Emergence of amplitude and oscillation death in identical coupled oscillators

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We deduce rigorous conditions for the onset of amplitude death (AD) and oscillation death (OD) in a system of identical coupled paradigmatic Stuart-Landau oscillators. A nonscalar coupling and high frequency are beneficial for the onset of AD. In strong contrast, scalar diffusive coupling and low intrinsic frequency are in favor of the emergence of OD. Our finding contributes to clearly distinguish intrinsic geneses for AD and OD, and further substantially corroborates that AD and OD are indeed two dynamically distinct oscillation quenching phenomena due to distinctly different mechanisms.

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Coupled nonlinear oscillators constitute an excellent framework to enhance our understanding on the emerging collective behaviors of several natural systems. Oscillation quenching is such a fundamental and ubiquitous emergent phenomenon naturally occurring in diverse fields including physics, biology, and engineering [1–3], which have been an active area of intense investigation recently [4,5]. The phenomenon of suppression of oscillations has been structurally distinguished into two different quenching processes, namely, amplitude death (AD) and oscillation death (OD) [5].

For many years, these two types of oscillation quenching were often misinterpreted in the literature. Only very recently, clear boundaries between AD and OD have been established from differences in their manifestations and applications [5,6]. AD refers to the cessation of oscillations by stabilizing an already existing unstable steady state of the uncoupled systems through coupling, resulting in a stable homogeneous steady state (HSS) [4]. Initially it was believed that frequency mismatch and time-delayed coupling were the two main sources for the onset of AD [7–14], but later studies revealed that AD can also be induced by dynamic and conjugate couplings [15–17]. OD appears due to the birth of a new set of stable fixed points under coupling, which disrupts the system's symmetry, resulting in a stable inhomogeneous steady state (IHSS) [18–21]. Thus the manifestations of AD and OD are clearly different, as AD retains symmetry [4], but the emergence of OD induces asymmetry through the birth of stable inhomogeneous steady states in coupled (even homogeneous) systems [5,6]. In addition, both AD and OD have distinct practical applications. For example, AD serves as a desirable control technique to suppress harmful oscillations in lasers [22], neuronal systems [23], or a healthy cell signaling network [24]. On the other hand, OD has strong implications in synthetic genetic networks [25–27] and is especially revealed as a background mechanism for cellular differentiation [28,29].

Extensive investigations have been carried out both theoretically and experimentally on AD for more than a couple of decades, whereas investigations on OD are comparatively less and are in their infancy. However, there has been a recent burst of research activities devoted to OD, since Koseska *et al.* clearly distinguished the transition from AD to OD via Turing bifurcation in nonidentical oscillators [6]. In particular, it has been shown that an AD–OD transition occurs in delay-coupled Stuart-Landau oscillators with a low degree of heterogeneity and in identical oscillators with dynamic and conjugate couplings [30]. Such a transition has been also identified in nonlinear oscillators with symmetry-breaking repulsive coupling [31,32] and mean-field diffusive coupling [33]. Furthermore, the transition from AD to OD has been experimentally observed in electronic circuits with mean-field coupling [34]. Very recently, an interesting connection between the chimera state and OD, termed as chimera death, has been established in a network of Stuart-Landau oscillators with one-dimensional nonlocal coupling [35].

To robustly observe both AD and OD in a system of coupled identical nonlinear oscillators, the following basic questions remain as open problems. Under what circumstances are both AD and OD robustly observed in a given system of coupled oscillators? Can the known scenarios that can induce AD favor the onset of OD or not? Are the underlying dynamical mechanisms different or the same? In this paper, we address the above challenging problems by deducing rigorous conditions under which both AD and OD can be observed in a given system of coupled identical nonlinear oscillators. In particular, we find that both AD and OD are observed in quite strong contrast conditions and thereby different mechanisms. Specifically, one-dimensional diffusive coupling is in favor of OD, but is adverse to AD; and a low intrinsic frequency facilitates OD but inhibits AD.

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We consider the following system of two Stuart-Landau oscillators [1] with diffusive coupling:

$$\dot{x}_j = P_j x_j - w y_j + K[x_k(t-\tau) - x_j(t)],$$
$$\dot{y}_j = w x_j + P_j y_j + \alpha K[y_k(t-\tau) - y_j(t)], \quad (1)$$

where $P_j = 1 - x_j^2 - y_j^2$, $j,k = 1,2$, and $j \neq k$. The value of K ($K \geqslant 0$) governs the diffusive coupling strength, and τ ($\tau \geqslant 0$) is the propagation time delay. The parameter α ($0 \leqslant \alpha \leqslant 1$) determines the nature of the coupling rate between the x and y components, where $\alpha = 1$ corresponds to a nonscalar coupling and $\alpha = 0$ to a scalar one [7]. Systems of coupled Stuart-Landau oscillators serve as a paradigmatic model for exploring AD by both analytical and numerical means [4]. However, this model has been only recently employed to reveal the OD phenomenon. The Stuart-Landau oscillator represents a normal form near a supercritical Hopf bifurcation. It has a stable limit-cycle motion $Z = x + iy = e^{iwt}$ and an unstable focus at the origin $Z = 0$. In the Cartesian coordinates, it is given by $\dot{x} = Px - wy$ and $\dot{y} = wx + Py$, where $P = 1 - |Z|^2 = 1 - x^2 - y^2$.

For $\alpha = 1$, Reddy *et al.* [9] have proved that the coupled system (1) experiences AD only for $\tau > 0$. For $\alpha = 0$, Koseska *et al.* [6] recently showed that OD is observed in the coupled system (1) even for $\tau = 0$. The presence of time delay $\tau > 0$ modulates the threshold value of K for the onset of OD, and even renders both AD and OD to occur in the same system for a large value of the frequency w when $\alpha = 0$ [30]. Clearly, there exists a gap in the study of AD and OD in the coupled system (1) from scalar ($\alpha = 0$) to nonscalar ($\alpha = 1$) couplings. In the following, we unveil rigorous conditions for the emergence of both AD and OD in system (1) of coupled identical oscillators.

The fixed points of Eq. (1) can be obtained as (i) a trivial HSS at the origin (0,0,0,0) and (ii) an IHSS (x_1^*, y_1^*,$-x_1^*$, $-y_1^*$) with $x_1^* = \pm w\sqrt{(1-p)/[(p-2K)^2 + w^2]}$ and $y_1^* = \mp\sqrt{[(1-p)(p-2K)^2]/[(p-2K)^2 + w^2]}$, where $p = (1+\alpha)K - \sqrt{(1-\alpha)^2K^2 - w^2}$. The onset conditions for AD and OD are determined by the characteristic eigenvalue equations obtained from the standard linear stability analysis of the coupled system at its HSS and IHSS, respectively.

It is noteworthy that the presence of a time delay $\tau > 0$ in the coupling does not generate any new steady-state solutions of coupled system (1), but rather influences the stability of the steady states. We first treat the case of $\tau = 0$. If $\alpha = 0$, it was established by the authors of Ref. [6] that an IHSS appears at $K = (w^2 + 1)/2$ via a pitchfork bifurcation and is stabilized for $K > w^2 + 1/4$, i.e., OD occurs. When $\alpha > 0$, we deduce that the IHSS emerges in an intermediate interval of K: $(1+\alpha-\sqrt{(1-\alpha)^2 - 4\alpha w^2})/(4\alpha) < K < (1+\alpha+\sqrt{(1-\alpha)^2 - 4\alpha w^2})/(4\alpha)$ if $w^2 < (1-\alpha)^2/(4\alpha)$, which analytically defines the emergence conditions for IHSS. Such an IHSS is stable for $(1+\alpha-\sqrt{(1-\alpha)^2 - 16\alpha w^2})/(8\alpha) < K < (1+\alpha+\sqrt{(1-\alpha)^2 - 16\alpha w^2})/(8\alpha)$ if $w^2 < (1-\alpha)^2/(16\alpha)$. This stable OD interval vanishes for $\alpha > \alpha_{\max} = 1 + 8w^2 - \sqrt{(1+8w^2)^2 - 1}$. To validate the above analysis, Figs. 1(a) to 1(d) show the bifurcation diagrams of the steady states of the coupled system (1) with $w = 10$, where the thick (red) lines denote the stable steady states and the thin (black) lines correspond to the unstable steady states. The OD is stabilized in a broad range of K for $K > 100.25$ and $\alpha = 0$ as illustrated in Fig. 1(a), whereas OD occurs only in a limited interval of coupling strength for $\alpha = 0.005$ [Fig. 1(b)]. The stable IHSS (OD) disappears if $\alpha > \alpha_{\max} \approx 0.0006$ [see Figs. 1(c) and 1(d) for $\alpha = 0.001$ and 0.002]. Note that HSS is always unstable for all $0 \leqslant \alpha \leqslant 1$, implying that AD is impossible in identical coupled oscillators with $\tau = 0$.

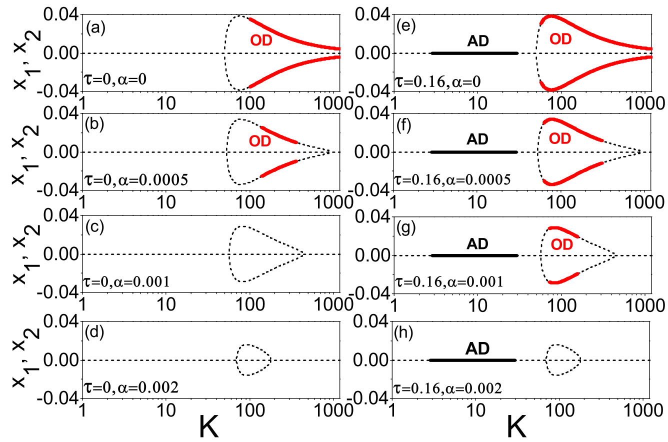

FIG. 1. (Color online) Bifurcation diagrams of steady states of the coupled system (1) with instantaneous coupling [left column (a)–(d) with $\tau = 0$] and time-delayed coupling [right column (e)–(h) with $\tau = 0.16$] for $\alpha = 0, 0.0005, 0.001$, and 0.002, respectively. $w = 10$. Thin dashed lines correspond to unstable steady states, whereas solid black (dark gray) and solid red (light gray) lines to stable HSS (AD) and stable IHSS (OD), respectively.

The bifurcation diagrams of the steady states of the coupled system (1) are depicted in Figs 1(e) to 1(h) in the presence of the propagation delay $\tau = 0.16$, $w = 10$, and for the corresponding values of α as in Figs. 1(a) to 1(d), respectively. It is evident from these figures that AD always occurs in a finite interval of the coupling strength for all $0 \leqslant \alpha \leqslant 1$.

It is to be noted that time-delayed coupling stabilizes IHSS (OD) with a much lower threshold value of the coupling strength at $K_c = 57.544 \ll w^2 + 1/4$ for $\alpha = 0$ [comparing Figs. 1(a) and 1(e)], and facilitates OD in a much larger range of coupling strength for $\alpha = 0.0005$ [comparing Figs. 1(e) and 1(f) to Figs. 1(a) and 1(b)]. More interestingly, the propagation delay can induce OD in a pronounced interval of the coupling strength for certain values of α [see Fig. 1(g) with $\alpha = 0.001$], where the IHSS is unstable without the propagation delay for the same value of α [see Fig. 1(c) with $\alpha = 0.001$]. However, if the coupling rate α is beyond a certain value $\alpha_{\max}$, the IHSS becomes unstable and OD is impossible even in the presence of time delay in the coupling as illustrated in Fig. 1(h) for $\alpha = 0.002$. Note that both AD and OD are found in the same system (1) of coupled identical oscillators in the presence of the propagation delay as shown in Figs. 1(e) to 1(g) for $\tau = 0.16$, whereas only OD is observed in the absence of the propagation delay as depicted in Figs. 1(a) to 1(b).

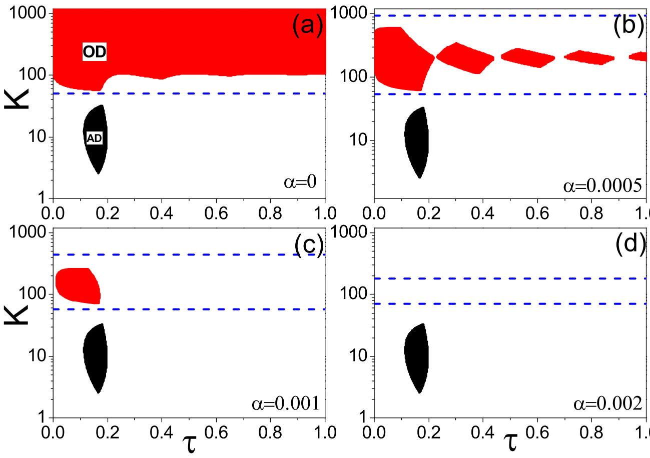

FIG. 2. (Color online) AD and OD regimes of the coupled system (1) in the parameter space of (τ, K) for different coupling rates α: (a) $\alpha = 0$, (b) $\alpha = 0.0005$, (c) $\alpha = 0.001$, (d) $\alpha = 0.002$. $w = 10$. The black (dark gray) and red (light gray) regions represent the stable HSS (AD) and stable IHSS (OD), respectively. The dashed blue lines indicate the limit of K for the emergence of IHSS.

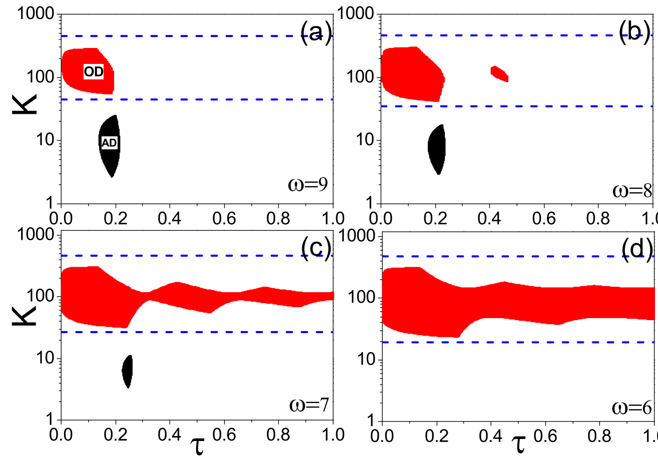

FIG. 3. (Color online) AD and OD regimes of a system of coupled identical oscillators (1) with $\alpha = 0.001$ on the (τ, K) plane for different values of the frequency w: (a) $w = 9$, (b) $w = 8$, (c) $w = 7$, and (d) $w = 6$. The black (dark gray) and red (light gray) regions represent the stable HSS (AD) and stable IHSS (OD), respectively.

To get a complete picture of the effect of propagation delay on AD and OD in the coupled system (1), Figs. 2(a) to 2(d) depict the regimes of stable HSS and IHSS in the parameter space of (τ, K) for $\alpha = 0, 0.0005, 0.001$, and 0.002, respectively, for the same value of w as in Fig. 1. The black (dark gray) and red (light gray) regions indicate the stable HSS (AD) and IHSS (OD), respectively. The dashed blue lines demarcates the boundary of K for the emergence of IHSS for a given α. For $\alpha = 0$, the propagation delay modulates the threshold value of K for the onset of OD and the OD regime is unbounded and connected along both directions of τ and K. To our surprise, upon increasing α meticulously, the OD region splits into several unconnected and bounded islands as shown in Fig. 2(b) for $\alpha = 0.0005$. Increasing α further results in decrease in the number of these OD islands. For example, only a single island of stable IHSSs is found in Fig. 2(c) for $\tau > 0$ and $\alpha = 0.001$. The OD island is completely revoked for α beyond a certain threshold $\alpha_{\max} \approx 0.0016$, which is confirmed in Fig. 2(d) for $\alpha = 0.002$, where the IHSSs are unstable in the entire parameter space.

Thus it is established that increasing α eliminates the onset of OD. This can be intuitively understood in the following way: OD is induced through symmetry breaking of the coupled dynamical system. When $\alpha = 1$, the coupling perfectly preserves the system's rotating symmetry, while for $\alpha = 0$, it clearly destroys the rotating symmetry which is the mechanism for the emergence of IHSS [35]. Hence, it is clear that increasing α from zero is detrimental for the onset of OD.

In strong contrast, we clearly observe that the parametric sets of AD constitute pronounced islands on the (τ, k) plane in Fig. 2. In fact, the AD island increases gradually as α is increased. This is due to that the time-delayed contributions of the coupled system are enhanced when α is increased. Hence, a large value of α is beneficial for facilitating the occurrence of AD in a large parameter space. The coupling rate α plays two completely contrary roles in the formation of AD and OD in the coupled identical system (1).

For the nonscalar coupling case ($\alpha = 1$), Reddy *et al.* demonstrated that the size of the AD island decreases with decreasing w, which vanishes below the threshold of $w_{\min} \approx 4.812$ [9] and more than one AD island appear for high values of $w > 14.438$ [10]. At this point, it is natural to wonder if the effect of w on OD and whether multiple OD islands can be formed in the parameter space of (τ, K) by tuning w. The answer to this question is depicted in Figs. 3(a) to 3(d), displaying the distribution of AD and OD regions on the (τ, K) plane for $w = 9$, 8, 7, and 6, respectively, for fixed $\alpha = 0.001$. Surprisingly, only a single OD island appears for $w = 9$ [Fig. 3(a)], but two OD islands are found for a slightly lower value of $w = 8$ [Fig. 3(b)]. Further decreasing w, multiple OD islands emerge and become a single connected region, which is unbounded along the τ direction as illustrated in Figs. 3(c) and 3(d) for $w = 7$ and 6. This effect is more pronounced for much smaller w. However, the corresponding AD island monotonically decreases [see Figs. 3(a) to 3(c)] and disappears below a certain threshold $w_{\min}$ [Fig. 3(d)]. Thus, AD and OD depend on the frequency w in quite different ways: low frequencies restrain AD but facilitate OD.

From the above analyses, it is evident that the stability regions of both AD and OD critically depend on the values of both α and w. There exists a threshold value $\alpha_{\max}$ such that if $\alpha > \alpha_{\max}$ OD does not occur for any combinations of τ and K for a fixed w [refer to Fig. 2]. For all $0 \leqslant \alpha \leqslant 1$, there exists a minimum value $w_{\min}$, below which $w < w_{\min}$ AD is impossible on the (τ, K) plane, as confirmed in Fig. 3. For a global picture, the dependence of $w_{\min}$ on α for AD [black triangles] and $\alpha_{\max}$ on w for OD [red squares] is shown in Fig. 4. The larger α, the smaller $w_{\min}$ is for AD; and the smaller w, the larger $\alpha_{\max}$ is for OD. Interestingly, we find that the coupled system (1) can indeed experience both AD and OD for a certain parametric set of (τ, K) when α is sufficiently small and w is large enough [the upper-left corner of Fig. 4].

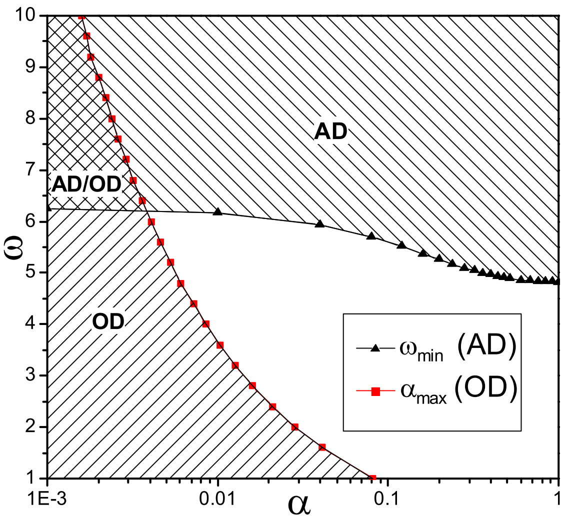

FIG. 4. (Color online) Minimum value $w_{\rm min}$ (black triangles) of w for AD and maximum value $\alpha_{\rm max}$ (red squares) of α for OD. Both AD and OD occur in the same system of coupled identical oscillators when α and w are located in the upper-left corner.

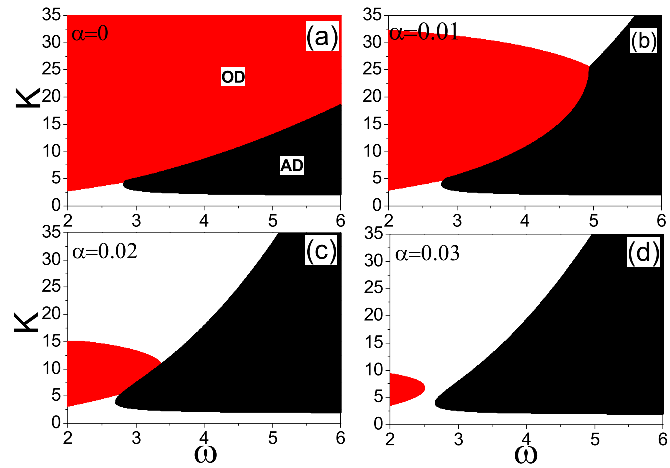

FIG. 5. (Color online) AD and OD of the dynamically coupled system (2) in the parameter space of (w,K) for different coupling rates α: (a) $\alpha = 0$, (b) $\alpha = 0.01$, (c) $\alpha = 0.02$, (d) $\alpha = 0.03$. The black (dark gray) and red (light gray) regions represent the stable HSS (AD) and stable IHSS (OD), respectively.

The above intricate dependencies of w and α for the emergence of AD and OD are generically valid in a system of identical oscillators with other coupling scenarios that are known to induce AD. To verify this, we illustrate our results in two identical Stuart-Landau oscillators with dynamic coupling represented as

$$\begin{aligned} \dot{x}_j &= P_j x_j - w y_j + K(u_j - x_j), \\ \dot{y}_j &= w x_j + P_j y_j + \alpha K(v_j - y_j), \\ \dot{u}_j &= -u_j + x_k, \\ \dot{v}_j &= -v_j + y_k. \end{aligned} \tag{2}$$

It is notable that the fixed points of dynamically coupled system (2) are the same as those of the delay-coupled system (1). The coupled system (2) with $\alpha = 0$ was investigated previously in Refs. [16,30]. Only AD has been extensively investigated in Ref. [16], while the transition from AD to OD has been reported in Ref. [30]. However, the intricate dependencies of w and α for the onset of AD and OD was not yet considered. Now, by introducing α, we elucidate the influences of both w and α on AD and OD. Figures 5(a) to 5(d) illustrate AD and OD regimes in the parameter space of (w,K) for $\alpha = 0$, 0.01, 0.02, and 0.03, respectively. Transitions from AD to OD as the coupling strength K increases can be observed for small values of α in a wide range of w [Figs. 5(a) to 5(c)]. It is clear from these figures that upon decreasing w, the spread of the coupling strength K for stable IHSSs (OD) increases, whereas that of stable HSS (AD) decreases. Upon increasing α, the spread of the OD region shrinks but that of the AD region enlarges in the (w,K) space. Therefore, a small w is in favor of OD but is detrimental to AD; a large α enhances AD but inhibits OD, which are the same effects as observed in the delay-coupled system (1).

To conclude, we have systematically explored the emergence of both AD and OD in a single system of coupled identical Stuart-Landau oscillators, with both time-delayed and dynamic couplings. By tuning the coupling rate α from 0 to 1, the coupling transits from scalar to nonscalar type. We found that large α is conducive to the occurrence of AD, but is adverse to the onset of OD. Generally, OD is favored for very small values of $\alpha \ll 1$. The parametric region of OD monotonically decreases by increasing α from zero, and completely vanishes if $\alpha > \alpha_{\rm max}$. The effects of the intrinsic frequency w on AD and OD are also examined meticulously. It is uncovered that larger values of w facilitates the spread of AD to a larger region when $w > w_{\rm min}$. The minimum value of w ($w_{\rm min}$) increases as decreasing α from 1. In sharp contrast to this scenario, a smaller value of w facilitates OD to occur in a larger domain of parameters, which also results in a larger threshold value for $\alpha_{\rm max}$. It is also established that the same system of coupled identical oscillators can produce both AD and OD for sufficiently small values of α and large enough values of w. Our results further corroborate that AD and OD are indeed two dynamically distinct oscillation quenching phenomena with significantly different dynamical mechanisms. It is to be noted that we have carried out our analysis by employing the paradigmatic Stuart-Landau oscillator, which represents a normal form describing dynamics near a supercritical Hopf bifurcation. Thus, our findings are expected to essentially characterize generic features of coupled systems near Hopf bifurcation. Finally, we firmly believe that our study serves as a genesis and a benchmark for future investigations of AD and OD in more complex systems, in particular, in electronic circuits, lasers, and neuronal networks.

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