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Response of scale-free networks with community structure to external stimuli

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ABSTRACT

The response of scale-free networks with community structure to external stimuli is studied. By disturbing some nodes with different strategies, it is shown that the robustness of this kind of network can be enhanced due to the existence of communities in the networks. Some of the response patterns are found to coincide with topological communities. We show that such phenomena also occur in the cat brain network which is an example of a scale-free like network with community structure. Our results provide insights into the relationship between network topology and the functional organization in complex networks from another viewpoint.

1. Introduction

Many natural systems are found, on one hand, to be able to react to small selected stimuli with large alterations, whereas, on the other hand, they can withstand large environmental variations (sometimes even unpredictable ones) with minimal changes or loss of functionality. This implies two complementary attributes of dynamical systems: sensitivity and robustness. Sensitivity implies the possibility of a large response to small stimuli and robustness implies the possibility of a small response to large stimuli. Not only biological systems but also several man-made complex systems, such as power grids or communication systems require this combination of traits to optimize the system's performance. Recently, the focus on understanding the interplay between dynamical behavior and their topologies has attracted a lot of interest [1–7]. Recent research [8] shows that the observed network topologies which are often scale-free or scale-free like [9] are not necessarily optimal in their connectivity and connectivity-related attributes. Moreover, it is manifest [10] that scale-free networks are fragile to intentional attack but resilient to random failures, in the face of node removal. We ask why so many networks found in nature have a scale-free (like) architecture with a lack of optimal network connectivity? In this paper, we study the properties which determine the efficiency of networks by analyzing the response of such systems to external perturbations.

To describe a complex system, one can take the units of response as nodes and the interactions between them as edges and then generate a network model. It is well known that many complex networks exhibit not only short average distances, but also a high clustering coefficient, the 'small-world' property [11]. Moreover, several of them can be approximated well

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by a power-law degree distribution, $P(k) \sim k^{-\gamma}$, the 'scale-free' property [9]. Many real-world networks exhibit not only the 'small-world' and 'scale-free' property, but also have a community structure which is defined as collections of nodes within which the connections are dense, but between the communities the connections are sparse. Community structures are supposed to play an important role in many real networks. For example, communities in a citation network might represent related papers on a single topic [12]; communities on the web might represent pages on related topics [13]; communities in a biochemical network or neuronal system might correspond to special functional units [14,15]. Therefore, it is important to study the response of a scale-free network with community structure.

In this paper, we use the dynamical model presented by Bar-Yam and Epstein [16] to study the response of scale-free networks with community structure to external stimuli. Our investigation reveals that the community characteristic of the networks is crucial to enhance its robustness. Some of the response patterns are found to coincide with the topology communities. As an example of scale-free like networks with community structure, such phenomena also occur in mammal brain networks.

This paper is organized as follows: In Section 2, the model of the network is introduced. The dynamical attractor network model is presented in Section 3. Numerical simulations and a detailed analysis are presented in Section 4. In Section 5, we study the response of the cat brain network to stimuli as an example. Finally, our conclusions are given in Section 6.

2. The model of the network

In order to create a scale-free network with community structure, we use a modified procedure of the algorithm proposed in [17]. We assume that there are $M (M \geq 2)$ communities in the network. This model is defined by the following scheme:

Step 1: Initialization: Start from a small number $m_0 (m_0 > 1)$ of fully connected nodes in each community. There are n random links between every two communities.

Step 2: Growth: At each time step, a new node is added to the network. We assume that the probability $P(I)$ of which community I the new node is added to depends on the number of nodes in the communities n_I, i.e.:

$$P(I) = \frac{n_I}{\sum_I n_I}. \tag{1}$$

The new node will be connected to m $(m_0 \geq m \geq 1)$ nodes inside the same community I through m intra-community links (defined as the links that connect nodes in the same community), and with probability α connected to n $(m \geq n \geq 1)$ nodes (none with probability $1 - \alpha$) to the other $M - 1$ communities through inter-community links (defined as the links that connect nodes among different communities). We assume that the probability $P(i, I)$ that a new node will be connected to node i in community I which is selected before depends on the inner-degree s_{iI} (define as the number of intra-links connected to node i) of that node, i.e.:

$$P(i, I) = \frac{s_{iI}}{\sum_k s_{kI}}. \tag{2}$$

We also assume that the probability $P(j, K)$ that a new node will be connected to node i in community $K (K \neq I)$ depends on the inter-degree l_{jK} (defined as the number of inter-links connected to the node), i.e.:

$$P(j, K) = \frac{l_{jK}}{\sum_{m,N,N \neq K} l_{mN}}. \tag{3}$$

We call this network a community-scale-free (CSF) network (compare with scale-free network (SF)). It is shown in Fig. 1 that such a CSF network has also a power-law degree distribution $P(k) \sim k^{-\gamma}$.

3. The dynamic model

To investigate the response of CSF networks, we choose a conventional multi-attractor network model; which describes a dynamical system. The node states $s_i = \pm 1, i \in 1, 2, \ldots, N$ are binary. The states of the network system are then composed by the set of the node states s_i. The dynamical attractor system evolves as follows:

$$s_i(t + 1) = \operatorname{sign}\left(\sum_{j=1}^{N} A_{ij} s_j(t)\right) \tag{4}$$

where $A = (A_{ij})_{N \times N}$ is the connection matrix whose elements A_{ij} are positive if there is a link going from node i to node j with $i \neq j$ and zero otherwise $(A_{ii} = 0)$ (A is symmetric if the network has no weights and no directions). This model can also be interpreted as social opinion models of binary states, such as yes $(+1)$ or no (-1). It is known [18] that there are multiple attractors generated by this model. Any attractor with a non-empty attracting basin is stable to perturbation and thus can represent a functional state of the system.

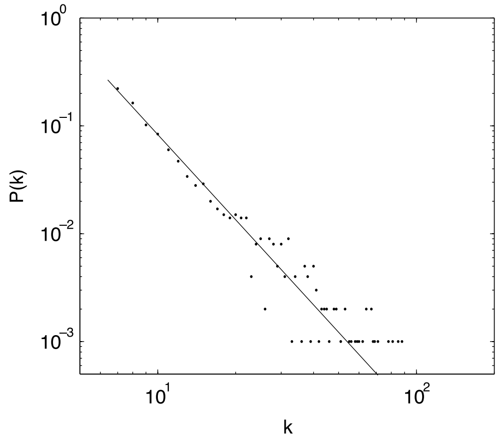

Fig. 1. The degree distribution of CSF. $N = 1000$, $M = 5$, $n = 6$, $m = 1$, $\alpha = 1$, $\gamma = 3$.

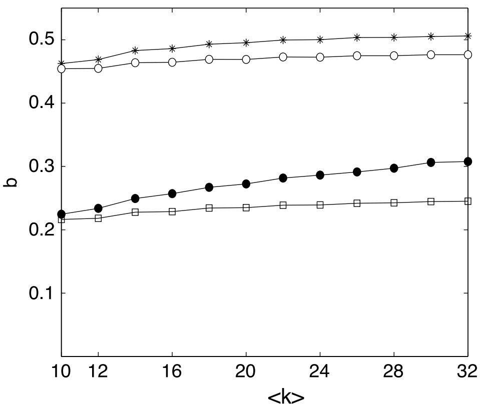

Fig. 2. Size of the basin of attraction (fraction of total nodes, b) as a function of the average degree $\langle k\rangle$, * CSF network and random stimulus, ∘ SF network and random stimulus, • CSF network and directed stimulus, □ SF network and directed stimulus, $\langle k\rangle = m$ in SF, $\langle k\rangle = m + \alpha n$ in CSF, $N = 1000$, $M = 4$.

External stimuli are modeled by flipping the signs of a specified set of nodes. When the states of some nodes are changed, the system either evolves back to its initial state or switches to other attractors. The response of the network system is described as a process of switching between the attractors. The size of the basin of attraction, i.e., the number of nodes whose states can be changed before the dynamics of the network fails to bring the systems back to its original states, indicates the degree of stability of the system. The system is said to be sensitive to a certain disturbance if it changes its current state to another one, and vice versa. We calculate the size of the basin of attraction in different cases of stimuli to reveal the sensitivity and robustness of the network [19,20].

4. Numerical simulations and analysis

We study the CSF networks mentioned in Section 2 and the Barabási–Albert model of SF networks. Without loss of generality, we can randomly choose two states to represent the functional states of the system. To ensure that these states are stable, we adopt the Hebbing imprinting rule $J_{ij} = \sum_\alpha s_i^\alpha s_j^\alpha$ to construct the desired attractors. For sufficiently many links and for a broad range of network topologies, this form of non-zero links will make the pre-selecting functional states into stable attractors of the network dynamics [18]. For the sake of convenience, we consider two attractors $S1$ and $S2$ ($S1 = (+1, +1, \dots, +1)$, $S2 = (-1, -1, \dots, -1)$) as the stable states between which the system can switch. Initially, all the nodes are set to be in the state $+1$, that is, $s_i = +1$, $i = 1, 2, \dots, N$. We suppose, at some time t, such environment changes or new information arises, which induce the states of some nodes being selected to flip to the opposite state $s_{k_i} = -1$, $i = 1, 2, \dots, r_s$, where r_s is the number of the nodes whose states are flipped. Then after a period of transient time, the system evolves into a stable state which is either its original state or another attractor. To explore the changes of the system state, 100 simulations are performed for each different average degrees.

We find (Fig. 2) that for the two basic types of stimuli: direct stimuli (changes are made to the most highly connected nodes, the hubs) and random stimuli (changes are made to randomly chosen nodes), CSF networks are more robust than SF networks. Therefore, the community structure enhances the robustness of the networks. In order to see how the community

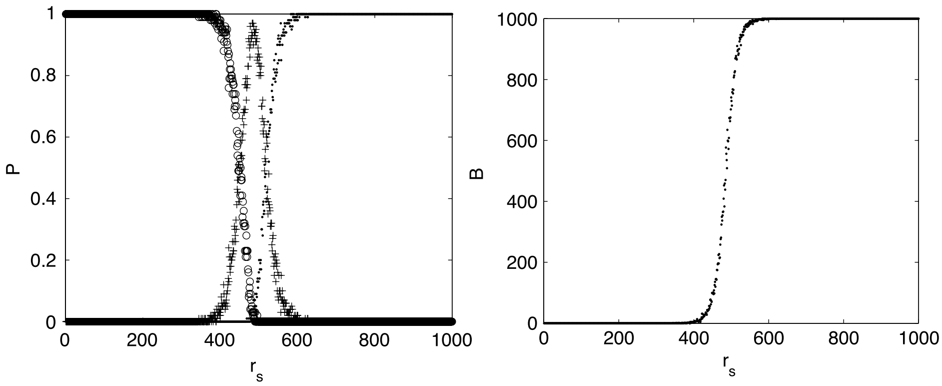

Fig. 3. CSF networks: $N = 1000$, $M = 4$. (a) The probability P that the system evolves into S1, S2 or other steady states under different disturbances r_s: o all the nodes were in status $+1$, i.e. the system evolves back to its initial state. $+$ some of the nodes were in status $+1$, others were in status -1, $\cdot$ all the nodes were in status -1, i.e. all the nodes were influenced. (b) The average number of influenced nodes B in the long-time behavior for different r_s.

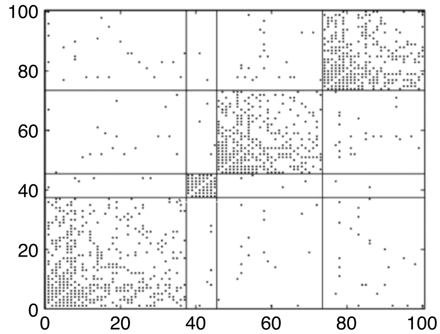

Fig. 4. Connection matrix A of the CSF network, $\cdot\, A_{ij} = 1$, blank: $A_{ij} = 0$. The parameters of the CSF network $N = 100$, $M = 4$, $m = 6$, $n = 1$.

structure influences the robustness of the network, we randomly chose a CSF network to investigate the probability P that the system evolves into S1, S2 or other steady states under different random disturbances.

It is shown in Fig. 3 that the CSF network maintains the original states under a small perturbation, which illustrates its robustness to some extent, until the strength of perturbation (numbers of flips r_s) exceeds a critical value. For intermediate stimuli, the system converges to mixed steady states in which some node states are $+1$ but the others are -1. By increasing the stimuli continuously, all of the nodes are influenced and the system evolves into an all -1 state (S2), which is an extreme response. It should be noted that there are two phase transition points in the process of response. The first one corresponds to the transition of the system from a normal state to partial destruction. The second one corresponds to the transition from partial destruction to complete destruction.

It is interesting to ask: why is the response of a CSF network different from a SF network without modular structure, though the degree distribution of the CSF is also scale-free? In the following, we explore the response patterns for r_s in the intermediate region i.e. in the region between both phase transitions. We randomly choose some nodes to be flipped and identify whose states are changed in the network as the system evolves into a steady state, which illustrates the response to stimuli. To describe the response patterns, a new variable is defined as follows:

$$R_{ij} = \begin{cases} -1, & \text{if } A_{ij} > 0 \text{ and } s_i = -1, s_j = -1 \\ +1, & \text{if } A_{ij} > 0 \text{ and } s_i, s_j \text{ are not in status-1 at the same time} \\ 0, & \text{if } A_{ij} = 0. \end{cases} \tag{5}$$

R indicates which part of the network was influenced. In order to see the patterns clearly, we use a small size network here as an example.The connection matrix is shown in the Fig. 4. We have also found similar results in much larger CSF networks and smaller networks. It is shown in Fig. 5 that for a certain r_s in the intermediate region, some patterns of R appear with large probability, which almost correspond to the topological communities. Clearly, the community structure plays a crucial role in response to the stimuli. When some segments of the network are destroyed, the community structure can prevent the damage from spreading to other segments. A similar impact of clustering has been also found in Ref. [21]. This phenomenon can be explained by the underlying structure of communities which has a high density of connections inside the communities and sparse connections with the outside nodes. Therefore, those nodes which connect two communities can hardly be affected by the negative nodes with respect to our majority opinion-like model. In this sense, the network

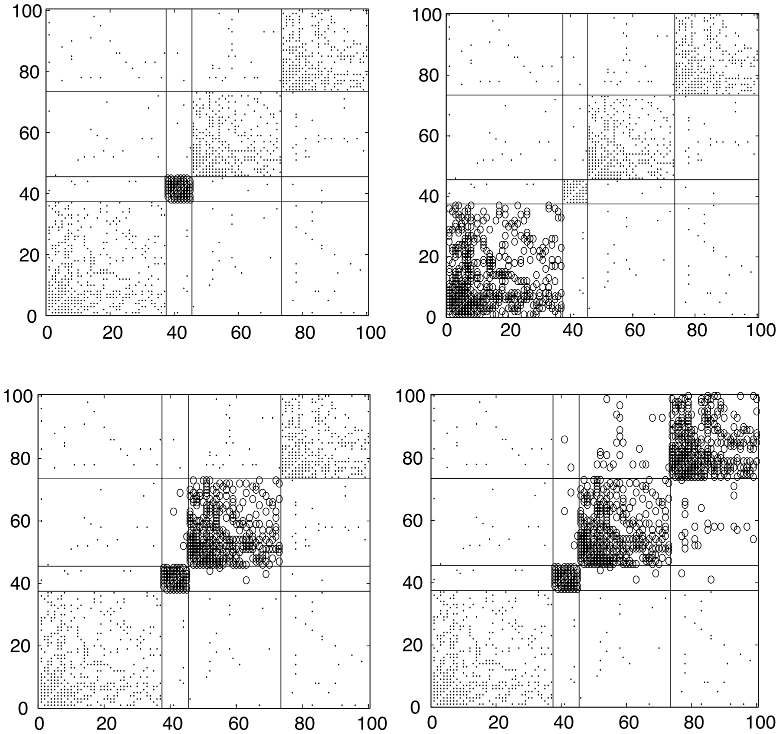

Fig. 5. The pattern of R with different initial conditions: o $R_{ij} = -1$, $\cdot$ $R_{ij} = 1$, blank: $R_{ij} = 0$. The parameters of the CSF network $N = 100, M = 4, m = 6, n = 1, \alpha = 1. r_s = 48$.

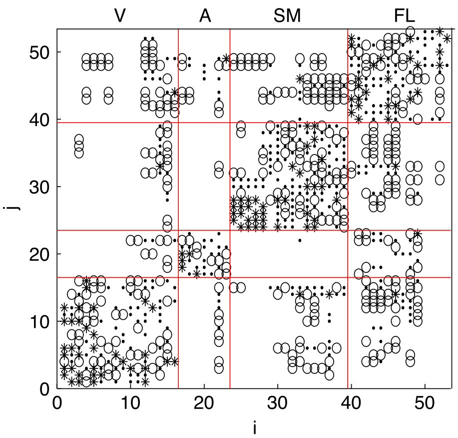

Fig. 6. Connection matrix A of the cortical network of cat brain. The different symbols represent different connection weights: 1 (o sparse), 2 (. intermediate), 3 (* dense).

structure coincides well with the dynamical pattern. By detecting some special patterns appearing with high probability, our results are expected to provide a new approach for community detection.

5. Response of cat brain networks to stimuli

As an example, the response of cat brain networks to stimuli is analyzed here. The cerebral cortex of a cat can be parcellated into 53 areas, linked by about 830 fibres of different densities into a weighted complex network as shown in Fig. 6 [22]. This network displays a heterogeneous structure, where some nodes have only 2 links while others have up to 35 connections. It is clear that the size of the network is too small to claim that the degree distribution is scale-free. Nevertheless, the distribution is very close to that of networks of the same size and density generated by scale-free models. Moreover, the cortical network of cats exhibits a hierarchically clustered organization. There exists a small number of topological clusters that broadly agree with four functional cortical sub-divisions: visual cortex (V,16 areas), auditory (A, 7 areas), somato-motor

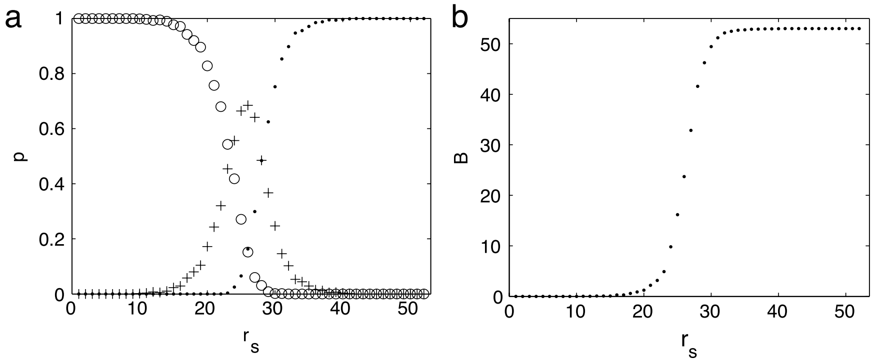

Fig. 7. Cat brain network (a) Probability P that the system evolves into S1, S2 or other steady states under different disturbances r_s: o all the nodes were in status $+1$, i.e. the system evolves back to its initial states. $+$ some of the nodes were in status $+1$, others were in status -1, $\cdot$ all the nodes were in status -1, i.e. all the nodes were influence. (b) The average number of influenced nodes B in the long-time behavior for different r_s.

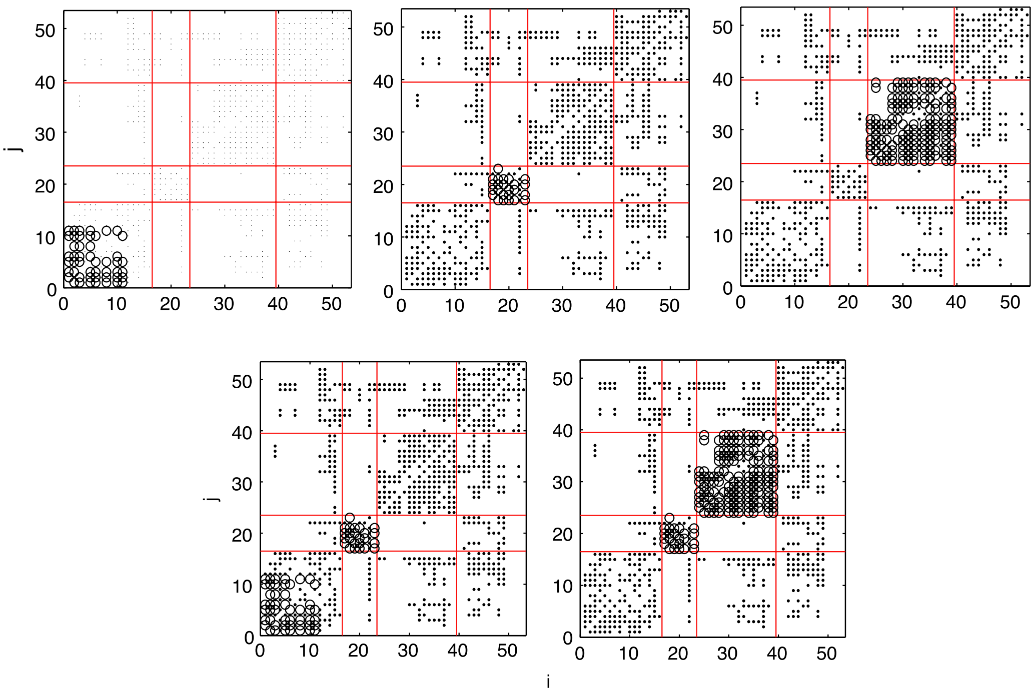

Fig. 8. Cat brain network. The pattern of R with different initial conditions: o $R_{ij} = -1$, $\cdot$ $R_{ij} = 1$, blank: $R_{ij} = 0$. $r_s = 25$.

(SM, 16 areas) and fronto-limbic (FL, 14 areas). In addition, this network also displays typical small-world properties, i.e. short average path length and high clustering coefficient [23–26].

We perform simulations on this network by applying the same dynamics as in our CSF model (Section 3). The responsivity under different external perturbations can be found in Figs. 7 and 8, which is similar to the case of our network model. It can provide insights into the relationship between network topology and functional organization of the cat brain networks from another viewpoint [27]. Furthermore, we make the stimuli acting on large-intensity nodes or on small-intensity nodes (directed stimuli) other than random stimuli and then see the response pattern of the network. We define the intensity c_i of node i as follows:

$$c_i = \sum_{j=1, j\neq i}^{N} A_{ij}. \tag{6}$$

In particular, for unweighted networks, the intensity of a node is the degree of this node. It is known that scale-free networks are more robust to random attacks, while more sensitive to directed disturbance to the large-degree nodes. As shown in Fig. 8, the brain network can be partially disturbed when $30 > r_s > 16$ directed stimuli are acting on small-degree nodes (shown in Fig. 9(b)). On the other hand, the system can be entirely disturbed when $r_s > 24$ for large-node perturbation (shown in Fig. 9(a)). It is manifest that the robustness for directed stimuli acting on large-degree nodes is stronger than the

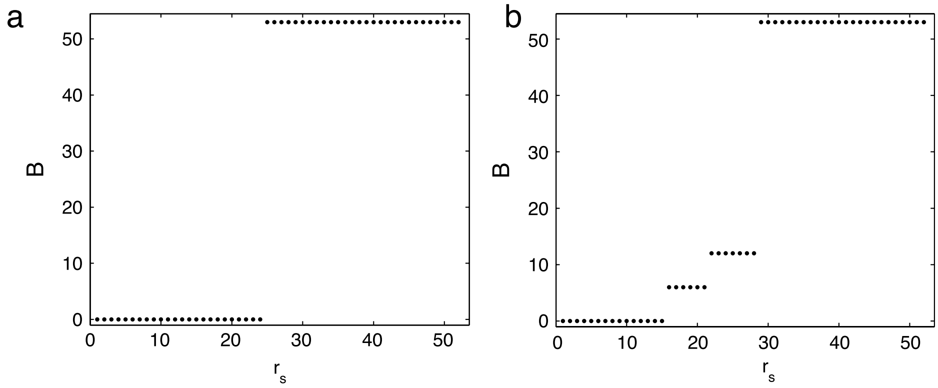

Fig. 9. The number of nodes (B) whose states are changed in long-time behavior for different stimuli. (a) The responses for directed stimuli acting on large-degree nodes. (b) The responses for directed stimuli acting on small-degree nodes.

case for directed stimuli on small-degree nodes. This enhanced robustness is also better than the case for random stimuli, which is different from the result in Refs. [16,28] for scale-free networks.

Compared with the robustness of the classical scale-free network model, a complex brain network is likely to be more robust for direct stimuli. It should be noted that the brain network displays not only heterogeneity on the degree distribution but also hierarchical clustering characteristic. These special properties may play an important role in the response to stimuli. However, more evidence should be presented in future studies.

6. Conclusion

In this letter, we investigate the relationship between dynamics of complex networks and their topology properties by studying the response of the whole system. An adaptive system should be robust for large stimuli, which makes the system stable. Additionally, it also should be sensitive for small stimuli, which makes the system react rapidly on the new external changes. According to the analysis of the response of scale-free networks with community structure, we find that the hierarchical characteristic of the networks enhances their robustness to external stimuli. Switching patterns are found to coincide with the topology communities. We verify our results in a real-world -cat brain network which has similar topological properties as our model. We show that the robustness for directed stimuli acting on large-degree nodes is better than in the case of directed stimuli on small-degree nodes, which is different from the response in the scale-free networks. Our results provide new insights into the relationship between network topology and the functional organization of the cat brain networks from another viewpoint.

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