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# Enhancing dynamical robustness in aging networks of coupled nonlinear oscillators

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**Abstract** – We propose an effective method to enhance the dynamical robustness of networks of diffusively coupled oscillators experiencing *aging transition*. By introducing a new control parameter into the normal diffusive coupling, we demonstrate that the dynamical robustness of coupled oscillator networks can be efficiently improved by enhancing the tolerance of dynamic activity in the network to inactivation or deterioration of the individual oscillators. Even a tiny deviation from the normal diffusive coupling greatly strengthens the robustness of networks. Particularly, the strong coupling in our scheme is shown to be in favor of the dynamic activity, which is in sharp contrast to the normal form of diffusive coupling with the tendency to spoil the dynamical robustness. Our proposed approach serves as a rather simple and efficient way to recover dynamic activity in networks of diffusively coupled oscillators that has been lost due to some inactivated or damaged oscillator components.

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**Introduction.** – Collective behavior in a large ensemble of coupled oscillators has attracted great interest during the past decades [1–5]. The reason comes from the fact that modelling coupled oscillators serves as a simple but efficient way for understanding basic self-organized phenomena in a variety of disciplines of science. Emergent dynamics of coupled oscillators crucially depends on both the intrinsic nature of individual elements and the manner of coupling between them. Recently, much attention has been paid to the dynamics of diffusively coupled oscillators composed of mixed populations with distinctly different individuality of constituent units [6–8]. Such studies are deemed to be instructive in illuminating the robustness of the function of physical, biological, and engineering systems [9–13].

In their pioneering work [6], Daido and Nakanishi proposed a fascinating framework to investigate the robustness of the dynamic activity of globally and diffusively coupled networks, where the oscillator nodes turn from active to inactive progressively. It is established that with the increase of the ratio  $p$  of inactivated elements, referred to as *aging*, the global oscillation of coupled oscillator networks, measured by the amplitude of a macroscopic order parameter, is weakened and completely vanishes at a certain critical threshold of  $p_c$  for sufficiently large coupling strengths. This intriguing emergent phenomenon is termed as *aging transition* (AT) [6]. The magnitude of  $p_c$  can be regarded as an index of the dynamical robustness of the coupled networks; the higher the value of  $p_c$ , the more dynamically robust the network is. Employing this index, the robustness of dynamic activity in networks of diffusively coupled oscillators can be

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quantitatively characterized, which has been intensively discussed during the last decade [14–27]. One peculiar and curious aspect of the AT is that the strong diffusive coupling spoils the robustness of dynamic activity of coupled systems commonly made in refs. [6,14–27], which is in stark contrast with the intuitive understanding of its effect in facilitating the coherent behaviors of systems [28,29].

In many natural or artificial systems, stable and robust activity is a prerequisite for their proper functioning. Thus, the onset of AT could lead to an irreversible malfunction of these systems, such as neural networks [30], cardiac and respiratory systems [31], power grids [32], etc. It is therefore of high practical importance to propose general techniques to enhance the robustness of dynamic activity of the coupled systems against aging or deterioration of individual elements caused by unexpected accidents and diseases. Quite recently, by introducing a proper feedback parameter to control the rate of diffusion is proved to serve as a novel technique for restoration of rhythmicity from amplitude death in networks of diffusively coupled nonlinear oscillators [33], which has been not only theoretically explored in distinct coupled systems, but also experimentally realized in electrochemical reactions [33] and nonlinear circuits [34].

In this work, we reveal that a minute deviation from the normal diffusive coupling greatly enhances the dynamical robustness of diffusively coupled networks of nonlinear oscillators. Remarkably, in contrast to the normal form of diffusive coupling, the strong coupling in our scheme is in favor of the dynamical robustness of the network's activity. Our findings in the present studies could in many cases lead to a better reconciliation of theoretical and experimental studies with the realistic observations of effects of diffusive coupling in living tissues and organs, where the strong coupling is generally believed to maintain the system's coherent activity [35]. Hence, our results may be expected to provide a new clue to understanding the role of diffusive coupling in generating robust rhythmicity in real coupled-oscillator systems.

**Theories and results.** – Following the framework proposed in ref. [6], let us consider the paradigmatic system of  $N$  globally and diffusively coupled Stuart-Landau oscillators as

$$\dot{Z}_j = (A_j + iw - |Z_j|^2)Z_j + \frac{K}{N} \sum_{k=1}^N (Z_k - \alpha Z_j), \quad (1)$$

where  $Z_j$  and  $A_j$  ( $j = 1, 2, \dots, N$ ) are the complex amplitude and the intrinsic parameter describing the distance from the Hopf bifurcation of the  $j$ -th oscillator, and  $w$  is the natural frequency of a single oscillator. The parameter  $K > 0$  quantifies the overall coupling strength. For  $K = 0$ , the  $j$ -th Stuart-Landau oscillator exhibits a stable limit cycle  $\sqrt{A_j}e^{wt}$  if  $A_j > 0$ , and settles down at the stable trivial fixed point  $Z_j = 0$  if  $A_j < 0$ , where  $|A_j|$  specifies the strength of attraction to the stable attractors. The

aging of the coupled system (1) proceeds in such a way that an active oscillator with  $A_j = a > 0$  turns inactive with  $A_j = -b < 0$ . Without loss of generality, one can set the group of active elements to  $j \in \{1, 2, \dots, N(1-p)\}$  and that of inactive elements to  $j \in \{N(1-p)+1, \dots, N\}$ . The parameter  $p$  is the fraction of inactive elements, which characterizes the level of aging or deterioration of the coupled system. We fix the system size at  $N = 1000$ , which is supposed to be sufficiently large to treat the ratio  $p$  virtually as a continuous parameter. The parameters  $a = 2$  and  $w = 3$  are used throughout the whole paper.

The coupling type in eq. (1) is different from that adopted in the previous studies about AT in networks of diffusively coupled oscillators [6,14–27], as a new feedback factor  $\alpha$  ( $0 \leq \alpha \leq 1$ ) is introduced in the normal form of diffusive coupling. In physics, the parameter  $\alpha$  plays a role to control the degree of diffusion, which recovers to symmetrical diffusion for  $\alpha = 1$  and direct coupling for  $\alpha = 0$ . The intermediate value of  $0 < \alpha < 1$  serves as a bridge linking direct coupling and normal diffusive interaction, thus it is more appropriate in natural circumstances, which may better characterize the diffusion process of a wide spectrum of real-world contexts such as in electrical synapses or gap junctions in the brain, neuronal networks, and power flows in electrical networks [33,33]. Here, our aim is to corroborate even a feeble deviation of  $\alpha$  from unity efficiently enhances the dynamical robustness of coupled oscillator networks, thus recovering their dynamic activity.

The effect of aging in the coupled system (1) can be checked by studying the behavior of the order parameter  $|Z|$ :  $Z = N^{-1} \sum_{k=1}^N Z_k$ , whose magnitude reflects the intensity of macroscopic oscillation of the whole network. For the diffusively coupled system (1) with  $\alpha = 1$ , Daido and Nakanishi analytically showed that the AT occurs at  $p_c = a(b+K)/[(a+b)K]$  if  $K > a$  [6]. This prediction is confirmed again in fig. 1(a) by numerically calculating the dependence of  $|Z|$  on  $p$ , where  $b = 1$  and  $K = 8$  are used. Figure 1(a) also plots the behavior of  $|Z|$  against the inactivation ration  $p$  for  $\alpha = 0.95, 0.9, 0.88$ , and  $0.87$ , respectively. With increasing  $p$  from zero,  $|Z|$  monotonically decreases, which changes from positive to zero at  $p = p_c < 1$  if  $\alpha > \alpha_c = 0.88$ . A larger value of  $p_c$  indicates a larger ratio  $p$  of inactive oscillators for the coupled network transiting from the global oscillatory behavior to a quiescent state. With the numerically obtained values of  $p_c$ , fig. 1(b) further depicts the dependence of  $p_c$  on  $\alpha$  (the black squares). As can be seen,  $p_c$  strongly increases with a tiny decrease of  $\alpha$  from unity, and rapidly reaches  $p_c = 1$  for  $\alpha_c = 0.88$ . Clearly, the presence of  $\alpha$  in the coupling strengthens the dynamical robustness of the network. With further decreasing  $\alpha$  below  $\alpha_c$ , surprisingly,  $|Z|$  is positive even for  $p = 1$ , which implies that the global oscillatory activity of the coupled network survives even for all the local elements turning from active to inactive. The dynamic activity of coupled network is robust to any level of inactivation of units with  $0 \leq p \leq 1$  once  $\alpha$  is below  $\alpha_c$ .

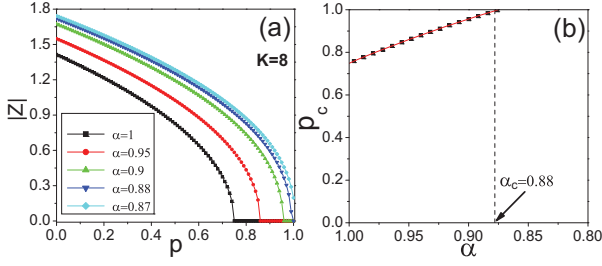


Fig. 1: (Color online) The order parameter  $|Z|$  against the inactivation ratio  $p$  for various values of  $\alpha$  in the coupled network (1) with  $N = 1000$  oscillators for  $a = 2$ ,  $b = 1$ ,  $w = 3$ , and  $K = 8$ . (b) The critical inactivation ratio  $p_c$ , at which  $|Z|$  vanishes ( $|Z| = 0$ ), vs. the value of  $\alpha$ . The value of  $p_c$  monotonically increases to unity with decreasing  $\alpha$  to the critical value  $\alpha_c = 0.88$ . The black squares and the red line indicate the numerical results and the theoretical predictions, respectively. The coupling strength is fixed at  $K = 8$ .

When the AT occurs, the global oscillation collapses at  $p_c$ , meanwhile the trivial fixed point  $Z_j = 0$  is stabilized. Following the method of analysis in ref. [6], the formula of  $p_c$  for  $0 \leq \alpha \leq 1$  can be analytically derived. Assuming that the coupled system (1) is divided into two subgroups, where all elements are identical in each one. Employing similar notations as in ref. [6], by setting  $Z_j = A$  for all the active elements and  $Z_j = I$  for all the inactive oscillators, the original coupled system (1) can be reduced as

$$\begin{aligned} \dot{A} &= (a + iw - \alpha K - pK + K - |A|^2)A + pKI, \\ \dot{I} &= (-b + iw - \alpha K + pK - |I|^2)I + (1 - p)KA. \end{aligned} \quad (2)$$

To obtain analytical expression for  $p_c$ , a linear stability analysis of the reduced system (2) around the origin ( $A = I = 0$ ) can be carried out, and the following Jacobian matrix is obtained:

$$\begin{pmatrix} a + iw - \alpha K + (1 - p)K & pK \\ (1 - p)K & -b + iw - \alpha K + pK \end{pmatrix}. \quad (3)$$

The origin ( $A = I = 0$ ) is stabilized if all the real part of the eigenvalues of the Jacobian matrix (3) are negative. The critical inactivation ratio  $p_c$  can be determined when two complex conjugate eigenvalues cross the imaginary axis. After some simplifications, one can directly arrive at the equation

$$p_c = \frac{a(b + K) + \alpha(1 - \alpha)K^2 + (1 - \alpha)(b - a)K}{(a + b)K}, \quad (4)$$

which degenerates to the same result obtained in ref. [6] for  $\alpha = 1$ . The validity of the above theoretical prediction for  $p_c$  with  $\alpha < 1$  is nicely confirmed by the red line plotted in fig. 1(b) for  $K = 8$ , which agrees well with the numerical simulations represented by the black squares.

The occurrence of AT is featured by the existence of two critical parameters  $p_c$  and  $K_c$ . For the normal diffusive coupling with  $\alpha = 1$ , the AT is observed for all

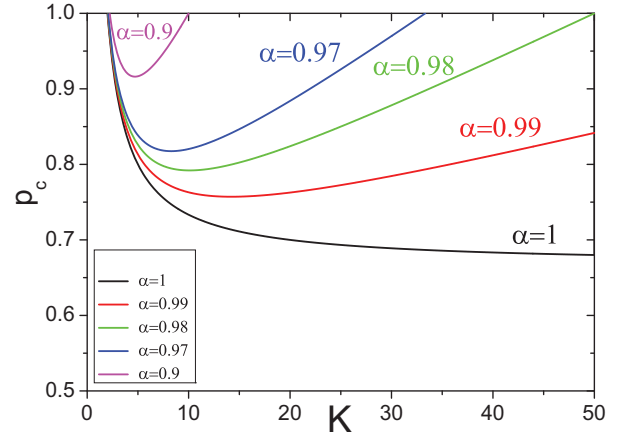


Fig. 2: (Color online) The critical ratio  $p_c$  vs. the coupling strength  $K$  for different values of  $\alpha$  in the coupled system (1) as treated in fig. 1. The AT occurs at  $p_c$  ( $p_c < 1$ ) for  $K > a$  if  $\alpha = 1$ , where  $p_c$  monotonically decreases for increasing  $K$ . The AT is observed only for a finite coupling interval if  $\alpha < 1$ , where  $p_c$  firstly decreases from unity to its minimal value, and then increases to unity. The smaller the value of  $\alpha$ , the larger the value of  $p_c$ . The dynamical robustness of the coupled system (1) is clearly enhanced with decreasing  $\alpha$ .

$K > K_c = a$  and  $p_c$  decreases as increasing  $K$  [6]. To unveil the impact of  $\alpha$  on the two key parameters of  $p_c$  and  $K_c$ , fig. 2 depicts  $p_c$  as a function of  $K$  for  $\alpha = 1$ , 0.99, 0.98, 0.97, and 0.9, respectively. Intriguingly, we find that when  $\alpha < 1$ , the AT takes place only for a finite interval of coupling strength. Within this interval  $p_c$  firstly decreases from unity to its minimal value of  $p_c = [\sqrt{(1 - \alpha)b} + \sqrt{\alpha a}]^2 / (a + b)$  at  $K = \sqrt{ab/[\alpha(1 - \alpha)]}$ , then increases from this minimum to unity. The coupled system loses its dynamic activity with the minimal ratio  $p$  of inactive oscillators at an intermediate coupling strength if  $\alpha < 1$ , at which the dynamic activity of the network is most vulnerable to deterioration of the individual oscillators. The AT is impossible if the coupling strength is large enough once  $\alpha$  deviating from unity. The above observation suggests that the large coupling strength is favorable for a dynamical robustness of the network against aging if  $\alpha < 1$ , which is sharp contrast to the case of  $\alpha = 1$ , where the strong coupling strength has been reported to spoil the robustness of the network's activity [6,14–27].

The constraint of the coupling strength for the AT can be summarized from the condition of  $p_c = 1$  in eq. (4) as

$$\begin{cases} K > a, & \text{for } \alpha = 1, \\ \frac{a}{\alpha} < K < \frac{b}{1 - \alpha}, & \text{for } \alpha < 1. \end{cases} \quad (5)$$

Upon decreasing  $\alpha$  from unity, the coupling interval monotonically decreases and vanishes if  $\alpha < a/(a + b)$ . Figure 3(a) shows the phase diagram of the coupled system (1) with respect to parameters of  $\alpha$  and  $K$ , where the upper-right region bounded by the black bold line

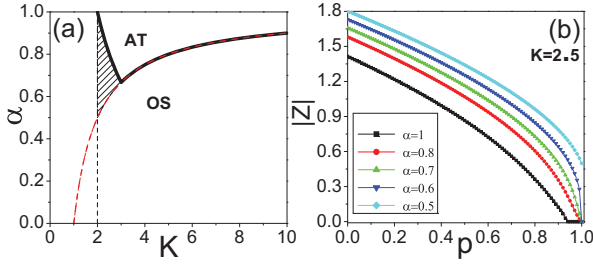


Fig. 3: (Color online) (a) Phase diagram in the parameter space of  $(\alpha, K)$  for the coupled system (1) as treated in fig. 1. The upper-right region encompassed by the black bold line confines the interval of coupling strength within which the AT is observed. The lower-right corner bounded by the red dashed line denotes the parametric regime for the oscillatory state (OS) in the fully damaged system (1) with  $p = 1$ . The shaded area marks the transition zone within which  $p_c$  remains at unity even with decreasing  $\alpha$ . (b) The order parameter  $|Z|$  against the inactivation ratio  $p$  for various values of  $\alpha$  with  $K = 2.5$  corroborating the transition zone in (a). The critical ratio  $p_c$  increases from 0.94 for  $\alpha = 1$  to 1 for  $\alpha = 0.8$ , and keeps at unity with further decreasing  $\alpha$  from 0.8 to 0.6. The value of  $|Z|$  is positive even for  $p = 1$  if  $\alpha < 0.6$ .

displays the interval of coupling strength for the AT. The global oscillation vanishes ( $|Z| = 0$ ) only if all the elements are inactivated ( $p = 1$ ) for the values of  $\alpha$  and  $K$  located on the black bold line in fig. 3(a). The strong coupling strength with  $K > b/(1 - \alpha)$  for  $\alpha < 1$  makes the coupled oscillator networks robust to any deterioration level ( $0 < p \leq 1$ ) of the individual components.

With further decreasing  $\alpha$  away from the boundary of the AT in fig. 3(a), the behavior of  $|Z|$  can be analyzed from a linear stability analysis about the trivial equilibrium  $Z_j = 0$  of the coupled system (1) with  $p = 1$ , which is destabilized if  $\alpha$  is below the critical value  $\alpha_{min}$ ,

$$\alpha_{min} = 1 - b/K. \quad (6)$$

The lower-right corner of fig. 3(a) divided by the dashed red line  $\alpha_{min}$  represents the parameters of  $\alpha$  and  $K$  for the oscillatory state in the coupled system (1) with  $p = 1$ . For  $a + b \leq K < b/(1 - \alpha)$ ,  $p_c$  increases to 1 as  $\alpha$  decreases to  $1 - b/K$ ; and with decreasing  $\alpha$  further results in  $|Z| > 0$  even for  $p = 1$ . This theoretical prediction explains the numerical observations of  $|Z|$  in fig. 1(a). Interestingly, for  $a < K < a + b$ , after  $p_c$  reaching at unity as decreasing  $\alpha$  to  $a/K$ , it remains at unity for  $\alpha$  further reducing from  $a/K$  to  $1 - b/K$ . This transition zone with  $|Z| = 0$  at  $p_c = 1$  is illustrated by the shaded region in fig. 3(a). By decreasing  $\alpha$  below  $1 - b/K$ ,  $|Z| > 0$  holds even for  $p = 1$ . The above analysis is numerically corroborated in fig. 3(b) by plotting the behavior of  $|Z|$  against  $p$  for various values of  $\alpha$  with  $K = 2.5$ . The order parameter  $|Z|$  is positive for  $0 \leq p \leq 1$  with  $\alpha = 0.5$ , which means that the global oscillation persists for any level of inactivation of the individual components in the network. Thus, the

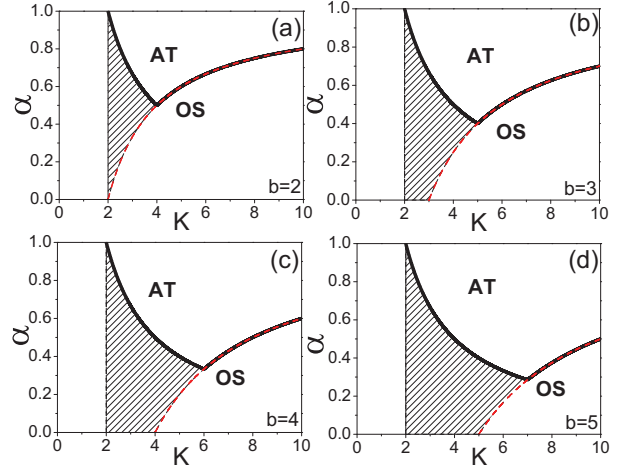


Fig. 4: (Color online) Similar to fig. 3(a) depicting phase diagrams of the coupled system (1) with (a)  $b = 2$  and (b)  $b = 4$ . The shaded region extends downwards to the  $\alpha = 0$  axis as  $b > a$ , which implies that  $p_c$  always stays at unity for  $a/K \geq \alpha \geq 0$  if  $a < K < b$ . All other parameters are the same as used in fig. 3(a).

presence of the factor  $\alpha$  in the diffusive coupling plays a constructive role in enhancing the dynamical robustness of the network against aging or deterioration of oscillatory nodes.

The AT takes place as a result of an intrinsic competition between the forces from the two groups of active elements with  $A_j > 0$  and inactive ones with  $A_j < 0$ . The active oscillators facilitate the global oscillation of the network, while the inactive ones suppress the global oscillation. The facilitation or suppression of dynamic activity is more pronounced with a larger value of  $|A_j|$ . Figures 4(a) and (b) further show the phase diagrams of the coupled system (1) in the parameter space of  $\alpha$  and  $K$  with  $b = 2$  and 4, respectively. Clearly, the AT is more prominent for larger  $b$ , and the shaded region touches the  $\alpha = 0$  axis for  $a < K < b$  when  $b > a$ . If the coupling strength is weak ( $a < K < b$ ), the global oscillation of the network with fully inactivated nodes ( $p = 1$ ) cannot be excited even for  $\alpha = 0$ . These observations are in harmony with the intuition that the dynamic activity of coupled oscillator networks is harder to recover when the strength of attraction of inactive oscillators becomes strong. Note that all the behaviors of  $|Z|$  similar to those observed in fig. 3(a) are reproduced when  $K > b$ . The strong coupling with  $\alpha < 1$  facilitates the dynamical robustness of the coupled system, which is manifested by improving the tolerance of the oscillator network to aging or deterioration of the individual nodes.

Figure 5(a) further depicts the variations of  $|Z|$  against  $p$  for different values of  $\alpha$  with  $b = 4$  ( $b > a$ ) and  $K = 3$ .  $p_c = 0.78$  is found for  $\alpha = 1$ , which monotonically increases to unity for  $\alpha = 0.67$  and remains at unity for all  $\alpha < 0.67$ . The above numerical observation of  $p_c$  is in good agreement with our theoretical predictions. The



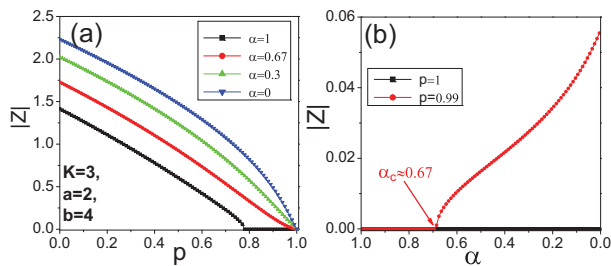


Fig. 5: (Color online) (a) The order parameter  $|Z|$  against the inactivation ratio  $p$  for various values of  $\alpha$  in the coupled network (1) with  $K = 3$  and  $b = 4$ . The critical ratio  $p_c$  increases from 0.78 for  $\alpha = 1$  to 1 for  $\alpha = 0.67$ , and keeps at unity with further decreasing  $\alpha$  from 0.8 to 0. (b) The plots of the order parameter  $|Z|$  with decreasing  $\alpha$  from unity to zero for  $p = 1$  (the black squares) and  $p = 0.99$  (the red circles). All the other parameters are used as in (a). The value of  $|Z|$  stays at zero for all  $1 \geq \alpha \geq 0$  if  $p = 1$ . The order parameter  $|Z|$  becomes positive and increases as  $\alpha < \alpha_c = 0.67$  for  $p = 0.99$ .

magnitude of the order parameter  $|Z|$  is larger for smaller values of  $\alpha$  with a given inactivation ratio  $p < 1$ , which illustrates that a decrease in  $\alpha$  efficiently intensifies the global oscillation of the oscillator network. The global activity cannot be triggered in the fully damaged network with  $p = 1$  for all  $0 \leq \alpha \leq 1$ , which is also verified in fig. 5(b) as indicated by the black squares. Interestingly, the global dynamic activity of coupled systems can be effectively recovered even in the presence of a negligible amount of active oscillators ( $p \rightarrow 1$ ) once  $\alpha < a/K$ . Figure 5(b) shows the dependence of  $|Z|$  on  $\alpha$  with  $p = 0.99$  denoted by the red circles, where  $|Z|$  transits from zero to positive at  $\alpha \approx 0.67$ , and grows as further reducing  $\alpha$ . The control parameter  $\alpha < 1$  has a constructive role in sustaining dynamic activity of the weakly coupled oscillator networks composing of nearly all inactivated oscillators with  $p \rightarrow 1$ . The introduction of  $\alpha < 1$  can efficiently maintain dynamic activity even when the inactive oscillators have a much larger distance from the Hopf bifurcation than the active ones.

**Conclusion and discussions.** — In summary, we have exclusively elucidated that introducing a control parameter  $\alpha$  into the traditional diffusive coupling can efficiently enhance the dynamical robustness of coupled oscillator networks, which is manifested by improving the tolerance of the network to aging or deterioration of the individual oscillators. We have found that the AT point  $p_c$  rapidly increases to unity with a tiny decrease of  $\alpha$  from unity, which implies that the network becomes more robust to the effect of elements becoming inactive. In strong contrast to the cases of the normal form of diffusive coupling with  $\alpha = 1$ , we have revealed that the strong coupling of our scheme with  $\alpha < 1$  is in favor of the dynamical robustness of the coupled oscillator networks. A tiny modification of the ordinary diffusive coupling makes the coupled systems more dynamically robust, which is valid for both small

and large coupling strengths. The enhancing effect of  $\alpha$  is more pronounced for strong coupling. The AT boundary  $p = p_c$  in the  $(K, p)$  phase diagram sharply shrinks even for a minute deviation of  $\alpha$  from unity, which is attributed to a symmetry breaking of diffusion induced by the presence of  $\alpha$ . The introduction of  $\alpha$  in the coupling provides a very simple but highly efficient technique to recover dynamic activity in networks of diffusively coupled oscillators, whose oscillatory behavior has been lost due to inactivation or deterioration of partial elements. In fact, the global oscillations can be even excited by  $\alpha$  in strongly coupled networks of purely non-oscillatory units.

It is worthwhile to note that the new parameter  $\alpha$  ( $0 \leq \alpha \leq 1$ ) is introduced to modify the normal form of diffusive coupling, which multiplies the self-coupling term of the diffusive interaction. Intuitively, the effect of  $\alpha$  in enhancing the dynamical robustness of networks can be mathematically explained as the following: Since the self-coupling term enters into the equations with a negative sign, decreasing the value of  $\alpha$  from unity tends to enlarge the amplitude of the individual oscillators, and thus favors the global oscillations of the ensembles. Therefore, when  $\alpha$  is reduced from unity, the coupled network is expected to have a lesser degree of aging, and the dynamical robustness is reasonably improved. It should be emphasized that the AT behaviors of the coupled systems with  $\alpha = 1$  and  $\alpha < 1$  are completely different: AT occurs once the coupling strength is beyond a certain threshold  $K > a$  for  $\alpha = 1$ , whereas AT is observed for only a finite coupling interval if  $\alpha < 1$ . Besides that, for the normal diffusive coupling with  $\alpha = 1$ , the larger the coupling strength  $K$ , the smaller value of the critical inactivation ratio  $p_c$  is; however, for the case of  $\alpha < 1$ , AT is impossible if the coupling strength  $K$  is strong enough. One of the most intriguing findings is that even a feeble deviation of  $\alpha$  from unity can strongly enhance the dynamical robustness of diffusively coupled damaged networks.

For the sake of mathematical tractability and numerical simulations, we have employed networks of globally and diffusively coupled Stuart-Landau oscillators, where the uncoupled Stuart-Landau oscillator is a normal form describing dynamical behavior near a Hopf bifurcation. It is well known that the Hopf bifurcation is established as one of the most fundamental bifurcations in physics and biology. Moreover, Hopf systems are prototypes in many biological contexts. On the other hand, diffusive coupling is a natural type of interaction that has attracted general interests in a wide range of disciplines. Thus, emergent behaviors in some realistic circumstances could be both quantitatively and qualitatively described and understood by investigating collective dynamics in diffusively coupled Hopf systems. We hope that our proposed scheme is applicable to other biologically relevant systems, whose oscillatory activity arises as a result of a Hopf bifurcation. In fact, the generality of our results can be extended to diffusively coupled Stuart-Landau oscillators with a variety of intricate scenarios, such as considering

the population heterogeneity, time-delayed coupling, complex networks, etc.

The designed coupling strategy is indeed plausible in diffusively coupled networks, which has been experimentally implemented in chemical reactions and electronic circuits quite recently in studying restoration of oscillatory behavior from amplitude death [33,34]. We believe both the proposed form of diffusive coupling and the established effects in the AT phenomenon can be corroborated and observed in many other nonlinear systems, such as chemical oscillators, excitable and oscillatory units, and circadian oscillator neurons in the suprachiasmatic nucleus of mammals [36,37]. On the other hand, one should notice that both the modification of the coupling and conservation of the intensity of coupling at a large level generally produce additional costs. A remaining important subject is to examine whether the proposed coupling strategy is rewarding enough for compensating such costs, which constitutes our future studies. Finally, we expect that our findings are beneficial to the development of designing a more robust network for a better functional performance of systems in biology, ecology, neuroscience, and engineering.

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