Annual Reviews in Control 38 (2014) 184–198

Contents lists available at ScienceDirect

Annual Reviews in Control

journal homepage: www.elsevier.com/locate/arcontrol

Review

Synchronization in complex networks and its application – A survey of recent advances and challenges ☆

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ARTICLE INFO

Article history:
Received 10 March 2014
Accepted 2 August 2014
Available online 18 October 2014

ABSTRACT

Complex networks have, in recent years, brought many innovative impacts to large-scale systems. However, great challenges also come forth due to distinct complex situations and imperative requirements in human life nowadays. This paper attempts to present an overview of recent progress of synchronization of complex dynamical networks and its applications. We focus on robustness of synchronization, controllability and observability of complex networks and synchronization of multiplex networks. Then, we review several applications of synchronization in complex networks, especially in neuroscience and power grids. The present limitations are summarized and future trends are explored and tentatively highlighted.

Contents

☆ Part of the paper was presented as a plenary talk at the 4th IFAC Workshop on Distributed Estimation and Control in Networked Systems, September 25–26, 2013, Koblenz, Germany.

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http://dx.doi.org/10.1016/j.arcontrol.2014.09.003

1. Introduction

Synchronization of a number of coupled systems has been widely observed in numerous distinct scenarios such as neuroscience, systems biology, electrochemistry, earth science, social societies and engineering (Arenas, Guilera, Kurths, Moreno, & Zhou, 2008; Dahlem et al., 2013; Gao, Chen, & Lam, 2008; Gu, Pasqualetti, Cieslak, Grafton, & Bassett, 2014; Huang, Ho, & Lu, 2012; Jadbabaie, Lin, & Morse, 2003; Li, Ho, & Lu, 2013; Lu, Kurths, Cao, Mahdavi, & Huang, 2012; Maraun & Kurths, 2005; Pikovsky, Rosenblum, & Kurths, 2001; Ren & Beard, 2008; Saber & Murray, 2004; Wielanda, Sepulchre, & Allgöwer, 2011; Zamora-López, Zhou, & Kurths, 2010). The analysis of synchronization is strengthened due to the fact that natural systems, which we intend to understand and exploit, are often interacted closely from different perspectives, determining the complex dynamics of system's properties. For instance, in Uhlhaas and Singer (2006), it is experimentally verified that synchronization plays an important role in the pathogenesis of several neurological diseases, such as Parkinson's disease, Alzheimer's disease and essential tremor (Arenas et al., 2008). In Machowski, Bialek, and Bumby (2008), Rohden, Sorge, Timme, and Witthaut (2012), power grid networks need to attain synchronization to make the entire smart grid operate in a steady state.

Synchronization is a widely studied topic in physics, while the consensus problem of multi-agent systems is an important research problem in engineering (Bakule, 2014; Lovisari & Zampieri, 2012; Sepulchre, 2012). Mathematically, the definitions for synchronization and consensus are quite similar (Cao, Yu, Ren, & Chen, 2013; Wielanda et al., 2011). The main difference is that synchronization focuses on networks with self-dynamics (linear or nonlinear dynamics) and therefore the final agreement state could be time-varying. Nevertheless, in multi-agent systems, the self-dynamics of each agent is usually neglected and thus the asymptotic consensus state is in general a constant (Cao et al., 2013; Wielanda et al., 2011). Recently, more and more researchers borrow ideas from interdisciplinary areas to study issues they care about of complex networks.

Reviews on the advances made in synchronization of complex networks or coordination of multi-agent systems never cease. Some summaries have been presented with various foci in different phases such as synchronization in complex networks (Arenas et al., 2008), synchronization in complex oscillator networks (Döfler & Bullo, 2014), coordination of multi-agent systems (Cao et al., 2013; Saber, Fax, & Murray, 2007), collective motions (Vicsek & Zafeiris, 2012), regulatory networks (Fiedler, Mochizuki, Kurosawa, & Saito, 2013; Mochizuki, Fiedler, Kurosawa, & Saito, 2013) and oscillation death versus amplitude death (Koseska, Volkov, & Kurths, 2013; Saxena, Prasad, & Ramaswamy, 2012; Zou, Senthilkumar, Zhan, & Kurths, 2013).

During the past decades, extensive studies on synchronization in complex networks have been carried out by both physical and control communities assuming different contexts, and various approaches have been proposed on how to deal with synchronization in complex networks. Many systematic results in this regard have unfolded with respect to the models, the methods and the different approaches for handling synchronization of complex networks. Here, we list some recent important topics in the area of synchronization or related ones:

(1) robustness of synchronization in complex networks;
(2) controllability of complex networks;
(3) observability of complex networks;
(4) synchronization of multiplex networks;
(5) explosive synchronization of complex networks;
(6) chimera states of complex networks;
(7) oscillation death and/or amplitude death of complex networks.

Since explosive synchronization and chimera states do not generally fall into the scope of control-oriented investigations (actually within the scope of statistical physics and nonlinear physics) and some reviews on oscillation death or amplitude death of complex networks have been reported (Koseska et al., 2013; Saxena et al., 2012), the state of art on them will not be pursued here. In this survey, our main focus is on synchronization in complex networks related to both control theory and physics, and review related advances by paying special attention to those which previous surveys did not refer to. Our purpose is to establish a connection between physics and engineering by drawing the attention from both areas to circumvent the above mentioned problems by developing appropriate control theories and approaches.

We try to present a survey on recent important results in synchronization of complex networks here. While covering all the contributions seems to be impossible, we devote ourselves to discussing explicit research lines and helping to categorize problems and methodologies. The survey is organized as follows. In Sections 2.1–2.4, we overview the robustness of synchronization in complex networks, controllability and observability of complex networks and synchronization of multiplex networks, respectively. In particular, the topics of controllability of complex networks are categorized into three classes. In Section 3 we focus on the applications of synchronization in complex networks, ranging from cancer therapy and power grids to neuroscience. Finally, a brief summary and outlook are presented in Section 4.

Basic Notations: In this paper, the concept of "controllability" is based on typical works in complex networks (Liu, Slotine, & Barabási, 2011) and control theory (Kalman, 1963; Rugh, 1996). $l \in [1, N]$ represents the number of driver nodes of a network, where N is the network size. $\delta_{\mathcal{D}}(\cdot)$ denotes the characteristic function of the set $\mathcal{D}$, i.e., $\delta_{\mathcal{D}}(i) = 1$ if $i \in \mathcal{D}$; otherwise, $\delta_{\mathcal{D}}(i) = 0$. Define a graph by $\mathcal{G} = [\mathcal{V}, \mathcal{E}]$, where $\mathcal{V} = \{1, \cdots, N\}$ and $\mathcal{E} = \{e(i,j)\}$ are the vertex set and the edge set, respectively. The graph $\mathcal{G}$ is assumed to be directed, weighted and simple. Let the weighted and directed matrix $L = [l_{ij}]_{i,j=1}^{N}$ be the Laplacian matrix of graph $\mathcal{G}$, which is defined as follows: for any pair $i \neq j$, $l_{ij} < 0$ if $e(i,j) \in \mathcal{E}$; otherwise, $l_{ij} = 0$. $l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}$ $(i = 1, 2, \cdots, N)$.

2. Main survey

This part is divided into four such parts including robustness in synchronization, controllability of complex networks, observability of complex networks and synchronization of multiplex networks. In discussions for each topic, we shall first make a review on the main achievements and present some limitations of current research.

2.1. Robustness in synchronization

Consider a network of N identical systems governed by the following equation:

$$\dot{x}_i(t) = f(x_i, t) - c\sum_{j=1}^{N} l_{ij} h(x_j(t)), \quad i = 1, \cdots, N, \tag{1}$$

$x_i(t) = [x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t)]^T \in \mathbb{R}^n$ $(i = 1, 2 \cdots, N)$ is the state vector of the ith node; c is the global coupling strength of the network; and $f(x_i, t) = [f_1(x_i, t), \cdots, f_n(x_i, t)]^T$ is a vector function describing the evolution of each individual oscillator in the case of no coupling

$c = 0$. n denotes the dimensional size of each node. In the coupling term, the node is connected through a generic output function $h(x_i(t))$. There always exists a synchronous state $\mathcal{M} = \{x_1(t) = x_2(t) = \ldots = x_N(t) = \alpha(t)\}$ in the Nn-dimensional state space in which all individual oscillators follow the same trajectory $\alpha(t)$. Here, the Laplacian matrix L is assumed to be connected, undirected and unweighted. $\mathcal{M}$ being stable means that the network (1) is stable. According to linear stability adopted in Pecora and Carroll (1998), the synchronization analysis of (1) can be separated into two steps. $f(\cdot,\cdot)$ and $h(\cdot,\cdot)$ define a master stability function $\mathrm{MSF}_{f,h}$ that is independent of the network. Then, c and the network define a set of numbers at which $\mathrm{MSF}_{f,h}$ is computed to find out whether $\mathcal{M}$ is stable or not. It is worth mentioning that the zero-row-sum property of the Laplacian matrix can ensure the block diagonalization of the Jacobian, which makes the master stability approach in Pecora and Carroll (1998) be powerful to determine the stability of a synchronized solution by reducing the dimensionality of the synchronization problem. From the framework presented in Pecora and Carroll (1998), $\mathcal{M}$ is table if c and the eigenvalues $0 = \lambda_1 \leqslant \lambda_2 \leqslant \ldots \leqslant \lambda_N$ of the Laplacian matrix L satisfy $\mathrm{MSF}_{f,h}(c\lambda_i) < 0, \forall i = 2, \ldots, N$. This condition is equivalent to demanding that all transverse eigenmodes of $\mathcal{M}$ have a negative Lyapunov exponent (Pecora & Carroll, 1998). Note that many choices of f and h yield a function $\mathrm{MSF}_{f,h}$ that is negative only in an interval (α_1, α_2) so that $\mathcal{M}$ is stable if $\alpha_1 < c\lambda_i < \alpha_2$, $\forall i = 2, \ldots, N$. Therefore, a very useful condition to ensure stability is proposed:

$$R = \frac{\lambda_N}{\lambda_2} < \frac{\alpha_2}{\alpha_1}. \tag{2}$$

By means of (2), the synchronization state $\mathcal{M}$ is stable if c belongs to the stability interval $I_s = \left(\frac{\alpha_1}{\lambda_2}, \frac{\alpha_2}{\lambda_N}\right)$. The smaller the ratio R is, the more the network is synchronizable. This useful condition has been widely adopted in Sorrentino, Bernardo, Garofalo, and Chen (2007), Arenas et al. (2008), Menck, Heitzig, Marwan, and Kurths (2013) and references therein. Although the condition (2) is easy to check, the level of R does not characterize how stable the synchronous state $\mathcal{M}$ is against even larger perturbations, which is an important topic in both physics (Arenas et al., 2008) and control theory (Khalil, 2002; Rugh, 1996). In order to answer this fundamental question, Menck et al. (2013) employs the concept of region of attraction to measure synchronizability versus robustness. It is worth mentioning that the estimation of region of attraction is a basic issue in nonlinear system theory, which is usually provided by using the Lyapunov function method (Khalil, 2002).

The concept of basin stability as proposed in Menck et al. (2013), is a measure related to the volume of the basin of attraction. Basin stability is nonlocal, nonlinear and easily applicable, even to high-dimensional systems, which circumvents the problem of traditional linearization-based approach to stability being too local.

The approach is used for Watts–Strogatz (WS) networks for paradigmatic Rössler systems, whose dynamics can be described as follows:

$$\begin{aligned} \dot{x}_{i1} &= -x_{i2} - x_{i3} - c\sum_{j=1}^{N} l_{ij}x_{j1} \\ \dot{x}_{i2} &= x_{i1} + ax_{i2} \\ \dot{x}_{i3} &= b + x_{i3}(x_{i1} - d) \end{aligned} \tag{3}$$

where $a = b = 0.2$ and $d = 7$. Every such network has a synchronous state in which all nodes follow the same trajectory. A network's synchronous state is stable if its synchronizability ratio $R < \frac{\alpha_2}{\alpha_1} = 37.85$ and $c \in I_s = \left(\frac{\alpha_1}{\lambda_2}, \frac{\alpha_2}{\lambda_N}\right)$, where $\alpha_1 = 0.1232$ and $\alpha_2 = 4.663$. Since the value of R does not quantify how stable the synchronous state $\mathcal{M}$ is against perturbations, the synchronous state's basin stability $S_{\mathcal{B}}$ for several $c \in I_s$ is computed and the mean $\bar{S}_{\mathcal{B}} = \mathrm{mean}\langle S_{\mathcal{B}}(c)\rangle_{c\in I_s}$ is then obtained. Finally, $\bar{S}_{\mathcal{B}}$ is averaged as $\langle \bar{S}_{\mathcal{B}} \rangle$ for different ensembles of Watts–Strogatz model. The volume of region of attraction $\mathcal{B}$ in a relative sense can be measured by

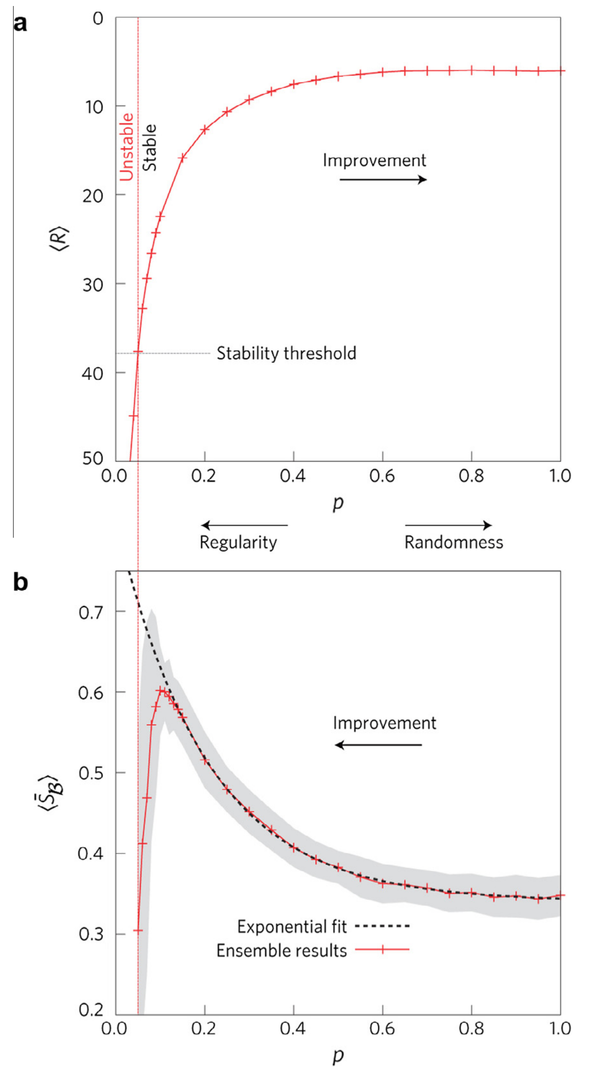

Fig. 1. Synchronizability and basin stability in Watts–Strogatz networks of chaotic Rössler oscillators. (a) Expected synchronizability $\langle R \rangle$ versus the Watts–Strogatz model's parameter p. The scale of the y axis was reversed to indicate improvement on increase in p. (b) Expected basin stability $\langle \bar{S}_{\mathcal{B}} \rangle$ versus p. The grey shading indicates ± one standard deviation. The dashed line shows an exponential curve fitted to the ensemble results for $p \geqslant 0.15$. Solid lines are guides to the eye. The plots shown were obtained for $N = 100$ oscillators of Rössler type, each having on average $k = 8$ neighbors. Choices of larger N and different k produce results that are qualitatively the same. The figure is taken from Menck et al. (2013).

$$S_{\mathcal{B}} = \frac{\mathrm{Vol}(\mathcal{B} \cap \mathcal{Q})}{\mathrm{Vol}(\mathcal{Q})} \in [0, 1],$$

where $\mathcal{Q}$ is a subset of state space that has finite volume. Specially, for the simplicity of calculation, the system equations for H initial conditions are drawn uniformly at random from $\mathcal{Q}$. $S_{\mathcal{B}}$ can be simply estimated as $\frac{J}{H}$, where J is the number of initial conditions that reach the synchronous state $\mathcal{M}$ (the other possible attractor being infinity).

Fig. 1(a) shows that for ensemble networks with too small WS rewiring probability p, the expected synchronizability R has not attained the threshold $\frac{\alpha_2}{\alpha_1}$ so that the synchronous state is not stable. The expected synchronizability improves rapidly, soon crossed the stability line and then improves even further, leading to the puzzle that the network becomes more synchronizable for more random topology. Fig. 1(b) displays that the expected mean basin stability $\langle \bar{S}_{\mathcal{B}} \rangle$ unveils a behavior quite different from that of the expected synchronizability shown in Fig. 1(a). After increasing very fast initially, the expected mean basin stability $\langle \bar{S}_{\mathcal{B}} \rangle$ decreases exponentially as the rewiring probability p increases. The behavior in Fig. 1(b) a network's mean basin stability $\langle \bar{S}_{\mathcal{B}} \rangle$ is determined primarily by the location of its stability interval I_s (Menck et al., 2013).

Actually, the concept of region of attraction has been used in controllability of complex networks (Cornelius, Kath, & Motter, 2013; Sun & Motter, 2013), in which the perturbations are considered as a tool to drive the states of a system to the region of attraction of a desired state in contrast to Menck et al. (2013). This way, the control objective of complex networks can be achieved. The robustness of complex power grids networks will be discussed in Section 3.1.

Although the method proposed in Menck et al. (2013) is easy to carry out and exactly characterizes the basin stability very well, the computational complexity can be expensive, which limits the results applied to large-scale networks. Therefore, how to exactly characterize basin stability by using tools from the nonlinear system theory (Khalil, 2002) poses a challenging and important topic.

2.2. Controllability of complex networks

This subsection is split into three parts, i. e., local controllability, global controllability-structural controllability and global controllability-Lyapunov function method.

2.2.1. Local controllability

The dynamics of natural and technical networks are intrinsically strongly nonlinear, making them complicated with respect to their topology and self-dynamics. Hence, nonlinearity is the main obstacle to control such systems, which has been well demonstrated in Khalil (2002).

Let a reference evolution/state (desired state) be as follows:

$$\dot{s}(t) = f(s(t)).$$

It is worth mentioning that the above differential equation is general to represent extensive real-world complex systems such as social networks, economic systems, biological systems and other natural complex systems (Wang & Su, 2014).

The complex network (1) considered here is a directed one and is composed of identical systems with several feedback controllers, which can be formulated as follows:

$$\dot{x}_i(t) = f(x_i,t) - c\sum_{j=1}^{N} l_{ij}h(x_j(t)) - c\delta_{\mathcal{M}}(i)\kappa_i(h(s(t)) - h(x_i(t))),\quad i = 1,\cdots,N, \tag{4}$$

where $x_i(t), c, f$ and h are given in (1). Let $\mu_p = \mu_p^r + j\mu_p^i$ $\left(j = \sqrt{-1}\right), (p = 1,2,\cdots,N)$, be the set of eigenvalues of L and assume that they are ordered by $\mu_1^r \leqslant \mu_2^r \leqslant \cdots \leqslant \mu_N^r$. κ_i are the feedback control gains between the vertex and the desired state. It is clear that $1 \leqslant \sum_{i=1}^{N}\delta_{\mathcal{M}}(i) \leqslant N$. The purpose of local controllability is to guide the network (4) towards the desired state $s(t)$, i. e., $x_1(t) = x_2(t) = \cdots = x_N(t) = s(t)$.

In order to examine the controllability of network (4), we consider an extended network of $N+1$ dynamical systems $y_i(t)$, where $y_i(t) = x_i(t)$ for $i = 1,2,\ldots,N$ and $y_{N+1}(t) = s(t)$. Then, (4) can be rewritten as follows Sorrentino et al. (2007):

$$\frac{dy_i(t)}{dt} = f(y_i,t) - c\sum_{j=1}^{N+1}\mathcal{W}_{ij}h(y_j(t)),\quad i = 1,\cdots,N+1,$$

where $\mathcal{W} = [\mathcal{W}_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ in the form of

$$\mathcal{W} = \begin{pmatrix} \mathcal{A}_1 & l_{12} & \ldots & l_{1N} & -\delta_{\mathcal{M}}(1)\kappa_1 \\ l_{21} & \mathcal{A}_2 & \ldots & l_{2N} & -\delta_{\mathcal{M}}(2)\kappa_2 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ l_{N1} & l_{N2} & \ldots & \mathcal{A}_N & -\delta_{\mathcal{M}}(N)\kappa_N \\ 0 & 0 & \ldots & 0 & 0 \end{pmatrix},$$

in which $\mathcal{A}_i = l_{ii} + \delta_{\mathcal{M}}(i)\kappa_i$. Let $\lambda_p = \lambda_p^r + j\lambda_p^i$ be the pth eigenvalue of $\mathcal{W}$ and assume that λ_p is sorted as $\lambda_1^r \leqslant \lambda_2^r \leqslant \cdots \leqslant \lambda_{N+1}^r$, where $\lambda_1^r = 0$. It has been well recognized that for a large class of systems (in terms of the dynamic function f and the output function h), there exists a bounded area of the complex plane centered on the real axis, for which the MSF is negative (Pecora & Carroll, 1998).

Similar to the analysis method of checking synchronizability of networks (Sorrentino et al., 2007), the controllability can be assessed in terms of

$$P = \frac{\lambda_{N+1}^r}{\lambda_2^r}, \tag{5}$$

and

$$\sigma = \max_p\left\{\lambda_p^i\right\}. \tag{6}$$

The smaller P and σ are, the easier the network is controllable (Sorrentino et al., 2007). By means of this method, the dynamical properties of the network have been decoupled from the factors encoded in the matrix $\mathcal{W}$. The *local controllability* of the network is related to the following three factors: (i) the original structure of the network topology; (ii) the choice of nodes injected with feedback controllers; and (iii) the values of control gains κ_i, where $\delta_{\mathcal{M}}(i) = 1$.

It should mentioned that $s(t)$ can be viewed as a leader in multi-agent systems. All the states $x_i(t)$ of the agent systems follow the trajectory of $s(t)$. Similar techniques of extension of the Laplacian matrix have been adopted in leader-following problems in the control area (Hong, Hu, & Gao, 2006), even in distributed containment control of multi-agent systems (Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008).

Based on this strategy, (Sorrentino et al., 2007) examined the controllability of undirected networks by assuming $\kappa_i = \kappa$ where two types of pinning control strategies are adopted: (i) Random pinning: The pinned nodes are randomly selected with uniform probability from all the network vertices; (ii) Selective pinning: The l pinned nodes are first sorted according to a certain property (e. g., the closeness, the importance, the degree or betweenness centrality), then the pinned nodes are chosen in that particular order. Obviously, the method is computationally simple, which will result in conservativeness due to the dissatisfactions of factors (ii) and (iii). In addition, only undirected networks are taken into account in Sorrentino et al. (2007).

In order to shorten such a gap, the controllability problem of networks is treated as a combinatorial and continuous optimization problem in the optimization field, which can also be viewed as a multimodal optimization problem (Tang, Gao, Kurths, & Fang, 2012a). Namely, the choice of pinned nodes is a combinatorial problem and the design of control gains is a continuous optimization problem. For example, for a network with N nodes and l pinned nodes allowed to input controllers. Therefore, there exist $\binom{N}{l}$ different combinations for choosing pinned nodes. However, the traditional easy enumeration method (Sorrentino et al., 2007) cannot be applied to tackle this problem well. Therefore, an evolutionary algorithm has been developed to enhance the controllability of networks. Simulations illustrate that the method in Tang et al. (2012a) substantially outperforms the approach in Sorrentino et al. (2007). Although (Tang et al., 2012a) investigated the controllability problem of complex networks, unfortunately, the network considered is undirected. Therefore, Tang, Gao, Zou, and Kurths (2012b) concentrates on the identification of controlling regions in neuronal networks of cats' brain, based on single-objective evolutionary computation methods, in which the network is directed. Then, one simple way to treat the controllability of directed networks is to consider the two measures of controllability P and σ, separately. Based on this treatment, the impact of the number of driver nodes on controllability is revealed and the properties of pinned nodes are shown in a statistical way. The pinned nodes are illustrated in microscopic, mesoscopic and macroscopic scales. It is revealed that the statistical properties of pinned nodes display a concave or convex shape with an increase of the allowed number of controlling nodes, indicating a clear transition in choosing driver nodes from the areas with a large degree to the areas with a low degree.

Evidently, the way regarding the objectives P and σ separately is unavoidable to induce conservativeness (Tang et al., 2012b). Additionally, σ is usually neglected due to the minor value of σ in most of coupling graphs and thus has trivial or minor impacts on synchronizability and controllability (Sorrentino et al., 2007). However, this assumption cannot reflect the actual synchronizability and controllability of networks. For instance, in some special networks, as normalized Laplacian matrix, the value of σ is comparable to that of P. Also, as shown in Tang et al. (2012b), σ is also comparable to P when the allowed number of pinned nodes is large. Based on these motivations, Tang, Wang, Gao, Swift, and Kurths (2012c) transformed the controllability of networks into a constrained optimization problem, in which optimizing P is regarded as an objective and minimizing σ is considered as a constraint, since P plays a more important role in controllability of networks than σ in most of scenarios. Based on an evolutionary constraint optimization method, i. e., an improved dynamic hybrid framework (IDyHF), the pinned nodes are detected in a microscopic and macroscopic way. The obtained results unveil the relationships among the locations of pinned nodes, the number of driver nodes l and the constraint r, which are closely related to in-degree and out-degree. When $r = +\infty$, the nodes with a large degree are important to control networks when l is small but the nodes with a small degree are useful to control networks when l increases. Similar observations are also presented in Liu et al. (2011), Yu, Chen, and Lü (2009). When $r = 0$, the mean degrees of the driver nodes increase as a function of l (Tang et al., 2012c).

Although the pinned nodes were identified under different levels of constraints in Tang et al. (2012c), it is inevitably to tune the constraint carefully. Therefore, it is necessary to take into account two measures of controllability of networks equally, i. e., P and σ, and identify the pinned regions under different levels of constraints at the same time. A natural approach is to formulate controllability of networks in a unified framework-a multiobjective optimization problem. Based on this consideration, by employing a differential evolution algorithm, a reference-point-based non-dominated sorting composite differential evolution (RP-NSCDE) has been developed to handle the multiobjective identification of pinned nodes in complex networks (Tang, Gao, & Kurths, 2013a). The proposed RP-NSCDE shows its competitive performance in terms of accuracy and convergence speed. The proposed evolutionary pinning technique has been also compared with other representative statistical methods in the complex network theory, single objective, and constraint optimization methods to validate its effectiveness and reliability. The results show that there exists a tradeoff between minimizing two objectives, and thus pareto fronts (PFs) have been presented (Tang et al., 2013a).

By modifying this model (4), the concept of spatial pinning control is introduced for a network of mobile chaotic agents (Frasca, Buscarino, Rizzo, & Fortuna, 2012). In a planar space, N agents move as random walkers and are connected according to a time-varying r-disk proximity graph. The controller is only activated when the agents fall into a given area, called control region. It has been shown that the control is effective in driving all the agents to a reference evolution and has better performance than pinning control on a fixed set of agents. Similar observations have been reported in stochastic resonance of complex networks, when partial noise and switching noise were considered (Tang, Gao, Zou, & Kurths, 2013c). The authors revealed analytically effects of the relative size of the control region, the agent density and the velocity on the global convergence of the system to the reference evolution.

Although extensive results have been reported in local controllability of networks by means of the enumeration method (Frasca et al., 2012; Sorrentino et al., 2007) or heuristic search methods Tang et al. (2012a, 2012b, 2012c, 2013a, 2014c), there are several important unsolved topics: (i) it is important to reduce the complexities of the enumeration method and heuristic search methods by utilizing some analytic methods, although they can be performed by parallel computing; and (ii) it also remains interesting to include some networked induced constraints such as time-delays, quantizations, actuator saturations, packet dropouts and sampling data (Gao et al., 2008; Hespanha, Naghshtabrizi, & Xu, 2007; Zhang, Gao, & Kaynak, 2013;), when studying local controllability of complex networks. The first step to address this problem is to develop intuitive new objectives when including such kind of factors.

2.2.2. Global controllability-structural controllability

Firstly, consider the following canonical linear, time-invariant system:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{7}$$

where $x(t) = (x_1(t), \ldots, x_N(t))^T$ is the state vector of a system of N nodes. $x_i(t)$ can represent different scenarios such as the position of robots, the amount of traffic that passes through a node i in a communication network or transcription factor concentration in a gene regulatory network (Liu et al., 2011; Ren & Beard, 2008). The matrix $A \in \mathbb{R}^{N\times N}$ denotes the coupling matrix of the system. $B \in \mathbb{R}^{N\times M} (M \leqslant N)$ is the input matrix needed to detect the nodes controlled by a controller $u(t) = (u_1(t), \ldots, u_M(t))^T$. Usually, if one aims to control a system, the set of nodes needs to be identified, which can help to control the entire network. In this survey, we call the nodes with controllers as either "pinned nodes" or "driver nodes", like Section 2.2.1. The minimum number of driver nodes N_D should be identified such that the entire network can be controlled.

According to Kalman's controllability rank condition (Kalman, 1963), system (7) is said to be controllable if it can be driven from any initial state to any desired final state in finite time, which is possible if and only if the $N \times NM$ controllability matrix

$$\mathcal{C} = (B, AB, A^2B, \ldots, A^{N-1}B), \tag{8}$$

has full rank, that is

$$\text{rank}(\mathcal{C}) = N. \tag{9}$$

Due to the fact that even if all weights are known, there exist 2^N distinct combinations for placing controllers by using a brute-force search. Hence, the so-called "structurally controllable" (Liu et al.,

2011) is utilized to control the network to overcome inherently incomplete knowledge of the link weights in A (Liu et al., 2011). The authors proved that the minimum number of driver nodes needed to maintain the full control of the network is determined by the 'maximum matching' in the network. Based on this method, the structural controllability problem can be converted into an equivalent geometrical problem on a network: full control over a directed network can be realized if and only if each unmatched node is directly controlled and there are directed paths from the input signals to all matched nodes (Liu et al., 2011). The maximum matching algorithm in directed networks can be identified numerically in at most $O(N^{0.5}S)$ steps, where S is the number of links/edges (Liu et al., 2011). Hence, it is efficient to detect driver nodes for an arbitrary directed network and N_D can be easily found.

Liu et al. (2011) found that for several real networks, the number of driver nodes is determined mainly according to the network's degree distribution. Sparse heterogeneous networks, are hard to control, while dense and homogeneous networks can be controlled using a rather small fraction of nodes. An interesting result is that in both model and practical systems the driver nodes have a tendency to avoid the high-degree nodes, which coincides with the findings in Yu et al. (2009), Tang et al. (2012c, 2014c). For example, denote $n_D = N_D/N$, for a directed Erdős-Rényi network, n_D decays as

$$n_D \approx \exp^{-\frac{\langle k\rangle}{2}},$$

where $\langle k\rangle$ is the mean degree of the network and $\langle k\rangle$ is large. For scale-free networks with the degree exponent $\gamma_{in} = \gamma_{out} = \gamma$ in the large-$\langle k\rangle$ limit, n_D decays as

$$n_D \approx \exp\left[-\frac{1}{2}(1-\frac{1}{\gamma-1})\langle k\rangle\right],$$

A difference of Liu et al. (2011) from the results in Tang et al. (2012c, 2014c) is that (Liu et al., 2011) aims to find the minimum number of nodes to guarantee the stability of networks. In Tang et al. (2012c, 2014c), the allowable number of driver nodes is gradually changed and, therefore, different sets of driver nodes are identified, which rely on the number l of driver nodes. Hence, a transition of driver nodes from areas with a large degree to areas with a low degree can be observed in Tang et al. (2012c, 2014c). In addition, Tang et al. (2012c, 2014c) examine the controllability of a specific neuronal network and discuss the driver nodes from the perspective of neuroscience. The results in Tang et al. (2012c, 2014c) do not show their generality in other networks, which is a little bit different from the statistical results in Liu et al. (2011).

Based on this seminal work Liu et al. (2011), there are intensive and extensive works focusing on controllability of complex networks (Jia & Barabási, 2013; Jia et al., 2013; Menichetti, Asta, & Bianconi, 2014; Nepusz & Vicsek, 2012; Pósfai, Liu, Slotine, & Barabási, 2013; Ruths & Ruths, 2014; Sun & Motter, 2013; Suweis, Simini, Banavar, & Maritan, 2013; Yan, Ren, Lai, Lai, & Li, 2012; Yuan, Zhao, Di, Wang, & Lai, 2013) and their applications (Csermely, Korcsmáros, Kiss, London, & Nussinov, 2013; Notarstefano & Parlangeli, 2013). For instance, a dynamical process of controllability defined on the edges of a network is discussed, and it is shown that the controllability properties of this process is significantly different from simple nodal dynamics (Nepusz & Vicsek, 2012), as shown in Liu et al. (2011). The experiments performed on real-world networks demonstrate that most of them are easier to control than their randomized counterparts. The analytic computations reveal that networks with scale-free degree distributions have better controllability properties than uncorrelated networks, and positively correlated in and out-degrees are helpful to increase the controllability of the edge dynamics. In Pósfai et al. (2013), the effect of three topological characteristics is investigated for controllability of complex networks including clustering, modularity and degree correlations. It is verified by simulations and analytic analysis that the clustering coefficient and the community structure (modularity) have no significant impact on the minimum number of driver nodes N_D. Interestingly, degree correlations demonstrate a robust effect on controllability, which manifest the findings in Nepusz and Vicsek (2012). In Menichetti et al. (2014), it is revealed that the number of driver nodes in the network depends on the density of nodes with in degree and out degree equal to one and two. In Ruths and Ruths (2014), it is unveiled that standard random network models do not replicate the types of control profiles found in real-world networks. The profiles of real networks form three well-defined clusters, which sheds light on the high-level organization and function of complex systems.

As shown in Liu et al. (2011), Jia and Barabási (2013), there may exist multiple minimum driver node sets (MDSs) to control the whole network. Therefore, it is eminently important to quantify the roles of nodes participating in control of a network. A control capacity is then introduced to measure the possibility that a node serves as a driver node. A random sampling algorithm is presented to supply a statistical estimate of the control capacity and shorten the gap between multiple microscopic control circumstances and macroscopic characteristics of the network under control. The likelihood of acting as a driver node decreases with a node's in-degree and is independent of its out-degree (Jia & Barabási, 2013; Liu et al., 2011; Tang et al., 2012a, 2012b, 2014c, 2012c; Yu et al., 2009). Moreover, Jia et al. (2013), Liu et al. (2011) classified each node in a network based on their role in control as three categories: *critical*, meaning that a node must always be as a system (it is part of all MDSs); *redundant*, indicating that it is never required for control (does not take part in any MDSs) and *intermittent*, meaning that it serves as driver node in some particular control situations. An analytical and efficient algorithm is proposed to identify the category of each node. It is shown that two control types in complex networks emerge when $\langle k\rangle$ exceeds a critical value k_c: *centralized* versus *distributed* control. That is, $P(n_r)$ (n_r is the fraction of redundant nodes for an arbitrary network) shows a bimodal behavior, indicating that networks with the same degree distribution $P(k)$ can exist in two modes: some have small n_r and for others n_r is large. For centralized control, one can realize control through a small fraction of all nodes ($n_c + n_i$, where n_c is the fraction of critical nodes for an arbitrary network). For distributed control, most nodes should serve as driver nodes in some MDSs. Therefore, this phenomenon can also be viewed as a bifurcation diagram.

Although it has been widely recognized that the structural controllability theory (Lin, 1974; Liu et al., 2011) delivers a useful and efficient framework to control any arbitrary directed network, it is still paramountly important to consider a universal framework for exploring the controllability of complex networks if exact link weights are given (Liu et al., 2011; Yuan et al., 2013). Yuan et al. (2013) presents an exact controllability paradigm on the basis of the maximum multiplicity to identify the minimum set of driver nodes for complex networks, even when the link weights are explicitly given. Based on the Popov–Belevitch–Hautus (PBH) rank condition (Hautus, 1969), (7) is controllable if and only if

$$\mathrm{rank}(\psi I_N - A, B) = N, \tag{10}$$

is satisfied for any complex number ψ. Full control can be ensured for (7), if and only if any eigenvalue λ of matrix A satisfies (10). N_D is determined by B as $N_D = \min(\mathrm{rank}(B))$. Equivalently, N_D can also be obtained by the maximum geometric multiplicity $\delta(\lambda_i)$ of the eigenvalue λ_i of A:

$$N_D = \max_i(\delta(\lambda_i)), \tag{11}$$

where $\delta(\lambda_i) = N - \text{rank}(\lambda_i I_N - A)$ and $\lambda_i (i = 1, \ldots, \imath)$ represents the nonidentical eigenvalues of A. The eigenvalue of maximizing (11) is denoted by $\lambda^M = \arg\max_i(\delta(\lambda_i))$. In particular, for undirected networks, N_D can be computed by the maximum algebraic multiplicity $\rho(\lambda_i)$ of λ_i:

$$N_D = \max_i(\rho(\lambda_i)), \tag{12}$$

where $\rho(\lambda_i)$ is the eigenvalue degeneracy of matrix A. For a large sparse network, with a small fraction of self-loops, N_D can be obtained by the following equation in terms of the rank of A (Yuan et al., 2013):

$$N_D = \max(1, N - \text{rank}(A)). \tag{13}$$

By means of these calculations, a general strategy is presented to find the driver nodes by the PBH condition (10), in which ψ is replaced by λ^M. The results in Yuan et al. (2013) are general and can be applied to directed or undirected networks, with or without link weights and self-loops. The method can be simplified for dense or sparse networks, which makes it universal due to the wide existence of sparse networks in real world. In summary, the "exact controllability" serves as a compensation for "structure controllability" when the link weights are explicitly known, while "structure controllability" is to evaluate the controllability of directed networks when the link weights are not exactly given (Liu et al., 2011; Yuan et al., 2013).

Another interesting topic to establishing a connection between complex networks and control theory is to investigate how much energy is required to control complex networks (Rugh, 1996; Yan et al., 2012). For the control system (7) from an arbitrary initial state $x_0 \in \mathbb{R}^n$ to an arbitrary desired state x_{T_r} for $t \in [0, T_r]$, assuming that system (7) is controllable and define the energy cost as follows Rugh (1996):

$$U(T_r) = \int_0^{T_r} \|u(t)\|^2 dt, \tag{14}$$

and the optimal control input is

$$u_{\text{opt}} = B^T e^{A^T(T_r - t)} W_{T_r}^{-1} v_{T_r}, \tag{15}$$

where $W_{T_r} = \int_0^{T_r} e^{At} BB^T e^{A^T t} dt$ and $v_{T_r} = x_{T_r} - e^{AT_r x_0}$ represents the difference vector between the desired state under control and the final state during free evolution. For the sake of simplicity, the desired state $x_{T_r} = 0$ and the energy cost is rewritten as

$$U(T_r) = x_0^T H^{-1} x_0, \tag{16}$$

where $H(T_r) = e^{-AT_r} W_{T_r} e^{-A^T T_r}$ is the symmetric Gramian matrix (Rugh, 1996). The normalized energy cost is

$$\mathcal{U}(T_r) = \frac{U(T_r)}{\|x_0\|^2} = \frac{x_0^T H^{-1} x_0}{x_0^T x_0}. \tag{17}$$

According to Yan et al. (2012), the normalized energy cost is

$$\frac{1}{\sigma_{\max}} = \mathcal{U}_{\min} \leqslant \mathcal{U}(T_r) \leqslant \mathcal{U}_{\max} = \frac{1}{\sigma_{\min}}, \tag{18}$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximal and minimal eigenvalues of the positive definite (PD) matrix H, respectively (H is PD, if the system is controllable).

Hence, for weighted undirected networks and $B \in \mathbb{R}^{N\times 1}$, Yan et al. (2012) presents the lower bound of $\mathcal{U}_{\min}$:

$$\mathcal{U}_{\min} \begin{cases} \approx T_r^{-1}, & \text{small } T_r, \\ \approx \frac{1}{[(A+A^T)^{-1}]_{cc}}, & \text{large } T_r, A \text{ is PD}, \\ \sim T_r^{-1} \to 0, & \text{large } T_r, A \text{ is semi PD}, \\ \sim e^{(2\lambda_N T_r)} \to 0, & \text{large } T_r, A \text{ is not PD}, \end{cases} \tag{19}$$

where c is the only node under direct control; $A = VZV^T$, where V is the orthonormal eigenvector matrix that satisfies $VV^T = V^T V = I, Z = \text{diag}\{\lambda_1, \ldots, \lambda_N\}$ with descending order $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_N$.

The upper bound of $\mathcal{U}_{\max}$ can be derived as follows Yan et al. (2012):

$$\mathcal{U}_{\max} \begin{cases} \approx T_r^{-\phi} (\phi \gg 1), & \text{small } T_r, \\ \approx \varepsilon(A, c), & \text{large } T_r, A \text{ is ND}, \\ \sim T_r^{-1} \to 0, & \text{large } T_r, A \text{ is semi ND}, \\ \sim e^{(2\lambda_1 T_r)} \to 0, & \text{large } T_r, A \text{ is not ND}, \end{cases} \tag{20}$$

where 'ND' represents negative definite; $\varepsilon(A, c)$ is a positive value that depends on the matrix A and the controlled node c. The results can be generalized for weighted and directed networks.

Then (Pasqualetti, Zampieri, & Bullo, 2014) proposed a metric to quantify the control problem as a function of the required control energy. Upper bounds of energy cost are provided to characterize the tradeoff between the control energy and the number of control nodes. Sun and Motter (2013) shows that numerical success rate increases abruptly from zero to nearly one as the number of control inputs is increased, in which the control trajectories are usually nonlocal in the phase space, and their lengths are anti-correlated with the numerical success rate and number of control inputs.

In addition, Cornelius et al. (2013) proposes compensatory perturbations to directly drive a *nonlinear* network to a desirable state even when the system stays at an undesirable state. The method is based on the idea of region of attraction, which is an important concept in nonlinear systems (Khalil, 2002). An optimization algorithm based on sequential quadratic programming is employed to realize the compensatory perturbation. It is worth mentioning that the method can be applied to nonlinear systems, while the computational complexity needs to be reduced.

Although enormous efforts have been made to study controllability of complex networks, system (7) is a linear time-invariant system only. As mentioned in Liu et al. (2011), the controllability of linear systems can give some insights to that of nonlinear systems. However, most real systems are driven by nonlinear processes (Gao et al., 2008; Hespanha et al., 2007; Zhang et al., 2013), which can exhibit more rich dynamics (Khalil, 2002). Therefore, one future topic is to develop control objectives for nonlinear systems. In addition, for most real networks are time dependent (for example Internet traffic and flocking of animals (Vicsek & Zafeiris, 2012)), it is promising to investigate controllability of networks with time-dependent connections. Additionally, optimal control theory can also be applied to characterize the relationship between convergence speed, energy cost and features of complex networks (Rugh, 1996), which has been neglected in Yan et al. (2012), Yuan et al. (2013), Pasqualetti et al. (2014). Finally, combining the complex network theory into linear system theory (Rugh, 1996), nonlinear system theory (Khalil, 2002) or even stochastic system theory (Mao, 2007) will result in richer results.

2.2.3. Global controllability-Lyapunov function method

In the last two subsection, we have summarized the recent advances in controllability by means of linearization (Sorrentino et al., 2007; Tang et al., 2012c; Wang & Su, 2014 or classical controllability theory Liu et al., 2011; Yuan et al., 2013). Although both of them can characterize the controllability in a clear way, linearization methods (local controllability) suffer from the controllability defined on a neighborhood of a point. Structural controllability (Liu et al., 2011) or exact controllability (Yuan et al., 2013) mainly focuses on linear time-invariant systems. However, it is of great importance to investigate controllability of networks under more complicated environments such as nonlinearities and stochastic disturbances. Therefore, the Lyapunov-based analysis technique emerges as a competent one to carry out controllability analysis for such kind of complex networks, although most of the results are only sufficient but not necessary.

In Yu et al. (2009), synchronization via pinning control (controllability) of complex dynamical networks is investigated for strongly connected networks, networks with a directed spanning tree, weakly connected networks, and directed forests. A synchronization criterion is provided for strongly connected networks. It is revealed that the vertices with very small in-degrees should be controlled first. Moreover, it is uncovered that the nodes with very large out-degrees may be controlled. The synchronization condition is extended for achieving synchronization for networks with a directed spanning tree. The results show that the strongly connected nodes with very few connections from other nodes should be controlled and the nodes with many connections from other nodes can achieve synchronization even without external controllers.

Tang, Gao, Lu, and Kurths (2014b, 2014a) investigates the problem of pinning distributed synchronization of nonlinear dynamical networks with multiple stochastic disturbances and nonlinearities. Two types of pinning strategies are considered: (i) driver nodes are fixed along the time evolution; and (ii) driver nodes are switched from time to time according to a set of binary switching stochastic variables. For the case of fixed pinned nodes, a novel mixed optimization method is proposed to select the driver nodes and find feasible solutions, which is made up of a semi-definite programming method (Tang, Gao, Zou, & Kurths, 2013b) and a constraint optimization evolutionary algorithm (Tang et al., 2012c). For the case of switching pinning scheme, upper bounds of the synchronization error and the mean control gain are derived theoretically, which indicates that the synchronization performance is closely related to systems' parameters, the second smallest eigenvalue of the Laplacian matrix and the expectations of Bernoulli stochastic variables. Actually, the idea of switching pinning is similar to spatial pinning control (Frasca et al., 2012) and switching noise (Tang et al., 2013c). Note that a general random switching subject to Markov chain for dynamic systems is proposed in Zhang and Boukas (2009) where the more practical switching phenomenon, i.e., only partial information on the transition probabilities is available a priori, is taken into account; the difference and relation between nondeterministic and random switching is also revealed. Such switching underscored in Zhang and Boukas (2009) can efficiently balance the conservatism of worst-case switching scenarios, like arbitrary switching and the necessity of priori knowledge on the statistics of switching probabilities. The above mentioned switching among driver nodes can be therefore further modeled as the pattern proposed in Zhang and Boukas (2009) as well, with different degrees of known information on modes transitions depending on different concrete applications. Besides, Wu, Feng, and Lam (2013) investigates the synchronization problem for discrete-time neural networks with switching parameters and time-varying delays. As a result of the novel ideas of average dwell time and piecewise Lyapunov function, the proposed stability and synchronization conditions in Wu et al. (2013) are much less conservative than most of the existing results like Wu, Feng, and Zheng (2010), thus are very important and significant. By proposing a novel method named "average impulsive interval", in Lu, Ho, and Cao (2010), the authors derived a unified framework for synchronization of impulsive dynamical networks, and the obtained results were theoretically proved to be less conservative than previous results.

Although the results in Tang et al. (2012b, 2012c), Yu et al. (2009), Liu et al. (2011), Porfiri and Bernardo (2008) are obtained under distinct frameworks or different approaches, they share a common feature: nodes with small degree play an important role in controlling the entire network.

2.3. Observability of complex networks

Observability has been verified its importance in biology and engineering. For example, in systems biology, experimental access is limited to only a subset of variables of metabolite concentrations in a cell, which will limit the observations of a complete description of a system's state (Liu et al., 2011; Liu, Slotine, & Barabási, 2013). Another example is the observability of power-grid networks, which is a well-recognized significant problem nowadays. In power-grid networks, the state of the (complex) voltage at all nodes can be obtained by phasor measurement units. Real-time monitoring of measurement from phasor measurement units could have prevented major recent blackouts (Yang, Wang, & Motter, 2012). Although it seems that a rather comprehensive review of observability of complex networks is out of the scope of this survey, we still introduce some advances in this field, since observability is a dual problem of controllability of complex networks in control theory (Khalil, 2002; Rugh, 1996).

For system (7) with output $y(t) = Cx(t)$, (7) is observable if and only if the observability matrix $\mathcal{D}$

$$\mathcal{D} = [C^T, (CA)^T, \ldots, (CA^{N-1})^T]^T \tag{21}$$

satisfies $\text{rank}(\mathcal{D}) = N$. Similar to the controllability problem, a brute-force search for a minimum sensor set requires to check the $\text{rank}(\mathcal{D})$ for about 2^N sensor combinations, which is a computationally prohibitive task for large-scale complex systems (Liu et al., 2013). Hence, a fundamental problem is posed here: identify the minimum set of sensors such that one can reconstruct all other state variables from whose measurements. In Liu et al. (2013), a graphical approach is presented to determine the sensors that are necessary to estimate the full internal state of a complex system. The theoretical framework can be used to find the necessary sensors for an arbitrary nonlinear dynamical system, aiming at providing the lower bound of the number of system variables required to observe. In Yang et al. (2012), a new type of percolation transition, named as a network observability transition, is proposed to describe the size of the network's largest observable component. The results have been validated by both simulations and analytical analysis. In Fiedler et al. (2013), Mochizuki et al. (2013), the authors reveal that the long-time dynamics of the entire network is determined by observations only from a feedback vertex set (FVS). The results offer a useful criterion to select key molecules to control a network. It is also unveiled that controlling the dynamics of the FVS is sufficient to change the dynamics of the whole network from one state to others, different from the original one.

Like for controllability of complex networks, networked induced constraints like measurement noise, measurement uncertainties, packet dropouts, time-delays, time-varying sampling intervals and quantization (Gao et al., 2008; Hespanha et al., 2007; Zhang et al., 2013) will possibly increase the number of sensors to guarantee its robustness and feasibility. In addition, various performance indices can also be included to investigate observability of complex systems like H_∞ and/or H_2 performance.

2.4. Synchronization of multiplex networks

The real-world is linked by a complex mixture of networks through which communication, people and goods flow. The different levels of networks are interdependent on each other, and present structural or/and dynamical properties different from those observed in isolated networks (Gao, Buldyrev, Stanley, & Havlin, 2012; Gómez et al., 2013; Radicchi & Arenas, 2013). Such kind of networks are called interacting, interdependent, multiplex networks or networks of networks (Radicchi & Arenas, 2013). Typical examples for multiplex networks are as follows: social networks (for example, Facebook, Youtube, Twitter, etc.) are coupled because they share the same users; transportation networks are made up of different layers (for example, bus, subway, etc.) that have the same places; the functioning of communication and power grid systems relies one on the other (Radicchi & Arenas, 2013). Therefore, it is important to analyze the statistical behavior of multiplex networks, especially the dynamics of multiplex networks (Gómez et al., 2013; Radicchi & Arenas, 2013).

Consider the following multiplex network composed of M layers (Gómez et al., 2013; Radicchi & Arenas, 2013):

$$\dot{x}_i^m = D_m \sum_{j=1}^{N} w_{ij}^m \left(x_j^m - x_i^m\right) + \sum_{h=1}^{M} D_{mh}\left(x_i^h - x_i^m\right), \tag{22}$$

where w_{ij}^m denotes the weight matrix at layer m ($w_{ij}^m = 0$ meaning that there is no link between nodes i and j in layer m); D_m is the diffusion constant; and among nodes in different layers m and h, in this case with a diffusion constant D_{mh}. Let $M = 2$ for the sake of simplicity, i. e., we consider a multiplex network composed of two-layer networks. The network at each layer is assumed to be connected, undirected and weighted. Therefore, a supra-Laplacian matrix $\mathcal{L}$ can be constructed (Gómez et al., 2013; Radicchi & Arenas, 2013):

$$\mathcal{L} = \begin{pmatrix} D_1L_1 + pI & -pI \\ -pI & D_2L_2 + pI \end{pmatrix}, \tag{23}$$

where L_1 and L_2 are the Laplacian matrices of each layer; I is the identity matrix; $p = D_{12} = D_{21}$. The Laplacian matrix of each layer m is just $L_m = S_m - W_m$, where W_m is the weight matrix of layer m and S_m is a diagonal matrix containing the strength of each node i at layer m, $(S_m)_{ii} = s_i^m = \sum_{j=1}^{N} w_{ij}^m$.

The second smallest eigenvalue of the supra-Laplacian matrix $\mathcal{L}$ is that given by

$$\lambda_2(\mathcal{L}) = \begin{cases} 2p, & \text{if } p \leqslant p^*, \\ \leqslant \frac{1}{2}\lambda_2(L_1 + L_2), & \text{if } p \geqslant p^*, \end{cases} \tag{24}$$

where p^* is the critical value at which the transition of the crossing between two distinct behaviors of λ_2 occurs (Radicchi & Arenas, 2013). This indicates that

$$p^* \leqslant \frac{1}{4}\lambda_2(L_1 + L_2). \tag{25}$$

The importance of discussing the second smallest eigenvalue of the supra-Laplacian matrix $\mathcal{L}$ is that it refers to the algebraic connectivity and is one of the most significant eigenvalues of the Laplacian to measure epidemics and synchronizationability. It is strictly larger than zero only if the graph is connected. Similar results can be extended to $M \geqslant 2$ by following the analytic analysis in Radicchi and Arenas (2013), Gómez et al. (2013).

3. Applications

Synchronization or controllability of complex networks has been widely observed in natural systems such as transportation systems, chemical reactions and communications (Arenas et al., 2008; Pikovsky et al., 2001; Vicsek & Zafeiris, 2012). In this section, we mainly focus on two typical applications of synchronization in engineering (e.g. power grids) and neuroscience domains. Although synchronization or controllability has been applied in other fields such as serving as a novel paradigm of drug discovery in molecular networks (Csermely et al., 2013), understanding cancer progression and to develop effective anti-cancer therapies (Cornelius et al., 2013), associate memory (Cornelius et al., 2013) and biochemical reaction systems (Liu et al., 2013), they are not discussed here since they are out of the scope of this survey.

3.1. Synchronization of power grid networks

3.1.1. Stability of power grid networks

The modern power grid faces various challenges due to increasingly complex interconnections at the continent size and environmental incentives (Giannakis et al., 2014). The ideal power grid is conceived to have unprecedented awareness and controllability over its services and infrastructure to offer rapid and accurate diagnosis/prognosis, operation resiliency upon contingencies and deliberate attacks (Pasqualetti, Döfler, & Bullo, 2013), as well as continuous integration of distributed renewable energy resources (Giannakis et al., 2014). In Giannakis et al. (2014), a comprehensive review is provided for grid monitoring and optimization, which is out of the scope of this survey. In this survey, we focus on the stability of power grids.

In fact, stability problems of power grids can be treated into synchronization ones (Chiang, Chu, & Cauley, 1995; Ribbens-Pavella & Evans, 1985). Synchronization of power grids has been a classic topic in engineering since more than three decades ago (Ribbens-Pavella & Evans, 1985) and is long-standing and still on-going research efforts nowadays (Chiang et al., 1995; Machowski et al., 2008), which is an imperative factor for the functioning of a power-grid network. Typically, the stability of power grids can be classified into two categories: *transient stability* and *steady-state (small signal) stability*. Transient stability is concerned with a power network's capability of attaining an acceptable steady-state (operating point) following an event disturbance, which may arise from event disturbances and load disturbances. Steady-state (small signal) stability is carried out for linearization of power grids at operating point.

A review is presented in 1985 for large-scale electric power systems for transient stability analysis with two distinct methodologies (Ribbens-Pavella & Evans, 1985). The first approach investigates application of the direct Lyapunov method to the conventional transient stability analysis. The second methodology concentrates on the derivation of stability indices, aiming for online monitoring, contingency evaluation and security control. Following this survey, a subsequent comprehensive review for transient stability analysis is provided in Chiang et al. (1995). In Ribbens-Pavella and Evans (1985), Chiang et al. (1995), the authors analyze the advantage and disadvantages of two classical methods: time-domain method (or numerical integration method in Ribbens-Pavella & Evans (1985)) and direct method. Due to the deficiencies of time-consuming and no measure of degree of the time-domain method, control scientists concentrate on the direct method in terms of energy function. The progress of direct methods in network-reduction models and network-preserving models is detailed reviewed (Chiang et al., 1995). For the latter one, network-preserving models can also be analyzed in terms of the singular perturbation theory (Khalil, 2002).

However, due to the maturity of complex network theory and urgency of renewable energy nowadays, it is important to prompt further synchronization analysis or stability of networks of power grids under the frameworks of complex network theory and nonlinear system theory (Khalil, 2002). The understanding of how synchronization relies on the topology of power grids and to derive feasible strategies is of great scientific, ecological and economic interest to establish new transmission lines. A recent survey on synchronization of complex oscillator networks is given in Döfler and Bullo (2014). Some advances for studying synchronization of power grids are presented in Döfler and Bullo (2013), Döfler, Chertkov, and Bullo (2013) by using non-uniform Kuramoto oscillators. We do not focus on Kuramoto models based results of synchronization of power grids, since they have been in detail discussed and reviewed in Döfler and Bullo (2014). The concentration is on some important results of synchronization of power grids in complex networks, which are missed in Döfler and Bullo (2014). Nevertheless, it is worth pointing out that the results of Döfler and Bullo (2013), Döfler et al. (2013) not only establish a strong connection between the control theory and the nonlinear physics, but also present more accurate results and relax several assumptions in previous results of transient stability of power grids (Chiang et al., 1995). Döfler and Bullo (2013), Döfler et al. (2013) do not suppose relative angular coordinate formulations accompanied by a uniform damping, and they do not assume the transfer conductances to be "sufficiently small". In addition, based on nonlinear consensus protocols (Saber et al., 2007; Sepulchre, 2012) and synchronization theory, the conditions are presented, which can be interpreted as "the network connectivity has to dominate the network's nonuniformity and the network's losses" (Döfler & Bullo, 2013; Döfler et al., 2013).

3.1.2. Transient stability analysis

In power grids, the networks operate in a steady state when the consumption in customer side matches the generation from generator side. In Rohden et al. (2012), a bifurcation simulation firstly shows that normal operation (a fixed point) and power outage (the limit cycle) coexist in elementary model grids. The results on British power grid illustrates decentralization, by its more distributed nature, which promotes synchrony. When cascade failure happens in power grid networks, the distributed structure increases the robustness of the power grid while decentralizing power sources may moderately be deleterious to the grids' dynamic stability.

Since increasing energy demands and more strongly distributing power sources, the question of how to add new connection lines to the already existing grid arises naturally (Rohden et al., 2012; Witthaut & Timme, 2012). The effect of additional individual links on the emergence of synchrony is investigated in oscillator networks that model power grids on coarse scales (Witthaut & Timme, 2012). Direct simulations show that adding new links have two aspects at the same time: not only enhance but also be detrimental to synchrony, which can be treated as counter-intuitive phenomenon to Braess's paradox known for traffic networks. The underlying mechanism is mathematically analyzed in an elementary grid model and illustrates that it indeed happens across a wide range of systems. Therefore, upgrading the grid or adding new connections needs particular care owing to the potentially desynchronization of the grid and thus inducing power outages (Rohden et al., 2012; Witthaut & Timme, 2012).

In Menck, Heitzig, Kurths, and Schellnhuber (2014), based on basin stability proposed in Menck et al. (2013), synchronization of power grids networks has also been analyzed for maintaining robustness. In fact, basin stability falls into the scope of transient stability, which aims to characterize the region of attraction. Actually, the method used for basin stability in Menck et al. (2013), Menck et al. (2014) is the time-domain method in Chiang et al. (1995) or the numerical integration method in Ribbens-Pavella and Evans (1985). Based on the graph theory, it is found that dead ends and dead trees heavily diminish stability of power grids. The basin stability theory is applied to the Northern European power system, which confirms this result and verifies that the inverse also holds: repairing dead ends by adding a few transmission lines is conducive to stability of power grids. In Cornelius et al. (2013), a compensatory perturbation method is proposed for synchronizing power grid networks when a link in real power grid networks suffers from disconnection, which also utilizes the concept of the region of attraction. Therefore, it is very promising to revisit the transient stability (synchronization) of power grids by integrating the graph theory, the complex network theory and the nonlinear control systems theory. It is worth mentioning that the time-consuming issue of the time-domain method seems not to be as important as 30 years ago due to the rapid development of parallel computing. Consequently, the time-domain method can serve as a good complement for the direct method, as manifested in Menck et al. (2013), Menck et al. (2014).

3.1.3. Small signal stability (local synchronization)

In Motter, Myers, Anghel, and Nishikawa (2013), easily-to-verified conditions are derived for spontaneous synchrony in power-grid networks by a linearization method, which is actually *small signal* stability of power grids (Machowski et al., 2008). However, due to the introduction of the graph theory and the master stability function, new synchronization conditions are developed and the optimization of synchronization performance is also investigated (Motter et al., 2013). Consider the following swing equation to describe the dynamics of generator i:

$$\frac{2Q_i}{w_R}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei} \tag{26}$$

where δ_i is the rotational phase of the ith generator, Q_i is the inertia constant of the generator, w_R is the reference frequency of the system, P_{mi} is the mechanical power provided by the generator i and P_{ei} is the power demanded of the generator by the network. In the equilibrium, $P_{mi} = P_{ei}$ and frequency $w_i = \dot{\delta}_i$ remains equal to a common constant for all i.

By linearizing (26) around an equilibrium (synchronous) state, associated with the electrical power P^*_{ei} and mechanical power P^*_{mi} and represented by δ^*_i and w^*_i, and assuming that $\delta_i = \delta^*_i + \delta'_i, P_{ei} = P^*_{ei} + P'_{ei}$ and $P_{mi} = P^*_{mi} + P'_{mi}$, we get

$$\frac{2Q_i}{w_R}\frac{d^2\delta'_i}{dt^2} = \frac{\partial P_{mi}}{\partial w_i}w'_i - \frac{\partial P_{ei}}{\partial w_i}w'_i - \sum_{j=1}^{N}\frac{\partial P_{ei}}{\partial \delta_j}\delta'_j, \tag{27}$$

where the dependence of the mechanical power on changes in the phase δ_i, is denoted by $w'_i = \dot{\delta}'_i$. For the first term in the right hand side (RHS) of (27), the droop equation is $\frac{\partial P_{mi}}{\partial w_i} = -\frac{1}{w_R R_i}$, where $R_i > 0$ is a regulation parameter. For the second term of (27), suppose that there is a constant damping coefficient $D_i > 0$ such that $\frac{\partial P_{ei}}{\partial w_i} = \frac{D_i}{w_R}$. The third term of RHS of (27) can be obtained from

$$P'_{ei} = \frac{D_i w'_i}{w_R} + \sum_{j=1}^{N} E_i E_j \left(B_{ij}\cos\delta^*_{ij} - C_{ij}\sin\delta^*_{ij}\right)\delta'_{ij},$$

where $\delta^*_{ij} = \delta^*_i - \delta^*_j$ and $\delta'_{ij} = \delta'_i - \delta'_j, C_{ij}$ and B_{ij} are the real and imaginary components of Y_{ij} constituting the admittance Y, E_i is the internal-voltage magnitude of the ith generator. By reformulating the n-dimensional vectors of δ'_i and $\dot{\delta}'_i$ by X_1 and X_2, respectively, yields the following $2n$ first-order equations (Motter et al., 2013):

$$\dot{X}_1 = X_2,$$
$$\dot{X}_2 = -\mathcal{P}\dot{X}_1 - \mathcal{H}\dot{X}_2 \tag{28}$$

where $\mathcal{P} = (P_{ij})$ is the zero row sum matrix:

$$P_{ij} = \begin{cases} \frac{w_R E_i E_j}{2Q_i}(C_{ij}\sin\delta_{ij}^* - B_{ij}\cos\delta_{ij}^*), & i \neq j, \\ -\sum_{k\neq i} P_{ik}, & i = j, \end{cases} \tag{29}$$

and $\mathcal{H}$ is the diagonal matrix of elements $\beta_i = \frac{\left(D_i + \frac{1}{R_i}\right)}{2Q_i}$. Assuming $\beta_i = \beta$, (28) can be diagonalized using $\mathcal{Z}_1 = \mathcal{Q}^{-1}X_1$ and $\mathcal{Z}_2 = \mathcal{Q}^{-1}X_2$, where $\mathcal{J} = \mathcal{Q}^{-1}\mathcal{P}\mathcal{Q}$. The transformation leads to $\dot{\mathcal{Z}}_1 = \mathcal{Z}_2$ and $\dot{\mathcal{Z}}_2 = -\mathcal{J}\mathcal{Z}_1 - \beta\mathcal{Z}_2$.

Then, system (28) can be decoupled into $2N$-dimensional systems of the form:

$$\dot{\xi}_i = \begin{pmatrix} 0 & 1 \\ -\alpha_i & -\beta \end{pmatrix} \xi_i, \tag{30}$$

where $\xi_i = (\mathcal{Z}_{1j}, \mathcal{Z}_{2j})^T, \alpha_i$ is the ith eigenvalue of the coupling matrix $\mathcal{P}$.

The stability of the synchronous state is determined by the Lyapunov exponents of (30):

$$\lambda_{i\pm}(\alpha_i, \beta) = -\frac{\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 - 4\alpha_i}. \tag{31}$$

The synchronization state is stable if and only if the following conditions is satisfied:

$$\max_{\{\pm\}} \mathrm{Re}\lambda_{i\pm} \leqslant 0, \forall i = 2, \ldots, N.$$

Equivalently, the synchronization stability is solely determined by the master stability function $\mathrm{MSF}_\beta(\alpha) = \max_{\{\pm\}}\mathrm{Re}\lambda_\pm(\alpha, \beta)$, in which α is tunable.

The synchronization stability relies on the second smallest eigenvalue α_2 of $\mathcal{P}$. The optimum of $\mathrm{MSF}_\beta(\alpha_2)$ can be achieved at

$$\beta = \beta_{\mathrm{opt}} = 2\sqrt{\alpha_2}. \tag{32}$$

For engineering adjustment of the optimum, one can change the droop parameter R_i to

$$R_i = \frac{1}{4Q_i\sqrt{\alpha_2} - D_i}, i = 1, 2, \ldots, N,$$

or the damping coefficient D_i to

$$D_i = 4Q_i\sqrt{\alpha_2} - \frac{1}{R_i}, i = 1, 2, \ldots, N.$$

As suggested in Motter et al. (2013), the tuning of R_i is suitable for off-line optimization of stability, because the timescales of R_i are usually larger than those associated with typical instabilities. D_i is adjusted dynamically at very short timescales and hence is suitable for online adjustment and fine-tuning under varying operating conditions.

The results in Menck et al. (2013), Cornelius et al. (2013), Motter et al. (2013), Rohden et al. (2012), Witthaut and Timme (2012) confirm that synchronization of power grid networks is a challenge for ongoing research on smart grids, which could establish a bridge between physics, control theory and engineering. In addition, the understanding of robustness and optimization of power grid networks will give insights into design of more robust power grid networks (Bolognani & Zampieri, 2013; Giannakis et al., 2014; Motter et al., 2013), even provide rules for useful controllers when failures occur and further understand optimization and control of power grid dynamics (Cornelius et al., 2013; Guerrero, Vasquez, Matas, Vicuña, & Castilla, 2011; Schiffer, Ortega, Astolfi, Raisch, & Sezi, 2014; Simpson-Porco, Döfler, & Bullo, 2013).

3.2. Synchronization in neuroscience

Recent advances in structural and functional magnetic resonance imaging, diffusion tensor imaging, magnetoencephalography and electroencephalography and recent methods of complex network theory, promote the investigation of the brain's structural and functional systems. It has been shown that brain network has a spatial topology and representative properties of complex networks, such as the existence of highly connected hubs, small-world topology, and modularity-both at a whole-brain (a macroscopic level) and a cellular scale (a microscopic level) (Bullmore & Sporns, 2009; Bullmore & Sporns, 2012; Gu et al., 2014; Tang et al., 2013a; Zamora-López et al., 2010). It has been verified that synchronization of distributed brain activity plays a key role in neural information processing and coordination (Engel, Fries, & Singer, 2001; Palva, Monto, Kulashekhar, & Palva, 2010; Uhlhaas & Singer, 2006). From experiments, abnormal neural synchronization is found to be closely related to schizophrenia, epilepsy, autism, Alzheimer's disease, and Parkinson's disease. As illustrated in Palva et al. (2010), Tang et al. (2012c, 2012b, 2013a), Dahlem et al. (2013), it remains of great significance to study the synchronization or controllability of a neuronal network, which can not only offer a deep understanding of intrinsic features of synchronization or control weighted and directed networks, but also be beneficial to supply some suggestions to avoid abnormal synchronization to suppress neural diseases such as schizophrenia, epilepsy, autism, Alzheimer's disease, and Parkinson's disease (Uhlhaas & Singer, 2006).

Several recent developments in synchronization or controllability of neuronal networks can be referred to Bullmore and Sporns (2012), Tang et al. (2012c, 2012b, 2013a), Liu et al. (2011), Tang et al. (2014c) and references therein. Since some of them have been discussed in previous sections, we elaborate on the results in Tang et al. (2014c), which focuses on the controllability of neuronal networks with constraints on the average value of the control gains injected in driver nodes. The controllability problem with saturations is considered on a neuronal network of cats' brain, where nodes usually stand for brain regions with coherent patterns of extrinsic anatomical or functional connections, while links represent anatomical, functional, or effective connections and are differentiated on the basis of their weight and directionality (Tang et al., 2013a, 2012b, 2014c, 2012c). The connection matrix is academically extracted from several subtle steps including cortical parcellation, thalamic parcellation, collation of connection data and translation from database to connection matrix (Tang et al., 2014c). The cerebral cortex of a cat can be divided into 53 areas ($N = 53$), connected by about 830 links with different densities, as shown in Fig. 2. There are four topological clusters that are in accordance with four functional cortical communities: visual cortex (16 areas), auditory (7 areas), somato-motor (16 areas) and fronto-limbic (14 areas).

In Tang et al. (2012c, 2012b, 2013a), the controllability problem of neuronal networks is investigated without the consideration of saturations. However, actually, as demonstrated in Tang et al. (2014c), constraints on control gains should be involved for the study of controllability of neuronal networks. The significance of such considerations lies in twofold: (1) The first one is from the constraint on implementation of engineering equipment and biological background. Saturations in actuator exist widely in practical control systems like model predictive control and networked control systems, since a physical actuator can only produce bounded signals (Khalil, 2002); and (2) The second one is that only an appropriate control input could produce an ideal control performance. For example, in therapy, the patient's recovery largely depends on the dosage of antibiotics, where the input of dosage can be treated as control gains. On one hand, the excessive injection of dosage

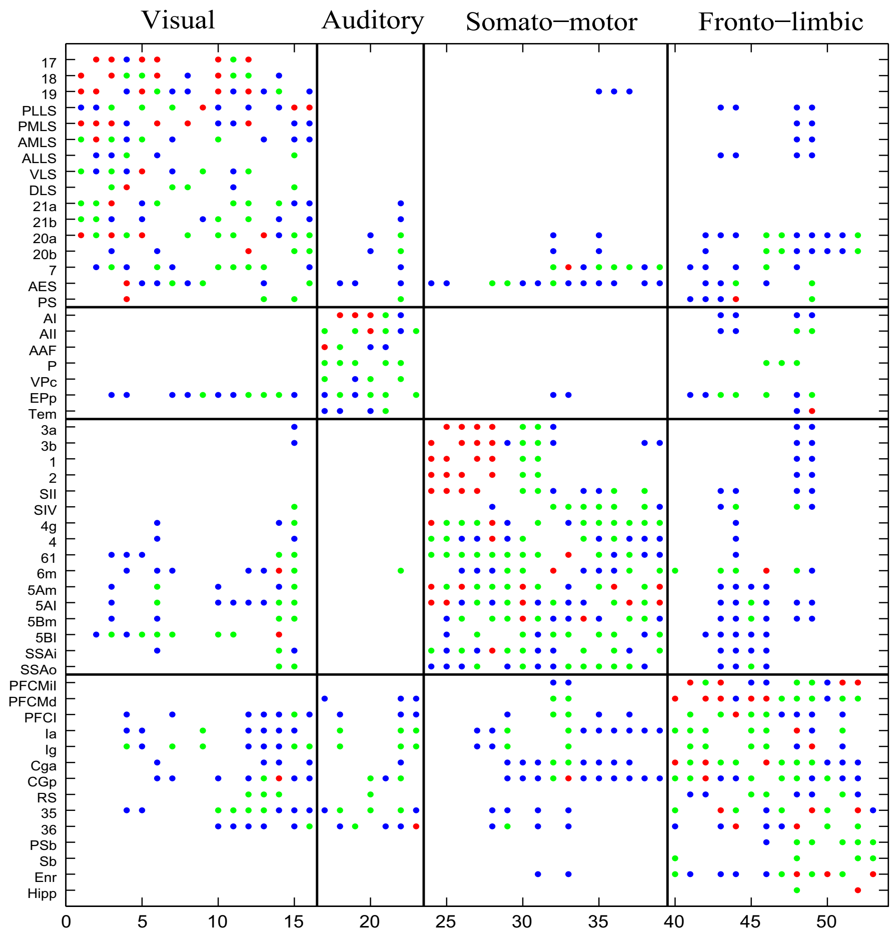

Fig. 2. The brain cortical network of the cat. The weighted adjacency matrix is shown (red: 3-dense, green: 2-intermediate, blue: 1-sparse). The matrix shows the partition of the network into four main modules (communities) of modally-related areas: visual, auditory, somatosensory-motor and fronto-limbic. The figure is from Tang et al. (2012c). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

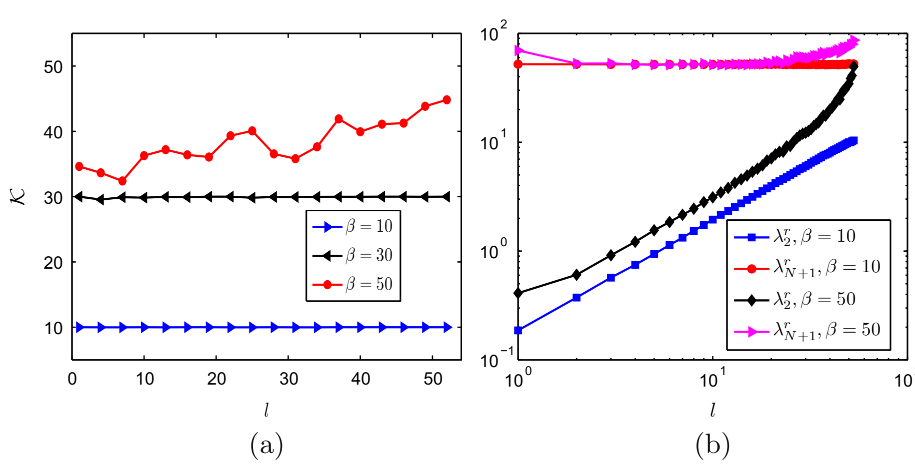

Fig. 3. Optimizing P by the MDyHF with different β as a function of l, when $\alpha = +\infty$. (a) Comparison of σ with different β as a function of l, when $\alpha = +\infty$; (b) comparison of λ_2^r and λ_{N+1}^r with different β as a function of l, when $\alpha = +\infty$. The figure is from Tang et al. (2014c).

of drugs will give rise to the creation of multidrug-resistant bacteria and finally no efficient antibiotics are available in various severe cases. Misuse of antibiotics can also have some damages on the beneficial bacteria and lead to immune system disorders in human body. On the other hand, a small injection of dosage will not be beneficial to patients' recovery and prolong the recovery time of patients. Based on these two respects, a suitable dosage should be injected to help therapy and not to upset the normal mechanism (Tang et al., 2014c).

In order to consider saturations in controllability of neuronal networks, $\mathcal{K}$ is used to denote the average of the control gains, which is formulated as follows:

$$\mathcal{K} = \text{mean}(\kappa_i), (i \in \mathcal{D}), \tag{33}$$

where $\text{mean}(\cdot)$ is the mean value operator. We transform the problem of controllability of a neuronal network into a constraint optimization problem, where P in (5) is the objective to be minimized and both σ in (6) and $\mathcal{K}$ in (33) are the constraints.

As in Tang et al. (2014c), two cases are investigated:

(i) The first case is as follows:

$$\begin{aligned} &\min\ P = \frac{\lambda^r_{N+1}}{\lambda^r_2}, \\ &\text{subject to:}\ q_1(y) \leqslant 0, \\ &\text{subject to:}\ q_2(y) \leqslant 0, \end{aligned} \tag{34}$$

where $q_1(y) = \sigma - \alpha, q_2(y) = \mathcal{K} - \beta, \alpha \in [0, +\infty)$ and $\beta \in [0, +\infty)$. If $\alpha = +\infty$, the problem considered here only focuses on the constraint on $\mathcal{K}$ and minimizes P. If $\alpha \neq +\infty$, the controllability problem concentrates on the constraint on $\mathcal{K}$ and σ simultaneously and minimizes the objective P.

(ii) The second case is:

$$\begin{aligned} &\min P = \frac{\lambda^r_{N+1}}{\lambda^r_2}, \\ &\text{subject to:}\ h_1(y) = 0, \\ &\text{subject to:}\ q_1(y) \leqslant 0, \end{aligned} \tag{35}$$

where $h_1(y) = \sigma - \alpha, \alpha = 0$ and $q_1(y) = \mathcal{K} - \beta$. In this case, the constraint $\sigma = \alpha = 0$ implies that the controllability of networks is only lying on P.

In Tang et al. (2014c), a modified dynamic hybrid framework (MDyHF) is proposed to deal with the controllability problem of neuronal networks with constraints on the average of control gains. The main results are shown in Fig. 3. Fig. 3(a) shows that when $\beta = 50$, the allowable control gains are sufficient, i. e. $\mathcal{K} < 50$. However, different from $\beta = 50$, when $\beta = 10$ and $\beta = 30$, the control gains should be used completely to enhance the controllability of the neuronal network, i. e. $\mathcal{K} = 10$ or $\mathcal{K} = 30$, respectively. Meanwhile, it is necessary to allocate the dosages to each node to make the controllability maximal. The observations unveil that there exists an intermediate control cost to maximize controllability of neuronal networks, which demonstrates other similar phenomena in biological observations and engineering background.

In addition, in Gu et al. (2014), it is shown that densely connected areas, particularly in the default mode system, are beneficial to the movement of the brain to various easily-reachable states. Weakly connected regions, particularly in cognitive control systems, are conductive to the movement of the brain to difficult-to-reach states. Regions located on the borders between network communities, particularly in attentional control systems, contribute to the integration or segregation of diverse cognitive systems.

4. Conclusions

In this paper, investigations on recent several important topics in synchronization of complex networks are mainly surveyed, including robustness in synchronization, controllability of complex networks, observability of complex networks, synchronization of multiplex networks and two important fields of applications of synchronization in power grids and neuroscience. In Section 2.1, we discuss the importance of the region of attraction in complex networks and show that there may exist a trade-off between synchronizability and "basin ability". In Section 2.2, we propose to classify recent works in controllability of complex networks into three categories: local controllability, global controllability-structural controllability and global controllability-Lyapunov function method. Extensive works in enhancing controllability or related topics have been reviewed and the connection between control theory and complex theory has been discussed. The applicability or the limitations of the developed approaches have not been commented on to some extent. In Section 2.3 and 2.4, observability of complex networks and synchronization of multiplex networks have been surveyed, respectively. Finally, we are aware that it is hardly possible to cover all contributions in applications of synchronization of complex networks; therefore, attention is paid to synchronization of power grids and neuronal networks and related control issues of the vast literature, as shown in Section 3.

Despite diverse results have been reported, there are several challenges that should be investigated in future research. We highlight some of them as follows:

(i) For studies of synchronization of complex networks, it is interesting to introduce distinct networked-induced constraints (Gao et al., 2008; Hespanha et al., 2007; Zhang et al., 2013) into the frameworks of synchronization such as time delays, time-varying sampling intervals, packet dropouts, saturations, communication noises and quantization errors, where the networked-induced constraints can be modeled either in a deterministic or a stochastic way.

(ii) It is promising to apply methods from control theory to synchronization of complex networks such as linear system theory (Rugh, 1996), nonlinear system theory (Khalil, 2002) and stochastic system theory (Mao, 2007). Further, synchronization of complex networks can be considered under various performance indices such as H_∞, H_2 and L_p, etc. In addition, statistical information in different scales (microscopic, mesoscopic and macroscopic scales) can also be introduced into the traditional control theory to capture the key points of complex systems, thus facilitating the in-depth understanding of large-scale networked systems. The usage of these tools would provide solutions that are not only capable of dealing with different kinds of complexities in complex networks but also provide a more exact way to understand the synchronization of real world complex networks.

(iii) In contrast to model-based approaches, data-driven control employs the information obtained from the available measurements to describe various complex behaviors (Yin, Ding, Xie, & Luo, 2014), thus has provided an efficient strategy for control issues in complex engineering applications. As for *heterogenous* complex networks (Zhang, Tang, Wu, & Fang, 2014), the self-dynamics of some nodes may be unknown and time-varying under complicated circumstances. Hence, it is meaningful to design controllers to realize synchronization based on the analysis of historical data.

(iv) Intelligent methods, such as neural networks and fuzzy systems, can be adopted for modeling dynamics of complex networks. Additionally, single objective/multiobjective evolutionary algorithms and constraint evolutionary algorithms are promising to serve as a candidate to handle complicated optimization problems in complex networks like controllability problems.

(v) It should be mentioned that the comprehensive studies integrating some topics above are not sufficient yet, especially for controllability of interdependent complex networks and robustness in controllability of complex networks (Bakule, 2014). It would be challenging and promising to simultaneously consider controllability, robustness, multiple layers, especially applying the results in robotic systems, neuroscience and power grids.

(vi) Power grids are becoming more distributed, intelligent, and flexible (Bolognani & Zampieri, 2013; Guerrero et al., 2011). Nowadays, since small distributed power generators and dispersed energy-storage devices are required to be added into the power network, smart grids are proposed to deliver electricity from suppliers to consumers to save energy, thus lessening cost and improving reliability and transparency. Owing to the emergence of microgrids in nowaday power grids, it becomes more important to utilize the droop controller to avoid circulating currents among the converters without using any critical communication (Guerrero et al., 2011; Schiffer et al., 2014; Simpson-Porco et al., 2013). Hence, it is imperative to utilize complex network theory and control theory to enhance the controllability of power grids by employing hierarchical control (Guerrero et al., 2011; Schiffer et al., 2014; Simpson-Porco et al., 2013).

Acknowledgements

The authors are grateful to the Editor and the anonymous reviewer for their helpful comments. This research is supported by the National 973 Project (2012CB720500), the National Natural Science Foundation of China (61203235, 61333012, 61333010, 61273201), the Key Laboratory of Integrated Automation for the Process Industry (Northeastern University), IRTG 1740 (DFG) and the Alexander von Humboldt Foundation of Germany.

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