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Generalized synchronization between two different complex networks

Yongqing Wu [a,*], Changpin Li [b], Yujiang Wu [a], Jürgen Kurths [c]

[a] School of Mathematics and Statistics, Lanzhou University, Lanzhou 730000, China
[b] Department of Mathematics, Shanghai University, Shanghai 200444, China
[c] Potsdam Institute for Climate Impact Research, Potsdam, Germany

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ABSTRACT

In this paper, generalized synchronization (GS) between two coupled complex networks is theoretically and numerically studied, where the node vectors in different networks are not the same, and the numbers of nodes of both networks are not necessarily equal. First, a sufficient criterion for GS, one kind of outer synchronizations, of two coupled networks is established based on the auxiliary system method and the Lyapunov stability theory. Numerical examples are also included which coincide with the theoretical analysis.

1. Introduction

Since 1990 chaos synchronization between two coupled systems has been a topic of great attention (see Refs. [1,2] and many references cited therein). After about ten years, synchronization was introduced into a complex network of coupled dynamical systems, which is relevant to the wide variety of physical, biological, social and engineering context. Since then, synchronization inside a network, or "inner synchronization" for brevity, has become a very hot topic. Till now there have been a huge number of publications [3].

Very recently, synchronization has been generalized to two coupled continuous-time networks [4]. Such a synchronization is called "outer synchronization" for simplicity. Shortly after, outer synchronization between two complex networks with nonidentical topological structures was investigated under the application of adaptive controllers [5]. However, in practice, it is necessary to consider the synchronization between different networks with different dynamical behaviours. In [6], under the precondition that the driving network gets inner synchronization, the authors studied outer synchronization between two different networks. Outer synchronization for two interacting discrete-time networks has been studied in [7]. In [8], linear generalized synchronization between two complex networks was investigated by using the nonlinear control method. Wu et al. considered generalized synchronization (GS) between two complex networks, where the transform function from driving network to the response one appeared in a controller, which made their consideration very special [9]. Furthermore, Shang et al. considered GS of complex networks where the response network was constructed from the drive network [10]. On the other hand, phase synchronization between two identical oscillators was also studied [11], which is the early work on synchronization between two coupled networks. The onset of synchronization in systems of interacting populations of heterogeneous oscillators was studied by Barreto et al. [12]. And synchronization in modular [13] or clustered networks [14,15] is also the similar works as one can regard each cluster as a separate network and the connection between clusters as the linkage between different networks.

* Corresponding author. Tel.: +86 931 8636424.
E-mail address: wuyongqing1001@yahoo.com.cn (Y. Wu).

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In this paper, the main goal is to further develop a general theory for GS between two different networks. The outline is organized as follows. In Section 2, the theoretical criterion of GS between two different networks are analyzed. Illustrate numerical examples are presented in Section 3. The last section includes the conclusions.

2. Theoretical criterion

We take a driving-response (or master–slave) configurations as follows

$$\begin{aligned} \dot{x}_i &= f(x_i(t)) + \mu \sum_{j=1}^{N_1} a_{ij} \Gamma_1 x_j, \quad i = 1, \ldots, N_1, \\ \dot{y}_i &= g(y_i(t)) + D(y_i, x_j) + \varepsilon \sum_{k=1}^{N_2} b_{ik} \Gamma_2 y_k, \\ i &= 1, \ldots, N_2, \quad j \in \{1, \ldots, N_1\}, \end{aligned} \tag{1}$$

where $D(y_i, x_j) = \dot{x}_j - g(x_j) + (H - \partial g(x_j)/\partial x_j)(y_i - x_j)$. Here x_i, $y_i \in R^n$; f, g:$R^n \to R^n$ are continuously differentiable. μ, $\varepsilon > 0$ are coupling strengths, Γ_1, Γ_2 are matrices (with order n) linking coupled variables. $A = (a_{ij})_{N_1 \times N_1}, B = (b_{ij})_{N_2 \times N_2}$ are respectively the inner connection matrices of the driving (or master) network $\mathbb{X}$ (corresponding to the first system of (1)) and response (or slave) network $\mathbb{Y}$ (corresponding to the second system of (1)). The matrix H is an arbitrary constant Hurwitz one (whose eigenvalues lie in open left semi-plane). In general, the elements in H can be chosen as simple as possible, e.g., if $(\partial g(x)/\partial x)_{ik}$ is a constant, we can choose $h_{ik} = (\partial g(x)/\partial x)_{ik}$ such that $(H - \partial g(x)/\partial x)_{ik}$ is zero; otherwise, we can introduce one or more parameters in H guaranteeing that H is a Hurwitz matrix.

The chosen interaction (1) is based on the open-plus-closed-loop (OPCL) method [16–18]. It is well-known that the closed-loop ("feedback") and open-loop control methods have their own advantages and disadvantages. To overcome their shortcomings, a new control method-OPCL method has been derived and successfully applied to synchronization of various chaotic systems [17,19], which show that the OPCL method is robust. A more illustrative fact is that the OPCL method was applied to complete synchronization of coupled complex dynamical networks [4,6,7]. Here we focus on studying GS of the typical master–slave configurations (1). We say networks $\mathbb{X}$ and $\mathbb{Y}$ possess the property of GS between $x \in R^{n \times N_1}$ and $y \in R^{n \times N_2}$ if there exist a transformation $\Phi : R^{n \times N_1} \to R^{n \times N_2}$, a manifold $M = \{(x,y){:}y = \Phi(x)\}$, and a subset $N = N_x \times N_y \subset R^{n \times N_1} \times R^{n \times N_2}$ with $M \subset N$ such that all trajectories of (1) with initial conditions in the attractive basin N approach the manifold M as time goes to $+\infty$.

Let $\phi_x^t : R^{n \times N_1} \to R^{n \times N_1}$ be the flow of network $\mathbb{X}$ in (1), $\phi^t = (\phi_x^t, \phi_y^t)$ be the flow of system (1) with $\phi_y^t : R^{n \times (N_1 + N_2)} \to R^{n \times N_2}$. We find that the map Φ relates to the flow ϕ_y^t. To more accurately characterize the conditions of the occurrence of GS for (1), the following criterion is presented.

GS Criterion: GS occurs in (1) if and only if for all $(x_0, y_0) \in N$ the response network $\mathbb{Y}$ is uniformly asymptotically stable, i.e., for arbitrarily given $x_0 \in N_x$, and $\forall y_{10}$, $y_{20} \in N_y$, $\lim_{t \to +\infty} \| y(t, x_0, y_{10}) - y(t, x_0, y_{20}) \| = 0$.

Proof. Necessity. Because of the occurrence of GS in (1), for any given y_{10}, $y_{20} \in N_y$ and $x_0 \in N_x$, there exists a transform Φ such that $\lim_{t \to +\infty} \| y(t, x_0, y_{10}) - \Phi(x(t, x_0)) \| = 0$ and $\lim_{t \to +\infty} \| y(t, x_0, y_{20}) - \Phi(x(t, x_0)) \| = 0$. It immediately follows that

$\lim_{t \to +\infty} \| y(t, x_0, y_{10}) - y(t, x_0, y_{20}) \| = 0$.

Sufficiency. In the following, we construct the transform function Φ satisfying $y_0 = \Phi(x_0) \in N_y$ for $x_0 \in N_x$. Since all states $y \in N_y$ of the response network $\mathbb{Y}$ converge to the manifold M, we consider the orbits starting in the past time at the point $(\phi^{-t}(x_0), y_0)$. When the trajectory passes the point x_0, the time t has elapsed and the point $(x_0, \phi^t(y_0))$ is the closer to the manifold M the larger time t is. By this reasoning, we set $\widetilde{\Phi}(x_0, y_0) = \lim_{t \to +\infty} \phi_y^t(\phi_x^{-t}(x_0), y_0)$. The asymptotic stable condition implies

$$\lim_{t \to +\infty} \left\| \phi_y^t(\phi_x^{-t}(x_0), y_{10}) - \phi_y^t(\phi_x^{-t}(x_0), y_{20}) \right\| = 0$$

for arbitrary y_{10}, $y_{20} \in N_y$. Hence, $\widetilde{\Phi}(x_0, y_0)$ does not depend upon y_0. The map Φ defining the GS manifold M is thus given by $\Phi(x_0) = \widetilde{\Phi}(x_0, y_0)$ for any $y_0 \in N_y$. Furthermore, asymptotic stability indicates that M is an attracting manifold. □

Remark. The above criterion still holds for general master–slave networks. It is also a generalized result of Ref. [20]. The main reason of choosing scheme (1) based on the OPCL technique is that OPCL is robust for highly complicated networks.

The most powerful (both numerical and analytical) tool for detecting GS is the *auxiliary system approach* [21]. In the following, in order to study the asymptotic stability of the network $\mathbb{Y}$, we construct an auxiliary network $\mathbb{Z}$ as follows,

$$\dot{z}_i = g(z_i(t)) + D(z_i, x_j) + \varepsilon \sum_{k=1}^{N_2} b_{ik} \Gamma_2 z_k, \quad i = 1, \ldots, N_2, \quad j \in \{1, \ldots, N_1\}, \tag{2}$$

where $D(z_i, x_j) = \dot{x}_j - g(x_j) + (H - \partial g(x_j)/\partial x_j)(z_i - x_j)$.

Letting $e_i = y_i - z_i$, and linearizing the error system around x_j, one has

$$\dot{e}_i = He_i + \varepsilon \sum_{k=1}^{N_2} b_{ik}\Gamma_2 e_k, \quad i = 1, \ldots, N_2. \tag{3}$$

If we set $e = (e_1, \ldots, e_{N_2}) \in R^{n\times N_2}$, then (3) can be rewritten in the compact form,

$$\dot{e} = He + \varepsilon\Gamma_2 e B^T, \tag{4}$$

where ^{T} denotes the transpose. For a given matrix B^T, there exist an invertible matrix P with order N_2 such that $B^T = PJP^{-1}$ in which J is a Jordan form, $J = \text{diag}(J_1, \ldots, J_\ell)$ where J_k is a Jordan block corresponding to the $m_k(\in Z^+)$ multiple eigenvalue $\lambda_k \in C$ of B^T,

$$J_k = \begin{pmatrix} \lambda_k & 1 & 0 & \cdots & 0 \\ 0 & \lambda_k & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_k & 1 \\ 0 & 0 & \cdots & 0 & \lambda_k \end{pmatrix}_{m_k\times m_k}, \quad k = 1, \ldots, \ell, \quad \sum_{k=1}^{\ell} m_k = N_2.$$

If we set $\eta = eP \in C^{n\times N_2}$, (4) can be written as another form,

$$\dot{\eta} = H\eta + \varepsilon\Gamma_2\eta J. \tag{5}$$

Now setting

$$\eta = (\eta_1, \ldots, \eta_{m_1}; \eta_{m_1+1}, \ldots, \eta_{m_1+m_2}; \ldots; \eta_{m_1+\cdots+m_{\ell-1}+1}, \ldots, \eta_{N_2}) \in C^{n\times N_2},$$

(5) gives,

$$\begin{aligned}
&\dot{\eta}_1 = (H + \varepsilon\lambda_1\Gamma_2)\eta_1, \\
&\dot{\eta}_2 = (H + \varepsilon\lambda_1\Gamma_2)\eta_2 + \varepsilon\Gamma_2\eta_1, \\
&\ldots \\
&\dot{\eta}_{m_1} = (H + \varepsilon\lambda_1\Gamma_2)\eta_{m_1} + \varepsilon\Gamma_2\eta_{m_1-1}; \\
&\dot{\eta}_{m_1+1} = (H + \varepsilon\lambda_2\Gamma_2)\eta_{m_1+1}, \\
&\dot{\eta}_{m_1+2} = (H + \varepsilon\lambda_2\Gamma_2)\eta_{m_1+2} + \varepsilon\Gamma_2\eta_{m_1+1}, \\
&\ldots \\
&\dot{\eta}_{m_1+m_2} = (H + \varepsilon\lambda_2\Gamma_2)\eta_{m_1+m_2} + \varepsilon\Gamma_2\eta_{m_1+m_2-1}; \\
&\ldots \\
&\dot{\eta}_{m_1+\cdots+m_{\ell-1}+1} = (H + \varepsilon\lambda_\ell\Gamma_2)\eta_{m_1+\cdots+m_{\ell-1}+1}, \\
&\dot{\eta}_{m_1+\cdots+m_{\ell-1}+2} = (H + \varepsilon\lambda_\ell\Gamma_2)\eta_{m_1+\cdots+m_{\ell-1}+2} + \varepsilon\Gamma_2\eta_{m_1+\cdots+m_{\ell-1}+1}, \\
&\ldots \\
&\dot{\eta}_{N_2} = (H + \varepsilon\lambda_\ell\Gamma_2)\eta_{N_2} + \varepsilon\Gamma_2\eta_{N_2-1}.
\end{aligned}$$

Without loss of generality, we only study the systems corresponding to λ_1. If $\lambda_1 = \alpha_1 + j\beta_1$, $\alpha_1, (0 \neq)\beta_1 \in R$, j is the imaginary part, we set $\eta_1 = u_1 + jv_1$, the above first sub-system can be then changed into

$$\begin{aligned}
&\dot{u}_1 = (H + \varepsilon\alpha_1\Gamma_2)u_1 - \varepsilon\beta_1\Gamma_2 v_1, \\
&\dot{v}_1 = (H + \varepsilon\alpha_1\Gamma_2)v_1 + \varepsilon\beta_1\Gamma_2 u_1.
\end{aligned}$$

It is easy to show that its zero solution is asymptotically stable if $(H^T + H) + \varepsilon\alpha_1\left(\Gamma_2^T + \Gamma_2\right) < 0$. This condition also guarantees that $\eta_2, \ldots, \eta_{m_1}$ approach to zero as $t \to +\infty$. If $\lambda_1 \in R$, i.e., $\beta_1 = 0, \eta_1, \ldots, \eta_{m_1}$ approach to zero as $t \to +\infty$ when the real parts of all eigenvalues of $H + \varepsilon\alpha_1\Gamma_2$ are negative. So we have following criterion.

Asymptotical stability criterion of zero to (4): Assume that B has m_k multiple eigenvalues $\lambda_k \in R$ where $k = 1, \ldots, \ell_0$, $\lambda_k = \alpha_k + j\beta_k \in C$ $(\alpha_k, \beta_k \in R, \beta_k \neq 0)$ where $k = \ell_0 + 1, \ldots, \ell$, and $\sum_{k=1}^{\ell} m_k = N_2$. If the real parts of all eigenvalues of $H + \varepsilon\lambda_k\Gamma_2$ (for $k = 1, \ldots, \ell_0$) are negative, and $(H^T + H) + \varepsilon\alpha_k\left(\Gamma_2^T + \Gamma_2\right) < 0$ for $k = \ell_0 + 1, \ldots, \ell$, then the zero solution to the matrix Eq. (4) is asymptotically stable.

Combining the *Asymptotical stability criterion of zero to (4)* and the *GS Criterion*, we get the following main result:

GS Criterion for (1): Assume that B has m_k multiple eigenvalues $\lambda_k \in R$ where $k = 1, \ldots, \ell_0$, $\lambda_k = \alpha_k + j\beta_k \in C$ $(\alpha_k, \beta_k \in R, \beta_k \neq 0)$ where $k = \ell_0 + 1, \ldots, \ell$, and $\sum_{k=1}^{\ell} m_k = N_2$. If the real parts of all eigenvalues of $H + \varepsilon\lambda_k\Gamma_2$ (for $k = 1, \ldots, \ell_0$) are negative, and $(H^T + H) + \varepsilon\alpha_k(\Gamma_2^T + \Gamma_2) < 0$ for $k = \ell_0 + 1, \ldots, \ell$, then GS occurs in (1).

3. Numerical examples

In this section, we present numerical experiments. In the network $\mathbb{X}$, the dynamics at every node follows the Rössler system

$$\begin{aligned} \dot{x}_{i1} &= 2 + x_{i1}(x_{i2} - 4), \\ \dot{x}_{i2} &= -x_{i1} - x_{i3}, \\ \dot{x}_{i3} &= x_{i2} + 0.45x_{i3}, \quad i = 1, \ldots, 10. \end{aligned} \tag{6}$$

The inner-coupling matrix is as follows,

$$A = \begin{pmatrix} -3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -4 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & -3 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -3 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & -5 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & -4 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & -3 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & -4 \end{pmatrix}.$$

We may simply set $\Gamma_1 = \mathrm{diag}\,(1,0,0)$, $\mu = 1$.

In the network $\mathbb{Y}$, the dynamics of the individual nodes is described by the Lorenz system

$$\begin{aligned} \dot{y}_{i1} &= \sigma(y_{i2} - y_{i1}), \\ \dot{y}_{i2} &= \gamma y_{i1} - y_{i1}y_{i3} - y_{i2}, \\ \dot{y}_{i3} &= y_{i1}y_{i2} - by_{i3}, \quad i = 1, \ldots, 12, \end{aligned} \tag{7}$$

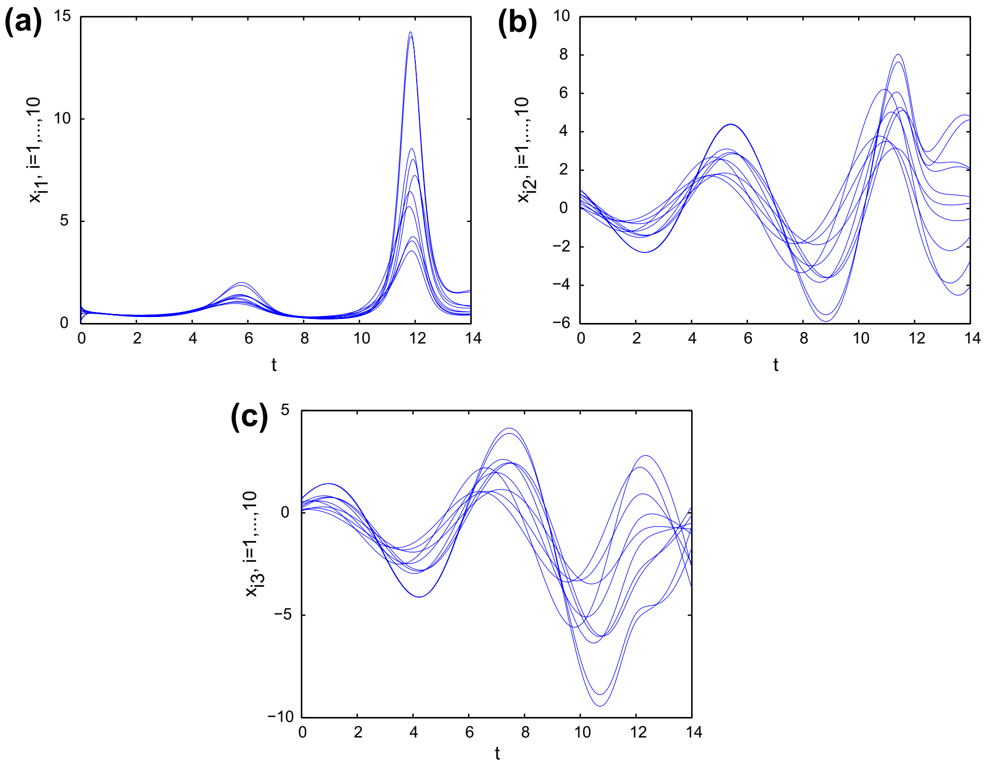

Fig. 1. Diagrams of the state variables in network $\mathbb{X}$ of system (1). (a) Evolution of state variables x_{i1}, (b) evolution of state variables x_{i2} and (c) evolution of state variables x_{i3}, $i = 1, \ldots, 10$.

where $\sigma = 10$, $\gamma = 28$, $b = 8/3$. The inner-coupling matrix is chosen as below,

$$B = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

We may set $\Gamma_2 = \mathrm{diag}\,(1,1,1)$, $j = 1$ in $D(y_i, x_j)$. The Hurwitz matrix H for the Lorenz system is

$$\begin{pmatrix} -\sigma & \sigma & 0 \\ \gamma + p_1 & -1 & p_2 \\ p_3 & p_4 & -b \end{pmatrix},$$

where $p_1, \ldots, p_4$ are parameters. A suitable choice of p_k is $p_1 < 1 - \gamma$, $p_2 = p_3 = p_4 = 0$. Here we let $p_1 = -29 < 1 - \gamma$ for the current simulation.

If $\varepsilon = 0.3$, it is easy to find that the conditions of *GS Criterion for (1)* hold. In the numerical simulations throughout this paper, the initial values are randomly chosen in the interval (0,1). Figs. 1 and 2 show the simulation results of the driving-response networks with OPCL configurations. The evolution of state variables $x_i = (x_{i1}, x_{i2}, x_{i3})^T$ and $y_i = (y_{i1}, y_{i2}, y_{i3})^T$ are shown in Figs. 1 and 2 (a)–(c), respectively. Obviously, networks $\mathbb{X}$ and $\mathbb{Y}$ do not reach complete synchronization.

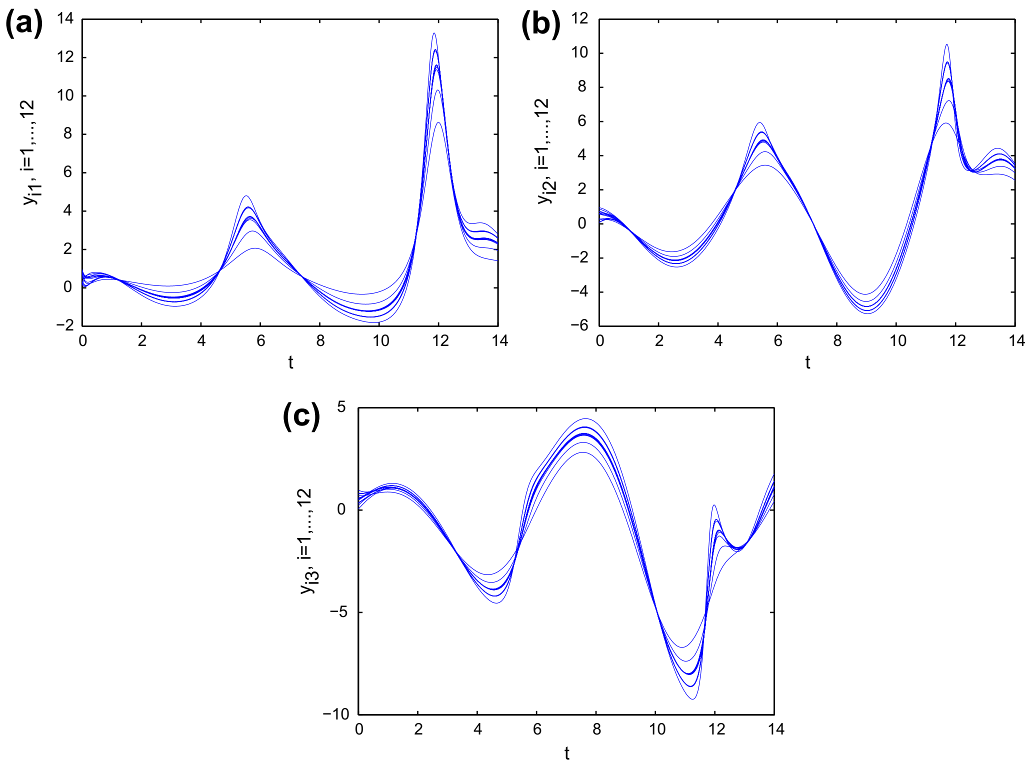

Fig. 2. Diagrams of the state variables in network $\mathbb{Y}$ of system (1). (a) Evolution of state variables y_{i1}, (b) evolution of state variables y_{i2} and (c) evolution of state variables y_{i3}, $i = 1, \ldots, 12$.

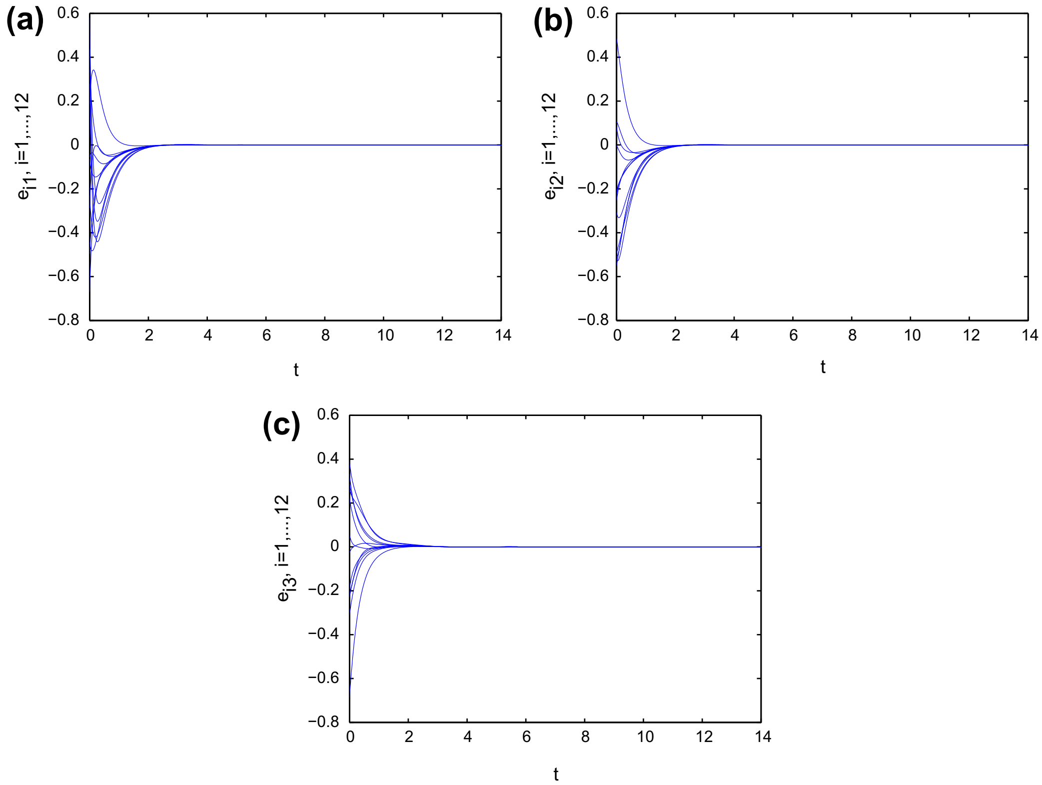

Fig. 3. Synchronization between networks $\mathbb{Y}$ and $\mathbb{Z}$ of the second system of (1) and system (2). In (a)–(c), $(e_{i1}, e_{i2}, e_{i3})^T = (y_{i1} - z_{i1}, y_{i2} - z_{i2}, y_{i3} - z_{i3})^T$, $i = 1, \ldots, 12$.

Now we introduce an auxiliary network $\mathbb{Z}$ which is a replica of the response network $\mathbb{Y}$. Fig. 3 displays $e_i = (e_{i1}, e_{i2}, e_{i3})^T = (y_{i1} - z_{i1}, y_{i2} - z_{i2}, y_{i3} - z_{i3})^T$. The appearance of complete synchronization between $\mathbb{Y}$ and $\mathbb{Z}$ implies that GS between $\mathbb{X}$ and $\mathbb{Y}$ of system (1) occurs.

Next we consider the case that the conditions of *GS Criterion for (1)* are not met. It is easy to find that $\lambda = 3.4515$ is an eigenvalue of B. If we rechoose $\varepsilon = 1$, then eigenvalues of $H + \varepsilon\lambda\Gamma_2$ are -5.2501, 1.1531 and 0.7848. Here, synchronization error between networks $\mathbb{Y}$ and $\mathbb{Z}$ of the second system of (1) and system (2) is measured by

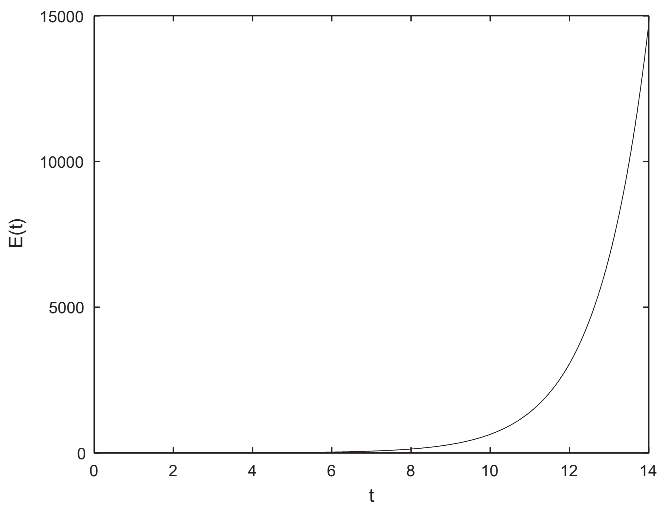

Fig. 4. The curve of $E(t)$ where $\varepsilon = 1$.

$$E(t) = \sqrt{\sum_{i=1}^{12}\sum_{j=1}^{3}(y_{ij}(t) - z_{ij}(t))^2}, \ for\ t \in [0, +\infty).$$

Fig. 4 displaces the evolution of the error $E(t)$, showing $E(t) \to \infty$, as $t \to \infty$. Thereby, GS between $\mathbb{X}$ and $\mathbb{Y}$ of system (1) does not occur.

4. Conclusion

In conclusion, we have presented criteria for the occurrence of GS in master–slave networks with OPCL configurations. The theoretical criterion is based on the uniform asymptotical stability of the response network, which can be verified by utilizing the Lyapunov stability theorem. Some possible applications of synchronization between two (or more) networks have been introduced in introductions of Refs. [4–7] and [9]. Other possible applications of GS between two networks may be in systems biology, e.g., investigating the interactions of protein network and gene network may disclose evolution process [22], and/or the climate inspired research by combining ecological with social-economic networks makes it possible to change climate.

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