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Reviving oscillation with optimal spatial period of frequency distribution in coupled oscillators

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The spatial distributions of system's frequencies have significant influences on the critical coupling strengths for amplitude death (AD) in coupled oscillators. We find that the left and right critical coupling strengths for AD have quite different relations to the increasing spatial period m of the frequency distribution in coupled oscillators. The left one has a negative linear relationship with m in log-log axis for small initial frequency mismatches while remains constant for large initial frequency mismatches. The right one is in quadratic function relation with spatial period m of the frequency distribution in log-log axis. There is an optimal spatial period m_0 of frequency distribution with which the coupled system has a minimal critical strength to transit from an AD regime to reviving oscillation. Moreover, the optimal spatial period m_0 of the frequency distribution is found to be related to the system size $\sqrt{N}$. Numerical examples are explored to reveal the inner regimes of effects of the spatial frequency distribution on AD. *Published by AIP Publishing.*
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Oscillation suppression, as one of the common collective behaviors in coupled nonlinear oscillators, has been a hot topic in nonlinear science, since it is related to many inner regimes of self-organization and engineering applications such as vibration control. Various kinds of oscillation quenching, as well as the transition processes, have been observed in coupled nonidentical oscillators with different kinds of coupling regimes. However, an inverse issue of reviving oscillation from such a quenching state has attracted a new focus especially in biology systems such as rescuing patient whose heart suddenly stops working properly due to cardiac arrest caused by the heart's electrical system malfunctions. Here, we study how the frequency distributions with different spatial periods influence the efficiency of reviving oscillation in coupled nonidentical oscillators. We mainly find that there is a size-related optimal spatial period with which the coupled system can be rather easily revived from the oscillation quenching states. The birth of this optimal spatial period is found to be the outcome of the competition between the coupling and the spatial heterogeneity, as well as the competition between the synchronization and the oscillation quenching. We expect that our work will contribute to better understanding of chaos control and engineering applications such as vibration reduction of buildings.

I. INTRODUCTION

Coupled dynamical systems are widely applied to explore various forms of self-organized behaviors,[1–3] such as synchronization[4–7] and oscillation quenching. The competition between synchronization and oscillation quenching may even generate rich patterns.[8–11] Synchronization dynamics has been found to be related to the inner regimes of many emergence phenomena since its original observation by Huygens. Meanwhile, oscillation quenching[12–14] refers to the suppression of oscillation under various types of interactions or intentional control and has attracted many researchers' interests because of its important application on the control of chaotic oscillations and stabilization of various unstable dynamics from the aspects of mechanical engineering,[15] synthetic genetic networks,[16,17] or laser systems.[18,19] Generally, two kinds of oscillation quenching have been reported as amplitude death (AD) and oscillation death (OD). AD refers to the stabilization of an already existing homogeneous steady state (HSS) of zero, while OD is manifested as a newly born inhomogeneous steady state (IHSS) from the symmetry breaking of the coupling

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interaction. Moreover, AD has been recently observed to be capable of transiting to OD via a Turing-type bifurcation due to rich interactions.[14,20,21] Based on the concepts of control, various coupling schemes are found to be available for the coupled system to transit to the oscillation quench state, such as repulsive coupling,[20,21] dissimilar (or conjugate) variables coupling,[12,22,23] dynamical coupling,[24,25] delayed time coupling,[26–29] indirect coupling,[30] mean field diffusion[31] and environmental coupling,[32] and amplitude-dependent coupling.[33]

Parameter mismatches are common between the interacting units due to the diversity of the natural world. The occurrence of oscillation quenching in coupled nonidentical oscillator systems has been analyzed in many interacting networks such as global (all-to-all) networks,[34] small-world networks,[35] or scale-free networks.[36,37] Oscillation quenching is observed as a result of a competition of parameter mismatches and topologies in interacting networks. Even in locally coupled networks,[38,39] transition processes from partial amplitude death (PAD) to AD are often exhibited because of a competition between frequency mismatches and coupling-induced synchronization clusters in coupled nonidentical oscillators with randomly distributed frequencies. Zou *et al*.[40,41] and Ghosh *et al.*[42] proposed the notion of reviving oscillations of coupled nonidentical oscillators from AD theoretically and experimentally, which is meaningful in control of dynamics in many biological systems such as the cardiac arrest due to cessation of the normal sinus rhythm of pacemaker cells,[43] or the mimic brain death involving a temporary loss of parts of brain function.[44] By considering the control of AD on the coupled oscillators, we extend the research on exploring effects of spatial period of frequency distributions on AD and come to a conclusion that two critical coupling strengths (lower or upper-bounded value) of complete AD exhibit a universal distribution (power law or a log-normal) for all possible spatial distributions of frequencies.[45,46] However, in some cases, the spatial distributions of frequency are not in random forms but in regular forms such as the linearly[39] or periodically distributed cases as numerically discussed in Ref. 47. It was found that the desynchronization-induced AD in coupled oscillators with a linear trend of frequency distribution can be eliminated by inducing a small deviation of the frequency distributions. Wu *et al*. found that the synchronization is easier to realize when each pair of neighbor nodes has large frequency mismatches in coupled oscillators with periodical frequency distribution.[48] Moreover, the distributions of the initial frequency have obvious effects on the patterns of the pendulum wave[49] which is a simple but rich phenomenon in pendulum. Since the spatial period of the frequency distribution is a basic character of the periodical frequency distribution, it is therefore meaningful to raise the question whether and how the spatial period of the frequency distribution influences the AD regimes in coupled nonidentical oscillators. In this contribution, we try to explore what is the optimal spatial period of the frequency distribution with which the coupled nonidentical oscillators can be revived to oscillate from the AD state.

II. MODEL

In this paper, we study a coupled nonidentical oscillator system consisting of N Stuart-Landau oscillators as follows:

$$\begin{aligned}\dot{z}_j(t) = (1 + i\omega_j - |z_j(t)|^2)z_j(t) + \epsilon(z_{j+1}(t) \\ + z_{j-1}(t) - 2z_j(t)), \quad j = 1, \ldots, N,\end{aligned} \tag{1}$$

where i is the imaginary and $z_j(t)$ is a complex variable. The coupled oscillators initially have a regular monotonic trend of the natural frequency distribution w_j

$$\begin{aligned}&\omega_j = \omega_1 + (j-1)\delta\omega, \quad j = 1, 2, \ldots, N, \\ &\omega_0 = \omega_N, \quad \omega_{N+1} = \omega_1,\end{aligned} \tag{2}$$

where ω_1 is arbitrarily set as 1 and $\delta\omega$ is the frequency mismatch of neighbor oscillators. Without coupling ($\epsilon = 0$), each oscillator has a limit cycle with a different oscillating frequency ω_j. In a periodical boundary condition of $z_{N+1}(t) = z_1(t), z_0(t) = z_N(t)$, the coupled oscillation system with a frequency distribution as in Eq. (2) can be taken as a spatial frequency distribution with period m ($m = 1$). Since the coupled system has V-shaped AD domains in the parameter space of $\delta\omega \sim \epsilon$,[45] the coupled system transits from oscillation regimes to AD and returns to oscillation again with the increment of coupling strength ϵ when the frequency mismatch is properly presented larger than a critical value $\delta\omega_c$ (related to the system size N). There are two critical coupling strength values as ϵ_{c1} and ϵ_{c2} ($\epsilon_{c2} > \epsilon_{c1}$), respectively. To explore effects of the spatial period m of the frequency distribution on the AD domain, we calculate the critical values of ϵ_{c1} and ϵ_{c2} for all different possible spatial periods m. To generate frequency distribution with a given spatial period m, the nodes in the coupled chain initially with the spatial period $m = 1$ are firstly divided into N/m groups in sequence (note that we choose m to be submultiple of N in order to form N/m groups with equal numbers of nodes in each group), then each group has m nodes numbered as j = 1,2,…,m. Regroup the nodes into m groups by selecting the nodes with the same index into the *jth* group. Figures 1(a) and 1(b) present a schematic diagram of spatial frequency distribution in the coupled oscillators with $N = 16$, where the spatial frequency distribution periods m are 1, 2, 3, respectively.

III. THEORETICAL ANALYSIS

The AD states of system (1) can be predicted by analyzing the stability of the fixed points $|z_j| = 0, j = 1, 2, \ldots, N$ in the coupled oscillators. By introducing a perturbation $\eta_j(t)$ into these fixed points $|z_j| = 0, j = 1, 2, \ldots, N$, the evolution of the perturbations is governed by

$$\dot{\eta}_j(t) = (1 - 2\epsilon + i\omega_j)\eta_j(t) + \epsilon\eta_{j+1}(t) + \epsilon\eta_{j-1}(t). \tag{3}$$

With the definition of the column vector $\eta(t) = (\eta_1(t), \eta_2(t), \ldots, \eta_n(t))'$ (where $'$ is the transpose symbol), Eq. (3) can be rewritten as

$$\dot{\eta}(t) = B\eta(t), \tag{4}$$

where the symmetric B is described as

$$B = \begin{pmatrix} 1-2\epsilon+i\omega_1 & \epsilon & 0 & \dots & \dots & 0 & \epsilon \\ \epsilon & 1-2\epsilon+i\omega_2 & \epsilon & 0 & \dots & \dots & 0 \\ 0 & \epsilon & 1-2\epsilon+i\omega_3 & \epsilon & 0 & \dots & 0 \\ & \dots & \dots & \dots & & & \\ \epsilon & 0 & \dots & \dots & 0 & \epsilon & 1-2\epsilon+i\omega_N \end{pmatrix}.$$

Assume that B can be diagonalized by a matrix P

$$P^{-1}BP = \mathrm{diag}(\lambda_0, \lambda_1, \dots, \lambda_{N-1}), \qquad (5)$$

where $\lambda_k, k = 0, 1, \dots, N-1$ are the eigenvalues of B. A necessary condition for stable AD of Eq. (2) is that all real parts of the eigenvalues $Re(\lambda_k) < 0, k = 0, 1, \dots, N-1$. Then, the AD domain is completely determined by the critical lines of all $Re(\lambda_k) \le 0, k = 0, 1, \dots, N-1$.

IV. NUMERICAL ANALYSIS

Based on the above theoretical analysis, it is difficult to work out the expression of the eigenvalue λ of matrix B when the system size N is large. Therefore, we have to numerically get the AD domain for different spatial periods m of frequency distribution. Let us first consider AD in coupled oscillators with a spatial frequency distribution in period m ($m = 1$). The maximal real parts of the eigenvalues of matrix B versus the coupling strength ϵ are presented in Fig. 2(a) with the frequency mismatches being $\delta\omega = 0.1$, 0.2, 0.3, 0.4, respectively, for arbitrary $N = 30$. Notice that there is no AD when the frequency mismatches $\delta\omega$ are small (for example, $\delta\omega < 0.12$ as $N = 30$ and $m = 1$), since the maximal real parts of the eigenvalues of B are all positive for all coupling strength ϵ. However, the V-shaped curves of $Re\lambda$ versus ϵ move down and intersect with the horizontal axis at two critical coupling strengths with the increment of the frequency mismatches $\delta\omega$, which results in an enlargement of the AD domain. In order to explore the effects of the spatial frequency distribution on the two critical coupling strengths, we have to consider the situation of large frequency mismatches $\delta\omega$, so that we can record all left and right critical coupling strengths ϵ_{c1} and ϵ_{c2} for all possible spatial periods m of frequency distribution. Figs. 2(b) and 2(c) present the relationship curve between the critical coupling strengths (ϵ_{c1} and ϵ_{c2}) of the AD domain and the frequency mismatches $\delta\omega$. The left critical coupling strength ϵ_{c1} decreases with the increment of $\delta\omega$ for $\epsilon_{c1} = C_1\delta\omega^{\gamma}$, $C_1 = 0.75$, $\gamma = -2.5$, while the right critical coupling strength ϵ_{c2} increases with the increment of $\delta\omega$ for $\epsilon_{c2} = C_2\delta\omega^{\gamma}$, $C_2 = 20$, $\gamma = 2$.

It was found that the frequency distribution had great influences on the critical coupling strengths of AD in a ring of coupled nonidentical oscillators, where the left (right) critical coupling strengths for AD obey a power-law (a log-normal) distribution for all possible frequency distributions.[45] Here, we focus on AD domain of a special frequency distribution which has a spatial period m. To figure out how the spatial period m influences the two critical coupling strengths of the AD domain, we rearrange the frequency of the coupled oscillators in different spatial periods m and calculate the maximal real part of the eigenvalues ($Re\lambda$) of B versus ϵ for all frequency distribution with spatial period m. Fig. 3(a) presents examples of $Re\lambda$ versus ϵ for $m = 1$, 4 with initial frequency mismatch $\delta\omega = 1$ and system size $N = 16$. ϵ_{c1} remains to be 0.5, while the values of ϵ_{c2} are quite different with $\epsilon_{c2} = 2.17$ for $m = 1$ and $\epsilon_{c2} = 19.68$ for $m = 4$. To make

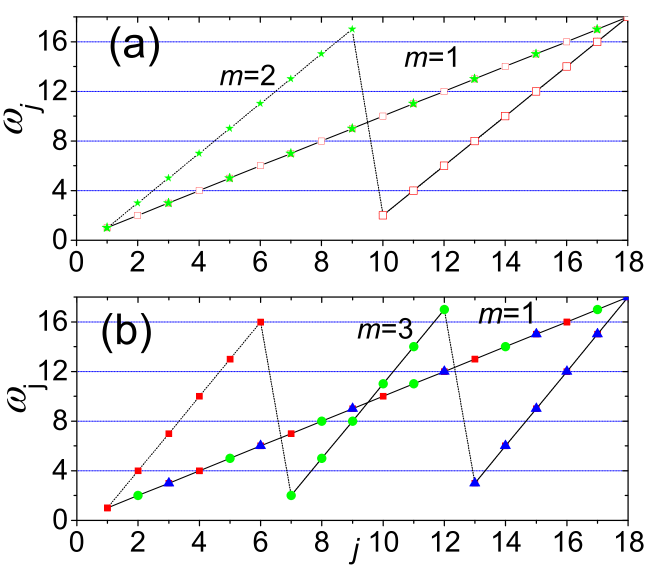

FIG. 1. The spatial frequency distribution of $N = 16$ oscillators (a) for $m = 1$ and $m = 2$, respectively, (b) for $m = 1$ and $m = 4$, respectively.

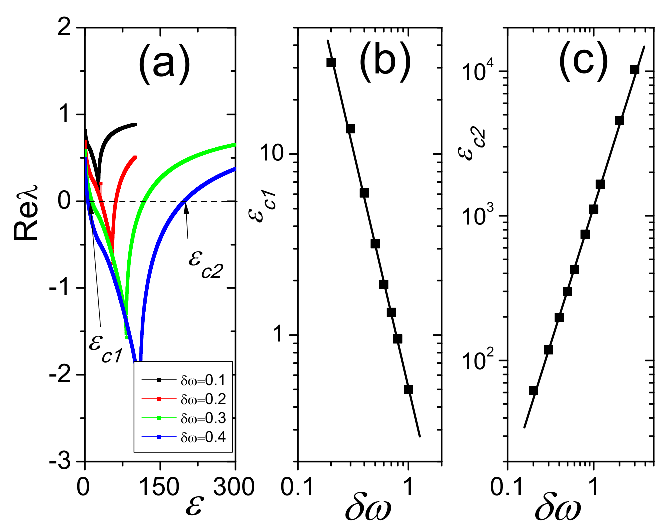

FIG. 2. (a) The real part of the eigenvalue of the matrix B ($Re\lambda$) for the frequency mismatches between neighbor oscillators $\delta\omega = 0.1$, 0.2, 0.3, 0.4, respectively, with $N = 30$ and $m = 1$. (b) and (c) The relation between the frequency mismatches between neighbor oscillators $\delta\omega$ and the critical values for AD ϵ_{c1} (left), ϵ_{c2} (right) in log-log axis, respectively.

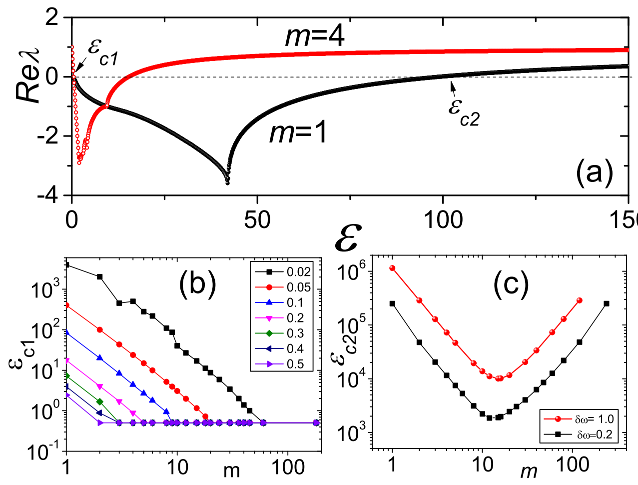

FIG. 3. (a) The real part of eigenvalue of the matrix B ($Re\lambda$) versus the coupling strength ϵ for $m=1$ (black solid dots) and $m=4$ (red circle), respectively, with $N=16$, $\delta\omega=1$. (b) The left critical value ϵ_{c1} versus the spatial period m for $\delta\omega=0.02$, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, respectively, and $N=360$. (c) The right critical value ϵ_{c2} versus the spatial period m in double log axis for $N=240$ and $\delta=0.2$ (black line), 1 (red line).

it clearer, we record all the critical values of ϵ_{c1} and ϵ_{c2} for different spatial periods m with different initial frequency mismatches. Figure 3(b) gives the dependence of the left critical values of ϵ_{c1} versus m (note that all m are selected being the submultiple of the number N) for arbitrarily selected large number $N=360$ and $\delta\omega=0.02$, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, respectively. Obviously, with small initial frequency mismatches (for example, $\Delta\omega=0.02$), the left critical coupling strength ϵ_{c1} linearly decreases in log-log axis to a constant value 0.5 with the increment of m. For larger initial frequency mismatches (for example, $\Delta\omega=0.5$), ϵ_{c1} decreases to 0.5 when m is larger than 2. Therefore, the spatial period m has no obvious influence on ϵ_{c1} when the initial frequency is larger than a certain value, since AD is reached when the coupling is larger than 0.5 (this value is theoretically presented in Ref. 38) for all m. However, the right critical coupling strength ϵ_{c2} first decreases then increases with the increment of m. There is a minimal value of ϵ_{c2} at $m=15$ in $N=200$ coupled oscillators, which indicates that the spatial period m of the frequency distribution significantly influences ϵ_{c2}. ϵ_{c2} has a relationship of quadratic function with the spatial period m of the frequency distribution in log-log axis. There is an optimal spatial period m_0 of the frequency distribution with which the coupled system has a minimal critical strength to transit from an AD regime to reviving oscillation. Moreover, this relationship keeps for different initial frequency mismatches. Increasing initial frequency mismatch tends to move the whole curve up in ϵ_{c2} versus ϵ diagram as shown in Fig. 3(c) for $\Delta\omega=0.2$, 1, respectively.

Considering size effects of the coupled system as mentioned in our previous work,[39] we calculated the curves of ϵ_{c2} versus m for different system sizes of $N=30$, 60, 90, 240, respectively, as shown in Fig. 4(a). The curves in log-log axis move up with the increment of size N while maintaining a v-shape. There are minimal values of ϵ_{c2} when $m=m_0$ with m_0 being related to the system size N for all different system sizes N. Fig. 4(b) presents all m_0 for different system sizes N which fit well with the curve $m_0=\sqrt{N}$. By rescaling the variables ϵ_{c2} and m to $\epsilon'_{c2}=\epsilon_{c2}/N^3$ and $m'=m/\sqrt{N}$, we find that the curves of ϵ'_{c2} versus m' for all N in Fig. 4(a) coincide well with each other which can be described as follows:

$$\log(\epsilon'_{c2}) = \epsilon_0 + Ae^{-\frac{\log(m')^2}{2\sigma^2}} \tag{6}$$

with $\sigma=0.662$, $\epsilon_0=0.63$, and $A=-2.47$. There is an optimal rescaled spatial period $m'=1$ (i.e., $m=\sqrt{N}$) of the

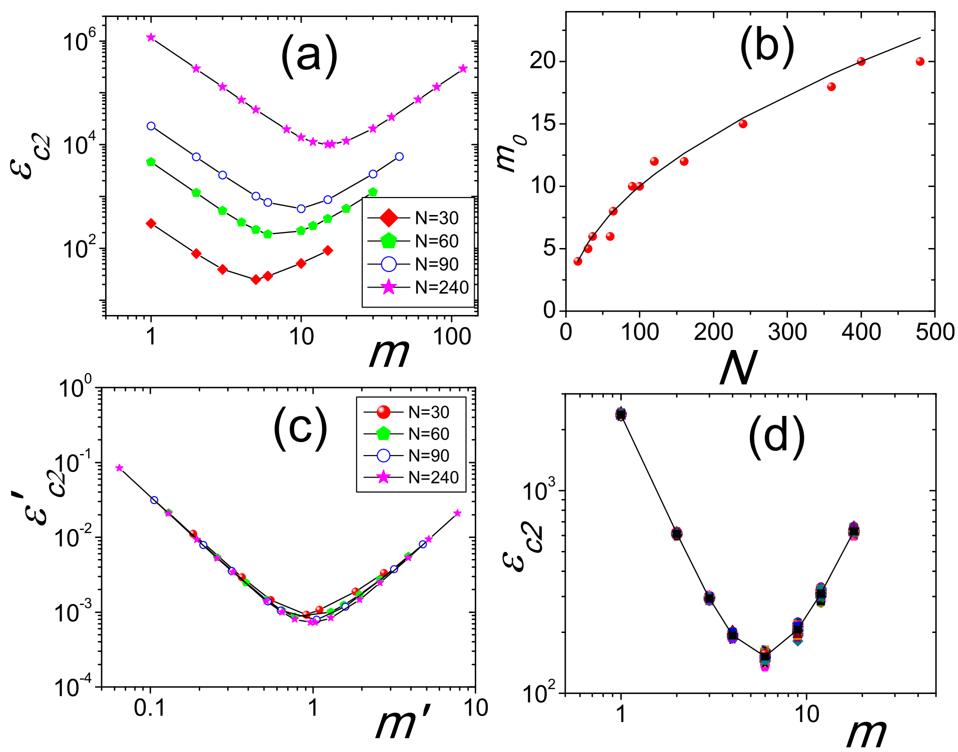

FIG. 4. (a) The right critical value ϵ_{c2} versus the spatial period m for $N=30$, 60, 90, 240, respectively, for $\delta\omega=0.5$. (b) The value of m_0 versus the system size N, where m_0 is the spatial frequency distribution period with which the system has minimal value of ϵ_{c2}. The fitted line has a function of $m_0=\sqrt{N}$. (c) The rescaled variable ϵ'_{c2} versus the rescaled variable m' for $N=30$, 60, 90, 240, respectively. (d) A total of 200 samples of right critical values ϵ_{c2} versus the spatial period m for coupled oscillators with random variations on the initial frequency distributions for $N=36$.

frequency distribution with which the coupled system has a minimal value of the ϵ_{c2}. That is to say, when the spatial period m of the frequency distribution is $\sqrt{N}$, the coupling strength needed for reviving oscillation from AD is minimal.

For simplicity, the initial frequency mismatches between each neighbor nodes are all set constantly as $\delta\omega$ in Eq. (2). It is natural to explore the question whether the existence of the optimal spatial frequency distribution can be robustly maintained to random initial frequency distributions. By applying random variations ξ on the initial frequencies of each node as described in Eq. (7), we change the frequency mismatch $\delta\omega_j$ between each pair of neighbor nodes

$$w_j = \omega_0 + (j-1)\delta\omega + \xi, \quad j = 1, 2, \ldots, N, \\ \xi \in [-\delta\omega/2, \delta\omega/2]. \tag{7}$$

Let us consider an example of $N = 36$ coupled oscillators. The values of ϵ_{c2} are presented in Fig. 4(d) corresponding to 200 sets of different random noise ξ for each given spatial period m of frequency distribution. Though the critical values of ϵ_{c2} needed to revive the AD change according to the noises ξ for each given m, there is still an optimal spatial period $m_0 = 6$ with which the coupled oscillator has the smallest ϵ_{c2} to revive the AD.

V. REGIMES ANALYSIS BASED ON THE NUMERICAL RESULTS

It is interesting to reveal the regimes of how the spatial period of the frequency distribution influences the left and right critical coupling strength of AD. According to the analysis presented in Ref. 51, it is well known that coupled oscillators with mismatches $\delta\omega$ have a v-shaped curve in the parameter space of $\delta\omega$ versus ϵ. There are two critical values of coupling constant, ϵ_{c1} and ϵ_{c2}, when the parameter mismatch $\delta\omega$ is larger than a critical value $\delta\omega_c$.[45] The coupled oscillators may approach AD when $\epsilon > \epsilon_{c1}$ and leave the AD state due to the realization of synchronization when $\epsilon > \epsilon_{c2}$. First, let us explore how the spatial periods m influence the left critical coupling strength ϵ_{c1}. Since the coupling strength is not strong enough to form synchronous clusters when the coupling strength is near ϵ_{c1}, the critical coupling strength for AD is then determined by the frequency mismatch between two neighbor oscillators. Initially, the mismatch between two neighbor oscillators (the boundary oscillators are excluded) is $\delta\omega$ when $m = 1$. As m increases, the mismatch between two neighbors will increase to be $m * \delta\omega$ (based on the spatial frequency distribution). Therefore, the mismatch between two arbitrary neighbor nodes keeps increasing with the increment of spatial period m which leads to linear decrement of ϵ_{c1} in log-log axis (based on the results in Fig. 2(b)). Moreover, when $\delta\omega$ is larger than a critical value $\delta\omega_c$ (related to number of coupled oscillators), the coupled system will become AD for all $\epsilon > 0.5$.[51] Therefore, the values of ϵ_{c1} remain constant of 0.5 when $m * \delta\omega$ is larger than the critical value $\delta\omega_c$ for all m.

Now let us focus on the effects of the spatial period of the frequency distribution on ϵ_{c2} which is determined by the effects of spatial period of frequency distribution on synchronizability. To make the picture clearer, we consider an example of coupled Landau-Stuart oscillators with an arbitrary system number $N = 16$. Figures 5(a)–5(d) present the average frequencies versus ϵ of the coupled oscillators with different spatial frequency periods $m = 1, 2, 4, 8$ for an arbitrarily given small frequency mismatch $\delta\omega = 0.2$. Obviously, the critical coupling constants ϵ_{c2} to realize synchronization are $\epsilon_{c2} = 4.25$, 1.60, 1.10, 1.90 for $m = 1, 2, 4, 8$, respectively. The coupled oscillators with the frequency distribution period $m = 4$ have a minimal value of ϵ_{c2}. By carefully checking the transition process to synchronization according to the average frequency of the oscillators, we find that the final synchronization is realized by combining the smaller synchronous clusters into larger ones. The initial frequency distribution dramatically influences the numbers and the distributions of the small synchronous clusters. When $m = 1$, the coupled oscillators tend to form four synchronous clusters with different synchronous frequencies as shown in Fig. 6(a). Each of them is a collection of four neighbor oscillators. Moreover, the synchronous frequency of each cluster increases in the sequence from left to right, which makes it difficult to be combined into larger cluster with the competition of each cluster, since the left and the right neighbor of each cluster have different frequencies (the left one is smaller, while the right one is larger than itself). Therefore, it needs a rather large coupling strength to transit from four clusters to three, then to two clusters till one cluster. However, when $m = 2$, the coupled oscillators first form 4 synchronous clusters with synchronous frequencies being high and low values alternately which gradually combine to three and then 2 clusters till the full synchronous state as shown in Fig. 6(b). Compared to $m = 1$, the left and right neighbors of each cluster have similar frequencies, which make it easier to transit to a larger synchronous cluster. For $m = 4$, it is easy to form 8 synchronous clusters with their synchronous frequencies being in high and low values alternately. Compared to $m = 2$, each cluster has a smaller size, which makes it easier to form larger clusters which have smaller mismatches of the synchronous frequencies as shown in Fig. 6(c). For $m = 8$, the initial frequencies of each pair of

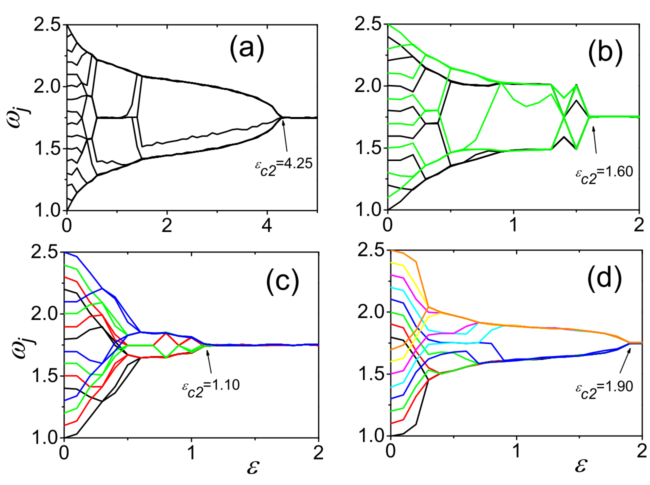

FIG. 5. (a)–(d) The average frequency versus coupling constant ϵ for spatial frequency distribution period (a) $m = 1, 2, 4, 8$, respectively. The nodes with the same color are within the same spatial period.

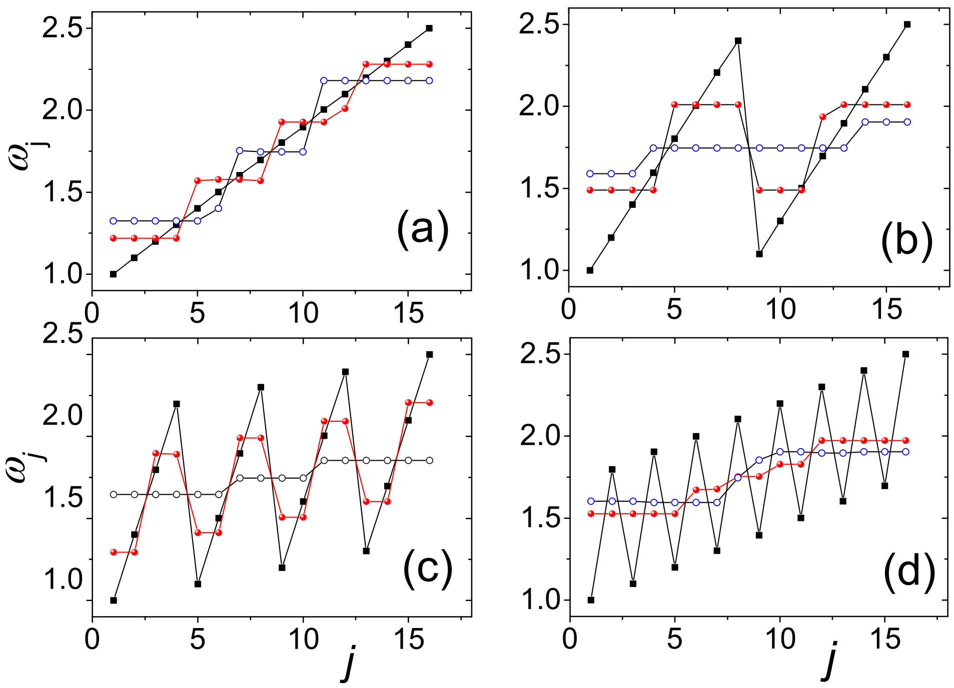

FIG. 6. The average frequency distribution of each node i for different coupling strengths of coupled Stuart-Landau. (a) $m=1$, $\epsilon=0$ (black squares), $\epsilon=0.4$ (red dots), $\epsilon=0.9$ (blue circles), (b) $m=2$, $\epsilon=0$ (black square), $\epsilon=1.3$ (red dots), $\epsilon=1.4$ (blue circles), (c) $m=4$, $\epsilon=0$ (black square), $\epsilon=0.3$ (red dots), $\epsilon=0.4$ (blue circles), (d) $m=8$, $\epsilon=0$ (black square), $\epsilon=0.5$ (red dots), $\epsilon=0.8$ (blue circles).

nodes have high and low values alternately; the coupling can easily make the system form two large clusters. However, compared to $m=4$, the two clusters have larger frequency mismatch than that of $m=4$ as shown in Fig. 6(d). Totally, we find that when $m=4$ ($\sqrt{N}$ with $N=16$), the coupled oscillators first form many small synchronous clusters with each pair of neighbor clusters being in high and low frequency alternatively, which makes it the easiest (need the lowest coupling strength) to form the larger synchronous clusters. Therefore, the smallest critical coupling constant is needed to form a whole synchronous cluster.

Moreover, the transition process to synchronization of the coupled chaotic oscillators is similar to that of coupled period oscillators, such as the coupled Rossler oscillators $dx/dt = -wy - z, dy/dt = wx + ay, dz/dt = f + z(x - c)$ ($a = 0.165, f = 0.2, c = 10, \delta\omega = 0.02$, the coupling term is on the variable x). Figures 7(a)–7(d) present the average frequencies of the coupled Rossler oscillators for $m = 1, 2, 4, 8$, respectively, which have the similar transition process as in Figs. 6(a)–6(d). For $m = 1$, the coupled chaotic oscillators first form four synchronous clusters with each containing different increasing synchronous frequencies in the sequence from left to right which is similar to that in Fig. 6(a). However, 8 nearly synchronous clusters are formed with each containing the synchronous frequency in high and low values alternatively for $m = 4$ which is right similar to that in Fig. 6(c). Therefore, the effects of spatial frequency period m on the ϵ_{c2} are common in either coupled period or chaotic oscillators.

VI. DISCUSSION AND CONCLUSION

Totally, we have discussed the oscillation quenching and reviving dynamics in coupled oscillators with special periodical frequency distribution. Although the frequency distribution in our model is an idealized one, there are still some nature systems and application in this aspect, for example, Ref. 50 has experimentally explored the effects of frequency distribution on chimera state in chemical oscillators. Moreover, when it is extended to more general cases of random frequency distribution,[45] a physical quantity called "roughness" R of a given frequency configuration can effectively be defined (Eq. (8)) and used to exhibit the effects of frequency distribution on synchronization[48] and AD regimes.

$$R = \frac{1}{N}\sum_{i=1}^{N}(\omega_{j+1} - \omega_j)^2. \tag{8}$$

Since there are variations of critical coupling strengths for all random frequency distributions and corresponding

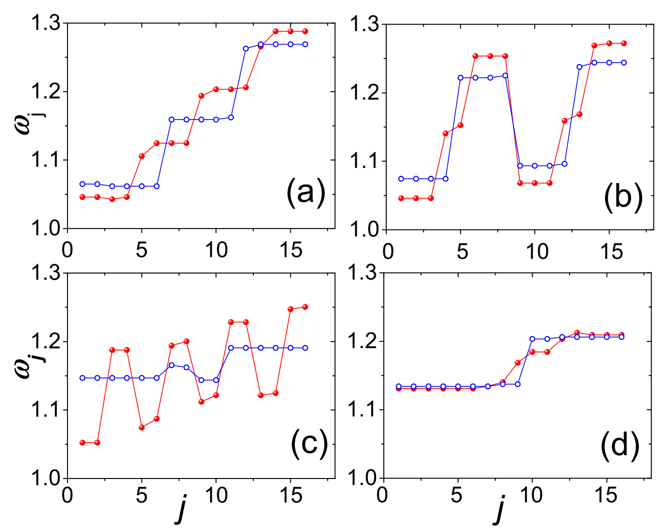

FIG. 7. The average frequency distribution of each node i for different coupling strengths of coupled Rossler. (a) $m=1$, $\epsilon=0.1$ (red dots), $\epsilon=0.2$ (blue circles), (b) $m=2$, $\epsilon=0.1$ (red dots), $\epsilon=0.2$ (blue circles), (c) $m=4$, $\epsilon=0.1$ (red dots), $\epsilon=0.2$ (blue circles), and (d) $m=8$, $\epsilon=0.22$ (red dots), $\epsilon=0.25$ (blue circles).

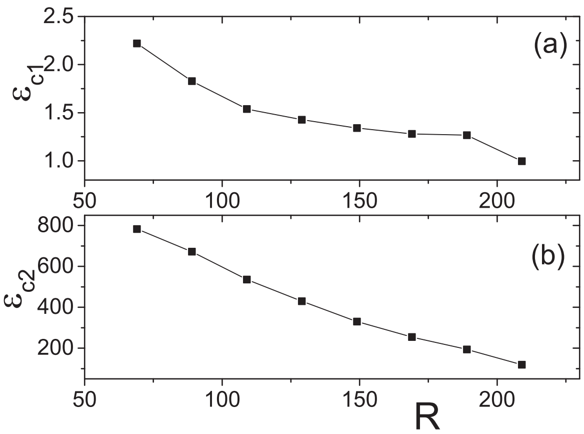

FIG. 8. (a) The average critical coupling strength $\langle \epsilon_{c1} \rangle$ versus R. (b) The average critical coupling strength $\langle \epsilon_{c2} \rangle$ versus R. The system size $N = 60$ with initial $\delta\omega = 0.5$.

roughness R. By averaging the critical coupling strengths ϵ_{c1} and ϵ_{c2} of AD domain for a certain range of roughness R, we find the two averaged critical coupling strengths both decrease with the increment of average roughness R as shown in Figs. 8(a) and 8(b). The spatial frequency distributions still obviously influence the critical coupling strengths of the AD domain. This work has extended our formal results[45] by considering the effects of more detailed spatially periodical frequency distribution on AD regimes.

In conclusion, the spatial frequency distributions in coupled oscillators have dramatic influences on the critical coupling strength of the AD. Considering a kind of periodically distributed parameter of frequency in the coupled Landau-Stuart oscillators, we find that the spatial periods m of the frequency distribution have different effects on the left and right critical coupling strength for AD. (1) The left one has a power law relationship with the spatial period m for small initial frequency mismatches but remain constant for large initial frequency mismatches. (2) The right one has a quadratic function relation with the spatial period m in log-log axis. There is an optimal period $m_0 = \sqrt{N}$ for which the coupled oscillators have the smallest right critical coupling strength ϵ_{c2} for reviving oscillation from AD. Among the regimes for the effects of the spatial period m on the left critical coupling strength ϵ_{c1} is that the increment of spatial period m tends to enlarge the frequency mismatches between two neighbor nodes which then decrease the critical coupling needed to realize AD. However, the regimes of the effects of spatial period m on the right critical coupling strength are the outcome of competition between the synchronization clusters and the frequency mismatches. With the optimal period of $m_0 = \sqrt{N}$, the coupled system easily forms small synchronous clusters which have two neighbors with a similar frequency. The coupling from the two neighbors makes it easier to form large synchronous clusters which revives the coupled oscillators from AD regimes to oscillating regimes. Our results may have potential applications in physics and biology; for example, rich pattern formations are expected in the coupled pendulum by setting up proper length of each pendulum in a coupled ring.

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[1]Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence* (Springer, Berlin, 1984).

[2]M. Lakshmanan and D. V. Senthilkumar, *Dynamics of Nonlinear Time-Delay Systems* (Springer, Berlin, 2010).

[3]S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, and C. Zhou, "The synchronization of chaotic systems," Phys. Rep. **366**, 1 (2002).

[4]L. M. Pecora and T. L., Carroll, "Synchronization in chaotic systems," Phys. Rev. Lett. **64**, 821 (1990).

[5]M. G. Rosenblum, A. S. Pikovsky, and J. Kurths, "Phase synchronization of chaotic oscillators," Phys. Rev. Lett. **76**, 1804 (1996).

[6]W. Liu, J. Xiao, X. Qian, and J. Yang, "Antiphase synchronization in coupled chaotic oscillators," Phys. Rev. E **73**, 057203 (2006).

[7]F. A. Rodrigues, T. K. DM. Perin, P. Ji, and J. Kurths, "The Kuramoto model in complex networks," Phys. Rep. **610**(26), 1–98 (2016).

[8]A. Zakharova, M. Kapeller, and E. Schöl, "Chimera death: Symmetry breaking in dynamical networks," Phys. Rev. Lett. **112**, 154101 (2014).

[9]C. Fu, W. Lin, L. Huang, and X. Wang, "Synchronization transition in networked chaotic oscillators: The viewpoint from partial synchronization," Phys. Rev. E **89**, 052908 (2014).

[10]Z. Song, A. Karma, J. N. Weiss, and Z. Qu, "Long-lasting sparks: Multi-metastability and release competition in the calcium release unit network," PLoS Comput. Biol. **12**(1), e1004671 (2016).

[11]Z. Song, C. Y. Ko, M. Nivala, J. N. Weiss, and Z. Qu, "Calcium-voltage coupling in the genesis of early and delayed after depolarizations in cardiac myocytes," Biophys. Soc. **108**, 1908 (2015).

[12]G. Saxena, A. Prasad, and R. Ramaswamy, "Amplitude death: The emergence of stationarity in coupled nonlinear systems," Phys. Rep. **521**, 205 (2012).

[13]A. Koseska, E. Volkov, and J. Kurths, "Oscillation quenching mechanisms: Amplitude vs. oscillation death," Phys. Rep. **531**, 173–199 (2013).

[14]A. Koseska, E. Volkov, and J. Kurths, "Transition from amplitude to oscillation death via turning bifurcation," Phys. Rev. Lett. **111**, 024103 (2013).

[15]G. Song, N. V. Buck, and B. N. Agrawal, "Spacecraft vibration reduction using pulse-width pulse-frequency modulated input shaper," Control Dyn. **22**(3), 433–440 (1999).

[16]E. Ullner, A. Zaikin, E. I. Volkov, and J. Garcia-Ojalvo, "Multistability and clustering in a population of synthetic genetic oscillators via phase repulsive cell-to-cell communication," Phys. Rev. Lett. **99**, 148103 (2007).

[17]A. Koseska, E. Volkov, and J. Kurths, "Detuning-dependent dominance of oscillation death in globally coupled synthetic genetic oscillators," Europhys. Lett. **85**, 28002 (2009).

[18]M. Y. Kim, R. Roy, J. L. Aron, T. W. Carr, and I. B. Schwartz, "Scaling behavior of laser population dynamics with time-delay coupling: Theory and experiment," Phys. Rev. Lett. **94**, 088101 (2005).

[19]A. Prasad, Y. C. Lai, A. Gavrielides, and V. Kovanis, "Amplitude modulation in a pair of time-delay coupled external-cavity semiconductor lasers," Phys. Lett. A **318**, 71 (2003).

[20]W. Liu, E. Volkov, J. Xiao, W. Zou, M. Zhan, and J. Yang, "Inhomogeneous stationary and oscillatory regimes in coupled chaotic oscillators," Chaos **22**, 033144 (2012).

[21]C. R. Hens, O. I. Olusola, P. Pal, and S. K. Dana, "Oscillation death in diffusively coupled oscillators by local repulsive link," Phys. Rev. E **88**, 034902 (2013).

[22]R. Karnatak, R. Ramaswamy, and A. Prasad, "Amplitude death in the absence of time delays in identical coupled oscillators," Phys. Rev. E **76**, 035201(R) (2007).

[23]Y. Zhu, X. Qian, and J. Yang, "A study of phase death states in a coupled system with stable equilibria," Europhys. Lett. **82**, 40001 (2008).

[24]K. Konishi, "Amplitude death induced by dynamic coupling," Phys. Rev. E **68**, 067202 (2003).

[25]K. Konishi, "Amplitude death induced by a global dynamic coupling," Int. J. Bifurcation Chaos **17**, 2781–2789 (2007).
[26]D. V. RamanaRedy, A. Sen, and G. L. Johnston, "Time delay induced death in coupled limit cycle oscillators," Phys. Rev. Lett. **80**, 5109 (1998).
[27]F. M. Atay, "Distributed delays facilitate amplitude death of coupled oscillators," Phys. Rev. Lett. **91**, 094101 (2003).
[28]W. Zou, D. V. Senthilkumar, A. Koseska, and J. Kurths, Phys. Rev. E **88**, 050901(R) (2013).
[29]S. H. Strogatz, "Nonlinear dynamics: Death by delay," Nature **394**, 316–317 (1998).
[30]A. Prasad, P. R. Sharma, and M. Shrimali, "Amplitude death in nonlinear oscillations with indirect coupling," Phys. Lett. A **376**, 1562–1566 (2012).
[31]A. Sharma and M. D. Shrimali, "Amplitude death with mean field diffusion," Phys. Rev. E **85**, 057204 (2012).
[32]V. Resmi, G. Ambika, R. E. Amritkar, and G. Rangarajan, "Amplitude death in complex networks induced by environment," Phys. Rev. E **85**, 046211–046217 (2012).
[33]W. Liu, G. Xiao, Y. Zhu, M. Zhan, J. Xiao, and J. Kurths, "Oscillator death induced by amplitude-dependent coupling in repulsively coupled oscillators," Phys. Rev. E **91**, 052902 (2015).
[34]G. B. Ermentrout, "Oscillator death in populations of "all to all"," coupled nonlinear oscillators, Physica (Amsterdam) D **41**, 219 (1990).
[35]Z. Hou and H. Xin, "Oscillator death on small-world networks," Phys. Rev. E **68**, 055103 (2003).
[36]W. Liu, X. Wang, S. Guan, and C. H. Lai, "Transition to amplitude death in scale-free networks," New J. Phys. **11**, 093016 (2009).
[37]H. Bi, X. Hu, X. Zhang, Y. Zou, Z. Liu, and S. Guan, "Explosive oscillation death in coupled Stuart-Landau oscillators," Europhys. Lett. **108**, 50003 (2014).
[38]J. Yang, "Transitions to amplitude death in a regular array of nonlinear oscillators," Phys. Rev. E **76**, 016204 (2007).
[39]W. Liu, J. Xiao, L. Li, Y. Wu, and M. Lu, "Effects of gradient coupling on amplitude death in nonidentical oscillators," Nonlinear Dyn. **69**, 1041–1050 (2012).
[40]W. Zou, D. V. Senthilkumar, M. Zhan, and J. Kurths, "Reviving oscillations in coupled nonlinear oscillators," Phys. Rev. Lett. **111**, 014101 (2013).
[41]W. Zou, D. V. Senthilkumar, R. Nagao, I. Z. Kiss, Y. Tang, A. Koseska, J. Duan, and J. Kurths, "Restoration of rhythmicity in diffusively coupled dynamical networks," Nat. Commun. **6**, 7709 (2015).
[42]D. Ghosh, T. Banerjee, and J. Kurths, "Revival of oscillation from mean-field-induced death: Theory and experiment," Phys. Rev. E **92**, 052908 (2015).
[43]J. Jalife, R. A. Gray, G. E. Morley, and J. M. Davidenko, "Self-organization and the dynamical nature of ventricular fibrillation," Chaos **8**, 79 (1998).
[44]M. Sara, S. Sacco, F. Cipolla, P. Onorati, C. Scoppetta, G. Albertini, and A. Carolei, "An unexpected recovery from permanent vegetative state," Brain Inj.: BI **21**, 101 (2007); *Textbook of Therapeutic Cortical Stimulation*, edited by S. Canavero (Nova Science, New York, 2009).
[45]Y. Wu, W. Liu, J. Xiao, W. Zou, and J. Kurths, "Effects of spatial frequency distributions on amplitude death in an array of coupled Landau-Stuart oscillators," Phys. Rev. E **85**, 056211 (2012).
[46]H. Ma, W. Liu, Y. Wu, M. Zhan, and J. Xiao, "Ragged oscillation death in coupled nonidentical oscillators," Commun. Nonlinear Sci. Numer. Simul. **19**, 2874–2882 (2014).
[47]L. Rubchinsky and M. Sushchik, "Disorder can eliminate oscillator death," Phys. Rev. E **62**, 6440 (2000).
[48]Y. Wu, J. H. Xiao, G. Hu, and M. Zhan, "Synchronizing large number of nonidentical oscillators with small coupling," Europhys. Lett. **97**, 40005 (2012).
[49]J. A. Flaten and K. A. Parendo, "Pendulum waves: A lesson in aliasing," Am. J. Phys. **69**, 778 (2001).
[50]M. Wickramasinghe and I. Z. Kiss, "Spatially organized partial synchronization through the chimera mechanism in a network of electrochemical reactions," Phys. Chem. Chem. Phys. **16**, 18360 (2014).
[51]D. G. Aronson, G. B. Ermentrout, and N. Kopell, "Amplitude response of coupled oscillators," Physica D **41**, 403 (1990).