Leader-Following Consensus of a Class of Stochastic Delayed Multi-Agent Systems with Partial Mixed Impulses ⋆

Yang Tang [a,b], Huijun Gao [c,d], Wenbing Zhang [e], Jürgen Kurths [a,b,f]

[a] *Institute of Physics, Humboldt University Berlin, Berlin 12489, Germany*

[b] *Potsdam Institute for Climate Impact Research, Potsdam 14473, Germany*

[c] *Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150080, China*

[d] *King Abdulaziz University, Jeddah 21589, Saudi Arabia*

[e] *Department of Mathematics, Yangzhou University, Jiangsu 225002, China*

[f] *Department of Control Theory, Nizhny Novgorod State University, Gagarin Avenue 23, Nizhny Novgorod 606950, Russia*

Abstract

In this paper, the exponential leader-following consensus problem is investigated for a class of nonlinear stochastic networked multi-agent systems with partial mixed impulses and unknown time-varying but bounded delays. The main feature of partial mixed impulses is that time-varying impulses are not only composed of synchronizing and desynchronizing impulses simultaneously but they are also injected into a fraction of nodes in multi-agent systems. Three kinds of partial mixed impulses are proposed: fixed partial mixed impulses, periodic partial mixed impulses, and try-once-discard-like partial mixed impulses. By means of the Lyapunov function theory and the comparison principle, conditions are derived for ensuring global exponential leader-following consensus under the presented three kinds of partial mixed impulses. Simulations of leader-following consensus of robotic systems are provided to validate the effectiveness of the proposed results and to show the advantages of the proposed partial mixed impulses.

Key words: Leader-following consensus, Tracking control, Multi-agent Systems, Complex networks, Impulsive Effects, Pinning control.

1 Introduction

Coordination, as a very important topic in collective cooperative motion, which is one of the most common and spectacular manifestation of coordinated behavior in nature and plays an important role in various contexts, such as biological networks, power networks, transportation networks, climate networks, social networks and technical networks [8, 18, 21, 24]. In modelling complex networks and multi-agent systems with self-dynamics, inherent time-delays [2, 6, 19, 20] and stochastic disturbances [14, 21] are widely observed in implementations of electronic networks, and genetic regulatory networks and injections of control inputs in large-scale networked systems. On the other hand, the states of various dynamical networks and/or multi-agent systems such as communication networks, large-scale chemical process networks and biological networks often suffer from instantaneous disturbances and undergo abrupt changes at certain instants, which may arise from switching phenomena, control requirements or frequency change, i.e., systems exhibit impulsive effects including stabilizing and destabilizing effects [5, 10, 22, 25, 26, 28].

In recent years, pinning control, like leader-following consensus or controllability [21], has sparked interests of many researchers, since there exists a common requirement to regulate the behavior of large ensembles of interacting units from engineering, social and biological systems by a rather small effort [1, 7, 11, 14, 18]. Pinning control has been investigated by utilizing various control techniques, such as state feedback control [1], adaptive control [1, 20, 21] and impulsive control [11, 14]. Unfortunately, up to now, coordination results for networked multi-agent systems or dynamical networks with both synchronizing and desynchronizing impulses, in which only a part of the nodes experience impulsive effects, have been widely overlooked in the literature primarily due to the difficulty in a mathematical derivation. This remains

⋆ This paper was not presented at any IFAC meeting. This research is supported by the National Natural Science Foundation of China (61203235, 61333012, 61273201), the Key Laboratory of Integrated Automation for the Process Industry (Northeastern University), IRTG 1740 (DFG) and the Alexander von Humboldt Foundation of Germany.
Corresponding author Y. Tang. Tel. +4933128820768. Fax +4930209399188.

Email addresses: tangtany@gmail.com (Yang Tang), hjgao@hit.edu.cn (Huijun Gao), zwb850506@126.com (Wenbing Zhang), Juergen.Kurths@pik-potsdam.de (Jürgen Kurths).

an important challenge in modelling time-varying properties of networked multi-agent systems and its generality of including the impulsive pinning strategy as a special case.

Actually, for a class of nonlinear stochastic networked multi-agent systems with partial time-varying impulses and unknown time-varying delays, it is theoretically challenging and practically difficult to establish easy-to-verify criteria for ensuring consensus. The inherent features of the leader-following consensus (tracking control) problem for stochastic delayed networked multi-agent systems with partial mixed impulses pose some fundamental difficulties: 1) How can we properly define partial time-varying/mixed impulses, in which the time-varying features reside in two aspects: only a fraction of nodes are subjected to impulses and the set of nodes injected with impulses is also time-varying? 2) Is it possible for us to establish a connection of partial mixed impulses with networked-induced constraints in networked control systems such as time-varying sampling intervals and competition of multiple nodes [3,16,27]? 3) How can we develop an effective technique to obtain mathematically verifiable leader-following consensus criteria and quantify consensus regions against such kind of partial mixed impulses? The answers to these questions may well explain why the tracking control problem for networked multi-agent systems with or without partial mixed impulses is still open.

Motivated by the above discussion, three kinds of partial mixed impulses are proposed and studied here, i. e., fixed partial mixed impulses, periodic partial mixed impulses, and try-once-discard-like partial mixed impulses are presented and discussed in detail. Based on the proposed three types of partial mixed impulses, the global mean square leader-following consensus problem is investigated for a class of stochastic networked multi-agent systems with unknown time-varying delays. Compared with the works of impulsive effects in uncoupled dynamical systems [13, 26], complex networks [4, 10, 11, 14, 28] and networked control systems [3, 16, 27], the main contributions of this paper are mainly threefold: (1) a novel concept of partial mixed impulses is proposed for the first time, which can encompass several well-known impulses [4,10,13,14,26]; (2) we establish a connection between the proposed impulses and networked-induced constraints like time-varying sampling intervals and competition of multiple nodes in networked control systems [3, 16, 27]; and (3) three different kinds of partial mixed impulses are investigated and compared; meanwhile, effects of systems' parameters on the size of consensus regions are characterized. This paper is organized as follows. In Section II, some preliminaries regarding the model, partial mixed impulses and assumptions are briefly outlined. In Section III, leader-following consensus conditions are presented by means of the comparison principle. In Section IV, simulations are exploited to show the effectiveness of the obtained results. The conclusions are given in Section V.

Notations: Let $\mathbb{N}_+ = \{1, 2, 3, ...\}$. $\|\cdot\|$ is the Euclidean vector norm in $\mathbb{R}^n$. $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of a matrix. $\#\mathcal{D}$ denotes the element number of the finite set $\mathcal{D}$ composed of the vertices to be controlled. $PC(m)$ denotes the class of piecewise right continuous function $\varphi : [t_0 - \tau, +\infty) \to \mathbb{R}^m$ with the norm defined by $\|\varphi(t)\|_\tau = \sup_{-\tau \le s \le 0} \|\varphi(t + s)\|$. For $x : \mathbb{R}^n \to \mathbb{R}^n$, denote $x(t^+) = \lim_{s\to 0^+} x(t + s)$ and $x(t^-) = \lim_{s\to 0^-} x(t + s)$. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e., the filtration contains all $\mathcal{P}$-null sets and is right continuous). Denote by $L^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ the family of all $\mathcal{F}_0$-measurable $PC([-\tau, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(s) : -\tau \le s \le 0\}$ such that $\sup_{-\tau\le s\le 0} \mathbb{E}\|\xi(s)\|^2 < \infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to a given probability measure $\mathcal{P}$. $A\backslash B$ represents the set difference from set A to set B.

2 Preliminaries

In this section, some preliminaries about the model and necessary assumptions are given. The problem formulation is briefly outlined. Three kinds of partial mixed impulses are proposed.

Consider the following reference state or the leader state:

$$\begin{aligned} ds(t) =&[As(t) + B\tilde{f}_1(s(t), t) + C\tilde{f}_2(s(t - \tau(t)))]dt \\ &+ f_3(s(t), s(\tilde{t} - \tau(t)), t)dw(t), \end{aligned}$$

where A, B and C are constant matrices which are defined on $\mathbb{R}^{n\times n}$; $\tilde{f}_1(s(t), t) = [\tilde{f}_{11}(s(t), t), ..., \tilde{f}_{1n}(s(t), t)]^T$ and $\tilde{f}_2(s(t-\tau(t))) = [\tilde{f}_{21}(s(t - \tau(t))), ..., \tilde{f}_{2n}(s(t - \tau(t)))]^T$ are continuous nonlinear vector functions; $\tilde{f}_3(.,.,.)$ is the noise intensity function; $w(t)$ is a scalar Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$ satisfying $\mathbb{E}\{dw(t)\} = 0$ and $\mathbb{E}\{[dw(t)]^2\} = dt$; $\tau(t)$ is an unknown but bounded time-varying delay satisfying $0 < \tau(t) \le \tau$, which is named by internal time-delay.

We consider the following nonlinear networked multi-agent system with stochastic disturbances and unknown time-varying delays, which can be forced to the leader state $s(t)$:

$$\begin{aligned} dx_i(t) =&[Ax_i(t) + B\tilde{f}_1(x_i(t), t) + C\tilde{f}_2(x_i(t - \tau(t)))]dt \\ &- dk_i(x_i(t) - s(t))dt + d\sum_{j=1}^{N} g_{ij}x_j(t)dt \\ &+ \tilde{f}_3(x_i(t), x_i(t - \tau(t)), t)dw(t), \\ &i = 1, 2, ..., N, \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector of the ith node; d stands for the control gain; $L = (g_{ij})_{N\times N}$ is the undirected coupling matrix representing the coupling topology, which is defined as follows: if there is a connection between nodes i and j $(i \ne j)$, $g_{ij} = g_{ji} = 1 > 0$; otherwise $g_{ij} = g_{ji} = 0$ $(i \ne j)$. For diagonal elements of L, $g_{ii} = -\sum_{j=1,j\ne i}^{N} g_{ij}, i = 1, 2, ..., N$. Assume that only one node $\jmath$ in the network has the information from the reference state $s(t)$, i. e., $k_\jmath = 1, \jmath \in \{1, 2, ..., N\}$ and $k_i = 0$ for $i \in \{1, 2, ..., N\}\backslash\{\jmath\}$. Here, L is assumed to be connected. According to [1, 7], all the eigenvalues of $M = L - K = (m_{ij})_{N\times N}$ are negative, where K is a $N \times N$ diagonal matrix whose the $\jmath$th diagonal element is $k_\jmath$ and the others are zero. For multiple nodes having the information from the reference state $s(t)$, the results in the following still hold. The initial conditions of system (1) are assumed to be $x_i(t) = \vartheta_i(t), -\tau \le t \le 0, i = 1, 2, \ldots, N$, where $\vartheta_i(t) \in L^2_{\mathcal{F}_0}([-\tau, 0], \mathbb{R}^n)$. In our model, only the internal delay is considered, which is widely observed in natural and engineering systems such as regulatory networks and milling process, etc. In multi-agent systems, communications are also often subjected to transmission delays, which may deteriorate the coordination performance. Therefore, it is interesting and challenging to extend our results to the case with transmission delays in the near future.

Let $e_i(t) = x_i(t) - s(t)$ be the tracking error of node i between the current state $x_i(t)$ and the reference state $s(t)$. By including impulsive effects on a fraction of nodes into system (1), the following impulsive effects are considered:

$$I_i(t) = \begin{cases} \sum_{k=1}^{+\infty} \rho_k e_i(t)\sigma(t - t_k), i \in \mathcal{D}(t_k), \\ 0, i \notin \mathcal{D}(t_k), \end{cases}$$

where ρ_k represent the time-varying strengths of impulsive effects, $\sigma(.)$ is the Dirac delta function, the time series

$\{t_1, t_2, t_3, ...\}$ is a sequence of strictly increasing impulsive instants satisfying $\lim_{k\to\infty} t_k = +\infty$; $\#\mathcal{D}(t_k) = l$, which means that the locations of impulses are time-varying.

By considering the fact that $d\sum_{j=1}^{N} g_{ij}s(t) = 0$, it yields from the impulsive effects $I_i(t)$ that the tracking error system between $x_i(t)$ and $s(t)$ is

$$\begin{cases} de_i(t) = [Ae_i(t) + Bf_1(e_i(t), t) \\ \qquad\quad +Cf_2(e_i(t-\tau(t)))]dt \\ \qquad\quad +d\sum_{j=1}^{N} m_{ij}e_j(t)dt \\ \qquad\quad +f_3(e_i(t), e_i(t-\tau(t)), t)dw(t), \\ \qquad\quad i = 1, 2, ..., N, \\ e_i(t_k^+) = e_i(t_k^-) + \rho_k e_i(t_k^-), i \in \mathcal{D}(t_k). \end{cases} \tag{2}$$

where $f_1(e_i(t), t) = \tilde{f}_1(x_i(t), t) - \tilde{f}_1(s(t), t)$, $f_2(e_i(t-\tau(t))) = \tilde{f}_2(x_i(t-\tau(t))) - \tilde{f}_2(s(t-\tau(t)))$ and $f_3(e_i(t), e_i(t-\tau(t)), t) = \tilde{f}_3(x_i(t), x_i(t-\tau(t)), t) - \tilde{f}_3(s(t), s(t-\tau(t)), t)$. The error system in (2) without impulses is similar to [12], in which the stability analysis of uncertain stochastic neural networks with unbounded time-varying delays was investigated. Different from [12], the model here takes into account three kinds of impulsive protocols and the effects of these impulsive protocols on tracking performance are investigated.

In addition, inspired by the impulsive effects from networked control systems [16, 23] and complex networks [14], we propose the following three kinds of partial mixed impulsive effects to characterize the set $\mathcal{D}(t_k)$ of partial mixed impulses:

$$\mathcal{D}(t_k) = \begin{cases} \mathcal{R}_1, \\ \mathcal{R}_2(t_k), \\ \mathcal{R}_3(t_k), \end{cases}$$

where $\mathcal{R}_1 \subseteq \mathcal{V} := \{1, 2, ..., N\}$ is a fixed set and $\#\mathcal{R}_1 = l$, in which the locations of synchronizing and desynchronizing impulses are fixed along the time evolution; $\mathcal{R}_2(t_k)$ and $\mathcal{R}_3(t_k)$ are time-varying sets, in which the locations of impulsive effects are time-varying according to certain protocols. It should be mentioned that in [16,23], the protocols adopted are used to update the states in the controller and the plant. Here, the three kinds of partial mixed impulsive effects are more pertinent to reflect the stabilizing effects in [6,10].

Under $\mathcal{R}_1$, the updating equation at t_k in (2) can be rewritten as follows:

$$e_i(t_k^+) = e_i(t_k^-) + \delta_{\mathcal{R}_1}(i)\rho_k e_i(t_k^-), \tag{3}$$

where $\delta_{\mathcal{R}_1}(\cdot)$ is the characteristic function of the set $\mathcal{R}_1$, i. e., $\delta_{\mathcal{R}_1}(i) = 1$ if $i \in \mathcal{R}_1$; otherwise, $\delta_{\mathcal{R}_1}(i) = 0$. In $\mathcal{R}_1$, the nodes subjected to impulsive effects are known beforehand, which is similar to the usual pinning control, leader-following consensus or controllability problem [18, 21] and can be viewed as a *static* protocol. We refer this kind of impulses as *fixed partial mixed* impulses.

Under $\mathcal{R}_2(t_k)$, the updating equation at t_k in (2) can be formulated as follows:

$$e_i(t_k^+) = e_i(t_k^-) + \delta_{\mathcal{R}_2}(t_k, i)\rho_k e_i(t_k^-),$$

where $\delta_{\mathcal{R}_2}(t_k, \cdot)$ is the characteristic function of the set $\mathcal{R}_2(t_k)$ defined by:
i) when $s + l - 1 \le N$

$$\begin{cases} \delta_{\mathcal{R}_2}(t_k, i) = 1, \text{ if } i \in [s, s+l-1] \cap \mathcal{V}, \\ \delta_{\mathcal{R}_2}(t_k, i) = 0, \text{ otherwise}, \end{cases}$$

or

ii) when $s + l - 1 > N$

$$\begin{cases} \delta_{\mathcal{R}_2}(t_k, i) = 1, \text{ if } i \in ([s, N] \cup [1, l-\alpha]) \cap \mathcal{V}, \\ \delta_{\mathcal{R}_2}(t_k, i) = 0, \text{ otherwise}, \end{cases}$$

where $s = k - \text{INT}(k-0.1, N) * N$, $\alpha = N - s + 1$ and $\text{INT}(a, b)$ is a function to return the integer part of a divided by b. For example, if $N = 3, l = 2$, we get the following relationship:

$$\begin{cases} \mathcal{R}_2(t_k) = \{1, 2\}, \text{ if } k = 1, \\ \mathcal{R}_2(t_k) = \{2, 3\}, \text{ if } k = 2, \\ \mathcal{R}_2(t_k) = \{3, 1\}, \text{ if } k = 3, \\ ... \end{cases}$$

It is worth mentioning that this kind of impulses is related to the idea of intermittent impulsive synchronization technique [13]. Different from [13], we propose three kinds of impulses and study the tracking control of networked multi-agent systems, which presents a unified framework of impulsive control techniques.

Under $\mathcal{R}_3(t_k)$, the updating equation at t_k in (2) is written as:

$$e_i(t_k^+) = e_i(t_k^-) + \delta_{\mathcal{R}_3}(t_k, i)\rho_k e_i(t_k^-),$$

in which $\delta_{\mathcal{R}_3}(t_k, i)$ is defined as follows:
i) one can reorder the vectors $e_1(t), e_2(t), ..., e_N(t)$ in the following way $\|e_{p1}(t)\| \ge \|e_{p2}(t)\| \ge ... \ge \|e_{pl}(t)\| \ge ... \ge \|e_{pN}(t)\|$, when $\rho_k \in \mathfrak{M}$. Then, $\mathcal{R}_3(t_k) = \{p_1, p_2, ..., p_l\}$;
ii) one can reorder the vectors $e_1(t), e_2(t), ..., e_N(t)$ in the following way $\|e_{v1}(t)\| \le \|e_{v2}(t)\| \le ... \le \|e_{vl}(t)\| \le ... \le \|e_{vN}(t)\|$, when $\rho_k \in \mathfrak{B}$. Then, $\mathcal{R}_3(t_k) = \{v_1, v_2, ..., v_l\}$.

Here, we present an example for showing the real-world applications of the proposed three kinds of impulses. For a class of mechanical systems including robotic manipulators and rigid bodies in a network described by Euler-Lagrange equations [18]:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = u_i, i = 1, 2, ..., N,$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive-definite inertia matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^p$ is the vector of Coriolis and centrifugal torques and u_i is the vector of torques produced by the actuators associated with the i-th system. u_i is taken as $u_i = -\sum_{j=1}^{N} g_{ij}(q_i - q_j) - \sum_{j=1}^{N} g_{ij}(\dot{q}_i - \dot{q}_j) - S_i(q_i - q_0)$, where $S_i \in \mathbb{R}^{p\times p}$ is symmetric positive definite. The leader's vector of generalized coordinates and vector of generalized coordinate derivatives are denoted by, respectively, q_0 and $\dot{q}_0$. In order to track the leader under $\mathcal{R}_3(t_k)$, a scheduler knows the real-time errors between $\dot{q}_i - \dot{q}_0$ at t_k^- and then determines the agents to update their coordinate derivatives $\dot{q}_i$. According to the proposed impulses, the agents receive the decision information from the scheduler through a wireless network and then *smartly* update their $\dot{q}_i$ at t_k. For $\mathcal{R}_1$ and $\mathcal{R}_2(t_k)$, the proposed impulses can also be applied similarly.

Remark 1. ρ_k is a constant for each instant t_k and the strength of ρ_k is time-varying for different instants $t_k, k \in \mathbb{N}^+$ along the time evolution for all three types of impulses mentioned above. When $|1 + \rho_k| > 1$, i. e., $\rho_k \in \mathfrak{B} := (-\infty, -2) \cup (0, +\infty)$, impulsive effects will destroy consensus and can be referred as desynchronizing impulses. On the other hand, when $|1 + \rho_k| < 1$, i. e., $\rho_k \in \mathfrak{M} := (-2, 0)$, impulsive effects will be useful for achieving consensus and can be called synchronizing impulses. Specially, if $|1+\rho_k| = 1$ ($\rho_k \in \mathfrak{Z} := \{-2\} \cup \{0\}$), the impulsive effects are neither harmful nor useful for consensus. We only consider time-varying impulses $\rho_k \in \mathfrak{B} \cup \mathfrak{M}$ and it is not difficult to extend our results to the case of $\rho_k \in \mathfrak{Z}$.

Remark 2. Under $\mathcal{R}_2(t_k)$, this kind of partial mixed impulsive effects can also be regarded as a static protocol. Such a kind of impulsive effects is *periodic* and can be referred as round robin (RR) static protocol from the perspective of networked control systems [16, 23]. Under $\mathcal{R}_3(t_k)$, the impulses are similar to the try-once-discard (TOD) protocol, which is a dynamical protocol. Here, if there are error states that have the same norm under synchronizing and desynchronizing impulses, i. e., $\|e_{p\theta_1}\| = \|e_{p\theta_2}\| = ... = \|e_{p\theta_\nu}\|$ or $\|e_{v\theta_1}\| = \|e_{v\theta_2}\| = ... = \|e_{v\theta_\nu}\|$, we can randomly choose the nodes from them to satisfy $\#\mathcal{D}(t_k) = l$. In order to describe both synchronizing and desynchronizing impulses simultaneously, we assume that the strengths of desynchronizing impulses take values from a finite set $\mathcal{I} = \{\epsilon_1, \epsilon_2, \dots, \epsilon_r\}$, which contains r elements and $\mathcal{I} \subset \mathfrak{B}$. Meanwhile, the strengths of synchronizing impulses are chosen from $\mathcal{H} = \{\eta_1, \eta_2, \dots, \eta_q\}$, which involves q elements and $\mathcal{H} \subset \mathfrak{M}$.

Remark 3. In networked control systems, there are several networked-induced constraints such as time-varying sampling intervals and competition of multiple nodes accessing networks [6, 27]. The time-varying sampling and the competition of network nodes can be handled by RR and TOD protocols, which are special cases of the proposed impulses and have been successfully used in controlling real-world networked systems, such as batch reactors and disk drives [3, 16, 23]. In [16, 23], only one node is subjected to an impulse and its strength is $\rho_k = -1$ at all the instants. To the best of the authors' knowledge, this is the first time to introduce RR-like or TOD-like protocols for investigating leader-following consensus (tracking control) of multi-agent systems, which would promote the investigation of consensus by using techniques from networked control systems. In addition to this difference, there still exist two major differences with [16, 23]: it is allowed here that impulses exist in several nodes instead of a single one at t_k and both synchronizing and desynchronizing impulses are considered at the same time.

Remark 4. The comparison with existing results is divided into several parts: 1) Compared with several tracking control problems in [7, 24], nonlinearities, impulsive effects, time-delays and stochastic disturbances are taken into account here at the same time, which makes our model more practical. 2) Different from distributed consensus of agent systems via adaptive control [20], this work adopts three kinds of impulsive control techniques and studies leader-following consensus. 3) Different from previous works on synchronization or consensus of complex networks or impulsive systems [4, 10, 14], the *time-varying* features of impulsive effects reside in twofold: a) the locations of impulsive effects on nodes are time-varying; b) the strengths of impulsive effects are time-varying, which include synchronizing and desynchronizing effects simultaneously. Since only a fraction of nodes are subjected to mixed impulsive effects, we call this kind of impulses as *partial mixed impulses*. In this paper, inspired by [16, 23], we propose three kinds of partial mixed impulses. It is worth mentioning that partial mixed impulses are quite general. When $l = N$, all three types of partial mixed impulses reduce to the impulses in [28] and how a fraction of nodes with impulsive effects affects tracking performance is overlooked, which is now an important issue in control theory and natural science. In addition, we establish a close relationship between the proposed impulsive effects and networked-induced constraints like time-varying sampling intervals and competition of network nodes in networked control systems [16, 23, 27]. When $\rho_k \in \mathfrak{M}$, the third type of partial mixed impulses in this paper reduces to the impulses in [14] and a unified framework for impulses and scheduling policies is not considered. Therefore, our proposed impulses are quite general to encompass several well-studied impulses in [4, 10, 14, 16, 23, 28] as special cases and can describe more practical situations.

Remark 5. In [1] and [11], the pinning problems of networks were investigated under a single state feedback controller and a single impulsive controller, respectively. In [1], it is proved that, if the global coupling strength d is sufficiently large, a coupled complex network can be pinned to a target without assuming symmetry irreducibility, or linearity of the couplings. In [11], under the assumption of the network containing a spanning tree, the network can achieve consensus when the impulsive controller is injected on the root node with an appropriate impulsive strength and intervals. Different from [11], our model involves time-delays and stochastic disturbances. The importance of our results is to employ scheduling techniques inspired from networked control systems [16, 23] to investigate pinning problems. Although our impulsive controller renders a good tracking performance due to the consideration of scheduling techniques, the implementation requires more real-time information than [11]. Therefore, it is promising to adopt the techniques in [11] without monitoring tracking errors $e_i(t)$.

In the following, we assume that $e_i(t)$ is right continuous at $t = t_k$, i.e., $e_i(t_k) = e_i(t_k^+)$. Hence, the solution of (2) is a piecewise right-hand continuous function with discontinuities at $t = t_k$ for $k \in \mathbb{N}_+$. It is worth mentioning that we can still get the same results according to the impulsive control theory when $e_i(t)$ is left continuous at $t = t_k$ [9]. The proof is similar by following [9] and we do not repeat it any more.

The following assumptions and definitions are required to present our results.

Assumption 1. The nonlinearities $\tilde{f}_1(.,.)$ and $\tilde{f}_2(.)$ satisfy the following Lipschitz conditions for $\forall x, y \in \mathbb{R}^n$:

$$\|\tilde{f}_1(x,t) - \tilde{f}_1(y,t)\| \leq \phi_1 \|x - y\|,$$
$$\|\tilde{f}_2(x) - \tilde{f}_2(y))\| \leq \phi_2 \|x - y\|,$$

where ϕ_1 and ϕ_2 are positive constants.

Assumption 2. The noise intensity function $\tilde{f}_3$ satisfies the uniformly Lipschitz continuous conditions

$$\begin{aligned}
&\text{trace}[(\tilde{f}_3(x_1, y_1, t) - \tilde{f}_3(x_2, y_2, t))^T \\
&\times (\tilde{f}_3(x_1, y_1, t) - \tilde{f}_3(x_2, y_2, t))] \\
&\leq \|\Sigma_1(x_1 - x_2)\|^2 + \|\Sigma_2(y_1 - y_2)\|^2,
\end{aligned}$$

where Σ_1 and Σ_1 are matrices with appropriate dimensions.

Assumption 2 is quite popular in dealing with stochastic disturbances when analyzing the stability of stochastic differential equations (see [15] and the references therein). In addition, Assumption 2 is used to confine the intensities of stochastic disturbances via linear terms of states, which can facilitate a mathematical derivation.

Assumption 3. $\tilde{f}_1(0, t) = 0$, $\tilde{f}_2(0) = 0$ and $\tilde{f}_3(0, 0, t) = 0$.

Definition 1. The stochastic delayed networked multi-agent system in (1) with partial mixed impulses is said to globally exponentially track the leader state $s(t)$ in mean square if there exist $\lambda > 0$, $T_0 > 0$ and $M > 0$ such that for any initial values $\vartheta_i(.)$ $(i = 1, 2, ..., N)$,

$$\begin{aligned}
&\mathbb{E}\sum_{i=1}^{N} \|e_i(t)\|^2 = \mathbb{E}\sum_{i=1}^{N} \|x_i(t) - s(t)\|^2 \leq M\mathrm{e}^{-\lambda t}, \\
&\forall i = 1, 2, ..., N,
\end{aligned}$$

hold for all $t > T_0$.

Definition 2. The average impulsive interval of the synchronizing impulses is not larger than $\check{T}_i$ and the average impulsive interval of the desynchronizing impulses is not less than $\hat{T}_j$, if there exist positive numbers $\check{T}_i$ and $\hat{T}_j$ such that

$$\check{N}_i(T,t) \geq \frac{T-t}{\check{T}_i} - N_0,$$

and

$$\hat{N}_j(T,t) \leq \frac{T-t}{\hat{T}_j} + N_0,$$

hold for $N_0 \geq 0$, $\check{T}_i$, $\hat{T}_j$, $i = 1, 2, \ldots, q, j = 1, 2, \ldots, r$, where $\check{N}_i(T,t)$ and $\hat{N}_j(T,t)$ stand for the number of the synchronizing impulsive sequence with impulsive strengths η_i and desynchronizing impulsive sequence with impulsive strengths ϵ_j on the interval (t,T), respectively. Without loss of generality, $N_0 = 0$ is set throughout this paper.

Lemma 1. [26] Let $0 \leq \tau_i(t) \leq \tau$, $F(t, u, \bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m) : \mathbb{R}^+ \times \overbrace{\mathbb{R} \times \ldots \times \mathbb{R}}^{m+1} \to \mathbb{R}$ be nondecreasing in $\bar{u}_i$ for each fixed $(t, u, \bar{u}_1, \ldots, \bar{u}_{i-1}, \bar{u}_{i+1}, \ldots, \bar{u}_m)$, $i = 1, 2, \ldots, m$, and $I_k(u)$ be any monotonically nondecreasing function: $\mathbb{R} \to \mathbb{R}$ in u. Suppose that $u(t), v(t) \in PC(1)$ satisfy

$$\begin{cases} D^+u(t) \leq F(t, u(t), u(t-\tau_1(t)), \ldots, u(t-\tau_m(t))), \\ u(t_k^+) \leq I_k(u(t_k^-)), \qquad k \in \mathbb{N}_+, \end{cases}$$

and

$$\begin{cases} D^+v(t) > F(t, v(t), v(t-\tau_1(t)), \ldots, v(t-\tau_m(t))), \\ v(t_k^+) \geq I_k(v(t_k^-)), \qquad k \in \mathbb{N}_+, \end{cases}$$

where the upper-right Dini derivative $D^+y(t)$ is defined as $D^+y(t) = \overline{\lim}_{h\to 0^+}(y(t+h) - y(t))/h$. Then $u(t) \leq v(t)$, for $-\tau \leq t \leq 0$ implies that $u(t) \leq v(t)$, for all $t \geq 0$.

3 Main results

In this section, we will investigate the leader-following consensus problem of the nonlinear stochastic networked multi-agent system in (1) with partial mixed impulses and unknown time-varying delays by virtue of the comparison principle and stochastic analysis techniques. Firstly, we study tracking control of the nonlinear stochastic networked multi-agent system in (1) with partial mixed impulses under $\mathcal{R}_3(t_k)$. Next, we extend the results to the cases with $\mathcal{R}_1$ and $\mathcal{R}_2(t_k)$ and other kinds of specific situations.

Theorem 1. Under $\mathcal{D}(t_k) = \mathcal{R}_3(t_k)$, suppose that Assumptions 1 and 2 hold, the average impulsive interval of synchronizing impulses is not larger than $\check{T}_i, i = 1, 2, ..., q$ and the average impulsive interval of synchronizing impulses is not less than $\hat{T}_j, j = 1, 2, ..., r$. Then, the nonlinear stochastic networked multi-agent system in (1) with partial mixed impulses and unknown time-varying delays will globally exponentially track the leader $s(t)$ in mean square, if the following inequality holds

$$\mu - \upsilon > 0, \tag{4}$$

where

$$\begin{aligned} \mu &= -(\beta + \sum_{i=1}^{q} \frac{\ln \check{\Psi}_i}{\check{T}_i} + \sum_{j=1}^{r} \frac{\ln \hat{\Psi}_j}{\hat{T}_j}), \ \check{\Psi}_i = \check{\psi}_i, \\ \hat{\Psi}_j &= \begin{cases} \dfrac{N\hat{\psi}_j}{N-l}, \ \text{if } l/N \leq \dfrac{1}{2}\Big(1 - \dfrac{1}{(2\epsilon_j^2 + 4\epsilon_j + 1)}\Big), \\ (1+\epsilon_j)^2, \ \text{otherwise}, \end{cases} \\ \hat{\psi}_j &= \frac{N + l\epsilon_j(\epsilon_j + 2)}{N} \in (1, +\infty), j = 1, ..., r, \\ \check{\psi}_i &= \frac{N + l\eta_i(\eta_i + 2)}{N} \in (0, 1), i = 1, ..., q, \\ \beta &= 2d\lambda_{\max}(M) + \lambda_{\max}(A + A^T) + \lambda_{\max}(\Sigma_1^T\Sigma_1) \\ &\quad + 2\sqrt{\lambda_{\max}(B^TB)}\phi_1 + \sqrt{\lambda_{\max}(C^TC)}\phi_2, \\ \upsilon &= \sqrt{\lambda_{\max}(C^TC)}\phi_2 + \lambda_{\max}(\Sigma_2^T\Sigma_2). \end{aligned}$$

Then, the nonlinear stochastic networked multi-agent system in (1) with partial mixed impulses and unknown time-varying delays can globally exponentially track $s(t)$:

$$\mathbb{E}\sum_{i=1}^{N} \|e_i(t)\|^2 \leq \chi e^{-\lambda t},$$

where $\chi = \mathbb{E}\sum_{i=1}^{N} \sup_{-\tau\leq s\leq 0}\{\|\vartheta_i(s)\|^2\}$ and $\lambda > 0$ is a unique solution of $\lambda - \mu + \upsilon e^{\lambda\tau} = 0$.

Proof. Consider the following Lyapunov function:

$$V(t) = \sum_{i=1}^{N} e_i^T(t)e_i(t). \tag{5}$$

Then, for any $t \in [t_{k-1}, t_k)$, according to the Itô-differential formula [15], the operator $\mathscr{L}$ can be calculated as follows:

$$\begin{aligned} \mathscr{L}V(t) = &\sum_{i=1}^{N} 2e_i^T(t)[Ae_i(t) + Bf_1(e_i(t), t) + Cf_2(e_i(t-\tau(t)))] \\ &+ d\sum_{i=1}^{N}\sum_{j=1}^{N} 2e_i^T(t)m_{ij}e_j(t) \\ &+ \sum_{i=1}^{N} \text{trace}[f_3^T(e_i(t), e_i(t-\tau(t)), t) \\ &\times f_3(e_i(t), e_i(t-\tau(t)), t)]. \end{aligned} \tag{6}$$

By Assumption 1, we have $\sum_{i=1}^{N} e_i^T(t)Bf_1(e_i(t), t) \leq \sqrt{\lambda_{\max}(B^TB)}\phi_1 \sum_{i=1}^{N} e_i^T(t)e_i(t)$.

Similarly, one gets from Assumption 1 and $2ab \leq a^2 + b^2$,

$$\begin{aligned} &2\sum_{i=1}^{N} e_i^T(t)Cf_2(e_i(t-\tau(t))) \\ \leq &2\sum_{i=1}^{N} \sqrt{\|e_i(t)\|^2}\sqrt{\|Cf_2(e_i(t-\tau(t)))\|^2} \\ \leq &\sqrt{(\lambda_{\max}(C^TC))}\phi_2 \sum_{i=1}^{N}(\|e_i(t)\|^2 + e_i^T(t-\tau(t))e_i(t-\tau(t))). \end{aligned} \tag{7}$$

From Assumption 2, it can be checked that

$$\sum_{i=1}^{N}\text{trace}[f_3^T(e_i(t),e_i(t-\tau(t)),t) \times f_3(e_i(t),e_i(t-\tau(t)),t)] \leq \sum_{i=1}^{N}[e_i^T(t)\Sigma_1^T\Sigma_1 e_i(t)+e_i^T(t-\tau(t))\Sigma_2^T\Sigma_2 e_i(t-\tau(t))]. \tag{8}$$

Due to the fact that the connection matrix L is connected, it yields that

$$\begin{aligned}&2d\sum_{i=1}^{N}\sum_{j=1}^{N}m_{ij}e_i^T(t)e_j(t)\\ =&2d\sum_{i=1}^{N}\sum_{j=1}^{N}m_{ij}\sum_{k=1}^{n}e_{ik}(t)e_{jk}(t)\\ =&2d\sum_{k=1}^{n}\sum_{i=1}^{N}\sum_{j=1}^{N}m_{ij}e_{ik}(t)e_{jk}(t)\\ =&2d\sum_{k=1}^{n}(e^k(t))^TM(e^k(t))\\ \leq&2d\lambda_{\max}(M)\sum_{i=1}^{N}e_i^T(t)e_i(t),\end{aligned} \tag{9}$$

where $e^k(t)=(e_{1k}(t),...,e_{Nk}(t))^T$ for $k=1,...,n$. Substituting (7)-(9) into (6), we have

$$\begin{aligned}\mathbb{E}\mathscr{L}V(t)=&\mathbb{E}\sum_{i=1}^{N}\beta e_i^T(t)e_i(t)\\ &+\mathbb{E}\sum_{i=1}^{N}\upsilon e_i^T(t-\tau(t))e_i(t-\tau(t))\\ =&\beta\mathbb{E}V(t)+\upsilon\mathbb{E}V(t-\tau(t)).\end{aligned} \tag{10}$$

In the following, we aim to derive the relationship between $V(t_k^+)$ and $V(t_k^-)$ such that the results can fit into the framework of Lemma 1. When $t=t_k, k\in\mathbb{N}$, according to (2), one gets

$$\begin{aligned}V(t_k^+)=&\sum_{i=1}^{N}e_i^T(t_k^+)e_i(t_k^+)\\ =&\sum_{i\in\mathcal{D}(t_k)}e_i^T(t_k^+)e_i(t_k^+)+\sum_{i\notin\mathcal{D}(t_k)}e_i^T(t_k^+)e_i(t_k^+)\\ =&\sum_{i\in\mathcal{D}(t_k)}(1+\rho_k)^2e_i^T(t_k^-)e_i(t_k^-)+\sum_{i\notin\mathcal{D}(t_k)}e_i^T(t_k^-)e_i(t_k^-).\end{aligned} \tag{11}$$

Denote

$$\begin{cases}\Phi_1(t_k^-)=\max\{\|e_i(t_k^-)\|: i\in\mathcal{R}_3(t_k)\}, & \text{if } \rho_k\in\mathfrak{B},\\ \Phi_2(t_k^-)=\min\{\|e_i(t_k^-)\|: i\notin\mathcal{R}_3(t_k)\}, & \text{if } \rho_k\in\mathfrak{B},\\ \Phi_3(t_k^-)=\min\{\|e_i(t_k^-)\|: i\in\mathcal{R}_3(t_k)\}, & \text{if } \rho_k\in\mathfrak{M},\\ \Phi_4(t_k^-)=\max\{\|e_i(t_k^-)\|: i\notin\mathcal{R}_3(t_k)\}, & \text{if } \rho_k\in\mathfrak{M}.\end{cases} \tag{12}$$

First, we consider the situation of $\rho_k=\epsilon_k\in\mathfrak{B}$. For $\hat{\psi}_k=\dfrac{N+l\epsilon_k(\epsilon_k+2)}{N}\in(1,+\infty)$, one can verify that the following equation holds,

$$N-l=\frac{[\hat{\psi}_k-(1+\epsilon_k)^2]l}{1-\hat{\psi}_k}\geq 0. \tag{13}$$

Then, one has from (12)

$$\begin{aligned}&\sum_{i\in\mathcal{R}_3(t_k)}(1+\rho_k)^2e_i^T(t_k^-)e_i(t_k^-)\\ \leq&l(1+\epsilon_k)^2\Phi_1^2(t_k^-)\\ \leq&l(1+\epsilon_k)^2\Phi_2^2(t_k^-)\\ =&\frac{l(1+\epsilon_k)^2}{N-l}(N-l)\Phi_2^2(t_k^-)\\ \leq&\frac{l(1+\epsilon_k)^2}{N-l}\sum_{i\notin\mathcal{R}_3(t_k)}e_i^T(t_k^-)e_i(t_k^-).\end{aligned} \tag{14}$$

Therefore, substituting (14) and (13) into (11), it can be checked that

$$\begin{aligned}V(t_k^+)\leq&\Big[\frac{l(1+\epsilon_k)^2}{N-l}+1\Big]\sum_{i\notin\mathcal{R}_3(t_k)}e_i^T(t_k^-)e_i(t_k^-)\\ =&\frac{\hat{\psi}_k-\hat{\psi}_k(1+\epsilon_k)^2}{\hat{\psi}_k-(1+\epsilon_k)^2}\sum_{i\notin\mathcal{R}_3(t_k)}e_i^T(t_k^-)e_i(t_k^-)\\ =&\frac{\hat{\psi}_k(1-(1+\epsilon_k)^2)}{\frac{l}{N}\epsilon_k(\epsilon_k+2)-\epsilon_k(\epsilon_k+2)}\\ &\times\sum_{i\notin\mathcal{R}_3(t_k)}e_i^T(t_k^-)e_i(t_k^-)\\ \leq&\frac{N\hat{\psi}_k}{N-l}\sum_{i=1}^{N}e_i^T(t_k^-)e_i(t_k^-).\end{aligned} \tag{15}$$

Therefore, when $\rho_k=\epsilon_k$,

$$V(t_k^+)\leq\frac{N\hat{\psi}_k}{N-l}V(t_k^-). \tag{16}$$

Note that the following inequality also holds for $\rho_k=\epsilon_k$:

$$V(t_k^+)\leq(1+\epsilon_k)^2V(t_k^-). \tag{17}$$

Comparing (16) and (17), we define $\hat{\Psi}_k$ as follows:

$$\hat{\Psi}_k=\begin{cases}\dfrac{N\hat{\psi}_k}{N-l}, & \text{if } l/N\leq\dfrac{1}{2}(1-\dfrac{1}{(2\epsilon_k^2+4\epsilon_k+1)}),\\ (1+\epsilon_k)^2, & \text{otherwise.}\end{cases} \tag{18}$$

Similarly, we consider the case of $\rho_k=\eta_k$, getting $\check{\psi}_k=\dfrac{N+l\eta_k(\eta_k+2)}{N}\in(0,1)$ and $N-l=\dfrac{[\check{\psi}_k-(1+\eta_k)^2]l}{1-\check{\psi}_k}\geq 0$. Hence, it can be obtained that

$$\begin{aligned}&\sum_{i\notin\mathcal{R}_3(t_k)}e_i^T(t_k^-)e_i(t_k^-)\\ \leq&(N-l)\Phi_3^2(t_k^-)\\ \leq&(N-l)\Phi_4^2(t_k^-)\\ =&\frac{[\check{\psi}_k-(1+\eta_k)^2]l}{1-\check{\psi}_k}\Phi_4^2(t_k^-)\\ \leq&\frac{[\check{\psi}_k-(1+\eta_k)^2]}{1-\check{\psi}_k}\sum_{i\in\mathcal{R}_3(t_k)}e_i^T(t_k^-)e_i(t_k^-).\end{aligned} \tag{19}$$

Thus, we get the following inequality by some computations

$$\begin{aligned}V(t_k^+)\leq&\sum_{i\in\mathcal{R}_3(t_k)}(1+\eta_k)^2e_i^T(t_k^-)e_i(t_k^-)\\ &+\sum_{i\notin\mathcal{R}_3(t_k)}e_i^T(t_k^-)e_i(t_k^-)\\ \leq&\check{\psi}_kV(t_k^-).\end{aligned} \tag{20}$$

By considering (18) and (20) together, it yields

$$\mathbb{E}V(t_k^+) \le \Psi_k \mathbb{E}V(t_k^-).$$

where $\Psi_k = \begin{cases} \hat{\Psi}_k := \dfrac{N\hat{\psi}_k}{N-l}, & \text{if } \rho_k \in \mathfrak{B}, \\ \check{\Psi}_k := \check{\psi}_k, & \text{if } \rho_k \in \mathfrak{M}. \end{cases}$

Now, we are in a position to utilize the comparison principle to prove exponential tracking for the nonlinear stochastic networked multi-agent system in (2) under $\mathcal{R}_3(t_k)$ in mean square sense. For $\forall \varepsilon > 0$, let $v(t)$ be a unique solution of the following impulsive delayed system

$$\begin{cases} \dot{v}(t) = \beta v(t) + \upsilon v(t-\tau(t)) + \varepsilon, t \neq t_k, \\ v(t_k) = \Psi_k v(t_k^-), \quad t = t_k, \ k \in \mathbb{N}_+, \\ v(s) = \mathbb{E}\sum_{i=1}^N \|\vartheta_i(s)\|^2, \quad -\tau \le s \le 0. \end{cases} \tag{21}$$

From [15], it is true that $t \in [t_k, t_{k+1})$, $D^+\mathbb{E}V(t) = \mathbb{E}\mathscr{L}V(t)$. Then it follows from Lemma 1 that $\mathbb{E}V(t) \le v(t), \quad t \ge 0$. According to the formula for the variation of parameters [9], one obtains from (21) that

$$v(t) = Q(t,0)v(0) + \int_0^t Q(t,s)[\upsilon v(s-\tau(s)) + \varepsilon]ds, \tag{22}$$

where $Q(t,s)$ $(t > s \ge 0)$ is the Cauchy matrix of the following linear impulsive system

$$\begin{cases} \dot{z}(t) = \beta z(t), & t \neq t_k, k \in \mathbb{N}_+, \\ z(t_k^+) = \Psi_k z(t_k^-), & t = t_k, k \in \mathbb{N}_+. \end{cases}$$

According to the representation of the Cauchy matrix, the following inequality holds from Definition 1:

$$Q(t,s) \le e^{\beta(t-s)} \prod_{i=1}^{q} \check{\Psi}_i^{\frac{t-s}{\check{T}_i}} \prod_{j=1}^{r} \hat{\Psi}_j^{\frac{t-s}{\hat{T}_j}} = e^{-\mu(t-s)}, \tag{23}$$

where $\mu = -(\beta + \sum_{i=1}^{q} \frac{\ln \check{\Psi}_i}{\check{T}_i} + \sum_{j=1}^{r} \frac{\ln \hat{\Psi}_j}{\hat{T}_j})$. Define $\chi = \mathbb{E}\sum_{i=1}^N \sup_{-\tau \le s \le 0}\{\|\vartheta_i(s)\|^2\}$. It yields from (22) that

$$v(t) \le \chi e^{-\mu t} + \int_0^t e^{-\mu(t-s)}[\upsilon v(s-\tau(s)) + \varepsilon]ds, \tag{24}$$

$\forall t \ge 0$. Let $P(\lambda) = \lambda - \mu + \upsilon e^{\lambda\tau}$. According to inequality (4), one has $P(0) = -\mu + \upsilon < 0$. Since $P(+\infty) = +\infty$ and $\dot{P}(\lambda) = 1 + \tau\upsilon e^{\lambda\tau} > 0$, there exists a unique solution $\lambda > 0$, such that the following equality holds $P(\lambda) = \lambda - \mu + \upsilon e^{\lambda\tau} = 0$.

From (4), $\mu - \upsilon > 0$ holds and thus one gets for $t \in [-\tau, 0]$ that $v(t) = \mathbb{E}\sum_{i=1}^N\{\|\vartheta_i(t)\|^2\} \le \chi < \chi e^{-\lambda t} + \frac{\varepsilon}{\mu-\upsilon}$. Now we are in a position to prove that the following inequality holds for any $t \ge 0$

$$v(t) < \chi e^{-\lambda t} + \frac{\varepsilon}{\mu - \upsilon}. \tag{25}$$

Suppose that (25) does not hold, there exists a $t^* > 0$ such that $v(t^*) \ge \chi e^{-\lambda t^*} + \frac{\varepsilon}{\mu-\upsilon}$ and

$$v(t) < \chi e^{-\lambda t} + \frac{\varepsilon}{\mu - \upsilon}, \ t < t^*. \tag{26}$$

It is true from (24) and (26) that

$$\begin{aligned} v(t^*) \le& \chi e^{-\mu t^*} + \int_0^{t^*} e^{-\mu(t^*-s)}[\upsilon v(s-\tau(s)) + \varepsilon]ds \\ <& e^{-\mu t^*}\Big\{\chi + \frac{\varepsilon}{\mu-\upsilon} + \int_0^{t^*} e^{\mu s}[\upsilon(\chi e^{-\lambda(s-\tau(s))} \\ &+ \frac{\varepsilon}{\mu-\upsilon}) + \varepsilon]ds\Big\} \\ =& \chi e^{-\lambda t^*} + \frac{\varepsilon}{\mu-\upsilon}, \end{aligned} \tag{27}$$

which contradicts $v(t^*) \ge \chi e^{-\lambda t^*} + \frac{\varepsilon}{\mu-\upsilon}$. Therefore, (25) holds. Let $\varepsilon \to 0$ and from $\mathbb{E}V(t) \le v(t)$, $\mathbb{E}V(t) = \mathbb{E}\sum_{i=1}^N e_i^T(t)e_i(t) \le v(t) \le \chi e^{-\lambda t}, t \ge 0$ holds. Consequently, this completes the proof. ■

The following corollary is presented to show the tracking results under $\mathcal{R}_1$ and $\mathcal{R}_2(t_k)$.

Corollary 1. Under $\mathcal{D}(t_k) = \mathcal{R}_1$ or $\mathcal{D}(t_k) = \mathcal{R}_2(t_k)$, suppose that Assumptions 1 and 2 hold, the average impulsive interval of synchronizing impulses is not larger than $\check{T}_i, i = 1, 2, ..., q$ and the average impulsive interval of synchronizing impulses is not less than $\hat{T}_j, j = 1, 2, ..., r$. Then, the nonlinear stochastic networked multi-agent system in (1) with partial mixed impulses and unknown time-varying delays will globally exponentially track $s(t)$ in mean square, if the following inequality holds

$$\mu - \upsilon > 0,$$

where $\mu = -(\beta + \sum_{i=1}^{q} \frac{\ln \check{\Psi}_i}{\check{T}_i} + \sum_{j=1}^{r} \frac{\ln \hat{\Psi}_j}{\hat{T}_j})$, $\hat{\Psi}_j = (1+\epsilon_j)^2$ $(j = 1, ..., r)$, $\check{\Psi}_i = 1$ $(i = 1, ..., q)$, $\beta = 2d\lambda_{\max}(M) + \lambda_{\max}(A + A^T) + \lambda_{\max}(\Sigma_1^T\Sigma_1) + 2\sqrt{\lambda_{\max}(B^TB)}\phi_1 + \sqrt{\lambda_{\max}(C^TC)}\phi_2$, $\upsilon = \sqrt{\lambda_{\max}(C^TC)}\phi_2 + \lambda_{\max}(\Sigma_2^T\Sigma_2)$. Then, the nonlinear stochastic networked multi-agent system in (1) with partial mixed impulses can globally exponentially track $s(t)$.

Remark 6. From Theorem 1, it can be shown that under synchronizing impulses, an increasing of the portion of $\frac{l}{N}$ will make the conditions in Theorem 1 more feasible. Conversely, under desynchronizing impulses, if the portion of $\frac{l}{N}$ increases, the conditions presented in Theorem 1 will be more difficult to satisfy. One can also infer from Theorem 1 a relationship among the system's parameters: the portion of nodes subjected to impulsive effects, the average impulsive intervals and strengths of impulsive effects. It is also worth mentioning that it might be unrealistic to locate impulsive disturbances to minimize consensus errors. However, the derived results exhibit the most ideal environment for consensus if all the parameters are fixed except the determination of $\mathcal{D}(t_k)$. In addition, the results in Corollary 1 are a little bit conservative and perhaps one can utilize the idea from a *finite-horizon* interval viewpoint to present less conservative results, which will inevitably make the algorithm more complicated.

4 Simulations

In this section, one example is given to illustrate the effectiveness of the obtained theoretical results. We consider system (1), in which each node is to describe the dynamics of robotic arms [17]. The dynamics of robotic arms is composed of two states $x_i = [x_{i1}, x_{i2}]^T$, where x_{i1} is the angle and x_{i2} is the rotational velocity. According to [17], $\tilde{f}_1(x_i(t), t) = [x_{i2}, -\theta_1 \sin(x_{i1}) + \theta_2 u_{\text{IN}}]^T$, where u_{IN} is the input torque and θ_1, θ_2 are fixed parameters. Hence, $A =$

$C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\phi_1 = \theta_1$. Let $\theta_1 = 9.81 \times 0.5$ and $\theta_2 = 2$ with $u_{\mathrm{IN}} = 10\sin(50t)$. The noise intensity function is $\tilde{f}_3(x_i(t)) = 0.5x_i(t) + 0.5x_i(t-\tau(t))$ and therefore $\Sigma_1 = \Sigma_2 = 0.5$, where $\tau(t) = \frac{e^t}{e^t+1}$. The connection matrix is generated by a scale-free network with $N = 100$ nodes. The growth of the scale-free network starts from three nodes and no edges. At each step, a new node with two edges will be added to the existing network. Calculating $\lambda_{\max}(M)$, we obtain $\lambda_{\max}(M) = -0.0094$. The global coupling is $d = 0.1$. The following equation is used to measure the tracking errors with different kinds of impulses $E(t) = \frac{1}{N}\sum_{i=1}^{N}\sqrt{(x_i(t)-s(t))^2}$.

Here, we only consider one value of synchronizing impulse and one value of desynchronizing impulse for the sake of simplicity, i. e., $r = q = 1$. In order to show the advantages of TOD-like partial mixed impulses over the other two kinds of partial mixed impulses, the comparison result is shown in Fig. 1, when $l = 50$, $\check{T}_1 = 0.025$, $\eta_1 = -0.46$, $\epsilon_1 = 0.15$ and $\hat{T}_1 = 0.05$. For making a fair comparison, the instants of impulsive effects of the three kinds of partial mixed impulses are the same. The only difference here is to identify the nodes subjected to impulsive effects. We find that Theorem 1 can be satisfied under such kind of parameters. It is shown that the error state $E(t)$ of the network converges most quickly when TOD-like partial mixed impulses are used, since they can efficiently identify the most important nodes to be controlled and the least important nodes to be injected with impulsive disturbances. The fixed partial mixed impulses work better than the network without impulses, where both cases can not track the leader $s(t)$ successfully. The error state achieved by periodic partial mixed impulses converges much faster than the one achieved by fixed partial mixed impulses. The line of periodic partial mixed impulses is close to that of TOD-like partial mixed impulses. To sum up, TOD-like partial mixed impulses work best among the three kinds of impulses and periodic partial mixed impulses rank second. The reason is that the protocols in $\mathcal{R}_2$ and $\mathcal{R}_3$ can efficiently use synchronizing impulses and reduce the effects of desynchronizing impulses according to their scheduling mechanisms. TOD-like impulses work better than periodic impulses in that TOD-like impulses require more real-time information than periodic impulses.

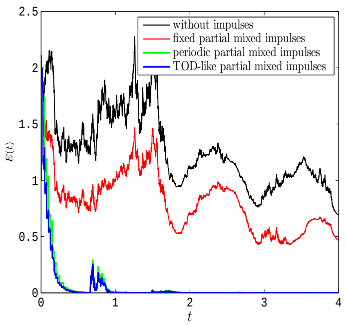

Fig. 1. Comparison of the three kinds of partial mixed impulses for system (1) when $\lambda_{\max}(M) = -0.0094$, $d = 0.1$, $l = 50$, $\check{T}_1 = 0.025$, $\eta_1 = -0.46$, $\epsilon_1 = 0.15$ and $\hat{T}_1 = 0.05$.

5 Conclusions

In this paper, the leader-following consensus problem has been investigated for a class of nonlinear stochastic networked multi-agent systems with partial mixed impulses and unknown time-varying delays. The main feature of partial mixed impulses is that the impulsive effects are time-varying and include synchronizing and desynchronizing impulses, in which the locations of the impulses are time-varying according to certain protocols. In order to characterize partial mixed impulses, we have proposed three types of impulses in the light of networked control systems, named as fixed partial mixed impulses, periodic partial mixed impulses and try-once-discard-like partial mixed impulses, respectively. Based on the stability theory, the impulsive control theory and stochastic analysis techniques, theoretical conditions have been derived for ensuring global exponential tracking control of nonlinear stochastic networked multi-agent systems with partial mixed impulses and unknown time-varying delays. A simulation example was provided to compare the three types of partial mixed impulses and to show the effectiveness of the obtained results. In the end, we give some valuable future topics. Firstly, it is necessary to extend the results to more general topologies. Secondly, it is interesting to reduce the conservativeness of the obtained results. Finally, it also remains promising to investigate tracking control with the proposed impulses and coupling delays.

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