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Anti-synchronization of coupled memristive neutral-type neural networks with mixed time-varying delays via randomly occurring control

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Abstract In this paper, a class of coupled memristive neural networks of neutral type with mixed time-varying delays via randomly occurring control is studied in order to achieve anti-synchronization. The model of the coupled memristive neural networks of neutral type with mixed time-varying delays is less conservative than those of traditional memristive neural networks. Some criteria are obtained to guarantee the anti-synchronization between the drive system and the response system. Two kinds of randomly occurring memristor-based controllers are designed. The analysis in this paper employs the differential inclusions theory, linear matrix inequalities, and the Lyapunov functional method. In addition, the new proposed results here are very easy to verify and also extend the results of earlier publications. Numerical examples are given to show the effectiveness of our results.

Keywords Memristive neural networks · Neutral-type · Mixed delays · Randomly occurring control

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1 Introduction

In the past decades, in order to process information intelligently, artificial neural networks haven been proposed to simulate the function of human brain. Traditional artificial neural networks have been implemented with a circuit, and the connection between neural processing units is realized with a resistor. The resistance is equal to the strength of synapses between neurons. The strength of the synapses is a variable, while the resistance is invariable. Combining the memory characteristic of memristor, the resistor is replaced by a memristor in order to simulate artificial neural network better, and a memristor eventually may be used in artificial neural networks. Recently, the authors in [1–4] have concentrated on the dynamical nature of memristor-based neural networks in order to use it in applications, such as pattern recognition, associative memories, and learning, in a way that mimics the human brain.

It is well known that time delays generate complex and unpredictable behaviors in practice and are often caused by finite switching speeds of the amplifiers. Therefore, much effort has been paid in recent years for analyzing dynamic behaviors of neural networks with various types of time delays (see [5–8]). The constant time delays, and the time-varying delays have been studied in [9,10]. Note that the neural signal propagation is often distributed during a certain time period in the presence of an amount of parallel pathways with a variety of axon sizes and lengths. Hence, the authors in [11–13] have concentrated on the discrete and distributed delays. In addition, anti-synchronization control of neural networks plays an important role in many potential applications, e.g., nonvolatile memories, neuromorphic devices to simulate learning, adaptive, and spontaneous behaviors. However, up to now, there are few studies on the anti-synchronization control of memristive neural networks with distributed delays and discrete delays.

On the other hand, due to the complicated dynamic properties of the neural cells in the real world, there exist many neural network models such as recurrent neural networks [14], fuzzy neural networks [15], and bidirectional associative memory neural networks [16,17] that cannot characterize the properties of a neural reaction process precisely. It is natural and important that systems will contain some information about the derivative of previous state. This kind of neural network is termed as neutral-type neural network. In recent years, there has been a growing research interest in the study of delayed neural networks of neutral type (see [18–22]). However, there were few studies to analyze the anti-synchronization control of coupled memristive neutral-type neural networks with discrete and distributed time-varying delays (mixed delays).

And due to the fact that signals in networked systems are not transmitted perfectly or the control is not available, as in the cases of packet dropouts, random failures, and repairs of actuators, control should be suspended from time to time. Therefore, control activation and networked systems may occur in a probabilistic or switching way and are randomly changeable in terms of their types or intensity. So the main contributions to this paper are as follows:

1. we fill the gap on anti-synchronization control of coupled neutral-type memristive neural networks with mixed delays;
2. we propose a less conservative model of the coupled neutral-type memristive neural networks with mixed time-varying delays;
3. we propose two kinds of memristor-based randomly occurring controllers, i.e., memristor-based delay-independent controller and memristor-based delay-dependent controller. The memristor-based delay-dependent controller is less conservative than the other one. Some criteria are obtained to guarantee the anti-synchronization between coupled memristive neural networks of neutral type with mixed delays.

2 Preliminaries

Based on the physical properties of memristor, the memristor-based neural networks of neutral type with time-varying mixed delays are described by ($i = 1, 2, \ldots, n$)

$$\begin{aligned} d\left[x_i(t) - d_i x_i(t-\tau_1(t))\right] &= \Bigg[-c_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))\tilde{f}(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(x_i(t))\tilde{g}(x_j(t-\tau_2(t))) \\ &\quad + \sum_{j=1}^{n} e_{ij}(x_i(t)) \int_{t-\tau_3(t)}^{t} \tilde{h}(x_j(s))\mathrm{d}s\Bigg]\mathrm{d}t, \end{aligned} \tag{1}$$

where $x_i(t)$ is the voltage of capacitor $\mathcal{C}_i$. $a_{ij}(x_i(t))$, $b_{ij}(x_i(t))$, $e_{ij}(x_i(t))$ represent memristor-based weights, $a_{ij}(x_i(t)) = \frac{W_{(1)ij}}{\mathcal{C}_i} \times sgin_{ij}$, $b_{ij}(x_i(t)) = \frac{W_{(2)ij}}{\mathcal{C}_i} \times sgin_{ij}$, $e_{ij}(x_i(t)) = \frac{W_{(3)ij}}{\mathcal{C}_i} \times sgin_{ij}$, and

$$sgin_{ij} = \begin{cases} 1, & i \neq j \\ -1, & i = j, \end{cases}$$

in which $W_{(1)ij}$, $W_{(2)ij}$, $W_{(3)ij}$ denote the memductances of memristors R_1, R_2, R_3. R_1, R_2, R_3 represent the memristors.

Combining with the physical structure of a memristor device, one can see that $W_{(1)ij} = \frac{\mathrm{d}q_{(1)ij}}{\mathrm{d}\sigma_{(1)ij}}$, $W_{(2)ij} =$

$\frac{\mathrm{d}q_{(2)ij}}{\mathrm{d}\sigma_{(2)ij}}$, $W_{(3)ij} = \frac{\mathrm{d}q_{(3)ij}}{\mathrm{d}\sigma_{(3)ij}}$, where $q_{(1)ij}, q_{(2)ij}, q_{(3)ij}$ and $\sigma_{(1)ij}, \sigma_{(2)ij}, \sigma_{(3)ij}$ denote charge and magnetic flux of the memristors R_1, R_2, R_3, respectively.

Many studies show that pinched hysteresis loop is the fingerprint of memristive devices. Under different pinched hysteresis loops, the evolutionary process of memristive systems evolves into different forms. It is generally known that the pinched hysteresis loop is due to the nonlinearity of the memductance function. The memductance functions $W_{(1)ij}$, $W_{(2)ij}$, and $W_{(3)ij}$ are given by:

$$W_{(1)ij} = l_{(1)ij} + 3l_{(2)ij}\sigma_{(1)ij}^2,$$
$$W_{(2)ij} = l_{(3)ij} + 3l_{(4)ij}\sigma_{(2)ij}^2,$$
$$W_{(3)ij} = l_{(5)ij} + 3l_{(6)ij}\sigma_{(3)ij}^2,$$

where $l_{(1)ij}$, $l_{(2)ij}$, $l_{(3)ij}$, $l_{(4)ij}$, $l_{(5)ij}$, $l_{(6)ij}$ are constants, $i, j = 1, 2, \ldots, n$. According to the feature of a memristor, $a_{ij}(x_i(t))$, $b_{ij}(x_i(t))$, $e_{ij}(x_i(t))$ are continuous functions, $\hat{a}_{ij} \le a_{ij}(x_i(t)) \le \check{a}_{ij}$, $\hat{b}_{ij} \le b_{ij}(x_i(t)) \le \check{b}_{ij}$, and $\hat{e}_{ij} \le e_{ij}(x_i(t)) \le \check{e}_{ij}$, for $i, j = 1, 2, \ldots, n$, where $\hat{a}_{ij}, \check{a}_{ij}, \hat{b}_{ij}, \check{b}_{ij}, \hat{e}_{ij}, \check{e}_{ij}$ are constants. $A(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$ and $B(x_i(t)) = (b_{ij}(x_i(t)))_{n\times n}$ are memristive connection weights, which represent the neuron interconnection matrix and the delayed neuron interconnection matrix, respectively. In the artificial neural networks, the memristors worked as synaptic weights. The connection weights

$$A(x_i(t)) = \left(a_{ij}(x_i(t))\right)_{n\times n},$$
$$B(x_i(t)) = \left(b_{ij}(x_i(t))\right)_{n\times n},$$
$$E(x_i(t)) = \left(e_{ij}(x_i(t))\right)_{n\times n},$$

change according to the state of each subsystem. If $A(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$, $B(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$ and $E(x_i(t)) = (e_{ij}(x_i(t)))_{n\times n}$are constants, the system (1) will reduce to a general network. $D = diag(d_1, \ldots, d_n) > 0$ and $C = diag(c_1, \ldots, c_n) > 0$ are self-feedback connection matrices. $\tilde{f}(x(t)) = [\tilde{f}(x_1(t)), \ldots, \tilde{f}(x_n(t))]^{\mathrm{T}}$, $\tilde{g}(x(t)) = [\tilde{g}(x_1(t)), \ldots, \tilde{g}(x_n(t))]^{\mathrm{T}}$, and $\tilde{h}(x(t)) = [\tilde{h}(x_1(t)), \ldots, \tilde{h}(x_n(t))]^{\mathrm{T}}$ are the neuron activation functions. $\tau_1(t)$, $\tau_2(t)$, $\tau_3(t)$ corresponds to the time-varying transmission delays.

When N memristor-based neural networks of neutral type with time-varying mixed delays are coupled by a network, we obtain

$$\begin{aligned} d[x_i(t) - d_i x_i(t-\tau_1(t))] &= \Bigg[-c_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))\tilde{f}(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(x_i(t))\tilde{g}(x_j(t-\tau_2(t))) \\ &\quad + \sum_{j=1}^{n} e_{ij}(x_i(t)) \int_{t-\tau_3(t)}^{t} \tilde{h}(x_j(s))\mathrm{d}s \\ &\quad + \sum_{j=1}^{N} \beta m_{ij}\Gamma x_j(t)\Bigg]\mathrm{d}t, \end{aligned} \tag{2}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\mathrm{T}}$ is the state variable of the ith memristive neural network. Suppose each memristive neural network is a node, and the information between two nodes is transmitted via an edge. $M = (m_{ij})_{N\times N}$ represents the coupling matrix, and if there is an edge from memristive neural network j to i, then $m_{ij} = 1$; otherwise, $m_{ij} = 0 (i \ne j)$. And $m_{ii} = -\sum_{j=1, j\ne i}^{N} m_{ij}$. β represents the coupling strength. The positive definite diagonal matrix Γ stands for the inner coupling between two connected memristive neural networks.

In this paper, we use the following assumptions and definitions.

Assumption 1 In this paper $\tau_1(t), \tau_2(t), \tau_3(t)$ are differential functions with $\dot{\tau}_1(t) < \mu_1 < 1$, $\dot{\tau}_2(t) < \mu_2 < 1$, $\dot{\tau}_3(t) < \mu_3 < 1$, and $\tau_1(t) < \tau_1$, $\tau_2(t) < \tau_2$, $\tau_3(t) < \tau_3$, where $\tau_1, \tau_2, \tau_3, \mu_1, \mu_2, \mu_3$ are positive constants.

Assumption 2 The functions $\tilde{f}_i$, $\tilde{g}_i$, and $\tilde{h}_i$ are bounded and odd functions.

Assumption 3 For $i, j = 1, 2, \ldots, n$,

$$co\{\hat{a}_{ij}, \check{a}_{ij}\}\tilde{f}_j(x_j(t)) + co\{\hat{a}_{ij}, \check{a}_{ij}\}\tilde{f}_j(y_j(t)) \subseteq co\{\hat{a}_{ij}, \check{a}_{ij}\}(\tilde{f}_j(x_j(t)) + \tilde{f}_j(y_j(t))),$$

$$co\{\hat{b}_{ij}, \check{b}_{ij}\}\tilde{g}_j(x_j(t-\tau_2(t))) + co\{\hat{b}_{ij}, \check{b}_{ij}\}\tilde{g}_j(y_j(t-\tau_2(t))) \subseteq co\{\hat{b}_{ij}, \check{b}_{ij}\}(\tilde{g}_j(x_j(t-\tau_2(t))) + \tilde{g}_j(y_j(t-\tau_2(t)))).$$

$$co\{\hat{b}_{ij}, \check{b}_{ij}\}\tilde{h}_j(x_j(t-\tau_3(t))) + co\{\hat{b}_{ij}, \check{b}_{ij}\}\tilde{h}_j(y_j(t-\tau_3(t))) \subseteq co\{\hat{b}_{ij}, \check{b}_{ij}\}(\tilde{h}_j(x_j(t-\tau_3(t))) + \tilde{h}_j(y_j(t-\tau_3(t)))).$$

Lemma 1 (see [23]) *For vectors a and b, the inequality* $\pm 2a^{\mathrm{T}}b \le a^{\mathrm{T}}Sa + b^{\mathrm{T}}S^{-1}b$ *holds, in which S is any matrix with* $S > 0$.

Lemma 2 (see [24]) *For any positive definite matrix W, a scalar $r > 0$ and a vector function $\eta : [0, r] \to R^n$ such that the integration concerned are well defined, then*

$$\left(\int_0^r \eta(s)\mathrm{d}s\right)^{\mathrm{T}} W \left(\int_0^r \eta(s)\mathrm{d}s\right) \le r \int_0^r \eta^{\mathrm{T}}(s) W \eta(s)\mathrm{d}s.$$

Definition 1 (*see* [25]) Suppose $E \subset \Re^n$. Then, $x \mapsto F(x)$ is called as a *set-valued* map defined on E, if for each point x of E, there exists a corresponding nonempty set $F(x) \subset \Re^n$. A set-valued map F with nonempty values is said to be *upper semicontinuous* at $x_0 \epsilon E$ if, for any open set N containing $F(x_0)$, there exits a neighborhood M of x_0 such that $F(M) \subset N$. $F(x)$ is said to have a *closed image* if for each $x \epsilon E$, $F(x)$ is closed.

In this paper, solutions of all the systems considered in the following are intended in the Filippov's sense, where $[\cdot, \cdot]$ represents the interval. Let $\bar{a}_{ij} = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\underline{a}_{ij} = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\bar{b}_{ij} = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\underline{b}_{ij} = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\bar{e}_{ij} = \max\{\hat{e}_{ij}, \check{e}_{ij}\}$, $\underline{e}_{ij} = \min\{\hat{e}_{ij}, \check{e}_{ij}\}$, $\bar{A} = (\bar{a}_{ij})_{n\times n}$, $\bar{B} = (\bar{b}_{ij})_{n\times n}$, $\bar{E} = (\bar{e}_{ij})_{n\times n}$, $\underline{A} = (\underline{a}_{ij})_{n\times n}$, $\underline{B} = (\underline{b}_{ij})_{n\times n}$, $\underline{E} = (\underline{e}_{ij})_{n\times n}$, for $i = 1, 2, \ldots, n$. $co\{u, v\}$ denotes the closure of a convex hull generated by real numbers u and v or real matrices u and v.

Based on Definition 1, by applying the theory of differential inclusion, the memristor-based neural networks of neutral type with mixed delays can be written as the following differential inclusion:

$$\begin{aligned} &d[x(t) - Dx(t-\tau_1(t))] \\ &\epsilon\Bigg[-Cx(t) + \sum_{j=1}^n co\{a_{ij}(x_i(t))\}\tilde{f}(x(t)) \\ &\quad + \sum_{j=1}^n co\{b_{ij}(x_i(t))\}\tilde{g}(x(t-\tau_2(t))) \\ &\quad + \sum_{j=1}^n co\{e_{ij}(x_i(t))\} \\ &\quad \times \int_{t-\tau_3(t)}^t \tilde{h}(x(s))\mathrm{d}s + \beta M\Gamma x(t)\Bigg]\mathrm{d}t. \end{aligned} \tag{3}$$

The differential inclusion (3) means that there exist $a_{ij}(t)\epsilon a_{ij}(x_i(t))$, $b_{ij}(t)\ \epsilon b_{ij}(x_i(t))$, $e_{ij}(t)\epsilon e_{ij}(x_i(t))$, and $\tilde{A}(t) = (a_{ij}(t))_{n\times n}$, $\tilde{B}(t) = (b_{ij}(t))_{n\times n}$, $\tilde{E}(t) = (e_{ij}(t))_{n\times n}$, such that

$$\begin{aligned} d[x(t) - Dx(t-\tau_1(t))] &= \Big[-Cx(t) + \tilde{A}(t)\tilde{f}(x(t)) \\ &\quad + \tilde{B}(t)\tilde{g}(x(t-\tau_2(t))) \\ &\quad + \tilde{E}(t)\int_{t-\tau_3(t)}^t \tilde{h}(x(s))\mathrm{d}s + \beta M\Gamma x(t)\Big]\mathrm{d}t. \end{aligned} \tag{4}$$

Let system (4) be the drive system, and the response system is as follows

$$\begin{aligned} d[y(t) - Dy(t-\tau_1(t))] &= \Big[-Cy(t) + \tilde{A}(t)\tilde{f}(y(t)) \\ &\quad + \tilde{B}(t)\tilde{g}(y(t-\tau_2(t))) \\ &\quad + \tilde{E}(t)\int_{t-\tau_3(t)}^t \tilde{h}(y(s))\mathrm{d}s + \beta M\Gamma y(t)\Big]\mathrm{d}t. \end{aligned} \tag{5}$$

According to Assumptions 3, let $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^{\mathrm{T}}$ be the anti-synchronization error, where $e_i(t) = x_i(t) + y_i(t)$, $e_i(t-\tau_2(t)) = x_i(t-\tau_2(t)) + y_i(t-\tau_2(t))$, $e_i(t-\tau_3(t)) = x_i(t-\tau_3(t)) + y_i(t-\tau_3(t))$. Using the theories of set-valued maps and differential inclusions, we can get the following anti-synchronization error system

$$\begin{aligned} &d[e(t) - De(t-\tau_1(t))]\epsilon \\ &\Bigg[-Ce(t) + \sum_{j=1}^n co\{a_{ij}(e_i(t))\}f(x(t)) \\ &\quad + \sum_{j=1}^n co\{b_{ij}(e_i(t))\}g(e(t-\tau_2(t))) \\ &\quad + \sum_{j=1}^n co\{e_{ij}(e_i(t))\}\int_{t-\tau_3(t)}^t h(e(s))\mathrm{d}s \\ &\quad + \beta M\Gamma e(t)\Bigg]\mathrm{d}t. \end{aligned} \tag{6}$$

Or equivalently, there exist $a_{ij}(t)\epsilon a_{ij}(e_i(t))$, $b_{ij}(t)\ \epsilon b_{ij}(e_i(t))$, $e_{ij}(t)\epsilon e_{ij}(e_i(t))$, and $\tilde{A}(t) = (a_{ij}(t))_{n\times n}$, $\tilde{B}(t) = (b_{ij}(t))_{n\times n}$, $\tilde{E}(t) = (e_{ij}(t))_{n\times n}$, such that

$$\begin{aligned} d[e(t) - De(t-\tau_1(t))] &= [-Ce(t) + \tilde{A}(t)f(e(t)) \\ &\quad + \tilde{B}(t)g(e(t-\tau_2(t))) \\ &\quad + \tilde{E}(t)\int_{t-\tau_3(t)}^t h(e(s))\mathrm{d}s + \beta M\Gamma e(t)]\mathrm{d}t, \end{aligned} \tag{7}$$

where $f_j(e_j(t)) = \tilde{f}_j(x_j(t)) + \tilde{f}_j(y_j(t))$, $g_j(e_j(t - \tau_2(t))) = \tilde{g}_j(x_j(t - \tau_2(t))) + \tilde{g}_j(y_j(t - \tau_2(t)))$, $h_j(e_j(t - \tau_3(t))) = \tilde{h}_j(x_j(t - \tau_3(t))) + \tilde{h}_j(y_j(t - \tau_3(t)))$.

3 Main results

In this section, we investigate the anti-synchronization control for the coupled memristor-based neural networks with mixed time-varying delays under the randomly occurring control.

Theorem 1 *Under Assumptions 1–3, the error system (7) of the coupled memristor-based neural networks of neutral type with mixed time-varying delays will be convergent, if there exist a positive diagonal matrix $P = diag(P_1, P_2, \ldots, P_n)$, positive matrices Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, S_1, S_2, and a positive scalar λ such that the following LMIs (Linear Matrix Inequality) hold:*

$$P \leq \lambda I, \tag{8}$$

$$\tau_3(S_1 + S_2) < Q_6,$$

$$\psi = \begin{pmatrix} \psi_{11} & \psi_{12} & 0 & 0 & \psi_{15} & 0 & 0 & \psi_{18} & 0 \\ * & \psi_{22} & 0 & 0 & \psi_{25} & 0 & 0 & \psi_{28} & 0 \\ * & * & \psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \psi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \psi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \psi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \psi_{88} & 0 \\ * & * & * & * & * & * & * & * & \psi_{99} \end{pmatrix}, \tag{9}$$

where

$$\psi_{11} = -2PC + 2\beta M\Gamma P - 2\rho P\underline{K} + P\bar{E}S_1^{-1}\bar{E}^{\mathrm{T}}P^{\mathrm{T}} + Q_1 + Q_2 + Q_3,$$
$$\psi_{12} = D^{\mathrm{T}}PC - \beta D^{\mathrm{T}}PM\Gamma + \rho D^{\mathrm{T}}P\underline{K},\ \psi_{15} = P\bar{A},\ \psi_{18} = P\bar{B},$$
$$\psi_{22} = PD^{\mathrm{T}}\bar{E}S_2^{-1}\bar{E}^{\mathrm{T}}DP^{\mathrm{T}} + (\mu_1 - 1)Q_1,$$
$$\psi_{25} = -D^{\mathrm{T}}\bar{A}P,\ \psi_{28} = -D^{\mathrm{T}}\bar{B}P,$$
$$\psi_{33} = (\mu_2 - 1)Q_2,\ \psi_{44} = (\mu_3 - 1)Q_3,$$
$$\psi_{55} = \tau_1 Q_7 + \tau_2 Q_8,\ \psi_{66} = Q_4,$$
$$\psi_{77} = Q_5 + \tau_3 Q_6,\ \psi_{88} = (\mu_2 - 1)Q_4,$$
$$\psi_{99} = (\mu_3 - 1)Q_5.$$

And the randomly occurring memristor-based controller is designed as

$$u(e(t)) = -\rho(t)K(e(t))e(t), \tag{10}$$
$$\underline{K}e(t) \leq K(e(t))e(t) \leq \bar{K}e(t), \tag{11}$$

where $\tilde{K}(t)\epsilon co\{K(e(t))\}$ and $\rho(t)$ is a stochastic variable that describes the following random events for system (7),

$$\begin{cases} \text{Event } 1: & (7) \text{ experiences } (10) \\ \text{Event } 2: & (7) \text{ does not experience } (10) \end{cases} \tag{12}$$

Let $\rho(t)$ be defined by

$$\rho(t) = \begin{cases} 1, & \text{if Event 1 occurs,} \\ 0, & \text{if Event 2 occurs,} \end{cases} \tag{13}$$

where $E[\rho(t)] = \rho\epsilon[0, 1]$.

Proof Construct the following Lyapunov–Krasovskii function

$$V(t, e(t)) = \sum_{i=1}^{9} V_i(t, e(t)). \tag{14}$$

Along the trajectory of system (7), we define an operator LV by

$$\begin{aligned} LV(t, e(t)) &= V_t(t, e(t)) + V_e(t, e(t))[-Ce(t) \\ &\quad + \tilde{A}(t)f(e(t)) + \tilde{B}(t)g(e(t - \tau_2(t))) \\ &\quad + \tilde{E}(t)\int_{t-\tau_3(t)}^{t} h(e(s))\mathrm{d}s \\ &\quad + \beta M\Gamma e(t) + u(t)], \end{aligned} \tag{15}$$

where $V_t(t, e(t)) = \frac{\partial V(t,e(t))}{\partial t}$, $V_e(t, e(t)) = (\frac{\partial V(t,e(t))}{\partial e_1}, \frac{\partial V(t,e(t))}{\partial e_2}, \ldots, \frac{\partial V(t,e(t))}{\partial e_n})$, $V_{ee}(t, e(t)) = (\frac{\partial^2 V(t,e(t))}{\partial e_j \partial e_j})_{n\times n}$, and the controller $u(t)$ is designed to achieve the synchronization of coupled memristor-based recurrent neural network. And

$$\begin{aligned} V_1(t, e(t)) = {} & [e(t) - De(t - \tau_1(t))]^{\mathrm{T}} P \\ & [e(t) - De(t - \tau_1(t))]. \end{aligned} \tag{16}$$

From (7) and (16), we get

$$\begin{aligned} LV_1(t, e(t)) &= 2[e(t) - De(t - \tau_1(t))]^{\mathrm{T}} P[-Ce(t) \\ &\quad + \tilde{A}(t)f(e(t)) + \tilde{B}(t)g(e(t - \tau_2(t))) \\ &\quad + \tilde{E}(t)\int_{t-\tau_3(t)}^{t} h(e(s))\mathrm{d}s + \beta M\Gamma e(t) \\ &\quad - \rho(t)\tilde{K}(t)e(t)] \\ &= e^{\mathrm{T}}(t)[-2PC]e(t) + e^{\mathrm{T}}(t)[2P\tilde{A}(t)]f(e(t)) \end{aligned}$$

$$
\begin{aligned}
&+e^{\mathrm{T}}(t)[2P\tilde{B}(t)]g(e(t-\tau_2(t)))\\
&+e^{\mathrm{T}}(t)[2P\tilde{E}(t)]\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\\
&+e^{\mathrm{T}}(t)[2\beta M\Gamma P]e(t)\\
&+e^{\mathrm{T}}(t)[2P(-\rho(t))\tilde{K}(t)]e(t)\\
&+e^{\mathrm{T}}(t-\tau_1(t))[2D^{\mathrm{T}}PC]e(t)\\
&+e^{\mathrm{T}}(t-\tau_1(t))[-2D^{\mathrm{T}}\tilde{A}(t)P]f(e(t))\\
&+e^{\mathrm{T}}(t-\tau_1(t))[-2D^{\mathrm{T}}P\tilde{B}(t)]g(e(t-\tau_2(t)))\\
&+e^{\mathrm{T}}(t-\tau_1(t))[-2D^{\mathrm{T}}P\tilde{E}(t)]\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\\
&+e^{\mathrm{T}}(t-\tau_1(t))[-2\beta D^{\mathrm{T}}PM\Gamma]e(t)\\
&+e^{\mathrm{T}}(t-\tau_1(t))[2D^{\mathrm{T}}P\rho(t)\tilde{K}(t)]e(t).
\end{aligned}
\tag{17}
$$

From Lemma 1, we get

$$
\begin{aligned}
&e^{\mathrm{T}}(t)[2P\tilde{E}(t)]\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\\
&\quad\le e^{\mathrm{T}}(t)[P\bar{E}S_1^{-1}\bar{E}^{\mathrm{T}}P^{\mathrm{T}}]e(t)\\
&\qquad+\left[\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\right]^{\mathrm{T}}S_1\left[\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\right],
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
&e^{\mathrm{T}}(t-\tau_1(t))[-2D^{\mathrm{T}}P\tilde{E}(t)]\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\\
&\quad\le e^{\mathrm{T}}(t-\tau_1(t))[PD^{\mathrm{T}}\bar{E}S_2^{-1}\bar{E}^{\mathrm{T}}DP^{\mathrm{T}}]e(t-\tau_1(t))\\
&\qquad+\left[\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\right]^{\mathrm{T}}S_2\left[\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\right].
\end{aligned}
\tag{19}
$$

Utilizing Lemma 2 yields

$$
\begin{aligned}
&\left[\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\right]^{\mathrm{T}}(S_1+S_2)\left[\int_{t-\tau_3(t)}^{t}h(e(s))\mathrm{d}s\right]\\
&\quad\le\tau_3(t)\int_{t-\tau_3(t)}^{t}h^{\mathrm{T}}(e(s))(S_1+S_2)h(e(s))\mathrm{d}s\\
&\quad\le\int_{t-\tau_3(t)}^{t}h^{\mathrm{T}}(e(s))[\tau_3(S_1+S_2)]h(e(s))\mathrm{d}s.
\end{aligned}
\tag{20}
$$

According to Assumption 1, we have

$$
\begin{aligned}
V_2(t,e(t))&=\int_{t-\tau_1(t)}^{t}e^{\mathrm{T}}(s)Q_1e(s)\mathrm{d}s.\\
LV_2(t,e(t))&=e^{\mathrm{T}}(t)Q_1e(t)\\
&\quad-(1-\dot{\tau}_1(t))e^{\mathrm{T}}(t-\tau_1(t))Q_1e(t-\tau_1(t))\\
&\le e^{\mathrm{T}}(t)Q_1e(t)-e^{\mathrm{T}}(t-\tau_1(t))\\
&\quad\times[(1-\mu_1)Q_1]e(t-\tau_1(t)).
\end{aligned}
\tag{21}
$$

And we set

$$
\begin{aligned}
V_3(t,e(t))&=\int_{t-\tau_2(t)}^{t}e^{\mathrm{T}}(s)Q_2e(s)\mathrm{d}s.\\
LV_3(t,e(t))&=e^{\mathrm{T}}(t)Q_2e(t)\\
&\quad-(1-\dot{\tau}_2(t))e^{\mathrm{T}}(t-\tau_2(t))Q_2e(t-\tau_2(t))\\
&\le e^{\mathrm{T}}(t)Q_2e(t)-e^{\mathrm{T}}(t-\tau_2(t))\\
&\quad\times[(1-\mu_2)Q_2]e(t-\tau_2(t)).
\end{aligned}
\tag{22}
$$

By Ito's differential formula studied in [26], we could infer that

$$
\begin{aligned}
V_4(t,e(t))&=\int_{t-\tau_3(t)}^{t}e^{\mathrm{T}}(s)Q_3e(s)\mathrm{d}s.\\
LV_4(t,e(t))&=e^{\mathrm{T}}(t)Q_3e(t)-(1-\dot{\tau}_3(t))\\
&\quad\times e^{\mathrm{T}}(t-\tau_3(t))Q_3e(t-\tau_3(t))\\
&\le e^{\mathrm{T}}(t)Q_3e(t)-e^{\mathrm{T}}(t-\tau_3(t))\\
&\quad\times[(1-\mu_3)Q_3]e(t-\tau_3(t)).
\end{aligned}
\tag{23}
$$

And

$$
\begin{aligned}
V_5(t,e(t))&=\int_{t-\tau_2(t)}^{t}g^{\mathrm{T}}(e(s))Q_4g(e(s))\mathrm{d}s.\\
LV_5(t,e(t))&=g^{\mathrm{T}}(e(t))Q_4g(e(t))-(1-\dot{\tau}_2(t))\\
&\quad\times g^{\mathrm{T}}(e(t-\tau_2(t)))Q_4g(e(t-\tau_2(t)))\\
&\le g^{\mathrm{T}}(e(t))Q_4g(e(t))-g^{\mathrm{T}}(e(t-\tau_2(t)))\\
&\quad\times[(1-\mu_2)Q_4]g(e(t-\tau_2(t))).
\end{aligned}
\tag{24}
$$

According to Assumption 1, we get

$$
\begin{aligned}
V_6(t,e(t))&=\int_{t-\tau_3(t)}^{t}h^{\mathrm{T}}(e(s))Q_5h(e(s))\mathrm{d}s.\\
LV_6(t,e(t))&=h^{\mathrm{T}}(e(t))Q_5h(e(t))-(1-\dot{\tau}_3(t))h^{\mathrm{T}}\\
&\quad\times(e(t-\tau_3(t)))Q_5h(e(t-\tau_3(t)))\\
&\le h^{\mathrm{T}}(e(t))Q_5h(e(t))-h^{\mathrm{T}}(e(t-\tau_3(t)))\\
&\quad\times[(1-\mu_3)Q_5]h(e(t-\tau_3(t))).
\end{aligned}
\tag{25}
$$

We set

$$
\begin{aligned}
V_7(t,e(t))&=\int_{-\tau_3(t)}^{0}\int_{t+r}^{t}h^{\mathrm{T}}(e(s))Q_6h(e(s))\mathrm{d}s\mathrm{d}r.\\
LV_7(t,e(t))&=\tau_3(t)h^{\mathrm{T}}(e(t))Q_6h(e(t))\\
&\quad-\int_{t-\tau_3(t)}^{t}h^{\mathrm{T}}(e(s))Q_6h(e(s))\mathrm{d}s\\
&\le h^{\mathrm{T}}(e(t))[\tau_3Q_6]h(e(t))\\
&\quad-\int_{t-\tau_3(t)}^{t}h^{\mathrm{T}}(e(s))Q_6h(e(s))\mathrm{d}s.
\end{aligned}
\tag{26}
$$

From condition (9), we obtain

$$\int_{t-\tau_3(t)}^{t} h^{\mathrm{T}}(e(s))[\tau_3(S_1+S_2)]h(e(s))\mathrm{d}s - \int_{t-\tau_3(t)}^{t} h^{\mathrm{T}}(e(s))Q_6 h(e(s))\mathrm{d}s \le 0. \tag{27}$$

We set

$$V_8(t,e(t)) = \int_{-\tau_1(t)}^{0}\int_{t+r}^{t} f^{\mathrm{T}}(e(s))Q_7 f(e(s))\mathrm{d}s\mathrm{d}r.$$

$$\begin{aligned} LV_8(t,e(t)) &= \tau_1(t) f^{\mathrm{T}}(e(t))Q_7 f(e(t)) - \int_{t-\tau_1(t)}^{t} f^{\mathrm{T}}(e(s))Q_7 f(e(s))\mathrm{d}s \\ &\le f^{\mathrm{T}}(e(t))[\tau_1 Q_7] f(e(t)). \end{aligned} \tag{28}$$

And

$$V_9(t,e(t)) = \int_{-\tau_2(t)}^{0}\int_{t+r}^{t} f^{\mathrm{T}}(e(s))Q_8 f(e(s))\mathrm{d}s\mathrm{d}r.$$

$$\begin{aligned} LV_9(t,e(t)) &= \tau_2(t) f^{\mathrm{T}}(e(t))Q_8 f(e(t)) - \int_{t-\tau_2(t)}^{t} f^{\mathrm{T}}(e(s))Q_8 f(e(s))\mathrm{d}s \\ &\le f^{\mathrm{T}}(e(t))[\tau_2 Q_8] f(e(t)). \end{aligned} \tag{29}$$

Substituting inequalities (16)–(29) into (15), we obtain

$$\begin{aligned} E[LV(t,e(t))] &\le e^{\mathrm{T}}(t)[-2PC + 2\beta M\Gamma P - 2\rho P\underline{K} \\ &\quad + P\bar{E}S_1^{-1}\bar{E}^{\mathrm{T}}P^{\mathrm{T}} \\ &\quad + Q_1 + Q_2 + Q_3]e(t) + e^{\mathrm{T}}(t)[2P\bar{A}]f(e(t)) \\ &\quad + e^{\mathrm{T}}(t)[2P\bar{B}]g(e(t-\tau_2(t))) + e^{\mathrm{T}}(t-\tau_1(t)) \\ &\quad \times [2D^{\mathrm{T}}PC - 2\beta D^{\mathrm{T}}PM\Gamma + 2\rho D^{\mathrm{T}}P\underline{K}]e(t) \\ &\quad + e^{\mathrm{T}}(t-\tau_1(t))[-2D^{\mathrm{T}}\bar{A}P]f(e(t)) \\ &\quad + e^{\mathrm{T}}(t-\tau_1(t))[-2D^{\mathrm{T}}P\bar{B}]g(e(t-\tau_2(t))) \\ &\quad + e^{\mathrm{T}}(t-\tau_1(t))[PD^{\mathrm{T}}\bar{E}S_2^{-1}\bar{E}^{\mathrm{T}}DP^{\mathrm{T}} \\ &\quad + (\mu_1-1)Q_1]e(t-\tau_1(t)) + e^{\mathrm{T}}(t-\tau_2(t)) \\ &\quad + [(\mu_2-1)Q_2]e(t-\tau_2(t)) + e^{\mathrm{T}}(t-\tau_3(t)) \\ &\quad \times [(\mu_3-1)Q_3]e(t-\tau_3(t)) \\ &\quad + f^{\mathrm{T}}(e(t))[\tau_1 Q_7 + \tau_2 Q_8]f(e(t)) \\ &\quad + g^{\mathrm{T}}(e(t))[Q_4]g(e(t)) \\ &\quad + h^{\mathrm{T}}(e(t))[Q_5 + \tau_3 Q_6]h(e(t)) \\ &\quad + g^{\mathrm{T}}(e(t-\tau_2(t)))[(\mu_2-1)Q_4]g(e(t-\tau_2(t))) \\ &\quad + h^{\mathrm{T}}(e(t-\tau_3(t)))[(\mu_3-1)Q_5]h(e(t-\tau_3(t))) \\ &= \phi^{\mathrm{T}}\psi\phi, \end{aligned} \tag{30}$$

where

$$\phi^{\mathrm{T}} = \Big[e^{\mathrm{T}}(t), e^{\mathrm{T}}(t-\tau_1(t)), e^{\mathrm{T}} \times (t-\tau_2(t)), e^{\mathrm{T}} \times (t-\tau_3(t)), f^{\mathrm{T}}(e(t)), g^{\mathrm{T}}(e(t)), h^{\mathrm{T}}(e(t)), g^{\mathrm{T}} \times (e(t-\tau_2(t))), h^{\mathrm{T}}(e(t-\tau_3(t)))\Big]^{\mathrm{T}},$$

$$\psi = \begin{pmatrix} \psi_{11} & \psi_{12} & 0 & 0 & \psi_{15} & 0 & 0 & \psi_{18} & 0 \\ * & \psi_{22} & 0 & 0 & \psi_{25} & 0 & 0 & \psi_{28} & 0 \\ * & * & \psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \psi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \psi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \psi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \psi_{88} & 0 \\ * & * & * & * & * & * & * & * & \psi_{99} \end{pmatrix},$$

where

$$\begin{aligned} \psi_{11} &= -2PC + 2\beta M\Gamma P - 2\rho \\ &\qquad P\underline{K} + P\bar{E}S_1^{-1}\bar{E}^{\mathrm{T}}P^{\mathrm{T}} + Q_1 + Q_2 + Q_3, \\ \psi_{12} &= D^{\mathrm{T}}PC - \beta D^{\mathrm{T}}PM\Gamma + \rho D^{\mathrm{T}}P\underline{K}, \\ \psi_{15} &= P\bar{A},\ \psi_{18} = P\bar{B}, \\ \psi_{22} &= PD^{\mathrm{T}}\bar{E}S_2^{-1}\bar{E}^{\mathrm{T}}DP^{\mathrm{T}} + (\mu_1-1)Q_1, \\ \psi_{25} &= -D^{\mathrm{T}}\bar{A}P,\ \psi_{28} = -D^{\mathrm{T}}\bar{B}P, \\ \psi_{33} &= (\mu_2-1)Q_2,\ \psi_{44} = (\mu_3-1)Q_3, \\ \psi_{55} &= \tau_1 Q_7 + \tau_2 Q_8,\ \psi_{66} = Q_4, \\ \psi_{77} &= Q_5 + \tau_3 Q_6,\ \psi_{88} = (\mu_2-1)Q_4, \\ \psi_{99} &= (\mu_3-1)Q_5. \end{aligned}$$

If $\psi < 0$, then $LV(t,e(t)) < 0$, so the drive system and the response system get anti-synchronization. This completes the proof. □

Remark 1 In the above proof, we construct a novel Lyapunov function by employing a delay-fractionizing approach in order to reduce the possible conservatism induced by the Lyapunov function when dealing with time delays.

When $D = 0$, from Theorem 1, we obtain the following corollary.

Corollary 1 *Under Assumption 1, the error system of the coupled memristor-based neural networks with mixed time-varying delays can be described by*

$$d[e(t)] = \Big[-Ce(t) + \tilde{A}(t)f(e(t)) + \tilde{B}(t)g(e(t-\tau_2(t))) + \tilde{E}(t)\int_{t-\tau_3(t)}^{t} h(e(s))\mathrm{d}s + \beta M\Gamma e(t)\Big]\mathrm{d}t \tag{31}$$

and (31) will be convergent, if there exist a positive diagonal matrix $P = diag(P_1, P_2, \ldots, P_n)$, positive *matrices Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, S_1, S_2, and a positive scalar λ such that the following LMIs hold:*

$$P \le \lambda I, \tag{32}$$

$$\tau_3(S_1 + S_2) < Q_6, \tag{33}$$

$$\psi = \begin{pmatrix} \psi_{11} & 0 & 0 & 0 & P\bar{A} & 0 & 0 & P\bar{B} & 0 \\ * & \psi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \psi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \psi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \psi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \psi_{88} & 0 \\ * & * & * & * & * & * & * & * & \psi_{99} \end{pmatrix},$$

where

$$\psi_{11} = -2PC + 2\beta M\Gamma P - 2\rho P\underline{K} + P\bar{E}S_1^{-1}\bar{E}^{\mathrm{T}}P^{\mathrm{T}} + Q_1 + Q_2 + Q_3,$$
$$\psi_{22} = (\mu_1 - 1)Q_1, \psi_{33} = (\mu_2 - 1)Q_2,$$
$$\psi_{44} = (\mu_3 - 1)Q_3,$$
$$\psi_{55} = \tau_1 Q_7 + \tau_2 Q_8, \psi_{66} = Q_4,$$
$$\psi_{77} = Q_5 + \tau_3 Q_6, \psi_{88} = (\mu_2 - 1)Q_4,$$
$$\psi_{99} = (\mu_3 - 1)Q_5.$$

The randomly occurring memristor-based controller is designed as

$$u(e(t)) = -\rho(t)K(e(t))e(t), \tag{34}$$

$$\underline{K}e(t) \le K(e(t)) \le \bar{K}e(t), \tag{35}$$

where $K(t)\epsilon co\{K(e(t))\}$ and $\rho(t)$ is a stochastic variable that describes the following random events for system (31),

$$\begin{cases} \text{Event 1:} & \text{(31) experiences (34)} \\ \text{Event 2:} & \text{(31) does not experience (34)} \end{cases} \tag{36}$$

Let $\rho(t)$ be defined by

$$\rho(t) = \begin{cases} 1, & \text{if Event 1 occurs,} \\ 0, & \text{if Event 2 occurs,} \end{cases} \tag{37}$$

where $E[\rho(t)] = \rho\epsilon[0, 1]$.

Remark 2 When $D = 0$, the systems are no longer neutral-type neural networks. When $\tilde{E}(t) = 0$, the systems no longer have distributed time-varying delays. We can also get the synchronization results from Theorem 1 when $D = 0$ or $\tilde{E}(t) = 0$.

Next, we will give another controller which depends on the time-varying delays.

Theorem 2 *Under Assumptions 1, the error system (7) of the coupled memristor-based neural networks of neutral type with mixed time-varying delays will be convergent, if there exist a positive diagonal matrix $P = diag(P_1, P_2, \ldots, P_n)$, positive matrices Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, S_1, S_2, and a scalar $\lambda > 0$ such that the following LMIs hold:*

$$P \le \lambda I, \tag{38}$$

$$\tau_3(S_1 + S_2) < Q_6, \tag{39}$$

$$\psi = \begin{pmatrix} \psi_{11} & \psi_{12} & 0 & 0 & \psi_{15} & 0 & 0 & \psi_{18} & 0 \\ * & \psi_{22} & 0 & 0 & \psi_{25} & 0 & 0 & \psi_{28} & 0 \\ * & * & \psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \psi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \psi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \psi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \psi_{88} & 0 \\ * & * & * & * & * & * & * & * & \psi_{99} \end{pmatrix} \le 0,$$

where

$$\psi_{11} = -2PC + 2\beta M\Gamma P - 2\rho_1 P\underline{K}_1 + P\bar{E}S_1^{-1}\bar{E}^{\mathrm{T}}P^{\mathrm{T}} + Q_1 + Q_2 + Q_3,$$
$$\psi_{12} = D^{\mathrm{T}}PC - \beta D^{\mathrm{T}}PM\Gamma + \rho_1 D^{\mathrm{T}}P\underline{K}_1 - \rho_2 P\underline{K}_2,$$
$$\psi_{15} = P\bar{A}, \psi_{18} = P\bar{B},$$
$$\psi_{22} = PD^{\mathrm{T}}\bar{E}S_2^{-1}\bar{E}^{\mathrm{T}}DP^{\mathrm{T}} + (\mu_1 - 1)Q_1 + 2\rho_2 D^{\mathrm{T}}\underline{K}_2 P,$$
$$\psi_{25} = -D^{\mathrm{T}}\bar{A}P, \psi_{28} = -D^{\mathrm{T}}\bar{B}P,$$
$$\psi_{33} = (\mu_2 - 1)Q_2, \psi_{44} = (\mu_3 - 1)Q_3,$$
$$\psi_{55} = \tau_1 Q_7 + \tau_2 Q_8, \psi_{66} = Q_4,$$
$$\psi_{77} = Q_5 + \tau_3 Q_6, \psi_{88} = (\mu_2 - 1)Q_4,$$
$$\psi_{99} = (\mu_3 - 1)Q_5.$$

The randomly occurring memristor-based controller is designed as

$$u(e(t)) = -\rho_1(t)K_1(e(t))e(t) - \rho_2(t)K_2(e(t))e(t - \tau_1(t)), \tag{40}$$

$$\underline{K}_1 e(t) \le K_1(e(t)) \le \bar{K}_1 e(t), \tag{41}$$

$$\underline{K}_2 e(t) \le K_2(e(t)) \le \bar{K}_2 e(t), \tag{42}$$

where $\tilde{K}_1(t)\epsilon co\{K_1(e(t))\}$, $\tilde{K}_2(t)\epsilon co\{K_2(e(t))\}$, $\rho_1(t)$ and $\rho_2(t)$ are stochastic variables which describe the following random events for system (7),

$$\begin{cases} \text{Event 1 :} & \text{(7) experiences (40)} \\ \text{Event 2 :} & \text{(7) does not experience (40)} \end{cases} \tag{43}$$

Let $\rho_1(t)$, $\rho_2(t)$ be defined by

$$\rho_1(t) = \begin{cases} 1, & \text{if Event 1 occurs,} \\ 0, & \text{if Event 2 occurs,} \end{cases} \tag{44}$$

$$\rho_2(t) = \begin{cases} 1, & \text{if Event 1 occurs, ,} \\ 0, & \text{if Event 2 occurs, ,} \end{cases} \tag{45}$$

where $E[\rho_1(t)] = \rho_1 \epsilon [0, 1]$, $E[\rho_2(t)] = \rho_2 \epsilon [0, 1]$.

Proof Consider the Lyapunov–Krasovskii function as the same as in Theorem 1. By Ito's differential formula, we could infer that

$$\begin{aligned}
LV_1(t, e(t)) &= 2[e(t) - De(t - \tau_1(t))]^{\mathrm{T}} P \\
&\quad [-Ce(t) + \tilde{A}(t) f(e(t)) + \tilde{B}(t) g(e(t - \tau_2(t))) \\
&\quad + \tilde{E}(t) \int_{t-\tau_3(t)}^{t} h(e(s))\mathrm{d}s + \beta M \Gamma e(t) \\
&\quad - \rho_1(t) \tilde{K}_1(t) e(t) - \rho_2(t) \tilde{K}_2(t)(t - \tau_1(t))] \\
&= e^{\mathrm{T}}(t)[-2PC]e(t) + e^{\mathrm{T}}(t)[2P\tilde{A}(t)] f(e(t)) \\
&\quad + e^{\mathrm{T}}(t)[2P\tilde{B}(t)] g(e(t - \tau_2(t))) \\
&\quad + e^{\mathrm{T}}(t)[2P\tilde{E}(t)] \int_{t-\tau_3(t)}^{t} h(e(s))\mathrm{d}s \\
&\quad + e^{\mathrm{T}}(t)[2\beta M \Gamma P] e(t) \\
&\quad + e^{\mathrm{T}}(t)[2P(-\rho_1(t))\tilde{K}_1(t)]e(t) \\
&\quad + e^{\mathrm{T}}(t)[-2P\rho_2(t)\tilde{K}_2(t)]e(t - \tau_1(t)) \\
&\quad + e^{\mathrm{T}}(t - \tau_1(t))[2D^{\mathrm{T}}PC]e(t) + e^{\mathrm{T}}(t - \tau_1(t)) \\
&\quad \times [-2D^{\mathrm{T}}\tilde{A}(t)P] f(e(t)) + e^{\mathrm{T}}(t - \tau_1(t))[-2D^{\mathrm{T}}P\tilde{B}(t)] \\
&\quad \times g(e(t - \tau_2(t))) + e^{\mathrm{T}}(t - \tau_1(t))[-2D^{\mathrm{T}}P\tilde{E}(t)] \\
&\quad - 2DP\tilde{E}(t) \int_{t-\tau_3(t)}^{t} h(e(s))\mathrm{d}s \\
&\quad + e^{\mathrm{T}}(t - \tau_1(t))[-2\beta D^{\mathrm{T}} PM\Gamma]e(t) \\
&\quad + e^{\mathrm{T}}(t - \tau_1(t))[2D^{\mathrm{T}} P\rho_1(t)\tilde{K}_1(t)]e(t) \\
&\quad + e^{\mathrm{T}}(t - \tau_1(t))[2D^{\mathrm{T}}\rho_2(t)\tilde{K}_2(t)P]e(t - \tau_1(t)).
\end{aligned} \tag{46}$$

And

$$\begin{aligned}
E[LV(t, e(t))] &\leq e^{\mathrm{T}}(t)[-2PC + 2\beta M\Gamma P - 2\rho_1 P\underline{K}_1 \\
&\quad + P\bar{E}S_1^{-1}\bar{E}^{\mathrm{T}}P^{\mathrm{T}} + Q_1 + Q_2 + Q_3]e(t) \\
&\quad + e^{\mathrm{T}}(t)[-2\rho_2 P\underline{K}_2]e(t - \tau_1(t)) \\
&\quad + e^{\mathrm{T}}(t)[2P\bar{A}] f(e(t)) + e^{\mathrm{T}}(t)[2P\bar{B}] g(e(t - \tau_2(t))) \\
&\quad + e^{\mathrm{T}}(t - \tau_1(t))[2D^{\mathrm{T}}PC - 2\beta D^{\mathrm{T}}PM\Gamma \\
&\quad + 2\rho_1 D^{\mathrm{T}} P\underline{K}_1]e(t) \\
&\quad + e^{\mathrm{T}}(t - \tau_1(t))[-2D^{\mathrm{T}}\bar{A}P] f(e(t)) \\
&\quad + e^{\mathrm{T}}(t - \tau_1(t))[-2D^{\mathrm{T}}P\bar{B}] g(e(t - \tau_2(t))) \\
&\quad + e^{\mathrm{T}}(t - \tau_1(t))[PD^{\mathrm{T}}\bar{E}S_2^{-1}\bar{E}^{\mathrm{T}}DP^{\mathrm{T}} \\
&\quad + 2\rho_2 D^{\mathrm{T}}\underline{K}_2 P + (\mu_1 - 1)Q_1]e(t - \tau_1(t)) \\
&\quad + e^{\mathrm{T}}(t - \tau_2(t))[(\mu_2 - 1)Q_2]e(t - \tau_2(t)) \\
&\quad + e^{\mathrm{T}}(t - \tau_3(t))[(\mu_3 - 1)Q_3]e(t - \tau_3(t)) \\
&\quad + f^{\mathrm{T}}(e(t))[\tau_1 Q_7 + \tau_2 Q_8] f(e(t)) \\
&\quad + g^{\mathrm{T}}(e(t))[Q_4] g(e(t)) + h^{\mathrm{T}}(e(t))[Q_5 + \tau_3 Q_6] h(e(t)) \\
&\quad + g^{\mathrm{T}}(e(t - \tau_2(t)))[(\mu_2 - 1)Q_4] g(e(t - \tau_2(t))) \\
&\quad + h^{\mathrm{T}}(e(t - \tau_3(t)))[(\mu_3 - 1)Q_5] h(e(t - \tau_3(t))) \\
&= \phi^{\mathrm{T}}\psi\phi,
\end{aligned} \tag{47}$$

where $\phi^{\mathrm{T}} = [e^{\mathrm{T}}(t), e^{\mathrm{T}}(t - \tau_1(t)), e^{\mathrm{T}}(t - \tau_2(t)), e^{\mathrm{T}}(t - \tau_3(t)), f^{\mathrm{T}}(e(t)), g^{\mathrm{T}}(e(t)), h^{\mathrm{T}}(e(t)), g^{\mathrm{T}}(e(t - \tau_2(t))), h^{\mathrm{T}}(e(t - \tau_3(t)))]^{\mathrm{T}}$. If $\psi \leq 0$ in Theorem 2, then $LV(t, e(t)) < 0$. So the coupled memristor-based neural networks with mixed time-varying delays will be convergent under the controller (40). This completes the proof. □

When the neural networks (7) are not coupled, then from Theorem 2, we obtain the following corollary.

$$\begin{aligned}
d[e(t) - De(t - \tau_1(t))] &= [-Ce(t) + \tilde{A}(t) f(e(t)) \\
&\quad + \tilde{B}(t) g(e(t - \tau_2(t))) + \tilde{E}(t) \int_{t-\tau_3(t)}^{t} h(e(s))\mathrm{d}s]\mathrm{d}t.
\end{aligned} \tag{48}$$

Remark 3 Theorem 1 makes the controller (10) a little conservative because it only depends on $e(t)$. In Theorem 2, we construct a different controller, which depends on both $e(t)$ and $e(t - \tau_1(t))$, such that it is less conservative than Theorem 1.

Remark 4 Compared with the randomly occurring controller in [27], the controllers (10) and (40) are based on memristor, which have memory characteristic.

Corollary 2 *Under Assumptions 1, the uncoupled memristor-based neural networks of neutral type with mixed time-varying delays (48) will be convergent, if there exist a positive diagonal matrix $P = diag(P_1, P_2, \ldots, P_n)$, positive matrices $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, S_1, S_2$, and a scalar $\lambda > 0$ such that the following LMIs hold:*

$$P \leq \lambda I, \tag{49}$$

$$\tau_3(S_1+S_2) < Q_6, \tag{50}$$

$$\psi = \begin{pmatrix} \psi_{11} & \psi_{12} & 0 & 0 & \psi_{15} & 0 & 0 & \psi_{18} & 0 \\ * & \psi_{22} & 0 & 0 & \psi_{25} & 0 & 0 & \psi_{28} & 0 \\ * & * & \psi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \psi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \psi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \psi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \psi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \psi_{88} & 0 \\ * & * & * & * & * & * & * & * & \psi_{99} \end{pmatrix} \le 0,$$

where

$$\begin{aligned}
\psi_{11} &= -2PC - 2\rho_1 P\underline{K}_1 \\
&\quad + P\bar{E}S_1^{-1}\bar{E}^{\mathrm{T}}P^{\mathrm{T}} + Q_1 + Q_2 + Q_3, \\
\psi_{12} &= D^{\mathrm{T}}PC + \rho_1 D^{\mathrm{T}}P\underline{K}_1 - \rho_2 P\underline{K}_2, \\
\psi_{15} &= P\bar{A}, \psi_{18} = P\bar{B}, \\
\psi_{22} &= PD^{\mathrm{T}}\bar{E}S_2^{-1}\bar{E}^{\mathrm{T}}DP^{\mathrm{T}} \\
&\quad + (\mu_1 - 1)Q_1 + 2\rho_2 D^{\mathrm{T}}\underline{K}_2 P, \\
\psi_{25} &= -D^{\mathrm{T}}\bar{A}P, \psi_{28} = -D^{\mathrm{T}}\bar{B}P, \\
\psi_{33} &= (\mu_2 - 1)Q_2, \psi_{44} = (\mu_3 - 1)Q_3, \\
\psi_{55} &= \tau_1 Q_7 + \tau_2 Q_8, \psi_{66} = Q_4, \\
\psi_{77} &= Q_5 + \tau_3 Q_6, \psi_{88} = (\mu_2 - 1)Q_4, \\
\psi_{99} &= (\mu_3 - 1)Q_5.
\end{aligned}$$

The randomly occurring memristor-based controller is designed as

$$\begin{aligned} u(e(t)) &= -\rho_1(t)K_1(e(t))e(t) \\ &\quad - \rho_2(t)K_2(e(t))e(t-\tau_1(t)), \end{aligned} \tag{51}$$

$$\underline{K}_1 e(t) \le K_1(e(t)) \le \bar{K}_1 e(t), \tag{52}$$

$$\underline{K}_2 e(t) \le K_2(e(t)) \le \bar{K}_2 e(t), \tag{53}$$

where $\tilde{K_1}(t)\epsilon co\{K_1(e(t))\}$, $\tilde{K_2}(t)\epsilon co\{K_2(e(t))\}$, $\rho_1(t)$ *and* $\rho_2(t)$ *are stochastic variables which describe the following random events for system* (48),

$$\begin{cases} \text{Event 1}: & \text{(48) experiences (51)} \\ \text{Event 2}: & \text{(48) does not experience (51)} \end{cases} \tag{54}$$

Let $\rho_1(t), \rho_2(t)$ *be defined by*

$$\rho_1(t) = \begin{cases} 1, & \text{if Event 1 occurs,} \\ 0, & \text{if Event 2 occurs,} \end{cases} \tag{55}$$

$$\rho_2(t) = \begin{cases} 1, & \text{if Event 1 occurs,} \\ 0, & \text{if Event 2 occurs,} \end{cases} \tag{56}$$

where $E[\rho_1(t)] = \rho_1\epsilon[0,1]$, $E[\rho_2(t)] = \rho_2\epsilon[0,1]$.

4 Illustrative Example

In this section, several examples are presented to illustrate the effectiveness of the results obtained above. Consider a coupled two-dimensional memristor-based recurrent neural network model as follows:

$$\begin{aligned} &d\left[e(t) - De(t-\tau_1(t))\right] \\ &\quad = \big[-Ce(t) + \tilde{A}(t)f(e(t)) + \tilde{B}(t)g(e(t-\tau_2(t))) \\ &\qquad + \tilde{E}(t)\int_{t-\tau_3(t)}^{t} h(e(s))\mathrm{d}s + \beta Me(t)\big]\mathrm{d}t, \end{aligned} \tag{57}$$

where $e(t) = (e_1(t), e_2(t))$ is the state of the error system (57) and the time-varying delays are $\tau_1(t) = \tau_2(t) = \tau_3(t) = \frac{1}{2}sin(t)$. Take $f(e_i(t)) = g(e_i(t)) = h(e_i(t)) = \frac{1}{2}(\mid e_i(t)+1 \mid - \mid e_i(t)-1 \mid)$, and obviously, $f(e_i(t))$ is odd and bounded.

Other parameters of the error system are given as follows: $D = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix}, C = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.7 \end{pmatrix},$

$a_{11}(e_{i1}(t)) = 0.3sin(e_{i1}(t)), a_{12}(e_{i1}(t)) = -0.2sin(e_{i1}(t)),$
$a_{21}(e_{i2}(t)) = -0.2sin(e_{i2}(t)), a_{22}(e_{i2}(t)) = sin(e_{i2}(t)),$
$a_{31}(e_{i3}(t)) = 0.6sin(e_{i3}(t)), a_{32}(e_{i3}(t)) = 0.3sin(e_{i3}(t)),$
$b_{11}(e_{i1}(t)) = 0.4sin(e_{i1}(t)), b_{12}(e_{i1}(t)) = 0.3sin(e_{i1}(t)),$
$b_{21}(e_{i2}(t)) = 0.5sin(e_{i2}(t)), b_{22}(e_{i2}(t)) = 0.2sin(e_{i2}(t)),$
$b_{31}(e_{i3}(t)) = 0.4sin(e_{i3}(t)), b_{32}(e_{i3}(t)) = 0.2sin(e_{i3}(t)),$
$e_{11}(e_{i1}(t)) = 0.3sin(e_{i1}(t)), e_{12}(e_{i1}(t)) = 0.5sin(e_{i1}(t)),$
$e_{21}(e_{i2}(t)) = 0.5sin(e_{i2}(t)), e_{22}(e_{i2}(t)) = 0.3sin(e_{i2}(t)),$
$e_{31}(e_{i3}(t)) = 0.3sin(e_{i3}(t)), e_{32}(e_{i3}(t)) = 0.3sin(e_{i3}(t)),$
$K(e_{i1}(t)) = 0.5cos(e_{i1}(t)), K(e_{i2}(t)) = 0.6cos(e_{i2}(t)).$

We choose $\mu_1 = \mu_2 = \mu_3 = 0.1$, $\tau_1 = 0.1$, $\tau_2 = 0.2$, $\tau_3 = 0.3$, and

$$\bar{A} = \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 1 \end{pmatrix}, \bar{B} = \begin{pmatrix} 0.4 & 0.3 \\ 0.5 & 0.2 \end{pmatrix},$$

$$\bar{E} = \begin{pmatrix} 0.3 & 0.5 \\ 0.5 & 0.3 \end{pmatrix}, \underline{K} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$M = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}.$$

Using the LMI toobox in MATLAB, we obtain the following feasible solutions to LMIs in Theorem 1

$$P = \begin{pmatrix} 0.2628 & -0.0671 \\ -0.0671 & 0.2497 \end{pmatrix},$$

$$Q1 = \begin{pmatrix} 2.032 & 0.0068 \\ 0.0068 & 2.041 \end{pmatrix},$$

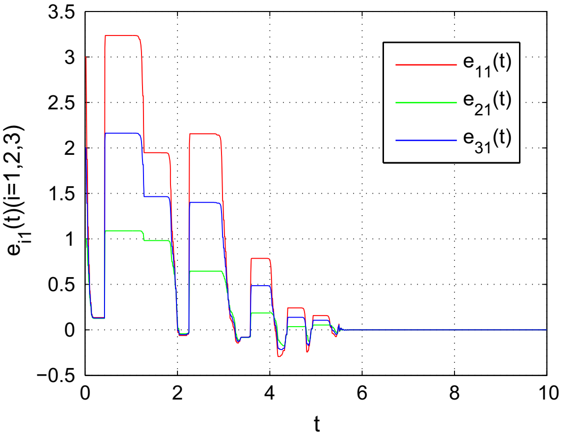

Fig. 1 (Color online) The curves of state error $e_{i1}(i = 1, 2, 3)$ for the memristive neural networks with neutral type (57) under the delay-independent randomly occurring controller (10), and the expectation of control probability is $E[\rho(t)] = 0.9$

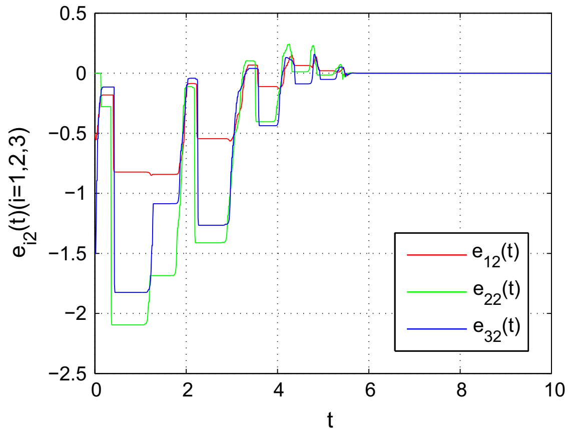

Fig. 2 (Color online) The curves of state error $e_{i2}(i = 1, 2, 3)$ for the memristive neural networks with neutral type (57) under the delay-independent randomly occurring controller (10), and the expectation of control probability is $E[\rho(t)] = 0.9$

$$Q2 = \begin{pmatrix} 0.4003 & 0.0002 \\ 0.0002 & 0.4004 \end{pmatrix},$$
$$Q3 = \begin{pmatrix} 0.1719 & 0.0002 \\ 0.0002 & 0.1719 \end{pmatrix},$$
$$Q4 = \begin{pmatrix} 0.5959 & 0 \\ 0 & 0.5959 \end{pmatrix},$$
$$Q5 = \begin{pmatrix} 79.5167 & 0 \\ 0 & 79.5167 \end{pmatrix},$$
$$Q6 = \begin{pmatrix} 503.6058 & 0 \\ 0 & 503.6058 \end{pmatrix},$$
$$Q7 = \begin{pmatrix} 147.9274 & 0 \\ 0 & 148.63 \end{pmatrix},$$
$$Q8 = \begin{pmatrix} 283.8615 & 0 \\ 0 & 283.5102 \end{pmatrix},$$
$$\lambda = \begin{pmatrix} 0.6853 & 0 \\ 0 & 0.6853 \end{pmatrix},$$
$$S_1 = \begin{pmatrix} 959.3976 & 0 \\ 0 & 959.0262 \end{pmatrix},$$
$$S_2 = \begin{pmatrix} 957.8386 & 0 \\ 0 & 958.21 \end{pmatrix}.$$

Furthermore, we consider the synchronization error system (57) under the controller (10), and $E[\rho(t)] = 0.9$. Figures 1 and 2 show the state error of system (57) with the controller (10) is synchronized. Thus, we verified Theorem 1.

In order to show the significant contribution to this paper, we compare with the existed results in [28], which is the memristive neural networks with mixed delays and without the neutral-type part. So we let $D = 0$ in the error system (57) without neutral type under the controller (10). Then, we get the error curves in Figs. 3 and 4. We summarize the comparisons between earlier works and the obtained results. It is concluded from this numerical example that the established results in this paper are new and the model is less conservative when compared to the existing results [28].

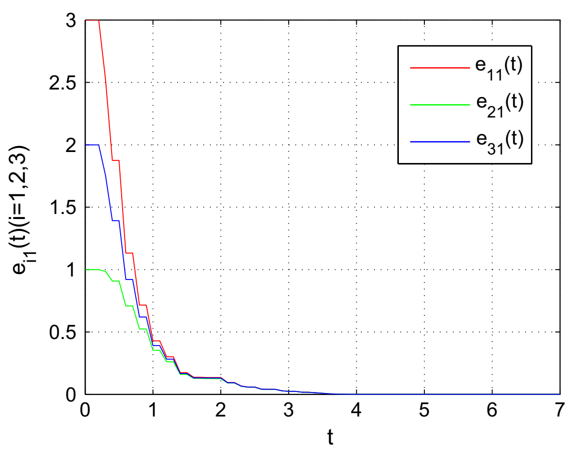

Fig. 3 (Color online) The curves of state error $e_{i1}(i = 1, 2, 3)$ of memristive neural networks with mixed delays and without neutral type under the memristive delay-independent randomly occurring controller (10), and the expectation of control probability is $E[\rho(t)] = 0.9$

In order to compare two different memristive controllers, we take the following example when the sys-

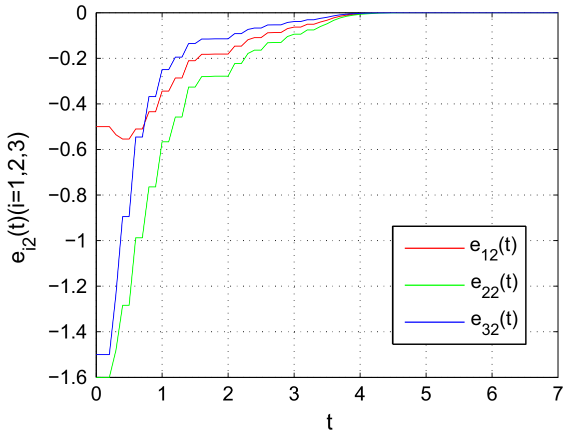

Fig. 4 (Color online) The curves of state error $e_{i2}(i = 1, 2, 3)$ of memristive neural networks with mixed delays and without neutral type under the memristive delay-independent randomly occurring controller (10), and the expectation of control probability is $E[\rho(t)] = 0.9$

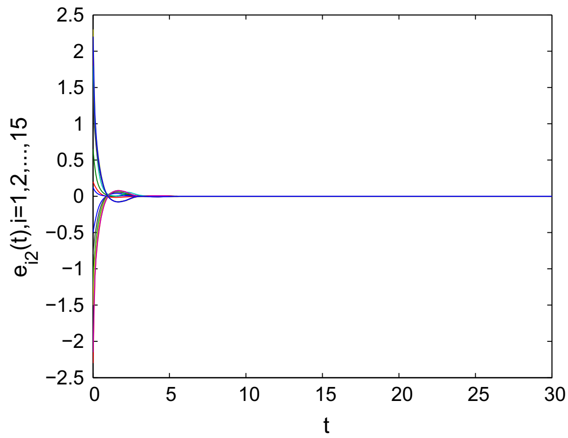

Fig. 6 (Color online) The curves of state error $e_{i2}(i = 1, 2, \ldots, 15)$ of memristive neural networks of neutral type with mixed delays under the memristive delay-dependent randomly occurring controller (40), $N = 15$, and the expectation of control probability is $E[\rho_1(t)] = 0.8$, $E[\rho_2(t)] = 0.7$

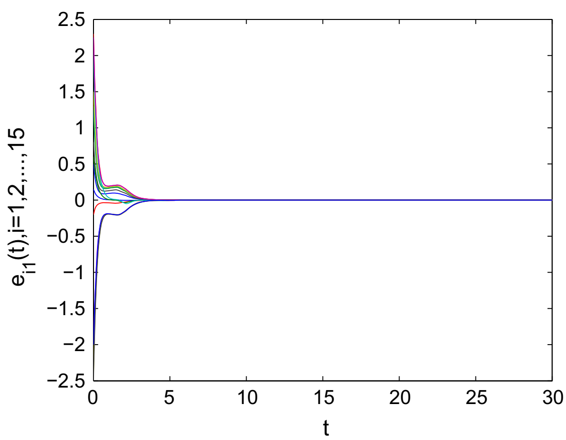

Fig. 5 (Color online) The curves of state error $e_{i1}(i = 1, 2, \ldots, 15)$ of memristive neural networks of neutral type with mixed delays under the memristive delay-dependent randomly occurring controller (40), $N = 15$, and the expectation of control probability is $E[\rho_1(t)] = 0.8$, $E[\rho_2(t)] = 0.7$

tem (57) works under the controller (40). The number of nodes in the networks is $N = 15$, and we take the same parameters as the above example. The adjacency matrix M is produced by a small-world network with rewiring probability 0.6 and the coupling strength $\beta = 1$. And $E[\rho_1(t)] = 0.8$, $E[\rho_2(t)] = 0.7$, $K_1(e_{i1}(t)) = 0.5cos(e_{i1}(t))$, $K_1(e_{i2}(t)) = 0.6cos(e_{i2}(t))$, $K_2(e_{i1}(t)) = 0.5sin(e_{i1}(t))$, $K_2(e_{i2}(t)) = 0.6sin(e_{i2}(t))$. Then, we get Figs. 5 and 6 which show the state error of the system. Thus, we verified Theorem 2.

Remark 5 Compared with the time-varying time delays in [29,30], the mixed probabilistic time-varying delays in [31] is less conservative. And in [31] authors studied the neural networks with random coupling strengths, then in this paper, we studied the neural networks under the random occurring controller.

5 Conclusion

This paper proposed two kinds of randomly occurring controllers in order to achieve the anti-synchronization of coupled neutral-type memristive neural network with mixed time-varying delays. According to the Lyapunov stability method, linear matrix inequalities, and the differential inclusion theory, these two kinds of control strategies are successful in ensuring the convergence of the system. It can well mimic the human brain in many applications, such as pattern recognition, associative memories, and learning. Finally, numerical examples are given to illustrate the effectiveness of the proposed theories. For further research topics, it is recommended that the synchronization of coupled memristive neutral-type neural networks with mixed time-varying delays via randomly occurring control should be studied. Also, it is important to extend our results to memristive neutral-type neural networks with multiple proportional delays.

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