Nonlinear Dyn (2018) 93:1315–1324
https://doi.org/10.1007/s11071-018-4261-7

ORIGINAL PAPER

Phase synchronization dynamics of coupled neurons with coupling phase in the electromagnetic field

Yong Zhao · Xiaoyan Sun · Yang Liu · Jürgen Kurths

Received: 12 January 2018 / Accepted: 2 April 2018 / Published online: 11 May 2018

Abstract Based on the law of electromagnetic theory, phase synchronization of coupled extended Hindmarsh–Rose neurons with magnetic and electrical couplings is discussed. It is found that the threshold for the coupling strength to reach phase synchronization is gradually smaller when the coupling phase is increased under the same stimulus current. Under the same coupling phase, the coupling strength to reach phase synchronization is almost increasing gradually with increasing the stimulus current, no matter in what state the neuron is. Our recent findings are significant and helpful for further understanding the collective behaviors of neuronal system including comprehensive physical mechanisms and information transmissions.

Y. Zhao (✉)
School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, China
e-mail: zhaoyong_54@163.com

X. Sun
The People's Hospital of Jiaozuo City, Jiaozuo, China

Y. Zhao · J. Kurths
Department of Physics, Humboldt University of Berlin, Berlin, Germany

Y. Zhao · Y. Liu · J. Kurths
Potsdam Institute for Climate Impact Research, Potsdam, Germany

Y. Liu
Department of Computer Science, Technische Universität Berlin, Berlin, Germany

Keywords Phase synchronization · Coupling phase · Magnetic coupling · Electrical coupling · Electromagnetic field

1 Introduction

Collective phenomena are ubiquitous in many complex physical, biological, sociological and neuronal systems [1–5]. The neuronal system is considered as one of the most complex systems, which are composed of billions of neurons. These neurons are linked through neuronal synapses. They form a complicated structure and also perform profound nonlinear collective dynamical behaviors [6]. Neuronal collective activities are considered to complete certain task and other diversity of cognitive functions, such as pattern recognition. Thus, analyzing the collective dynamics of such neuronal systems is very important. It can provide substantial help for practical applications, such as Parkinson's diseases, Alzheimer's diseases and epilepsies. Various studies show that brain pathologies and disorders are related to synchronization behaviors of neurons [2,7–9]. Buzsaki found that a lot of oscillatory behaviors exist in the brain. Especially, synchronization and desynchronization in neuronal system are very interesting phenomena [5,10,11]. The effect of the triplet-structure coordinated reset stimulations on destabilizing a strong synchronous state is considered in the basal ganglia-thalamocortical motor circuit [11]. They found that the desynchronization and

reliability are further improved within considering the closed-loop triplet-structure coordinated reset stimulations, which is helpful for us to understand the pathophysiology mechanism of Parkinson's diseases. Wang discussed that the effect of information transmission delays on the evolution of spatiotemporal dynamics and synchronization transition of small-world HH neuronal networks with channel noises [12]. Applying a physiologically corticothalamic circuit model, Zhang et al. explored dynamical mechanisms during seizure behavior. Synaptic connections, delays, conductance and slow variables have remarkable effects on the activity state of the brain [13]. These can be significant for controlling and even eliminating seizure spike waves in theoretical models.

Because of the diversity of neurons and the complexity in neuronal systems, it is difficult to achieve complete synchronization. In this case, it is significant to consider phase information. Oscillatory activity in a wide range of frequencies is modulated during working memory and long-term memory processes [14]. Beta waves (13–30 Hz) and gamma waves (> 40 Hz) are thought to play an import role in information encoding [15,16]. In recent years, neuronal phase synchronization has become a main field of investigating neuronal activities [7,17–24]. Phase synchronization between neurons is suggested to indicate the collective behaviors. It is widely assumed that it can provide a possible mechanism for understanding neuronal information encoding and decoding.

In neuronal systems, a single neuron is able to exhibit a diversity of dynamical behaviors, such as quiescent, spiking, bursting and even chaotic behaviors under external stimulus currents. Based on stability and bifurcation analysis, it was found that the excited neuron can exhibit periodic or chaotic spiking/bursting behaviors with the increase in external current [25]. Generally, a single neuron cannot finish signal transmissions. The synapse plays an important role in the information transmission. A 'presynaptic' neuron and a 'postsynaptic' neuron communicate through a synapse connect. Wang et al. considered the dependence of synapses on synchronization transitions in scale-free networks of bursting neurons. It was found that different probabilities of inhibitory synapses have different effects on neuronal synchronization [26]. Wang et al. also investigated the effect of delays on synchronization transitions of scale-free neuronal networks with attractive and repulsive coupling. Under the attractive and repulsive couplings, effects of delays always play a subtle role in either promoting or impairing synchronization. The attractive and repulsive couplings play a different role in the spatiotemporal synchrony, respectively [27]. Yu et al. studied the dependence of local and global synchronization transitions in small-world neuronal networks with chemical synapses on the information transmission delay and the other parameters [28]. Numerical results show that, for both excitatory and inhibitory coupling, the synaptic time delay can always induce synchronization transitions in small-world neuronal networks. It can either enhance or destroy the synchrony of neuronal activities [29,30]. Thus, considering the effect of synapses is essential to achieve the information communication. Generally, electrical and chemical synapses are considered in theoretical neuroscience, which are usually found in the nervous system, because ion concentrations of the intracellular and extracellular membranes of cells are exchanging continually, and can induce the complex electromagnetic flux. This can result in complex dynamics of neurons. So the intracellular and extracellular ions of a membrane are exchanging, and maybe there exists an electromagnetic field [24,31,32]. Recent studies show that the electromagnetic field can have effects on dynamical behaviors of neurons [31–36], in addition to external electrical field [37]. In the electromagnetic field, phase synchronization of coupled neurons with field couplings is considered, and they found that external electromagnetic fields can induce phase synchronization [24]. In this paper, we extend the subject by considering phase synchronization of coupled neurons with electrical and magnetic couplings. For both coupling types, we describe the coupling characteristic by applying a rational matrix. Based on an extended Hindmarsh–Rose neuron with an electromagnetic field, we discuss effects of the coupling phase and other parameters on phase synchronization of coupled neurons with magnetic and electrical couplings.

2 Model description and methods

Similar to [24,31], we consider the effect of electromagnetic inductions in the single neuron. A single neuron model will be described by the following four variable dynamical equations. In addition to considering the magnetic coupling, we also include the electrical

coupling at the same time. The response dynamical systems with coupling phase can be described as follows:

$$\begin{cases} \frac{dx_1}{dt} = y_1 - ax_1^3 + bx_1^2 - z_1 + I_{\text{ext}} - k\rho(\varphi_1)x_1 \\ \quad + g[b_{xx}(x_2 - x_1) + b_{x\varphi}(\varphi_2 - \varphi_1)] \\ \frac{dy_1}{dt} = c - dx_1^2 - y_1 \\ \frac{dz_1}{dt} = r[s(x_1 - \mu) - z_1) \\ \frac{d\varphi_1}{dt} = k_1x_1 - k_2\varphi_1 + g[b_{\varphi x}(x_2 - x_1) + b_{\varphi\varphi}(\varphi_2 - \varphi_1)] \end{cases}$$

$$\begin{cases} \frac{dx_2}{dt} = y_2 - ax_2^3 + bx_2^2 - z_2 + I_{\text{ext}} - k\rho(\varphi_2)x_2 \\ \quad + g[b_{xx}(x_1 - x_2) + b_{x\varphi}(\varphi_1 - \varphi_2)] \\ \frac{dy_2}{dt} = c - dx_2^2 - y_2 \\ \frac{dz_2}{dt} = r[s(x_2 - \mu) - z_2) \\ \frac{d\varphi_2}{dt} = k_1x_2 - k_2\varphi_2 + g[b_{\varphi x}(x_1 - x_2) + b_{\varphi\varphi}(\varphi_1 - \varphi_2)] \end{cases}$$

where x_i is the membrane potential of the ith neuron. y_i represents the recovery variable for slow currents. z_i is the adaption current, I_{ext} the external stimulus current, and φ the magnetic flux across the membrane. The memductance of the memristor is defined by $\rho(\varphi) = \alpha + 3\beta\varphi^2$ [6,31], $\alpha = 0.4$, $\beta = 0.02$. $a = 1, b = 3, c = 1, d = 5, r = 0.006, s = 4, \mu = -1.6, k = 0.4, k_1 = 0.9, k_2 = 0.5$.

As in [38,39], our systems contain not only $x - x$ and $\varphi - \varphi$ coupling, but also cross-coupling between the variables x and φ, which describes the interaction of the magnetic flux and membrane potential between neurons. For the sake of simplicity, the feature of coupling can be modeled by a rational coupling matrix

$$B = \begin{pmatrix} b_{xx} & b_{x\varphi} \\ b_{\varphi x} & b_{\varphi\varphi} \end{pmatrix} = \begin{pmatrix} \cos\emptyset & \sin\emptyset \\ -\sin\emptyset & \cos\emptyset \end{pmatrix}$$

depending on the coupling phase $\emptyset \in [-\pi, \pi)$. Thus, we can change the four parameters $I_{\text{ext}}, \emptyset, g, r$ and investigate the collective behavior of the coupled neurons with electrical couplings and magnetic couplings under different external stimulus currents.

More specially, we consider three cases of coupling terms:

(I) Direct coupling in the both variables x and φ ($\emptyset = 0$);
(II) Direct and cross-coupling in the both variables x and φ ($\emptyset = \frac{\pi}{4}$);
(III) Cross-coupling in the both variables x and φ ($\emptyset = \frac{\pi}{2}$);

In neuronal systems, lots of neurons are different, therefore it is difficult to achieve complete synchronization, and we analyze phase information. Thus, considering phase synchronization is very significant. Generally, we detect the phase information through the occurrence of electrical activities of membrane potentials. The phase is calculated by [1,24,28]

$$\theta_i(t) = 2\pi \frac{t - t_n}{t_{n+1} - t_n} + 2\pi n, \quad t_n < t < t_{n+1}$$

where t_n is the time of the nth firing of neuron i and $t_{n+1} - t_n$ is the inter-spike interval (ISI). Thus, it is assumed that the neuronal phase $\theta_i(t)$ increases linearly between the firing moments t_n and t_{n+1}.

In order to discuss phase synchronization between neurons, the phase difference between two neurons can be defined by

$$\Delta\theta(t) = |\theta_1(t) - \theta_2(t)|$$

At the same time, in order to measure whether it is complete synchronization under phase synchronization, the difference of the membrane potentials is calculated by

$$\Delta x(t) = x_2(t) - x_1(t)$$

3 Numerical results

The Runge–Kutta algorithm is used in our numerical results. The time step is $h = 0.01$, and the initial values are $x_1(0) = 0.01$, $y_1(0) = 0.02$, $z_1(0) = 0.029$, $\varphi_1(0) = 1.01$, $x_2(0) = 0.01$, $y_2(0) = 0.02$, $z_2(0) = 0.003$, $\varphi_2(0) = 0.001$. When $g = 0$, a single neuron shows different dynamical behaviors under the external stimulus current. We set the following external stimulus currents $I_{\text{ext}} = 1.20$, $I_{\text{ext}} = 1.35$, $I_{\text{ext}} = 1.75$, $I_{\text{ext}} = 1.90$, $I_{\text{ext}} = 2.50$, $I_{\text{ext}} = 4.90$, $I_{\text{ext}} = 5.00$, respectively. The response results are shown in the plots in Fig. 1a–g. From these time series, it is found that quiescent state, spike, bursting and periodical oscillation state can exist by selecting an appropriate external current. Increasing the external current, the bursting state is enhanced firstly, after that, when the external stimulus current $I_{\text{ext}} = 5.00$, the bursting state is lost and it is replaced by periodic oscillations (period spikes). Furthermore, the inter-spike interval (ISI) is calculated by changing the external stimulus current, as shown in Fig. 1h, which also confirmed dynamical behaviors under external stimulus currents discussed above. We also detected spikes increase gradually from both spikes to multiple spikes in intra-bursting when bursting occurs under an external stimulus current.

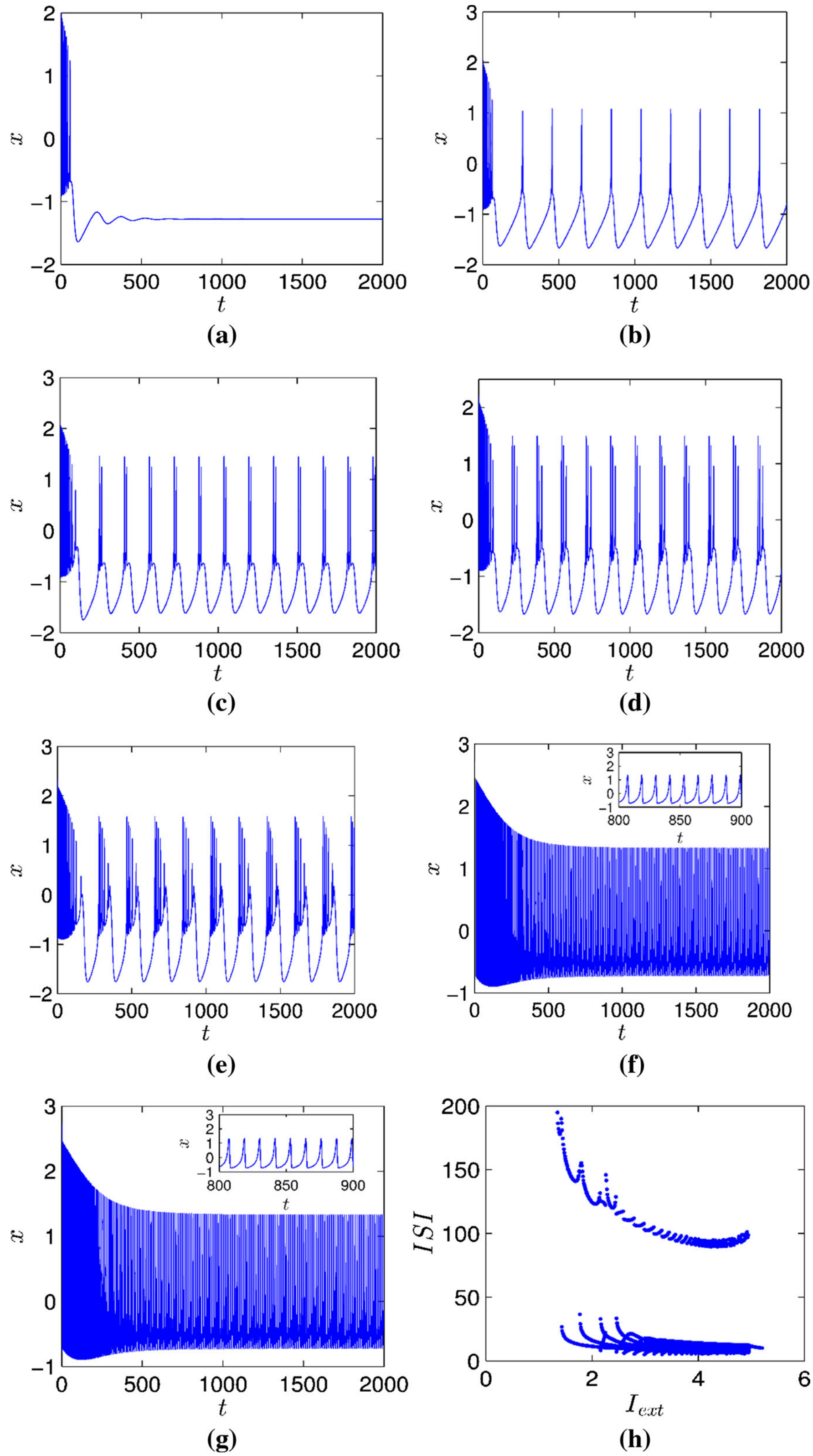

Fig. 1 **a**–**g** Time series under different external stimulus current I_{ext}, **h** ISI bifurcation diagram under external stimulus current I_{ext}. **a** $I_{\text{ext}} = 1.20$. **b** $I_{\text{ext}} = 1.35$. **c** $I_{\text{ext}} = 1.75$. **d** $I_{\text{ext}} = 1.90$. **e** $I_{\text{ext}} = 2.50$. **f** $I_{\text{ext}} = 4.90$. **g** $I_{\text{ext}} = 5.00$. **h** ISI vt I_{ext}

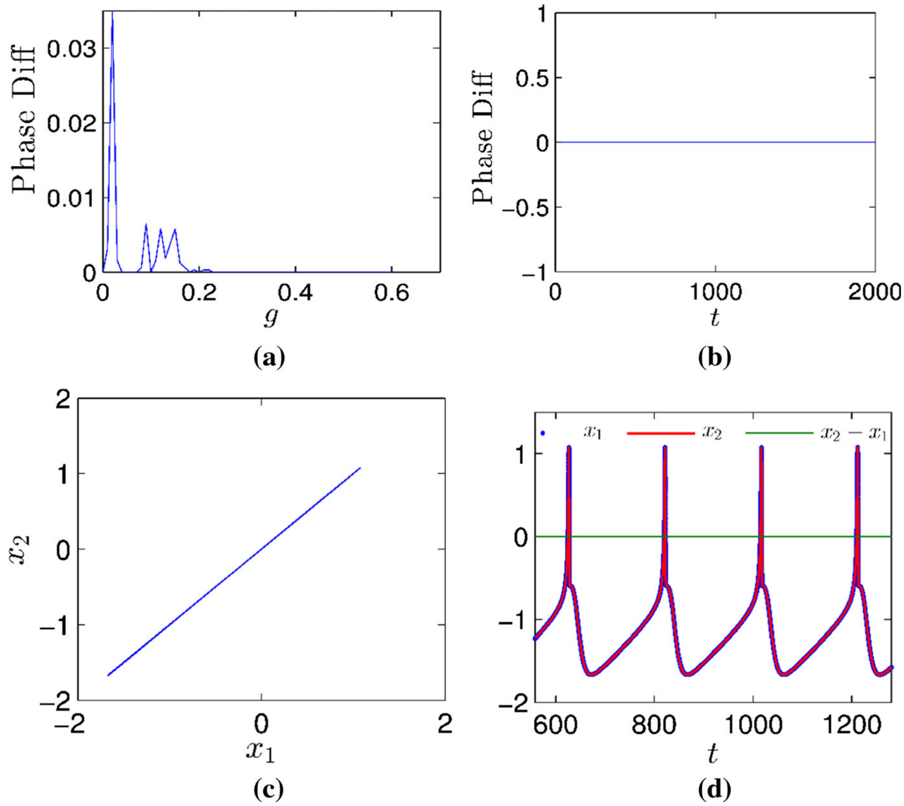

Fig. 2 Phase synchronization under direct coupling when $I_{\text{ext}} = 1.35$. **a** Phase difference versus g; when $g = 0.23$, **b** evolution of phase difference with t; **c** phase diagram x_1 and x_2; **d** time series of x_1, x_2 and $x_2 - x_1$, respectively

It is very interesting to investigate effects of the coupling strength on phase synchronization under different electromagnetic couplings.

For case (I) Direct coupling in the both variables x and φ ($\emptyset = 0$);

In the first case, for $I_{\text{ext}} = 1.35$, phase synchronization is considered. In Fig. 2a, the relationship between phase difference (phase diff) and the coupling strength g is described. We see that when $g \in [0.04, 0.07]$, an intermittent phase synchronization emerges, and when $g = 0.23$, complete phase synchronization is achieved because phase difference with t is always zero. When the coupling strength is beyond $g = 0.23$, the two neurons still stay in complete phase synchronization. (b) and (c) confirm phase synchronization of two neurons when $g = 0.23$; furthermore, from (d), we see that the membrane potentials of both neurons are nearly synchronized but not completely synchronized.

For $I_{\text{ext}} = 1.90$ and $g = 0.34$, complete phase synchronization will be achieved (Fig. 3). When the coupling strength is beyond $g = 0.34$, both neurons still stay in the complete phase synchronization. (b) and (c) confirm phase synchronization of both neurons when $g = 0.34$; from (d), we also investigate membrane potentials of both neurons and confirm both neurons is near-synchronization but not complete synchronization.

When external stimulus current $I_{\text{ext}} = 2.50$, and $g \in [0.25, 0.255]$, an intermittent phase synchronization emerges (Fig. 4a). When $g = 0.23$, we know complete phase synchronization is achieved from the evolution of phase difference with t. When the coupling strength is beyond $g = 0.285$, both neurons still stay in complete phase synchronizations. (b) and (c) confirm phase synchronization of two neurons when $g = 0.285$; meanwhile, from (d), it is shown that membrane potentials of two neurons are near-synchronization but not complete synchronization.

For $I_{\text{ext}} = 4.90$, the phase synchronization is detected. In Fig. 5a, the evolution of phase synchronization (phase diff) with coupling strength g is obtained. When $g = 0.45$, complete phase synchronization can be achieved. When the coupling strength is beyond $g = 0.45$, both neurons still stay in the complete phase

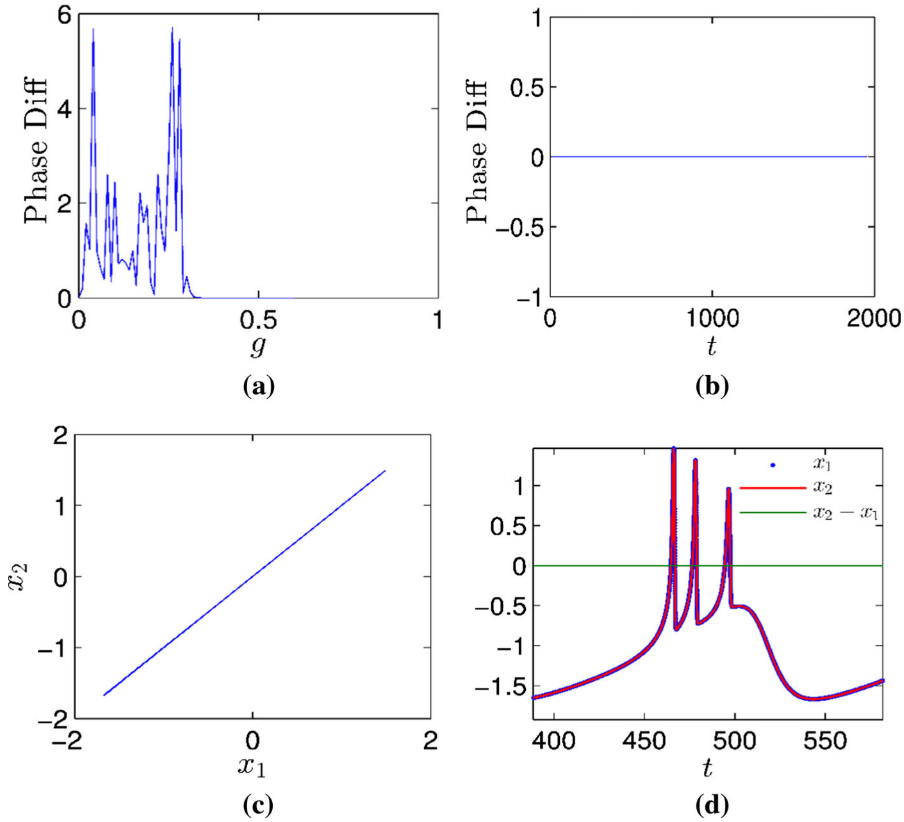

Fig. 3 Phase synchronization under direct coupling when $I_{ext} = 1.90$. **a** Phase difference versus g; when $g = 0.34$, **b** evolution of phase difference with t; **c** phase diagram x_1 and x_2; **d** time series of x_1, x_2 and $x_2 - x_1$, respectively

synchronization. (b) and (c) confirm phase synchronizations of both neurons when $g = 0.45$; furthermore, from (d), we find that membrane potentials of two neurons are nearly synchronized but not completely synchronized.

At last, $I_{ext} = 5.00$, phase synchronization is explored. When $g \in [0.44, 0.45]$, an intermittent phase synchronization emerges (Fig. 6a). When $g = 0.47$, complete phase synchronization is achieved. When the coupling strength is beyond $g = 0.47$, both neurons still stay in the complete phase synchronization. (b) and (c) confirm both neurons are in phase synchronizations when $g = 0.47$; furthermore, from (d), we obtained that membrane potentials of two neurons are nearly synchronized not completely synchronized.

For case (II) Direct and cross-coupling in the both variables x and φ ($\emptyset = \frac{\pi}{4}$);

In this case, for $I_{ext} = 1.35$. and $g = 0.16$, complete phase synchronization can be achieved. When the coupling strength is beyond $g = 0.16$, both neurons still stay in complete phase synchronization. Under the stimulus current $I_{ext} = 1.90$, when $g = 0.277$, complete phase synchronization can be reached. When $I_{ext} = 2.50$, phase synchronization is reached when the coupling strength is above $g = 0.278$. For $I_{ext} = 4.90$, and $g = 0.39$, complete phase synchronization can be achieved. When the coupling strength is beyond $g = 0.39$, both neurons are still in complete phase synchronization. Furthermore, for $I_{ext} = 5.00$, and $g = 0.41$, both neurons are in phase synchronization. When the coupling strength is more than $g = 0.41$, both neurons are robust for complete phase synchronization.

In what follows, similarly, we investigate phase synchronization of both neurons under the coupling phase $\emptyset = \frac{\pi}{2}$.

For case (III) Cross-coupling in the both variables x and φ ($\emptyset = \frac{\pi}{2}$);

In this case, for $I_{ext} = 1.35$, and $g = 0.135$, complete phase synchronization can be achieved. When the coupling strength is beyond $g = 0.135$, both neurons still stay in complete phase synchronization. When

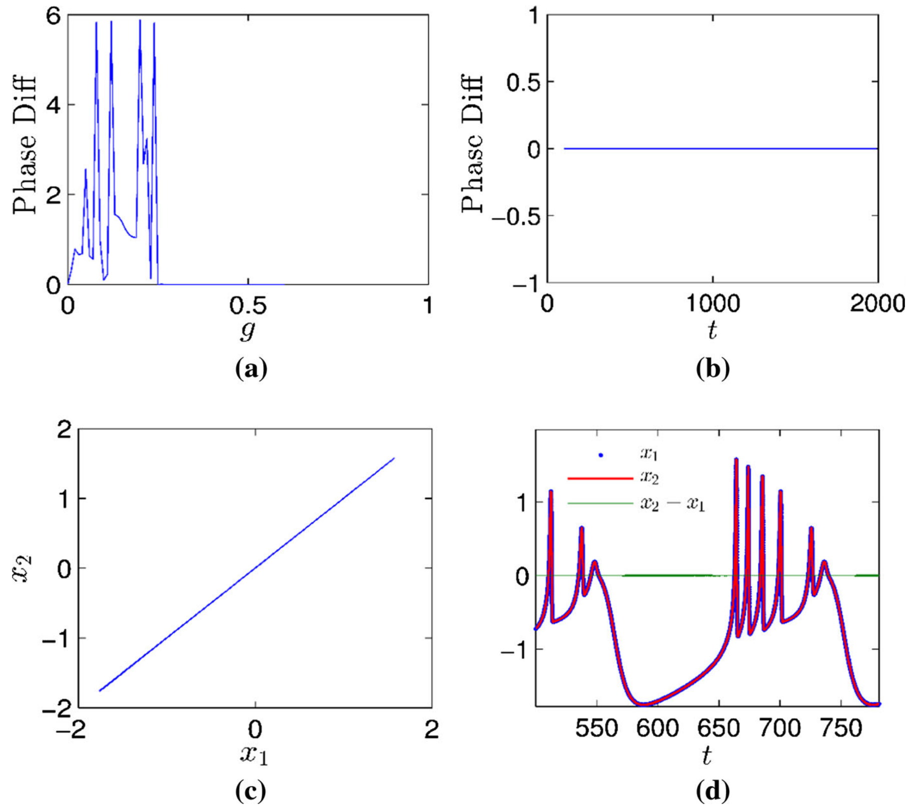

Fig. 4 Phase synchronization under direct coupling when $I_{\text{ext}} = 2.50$. **a** Phase difference versus g; when $g = 0.285$, **b** evolution of phase difference with t; **c** Phase diagram x_1 and x_2; **d** time series of x_1, x_2 and $x_2 - x_1$, respectively

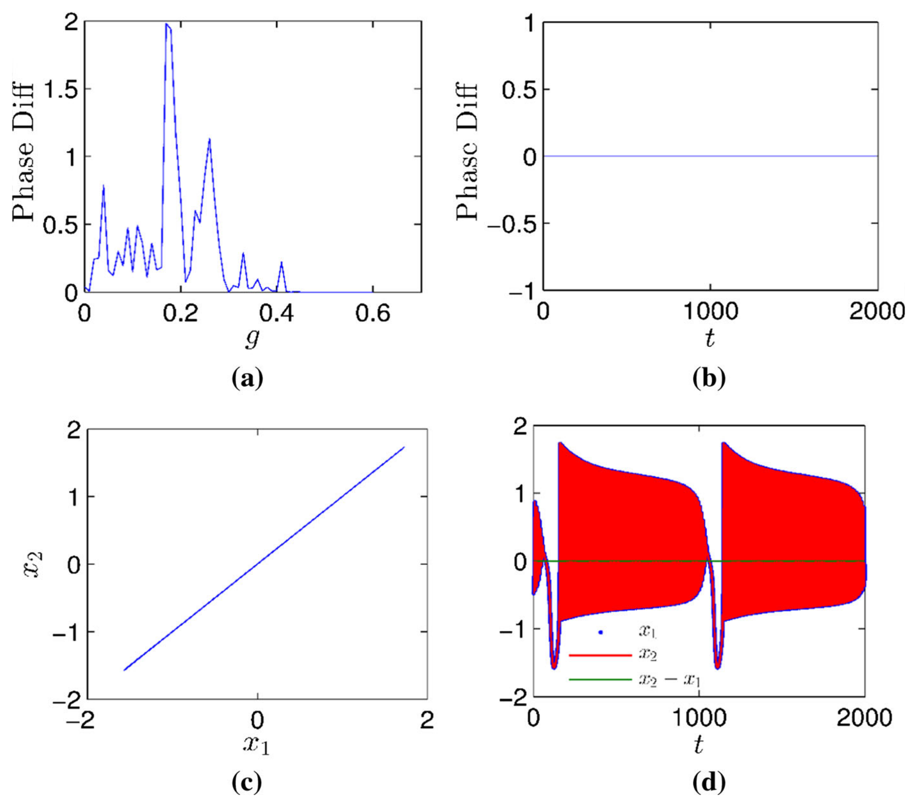

Fig. 5 Phase synchronization under direct coupling when $I_{\text{ext}} = 4.90$. **a** Phase difference versus g; when $g = 0.45$, **b** evolution of phase difference with t; **c** phase diagram x_1 and x_2; **d** time series of x_1, x_2 and $x_2 - x_1$, respectively

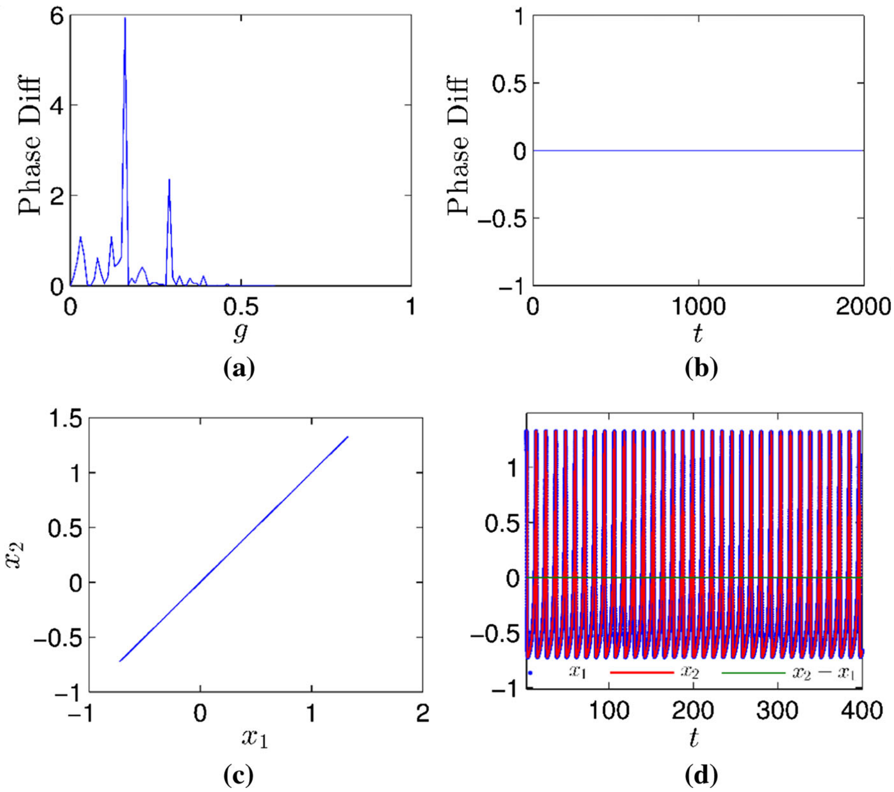

Fig. 6 Phase synchronization under direct coupling when $I_{\text{ext}} = 5.00$. **a** Phase difference versus g; when $g = 0.47$, **b** evolution of phase difference with t; **c** phase diagram x_1 and x_2; **d** time series of x_1, x_2 and $x_2 - x_1$, respectively

$I_{\text{ext}} = 1.90$, and $g = 0.24$, it has been complete phase synchronization. When the coupling strength is above $g = 0.24$, both neurons are still in complete phase synchronization. Under the stimulus current $I_{\text{ext}} = 2.50$, and $g = 0.26$, complete phase synchronization can be reached. When the coupling strength is more than $g = 0.26$, complete phase synchronization still exists. for $I_{\text{ext}} = 4.90$, and $g = 0.302$, complete phase synchronization can be achieved. When the coupling strength is beyond $g = 0.302$, both neurons still kept complete phase synchronization. When $I_{\text{ext}} = 5.00$, through calculations, it is shown that complete phase synchronization of both neurons exists when the coupling strength is more than $g = 0.31$.

Furthermore, we investigated the effect of external stimulus currents, coupling phases and coupling strengths on phase synchronization. We show the relationships of I_{ext}, Ø and g_{phasesyn} in Table 1. We mainly find that increasing the coupling phase under the same stimulus current, the threshold for the coupling strength to reach phase synchronization becomes smaller gradually. Under the same coupling phase, the coupling strength for phase synchronization is almost increasing gradually with increasing the stimulus current.

4 Discussions

In this paper, we describe three modes of electrical and magnetic couplings by applying a rational matrix. The coupling strength for the occurrence of phase synchronization in various coupling schemes under electromagnetic field is investigated. Furthermore, we find that even smaller coupling strength can achieve phase synchronization under the same external stimulus current, compared with the reference [24].

Recently, some researchers discussed dynamical behaviors of coupled neurons in electromagnetic fields [24,32], while dynamical behaviors of coupled neurons with both magnetic and electrical couplings were not discussed before. We believe that our recent findings are important and helpful for further investigating collective behaviors of neuronal systems in physical mechanisms. Furthermore, the results can be extended to the study of synchronization behaviors of large-scale net-

Table 1 Relationship table of I_{ext}, Ø and $g_{phasesyn}$

$g_{phasesyn}$ \ Ø / I_{ext}	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
1.35	0.23	0.16	0.135
1.77	0.26	0.24	0.215
1.90	0.34	0.277	0.24
2.50	0.285	0.278	0.26
3.00	0.27	0.31	0.265
4.00	0.32	0.33 or 0.34	0.29
4.50	0.38	0.36	0.30
4.90	0.45	0.39	0.302
5.00	0.47	0.41	0.31

works, even multilayer networks. We look forward for further promoting the development for neuronal information encoding and decoding.

Acknowledgements This work is support by the National Natural Science Foundation of China (Grant No. 11502073) and Scientific Research Fund of Henan Provincial Education Department (Grant No. 14A110004) and Doctoral Foundation of Henan Polytechnic University (Grant No. B2012-107).

Compliance with ethical standards

Conflicts of interest The authors declared that they have no conflict of interest.

References

1. Pikovsky, A., Rosenblum, M., Kurths, J.: Synchronization: a universal concept in nonlinear sciences. Cambridge University Press, Cambridge (2011)
2. Uhlhaas, P.J., Singer, W.: Neural synchrony in brain review disorders: relevance for cognitive dysfunctions and pathophysiology. Neuron **52**, 155–168 (2006)
3. Perc, M., Jordan, J.J., Rand, D.G., Wang, Z., Boccaletti, S., Szolnoki, A.: Statistical physics of human cooperation. Phys. Rep. **687**, 1–51 (2017)
4. Helbing, D., Brockmann, D., Chadefaux, T., Donnay, K., Blanke, U., Woolley-Meza, O., Moussaid, Mehdi, Johansson, A., Krause, J., Schutte, S., Perc, M.: Saving human lives: what complexity science and information systems can contribute. J. Stat. Phys. **158**, 735–781 (2015)
5. Buzsaki, G.: Rhythms of the Brain. Oxford University Press, Oxford (2011)
6. Volos, ChK, Kyprianidis, I.M., Stouboulos, I.N., Tlelo-Cuautle, E., Vaidyanathan, S.: Memristor: a new concept in synchronization of coupled neuromorphic circuits. J. Eng. Sci. Technol. Rev. **8**, 157–173 (2015)
7. Tass, P., Rosenblum, M.G., Weule, J., Kurths, J., Pikovsky, A., Volkmann, J., Schnitzler, A., Freund, H.J.: Detection of n: m phase locking from noisy data: application to magnetoencephalography. Phys. Rev. Lett. **81**, 3291 (1998)
8. Popovych, O.V., Hauptmann, C., Tass, P.A.: Effective synchronisation by nonlinear delayed feedback. Phys. Rev. Lett. **94**, 164102 (2005)
9. Schnitzler, A., Gross, J.: Normal and pathological oscillatory communication in the brain. Nat. Rev. Neurosci. **6**, 285–296 (2005)
10. Zhao, Y., Feng, Z.S.: Desynchronization in synchronous multi-coupled chaotic neurons by mix-adaptive feedback control. J. Biol. Dyn. **7**, 1–10 (2013)
11. Fan, D.G., Wang, Q.Y.: Improving desynchronization of parkinsonian neuronal network via triplet-structure coordinated reset stimulation. J. Theor. Biol. **370**, 157–170 (2015)
12. Wang, Q.Y., Shi, X., Chen, G.R.: Delay-induced synchronization transition in small-world Hodgkin–Huxley neuronal networks with channel blocking. Discrete Contin. Dyn. Syst. B **16**, 607–621 (2011)
13. Zhang, H.H., Zheng, Y.H., Su, J.Z., Xiao, P.C.: Seizures dynamics in a neural field model of cortical-thalamic circuitry. Sci. China Technol. Sci. **60**, 974–984 (2017)

14. Hanslmayr, S., Staudigl, T.: How brain oscillations form memories-a processing based perspective on oscillatory subsequent memory effects. Neuroimage **85**, 648–655 (2014)
15. Tallon-Baudry, C., Bertrand, O., Peronnet, F., Pernier, J.: Induced gamma-band activity during the delay of a visual short-term memory task in humans. J. Neurosci. **18**, 4244–4254 (1998)
16. Schneider, S.L., Rose, M.: Intention to encode boosts memory-related pre-stimulus EEG beta power. Neuroimage **125**, 978–987 (2016)
17. Tallon-Baudry, C., Bertrand, O., Fischer, C.: Oscillatory synchrony between human extrastriate areas during visual short-term memory maintenance. J. Neurosci. **21**(RC177), 1–5 (2001)
18. Kyrychko, Y.N., Blyuss, K.B., Schöll, E.: Amplitude and phase dynamics in oscillators with distributed-delay coupling. Phil. Trans. R. Soc. A **371**, 20120466 (2013)
19. Roux, F., Uhlhaas, P.J.: Working memory and neural oscillations: alpha–gamma versus theta–gamma codes for distinct WM information? Trends Cognit. Sci. **18**, 16–25 (2014)
20. Daume, J., Gruber, T., Engel, A.K., Friese, U.: Phase-amplitude coupling and long-range phase synchronization reveal frontotemporal interactions during visual working memory. J. Neurosci. **37**, 313–322 (2017)
21. Ivanchenko, M.V., Osipov, G.V., Shalfeev, V.D., Kurths, J.: Phase synchronization in ensembles of bursting oscillators. Phys. Rev. Lett. **93**, 134101 (2004)
22. Yu, H.T., Wang, J., Deng, B., Wei, X.l, Wong, Y.K., Chan, W.L., Tsang, K.M., Yu, Z.Q.: Chaotic phase synchronization in small-world networks of bursting neurons. Chaos **21**, 013127 (2011)
23. Sun, X.J., Perc, M., Kurths, J.: Effects of partial time delays on phase synchronization in Watts–Strogatz small-world neuronal networks. Chaos **27**, 053113 (2017)
24. Ma, J., Lv, M., Zhou, P., Xu, Y., Tasawar, H.: Phase synchronization between two neurons induced by coupling of electromagnetic field. Appl. Math. Comput. **307**, 321–328 (2017)
25. Wang, H.X., Wang, Q.Y., Zheng, Y.H.: Bifurcation analysis for Hindmarsh–Rose neuronal model with time-delayed feedback control and application to chaos control. Sci. China Technol. Sci. **57**, 872–878 (2014)
26. Wang, Q.Y., Chen, G.R.: Delay-induced intermittent transition of synchronization in neuronal networks with hybrid synapses. Chaos. **21**, 013123 (2011)
27. Wang, Q.Y., Chen, G., Perc, M.: Synchronous bursts on scale-free neuronal networks with attractive and repulsive coupling. PLoS ONE **6**, e15851 (2011)
28. Yu, H.T., Wang, J., Du, J.W., Deng, B., Wei, X.L.: Local and global synchronization transitions induced by time delays in small-world neuronal networks with chemical synapses. Cognit. Neurodyn. **9**, 93–101 (2015)
29. Wang, Q.Y., Duan, Z.S., Perc, M.: Synchronization transitions on small-world neuronal networks: effects of information transmission delay and rewiring probability. EPL **83**, 50008 (2008)
30. Wang, Q.Y., Perc, M., Duan, Z.S., Chen, G.R.: Synchronization transitions on scale-free neuronal networks due to finite information transmission delays. Phys. Rev. E **80**, 026206 (2009)
31. Lv, M., Ma, J.: Model of electrical activity in a neuron under magnetic flow effect. Nonlinear Dyn. **85**, 1479–1490 (2016)
32. Ren, G.D., Xu, Y., Wang, C.N.: Synchronization behavior of coupled neuron circuits composed of memristors. Nonlinear Dyn. **88**, 893–901 (2017)
33. Xu, Y., Jia, Y., Ma, J., Tasawar, H., Ahmed, A.: Collective responses in electrical activities of neurons under field coupling. Sci. Rep. **8**, 1349 (2018)
34. Xu, Y., Ying, H.P., Jia, Y., Ma, J., Tasawar, H.: Autaptic regulation of electrical activities in neuron under electromagnetic induction. Sci. Rep. **7**, 43452 (2017)
35. Guo, S., Xu, Y., Wang, C., Jin, W., Aatef, Hobiny, Ma, Jun: Collective response, synapse coupling and field coupling in neuronal network. Chaos Solitons Fractals **105**, 120–127 (2017)
36. Wang, C.N., Ma, J.: A review and guidance for pattern selection in spatiotemporal system. Int. J. Mod. Phys. B **32**, 1830003 (2018)
37. Wang, H.T., Chen, Y.: Spatiotemporal activities of neural network exposed to external electric fields. Nonlinear Dyn. **85**, 881–891 (2016)
38. Omelchenko, I., Omel'chenko, O.E., Hövel, P., Schöll, E.: When nonlocal coupling between oscillators becomes stronger: patched synchrony or multichimera states. Phys. Rev. Lett. **110**, 224101 (2013)
39. Hizanidis, J., Kanas, V.G., Bezerianos, A., Bountis, T.: Chimera states in networks of nonlocally coupled Hindmarsh–Rose neuron models. Int. J. Bifurc. Chaos **24**, 450030-1–450030-9 (2014)