Topological Identification of Weak Points in Power Grids

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***Abstract*—Stability of a power grid's synchronous operating mode is crucial to its reliable function. We quantify the stability of this operating mode in different humble power grid layouts that we numerically simulate employing a widely used electro-mechanical model. The method with which we quantify stability delivers a single number, called basin stability, for each node in a grid. A node with a poor basin stability is a weak point, as a rather small perturbation to this node would suffice to destroy the synchrony of the whole system and make it collapse. Using tools from the theory of complex networks, we statistically evaluate an ensemble of grids to identify topological classes of nodes whose members typically have the same (poor or large) value of basin stability.**

I. Introduction

Failure of a power grid can impose huge economic losses on society. Therefore research into understanding and preventing such failures is highly relevant. A power grid can only function properly if it is physically connected, i.e., if each of its *nodes* (generators, substations, buses) is linked to the others by at least one *edge* (transmission line). This observation motivated a series of studies that aim at identifying vital components whose loss would heavily impair connectivity, possibly by triggering cascading failures [1–3]. Whereas these studies focus mostly on (topological) network properties of the grid, there is also a vast literature on the electrodynamical aspects of its stability [4, 5]. The AC voltage at each node alternates in time with a certain *nodal frequency* that turns out to be a crucial quantity. Indeed, operators strive to run a power grid always as closely as possible to the *synchronous operating mode* $\mathcal{M}_s$ in which all nodal frequencies are equal to the grid's rated frequency (50 Hz in Europe). Deviation from $\mathcal{M}_s$ endangers the grid's function, as it might entail resonance damage to mechanical appliances and thereby provoke failure of components.

The synchronous operating mode $\mathcal{M}_s$ can only be maintained if it is stable against perturbations that persistently occur in the grid – either in the form of minor adjustments (e.g., a load being switched off) or in the form of major faults (e.g., a transmission line being tripped after a short-circuit). The problem whether a power grid returns to $\mathcal{M}_s$ after a particular perturbation is addressed by *transient stability analysis*. Basically, transient stability analysis deals with the *basin of attraction* $\mathcal{S}$ of $\mathcal{M}_s$. $\mathcal{S}$ is the set of points in state space from which the grid's dynamics converge to $\mathcal{M}_s$. If the state of the grid falls into $\mathcal{S}$ after a perturbation, $\mathcal{M}_s$ is said to be stable against this perturbation [5]. Note that due to its high-dimensionality, knowledge on $\mathcal{S}$ is quite hard to gather.

Only recently have researchers turned to the question how properties of $\mathcal{S}$ are related to the network characteristics of the power grid [6]. Indeed, are there kinds of network structures that extraordinarily enhance or deteriorate the stability of $\mathcal{M}_s$? *Filatrella et al* have compared the stability of different very small (3 to 6 nodes) power grid layouts with respect to perturbations of varying severity [7]. *Dörfler et al* have concentrated on idealized *all-to-all*-coupled yet weighted power grids, for which they have deduced rigorous analytical conditions that *qualify* $\mathcal{M}_s$ as stable against a certain group of perturbations [8].

In this paper we seek to *quantify* the stability of $\mathcal{M}_s$ in different power grids of N nodes. More specifically, for each node $i \in \{1, \dots, N\}$ in a grid we estimate the *basin stability* S_i, a number that captures the probability that the grid returns to $\mathcal{M}_s$ after a random perturbation that initially only affects node i [9]. A node with a rather poor value of S can be seen as a weak point, as a rather small perturbation to this single node would suffice to destroy the synchrony of the whole grid (without which it cannot function). To topologically categorize weak (and strong) points we apply the tools of complex network theory to an ensemble of randomly created power grids with $N = 64$ nodes [10].

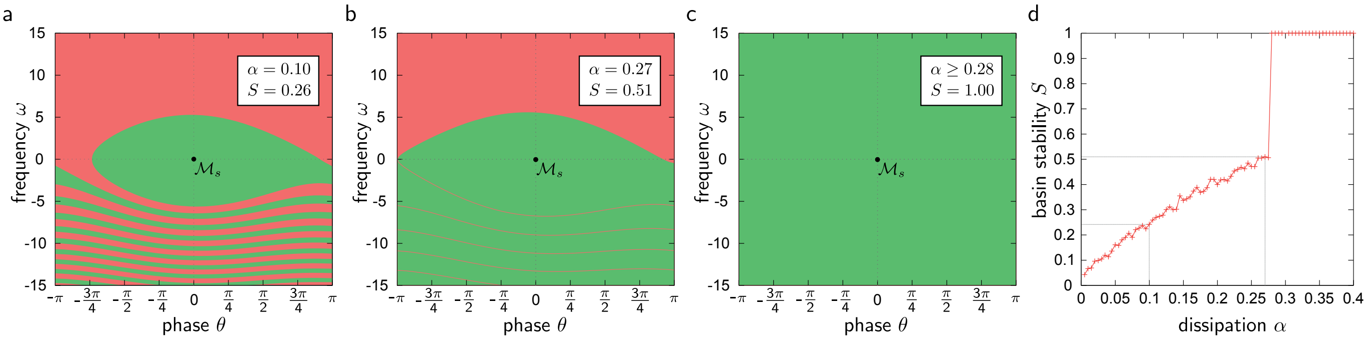

Fig. 1. **Results for the generator – infinite-busbar model. a-c:** the green colour indicates the basin of attraction $\mathcal{S}$ of the synchronous operating mode $\mathcal{M}_s$ for $\alpha = 0.1, 0.27, 0.3$ at $P = 1$ and $K = 8$. The red colour indicates non-synchronizing initial values. **d:** Basin stability S of $\mathcal{M}_s$ against α at $P = 1$ and $K = 8$. Recall that S is measured relative to $B := [\theta_s - \pi, \theta_s + \pi] \times [-100, 100]$.

II. The Swing Equation Model

Consider a power grid made up of N nodes. We represent the AC voltage voltage at node i by $V_i \exp(\mathrm{i}\theta_i)$ with magnitude V_i and phase θ_i. Furthermore, we describe a transmission line connecting nodes i and j by its impedance $Z_{ij} = R_{ij} + \mathrm{i}X_{ij}$. Here we concentrate on the transmission (as opposed to distribution) part of the grid where typically $R_{ij}/X_{ij} \ll 1$. Therefore we take all lines to be lossless ($R_{ij} = 0$). Then the power transferred from i to j is given by

$$\frac{V_i V_j}{X_{ij}} \sin(\theta_i - \theta_j).$$

This is one ingredient of the classic *swing equation model* that has been widely used as a tool to understand power grids. In a simplified version of this model, the dynamics of the generator at node i are given by

$$\dot{\theta}_i = \omega_i \tag{1}$$

$$\dot{\omega}_i = P_i - \alpha\omega_i + K \sum A_{ij} \sin(\theta_j - \theta_i). \tag{2}$$

Here, θ_i is the phase at node i measured in a reference frame that co-rotates with the grid's rated frequency Ω_r. Furthermore, ω_i denotes i's frequency *deviation* from Ω_r and P_i labels its *net power input*. A_{ij} is the adjacency matrix with $A_{ij} = 1$ if nodes i and j are connected and $A_{ij} = 0$ otherwise. For simplicity, we assume all generator properties, voltage magnitudes and line reactances to be the same. Therefore the *dissipation constant* α is the same at all i. Similarly, the *coupling constant* K is the same for all transmission lines. *Loads* do not appear in this model as individual nodes, but are treated as static power sinks connected directly to the generators whose *net* power inputs P_i they affect [11]. An equilibrium may only be reached if $\sum_i P_i = 0$. We refer to nodes with $P_i < 0$ as net *consumers*.

III. Basin Stability – An Illustration

As a first illustration, we study a textbook example: the *generator – infinite-busbar model*

$$\dot{\theta} = \omega \tag{3}$$

$$\dot{\omega} = P - \alpha\omega - K \sin(\theta_i - \theta_{\text{grid}}). \tag{4}$$

In this model, a single generator is connected to a grid that is *infinite* in the sense that it absorbs any amount of power $P > 0$ that the generator injects without deviating from the rated frequency Ω_r, i.e., $\omega_{\text{grid}} \equiv 0$. Hence also always $\theta_{\text{grid}} \equiv 0$. For any $K > P$ the synchronous mode $\mathcal{M}_s = (\theta_s = \arcsin(P/K), \omega_s = 0)$ is linearly stable against small perturbations (assuming $|\theta_s| \leq \pi/2$) [4]. To determine *how stable* it is, we calculate its basin stability [9]: We randomly draw $T = 500$ initial value vectors (θ^0, ω^0) from the box $B := [\theta_s - \pi, \theta_s + \pi] \times [-100, 100]$, integrate eqns. (3,4) for all of them and count the number I of initial value vectors from which the dynamics converge to $\mathcal{M}_s$. Then $I/T \in [0, 1]$ estimates the basin stability S of $\mathcal{M}_s$. In Fig. 1, a plot of the dependence of S on the dissipation α (panel **d**) is accompanied by a few impressions of the basin $\mathcal{S}$ in this two-dimensional system (panels **a-c**). Basin stability S increases quite monotonically from about 0.05 to 0.5 in the interval $\alpha \in [0.01, 0.27]$. Then S suddenly jumps to 1.0. Panels **a-c** reveal that the generator ($P > 0$) becomes much more stable against *negative* frequency perturbations as α increases towards 0.27 with no similar effect for positive frequency perturbations.

IV. Ensemble of Power Grids

We now apply the same technique to an ensemble of power grids. More specifically, we investigate the statistical properties of 5387 grids that consist of $N = 64$ nodes among which $E = 80$ edges are placed uniformly at random (no loops or double edges permitted). Note that this results in networks in which every node has on average 2.5 links, similar to what ref. [12] reports for real power grids. Half of the nodes are net generators with $P_i = +P$ and the other half are net consumers with $P_i = -P$. For each grid we identify the synchronous operating mode $\mathcal{M}_s$ in which the (differences of the) phases $\theta_{s,i}$ are such that $\dot{\omega}_i = 0$ for all i and $\omega_{s,i} = 0$ for all i. Then we calculate the N *single-node basin stabilities* S_i, $i \in \{1, \dots, N\}$ as follows [9]. We create $T = 500$ initial value vectors $(\theta_{s,1}, \omega_{s,1}, \dots, \theta_i^0, \omega_i^0, \dots, \theta_{s,N}, \omega_{s,N})$ with (θ_i^0, ω_i^0) drawn randomly from B, integrate eqns. (1,2) for all of them, count the number I_i of arrivals at $\mathcal{M}_s$ and estimate S_i by $I_i/T \in [0, 1]$. S_i quantifies the stability of $\mathcal{M}_s$ against perturbations that initially only affect node i while leaving the other state variables at their synchronous-mode values. If $S_i > S_j$ for two nodes i and j, we say that i is *more stable* than j. In the following we present statistics for the $5783 \cdot 64 = 370112$ realizations of S that we computed in the ensemble.

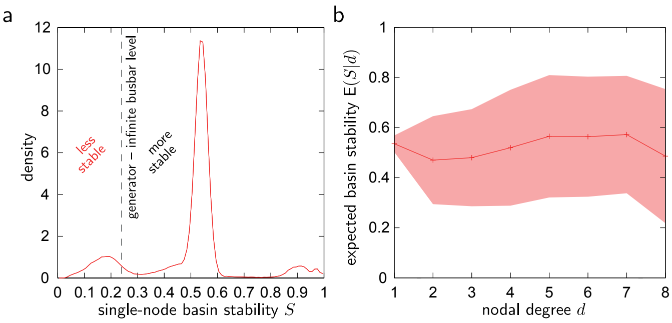

Fig. 2. **Ensemble Results I. a:** Histogram of single-node basin stability S. About 15% of the realizations reside in the left peak $[0, 0.3]$, about 78% in the middle peak $[0.3, 0.8]$ and about 7% in the right peak $[0.8, 1]$. The dashed line indicates the basin stability of a generator connected to an infinite busbar. **b:** Expected basin stability $\mathsf{E}(S|d)$ at nodal degree d. The shade indicates the standard deviation. Results were computed for $\alpha = 0.1$, $K = 8$ and $P = 1$.

Firstly, we look at the histogram of S that is shown in Fig. 2a. It turns out that about 85% of the nodes are *more stable* than the generator in the infinite-busbar model (cf. Fig. 1a,d). Indeed, about 78% of nodes reveal basin stability values close to 0.5 and also show single-node basins similar to Fig. 1b (though reversed for net consumers which are highly stable against positive frequency perturbations). Around 7% of the nodes even show the maximum basin stability 1.0. In the infinite-busbar model, such values of S could be achieved by strongly increasing the dissipation α. This supports the conlusion that the electromechanical interactions possible between nodes in power grids (and impossible in the infinite-busbar model) significantly enhance basin stability – a positive network effect.

Now that we witnessed some nodes being more and others less stable, can we tell them apart just by identifying their different *topological roles* in the network? A first guess at a nodal property that might be important to S is *degree*: d_i counts the number of edges that node i is an endpoint of. However, Fig. 2b shows that a node with higher d cannot be expected to have a larger S.

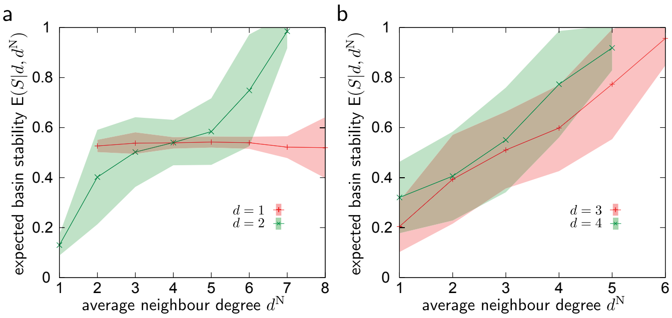

Fig. 3. **Ensemble Results II.** Expected basin stability $\mathsf{E}(S|d, d^{\mathsf{N}})$ of nodes with degree d whose neighbours' average degree is d^{N}. Shades indicate the standard deviation. **a** shows this for $d = 1, 2$, **b** for $d = 3, 4$.

A more insightful characteristic is shown in Fig. 3: The expected basin stability $\mathsf{E}(S|d, d^{\mathsf{N}})$ of a node of degree d whose neighbours have average degree d^{N}. Clearly, for nodes with $d = 1$ there is not much influence of d^N on $\mathsf{E}(S|d, d^{\mathsf{N}})$. But for $d \geq 2$, $\mathsf{E}(S|d, d^{\mathsf{N}})$ improves strongly as d^{N} increases; a node with degree $d = 3$ and average neighbour degree $d^{\mathsf{N}} = 6$ is expected to have a basin stability of 0.98!

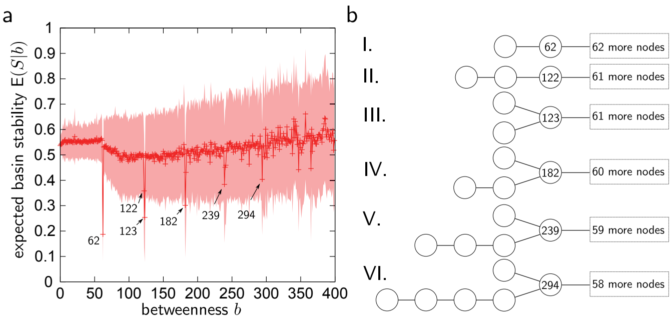

Fig. 4. **Ensemble Results III. a:** Expected basin stability $\mathsf{E}(S|b)$ of nodes with betweenness b. The shade indicates the standard deviation. **b:** Illustration of particular values of nodal betweenness.

Finally, we study the impact of a node's *shortest path betweenness* b on its expected stability. b_i is equal to the number of shortest paths between the grid's nodes that pass through i [10]. Fig. 4 displays that for most values of b there is no significant effect on $\mathsf{E}(S|b)$. Yet for certain b there is a strong effect: The illustrations in Fig. 4b reveal that nodes belonging to a mere *appendix* of the bulk grid have a low expected stability.

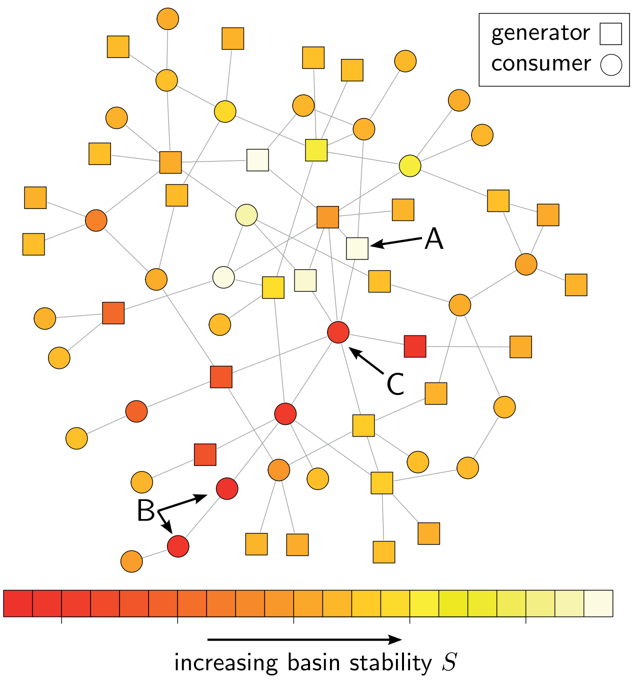

Fig. 5. **Image of one power grid from the ensemble.** Boxes represent net generators ($P_i = P$), circles net consumers ($P_i = -P$). A shape's colour indicates the basin stability of the represented node.

V. Conclusion & Outlook

Our statistical study of single-node basin stability in power grids revealed some interesting insights. Firstly, the comparison of generators in a grid to the generator – infinite-busbar model disclosed that the electromechanical interactions in the former significantly improve basin stabilty – a positive network effect. Secondly, we found that a node's expected stability is decisively influenced by its neighbours' average degree (but not by its own degree). This helps to understand why node A in Fig. 5 is so stable. Thirdly, the statistics on shortest path betweenness identified nodes residing in *appendices* to the bulk grid as rather unstable. Examples are indicated by B in Fig. 5. However, our results do not explain why node C is it so unstable. This demands further research. It might be interesting to incorporate the two types of nodes (generators, consumers) into the statistics, e.g., via a *similarity* measure [13]. Alternatively, one could extend single-node basin stability to two-node basin stability. This would be in the spirit of ref. [9]. Ultimately, all this should be applied to realistic power grid models.

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