On Controllability of Neuronal Networks With Constraints on the Average of Control Gains

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***Abstract*—Control gains play an important role in the control of a natural or a technical system since they reflect how much resource is required to optimize a certain control objective. This paper is concerned with the controllability of neuronal networks with constraints on the average value of the control gains injected in driver nodes, which are in accordance with engineering and biological backgrounds. In order to deal with the constraints on control gains, the controllability problem is transformed into a constrained optimization problem (COP). The introduction of the constraints on the control gains unavoidably leads to substantial difficulty in finding feasible as well as refining solutions. As such, a modified dynamic hybrid framework (MDyHF) is developed to solve this COP, based on an adaptive differential evolution and the concept of Pareto dominance. By comparing with statistical methods and several recently reported constrained optimization evolutionary algorithms (COEAs), we show that our proposed MDyHF is competitive and promising in studying the controllability of neuronal networks. Based on the MDyHF, we proceed to show the controlling regions under different levels of constraints. It is revealed that we should allocate the control gains economically when strong constraints are considered. In addition, it is found that as the constraints become more restrictive, the driver nodes are more likely to be selected from the nodes with a large degree. The results and methods presented in this paper will provide useful insights into developing new techniques to control a realistic complex network efficiently.**

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I. Introduction

THE PAST one decade has witnessed a tremendous upsurge in the research interest toward theoretical modeling, analysis, and application of complex networks from a variety of research communities [1]. The main reason lies in the fact that complex networks can describe many practical systems such as genetic networks, social networks, sensor networks, neuronal networks, electronic networks, or transportation networks. Among them, the modeling of neuronal networks of a brain can be viewed as a typical application of complex networks [2], [3]. In [4]–[6], the theoretical modeling, the tackling learning of neuronal networks, and the applications of neuronal networks to image processing have been investigated, respectively. Modern brain-mapping approaches, such as diffusion magnetic resonance imaging (MRI), functional MRI, electroencephalography, and magnetoencephalography, have constantly produced large datasets of anatomical and functional connection patterns. Complex network theory has been used to describe important properties of large connection datasets by quantifying structures of their respective network representations. It has been widely recognized that network characterization of structural and functional connectivity data of brain has attracted increasing attention due to its reliability and effectiveness [2]. Recently, the existence has been revealed with respect to the communities, hierarchy, centrality, and distribution of cortical hubs in anatomical connectivity of the mammalian brain [3], [7]–[10].

Complex networks, especially neuronal networks, have been investigated in the context of dynamical systems and have already become an interdisciplinary research area for mathematicians, computer scientists, and biologists to interpret functional information and explore network robustness and vulnerability, which are likely to become increasingly relevant in relation to neuroscience, physics, and engineering. Recently, as an emerging phenomenon of neuroscience and multiagent systems, synchronization has gained particular research attention for complex networks (neuronal networks) in various fields [11]–[15]. Synchronization of distributed brain activity has been revealed to serve a central role in high-level neural information processing [16]. Experimentally observed evidence has asserted that certain brain disorders, such as schizophrenia, epilepsy, autism, Alzheimer's disease, and Parkinson's disease, are highly relevant to abnormal neural synchronization [17]. In [7] and [18], synchronization in the cortical brain network of the cat is investigated by modeling each node (cortical area) with a subnetwork of interacting excitable neurons. In [15], the distributed synchronization problem is investigated for networks of agent systems with nonlinearities and controller failure subject to Bernoulli switchings and conditions that are given in terms of a semidefinite programming problem.

On another research frontier, controllability of complex networks has received considerable attention in the past 10 years. Controllability of complex networks can be referred to a set of nodes that are regarded as driver nodes/references and are used to control the dynamics of entire networks to a desired state, which is required for an engineering, medical, or biological purpose [19]–[24]. In particular, as illustrated in [24], the importance of investigating controllability of neuronal networks will not only help us to elucidate how to control an intricate system efficiently but also be beneficial to understand the processing of high-level information in brains [7] and dynamical properties of neuronal networks [10]. Recently, a variety of works have been proposed to realize pinning control or detection of controlling regions in complex networks. In addition to the mathematical methods for studying the controllability of complex networks, some efforts on choosing key nodes by utilizing evolutionary methods have been made [13], [22]–[24]. The problem of pinning control of complex networks has been converted into an unconstrained problem [11], a multiobjective problem [23], and a constrained problem [24], respectively. In particular, two measures of controllability of neuronal networks have been incorporated into one unified framework [24], where the more important measure is regarded as an objective and the other one is viewed as a constraint.

It should be noted that, up to now, almost all research efforts on controllability (pinning control) of complex networks have been devoted to the case of choosing effective nodes to control the entire network. However, in reality, constraints on control gains should be taken into account. The importance of such considerations resides in twofold. 1) The first one is from the constraint on implementation of engineering equipment and biological background. Saturation in actuator exists widely in practical control systems since a physical actuator can only generate bounded signals, and the control of plants with actuator saturations is also challenging [25]. 2) The second one is that only a suitable control input could result in an ideal control performance. For example, in therapy, the patient's recovery is closely related to the dosage of antibiotics, where the input of dosage can be viewed as control gains. The excessive injection of dosage of drugs will result in the creation of multidrug-resistant bacteria and finally no efficient antibiotics are available in some severe cases [26], [27]. Misuse of antibiotics can also destroy the beneficial bacteria and can cause immune system disorders in the human body. On the other hand, a small injection of dosage will not be conducive to patients' recovery and will prolong the recovery time of patients. Therefore, a dosage should be injected at an appropriate level that would work on the infected cells and would not upset the normal mechanism.

For the sake of simplicity, in [21] and [24], the controllability of complex networks and neuronal networks has been investigated, respectively, where the intrinsic constraint on the control gains on the dynamics of networks has been overlooked and only the boundary of the control gains is discussed. Despite the fact that many phenomena in nature are closely related to the constraints on control gains on controllability of complex networks, the gain constraint issue has, unfortunately, been largely neglected in the area primarily due to the complexity in optimizing and tackling the existence of gain constraints. It is, therefore, the main purpose of this paper to investigate how much the controllability of weighted and directed neuronal networks is affected in the presence of the constraints on control gains, which aims to improve our recent work [24] and does not consider the importance of gain constraints.

In this paper, we aim to make the first attempt to address the controllability of neuronal networks with several constraints on control gains. Such a controllability issue is later converted into a constrained optimization problem (COP). Due to the nature of combinatorial optimization problems in selecting controlling nodes, constrained optimization evolutionary algorithms (COEAs) [28]–[30] are promising candidates to solve this COP. COEAs are composed of two major parts: a search technique and a constraint-handling scheme. Their performance rests largely on these two components. The constraint-handling scheme can be categorized into several classes [29]. In addition, it is important to develop an effective search algorithm to refine solutions that can find global optimal solutions for COPs. Recently, multiobjective optimization-based constraint-handling schemes are used to tackle COPs, together with differential evolution (DE) due to its prospect and potential [31], [32]. Nonetheless, the search performance of these methods can still be further improved by introducing adaptive mechanisms in DEs.

In this paper, the controllability of neuronal networks with constraints on control gains is investigated. By adding an adaptive differential evolution (JaDE) [33] into a search scheme of a dynamic hybrid framework (DyHF), a modified dynamic hybrid framework (MDyHF) is proposed here to study controllability of neuronal networks. Compared with [24], the main contributions of this paper are threefold: 1) the average value of the control gains is considered as a constraint and the controllability of neuronal networks with the constraints on the control gains is then transformed into a COP; 2) based on an adaptive DE and Pareto dominance, a MDyHF is proposed to show its competitive performance by several experiments; and 3) the controlling regions of the neuronal network are identified by the proposed MDyHF and the relationship between controllability and control gains is presented, which will interpret the mechanism of controlling natural systems.

The organization of this paper is arranged as follows. Section II presents some preliminaries and problem formulation briefly. The MDyHF is presented in Section III. In

Section IV, comparisons and results are provided. Conclusions are given in Section V.

II. Problem Formulation

In this section, some notations, preliminaries, and problem formulation are provided.

A. Notations

Throughout this paper, let a graph be $\mathcal{G} = [\mathcal{V}, \mathcal{E}]$, where $\mathcal{V} = \{1, \cdots, N\}$ denotes the vertex set and $\mathcal{E} = \{e(i, j)\}$ stands for the edge set. The graph $\mathcal{G}$ is directed, weighted, and simple (without self-loops and multiple edges). Let $G = [g_{ij}]_{i,j=1}^{N}$ be the adjacency matrix of neuronal network of cat's brain $\mathcal{G}$, which is defined as follows: for any pair $i \neq j, g_{ij} < 0$ if $e(i, j) \in \mathcal{E}$; otherwise, $g_{ij} = 0$. $l \in [1, N]$ is the number of driver nodes of a network. $\phi_{\mathcal{P}}(\cdot)$ represents the characteristic function of the set $\mathcal{P}$, that is, $\phi_{\mathcal{P}}(i) = 1$ if $i \in \mathcal{P}$; otherwise, $\phi_{\mathcal{P}}(i) = 0$. Here, $\mathcal{P}$ denotes the set of nodes injected with controllers. $g_{ii} = -\sum_{j=1,j\neq i}^{N} g_{ij}$ $(i = 1, 2, \cdots, N)$. The adjacency matrix G can be converted into the Laplacian matrix L by neglecting the weights over the network. For any pair $i \neq j, l_{ij} = -1$ if $e(i, j) \in \mathcal{E}$; otherwise, $l_{ij} = 0$. $l_{ii} = -\sum_{j=1,j\neq i}^{N} l_{ij}$, $(i = 1, 2, \cdots, N)$.

B. Controllability of the Neuronal Network

Hereafter, a desired state can be described as follows:

$$\frac{ds(t)}{dt} = F(s(t)).$$

This differential equation is widely used to represent extensive real-world natural and technological systems.

In order to control the states of the neuronal network to the reference evolution $s(t)$, the dynamics of the neuronal network with output feedback controllers can be written as

$$\begin{aligned} \frac{dx_i(t)}{dt} &= F(x_i, t) - C\sum_{j=1}^{N} g_{ij}H(x_j(t)) \\ &\quad - C\phi_{\mathcal{P}}(i)\kappa_i(H(s(t)) - H(x_i(t))) \\ & i = 1, \cdots, N \end{aligned} \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \cdots, x_{id}(t)]^T \in \mathbb{R}^d$ $(i = 1, 2 \cdots, N)$ is the state vector of the ith node/brain region/cortical area, $F(x_i, t) = [F_1(x_i, t), \cdots, F_d(x_i, t)]^T$ is a continuous vector function, and $H(x_i(t))$ is the coupling continuous function. C is the global coupling gain of the neuronal network. Let $\mu_p = \mu_p^r + j\mu_p^m (j = \sqrt{-1}), (p = 1, 2, \cdots, N)$, be the set of eigenvalues of G that are sorted in such a way that $\mu_1^r \leq \mu_2^r \leq \cdots \leq \mu_N^r$. $\kappa_i, i \in \mathcal{P}$ is the control gain injected in driver nodes, where $\mathcal{P}$ denotes the set of nodes injected with controllers, that is, $\mathcal{P}$ contains the set of driver nodes. Apparently, $1 \leq \sum_{i=1}^{N} \phi_{\mathcal{P}}(i) \leq N$. The objective of controllability is to regulate the states of the neuronal network (1) toward the desired reference state $s(t)$, that is, $x_1(t) = x_2(t) = \cdots = x_N(t) = s(t)$.

For a demonstration purpose, we use cortical network as an example. Here, $G = [g_{ij}]_{i,j=1}^{N}$ is the adjacency matrix of neuronal network of cat's brain, where nodes usually represent brain regions with coherent patterns of extrinsic anatomical or functional connections, while links stand for anatomical, functional, or effective connections [34], [35] and are differentiated on the basis of their weight and directionality. Here, the version of a dataset presented in [35] and [36] is used. The construction of connection matrix and its analysis was provided in [35], which is extracted from several subtle steps including cortical parcellation, thalamic parcellation, collation of connection data, and translation from database to connection matrix. For further details regarding the construction of connection matrix, please refer to [35] and references therein.

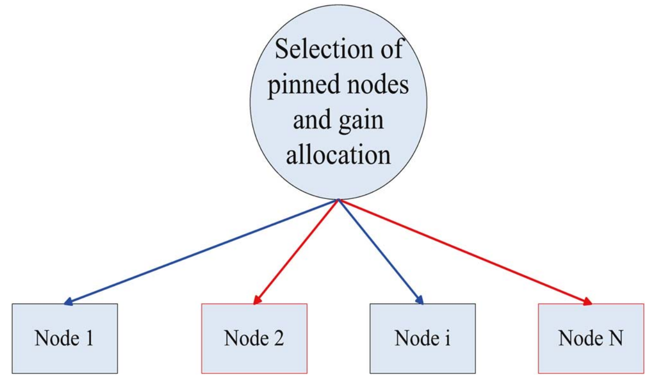

Fig. 1. Gain allocation of networks. Red rectangle and red line mean that the node is selected and gains should be allocated in $\mathcal{P}$; dark rectangle and blue line mean that the node is not selected and gains will not be allocated.

Following the way in [21], the extended network of $N + 1$ dynamical systems z_i is considered, where $z_i = x_i$ for $i = 1, 2, \ldots, N$ and $z_{N+1} = s(t)$. Hence, (1) is

$$\begin{aligned} \frac{dz_i(t)}{dt} &= F(z_i, t) - C\sum_{j=1}^{N+1} \mathcal{W}_{ij}H(z_j(t)) \\ & i = 1, \cdots, N + 1 \end{aligned} \tag{2}$$

where $\mathcal{H} = [\mathcal{W}_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ in the form of

$$\mathcal{H} = \begin{pmatrix} \mathcal{D}_1 & g_{12} & \ldots & g_{1N} & -\phi_{\mathcal{P}}(1)\kappa_1 \\ g_{21} & \mathcal{D}_2 & \ldots & g_{2N} & -\phi_{\mathcal{P}}(2)\kappa_2 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ g_{N1} & g_{N2} & \ldots & \mathcal{D}_N & -\phi_{\mathcal{P}}(N)\kappa_N \\ 0 & 0 & \ldots & 0 & 0 \end{pmatrix} \tag{3}$$

in which $\mathcal{D}_i = g_{ii} + \phi_{\mathcal{P}}(i)\kappa_i$. Let $\lambda_p = \lambda_p^r + j\lambda_p^m$ be the pth eigenvalue of $\mathcal{H}$ and suppose that λ_p is sorted as $\lambda_1^r \leq \lambda_2^r \leq \cdots \leq \lambda_{N+1}^r$, where $\lambda_1^r = 0$.

The controllability can be measured in terms of

$$R = \frac{\lambda_{N+1}^r}{\lambda_2^r}$$

and

$$\delta = \max_p\{\lambda_p^m\}.$$

In order to enhance controllability of neuronal networks, we should minimize R and δ as much as possible [21], [37].

C. Incorporation of the Constraints on the Control Gains in Controllability of Neuronal Networks

As stated in [25] and [38], the generation of signals is saturated in realistic networked control systems and artificial neural networks. Usually, in neural networks, the original network utilizes multiple layers of weight-sum units of the type $\mathscr{N} = h(w^T x + b)$, where $h(.)$ is a sigmoid function or logistic function to be bounded [38]. Also, in biological meaning [26], [27], although antibiotics are required to treat severe bacterial infections, misuse will give rise to bacterial resistance, and thereby inhibiting the treatment. Inadequate antibiotics will prolong the recovery of the patients. Therefore, the consideration of constraints on control gains is very important from the view of engineering and biology, as seen from Fig. 1. The control systems under saturation have been investigated in model predictive control [39] and networked control systems [25], [40], [41]. It should be noted that, in all the references mentioned earlier, control gains under consideration are assumed to be bounded, which would largely neglect the typical restrictions in applications. In this paper, we investigate the controllability with the constraints on the control gains in detail. Not only the case of control gains being bounded is studied but also the average of the control gains under a constraint is investigated.

In this paper, $\mathcal{K}$ is used to denote the average of the control gains, which is formulated as follows:

$$\mathcal{K} = \mathrm{mean}(\kappa_i), (i \in \mathcal{P}) \tag{4}$$

where mean(.) is the mean value operator. We convert the problem of controllability of a neuronal network into a COP, where R is the objective to be minimized and both δ and $\mathcal{K}$ are the constraints.

Remark 1: In previous works, the value of δ was usually ignored in measuring synchronizability and controllability of networks [42], since δ is small compared with R in most of coupling graphs. However, δ cannot be overlooked in some special cases, such as normalized Laplacian matrix or the number of driver nodes l is large, where the value of δ is comparable to that of R. Hence, the assumption of neglecting δ will inevitably cause conservativeness and cannot reflect actual controllability of networks. In [24], we combine these two measures into a unified framework to investigate controllability of networks, in which R is viewed as an objective and δ is regarded as a constraint. Although [24] presents a unified framework to include R and δ, it is worth mentioning that the role of the control gain κ_i has not been addressed despite its great importance in therapy, system biology, engineering, and nonlinear science, as mentioned in Sections I and II-C. Therefore, compared with [24], one main purpose of this paper is to investigate effects of the average of the control gains $\mathcal{K}$ on controllability. In addition, we also compare the newly proposed method with the improved dynamic hybrid framework (IDyHF) in [24] to show the advantage of the proposed method.

Remark 2: The problem of resource allocation widely exists in medicine and engineering. For example, in [43], the authors investigate the resource allocation in sensor networks, that is, how to allocate limited energy, radio bandwidth, and other resources to optimize the value of each node's contribution to the entire network. In [44], scarcity of resources, such as drugs, equipment, or time, make it difficult to supply the full measure of service and devotion. When circumstances of scarcity occur, it is necessary to face up to the tradeoffs in a fair and compassionate manner. In this paper, controllability of neuronal networks is investigated by taking both the boundary and entire costs of the control gains into account simultaneously.

D. Problem Transformation Into a COP

In the following, some preliminaries of the COP are given. The COP is formulated as follows: find the decision variables $y = (y_1, \cdots, y_D) \in \mathbb{R}^D$ to minimize the objective function

$$\min \quad f_j(y),\ y \in \Omega \subseteq \mathcal{S}$$

where Ω is the feasible region and $\mathcal{S}$ is the decision space defined by the parametric constraints $\bar{Y}_i \leq y_i \leq \bar{Z}_i, i = 1, 2, \cdots, D$. The decision variables y should satisfy $\bar{s}$ constraints including u inequality constraints

$$q_j(y) \leq 0, j = 1, 2, \cdots, u$$

and $\iota = \bar{s} - u$ equality constraints

$$h_j(y) = 0, j = u + 1, 2, \cdots, \bar{s}.$$

The degree of constraint violation of a vector y on the jth constraint is defined as

$$M_j(y) = \begin{cases} \max\{0, q_j(y)\}, & 1 \leq j \leq u \\ \max\{0, |h_j(y)|\}, & u + 1 \leq j \leq \bar{s}. \end{cases} \tag{5}$$

Then

$$\Psi(y) = \sum_{j=1}^{\bar{s}} M_j(y) \tag{6}$$

reflects the degree of constraint violation of the vector y.

In the following, we consider R as an objective and δ as a constraint. This formulation is based on three aspects.

1) *Generality*: R exists in both directed and undirected networks while δ is only observed in directed networks. Hence, our formulation can also easily be extended to undirected networks by removing the constraint on δ.
2) *Importance*: R plays a more important role than δ in most real-world networks [21], since R is larger than δ in most cases.
3) *Suppressibility*: As mentioned in [11], it is easy to control δ to 0 while it is hard to suppress R to a very small value.

Furthermore, we can also treat δ as an objective and R as a constraint by the MDyHF, which will not influence the results of this paper.

We consider the following two cases in this paper.

1) The first case is formulated as follows:

$$\begin{aligned} &\min R = \frac{\lambda_{N+1}^r}{\lambda_2^r} \\ &\text{subject to: } q_1(y) \leq 0 \\ &\text{subject to: } q_2(y) \leq 0 \end{aligned} \tag{7}$$

where $q_1(y) = \delta - \alpha$, $q_2(y) = \mathcal{K} - \beta$, $\alpha \in [0, +\infty)$, and $\beta \in [0, +\infty)$. If $\alpha = +\infty$, the problem considered here only focuses on the constraint on $\mathcal{K}$ and minimizes R. If $\alpha \neq +\infty$, the problem considered here focuses on the constraint on $\mathcal{K}$ and δ simultaneously and minimizes the objective R.

2) The second case can be written as follows:

$$\min R = \frac{\lambda^r_{N+1}}{\lambda^r_2}$$
$$\text{subject to: } h_1(y) = 0$$
$$\text{subject to: } q_1(y) \leq 0 \qquad (8)$$

where $h_1(y) = \delta - \alpha$, $\alpha = 0$ and $q_1(y) = \mathcal{K} - \beta$. The second case is to minimize R as well as make the inequality and equality constraints feasible. As mentioned in [21] and [37], the controllability of a directed network depends on both R and δ. In order to enhance controllability, both R and δ should be minimized as much as possible. Therefore, the constraint $\delta = \alpha = 0$ means that the controllability of networks purely relies on R. In this case, one can show the impact of the average of the control gains $\mathcal{K}$ on controllability more clearly.

Remark 3: Actually, there are two types of constraints on control gains considered here. The first one is to consider control gains to be bounded like actuator saturations [25], that is, $\kappa_i (i \in \mathcal{P}) \in [\kappa_{i,\min}, \kappa_{i,\max}]$. In addition to the boundary of the control gains, the constraint on entire costs is also included and the total costs injected in networks have to be allocated in an appropriate way to maximize the controllability of networks.

Remark 4: Note that the problem of choosing key nodes to control the dynamics of the entire network is a natural combinatorial problem and the design of control gains is a continuous optimization problem [11], [24], which can be solved by evolutionary algorithms. Different from the works in [11] and [24], limited costs will affect the selection of key nodes, which will increase the difficulty and the complexity of the problem, as its importance stated in Remark 2. To the best of the authors' knowledge, this is the first attempt to use COEAs to study controllability/pinning control of complex networks/neuronal networks with the constraint on the average of the control gains.

Remark 5: It is worth mentioning that the problem here can also be treated as a multiobjective optimization problem (MOP) [23], which can be solved by a multiobjective optimization evolutionary algorithm (MOEA). The reasons for considering the problem here as a COP are twofold. 1) The research problem in (7) and (8) has an objective R and two constraints δ and the average of the control gains $\mathcal{K}$. If we treat the problem here as a MOP, the problem will have three objectives, that is, R, δ, and $\mathcal{K}$, which would lead to an unclear visualization in objective space and the impact of $\mathcal{K}$ on controllability is not easy to show. By considering the problem as a COP, one can fix δ and then show the impact of the average of the control gains $\mathcal{K}$ on controllability in a clear way (please refer to the details in Figs. 3 and 4 and Table II), which is one main purpose of this paper. 2) Actually, in most cases, δ is less important than R, especially when l is small, as shown in Fig. 2. In addition, as shown in [11], δ is easy to be stabilized to 0, while it is difficult to control R to a small value. Therefore, the formulation as a COP allows us to pay more attention to minimization of R as well as illustrating the effect of $\mathcal{K}$.

III. MDyHF and Its Encoding Scheme

A. Dynamic Hybrid Framework

COEAs usually include a search algorithm for refining solutions and a constraint-handling technique to make solutions feasible. In [31], a DyHF was proposed, which includes global and local search schemes. The global search model is used to refine the solutions, while the local search model is to motivate the population to approach or enter the feasible region from different directions promptly. In order to fit the search environments adaptively, the global and the local search methods are switched according to the probabilities of proportion of feasible solutions in the population. In addition, traditional DE works as a search algorithm in global and local search schemes.

Different from other kinds of COEAs, the DyHF is inspired by multiobjective optimization [31] and therefore the COP is handled on the basis of a biobjective optimization problem $\overrightarrow{F}(y) = (f(y), \Psi(y))$. All the constraints are included in the degree of constraint violation $\Psi(y)$ in (6), which is regarded as an additional objective. For example, the constraints on $\mathcal{K}$ and σ are included into $\Psi(y)$ together. This way, the original objective function $f(y)$ and the degree of constraint violation $\Psi(y)$ can be considered simultaneously when comparing the solutions of individuals in the population. This approach can well balance the original objective and the degree of constraint violation $\Psi(y)$, which can make the algorithms find feasible solutions easily and maintain a satisfactory convergence speed. The performance of DyHF has been verified on 22 benchmark test functions and it is shown that DyHF has the capability to solve all the test functions successfully [31]. For more details regarding the usage of biobjective optimization for dealing with COPs by the DyHF, one can refer to [31] and [24] and references therein.

Although DyHF is an effective attempt to solve a COP, the search engine in DyHF is not adaptive to fit complicated search circumstances. In particular, the main purpose of a global search scheme in DyHF is exploited to detect more promising regions, where a simple mutation and a crossover scheme from the conventional DE is utilized. Unfortunately, the traditional DE suffers from a slow convergence speed, lack of ability to find the global optimum, and cannot tune itself to confront with complex optimization problems. Motivated by these points, we preserve the constraint-handling approach of the DyHF due to its efficiency from Pareto dominance and aim to improve the part of the global search algorithm by an adaptive DE. Note that the efficiency of JaDE was demonstrated in [33] and here JaDE is utilized to generate offspring to enhance the search ability of the global search scheme and exploit more promising areas, which can efficiently adjust the control parameters in DE and thus make DE adapt to various search situations. In the following, the numerical experiments will validate its performance on different dimensional controllability problem.

B. Modified Dynamic Hybrid Framework

1) *Adaptive Differential Evolution:* JaDE initializes a population of SP individuals/particles in a D-dimensional search space, which can be used to deal with our optimization problem. Each individual can be viewed as a chromosome, representing a potential solution. After initialization, mutation, crossover, and selection operators are carried out at each generation to guide its population toward the global optimum. The population with its individuals can be written as $P = (y_{1,n}, y_{2,n}, \ldots, y_{i,n}), i = 1, 2, \ldots, SP, n = 0, 1, 2, \ldots, n_{\max}$, and $y_{i,n} = (y^1_{i,n}, y^j_{i,n}, \ldots, y^D_{i,n}), j = 1, 2, \ldots, D$, where n is the generation counter.

JaDE is used to serve as the search engine in the global search of DyHF. In JaDE, a mutation strategy and an external archive are used to provide information of the progress direction. The DE/current-to-ϵbest strategy adopts multiple best solutions to balance the convergence speed and the diversity of the population, which is updated according to the following:

$$v_{i,n} = y_{i,n} + \mathcal{F}_i \cdot (y^{\epsilon}_{\text{best},n} - y_{i,n}) + \mathcal{F}_i \cdot (y_{r_1,n} - \tilde{y}_{r_2,n})$$

where $y^{\epsilon}_{\text{best},n}$ is randomly selected as one of the top $100\epsilon\%$ individuals of the current swarm with $\epsilon = 0.05$. $y_{i,n}$, $y^{\epsilon}_{\text{best},n}$, and $y_{r_1,n}$ are chosen from the current population P. $\tilde{y}_{r_2,n}$ is randomly selected from the union $P \cup \mathcal{A}$, where $\mathcal{A}$ is an archive and is used to store the recently explored inferior solutions. $\mathcal{F}_i$ and $\mathcal{C}_i$ are the scaling factors associated with the ith individual and crossover probability, respectively. $\mathcal{F}_i$ and $\mathcal{C}_i$ are updated dynamically at each generation according to a normal distribution and a Cauchy distribution, respectively

$$\mathcal{F}_i = \text{randc}_i(\varphi_{\mathcal{F}}, 0.1)\mathcal{C}_i = \text{randn}_i(\varphi_{\mathcal{C}}, 0.1)$$

where $\varphi_{\mathcal{F}}$ is the mean value of a normal distribution and $\varphi_{\mathcal{C}}$ is the mean value of a Cauchy distribution. The two parameters are initialized to be 0.5 and then adjusted at each generation according to

$$\varphi_{\mathcal{F}} = (1 - w) \cdot \varphi_{\mathcal{F}} + w \cdot \text{mean}_L(S_{\mathcal{F}})$$

$$\varphi_{\mathcal{C}} = (1 - w) \cdot \varphi_{\mathcal{C}} + w \cdot \text{mean}_A(S_{\mathcal{C}})$$

where $w = 0.1$ is a constant. $S_{\mathcal{F}}$ and $S_{\mathcal{C}}$ stand for the set of all successful mutation/crossover rates; $\text{mean}_A(\cdot)$ indicates the usual arithmetic mean and $\text{mean}_L(\cdot)$ the Lehmer mean

$$\text{mean}_L(S_{\mathcal{F}}) = \frac{\sum_{i=1}^{|S_{\mathcal{F}}|} \mathcal{F}_i^2}{\sum_{i=1}^{|S_{\mathcal{F}}|} \mathcal{F}_i}.$$

2) *Details of MDyHF:* It is worth mentioning that the major algorithmic structure of the DyHF, that is, the local search strategy and the constraint-handling technique, are retained in the MDyHF, the details can be referred to [31].

Remark 6: Note that DE/current–ϵbest strategy is adopted in JaDE, which means that $y^{\epsilon}_{\text{best},n}$ is randomly selected as one of the top $100\epsilon\%$ individuals of the current swarm with $\epsilon = 0.05$. Different from the single-objective optimization problem, the $100\epsilon\%$ best individuals cannot be measured by only considering objective values. In this paper, we adopt the method in [45] and sort the solutions according to dominance, which is shown in the following way.

A solution i is said to constrained dominate a solution j, if any of the following conditions is true.

1) Solution i is feasible but solution j is not.
2) Solutions i and j are both infeasible, but solution i has a smaller overall constraint violation.
3) Solutions i and j are feasible and solution i dominates solution j.

Based on JaDE and the above dominance mechanisms, the adaptive global search model is proposed that concentrates on exploring more promising regions and refining the overall objective values of the population. Based on multiobjective optimization, if u_i dominates y_i, the trial vector u_i will replace the target vector y_i according to $\mathcal{C}_i$, else no replacement take places.

By employing the trial vector u_i to remove the inferior target vector y_i, the population P is updated through Pareto dominance. Apparently, our modification of the DyHF is technically simple and can be easily implemented. Even so, the following experimental results will illustrate the encouraging and promising performance of MDyHF. Therefore, MDyHF follows these steps.

1) Set the generation counter $n = 0$, $f_e = 0$ and obtain an initial population P by uniformly and randomly generating from the search space, calculate the objective value f and the constraint violation Ψ for each individual i, and evaluate the number of feasible solutions (NOFS) in P.
2) Let $\chi = \frac{SP-\text{NOFS}}{SP}$ and if $\text{rand}(0, 1) \geq \chi$ (where rand(0, 1) is a uniformly distributed random number between 0 and 1), then the global search with JaDE is implemented to refine feasible solutions, which is equipped with adaptive mechanism; otherwise, the local search is used to detect potential areas of feasible solutions.
3) Compute NOFS in P and set $n = n + 1$. If the stopping criterion is met, stop and output the best solution in P, else go to Step 2.

Remark 7: Evolutionary algorithms with an elitism method (the best individual survives with probability one) such as MDyHF can be ensured to find the global optimum with probability 1 if the number of generations tends to infinity, by using the concept of nonhomogeneous Markov chains, as proved in [46]–[48].

C. Encoding Scheme of COEAs

In this section, an appropriate encoding scheme is used and can be referred to [24]. The encoding scheme consists of two parts with an equal dimension size l: the first one is an integer search space to denote the locations of the driver nodes and the second one is a continuous search space to represent their corresponding control gains. The encoding scheme follows [24].

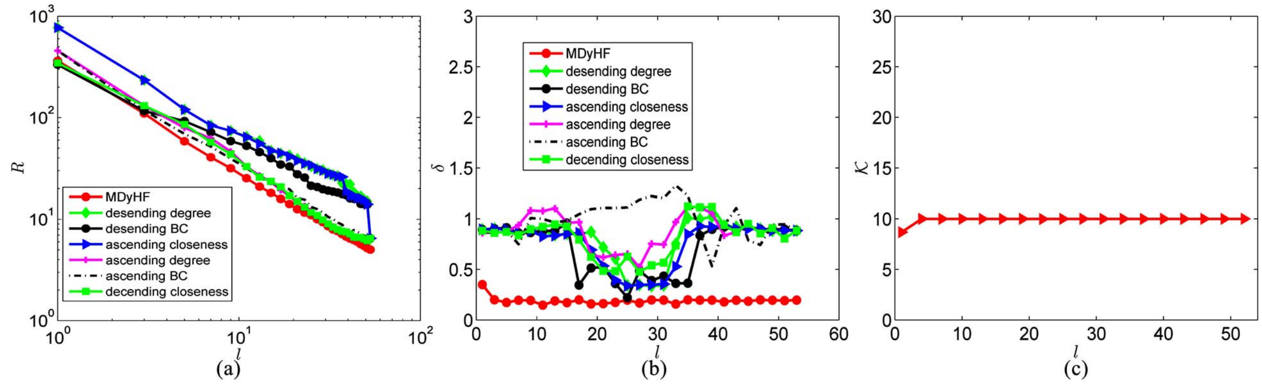

Fig. 2. Minimizing R using different schemes, α = 0.2 and β = 10, as a function of l. (a) Comparison of R with different schemes as a function of l. (b) Comparison of δ with different schemes as a function of l. (c) Average of the control gains $\mathcal{K}$ using MDyHF.

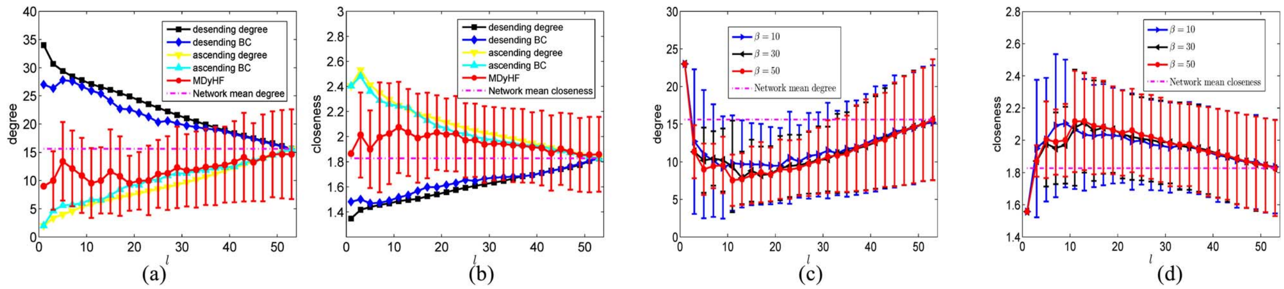

Fig. 3. Degree and closeness information of driver nodes with different schemes as a function of l. (a) Mean values of degree information of driver nodes with $\alpha = 0.2$, $\beta = 10$ as a function of l. (b) Mean values of closeness information of driver nodes with $\alpha = 0.2$, $\beta = 10$ as a function of l. (c) Mean values of degree of driver nodes under different β as a function of l, when $\alpha = +\infty$. (d) Mean values of closeness of driver nodes under different β as a function of l, when $\alpha = +\infty$.

TABLE I

SEARCH RESULT COMPARISONS AMONG FOUR ALGORITHMS FOR DIFFERENT l OF DRIVER NODES IN THE NEURONAL NETWORK WITH $N = 53$ (THE CALCULATION OF Q IS GIVEN IN (9); ALL THE ALGORITHMS ARE RUN 20 TIMES, $\eta = 12\,500$ AND $\alpha = 0$, $\beta = 30$; THE BEST RESULTS AMONG THE FOUR ALGORITHMS ARE SHOWN IN BOLD FONTS

		CMODE		DyHF		IDyHF		MDyHF	
		R	Ψ	R	Ψ	R	Ψ	R	Ψ
$l = 6$	Mean	46.4711	0	39.0959	0	36.6305	0	**35.0913**	0
	Best	38.6713	0	31.0939	0	**29.3161**	0	30.0323	0
	Q	42.3922	0	34.8661	0	32.7699	0	**32.4634**	0
$l = 12$	Mean	27.7295	0	21.3476	0	18.3069	0	**16.7967**	0
	Best	24.0568	0	16.0881	0	15.8778	0	**15.461**	0
	Q	25.8279	0	18.5322	0	17.0491	0	**16.115**	0
$l = 18$	Mean	19.9953	0	16.1677	0	12.1994	0	**11.5577**	0
	Best	17.9219	0	12.5422	0	10.899	0	**10.8525**	0
	Q	18.9302	0	14.2401	0	11.5309	0	**11.1995**	0
$l = 24$	Mean	18.1467	0	12.5788	0	8.5804	0	**8.4081**	0
	Best	15.4222	0	9.491	0	**7.6536**	0	7.7428	0
	Q	16.7291	0	10.9264	0	8.1038	0	**8.0686**	0
$l = 30$	Mean	14.0262	0	10.9395	0	6.8758	0	**6.577**	0
	Best	12.7251	0	7.3187	0	6.1767	0	**6.1259**	0
	Q	13.3598	0	8.9478	0	6.5168	0	**6.3474**	0
$l = 36$	Mean	12.9602	0	9.3311	0	5.6825	0	**5.5889**	0
	Best	11.0792	0	6.3573	0	5.0031	0	**4.9809**	0
	Q	11.9828	0	7.702	0	5.332	0	**5.2761**	0
$l = 42$	Mean	12.5129	0	9.7678	0	4.613	0	**4.5777**	0
	Best	8.7404	0	7.4653	0	4.2419	0	**4.1078**	0
	Q	10.4579	0	8.5393	0	4.4236	0	**4.3364**	0
$l = 48$	Mean	11.0912	0	7.6824	0	3.8948	0	**3.8361**	0
	Best	8.8865	0	5.9346	0	**3.1945**	0	3.2418	0
	Q	9.9278	0	6.7522	0	3.5273	0	**3.5265**	0

Remark 8: As stated in [24], the search range of each dimension is assumed to be the same and therefore can be written as $\triangle y = (\bar{Z}_i - \bar{Y}_i)$. In order to identify the driver nodes from $N = 53$ as a function of l, there are C_N^l distinct combinations, which is a natural NP-hard problem and it is difficult to adopt a Brute-force method to select the driver nodes. In addition, even if the locations of the driver nodes can be determined *a priori*, the problem is reduced into an l-dimensional continuous optimization problem. One effective method to handle a NP-hard problem is evolutionary computation algorithms. In this paper, we use MDyHF to study the controllability of neuronal networks.

IV. MAIN RESULTS

In this section, several examples are presented to verify the performance of the MDyHF in comparison with two COEAs and several methods from graph theory. The controlling regions are identified in microscopic and macroscopic ways.

A. Methods for Determining the Locations of Driver Nodes

In this section, the following methods are used for detecting the locations of driver nodes/controlling regions.

1) *Degree-Based Methods*: The controlling regions are selected according to out-degree in an ascending or a descending way, which are named the ascending and the descending degree-based methods, respectively.
2) *Betweenness Centrality (BC)-Based Methods*: Descending and ascending BC-based methods are used here.
3) *Closeness-Based Methods*: Two types of closeness-based methods, that is, descending and ascending closeness-based strategies are used.

4) *Evolutionary Algorithm-Based Methods*: COEAs are used to select driver nodes and design their control gains. Two evolutionary computation approaches, the combining multiobjective optimization with differential evolution (CMODE) [32] and the DyHF [31], are used to compare with the MDyHF. The CMODE and the DyHF have been recently developed and have shown their advantages over some well-known COEAs [31], [32].

B. Parameter Settings of COEAs

If not mentioned differently, the parameter setting of COEAs is adopted as follows. The maximum fitness evaluation $f_{e,\max}$ is set to $f_{e,\max} = \eta * D$ and $D = 2 * l$ is the dimension size. $\eta = 18\,750$ is a predefined constant. η is an adjustable parameter to get a balance between complexity and search accuracy. Usually, a large η is helpful to enhance search performance but leads to huge computation complexity. A small η can save the computation resources but may result in unsatisfactory search performance. In fact, similar results can also be obtained if we do not change η a lot, since in most cases the results are improved only a little after $\eta = 10\,000$. In order to make a balance between complexity and search accuracy, we choose $\eta = 18\,750$. Of course, one can choose a small η or a large η according to the personal requirement. When comparing the performance among COEAs, COEAs will be repeated 20 times independently for eliminating random discrepancy and terminated when COEAs algorithms attain $f_{e,\max}$. When showing the advantages of MDyHF over statistical methods, MDyHF will be repeated 10 times when MDyHF achieves $f_{e,\max}$. The parameter settings of the CMODE, the DyHF, and the IDyHF are given in [24], [31], and [32], respectively. The parameter setting of the MDyHF is according to [31] and [33]. Similar to [21], in the degree-based, the BC-based, and the closeness-based, the control gains in all the vertices are supposed to be identical and are gradually adjusted by a stepsize $\frac{N}{100}$.

C. Comparisons of the MDyHF With Evolutionary Algorithms and Statistical Methods

In this section, the performance of the proposed MDyHF is compared with other COEAs and statistical methods in Section IV-A. The COEAs used for comparison are the CMODE, the DyHF, and the IDyHF [24], [31], [32]. When compared with COEAs, we compare the objective value R and the constraint-handling results Ψ. When compared with statistical methods, the objective value R and the value of δ are compared, since the constraint on the average of the control gains in all the statistical methods are identical and satisfy the gain constraint.

As stated in [24], both the mean value and the best value of the solutions are important for measuring the reliability of the algorithm, the following measure is considered to incorporate them together:

$$Q = \sqrt{\text{Best} \times \text{Mean}}. \tag{9}$$

Obviously, Q should be made as small as possible.

TABLE II

CONTROLLING TIMES AND ITS COMMUNITY OF EACH NODE BELONGING TO WHEN OPTIMIZING R UNDER DIFFERENT β, WHEN $\alpha = \infty$ [ξ CAN BE SEEN FROM (10)]

$\beta = 50$			$\beta = 10$		
Name	ξ	Community	Name	ξ	Community
VPc	50	Auditory	VPc	53	Auditory
2	50	Somato-motor	36	53	Frontolimbic
AMLS	48	Visual	21a	50	Visual
21a	48	Visual	Hipp	49	Frontolimbic
21b	47	Visual	AMLS	48	Visual
PS	47	Visual	SII	48	Somato-motor
Sb	46	Frontolimbic	AAF	47	Auditory
ALLS	44	Visual	Sb	46	Frontolimbic
AAF	44	Auditory	2	45	Somato-motor
Hipp	44	Frontolimbic	PLLS	44	Visual
3a	42	Somato-motor	21b	44	Visual
Tem	41	Auditory	ALLS	42	Visual
SIV	41	Somato-motor	PS	41	Visual
1	40	Somato-motor	P	40	Auditory
DLS	38	Visual	19	39	Visual
PSb	37	Frontolimbic	3a	39	Somato-motor
P	36	Auditory	Tem	37	Auditory
SII	35	Somato-motor	1	37	Somato-motor
4	35	Somato-motor	PSb	35	Frontolimbic
RS	35	Frontolimbic	DLS	34	Visual
PLLS	34	Visual	SIV	33	Somato-motor
AII	34	Auditory	PMLS	32	Visual
PMLS	30	Visual	4	31	Somato-motor
PFCMiI	30	Frontolimbic	AII	29	Auditory
19	29	Visual	18	27	Visual
VLS	29	Visual	3b	27	Somato-motor
20b	27	Visual	35	27	Frontolimbic
SSAo	27	Somato-motor	Enr	27	Frontolimbic
3b	26	Somato-motor	RS	25	Frontolimbic
17	25	Visual	AES	24	Visual
Enr	25	Frontolimbic	20b	23	Visual
18	23	Visual	AI	22	Auditory
AI	21	Auditory	17	21	Visual
36	21	Frontolimbic	VLS	20	Visual
4g	20	Somato-motor	4g	20	Somato-motor
61	20	Somato-motor	PFCMiI	19	Frontolimbic
7	19	Visual	Ig	19	Frontolimbic
PFCI	19	Frontolimbic	7	15	Visual
Ig	16	Frontolimbic	SSAo	15	Somato-motor
5Bm	14	Somato-motor	EPp	13	Auditory
SSAi	14	Somato-motor	61	13	Somato-motor
6m	12	Somato-motor	Ia	12	Frontolimbic
Ia	12	Frontolimbic	SSAi	11	Somato-motor
PFCMd	11	Frontolimbic	5Bm	10	Somato-motor
AES	9	Visual	PFCI	10	Frontolimbic
EPp	8	Auditory	20a	9	Visual
Cga	7	Frontolimbic	6m	7	Somato-motor
5BI	6	Somato-motor	PFCMd	5	Frontolimbic
35	5	Frontolimbic	5Am	4	Somato-motor
5Am	4	Somato-motor	5BI	4	Somato-motor
20a	3	Visual	5AI	3	Somato-motor
CGp	2	Frontolimbic	Cga	3	Frontolimbic
5AI	1	Somato-motor	CGp	0	Frontolimbic

First, we show the comparison results of COEAs. Table I shows the comparison results of the CMODE, the DyHF, the IDyHF, and the MDyHF under a different dimension size. Table I reveals that Ψ achieves zero in all the four algorithms under different l, implying that all the four algorithms find feasible solutions. Hence, we only focus on the objective value R. Clearly, the MDyHF performs best among the four algorithms. the IDyHF ranks second among the four algorithms. The MDyHF performs better than the IDyHF. The DyHF performs better than the CMODE but works worse than the MDyHF and the IDyHF. The improvement of the MDyHF arises from the introduction of the JaDE into the global search scheme and retains other parts of efficient strategies in the MDyHF. After the local model finds possible feasible solutions, the adaptive global search method can well adjust itself to various search situations, thereby improving the performance of the DyHF. From the above observations, the MDyHF is the most powerful algorithm among the four COEAs. The adaptive global search method helps the MDyHF to refine solutions and maintain a good convergence speed, while the local search scheme is kept to explore feasible solutions. Due to the reliability of the MDyHF, it is used to carry out the following simulations.

Next, the MDyHF is compared with the statistical schemes in Section IV-A. The statistical methods only deal with minimizing R and δ is neglected, different from the MDyHF. Also, since the control gain of each node in neuronal networks is assumed to be identical in all the statistical methods, the search boundary of the control gains can be simply assumed to be $[0, \beta]$. In the following, we consider $\alpha = 0.2$ and $\beta = 10$. It is noteworthy that other parameters will yield similar results. The comparison of different β will be presented in Section IV-F.

Fig. 2(a) shows that the MDyHF performs better than the other methods in terms of R in most cases. The MDyHF offers poorer performance than a few methods in terms of R in several cases, since MDyHF consider the constraint on δ while the statistical methods neglect the effect of δ. Fig. 2 shows that the average of the control gains satisfies the constraint $\beta = 10$. Fig. 2(a) shows that, at the initial stage, the descending BC-based method performs best. When l increases, the descending degree-based, the descending BC-based, and the ascending closeness-based strategies become worse. Conversely, the ascending degree-based, the ascending BC-based, and the descending closeness-based strategies perform well in most cases, as depicted in Fig. 2(a).

In Fig. 2(b), although some statistical methods might work better than the MDyHF in terms of R, the MDyHF performs best among all the methods in terms of δ and the statistical methods cannot find feasible solutions in all the cases. In order to satisfy the constraints on δ and β, the MDyHF has to encounter a tradeoff between losing the performance of minimizing R to satisfy constraints. Also, when l crosses a threshold, the MDyHF always satisfies the constraint δ. From Fig. 2(a) and (b), it can be observed that the values of δ are much smaller than R, especially when l is small. This is consistent with the findings regarding synchronizability of complex networks [37]. However, when l increases, δ is turning more important since R is becoming smaller and the value of R is comparable to the value of δ.

D. Microscopic Identification of Controlling Regions Using the MDyHF Under the Gain Constraint

In Fig. 3(a) and (b), we show the mean values of the degree and closeness information of the driver nodes by various methods as a function of l, when $\alpha = 0.2$, $\beta = 10$. We find that the mean values of the driver nodes selected by the MDyHF are intermediate, belonging to the range of mean values of the ascending and the descending degree-based schemes, which are also less than the mean values of the network degree. This phenomenon shows that the nodes with neither a large nor a small degree are optimal to be selected as driver nodes. As l increases, the mean values of driver nodes selected by the MDyHF gradually increase and finally converge to the mean value of network degree. In summary, one should pick more nodes with a large degree as l increases, while the nodes with a small degree should also be chosen. The findings observed here, which largely depend on l, are different from the work in [20].

E. Macroscopic Identification of Controlling Regions Using the MDyHF Under the Gain Constraint

Due to the efficiency of the MDyHF, the identification of controlling regions of the neuronal network with different β is studied now. Denote

$$\xi_i = \sum_{l=1}^{N} \phi_{\mathcal{P}}(i) \tag{10}$$

which calculates the times of each node to serve as driver nodes as a function of l. The regions with a large ξ_i are more important to control the network. After control of the neuronal network with an increase of l (stepsize 1), ξ_i are sorted under $\beta = 10$, $\beta = 30$, and $\beta = 50$. The results are illustrated in Table II. The regions are sorted according to their importance in the neuronal networks. Table II shows that there exist some differences for the pinned times of each node in the three cases of β. The controlling regions are spread widely in four communities. It can be found that the regions such as VPc and 21a are important to control the neuronal network to a desired state, which are different from the usual hubs [9]. Meanwhile, the regions such as CGp and 5AI are unsuitable to serve as driver nodes.

F. Comparisons of the MDyHF With Different β

In this section, enhancing controllability of the neuronal network is examined using the MDyHF under different constraints β. The comparisons of R and $\mathcal{K}$ are presented in Fig. 4(a) and (b). Fig. 4(a) shows that when $\beta = 50$, the MDyHF performs best. However, the differences between the lines of $\beta = 30$ and $\beta = 50$ are close to each other. When l is large, the line of $\beta = 50$ decays faster than $\beta = 30$. Fig. 4(b) shows that when $\beta = 50$, there is no need to use the allowed control gains completely, that is, $\mathcal{K} < 50$. However, different from the case of $\beta = 50$, when $\beta = 10$ and $\beta = 30$, the allocations of the control gains should be used completely to enhance the controllability of the neuronal network, that is, $\mathcal{K} = 10$ or $\mathcal{K} = 30$, respectively. Also, one should carefully allocate the resources to each node to make the controllability maximal. This phenomenon shows that there exists an intermediate control cost to maximize controllability of neuronal networks, which verifies the phenomena in biological observations and engineering background [27], [44], as illustrated in Remark 2. In summary, the MDyHF can enhance the controllability of the neuronal network, while keeping the solutions in a feasible space.

In Fig. 3(c) and (d), the mean values of degree and closeness of the driver nodes under different l and β are shown. As l increases, the mean values of the degree of the driver nodes attain minimum and then increases, which shows a clear transition of the mean values of the degree of the driver nodes, that is, from nodes with a large degree to nodes with a small degree and again nodes with a large degree. As an increase of

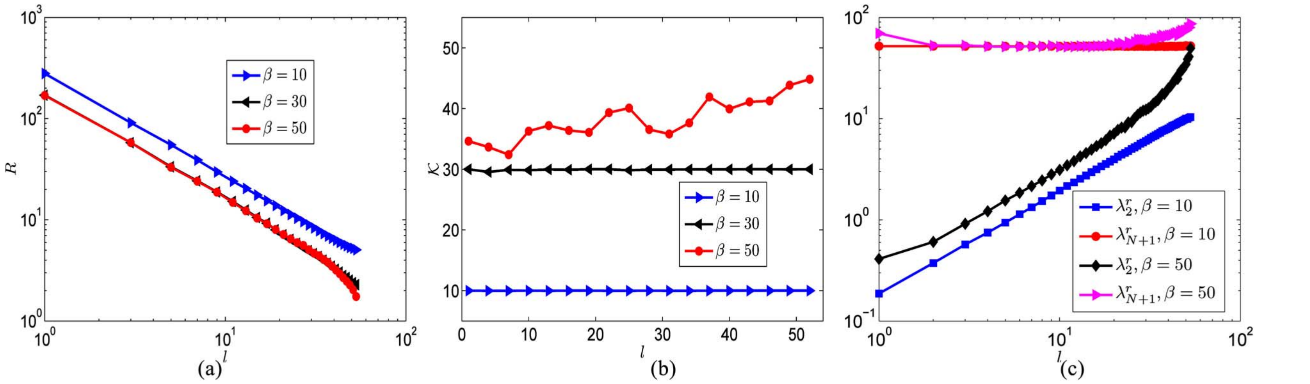

Fig. 4. Optimizing R by the MDyHF with different β as a function of l, when $\alpha = +\infty$. (a) Comparison of R with different β as a function of l. (b) Comparison of δ with different β as a function of l, when $\alpha = +\infty$. (c) Comparison of λ_2^r and λ_{N+1}^r with different β as a function of l, when $\alpha = +\infty$.

β (the constraint is more restrictive), the driver nodes tend to be chosen from the nodes with a large degree. The reason for this is that to enhance controllability of complex networks, the nodes with a small degree requires a large control input [24]. Therefore, when the constraint on the average of the control gains is considered, we have to control the nodes with a large degree and allocate control gains economically.

In the following, the dependence of R, λ_2^r, λ_{N+1}^r on l and β are investigated in Fig. 4(a) and (c). As plotted in Fig. 4(a), we get $R(l) \propto l^{-\gamma}$. In addition, in order to minimize R whenever $\beta = 10$ or $\beta = 50$, λ_2^r should be increased as much as possible, while λ_{N+1}^r should be suppressed as much as possible. Fig. 4 shows that the shape of R depends largely on λ_2^r when $\beta = 10$ and $\beta = 50$. When $l \to N$ and $\beta = 50$, $\lambda_2^r \approx \lambda_{N+1}^r$ and λ_2^r grow faster than λ_{N+1}^r, which leads to $R \approx 1$. However, when $\beta = 10$, the focus is on keeping λ_{N+1}^r stable and enlarge λ_2^r as much as possible. λ_2^r of $\beta = 50$ is larger than that of $\beta = 10$. This observation means that when more resources are allowed, it is desirable to enlarge λ_2^r and keep λ_{N+1}^r stable to enhance controllability. Therefore, when more resources are allowed, it is more efficient to find ways to enlarge λ_2^r. The above observations show that controlling λ_2^r plays a more important role in controllability than λ_{N+1}^r.

Remark 9: From the above results, the proposed method can not only solve the controllability of neuronal networks under the constraint on the average of the control gains but also deliver a better performance than other methods. The consideration of the constraint on the average of the control gains makes the controllability more practical, since saturations are widely observed in therapy, systems biology, and engineering. In addition, the results here reveal the relationship between l, R, $\mathcal{K}$ (or β), δ (or α), λ_2^r, and λ_2^{N+1}, which is of great importance for enhancing controllability. It can be found that some interesting results are presented for the locations of driver nodes under different l and $\mathcal{K}$. Although the method can be run on today's workstations, to make the algorithm solving the problem more quickly, it is important to develop more efficient methods based on our results. For example, it is promising to initialize the population based on our findings of driver nodes and control gains, or one can design a weight function to combine the optimization of R and δ into a single optimization problem based on our results, which can make them equally important as well as reduce computational costs. In summary, the research problem is more realistic due to the consideration of a directed neuronal network with the constraint on the average of the control gains. The limitations of our techniques will motivate us to develop simpler and more efficient approaches to identify driver nodes and design their corresponding control gains, thereby enhancing controllability of a realistic network, such as gene regulatory networks, power grids, large-scale chemical processes, or transportation networks.

V. Conclusion

In this paper, we investigate the problem of controllability of a realistic neuronal network of the cat under constraints on control gains by utilizing a MDyHF. The problem of detecting driver nodes under constraints on control gains is converted into a COP, in which two measures of controllability R and δ are viewed as an objective and a constraint, respectively, and the average of the control gains is regarded as a constraint, thereby the objective and the constraints are incorporated into one unified framework. By adding the JaDE with Pareto dominance into the DyHF, the MDyHF can fit the search circumstances adaptively. By comparing with two recent COEAs and statistical methods, the experimental results demonstrate the effectiveness of the MDyHF. By using the MDyHF, the controlling regions under gain constraints are identified. Some interesting findings about the constraint on the average of the control gains, the objective R, the number of driver nodes l, and the eigenvalues of the extended topology graph are illustrated by simulations. We show that there exist intermediate control costs to enhance controllability of neuronal networks and the control costs should be carefully allocated to maximize the controllability of neuronal networks. The effects of constraints on β on controllability of neuronal networks are also investigated and it is shown that the variation of β does affect the selection of controlling regions and the controllability of neuronal networks. We find that the controlling regions vary under different β.

Many extensions and refinements of this paper are possible. These include the analysis of data on other kinds of networks [49]–[52], the usage of advanced control algorithms [53]–[55], the development of more powerful COEAs or evolutionary-type optimization techniques to handle the controllability of neuronal networks, and the consideration of other types of real-world constraints. With the arrival of new methods, it will be feasible to apply our methods to more natural systems or analysis of their global controllability, and thereby further enhancing our understanding of how to control a directed and weighted complex network with a suitable control cost and a small number of driver nodes.

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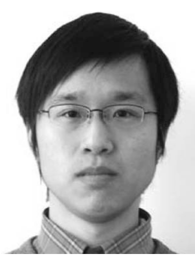

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