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Synchronization Control of Memristive Multidirectional Associative Memory Neural Networks and Applications in Network Security Communication

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ABSTRACT In this paper, we investigate the synchronization in the mean square sense of memristive multidirectional associative memory neural networks with mixed time-varying delays and stochastic perturbations. In the proposed approach, the mixed delays include time-varying delays and distributed time delays. Sufficient criteria guaranteeing the synchronization of the drive-response system are derived based on the drive-response concept, the stochastic differential theory and Lyapunov function. With the removal of certain strict conditions of weight parameters, less conservative results are generated. To illustrate the performance of the proposed synchronization criteria, a secure communication scheme to realize secure data transmission is designed. Meanwhile, the effectiveness of the proposed theories is validated with numerical experiments.

INDEX TERMS Memristive multidirection associative memory, mixed delays, secure communication, stochastic perturbations, synchronization control.

I. INTRODUCTION

With the rapid development of network communication, secure transfers of a large amount of data have become a challenging task. Therefore, secure communication technologies have become very important tools. Recently, secure communication technologies based on chaotic systems and synchronization have been widely studied in [1]–[4]. In [1], the authors proposed a secure communication scheme based on parameter modulation via the synchronization of hyperchaotic systems. In [2], in order to improve the physical realization of secure communication based on chaotic systems, the authors proposed a novel chaos masking encoding method. The authors presented a method based on the state observer design, which applied to secure communications in [3]. The secure communication method was designed based on exponential synchronization, masking method and parameter modulation in [4]. Although the secure communication methods based on chaotic systems and synchronization have attracted the attentions of many researchers, there are few studies for memristive neural networks (MNNs) [5]. Therefore, it is meaningful to analyze secure communication methods based on MNNs.

Traditional biological neural networks lack the capability of reflecting variable synaptic weights when simulating associative memory of human brains. Therefore, memristors instead of resistors were employed in artificial neural networks to form MNNs [6]–[8]. In recent years, the dynamic behaviors of MNNs were studied in [9]–[11], in which the dynamic behaviors have been widely applied to associative memory [12], medical image processing [13], [14], etc..

Meanwhile, memristive bidirectional associate memory neural networks (BAMNNs) have been extensively studied in [15]–[17]. In addition, as an extension of BAMNNs, MAMNNs were proposed by Hagiwara [18] and their dynamic behaviors have been widely studied in [19]–[21]. However, there are few studies for memristive MAMNNs. Thus, it is necessary to study the dynamic behaviors of memristive MAMNNs.

In the real world, the stability and synchronization of chaotic systems play an important role due to their potential applications to image encryption [22], secure communication [23], secure image transmission [24], intelligent data analysis [25], etc.. However, the stability and synchronization of memristive MAMNNs are rarely reported in literatures. Thus, the stability and synchronization of memristive MAMNNs desire more research attentions.

It is well known that time delays and stochastic perturbations are ubiquitous in practical applications. The time delays are inevitable in the hardware implementation due to the switching of amplifiers. Various types of time delays, such as time-varying delays [26], discrete time delays [27], distributed delays [28], leakage delays [29], additive delays [30] and mixed delays [31] are often considered. Moreover, in real nervous systems, synaptic transmission is a noisy process, which is caused by random fluctuations from the release of neurotransmitters and other probabilistic causes. Therefore, stochastic perturbations are also inevitable in the real world. The related literatures [32], [33] take into account the effect of stochastic perturbations. Since time delays and stochastic perturbations may induce the instability or oscillation of systems, we require appropriate control strategies to make systems stable. Over the past few years, various control methods have been utilized for stabilization and synchronization [34]–[36]. Hence, it is meaningful to analyze the dynamic behaviors of MNNs with time delays and stochastic perturbations.

In this paper, we propose a new model by combining the characters of both MAMNNs and MNNs, which can simulate the associative memory process of human brains more effectively. The main contributions of this paper can be summarized in the following:

(1) We propose a novel stochastic memristive MAMNNs model, which includes time-varying delays, distributed delays and stochastic perturbations.

(2) Sufficient criteria guaranteeing the synchronization of the chaotic drive-response system are derived, which based on the drive-response concept, the stochastic differential theory and Lyapunov function.

(3) With the removal of certain strict conditions on the weight parameters and discuss the cases in detail, we obtain some less conservative results for the synchronization of the drive-response system.

(4) To illustrate the performance of the proposed synchronization criteria, we design a secure communication scheme based on chaotic memristive MAMNNs, which can realize secure data transmission.

The rest of this paper is organized as follows. The proposed memristive MAMNNs model with mixed delays and stochastic perturbations is introduced with some preliminaries in Section 2. Based on the drive-response concept, the stochastic differential theory and some inequality techniques, some sufficient criteria for ensuring the synchronization of drive-response system are obtained in section 3. A secure communication scheme is designed in Section 4. Numerical examples are discussed in Section 5, while Section 6 concludes this paper with some insights provided.

II. PRELIMINARIES

In this section, we introduce the following memristive MAMNNs with mixed delays :

$$\begin{aligned}\frac{dx_{ki}(t)}{dt} &= I_{ki} - d_{ki}(x_{ki}(t))x_{ki}(t)\\ &\quad+\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p} a_{pjki}(x_{ki}(t))f_{pj}(x_{pj}(t))\\ &\quad+\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p} b_{pjki}(x_{ki}(t))f_{pj}(x_{pj}(t-\tau_{pjki}(t)))\\ &\quad+\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p} c_{pjki}(x_{ki}(t))\int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds, \qquad (1)\end{aligned}$$

where $x_{ki}(t)$ denotes the voltage of the ith neuron in the field k, m is the total number of fields and n_p corresponds to the number of neurons in the field p. $f_{ki}(x)$ shows activation functions. $d_{ki}(x_{ki}(t))$, $a_{pjki}(x_{ki}(t))$, $b_{pjki}(x_{ki}(t))$ and $c_{pjki}(x_{ki}(t))$ denote the synptic connection weights. The time delays $\tau_{pjki}(t)$ and $\rho(t)$ are time-varying delays and distributed delay, respectively. I_{ki} represents the external input constants.

Throughout this paper, $co[\underline{\xi}, \overline{\xi}]$ denotes the convex closure on $[\underline{\xi}, \overline{\xi}]$. R^n represents n-dimensional Euclidean space. A column vector is defined as $col(x_{ki}) = (x_{11}, x_{12}, \cdots, x_{mn_m})^T$. Besides, the initial values of system (1) are given as follows: $\phi(s) = (\phi_{11}(s), \phi_{12}(s), \cdots, \phi_{mn_m}(s))^T \in C([-\tau, 0], R^n)$, in which $\tau = \max\limits_{1\leq p\leq m, p\neq k} \max\limits_{1\leq j\leq n_p} \{\tau_{pjki}(t), \rho(t)\}$.

Some notations are defined as follows: $\overline{d}_{ki} = \max\{\acute{d}_{ki}, \grave{d}_{ki}\}$, $\underline{d}_{ki} = \min\{\acute{d}_{ki}, \grave{d}_{ki}\}$, $\overline{a}_{pjki} = \max\{\acute{a}_{pjki}, \grave{a}_{pjki}\}$, $\underline{a}_{pjki} = \min\{\acute{a}_{pjki}, \grave{a}_{pjki}\}$, $\overline{b}_{pjki} = \max\{\acute{b}_{pjki}, \grave{b}_{pjki}\}$, $\underline{b}_{pjki} = \min\{\acute{b}_{pjki}, \grave{b}_{pjki}\}$, $\overline{c}_{pjki} = \max\{\acute{c}_{pjki}, \grave{c}_{pjki}\}$, $\underline{c}_{pjki} = \min\{\acute{c}_{pjki}, \grave{c}_{pjki}\}$, $0 \leq \tau_{pjki}(t) \leq \tau_1$, $0 \leq \rho(t) \leq \rho_1$, $\dot{\tau}_{pjki}(t) \leq \tau_2 < 1$.

According to the features of memristors and the current-voltage characteristics, for convenience, we define the following formulas by applying the set-valued mapping theorem

and the stochastic differential inclusion theorem.

$$co(d_{ki}(x_{ki}(t))) = \begin{cases} \acute{d}_{ki}, & |x_{ki}(t)| < \Gamma_{ki}, \\ co\{\acute{d}_{ki}, \grave{d}_{ki}\}, & |x_{ki}(t)| = \Gamma_{ki}, \\ \grave{d}_{ki}, & |x_{ki}(t)| > \Gamma_{ki}, \end{cases}$$

$$co(a_{pjki}(x_{ki}(t))) = \begin{cases} \acute{a}_{pjki}, & |x_{ki}(t)| < \Gamma_{ki}, \\ co\{\acute{a}_{pjki}, \grave{a}_{pjki}\}, & |x_{ki}(t)| = \Gamma_{ki}, \\ \grave{a}_{pjki}, & |x_{ki}(t)| > \Gamma_{ki}, \end{cases}$$

$$co(b_{pjki}(x_{ki}(t))) = \begin{cases} \acute{b}_{pjki}, & |x_{ki}(t)| < \Gamma_{ki}, \\ co\{\acute{b}_{pjki}, \grave{b}_{pjki}\}, & |x_{ki}(t)| = \Gamma_{ki}, \\ \grave{b}_{pjki}, & |x_{ki}(t)| > \Gamma_{ki}, \end{cases}$$

$$co(c_{pjki}(x_{ki}(t))) = \begin{cases} \acute{c}_{pjki}, & |x_{ki}(t)| < \Gamma_{ki}, \\ co\{\acute{c}_{pjki}, \grave{c}_{pjki}\}, & |x_{ki}(t)| = \Gamma_{ki}, \\ \grave{c}_{pjki}, & |x_{ki}(t)| > \Gamma_{ki}, \end{cases}$$

where the switching jumps $\Gamma_{ki} > 0$, for $k = 1, 2, \cdots, m$ and $i = 1, 2, \cdots, n_k$. $\acute{d}_{ki} > 0$, $\grave{d}_{ki} > 0$, $\acute{a}_{pjki}$, $\grave{a}_{pjki}$, $\acute{b}_{pjki}$, $\grave{b}_{pjki}$, $\acute{c}_{pjki}$ and $\grave{c}_{pjki}$ are constants. Obviously, $co\{\acute{d}_{ki}, \grave{d}_{ki}\} = [\underline{d}_{ki}, \overline{d}_{ki}]$, $co\{\acute{a}_{pjki}, \grave{a}_{pjki}\} = [\underline{a}_{pjki}, \overline{a}_{pjki}]$, $co\{\acute{b}_{pjki}, \grave{b}_{pjki}\} = [\underline{b}_{pjki}, \overline{b}_{pjki}]$ and $co\{\acute{c}_{pjki}, \grave{c}_{pjki}\} = [\underline{c}_{pjki}, \overline{c}_{pjki}]$, for k, $p = 1, 2, \cdots, m, p \neq k, i = 1, 2, \cdots, n_k, j = 1, 2, \cdots, n_p$.

According to the above definitions, system (1) can be written as follows

$$\begin{aligned} \frac{dx_{ki}(t)}{dt} \in\; & I_{ki} - co(d_{ki}(x_{ki}(t)))x_{ki}(t) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} co(a_{pjki}(x_{ki}(t)))f_{pj}(x_{pj}(t)) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} co(b_{pjki}(x_{ki}(t)))f_{pj}(x_{pj}(t-\tau_{pjki}(t))) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} co(c_{pjki}(x_{ki}(t))) \int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds, \end{aligned} \tag{2}$$

or equivalently, for $k = 1, 2, \cdots, m, p \neq k, i = 1, 2, \cdots, n_k$, there exist $\hat{d}_{ki}(x_{ki}(t)) \in co(d_{ki}(x_{ki}(t)))$, $\hat{a}_{pjki}(x_{ki}(t)) \in co(a_{pjki}(x_{ki}(t)))$, $\hat{b}_{pjki}(x_{ki}(t)) \in co(b_{pjki}(x_{ki}(t)))$ and $\hat{c}_{pjki}(x_{ki}(t)) \in co(c_{pjki}(x_{ki}(t)))$, such that

$$\begin{aligned} \frac{dx_{ki}(t)}{dt} =\; & I_{ki} - \hat{d}_{ki}(x_{ki}(t))x_{ki}(t) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} \hat{a}_{pjki}(x_{ki}(t))f_{pj}(x_{pj}(t)) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} \hat{b}_{pjki}(x_{ki}(t))f_{pj}(x_{pj}(t-\tau_{pjki}(t))) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} \hat{c}_{pjki}(x_{ki}(t)) \int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds. \end{aligned} \tag{3}$$

In this paper, system (2) or (3) is considered as the drive system. Then the corresponding response system is described as

$$\begin{aligned} \frac{dy_{ki}(t)}{dt} \in\; & I_{ki} + \mu_{ki}(t) - co(d_{ki}(y_{ki}(t)))y_{ki}(t) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} co(a_{pjki}(y_{ki}(t)))f_{pj}(y_{pj}(t)) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} co(b_{pjki}(y_{ki}(t)))f_{pj}(y_{pj}(t-\tau_{pjki}(t))) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} co(c_{pjki}(y_{ki}(t))) \int_{t-\rho(t)}^{t} f_{pj}(y_{pj}(s))ds, \end{aligned} \tag{4}$$

or equivalently, for $k = 1, 2, \cdots, m, p \neq k, i = 1, 2, \cdots, n_k$, there exist $\hat{d}_{ki}(y_{ki}(t)) \in co(d_{ki}(y_{ki}(t)))$, $\hat{a}_{pjki}(y_{ki}(t)) \in co(a_{pjki}(y_{ki}(t)))$, $\hat{b}_{pjki}(y_{ki}(t)) \in co(b_{pjki}(y_{ki}(t)))$ and $\hat{c}_{pjki}(y_{ki}(t)) \in co(c_{pjki}(y_{ki}(t)))$, such that

$$\begin{aligned} \frac{dy_{ki}(t)}{dt} =\; & I_{ki} + \mu_{ki}(t) - \hat{d}_{ki}(y_{ki}(t))y_{ki}(t) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} \hat{a}_{pjki}(y_{ki}(t))f_{pj}(y_{pj}(t)) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} \hat{b}_{pjki}(y_{ki}(t))f_{pj}(y_{pj}(t-\tau_{pjki}(t))) \\ & + \sum_{\substack{p=1,\\ p\neq k}}^{m} \sum_{j=1}^{n_p} \hat{c}_{pjki}(y_{ki}(t)) \int_{t-\rho(t)}^{t} f_{pj}(y_{pj}(s))ds, \end{aligned} \tag{5}$$

where $\mu_{ki}(t)$ represent the appropriate control inputs and

$$co(d_{ki}(y_{ki}(t))) = \begin{cases} \acute{d}_{ki}, & |y_{ki}(t)| < \Gamma_{ki}, \\ co\{\acute{d}_{ki}, \grave{d}_{ki}\}, & |y_{ki}(t)| = \Gamma_{ki}, \\ \grave{d}_{ki}, & |y_{ki}(t)| > \Gamma_{ki}, \end{cases}$$

$$co(a_{pjki}(y_{ki}(t))) = \begin{cases} \acute{a}_{pjki}, & |y_{ki}(t)| < \Gamma_{ki}, \\ co\{\acute{a}_{pjki}, \grave{a}_{pjki}\}, & |y_{ki}(t)| = \Gamma_{ki}, \\ \grave{a}_{pjki}, & |y_{ki}(t)| > \Gamma_{ki}, \end{cases}$$

$$co(b_{pjki}(y_{ki}(t))) = \begin{cases} \acute{b}_{pjki}, & |y_{ki}(t)| < \Gamma_{ki}, \\ co\{\acute{b}_{pjki}, \grave{b}_{pjki}\}, & |y_{ki}(t)| = \Gamma_{ki}, \\ \grave{b}_{pjki}, & |y_{ki}(t)| > \Gamma_{ki}, \end{cases}$$

$$co(c_{pjki}(y_{ki}(t))) = \begin{cases} \acute{c}_{pjki}, & |y_{ki}(t)| < \Gamma_{ki}, \\ co\{\acute{c}_{pjki}, \grave{c}_{pjki}\}, & |y_{ki}(t)| = \Gamma_{ki}, \\ \grave{c}_{pjki}, & |y_{ki}(t)| > \Gamma_{ki}. \end{cases}$$

The initial values of system (4) are given as follows : $\Phi(s) = (\Phi_{11}(s), \Phi_{12}(s), \cdots, \Phi_{mn_m}(s))^T \in C([-\tau, 0], R^n)$, in which $\tau = \max\{\tau_1, \rho_1\}$.

Based on the discussions above, a class of memristive MAMNNs model with mixed time-varying delays and stochastic perturbations is proposed as follows:

$$\begin{aligned}dx_{ki}(t) = \Big[&I_{ki} - d_{ki}(x_{ki}(t))x_{ki}(t)\\&+ \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p} a_{pjki}(x_{ki}(t))f_{pj}(x_{pj}(t))\\&+ \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p} b_{pjki}(x_{ki}(t))f_{pj}(x_{pj}(t-\tau_{pjki}(t)))\\&+ \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p} c_{pjki}(x_{ki}(t))\int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds\Big]dt\\&+ \sigma_{ki}\Big(t, x_{pj}(t), x_{pj}(t-\tau_{pjki}(t))\Big)d\omega(t), \qquad (6)\end{aligned}$$

where $\sigma_{ki}\Big(t, x_{pj}(t), x_{pj}(t-\tau_{pjki}(t))\Big)$ is the noise intensity and $d\big(\omega_{11}(t), \omega_{12}(t), \cdots, \omega_{mn_m}(t)\big)$ represents a standard Brown notions defined on the probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$. $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ is a complete probability space with a natural filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $E(\cdot)$ denotes the expectation operator with the respect to the given probability measure P.

In this paper, system (6) is considered as the drive system. Then the corresponding response system is described as

$$\begin{aligned}dy_{ki}(t) = \Big[&I_{ki} - d_{ki}(y_{ki}(t))y_{ki}(t)\\&+ \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p} a_{pjki}(y_{ki}(t))f_{pj}(y_{pj}(t))\\&+ \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p} b_{pjki}(y_{ki}(t))f_{pj}(y_{pj}(t-\tau_{pjki}(t)))\\&+ \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p} c_{pjki}(y_{ki}(t))\int_{t-\rho(t)}^{t} f_{pj}(y_{pj}(s))ds\Big]dt\\&+ \sigma_{ki}\Big(t, y_{pj}(t), y_{pj}(t-\tau_{pjki}(t))\Big)d\omega(t). \qquad (7)\end{aligned}$$

In order to obtain our main results, we define the synchronization error of the system as follows:

$$e_{ki}(t) = y_{ki}(t) - x_{ki}(t),$$

where the initial values of error systems are defined as follows: $\Psi(s) = \Phi(s) - \phi(s) = (\Psi_{11}(s), \Psi_{12}(s), \cdots, \Psi_{21}(s), \cdots, \Psi_{mn_m}(s))^T \in C([-\tau, 0], R^n)$, in which $\tau = \max\{\tau_1, \rho_1\}$.

Assumption 1: For $k = 1, 2, \cdots, m, i = 1, 2, \cdots, n_k$, $\forall s_1, s_2 \in R$ and $s_1 \neq s_2$, the activation functions $f_{ki}(\cdot)$ are odd bounded and satisfy the Lipschitz condition

$$|f_{ki}(s_1) - f_{ki}(s_2)| \leq L_{ki}|s_1 - s_2|,$$
$$|f_{ki}(\cdot)| \leq F,$$

where L_{ki} and F are nonnegative constants.

Assumption 2: For $k = 1, 2, \cdots, m, i = 1, 2, \cdots, n_k$, the noise intensity $\sigma_{ki}\Big(t, x_{pj}(t), x_{pj}(t-\tau_{pjki}(t))\Big)$ satisfies the linear growth condition with $\sigma(t, 0, 0) = 0$ and there exist two nonnegative constants α and β, such that

$$\begin{aligned}&\Big|\sigma_{ki}\big(t, x_{pj}(t), x_{pj}(t-\tau_{pjki}(t))\big)\Big|^2\\&\qquad\leq \alpha|x_{pj}(t)|^2 + \beta|x_{pj}(t-\tau_{pjki}(t))|^2.\end{aligned}$$

Definition 1: For any initial values $\Psi(s) \in L^2_{\mathscr{F}_0}([-\tau, 0]; R^n)$, if there exist $\lambda > 0$ and $M > 0$ such that

$$E(|e_{ki}(t)|^2) \leq Me^{-\lambda t}\sup_{-\tau\leq s\leq 0} E(|\Psi(s)|^2), t \geq 0,$$

then the equilibrium point of system is exponentially stable in the mean square.

III. MAIN RESULTS

In this section, we will design proper controllers $\mu_{ki}(t)$ to realize exponential synchronization of memristive MAMNNs with mixed delays and stochastic perturbations. In the following, our main results are described:

A. SYNCHRONIZATION CONTROL OF MEMERISTIVE MAMNNS WITH MIXED DELAYS AND WITHOUT STOCHASTIC PERTURBATION

In this subsection, we investigate the memristive MAMNNs with mixed delays and without stochastic perturbations.

Theorem 1: Suppose that the Assumption 1 holds, the nonlinear controllers are designed as $\mu_{ki}(t) = h_{ki}e_{ki}(t) - \eta_{ki}sign(e_{ki}(t))$. If there exist constants h_{ki} and $\eta_{ki} > 0$, such that

$$\begin{aligned}h_{ki} \leq \min\Big\{&\acute{d}_{ki} - \frac{1}{2}\sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}[\acute{a}^2_{pjki}L^2_{pj} + \acute{b}^2_{pjki}L^2_{pj}\\&+ \rho_1\acute{c}^2_{pjki}L^2_{pj} + 1 + \frac{1}{1-\tau_2} + \rho_1],\\&\times\grave{d}_{ki} - \frac{1}{2}\sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}[\grave{a}^2_{pjki}L^2_{pj} + \grave{b}^2_{pjki}L^2_{pj}\\&+ \rho_1\grave{c}^2_{pjki}L^2_{pj} + 1 + \frac{1}{1-\tau_2} + \rho_1]\Big\}, \qquad (8)\end{aligned}$$

$$\begin{aligned}\eta_{ki} > &\big|\acute{d}_{ki} - \grave{d}_{ki}\big|\Gamma_{ki} + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\big|\acute{a}_{pjki} - \grave{a}_{pjki}\big|L_{pj}\Gamma_{pj}\\&+ F\sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\Big[\big|\acute{b}_{pjki} - \grave{b}_{pjki}\big| + \rho_1\big|\acute{c}_{pjki} - \grave{c}_{pjki}\big|\Big], \qquad (9)\end{aligned}$$

then the drive system (2) and response system (4) are mean square exponential synchronization.

Proof: Please see Appendix A.

Remark 1: There are some previous related works about synchronization of MNNs under the following

conditions [37]:

$$
\begin{aligned}
&co[\underline{d}_{ki}, \overline{d}_{ki}]y_{pj} - co[\underline{d}_{ki}, \overline{d}_{ki}]x_{pj} \\
&\subseteq co[\underline{d}_{ki}, \overline{d}_{ki}](y_{pj} - x_{pj}), co[\underline{a}_{pjki}, \overline{a}_{pjki}]f_{pj}(y_{pj}(t)) \\
&\quad - co[\underline{a}_{pjki}, \overline{a}_{pjki}]f_{pj}(x_{pj}(t)) \\
&\subseteq co[\underline{a}_{pjki}, \overline{a}_{pjki}](f_{pj}(y_{pj}(t)) - f_{pj}(x_{pj}(t))), \\
&\quad \times co[\underline{b}_{pjki}, \overline{b}_{pjki}]f_{pj}(y_{pj}(t - \tau_{pjki}(t))) \\
&\quad - co[\underline{b}_{pjki}, \overline{b}_{pjki}]f_{pj}(x_{pj}(t - \tau_{pjki}(t))) \\
&\subseteq co[\underline{b}_{pjki}, \overline{b}_{pjki}](f_{pj}(y_{pj}(t - \tau_{pjki}(t))) - f_{pj}(x_{pj}(t - \tau_{pjki}(t)))), \\
&\quad \times co[\underline{c}_{pjki}, \overline{c}_{pjki}] \int_{t-\rho(t)}^{t} f_{pj}(y_{pj}(s))ds - co[\underline{c}_{pjki}, \overline{c}_{pjki}] \\
&\quad \times \int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds \\
&\subseteq co[\underline{c}_{pjki}, \overline{c}_{pjki}](\int_{t-\rho(t)}^{t} f_{pj}(y_{pj}(s))ds - \int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds).
\end{aligned}
$$

It is easy to know that the above conditions hold when $x_{ki}(t)$ and $y_{ki}(t)$ have same signs, or $x_{ki}(t) = 0$ or $y_{ki}(t) = 0$. Meanwhile, the switching jumps Γ_{ki} are ignored in [37]. Compared with the results obtained in [37], with the removal of certain strict conditions and discuss the cases in detail, the results we obtained are less conservative.

Corollary 1: Suppose that the Assumption 1 holds, the nonlinear controllers are designed as $\mu_{ki}(t) = h_{ki}e_{ki}(t) - \eta_{ki}sign(e_{ki}(t))$. If there exist constants h_{ki} and $\eta_{ki} > 0$, such that

$$
\begin{aligned}
h_{ki}(t) \leq \min\Big\{&\acute{d}_{ki} - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\acute{a}_{pjki}^2 L_{pj}^2 + \acute{b}_{pjki}^2 L_{pj}^2 \\
&+ 1 + \frac{1}{1-\tau_2}), \grave{d}_{ki} - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\grave{a}_{pjki}^2 L_{pj}^2 \\
&+ \grave{b}_{pjki}^2 L_{pj}^2 + 1 + \frac{1}{1-\tau_2})\Big\},
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\eta_{ki} > &\left|\acute{d}_{ki} - \grave{d}_{ki}\right|\Gamma_{ki} + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\left|\acute{a}_{pjki} - \grave{a}_{pjki}\right|L_{pj}\Gamma_{pj} \\
&+ \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\left|\acute{b}_{pjki} - \grave{b}_{pjki}\right|F,
\end{aligned} \tag{11}
$$

then the drive system (2) and response system (4) are mean squre exponential synchronization.

Proof: Let the distributed delay $\rho(t) = 0$. The process of proof is similar to Theorem 1, so it is omitted here.

Corollary 2: Suppose that the Assumption 1 holds, the linear controllers are described as $\mu_{ki}(t) = h_{ki}e_{ki}(t)$. If there exists a constant h_{ki}, such that

$$
h_{ki}(t) \leq D_{ki} - \frac{1}{2}\sum_{\substack{p=1\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(A_{pjki}^2 L_{pj}^2 + B_{pjki}^2 L_{pj}^2 + 1 + \frac{1}{1-\tau_2}), \tag{12}
$$

where $D_{ki} = \min\{|\underline{d}_{ki}|, |\overline{d}_{ki}|\}$, $A_{kipj} = \max\{|\underline{a}_{kipj}|, |\overline{a}_{kipj}|\}$, $B_{pjki} = \max\{|\underline{b}_{kipj}|, |\overline{b}_{kipj}|\}$, then the drive system (2) and response system (4) are mean squre exponential synchronization.

Proof: The proof process is similar to the Corollary 1, so it is omitted here.

B. SYNCHRONIZATION CONTROL OF MEMERISTIVE MAMNNS WITH MIXED DELAYS AND STOCHASTIC PERTURBATION

In this subsection, we investigate the memristive MAMNNs with mixed delays and stochastic perturbations, in which the nonlinear controllers are showed as $\mu_{ki}(t) = h_{ki}e_{ki}(t) - \eta_{ki}sign(e_{ki}(t))$.

Theorem 2: Suppose that the Assumptions 1 and 2 hold. Then the drive system (6) and response system (7) are mean square exponential synchronization, if there exist constants h_{ki} and $\eta_{ki} > 0$, such that

$$
\begin{aligned}
h_{ki} \leq \min\Big\{&\acute{d}_{ki} - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}[\acute{a}_{pjki}^2 L_{pj}^2 + \acute{b}_{pjki}^2 L_{pj}^2 \\
&+ \rho_1 \acute{c}_{pjki}^2 L_{pj}^2 + 1 + \alpha + \frac{1+\beta}{1-\tau_2} + \rho_1], \\
&\times \grave{d}_{ki} - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}[\grave{a}_{pjki}^2 L_{pj}^2 + \grave{b}_{pjki}^2 L_{pj}^2 \\
&+ \rho_1 \grave{c}_{pjki}^2 L_{pj}^2 + 1 + \alpha + \frac{1+\beta}{1-\tau_2} + \rho_1]\Big\},
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
\eta_{ki} > &\left|\acute{d}_{ki} - \grave{d}_{ki}\right|\Gamma_{ki} + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\left|\acute{a}_{pjki} - \grave{a}_{pjki}\right|L_{pj}\Gamma_{pj} \\
&+ F\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\Big[\left|\acute{b}_{pjki} - \grave{b}_{pjki}\right| + \rho_1\left|\acute{c}_{pjki} - \grave{c}_{pjki}\right|\Big].
\end{aligned} \tag{14}
$$

Proof: Please see Appendix B.

Corollary 3: Suppose that the Assumptions 1 and 2 hold. Then the drive system (6) and response system (7) are mean square exponential synchronization, if there exist constants h_{ki} and $\eta_{ki} > 0$, such that

$$
\begin{aligned}
h_{ki}(t) \leq \min\Big\{&\acute{d}_{ki} - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}[\acute{a}_{pjki}^2 L_{pj}^2 + \acute{b}_{pjki}^2 L_{pj}^2 \\
&+ 1 + \alpha + \frac{1+\beta}{1-\tau_2}], \grave{d}_{ki} - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}[\grave{a}_{pjki}^2 L_{pj}^2 \\
&+ \grave{b}_{pjki}^2 L_{pj}^2 + 1 + \alpha + \frac{1+\beta}{1-\tau_2}]\Big\},
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\eta_{ki} > &\left|\acute{d}_{ki} - \grave{d}_{ki}\right|\Gamma_{ki} + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\left|\acute{a}_{pjki} - \grave{a}_{pjki}\right|L_{pj}\Gamma_{pj} \\
&+ \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\left|\acute{b}_{pjki} - \grave{b}_{pjki}\right|F.
\end{aligned} \tag{16}
$$

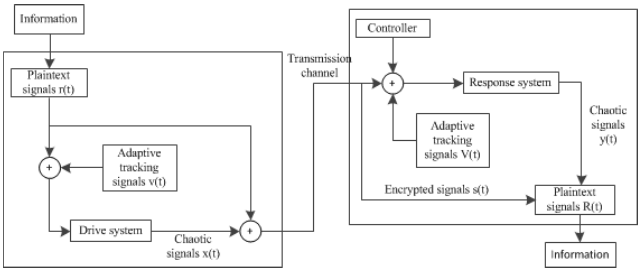


FIGURE 1. **The process of secure communication.**

Proof: Let the distributed delay $\rho(t) = 0$. The process of proof is similar to Theorem 2, so it is omitted here.

Remark 2: In Theorem 1, we only consider the effect of time delays on the synchronization of system. However, we consider the effects of time delays and stochastic perturbations on the synchronization of system in Theorem 2.

Remark 3: In practical applications, the effect of stochastic perturbations on the synchronization of system is inevitable. Therefore, we introduce stochastic perturbations to make the system more realistic in Theorem 2.

IV. DESIGN PROCESS OF SECRET COMMUNICATION SCHEME

Secure transfers of a large amount of data have always been a hot topic. In this section, we propose a secure communication scheme based on the synchronization criteria. The specific process of security communication is shown in Fig.1.

1) Based on the memristive MAMNNs model and the drive-response concept, we design a proper drive system X and the corresponding response system Y.
2) An error system is constructed and the system can be synchronized by designing suitable linear controllers based on Corollary 2.
3) Transmitter: suppose the plaintext signals to be sent are $r_{ki}(t)$. The plaintext signals $r_{ki}(t)$ and the adaptive tracking signals $v_{ki}(t)$ are introduced into the drive system X to generate the corresponding chaotic signals $x_{ki}(t)$. The chaotic signals $x_{ki}(t)$ are superimposed with the plaintext signals $r_{ki}(t)$ to produce encrypted transmission signals $s_{ki}(t) = x_{ki}(t) + r_{ki}(t)$.
4) Receiver: the received encrypted signals $s_{ki}(t)$, adaptive tracking signals $V_{ki}(t)$ and linear controllers are introduced into the response system Y to generate corresponding chaotic signals $y_{ki}(t)$. Then, according to the chaotic signals $y_{ki}(t)$ and received encrypted signals $s_{ki}(t)$, the decrypted plaintext signals $R_{ki}(t) = s_{ki}(t) - y_{ki}(t)$ are obtained.

Remark 4: The chaotic path of the memristive MAMNNs used in the secure communication scheme is more complicated, which improves the confidentiality of the network transmission. In the transmission process, only one transmission signal can be realized, the physical realization is more convenient and universal.

V. NUMERICAL SIMULATION

In this section, in order to illustrate the performance of the proposed synchronization criteria, several numerical examples are given to illustrate the effectiveness of our proposed results.

Example 1: We consider the following memristive MAMNNs with mixed delays and without stochastic perturbations, there are three fields and one neuron in each field.

$$\begin{aligned}\frac{dx_{k1}(t)}{dt} &= I_{k1} - d_{k1}(x_{k1}(t))x_{k1}(t) \\ &\quad + \sum_{\substack{p=1,\\ p\neq k}}^{3} a_{p1k1}(x_{k1}(t))f_{p1}(x_{p1}(t)) \\ &\quad + \sum_{\substack{p=1,\\ p\neq k}}^{3} b_{p1k1}(x_{k1}(t))f_{p1}(x_{p1}(t-\tau_{p1k1}(t))) \\ &\quad + \sum_{\substack{p=1,\\ p\neq k}}^{3} c_{p1k1}(x_{k1}(t))\int_{t-\rho(t)}^{t} f_{p1}(x_{p1}(s))ds, \qquad (17)\end{aligned}$$

where

$$d_{11}(x_{11}(t)) = \begin{cases}1.2, & |x_{11}| \le \Gamma_{11},\\ 1.3, & |x_{11}| > \Gamma_{11},\end{cases}$$

$$d_{21}(x_{21}(t)) = \begin{cases}0.3, & |x_{11}| \le \Gamma_{21},\\ 0.4, & |x_{11}| > \Gamma_{21},\end{cases}$$

$$d_{31}(x_{31}(t)) = \begin{cases}0.5, & |x_{31}| \le \Gamma_{31},\\ 0.7, & |x_{31}| > \Gamma_{31},\end{cases}$$

$$a_{1121}(x_{21}(t)) = \begin{cases}-0.92, & |x_{21}(t)| \le \Gamma_{21},\\ 0.33, & |x_{21}(t)| > \Gamma_{21},\end{cases}$$

$$a_{1131}(x_{31}(t)) = \begin{cases}0.24, & |x_{31}(t)| \le \Gamma_{31},\\ 0.37, & |x_{31}(t)| > \Gamma_{31},\end{cases}$$

$$a_{2111}(x_{11}(t)) = \begin{cases}-0.6, & |x_{11}(t)| \le \Gamma_{11},\\ 0.4, & |x_{11}(t)| > \Gamma_{11},\end{cases}$$

$$a_{2131}(x_{31}(t)) = \begin{cases}1.12, & |x_{31}(t)| \le \Gamma_{31},\\ 0.36, & |x_{31}(t)| > \Gamma_{31},\end{cases}$$

$$a_{3111}(x_{11}(t)) = \begin{cases}0.98, & |x_{11}(t)| \le \Gamma_{11},\\ 0.1, & |x_{11}(t)| > \Gamma_{11},\end{cases}$$

$$a_{3121}(x_{21}(t)) = \begin{cases}-0.58, & |x_{21}(t)| \le \Gamma_{21},\\ 0.12, & |x_{21}(t)| > \Gamma_{21},\end{cases}$$

$$b_{1121}(x_{21}(t)) = \begin{cases}0.36, & |x_{21}(t)| \le \Gamma_{21},\\ 0.21, & |x_{21}(t)| > \Gamma_{21},\end{cases}$$

$$b_{1131}(x_{31}(t)) = \begin{cases}-0.52, & |x_{31}(t)| \le \Gamma_{31},\\ 0.62, & |x_{31}(t)| > \Gamma_{31},\end{cases}$$

$$b_{2111}(x_{11}(t)) = \begin{cases}0.85, & |x_{11}(t)| \le \Gamma_{11},\\ 0.29, & |x_{11}(t)| > \Gamma_{11},\end{cases}$$

$$b_{2131}(x_{31}(t)) = \begin{cases} 0.85, & |x_{31}(t)| \le \Gamma_{31}, \\ -0.39, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$b_{3111}(x_{11}(t)) = \begin{cases} -0.98, & |x_{11}(t)| \le \Gamma_{11}, \\ -0.95, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$b_{3121}(x_{21}(t)) = \begin{cases} -0.65, & |x_{21}(t)| \le \Gamma_{21}, \\ -0.72, & |x_{21}(t)| > \Gamma_{21}. \end{cases}$$

$$c_{1121}(x_{21}(t)) = \begin{cases} -0.52, & |x_{21}(t)| \le \Gamma_{21}, \\ 0.47, & |x_{21}(t)| > \Gamma_{21}, \end{cases}$$

$$c_{1131}(x_{31}(t)) = \begin{cases} 0.44, & |x_{31}(t)| \le \Gamma_{31}, \\ 0.48, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$c_{2111}(x_{11}(t)) = \begin{cases} -0.46, & |x_{11}(t)| \le \Gamma_{11}, \\ -0.2, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$c_{2131}(x_{31}(t)) = \begin{cases} -0.55, & |x_{31}(t)| \le \Gamma_{31}, \\ -0.47, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$c_{3111}(x_{11}(t)) = \begin{cases} 0.68, & |x_{11}(t)| \le \Gamma_{11}, \\ 0.35, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$c_{3121}(x_{21}(t)) = \begin{cases} 0.81, & |x_{21}(t)| \le \Gamma_{21}, \\ -0.67, & |x_{21}(t)| > \Gamma_{21}. \end{cases}$$

Let $\Gamma_{11} = \Gamma_{21} = \Gamma_{31} = 1$. We set the action functions as $f_{ki}(x) = \tanh(x)$. The time-varying delays and distributed delays are $\tau_{pjki}(t) = 0.5\cos(t) + 0.5$ and $\rho(t) = 0.5\sin(t) + 0.5$, respectively. According to Assumption 1, we have $L_{ki} = L_{pj} = 1$, $F = 1$. By calculating, we get $\tau_1 = 1$, $\tau_2 = 0.5$ and $\rho_1 = 1$. The initial values are set as $[x_{11}(t), x_{21}(t), x_{31}(t)] = [1.05, 0.25, -0.75]$, $[y_{11}(t), y_{21}(t), y_{31}(t)] = [0.3, 0.45, 0.2]$.

Fig.2 represents the drive system (2) without stochastic perturbations. It has chaotic attractor with the initial values given above. Fig.3 depicts the state trajectories of the drive system (2) and the response system (4). According to the conditions of Theorem 1, the nonlinear controllers are set as $\mu_{11}(t) = -3e_{11}(t) - 4sign(e_{11}(t))$, $\mu_{21}(t) = -4e_{21}(t) - 5sign(e_{21}(t))$, $\mu_{31}(t) = -3e_{31}(t) - 4sign(e_{31}(t))$.

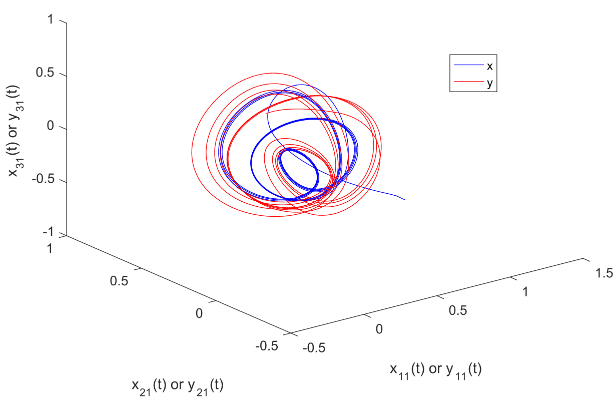


FIGURE 2. Phase trajectories of system (2) (corresponds to x) and system (4) (corresponds to y) with mixed delays and without stochastic perturbations.

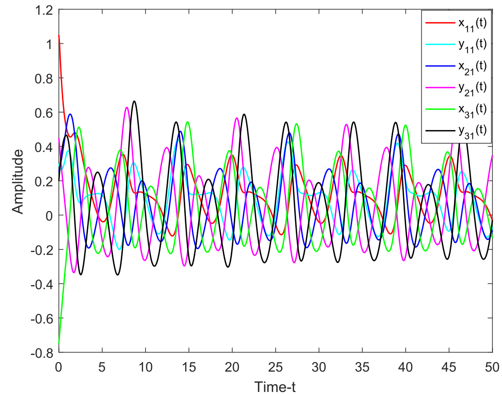


FIGURE 3. State trajectories of drive system (2) (corresponds to x) and response system (4) (corresponds to y).

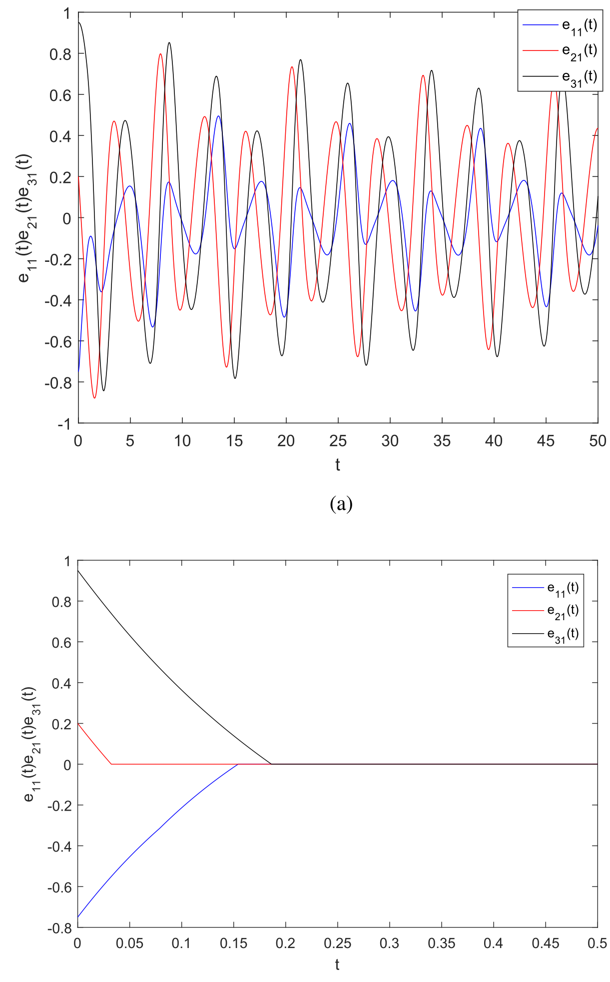


FIGURE 4. State trajectories of error between the drive system (2) and the response system (4) without stochastic perturbations. . (a) The error without controller. (b) The error with nonlinear controller.

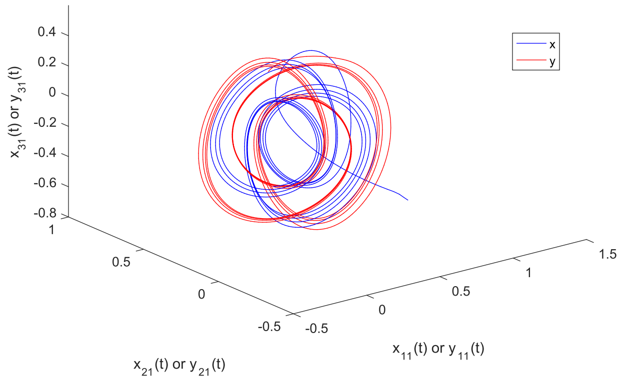


FIGURE 5. **Phase trajectories of system (2) (corresponds to x) and system (4)(corresponds to y) without distributed time delays and stochastic perturbations.**

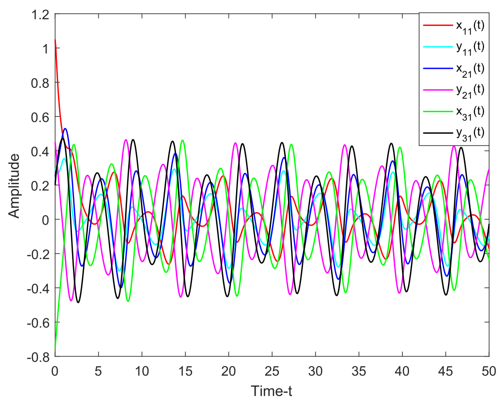
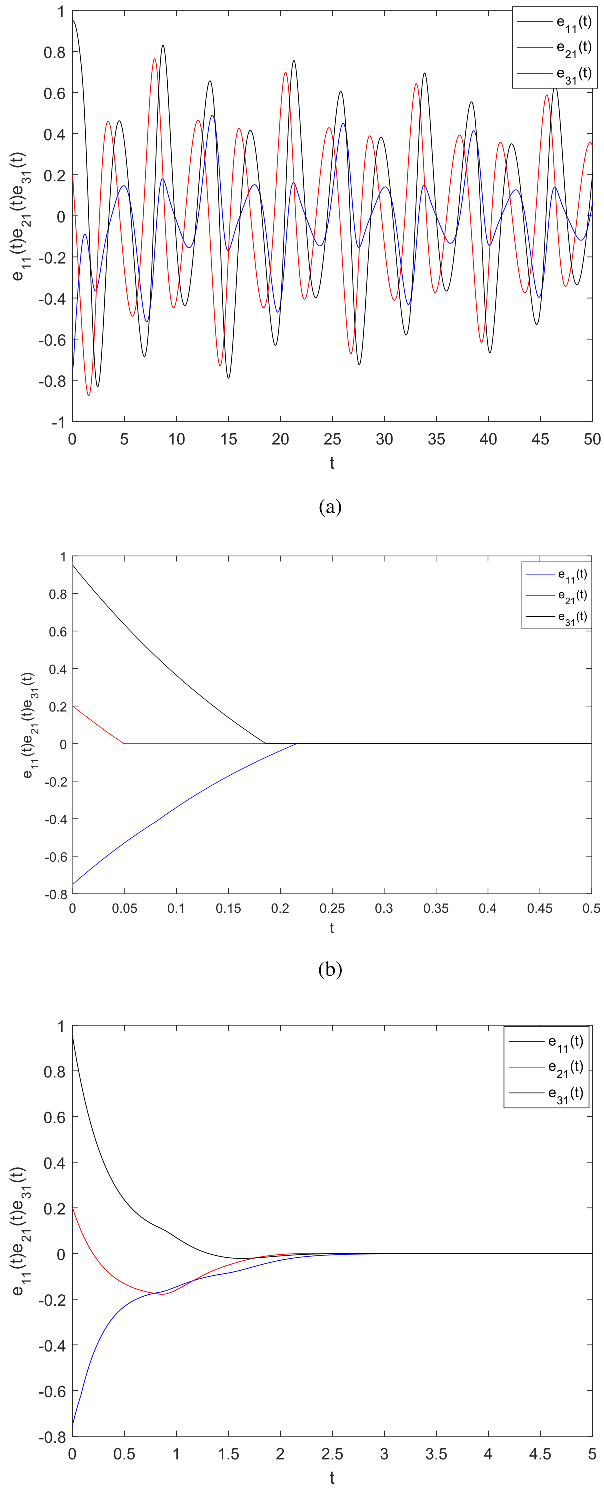


FIGURE 6. **State trajectories of drive system (2) (corresponds to x) and response system (4) (corresponds to y).**

Then (a) and (b) in Fig.4 describe the state trajectories of the error system without controllers and with nonlinear controllers, respectively. It implies that the drive system (2) and the corresponding response system (4) are mean square exponential synchronization.

Then we investigate system (2) and system (4) without distributed delays and stochastic perturbations. Under the same parameters, Fig.5 represents the drive system (2) without distributed delays and stochastic perturbations. It has chaotic attractor with the initial values given above. Fig.6 depicts the state trajectories of system (2) and system (4). According to the conditions of Corollary 1, the nonlinear controllers are set as $\mu_{11}(t) = -2e_{11}(t) - 3sign(e_{11}(t))$, $\mu_{21}(t) = -3e_{21}(t) - 3sign(e_{21}(t))$, $\mu_{31}(t) = -3e_{31}(t) - 4sign(e_{31}(t))$. According to the conditions of Corollary 2, the linear controllers are set as $\mu_{11}(t) = -2e_{11}(t)$, $\mu_{21}(t) = -3e_{21}(t)$, $\mu_{31}(t) = -3e_{31}(t)$. Then Fig.7 describes the state trajectories of the error system without controllers, with nonlinear controllers and with linear controllers, it implies that the drive system (2) and the corresponding response system (4) are mean square exponential synchronization.



(a)

(b)

(c)

FIGURE 7. **State trajectories of error between the drive system (2) and the response system (4) without distributed time delays and stochastic perturbations. (a) The error without controller. (b) The error with nonlinear controller. (c) The error with linear controller.**

Example 2: We consider the following stochastic memristive MAMNNs with mixed delays and stochastic perturbations, there are three fields and one neuron in

each field.

$$dx_{k1}(t) = \Big[I_{k1} - d_{k1}(x_{k1}(t))x_{k1}(t) + \sum_{\substack{p=1,\\ p\neq k}}^{3} a_{p1k1}(x_{k1}(t))f_{p1}(x_{p1}(t)) + \sum_{\substack{p=1,\\ p\neq k}}^{3} b_{p1k1}(x_{k1}(t))f_{p1}(x_{p1}(t-\tau_{p1k1}(t))) + \sum_{\substack{p=1,\\ p\neq k}}^{3} c_{p1k1}(x_{k1}(t))\int_{t-\rho(t)}^{t} f_{p1}(x_{p1}(s))ds\Big]dt + \sigma_{k1}\Big(t, x_{p1}(t), x_{p1}(t-\tau_{p1k1}(t))\Big)d\omega(t), \quad (18)$$

where

$$d_{11}(x_{11}(t)) = \begin{cases} 1.3, & |x_{11}| \le \Gamma_{11}, \\ 1.4, & |x_{11}| > \Gamma_{11}, \end{cases}$$

$$d_{21}(x_{21}(t)) = \begin{cases} 1.1, & |x_{11}| \le \Gamma_{21}, \\ 1.2, & |x_{11}| > \Gamma_{21}, \end{cases}$$

$$d_{31}(x_{31}(t)) = \begin{cases} 1.5, & |x_{31}| \le \Gamma_{31}, \\ 1.7, & |x_{31}| > \Gamma_{31}, \end{cases}$$

$$a_{1121}(x_{21}(t)) = \begin{cases} -0.25, & |x_{21}(t)| \le \Gamma_{21}, \\ 0.32, & |x_{21}(t)| > \Gamma_{21}, \end{cases}$$

$$a_{1131}(x_{31}(t)) = \begin{cases} 0.26, & |x_{31}(t)| \le \Gamma_{31}, \\ 0.36, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$a_{2111}(x_{11}(t)) = \begin{cases} -1.3, & |x_{11}(t)| \le \Gamma_{11}, \\ 1.4, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$a_{2131}(x_{31}(t)) = \begin{cases} 1.54, & |x_{31}(t)| \le \Gamma_{31}, \\ 0.32, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$a_{3111}(x_{11}(t)) = \begin{cases} 1.8, & |x_{11}(t)| \le \Gamma_{11}, \\ 1.1, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$a_{3121}(x_{21}(t)) = \begin{cases} -0.68, & |x_{21}(t)| \le \Gamma_{21}, \\ 0.12, & |x_{21}(t)| > \Gamma_{21}, \end{cases}$$

$$b_{1121}(x_{21}(t)) = \begin{cases} 0.32, & |x_{21}(t)| \le \Gamma_{21}, \\ 0.24, & |x_{21}(t)| > \Gamma_{21}, \end{cases}$$

$$b_{1131}(x_{31}(t)) = \begin{cases} -0.54, & |x_{31}(t)| \le \Gamma_{31}, \\ 0.42, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$b_{2111}(x_{11}(t)) = \begin{cases} 1.48, & |x_{11}(t)| \le \Gamma_{11}, \\ 1.19, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$b_{2131}(x_{31}(t)) = \begin{cases} 0.35, & |x_{31}(t)| \le \Gamma_{31}, \\ -0.49, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$b_{3111}(x_{11}(t)) = \begin{cases} -0.88, & |x_{11}(t)| \le \Gamma_{11}, \\ -0.75, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$b_{3121}(x_{21}(t)) = \begin{cases} -0.45, & |x_{21}(t)| \le \Gamma_{21}, \\ -0.22, & |x_{21}(t)| > \Gamma_{21}. \end{cases}$$

$$c_{1121}(x_{21}(t)) = \begin{cases} 0.72, & |x_{21}(t)| \le \Gamma_{21}, \\ 0.42, & |x_{21}(t)| > \Gamma_{21}, \end{cases}$$

$$c_{1131}(x_{31}(t)) = \begin{cases} -0.54, & |x_{31}(t)| \le \Gamma_{31}, \\ 0.68, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$c_{2111}(x_{11}(t)) = \begin{cases} -0.86, & |x_{11}(t)| \le \Gamma_{11}, \\ 0.8, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$c_{2131}(x_{31}(t)) = \begin{cases} 0.84, & |x_{31}(t)| \le \Gamma_{31}, \\ 0.72, & |x_{31}(t)| > \Gamma_{31}, \end{cases}$$

$$c_{3111}(x_{11}(t)) = \begin{cases} -0.58, & |x_{11}(t)| \le \Gamma_{11}, \\ 0.63, & |x_{11}(t)| > \Gamma_{11}, \end{cases}$$

$$c_{3121}(x_{21}(t)) = \begin{cases} -0.76, & |x_{21}(t)| \le \Gamma_{21}, \\ -0.82, & |x_{21}(t)| > \Gamma_{21}. \end{cases}$$

Let $\Gamma_{11} = \Gamma_{21} = \Gamma_{31} = 1$. We set the action functions as $f_{ki}(x) = \tanh(x)$. The time-varying delays and distributed delays are $\tau_{pjki}(t) = 0.5\cos(t) + 0.5$ and $\rho(t) = 0.5\sin(t) + 0.5$, respectively. According to Assumption 1, we have $L_{ki} = L_{pj} = 1$, $F = 1$. By calculating, we get $\tau_1 = 1$, $\tau_2 = 0.5$ and $\rho_1 = 1$. We set the stochastic perturbations are $\sigma_{ki}(t, x_{pj}(t), x_{pj}(t-\tau_{pjki}(t))) = -x_{pj}(t) - 0.2x_{pj}(t-\tau_{pjki}(t))$, then we have $\alpha = 2$, $\beta = 0.1$. The initial values are set as $[x_{11}(t), x_{21}(t), x_{31}(t)] = [1.05, 0.25, -0.75]$, $[y_{11}(t), y_{21}(t), y_{31}(t)] = [0.3, 0.45, 0.2]$.

Fig.8 represents the drive system (6) with mixed delays and stochastic perturbations. It has chaotic attractor with the initial values given above. Fig.9 depicts the state trajectories of the drive system (6) and the response system (7). According to the conditions of Theorem 2, the nonlinear controllers are set as $\mu_{11}(t) = -7e_{11}(t) - 7sign(e_{11}(t))$, $\mu_{21}(t) = -3e_{21}(t) - 3sign(e_{21}(t))$, $\mu_{31}(t) = -4e_{31}(t) - 5sign(e_{31}(t))$. Then Fig.10 describes the state trajectories of error system

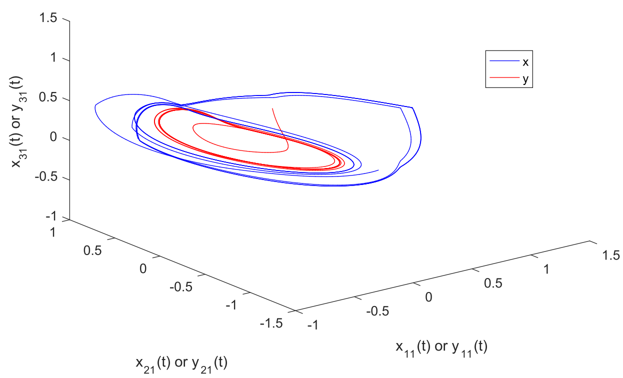


FIGURE 8. Phase trajectories of system (6) (corresponds to x) and system (7) (corresponds to y) with mixed delays and stochastic perturbations.

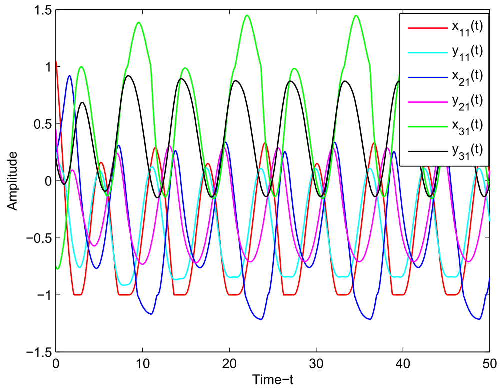


FIGURE 9. State trajectories of system (6) (corresponds to x) and system (7) (corresponds to y).

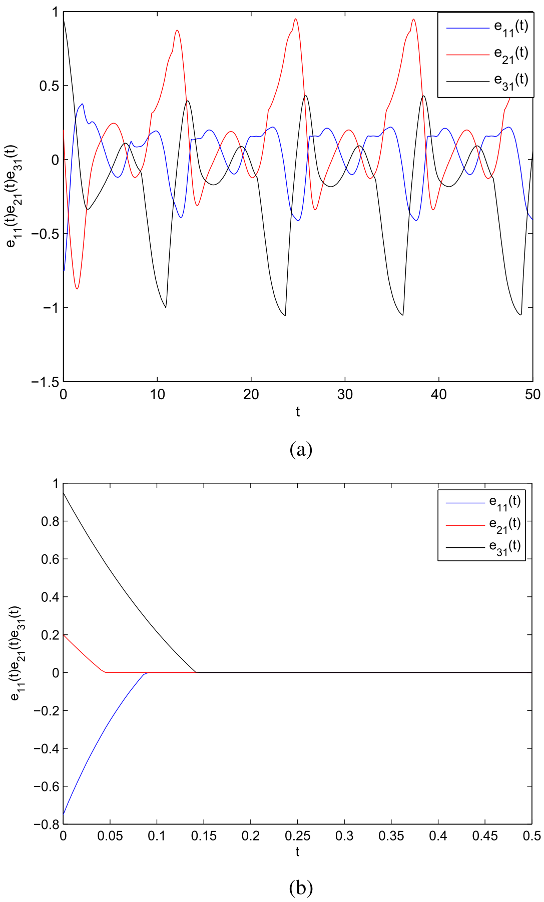


FIGURE 10. State trajectories of error between the drive system (6) and the response system (7) with mixed delays and stochastic perturbations. (a) The error without controller. (b) The error with nonlinear controller.

without controllers and with nonlinear controllers, it implies that the drive system (6) and the corresponding response system (7) are mean square exponential synchronization.

Then we investigate the drive system (6) and the response system (7) without distributed time delays. Under the same parameters, we set the stochastic perturbations are $\sigma_{ki}(t, x_{pj}(t), x_{pj}(t-\tau_{pjki}(t))) = -1.4x_{pj}(t)-0.2x_{pj}(t-\tau_{pjki}(t))$,

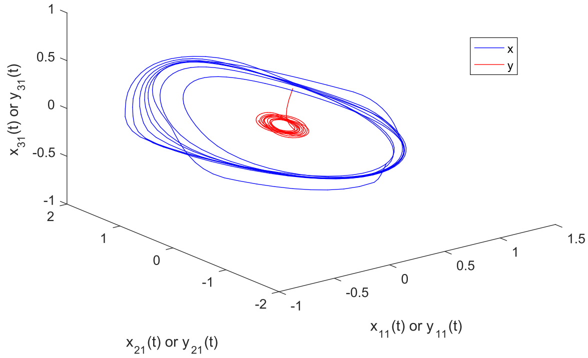


FIGURE 11. Phase trajectories of system (6) (corresponds to x) and system (7) (corresponds to y) without distributed time delays.

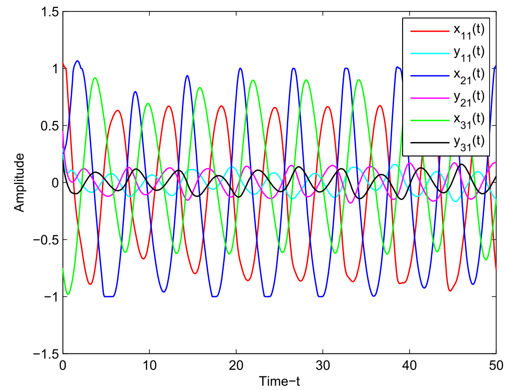


FIGURE 12. State trajectories of system (6) (corresponds to x) and system (7) (corresponds to y).

then we have $\alpha = 4$, $\beta = 0.1$. Then the drive system (6) has chaotic attractor with the initial values which can be seen in Fig.11. Fig.12 depicts the state trajectories of the drive system (6) and the response system (7). According to the conditions of Corollary 3, the nonlinear controllers are set as $\mu_{11}(t) = -7e_{11}(t) - 4sign(e_{11}(t))$, $\mu_{21}(t) = -3e_{21}(t) - 2sign(e_{21}(t))$, $\mu_{31}(t) = -4e_{31}(t) - 4sign(e_{31}(t))$. Then Fig.13 describes the state trajectories of error system without controllers and with nonlinear controllers, it implies that the drive system (6) and the corresponding response system (7) are mean square exponential synchronization under the controller.

Example 3: In this example, the secret communication based on the memristive MAMNNs is realized by hiding the signals in the chaotic system. We consider the memristive MAMNNs without distributed delays and stochastic perturbation, there are three fields and one neuron in each field. In the following, the specific steps are displayed:

1) Based on the drive-response concept, we choose the driver system and response system without distributed delays and stochastic perturbations.
2) An error system is constructed as follows:

$$e_{ki}(t) = y_{ki}(t) - x_{ki}(t),$$

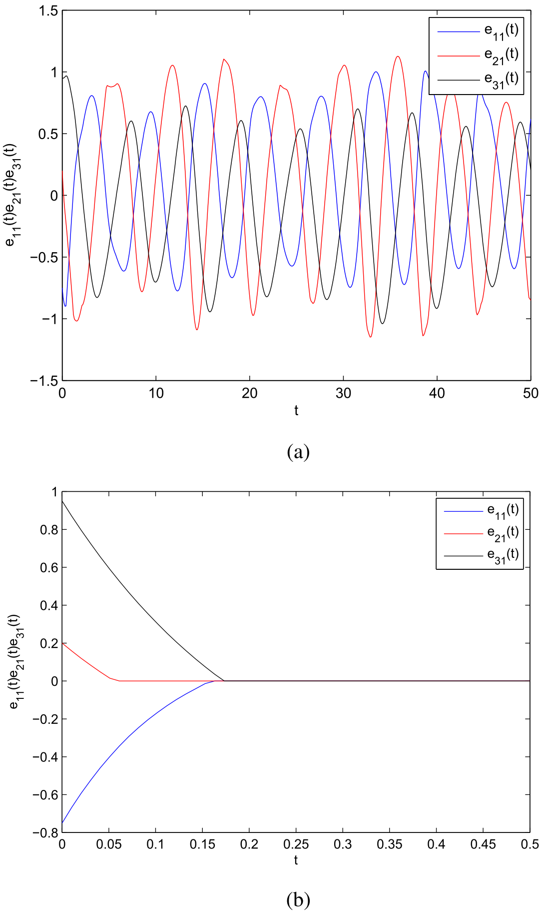


FIGURE 13. State trajectories of error between the drive system (6) and the response system (7) without distributed time delays. (a) The error without controller. (b) The error with nonlinear controller.

and according to the conditions of Corollary 2, the linear controllers are set as $\mu_{11}(t) = -3e_{11}(t)$, $\mu_{21}(t) = -2e_{21}(t)$, $\mu_{31}(t) = -3e_{31}(t)$.

3) Transmitter: suppose the plaintext signals to be sent are $r_{11}(t) = 0.05\sin(t)$, $r_{21}(t) = 0.1\cos(0.8t) + 0.06\sin(t)$ and $r_{31}(t) = 0.2\sin(t) - 2\tanh(t)$. The plaintext signals $r_{ki}(t)$ and the adaptive tracking signals $v_{ki}(t)$ are introduced into the drive system to generate the corresponding chaotic signal $x_{ki}(t)$ as follows:

$$\begin{cases} \dfrac{dx_{k1}(t)}{dt} & = I_{k1} - d_{k1}(x_{k1}(t))x_{k1}(t) \\ & \quad + \displaystyle\sum_{\substack{p=1,\\ p\neq k}}^{3} a_{p1k1}(x_{k1}(t))f_{p1}(x_{p1}(t)) \\ & \quad + \displaystyle\sum_{\substack{p=1,\\ p\neq k}}^{3} b_{p1k1}(x_{k1}(t))f_{p1}(x_{p1}(t - \tau_{p1k1}(t))) \\ & \quad + q_{k1} * (r_{k1}(t) - v_{k1}(t)), \\ \dfrac{dv_{k1}(t)}{dt} & = \hat{q}_{k1} * (r_{k1}(t) - v_{k1}(t)), \end{cases} \tag{19}$$

where the parameters are set the same as in Example 1, $q_{11} = q_{21} = q_{31} = 0.1$, $\hat{q}_{11} = \hat{q}_{31} = \hat{q}_{31} = 1$. The initial values of $v_{11}(t)$, $v_{21}(t)$ and $v_{31}(t)$ are $v_{11}(0) = v_{21}(0) = v_{31}(0) = 0$. The chaotic signals $x_{ki}(t)$ are superimposed with the plaintext signals $r_{ki}(t)$ to produce encrypted transmission signals $s_{ki}(t) = x_{ki}(t) + r_{ki}(t)$.

4) Receiver: the received encrypted signals $s_{ki}(t)$, adaptive tracking signals $V_{ki}(t)$ and linear controller are introduced into the response system to generate corresponding chaotic signals $y_{ki}(t)$. So the corresponding response system is defined as follows:

$$\begin{cases} \dfrac{dy_{k1}(t)}{dt} & = I_{k1} - d_{k1}(y_{k1}(t))y_{k1}(t) \\ & \quad + \displaystyle\sum_{\substack{p=1,\\ p\neq k}}^{3} a_{p1k1}(y_{k1}(t))f_{p1}(y_{p1}(t)) \\ & \quad + \displaystyle\sum_{\substack{p=1,\\ p\neq k}}^{3} b_{p1k1}(y_{k1}(t))f_{p1}(y_{p1}(t - \tau_{p1k1}(t))) \\ & \quad + \mu_{k1}(t) + q_{k1} * (s_{k1}(t) - y_{k1}(t) - V_{k1}(t)), \\ \dfrac{dV_{k1}(t)}{dt} & = \hat{q}_{k1} * (s_{k1}(t) - y_{k1}(t) - V_{k1}(t)), \end{cases} \tag{20}$$

where the initial values of $V_{11}(t)$, $V_{21}(t)$ and $V_{31}(t)$ are $V_{11}(0) = V_{21}(0) = V_{31}(0) = 0$. Then, according to the chaotic signals $y_{ki}(t)$ and received encrypted signals $s_{ki}(t)$, the decrypted plaintext signals $R_{ki}(t) = s_{ki}(t) - y_{ki}(t)$ are obtained.

The (a) in Fig.14 represents time response curves of the plaintext signals $r_{k1}(t)$, the decrypted signals $R_{k1}(t)$ and encrypted transmission signals $s_{k1}(t)$ of three fields, respectively. The (b) in Fig.14 represents time response curves of the plaintext signal $r(t)$, the decrypted signal $R(t)$,and encrypted transmission signal $s(t)$ of each field, respectively. The (c) in Fig.14 describes the error between the plaintext signals $r_{k1}(t)$ and the decrypted signals $R_{k1}(t)$ of three fields, respectively.

Remark 5: Compared with literature [5], we utilize the memristive MAMNNs model to encrypt the signals, which makes the chaotic characteristics of signals more complicated. Since three fields and one neuron in each field are considered, we can transmit three signals at the same time, which improves the utilization of the networks.

Remark 6: Compared with traditional methods of secure communication [1]–[4], we use memristors instead of resistances in artificial neural networks, which makes the system have the nonlinear and "memory" characteristics of memristors. At the same time, due to the complex structural characteristics of MAMNNs, we use memristive MAMNNs to realize secure communication, which will improve the security of information transmission.

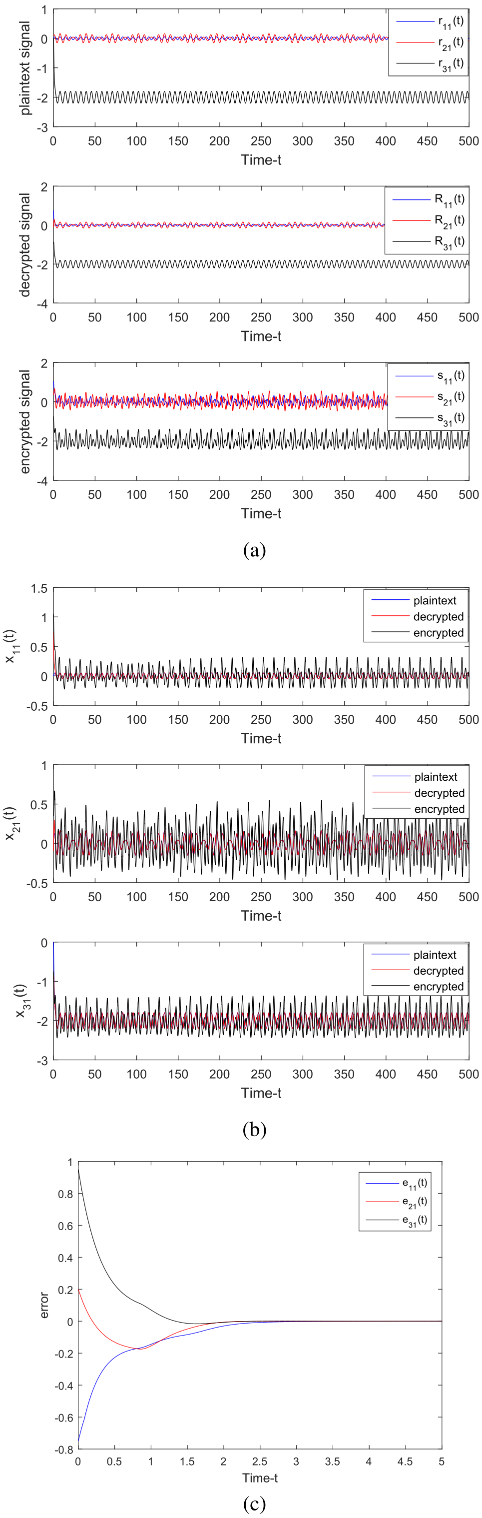


FIGURE 14. The implementation of secure communication. (a) Plaintext signals $r_{k1}(t)$, decrypted signals $R_{k1}(t)$ and encrypted transmission signals $s_{k1}(t)$. (b) Plaintext signal $r(t)$, decrypted signal $R(t)$ and encrypted transmission signal $s(t)$ of each field. (c) Error.

VI. CONCLUSION

Traditional biological neural networks can not reflect the variable synaptic weights when simulating associative memory. In this paper, we propose a novel memristive MAMNNs model, which considers time-varying delays, distributed time delays and stochastic perturbations. Then the synchronization of our proposed model are analyzed by creating appropriate controllers. In the proposed approach, we obtain some less conservative results by removing certain strict conditions. By constructing a suitable Lyapunov function, using the stochastic differential inclusions and some inequality techniques, some sufficient criteria for guaranteeing the synchronization of drive-response system are obtained. Based on the synchronization criteria and memristive MAMNNs, we design a secure communication scheme. Furthermore, some numerical simulations are given to demonstrate the effectiveness of our main results.

APPENDIX A
PROOF OF THEOREM 1

If the Eq.(8) in Theorem 1 holds, there exists a small enough constant $\lambda > 0$ such that

$$\begin{aligned} h_{ki} \le \min\Big\{ & \acute{d}_{ki} - \frac{1}{2}\lambda - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\big[\acute{a}^2_{pjki}L^2_{pj} + \acute{b}^2_{pjki}L^2_{pj} \\ & + \rho_1\acute{c}^2_{pjki}L^2_{pj} + 1 + \frac{e^{\lambda\tau_1}}{1-\tau_2} + \rho_1 e^{\lambda\rho_1}\big], \\ & \times \grave{d}_{ki} - \frac{1}{2}\lambda - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\big[\grave{a}^2_{pjki}L^2_{pj} + \grave{b}^2_{pjki}L^2_{pj} \\ & + \rho_1\grave{c}^2_{pjki}L^2_{pj} + 1 + \frac{e^{\lambda\tau_1}}{1-\tau_2} + \rho_1 e^{\lambda\rho_1}\big]\Big\}. \end{aligned} \tag{21}$$

Then we consider the following Lyapunov function:

$$\begin{aligned} V\big(e(t), t\big) &= e^{\lambda t}e^2_{ki}(t) \\ &\quad + \frac{1}{1-\tau_2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\int_{t-\tau_{pjki}(t)}^{t} e^2_{pj}(s)e^{\lambda(s+\tau_1)}ds \\ &\quad + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\int_{-\rho_1}^{0}\int_{t+s}^{t} e^2_{pj}(z)e^{\lambda(z+\rho_1)}dzds. \end{aligned} \tag{22}$$

Due to the features of memristor, Theorem 1 will be proved in nine cases.

① $|x_{ki}(t)| < \Gamma_{ki}$, $|y_{ki}(t)| < \Gamma_{ki}$.

The drive system (2) can be written as follows:

$$\begin{aligned} \frac{dx_{ki}(t)}{dt} &= I_{ki} - \acute{d}_{ki}x_{ki}(t) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{a}_{pjki}f_{pj}(x_{pj}(t)) \\ &\quad + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{b}_{pjki}f_{pj}(x_{pj}(t-\tau_{pjki}(t))) \\ &\quad + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{c}_{pjki}\int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds. \end{aligned} \tag{23}$$

The response system (4) can be written as follows:

$$\frac{dy_{ki}(t)}{dt} = I_{ki} - \acute{d}_{ki}y_{ki}(t) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{a}_{pjki}f_{pj}(y_{pj}(t)) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{b}_{pjki}f_{pj}(y_{pj}(t-\tau_{pjki}(t))) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{c}_{pjki}\int_{t-\rho(t)}^{t}f_{pj}(y_{pj}(s))ds + \mu_{ki}(t). \quad (24)$$

In the following, we will get the error system such that

$$\frac{de_{ki}(t)}{dt} = \mu_{ki}(t) - \acute{d}_{ki}e_{ki}(t) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{a}_{pjki}\tilde{f}_{pj}(e_{pj}(t)) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{b}_{pjki}\tilde{f}_{pj}(e_{pj}(t-\tau_{pjki}(t))) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{c}_{pjki}\int_{t-\rho(t)}^{t}\tilde{f}_{pj}(e_{pj}(s))ds, \quad (25)$$

where $\tilde{f}_{pj}(e_{pj}(t)) = f_{pj}(y_{pj}(t)) - f_{pj}(x_{pj}(t)), \tilde{f}_{pj}(e_{pj}(t-\tau_{pjki}(t))) = f_{pj}(y_{pj}(t-\tau_{pjki}(t))) - f_{pj}(x_{pj}(t-\tau_{pjki}(t))), \int_{t-\rho(t)}^{t}\tilde{f}_{pj}(e_{pj}(s))ds = \int_{t-\rho(t)}^{t}f_{pj}(y_{pj}(s))ds - \int_{t-\rho(t)}^{t}f_{pj}(x_{pj}(s))ds$.

According to It $\hat{o}$'s differential formula and Assumption 1, along the trajectory of system (25), we get

$$\begin{aligned}
&\mathscr{L}V\big(e(t),t\big) \\
&\leq e^{\lambda t}\Big\{(\lambda - 2\acute{d}_{ki} + 2h_{ki})e_{ki}^2(t) - 2\eta_{ki}|e_{ki}(t)| \\
&+ 2|e_{ki}(t)|\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{a}_{pjki}L_{pj}|e_{pj}(t)| \\
&+ 2|e_{ki}(t)|\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{b}_{pjki}L_{pj}|e_{pj}(t-\tau_{pjki}(t))| \\
&+, 2|e_{ki}(t)|\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\acute{c}_{pjki}L_{pj}\int_{t-\rho(t)}^{t}|e_{pj}(s)|ds \\
&+ \frac{e^{\lambda\tau_1}}{1-\tau_2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}e_{pj}^2(t) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}e_{pj}^2(t) \\
&\times \rho_1 e^{\lambda\rho_1} - \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}e_{pj}^2(t-\tau_{pjki}(t)) \\
&- \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\int_{t-\rho(t)}^{t}e_{pj}^2(s)ds\Big\}.
\end{aligned} \quad (26)$$

By using the mean-value inequality, then we have

$$\begin{aligned}
&2|e_{ki}(t)|\acute{a}_{pjki}L_{pj}|e_{pj}(t)| \\
&\leq \acute{a}_{pjki}^2L_{pj}^2e_{ki}^2(t) + e_{pj}^2(t), 2|e_{ki}(t)|\acute{b}_{pjki}L_{pj}|e_{pj}(t-\tau_{pjki}(t))| \\
&\leq \acute{b}_{pjki}^2L_{pj}^2e_{ki}^2(t) + e_{pj}^2(t-\tau_{pjki}(t)), 2|e_{ki}(t)|\acute{c}_{pjki}L_{pj} \\
&\quad\times\int_{t-\rho(t)}^{t}|e_{pj}(s)|ds \leq \rho_1\acute{c}_{pjki}^2L_{pj}^2e_{ki}^2(t) + \int_{t-\rho(t)}^{t}e_{pj}^2(s)ds.
\end{aligned}$$

Then we get

$$\begin{aligned}
\mathscr{L}V\big(e(t),t\big) &\leq e^{\lambda t}\Big\{\big[\lambda - 2\acute{d}_{ki} + 2h_{ki} + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\acute{a}_{pjki}^2L_{pj}^2 \\
&\quad + \acute{b}_{pjki}^2L_{pj}^2 + \rho_1\acute{c}_{pjki}^2L_{pj}^2)\big]e_{ki}^2(t) + \big[1 + \frac{e^{\lambda\tau_1}}{1-\tau_2} \\
&\quad + \rho_1 e^{\lambda\rho_1}\big]\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}e_{pj}^2(t)\Big\} \\
&= e^{\lambda t}\Big\{\big[\lambda - 2\acute{d}_{ki} + 2h_{ki} + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\acute{a}_{pjki}^2L_{pj}^2 \\
&\quad + \acute{b}_{pjki}^2L_{pj}^2 + \rho_1\acute{c}_{pjki}^2L_{pj}^2 + 1 + \frac{e^{\lambda\tau_1}}{1-\tau_2} \\
&\quad + \rho_1 e^{\lambda\rho_1})\big]e_{ki}^2(t)\Big\}.
\end{aligned} \quad (27)$$

According to Eq.(21), we obtain $\mathscr{L}V\big(e(t),t\big) \leq 0$. Furthermore, we have

$$E\big(V(e(t),t)\big) - E\big(V(0,0)\big) \leq E\int_0^t \mathscr{L}V\big(e(t),t\big)dt \leq 0. \quad (28)$$

Owing to

$$\begin{aligned}
E\big(V(e(0),0)\big) &\leq \max_{1\leq p\leq m, p\neq k}\max_{1\leq j\leq n_p}\Big\{1 + \frac{\tau_1 e^{\lambda\tau_1}}{1-\tau_2} \\
&\quad + \rho_1^2 e^{\lambda\rho_1}\Big\}\sup_{-\tau<s<0}E|e_{ki}(s)|^2 \\
&= M\sup_{-\tau<s<0}E|e_{ki}(s)|^2.
\end{aligned} \quad (29)$$

On the basis of Eq.(28), we get

$$E\big(V(e(0),0)\big) \geq E\big(V(e(t),t)\big) \geq e^{\lambda t}E|e_{ki}(t)|^2.$$

Therefore, we gain

$$E|e_{ki}(t)|^2 \leq Me^{-\lambda t}\sup_{-\tau<s<0}E|e_{ki}(s)|^2,$$

this implies drive system (2) and response system (4) are mean squre exponential synchronization.

② $|x_{ki}(t)| > \Gamma_{ki}, |y_{ki}(t)| > \Gamma_{ki}$.

The drive system (2) can be written as follows:

$$\frac{dx_{ki}(t)}{dt} = I_{ki} - \grave{d}_{ki}x_{ki}(t) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{a}_{pjki}f_{pj}(x_{pj}(t)) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{b}_{pjki}f_{pj}(x_{pj}(t-\tau_{pjki}(t))) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{c}\,pjki\int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds. \quad (30)$$

The response system (4) can be written as follows:

$$\frac{dy_{ki}(t)}{dt} = \mu_{ki}(t) - \grave{d}_{ki}y_{ki}(t) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{a}_{pjki}f_{pj}(y_{pj}(t)) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{b}_{pjki}f_{pj}(y_{pj}(t-\tau_{pjki}(t))) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{c}_{pjki}\int_{t-\rho(t)}^{t} f_{pj}(y_{pj}(s))ds + I_{ki}. \quad (31)$$

In the following, we will get the error system such that

$$\frac{de_{ki}(t)}{dt} = \mu_{ki}(t) - \grave{d}_{ki}e_{ki}(t) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{a}_{pjki}\tilde{f}_{pj}(e_{pj}(t)) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{b}_{pjki}\tilde{f}_{pj}(e_{pj}(t-\tau_{pjki}(t))) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{c}_{pjki}\int_{t-\rho(t)}^{t} \tilde{f}_{pj}(e_{pj}(s))ds. \quad (32)$$

Then we get

$$\mathscr{L}V\big(e(t),t\big) \leq e^{\lambda t}\Big\{\big[\lambda - 2\grave{d}_{ki} + 2h_{ki} + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}(\grave{a}^2_{pjki}L^2_{pj} + \grave{b}^2_{pjki}L^2_{pj} + \rho_1\grave{c}^2_{pjki}L^2_{pj} + 1 + \frac{e^{\lambda\tau_1}}{1-\tau_2} + \rho_1 e^{\lambda\rho_1})\big]e^2_{ki}(t)\Big\}. \quad (33)$$

According to Eq.(21) and Eq.(28), we know that drive system (2) and response system (4) are mean squre exponential synchronization.

③ $|x_{ki}(t)| < \Gamma_{ki}, |y_{ki}(t)| > \Gamma_{ki}$.

The drive system (2) can be written as system (23), the response system (4) can be written as system (31). In the following, we will get the error system such that

$$\frac{de_{ki}(t)}{dt} = \mu_{ki}(t) - \grave{d}_{ki}e_{ki}(t) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{a}_{pjki}\tilde{f}_{pj}(e_{pj}(t)) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{b}_{pjki}\tilde{f}_{pj}(e_{pj}(t-\tau_{pjki}(t))) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{c}_{pjki}\int_{t-\rho(t)}^{t} \tilde{f}_{pj}(e_{pj}(s))ds + (\acute{d}_{ki} - \grave{d}_{ki})x_{ki}(t) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}(\grave{a}_{pjki} - \acute{a}_{pjki})f_{pj}(x_{pj}(t)) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}(\grave{b}_{pjki} - \acute{b}_{pjki})f_{pj}(x_{pj}(t-\tau_{pjki}(t))) + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}(\grave{c}_{pjki} - \acute{c}_{pjki})\int_{t-\rho(t)}^{t} f_{pj}(x_{pj}(s))ds. \quad (34)$$

Then, according to Itô's differential formula and Assumption 1, along the trajectory of system (34), we get

$$\mathscr{L}V\big(e(t),t\big) \leq e^{\lambda t}\Big\{\big[\lambda - 2\grave{d}_{ki} + 2h_{ki} + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}(\grave{a}^2_{pjki}L^2_{pj} + \grave{b}^2_{pjki}L^2_{pj} + \rho_1\grave{c}^2_{pjki}L^2_{pj} + 1 + \frac{e^{\lambda\tau_1}}{1-\tau_2} + \rho_1 e^{\lambda\rho_1})\big]e^2_{ki}(t) + 2\big[(\acute{d}_{ki} - \grave{d}_{ki})\Gamma_{ki} + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}(\grave{a}_{pjki} - \acute{a}_{pjki})L_{pj}\Gamma_{pj} + \sum_{\substack{p=1,\\p\neq k}}^{m}\sum_{j=1}^{n_p}[(\grave{b}_{pjki} - \acute{b}_{pjki})F + (\grave{c}_{pjki} - \acute{c}_{pjki})\rho_1 F] - \eta_{ki}\big]|e_{ki}(t)|\Big\}. \quad (35)$$

According to Eq.(9) and Eq.(21), we obtain $\mathscr{L}V\big(e(t),t\big) \leq 0$. On the basis of Eq.(28) and Eq.(29), we know that drive system (2) and response system (4) are mean squre exponential synchronization.

④ $|x_{ki}(t)| > \Gamma_{ki}, |y_{ki}(t)| < \Gamma_{ki}$.

The drive system (2) can be written as system (24), the response system (4) can be written as system (30). In the following, we will get the error system

such that

$$\frac{de_{ki}(t)}{dt} = \mu_{ki}(t) - \grave{d}_{ki}e_{ki}(t) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{a}_{pjki}\tilde{f}_{pj}(e_{pj}(t))$$
$$+ \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{b}_{pjki}\tilde{f}_{pj}(e_{pj}(t-\tau_{pjki}(t)))$$
$$+ \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\grave{c}_{pjki}\int_{t-\rho(t)}^{t}\tilde{f}_{pj}(e_{pj}(s))ds + (\grave{d}_{ki}$$
$$- \acute{d}_{ki})y_{ki}(t) + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\acute{a}_{pjki} - \grave{a}_{pjki})f_{pj}(y_{pj}(t))$$
$$+ \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\acute{b}_{pjki} - \grave{b}_{pjki})f_{pj}(y_{pj}(t-\tau_{pjki}(t)))$$
$$+\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\acute{c}_{pjki} - \grave{c}_{pjki})\int_{t-\rho(t)}^{t}f_{pj}(y_{pj}(s))ds. \quad (36)$$

According to Itô's differential formula and Assumption 1, we have

$$\mathscr{L}V\big(e(t),t\big) \le e^{\lambda t}\Big\{\big[\lambda - 2\grave{d}_{ki} + 2h_{ki} + \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\grave{a}^2_{pjki}L^2_{pj}$$
$$+ \grave{b}^2_{pjki}L^2_{pj} + \rho_1\grave{c}^2_{pjki}L^2_{pj} + 1 + \frac{e^{\lambda\tau_1}}{1-\tau_2}$$
$$+ \rho_1 e^{\lambda\rho_1})\big]e^2_{ki}(t) + 2\big[(\grave{d}_{ki} - \acute{d}_{ki})\Gamma_{ki}$$
$$+ \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}(\acute{a}_{pjki} - \grave{a}_{pjki})L_{pj}\Gamma_{pj}$$
$$+ \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\big[(\acute{b}_{pjki} - \grave{b}_{pjki})F$$
$$+ (\acute{c}_{pjki} - \grave{c}_{pjki})\rho_1 F\big] - \eta_{ki}\big]|e_{ki}(t)|\Big\}. \quad (37)$$

According to Eq.(9) and Eq.(21), we obtain $\mathscr{L}V\big(e(t),t\big) \le 0$. On the basis of Eq.(28) and Eq.(29), we know that drive system (2) and response system (4) are mean squre exponential synchronization.

⑤ $|x_{ki}(t)| < \Gamma_{ki}$, $|y_{ki}(t)| = \Gamma_{ki}$.
⑥ $|x_{ki}(t)| = \Gamma_{ki}$, $|y_{ki}(t)| < \Gamma_{ki}$.
⑦ $|x_{ki}(t)| = \Gamma_{ki}$, $|y_{ki}(t)| = \Gamma_{ki}$.
⑧ $|x_{ki}(t)| = \Gamma_{ki}$, $|y_{ki}(t)| > \Gamma_{ki}$.
⑨ $|x_{ki}(t)| > \Gamma_{ki}$, $|y_{ki}(t)| = \Gamma_{ki}$.

The rest of five cases are similar to cases ③ and ④, and the process of proof is omitted here. To sum up, Theorem 1 is proved.

APPENDIX B
PROOF OF THEOREM 2

If the Eq.13 in Theorem 2 holds, there exists a small enough constant $\lambda > 0$ such that

$$h_{ki} \le \min\Big\{\acute{d}_{ki} - \frac{1}{2}\lambda - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\big[\acute{a}^2_{pjki}L^2_{pj} + \acute{b}^2_{pjki}L^2_{pj}$$
$$+ \rho_1\acute{c}^2_{pjki}L^2_{pj} + 1 + \alpha + e^{\lambda\tau_1}\frac{1+\beta}{1-\tau_2} + \rho_1 e^{\lambda\rho_1}\big],$$
$$\times\, \grave{d}_{ki} - \frac{1}{2}\lambda - \frac{1}{2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\big[\grave{a}^2_{pjki}L^2_{pj} + \grave{b}^2_{pjki}L^2_{pj}$$
$$+ \rho_1\grave{c}^2_{pjki}L^2_{pj} + 1 + \alpha + e^{\lambda\tau_1}\frac{1+\beta}{1-\tau_2} + \rho_1 e^{\lambda\rho_1}\big]\Big\}, \quad (38)$$

Then we consider the following Lyapunov function:

$$V\big(e(t),t\big) = e^{\lambda t}e^2_{ki}(t)$$
$$+ \frac{1+\beta}{1-\tau_2}\sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\int_{t-\tau_{pjki}(t)}^{t}e^2_{pj}(s)e^{\lambda(s+\tau_1)}ds$$
$$+ \sum_{\substack{p=1,\\ p\neq k}}^{m}\sum_{j=1}^{n_p}\int_{-\rho_1}^{0}\int_{t+s}^{t}e^2_{pj}(z)e^{\lambda(z+\rho_1)}dzds. \quad (39)$$

Due to the characteristics of memristor, the theorem will be proved in nine cases. The process of proof are similar to Theorem 1, so it is omitted here. To sum up, Theorem 2 is proved.

REFERENCES

[1] J. He, J. Cai, and J. Lin, "Synchronization of hyperchaotic systems with multiple unknown parameters and its application in secure communication," *Optik-Int. J. Light Electron Opt.*, vol. 127, no. 5, pp. 2502–2508, 2016, doi: 10.1016/j.ijleo.2015.11.055.

[2] G. Xu, J. Xu, C. Xiu, F. Liu, and Y. Zang, "Secure communication based on the synchronous control of hysteretic chaotic neuron," *Neurocomputing*, vol. 227, pp. 108–112, Mar. 2017, doi: 10.1016/j.neucom.2016.08.100.

[3] I. N'Doye, H. Voos, M. Darouach, "Observer-based approach for fractional-order chaotic synchronization and secure communication," *IEEE J. Emerg. Sel. Topics Circuits Syst.*, vol. 3, no. 3, pp. 442–450, Sep. 2013, doi: 10.1109/JETCAS.2013.2265792.

[4] B. Naderi and H. Kheiri, "Exponential synchronization of chaotic system and application in secure communication," *Optik-Int. J. Light Electron Opt.*, vol. 127, no. 5, pp. 2407–2412, 2016, doi: 10.1016/j.ijleo.2015.11.175.

[5] F. T. Duan and B. T. Cui, "Synchronization of memristor-based competitive neural networks with different time scales," *Appl. Mech. Mater.*, vol. 740, pp. 238–242, Mar. 2015, doi: 10.4028/www.scientific.net/AMM.740.238.

[6] Y. V. Pershin and M. Di Ventra, "Experimental demonstration of associative memory with memristive neural networks," *Neural Netw.*, vol. 23, no. 7, pp. 881–886, 2010, doi: 10.1016/j.neunet.2010.05.001.

[7] M. Itoh and L. O. Chua, "Memristor cellular automata and memristor discrete-time cellular neural networks," *Int. J. Bifurcation Chaos*, vol. 19, no. 11, pp. 3605–3656, Nov. 2009, doi: 10.1142/S0218127409025031.

[8] X. Huang, Y. Fan, J. Jia, Z. Wang, and Y. Li, ''Quasi-synchronisation of fractional-order memristor-based neural networks with parameter mismatches,'' *IET Control Theory Appl.*, vol. 11, no. 14, pp. 2317–2327, Sep. 2017, doi: 10.1049/iet-cta.2017.0196.

[9] W. Wang, L. Li, P. Haipeng, J. Kurths, J. Xiao, and Y. Yang, ''Anti-synchronization of coupled memristive neutral-type neural networks with mixed time-varying delays via randomly occurring control,'' *Nonlinear Dyn.*, vol. 83, pp. 2143–2155, Nov. 2015, doi: 10.1007/s11071-015-2471-9.

[10] P. Jiang, Z. Zeng, and J. Chen, ''On the periodic dynamics of memristor-based neural networks with leakage and time-varying delays,'' *Neurocomputing*, vol. 219, pp. 163–173, Jan. 2017, doi: 10.1016/j.neucom.2016.09.029.

[11] M. Zheng *et al.*, ''Finite-time stability and synchronization of memristor-based fractional-order fuzzy cellular neural networks,'' *Commun. Nonlinear Sci. Numer. Simul.*, vol. 59, pp. 272–291, Jun. 2018, doi: 10.1016/j.cnsns.2017.11.025.

[12] J. Yang, L. Wang, Y. Wang, and T. Guo, ''A novel memristive Hopfield neural network with application in associative memory,'' *Neurocomputing*, vol. 227, pp. 142–148, Mar. 2017, doi: 10.1016/j.neucom.2016.07.065.

[13] X. Hu, G. Feng, S. Duan, and L. Liu, ''A memristive multilayer cellular neural network with applications to image processing,'' *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 8, pp. 1889–1901, Aug. 2016, doi: 10.1109/TNNLS.2016.2552640.

[14] S. Zhu, L. Wang, and S. Duan, ''Memristive pulse coupled neural network with applications in medical image processing,'' *Neurocomputing*, vol. 227, pp. 149–157, Mar. 2017, doi: 10.1016/j.neucom.2016.07.068.

[15] C. Chen, L. Li, H. Peng, and Y. Yang, ''Fixed-time synchronization of memristor-based BAM neural networks with time-varying discrete delay,'' *Neural Netw.*, vol. 96, pp. 47–54, Sep. 2017, doi: 10.1016/j.neunet.2017.08.012.

[16] H. Li, H. Jiang, and C. Hu, ''Existence and global exponential stability of periodic solution of memristor-based BAM neural networks with time-varying delays,'' *Neural Netw.*, vol. 75, pp. 97–109, Mar. 2016, doi: 10.1016/j.neunet.2015.12.006.

[17] M. S. Ali, R. Saravanakumar, and J. Cao, ''New passivity criteria for memristor-based neutral-type stochastic BAM neural networks with mixed time-varying delays,'' *Neurocomputing*, vol. 171, pp. 1533–1547, Jan. 2016, doi: 10.1016/j.neucom.2015.07.101.

[18] M. Hagiwara, ''Multidirectional associative memory,'' in *Proc. Int. Joint Conf. Neural Netw.*, vol. 1, 1990, pp. 3–6.

[19] S.-C. Chen and H. Gao, ''Multivalued exponential multidirectional associative memory,'' *J. Softw.*, vol. 9, no. 5, pp. 397–400, 1998.

[20] M. Wang, T. Zhou, and X. Zhang, ''Global exponential stability of discrete-time multidirectional associative memory neural network with variable delays,'' *ISRN Discrete Math.*, vol. 2012, Sep. 2012, Art. no. 831715, doi: 10.5402/2012/831715.

[21] P. Basaras, G. Iosifidis, D, Katsaros, and L. Tassiulas, ''Identifying influential spreaders in complex multilayer networks: A centrality perspective,'' *IEEE Trans. Netw. Sci. Eng.*, to be published, doi: 10.1109/TNSE.2017.2775152.

[22] A. Akhavan, A. Samsudin, and A. Akhshani, ''A symmetric image encryption scheme based on combination of nonlinear chaotic maps,'' *J. Franklin Inst.*, vol. 348, pp. 1797–1813, Oct. 2011, doi: 10.1016/j.jfranklin.2011.05.001.

[23] O. M. Kwon, J. H. Park, and S. M. Lee, ''Secure communication based on chaotic synchronization via interval time-varying delay feedback control,'' *Nonlinear Dyn.*, vol. 63, pp. 239–252, Jan. 2011, doi: 10.1007/s11071-010-9800-9.

[24] H. Tirandaz and A. Karmi-Mollaee, ''Modified function projective feedback control for time-delay chaotic Liu system synchronization and its application to secure image transmission,'' *Optik-Int. J. Light Electron Opt.*, vol. 147, pp. 187–196, Oct. 2017, doi: 10.1016/j.ijleo.2017.08.103.

[25] X. Luo, J. Deng, J. Liu, W. Wang, X. Ban, and J.-H. Wang, ''A quantized kernel least mean square scheme with entropy-guided learning for intelligent data analysis,'' *China Commun.*, vol. 14, pp. 127–136, Jul. 2017.

[26] H. Zhao, L. Li, H. Peng, J. Xiao, and Y. Yang, ''Finite-time boundedness analysis of memristive neural network with time-varying delay,'' *Neural Process. Lett.*, vol. 44, pp. 665–679, Dec. 2016, doi: 10.1007/s11063-015-9487-5.

[27] A. Wu and Z. Zeng, ''Lagrange stability of memristive neural networks with discrete and distributed delays,'' *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 4, pp. 690–703, Apr. 2013, doi: 10.1109/TNNLS.2013.2280458.

[28] P. Jiang, Z. Zeng, and J. Chen, ''Almost periodic solutions for a memristor-based neural networks with leakage, time-varying and distributed delays,'' *Neural Netw.*, vol. 68, pp. 34–45, Aug. 2015, doi: 10.1016/j.neunet.2015.04.005.

[29] A. Chandrasekar, R. Rakkiyappan, and X. Li, ''Effects of bounded and unbounded leakage time-varying delays in memristor-based recurrent neural networks with different memductance functions,'' *Neurocomputing*, vol. 202, pp. 67–83, Aug. 2016, doi: 10.1016/j.neucom.2016.04.012.

[30] W. Wang, M. Yu, X. Luo, L. Liu, M. Yuan, and W. Zhao, ''Synchronization of memristive BAM neural networks with leakage delay and additive time-varying delay components via sampled-data control,'' *Chaos Solitons Fractals*, vol. 104, pp. 84–97, Nov. 2017, doi: 10.1016/j.chaos.2017.08.011.

[31] Y. Song and S. Wen, ''Synchronization control of stochastic memristor-based neural networks with mixed delays,'' *Neurocomputing*, vol. 156, pp. 121–128, May 2015, doi: 10.1016/j.neucom.2014.12.077.

[32] J. Lu and D. W. C. Ho, ''Stabilization of complex dynamical networks with noise disturbance under performance constraint,'' *Nonlinear Anal. B, Real World Appl.*, vol. 12, pp. 1974–1984, Aug. 2011, doi: 10.1016/j.nonrwa.2010.12.013.

[33] X. Li, J.-A. Fang, and H. Li, ''Exponential stabilisation of stochastic memristive neural networks under intermittent adaptive control,'' *IET Control Theory Appl.*, vol. 11, no. 15, pp. 2432–2439, Sep. 2017, doi: 10.1049/iet-cta.2017.0021.

[34] M. Yu, W. Wang, M. Yuan, X. Luo, and L. Liu, ''Exponential anti-synchronization control of stochastic memristive neural networks with mixed time-varying delays based on novel delay-dependent or delay-independent adaptive controller,'' *Math. Problems Eng.*, vol. 2017, Mar. 2017, Art. no. 8314757, doi: 10.1155/2017/8314757.

[35] S. Ding, Z. Wang, N. Rong, and H. Zhang, ''Exponential stabilization of memristive neural networks via saturating sampled-data control,'' *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3027–3039, Oct. 2017, doi: 10.1109/TCYB.2017.2711496.

[36] J. Gao, P. Zhu, A. Alsaedi, F. E. Alsaadi, and T. Hayat, ''A new switching control for finite-time synchronization of memristor-based recurrent neural networks,'' *Neural Netw.*, vol. 86, pp. 1–9, Feb. 2017, doi: 10.1016/j.neunet.2016.10.008.

[37] A. Wu and Z. Zeng, ''Anti-synchronization control of a class of memristive recurrent neural networks,'' *Commun. Nonlinear Sci. Numer. Simul.*, vol. 18, no. 2, pp. 373–385, 2013, doi: 10.1016/j.cnsns.2012.07.005.

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