

Synchronized pendula: From Huygens' clocks to chimera states

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Abstract. This topical issue collects contribution exemplifying the recent scientific progress in understanding the dynamics of coupled pendula. The individual papers focus on different questions of present day interest in theory and applications of systems of coupled oscillators. Both theoretical and experimental studies are presented.

In 1665 Ch. Huygens noticed the antiphase synchronization of two pendulum clocks mounted together on the same beam [1–3]. This was the beginning of nonlinear science and one of the first observations of the phenomenon of coupled harmonic oscillators, which has been since used to model various systems in nearly all branches of science. Synchronization has become a basic concept of the interdisciplinary field of nonlinear and complex systems science [4]. In the light of recent experimental and theoretical progress in this field there has been a great deal of interest in pendula systems in the last two decades leading to a new understanding of the classic phenomenon but also to new phenomena as chimera states and opening new directions of applications. In particular Huygens' experiment has been rediscussed by a few groups of researchers [5–24]. To explain his observations special experimental devices have been built. One of them consists of two interacting pendulum clocks hanged on a heavy support which was mounted on a low-friction wheeled cart [6]. The device moves by the action of reaction forces generated by the swing of two pendula and the interaction of the clocks occurs due to the motion of the clocks' base. It has been shown that to repeat Huygens' results, high precision (the precision that Huygens certainly could not achieve) is necessary. Another device, the so-called “coupled pendula of the Kumamoto University” [16], consists of two pendula whose suspension rods are connected by a weak spring and one of the pendula is excited by an external rotor. Finally it has been shown that two real mechanical clocks when mounted to a horizontally moving beam can synchronize both in-phase and antiphase [7, 10, 24]. Synchronization is achieved due to the energy transfer via the oscillating beam.

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Meanwhile, subsequent studies of large systems of coupled pendula identified new interesting phenomena, such as clustering [8,9,24] or chimera states. Particularly experimental studies with the use of metronomes are very promising and can lead to a full understanding of the observed phenomena [8,9,12,19,22,23]. Various types of coupling structures lead to different synchronous states. Its identification and understanding of the energy transfer between pendula is of prime interest.

This special issue on Synchronization of Pendula Systems consists of 15 original papers covering various aspects and recent achievements of the dynamics of coupled pendula and their applications. These papers can be grouped into 3 categories, namely, surveys [25–27], qualitative analysis of coupled pendula systems [28–34], and applications in mainly engineering systems [35–39].

In [25] the energy balance of synchronized pendula is discussed. It is shown how simple mechanical principle allows an identification of the synchronized states and derivation of the approximate analytical conditions for the energy transfer between coupled oscillating and rotating pendula. As an example a system of two coupled double pendula has been used.

The applications of metronomes in experimental studies of coupled pendula are presented in [26]. Metronomes are used for the description of order (spontaneously synchronized beats)-disorder (unsynchronized swings) type phase-transitions.

Methods of modeling coupled spherical pendula are discussed in [27]. Lagrange's multipliers have been used in the derivation of the equations of motion here. Two different synchronous states in which both pendula rotate in the same and opposite directions have been identified and observed in a simple experiment.

The results previously obtained for a model of the Huygens's two pendulum clocks, as well as a model for synchronization mediated by an elastic media are extended to arrays, finite or infinite, of conservative pendula coupled by elastic forces in [28]. Various types of synchronization are described for a finite number of pendula and in the infinite limit, the pattern of synchronization is described by a quasi-periodic longitudinal wave.

In [29] control techniques for noise self-sustained oscillators are studied. The author focuses on the uncertainty principle and shows how the reliability and coherence of noisy limit-cycle oscillators can be controlled by a general linear feedback. The obtained results are applied to a system of coupled pendula.

The properties of an iterative map for the study of coupled pendula are described in [30]. The author has developed a generic iterative map model of coupled oscillators based on simple physical processes common to many such systems. The model allows us to understand, from a unified perspective, the range of different outcomes in systems of coupled oscillators.

The effect of the coupling strength on the basin stability of the coupled metronomes is studied in [31]. For two coupled non-identical metronomes it has been found that the coupling strength linearly decreases the basin stability of in-phase synchronization and increases that of the anti-phase synchronization. This effect has been observed both numerically and experimentally.

The dynamics of co- and counter-rotating coupled spherical is considered in [32]. The existence of three rotating linear modes allows understanding of the nonlinear normal modes, which are visualized in frequency-energy plots. Additionally, the energy transfer between pendula is analyzed.

Although it was long thought that synchrony and disorder were mutually exclusive steady states for a network of identical oscillators, numerous theoretical studies in recent years have revealed the intriguing possibility of "chimera states", in which the symmetry of an oscillator population is broken into a synchronous part and an asynchronous part. Paper [33] shows the possibility of the occurrence of chimera states in two coupled populations of pendulum-like oscillators.

A theoretical multidimensional physical system modeled as a chain of nonlinearly coupled chaotic pendula is studied in [34]. In extensive numerical simulations various synchronous states have been identified and described.

A controlling technique which allows the synchronization of a network of dynamical systems described by second-order ordinary differential equations is developed in [35]. The efficiency of the method has been shown in experiments on pendula systems.

State estimation and synchronization of pendula systems over digital communication channels is discussed in [36]. Theoretical results as well as the quality of the state estimation are compared with experiments with multi-pendula systems.

In [37], the authors have studied experimentally control methods of a parametric pendulum excited harmonically to initiate and maintain a period-one rotation. This research is motivated by a possible application of the rotating pendula in energy harvesting devices.

Paper [38] shows that three motors located on the same rectangular plate can enter into synchronization with different phase differences depending on the physical characteristics of the motors and the plate. The motors are considered as non-ideal oscillators which act as an external excitation on a specific area of the plate. The analysis of the plate's oscillations indicates through numerical simulations that one can obtain a reduction of oscillations when the motors rotate in different directions.

A possible application of coupled rotating pendula in aerospace engineering (in modeling helicopters) is presented in [39] where a model of a nonlinear system composed of a hub with attached two pendula is studied. Complete, phase synchronization, as well as the bifurcation and the transition through resonances are analysed.

Thus, this special issue provides a wide spectrum of current research on coupled oscillators (particularly coupled pendula systems), and we hope that related researchers in this field will find it useful. We wish to express our appreciation to the authors of all the papers in this special issue for the excellent contributions as well as many reviewers for their high-quality work on reviewing the manuscripts.

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