. The global heat release rate values are then normalized with their respective instantaneous amplitudes obtained from the Hilbert transform of the signal [1]. Thus, four different time series, corresponding to each oscillator, are obtained for each combination of distances.

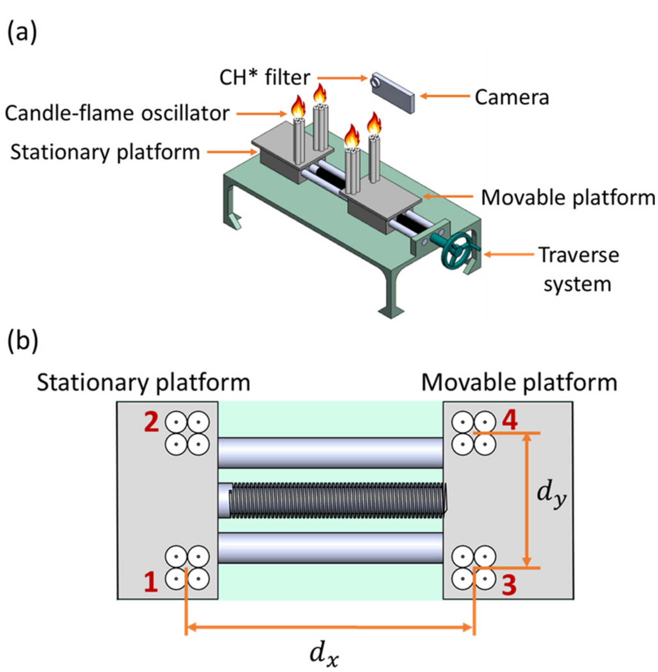

FIG. 1. (a) Isometric view and (b) top view of the experimental setup of four candle-flame oscillators.

III. RESULTS AND DISCUSSION

To gain an understanding of the various coupled dynamics exhibited by four candle-flame oscillators (explained hereafter) and SL oscillators (provided as Supplemental Material [39]), we examine the temporal variation of the normalized amplitude (I), wrapped instantaneous phase difference ($\Delta\phi$), and dominant frequencies (f) of each oscillator. The relative phase ($\Delta\phi$) is calculated for each combination of oscillators as the difference between the instantaneous phases, obtained from the Hilbert transform [1]. Oscillator pairs exhibiting synchronized behavior display a constant phase difference between them and oscillate at identical frequencies. On the other hand, desynchronized oscillators exhibit a phase-drifting behavior in time and oscillate at different frequencies. Various dynamical states observed for each combination of inter oscillator distances, shown by d_x and d_y in Fig. 1(b), are individually discussed below.

We witness the experimental evidence of weak chimera states in a coupled candle-flame oscillator system. A case of weak chimera is presented in Fig. 2(a), where a group of three synchronized oscillators {1, 3, and 4} coexists with a desynchronized oscillator 2, when $d_x = d_y = 7$ cm (see Supplemental Movie 1 [39]). Oscillators 1 and 3 are in-phase synchronized, and oscillator 4 exhibits an antiphase synchronization with them. In contrast, oscillator 2 separates itself as the desynchronized oscillator, exhibiting a phase-drifting behavior with the synchronized group of oscillators.

We report the discovery of a state of multiphase weak chimera, where the oscillators in a frequency-synchronized group exhibit different phase-locking behavior from the other group while retaining desynchrony between them. In our system, an antiphase synchronized oscillator pair coexists with an in-phase synchronized oscillator pair [Fig. 2(b)], while both pairs are desynchronized with each other (see

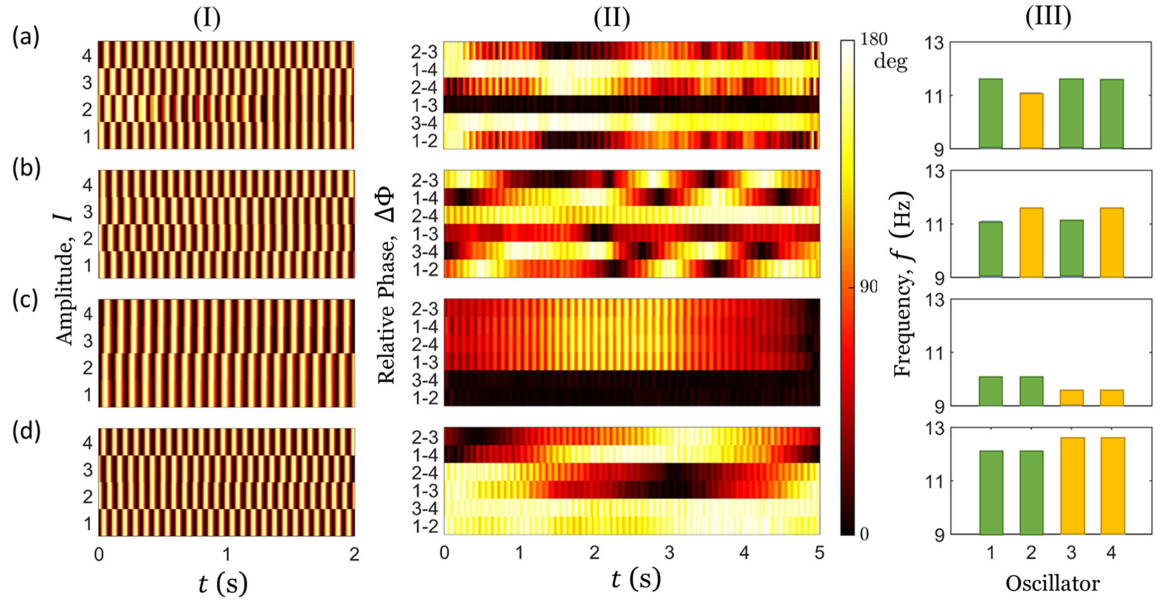

FIG. 2. The temporal variation of (I) amplitude of each candle-flame oscillator, (II) relative phase between all combinations of such oscillator pairs, and (III) the values of dominant frequencies of each oscillator for different states of coupled dynamics characterized as (a) weak chimera, (b) multiphase weak chimera, (c) in-phase chimera, and (d) antiphase chimera. Oscillators having the same color in (III) possess equal frequencies.

Supplemental Movie 2 [39]). The oscillators {1, 3} are in-phase synchronized, while the oscillators {2, 4} are antiphase synchronized, when $d_x = 5$ cm and $d_y = 7$ cm.

A recent theoretical study by Maistrenko *et al.* [10] showed the possibility of other states of weak chimera such as imperfect chimera, chaotic chimera, antiphase chimera, and in-phase chimera in a theoretical model of three pendulum-like nodes. Here we report the maiden experimental observation of such in-phase and antiphase chimera states in candle-flame oscillators. During in-phase chimera [Fig. 2(c)], the system divides into two pairs of in-phase synchronized oscillators, {1, 2} and {3, 4} while retaining desynchrony between those pairs (see Supplemental Movie 3 [39]) when $d_x = 7$ cm and $d_y = 4$ cm. The state of antiphase chimera [Fig. 2(d)] displays similarity to the state of in-phase chimera shown in Fig. 2(c) with the only difference being the synchronized pair of oscillators exhibiting an antiphase mode of synchronization. Such a state of antiphase chimera is observed for $d_x = 5$ cm and $d_y = 6$ cm (see Supplemental Movie 4 [39]).

Yet another specific case of weak chimera which is thoroughly investigated is the state of bare minimum chimera [8], where a system of four oscillators separates into pairs of synchronized and desynchronized oscillators. We observe the existence of such a chimera state at a distance of $d_x = d_y = 6$ cm (see Supplemental Movie 5 [39]). In Figs. 3(a) and 3(b), we see the oscillators {1, 3} that exhibit antiphase synchronization coexisting with the desynchronized pair of oscillators {2, 4}.

The chimera state found in candle-flame oscillators is observed to mediate between coherent states (clustering states or in-phase synchronized states), due to their low stability in small populations, commonly referred to as alternating chimera [40]. In Fig. 3(d) we observe that the system dynamics transition from the chimera state, with oscillators {3, 4} as the synchronized pair and oscillators {1, 2} as the desynchronized pair, to a clustering state where the oscillators separate into two clusters of synchronized oscillators (a detailed explanation is provided subsequently) and later return to the chimera state. During the chimera state, the oscillator pair {3, 4} exhibits nearly antiphase synchronization, and the other oscillators 1 and 2 remain desynchronized. As the dynamics transition to the clustering state, we observe that oscillators 1 and 2 are synchronized with oscillators 4 and 3, respectively, with a zero degree phase shift. The oscillator pair {1, 4} and {2, 3} form two clusters oscillating at a frequency of 11.5 Hz with antiphase synchronization between the clusters.

On the other hand, in Fig. 3(e), we observe that the chimera state is followed by the state of in-phase synchronization, which yet again transitions to the chimera states. The oscillator pair {1, 2} remains in-phase synchronized, having nearly zero degree phase shift throughout the time series. In contrast, oscillators 3 and 4 are desynchronized with themselves and with the synchronized pair during the chimera state. The oscillators are in-phase synchronized with themselves and the others and have an oscillation frequency of 10.8 Hz during the epoch of in-phase synchronization. We observe the existence of the chimera state for an average of 80 cycles in both cases of alternating chimera.

Hitherto, we have discussed the existence of symmetry-breaking states where we observe a combined existence of synchronized and desynchronized oscillators upon coupling. Hereon, we will discuss the presence of only synchronization states observed at various distances between coupled candle-flame oscillators. These states are observed when the oscillators are placed close to each other.

When the oscillators are very close to each other at $d_x = d_y = 2$ cm [Fig. 4(a)], we observe that every oscillator (oscillators 1 to 4) attains maximum and minimum amplitudes

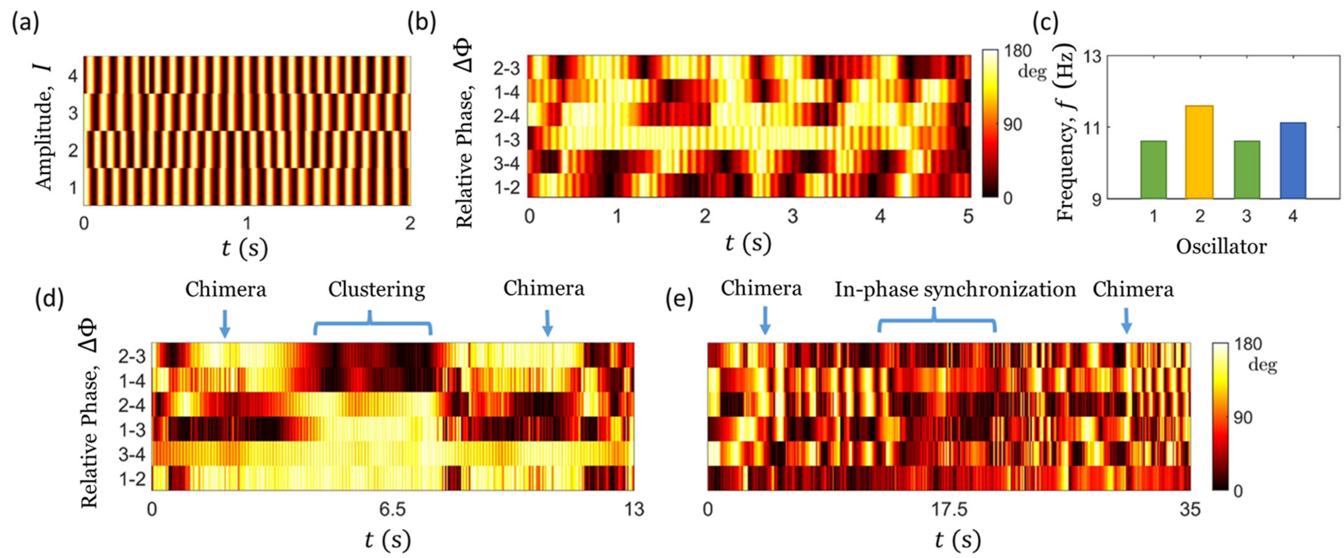

FIG. 3. The temporal variation in (a) the amplitude of each oscillator, (b) the relative phase between each pair of oscillators, and (c) frequencies of each oscillator. (d), (e) The plots of relative phase representing the states of alternating chimera states seen in the experiments where the existence of chimera is observed to mediate between the states of clustering and in-phase synchronization, respectively.

simultaneously, exhibiting in-phase synchronization [1] at a frequency of approximately 10.7 Hz (see Supplemental Movie 6 [39]). As the oscillators are positioned a small distance apart ($d_x = d_y = 4$ cm), the coupling between them leads to the simultaneous quenching of oscillations in all the oscillators (see Supplemental Movie 7 [39]). This state where all the oscillators occupy a homogeneous steady state due to mutual coupling is referred to as amplitude death [4]. The amplitude plot of this state [Fig. 4(b)] shows minor fluctuations around zero, highlighting the lack of oscillations in the system. The instantaneous Hilbert phases of these oscillators, although physically undefined due to the lack of narrow-band oscillations [1], do not show any particular trend for this state.

When the candle-flame oscillators are moved to a distance of $d_x = 1$ cm and $d_y = 4$ cm, the population of these oscillators separates into different clusters, depending on the values of instantaneous properties (amplitude and phase) of their signals [5,41]. Here the oscillators belonging to the same cluster exhibit equal instantaneous phases that are different from the other clusters; while all the oscillators in the population carry an identical frequency of approximately 11 Hz (see Supplemental Movie 8 [39]). In Fig. 5(a) the oscillator pairs {1, 2} and {3, 4} separate into two clusters which exhibit antiphase synchronization between them. An interchange in the distances d_x and d_y (i.e., $d_x = 4$ cm and $d_y = 1$ cm) results in the formation of a clustering state shown in Fig. 5(b). During this state of clustering, the oscillators belonging to the same clusters are now on different platforms.

Apart from the aforementioned types of clustering where adjacent oscillators on the rectangle group into a cluster, we also observe the occurrence of clustering between diagonal pair of oscillators ($d_x = 3$ cm and $d_y = 3$ cm). During diagonal clustering, the oscillators which exhibit in-phase synchronization would be on diagonally opposite ends of the rectangle (i.e., oscillator pairs {1,4} and {2,3} separate into two clusters), and the adjacent oscillators display antiphase synchronization [see Fig. 5(c)].

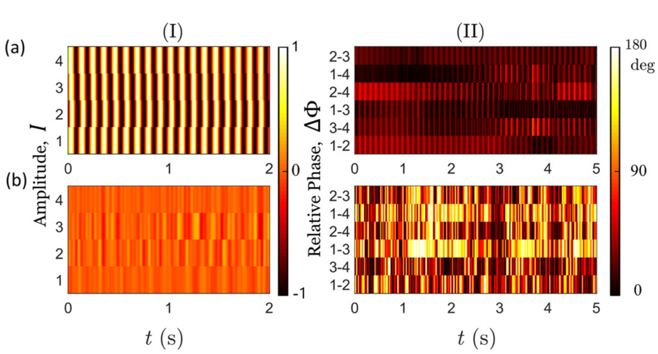

FIG. 4. Plots of temporal variation in (I) amplitude of each candle-flame oscillator and (II) instantaneous phase difference between all combinations of oscillator pairs for the states of (a) in-phase synchronization and (b) amplitude death.

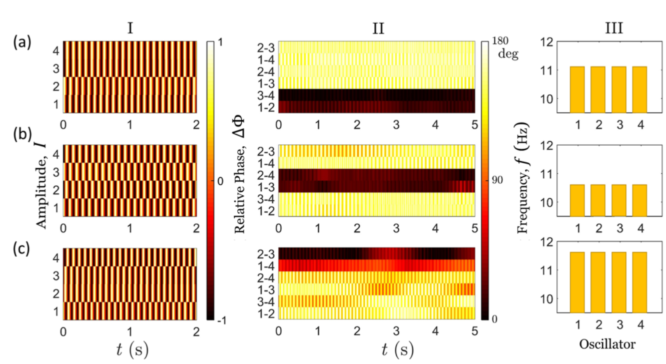

FIG. 5. Variation in (I) amplitude, (II) relative phase, and (III) dominant frequency of each candle-flame oscillator during different states of clustering. In all three types of clustering exhibited by four candle-flame oscillators, we observe the presence of two clusters, each consisting of two oscillators and antiphase synchronization between the clusters.

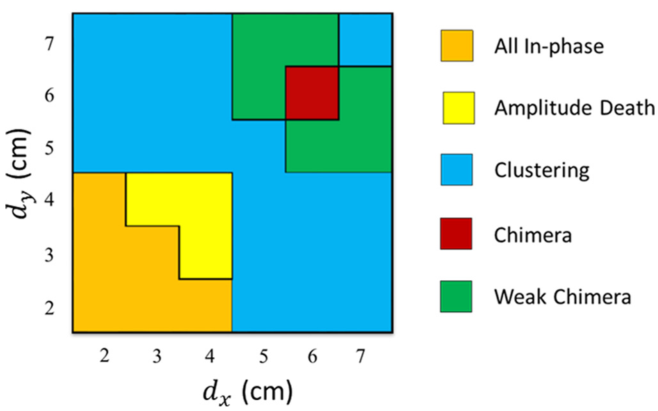

FIG. 6. Two-parameter bifurcation plots between distances d_x and d_y depicting the presence of various dynamical states observed in a system consisting of four candle-flame oscillators exhibiting limit cycle oscillations.

Having discussed all the observed prominent synchronization and symmetry-breaking states in an experimental four-oscillator system, let us now compare their behavior on a two-parameter bifurcation plot. Figure 6 shows the overall mapping of the coupled dynamics of candle-flame oscillators at various values of coupling parameters (d_x and d_y). This plot projects all the possible modes of coupled oscillations exhibited by four limit cycle oscillators considered in our study at a given combination of coupling parameters. It also helps us examine the various available routes to traverse from a given mode of oscillations to another and to smartly control the system dynamics by selection of the preferential route.

In nonlocally coupled candle-flame oscillators, the mutual interaction or coupling between them decreases with an increase in the distance [31]. In Fig. 6 we observe the exhibition of in-phase synchronization when the oscillators are placed very close to each other, due to the presence of strong interaction between them. When either d_x or d_y is increased keeping the other at a very low value, we observe various states of clustering due to the difference in the synchronization properties between pairs of oscillators. On the other hand, a slight increase in the values of both d_x and d_y leads to the exhibition of amplitude death in their dynamics. With further increase in the values of d_x and d_y, we observe the presence of desynchrony in the system, through the states of chimera and weak chimeras.

An interesting feature of the two-parameter bifurcation plot is its symmetry [Fig. 6]. We observe a spatial symmetry in the coupled dynamics of four candle-flame oscillators about the diagonal line of the bifurcation plot. An interchange in d_x and d_y would not make much significant change in the dynamics of the group of oscillators. This fact further confirms the repeatability of these experiments.

IV. CONCLUSION

To summarize, in the present study, we provide experimental evidence to the emergence of rich dynamical behaviors exhibited by a system with a minimal number ($N = 4$) of coupled candle-flame oscillators due to a change in their topological arrangement. Various coupled dynamics including in-phase synchronization, amplitude death, clustering, and chimeras are observed. We report the discovery of a state of multiphase weak chimera. Further, we provide experimental evidence of weak chimera states such as in-phase chimera and antiphase chimera, which were observed previously in theoretical studies. Hence, by varying the coupling between the oscillators, we bring forth various routes of transition from one dynamical state to another.

In many practical systems, one among these states is undesirable. For example, phase-locked (or synchronization) states leading to amplitude growth are unwanted in many oscillatory systems, such as thermoacoustic systems [42,43], pedestrians on the Millennium Bridge [44], ecological systems [45], the spread of epidemics [46], and epileptic seizures [47]. On the other hand, amplitude death proves dangerous in neural systems, causing the occurrence of Alzheimer's [48] and Parkinson's disease [49]. The difficulty in controlling the oscillatory states enhances during the partial synchronization states, such as chimera and weak chimera. Therefore, a transition from these undesirable states in such oscillatory systems to the desired state is possible with smart control of the coupling parameters.

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