Enhanced synchronizability in scale-free networks

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We introduce a modified dynamical optimization coupling scheme to enhance the synchronizability in the scale-free networks as well as to keep uniform and converging intensities during the transition to synchronization. Further, the size of networks that can be synchronizable exceeds by several orders of magnitude the size of unweighted networks.
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Works on synchronizability in networks with a given topology can be divided into two classes according to the coupling matrix. One class is the static mechanism, where the coupling matrix remains fixed during the transition to synchronization. This mechanism includes the degree and load based weighted networks. The other class is the dynamical mechanism, where the coupling matrix evolves in time by introducing adaptive strengths between connected oscillators. The adaptation process can enhance synchronization by modifying the coupling matrix, but the resulting networks have nonuniform intensities even for networks with homogeneous degrees. In this paper, we introduce a modified dynamical optimization mechanism to enhance the synchronizability in the scale-free networks as well as to keep uniform and converging intensities during the transition to synchronization. Further, the size of networks that can be synchronizable exceeds by several orders of magnitude the size of unweighted networks.

I. INTRODUCTION

In the past few years, the dynamics of complex networks has been extensively investigated.[1–4] As a typical dynamical process on networks, synchronization, especially the ability of networks to obtain synchronization (synchronizability), has attracted a lot of interest.[5–22] Recent studies have revealed that unweighted small-world and scale-free networks are more synchronizable than unweighted regular networks.[5,6] But the assumption that local units are symmetrically coupled with undirected couplings does not match the properties of real networks (such as unequal connection weights and asymmetry of the couplings).[7,8] Recent efforts have been focused on achieving efficient synchronization by introducing connection weights and directionality into networks.[9–15,18–21]

From Ref. 20, works on the synchronizability in networks with a given topology can be divided into two classes according to the coupling matrix. One class is the static mechanisms, where the coupling matrix is invariant.[6,9–17] For randomly enough unweighted and weighted networks, the synchronizability is controlled by $S_{\max}/S_{\min}$, where $S_{\max}$ and $S_{\min}$ are, respectively, the maximum and minimum of intensity S_i, which is defined by the sum of the coupling strengths of oscillator i.[14] For unweighted Barabási–Albert (BA) networks,[14] $S_{\max}/S_{\min}=k_{\max}/k_{\min}\sim N^{1/2}$, where $k_{\max}$ and $k_{\min}$ are the maximal and minimal degrees, respectively. Hence, the synchronizability can be enhanced if intensities become more homogeneous. From the degree based weighted networks,[11,13] one necessary condition for the optimal synchronizability R_{opt} is that the intensities become uniform.

The other class is the dynamical mechanisms, where the coupling matrix is variant by introducing adaptive strengths into networks of identical oscillators[18] and nonidentical oscillators.[19] The adaptation process can enhance the synchronization by modifying the coupling matrix, but the resulting networks have heterogeneous intensities due to heterogeneous degrees. For BA networks, after the adaptation, the synchronizability is characterized by $S_{\max}/S_{\min}\sim N^{\beta/2}$ with $\beta=1-\theta$ and $\theta\sim 0.5$.[18] Inspired by the static mechanisms,[11,13] one necessary condition for the optimal synchronizability is that intensities become uniform. However, even for networks with homogeneous degrees, the mechanisms[18,19] cannot ensure uniform intensities due to different initial conditions of oscillators.[20] Therefore, a problem naturally arises: *By the dynamical mechanism, how can we realize the synchronization in networks as well as ensure uniform intensities during the transition to synchronization and enhance the synchronizability, regardless of heterogeneous degrees and initial conditions of oscillators*?

II. THE MODIFIED DYNAMICAL OPTIMIZATION MECHANISM

Recently, we have already obtained some results on the above problem. For different variants of the Kuramoto model, we have proposed a dynamical gradient network (DGN) approach to realize phase synchronization.[21] It is shown that all the oscillators have uniform intensities during the transition to synchronization. However, the DGN approach is very special in two aspects. One is that it should assign a scale potential to each oscillator within any time interval, which depends on the extent of the local synchronization among itself and its neighbor oscillators. The other is

that the adjustment of the respective link by the DGN approach is often mostly ineffective. Inspired by the DGN approach,[21] we have further introduced the original dynamical optimization (DO) mechanism for small-world networks (SWNs).[20] The main idea in the original DO mechanism is to increase the coupling strength of only one incoming link of oscillator i by a small value in different intervals with a fixed length. It reflects the "winner-take-all" strategy, where the incoming link to be adjusted is always chosen as a pair of oscillators with the weakest synchronization. This means that the original DO mechanism is more effective than the DGN approach. We previously showed that the original DO mechanism has much better synchronizability in SWNs.[20]

Unfortunately, there exists one main shortcoming in the original DO mechanism.[20] That is, the coupling strength between two connected oscillators is an increasing function of time as well as the intensities are diverging to infinity. Basically, this means that full synchronization is trivially obtained for some kinds of networks, such as any variant of the Kuramoto model[21] and networks of Rössler oscillators coupled through full states. The above networks always converge to a fully synchronized regime if the couplings (or intensities) are sufficiently large. However, for some kinds of networks such as networks of Rössler oscillators coupled through partial states,[22] the synchronization cannot be realized if the couplings (or intensities) are largely enough. In our recent work,[20] we have to end the original DO mechanism provided that the synchronization error is small enough. If not, the couplings (or intensities) are so large that the synchronization can be destroyed and the synchronization error becomes large again. Obviously, it is reasonable to introduce one dynamical mechanism with limited couplings (or intensities) even if time increases to infinity. Here we modify the original DO mechanism such that the intensities are converging and the ultimate intensity can be adjusted.

We consider networks consisting of N coupled oscillators

$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i) + \sum_{j\neq i,j=1}^{N} G_{ij}(\mathbf{H}(\mathbf{x}_j) - \mathbf{H}(\mathbf{x}_i)), \quad 1 \leqslant i \leqslant N, \tag{1}$$

where $\mathbf{x}_i$ is the state, $\mathbf{F}$ is the dynamics of individual oscillator, $\mathbf{H}$ is the output function, and $G=(G_{ij})$ is the coupling matrix. $G_{ij}=A_{ij}W_{ij}$, where $A=(A_{ij})$ is the binary adjacency matrix, W_{ij} is the coupling strength of the incoming link (i,j) pointing from oscillator j to oscillator i if they are connected, $G_{ii}=-\sum_{j\in K_i}A_{ij}W_{ij}$, and K_i is the neighbor set of oscillator i. In unweighted networks, $W_{ij}=1$ is uniform for all the incoming links.

In the original DO mechanism,[20] we increase the coupling strength of only one incoming link of oscillator i by a small value, at the time step $t_n=t_0+n\tau$, where $n\geqslant 1$ is the positive integer, t_0 is the transient time, and $\tau>0$ is the duration time. This adaptation results from the competition between neighbor oscillators within the interval $[t_{n-1},t_n)$. For oscillator i and one neighbor $j\in K_i$, a total synchronization difference, i.e., $E_n(i,j)=\int_{t_{n-1}}^{t_n}\phi(\mathbf{x}_i,\mathbf{x}_j)dt$, within the interval $[t_{n-1},t_n)$ is evaluated, where ϕ is a non-negative error function, and satisfies $\phi(\mathbf{x}_i,\mathbf{x}_j)=0$ if oscillators i,j are synchronized. For oscillator i, the incoming link with the weakest synchronization, i.e., $(i,j^n_{\max})$, is the winner within the interval $[t_{n-1},t_n)$, where the index $j^n_{\max}$ is decided by the optimization problem

$$j^n_{\max} = \arg\max_{j\in K_i} E_n(i,j). \tag{2}$$

If several neighbors have the same synchronization difference, we choose only one randomly. In the original DO mechanism, the coupling strength is adjusted dynamically by[20]

$$\begin{aligned} W^{n+1}_{ij^n_{\max}} &= W^n_{ij^n_{\max}} + \varepsilon, \\ W^{n+1}_{ij} &= W^n_{ij}, j\neq j^n_{\max}, \end{aligned} \tag{3}$$

where the incremental coupling $\varepsilon>0$ is a small value, and W^n_{ij} is the coupling strength in the interval $[t_{n-1},t_n)$. Obviously, the intensities are diverging as time tends to infinity.

In order to ensure that the intensities converge to a limited value as time tends to infinity, we adjust the coupling strength by

$$\begin{aligned} W^{n+1}_{ij^n_{\max}} &= W^n_{ij^n_{\max}} + \chi_n, \\ W^{n+1}_{ij} &= W^n_{ij}, j\neq j^n_{\max}, \end{aligned} \tag{4}$$

where $\chi_n>0$ is the incremental coupling. Here we give some basic rules for choosing the incremental coupling χ_n, which make the ultimate intensities be uniform and convergent. (i) The incremental couplings χ_n for all oscillators are identical at the time step t_n, which make the intensities S_i be uniform during the transition to synchronization. (ii) The incremental coupling χ_n is limited by the fixed constant ε, which implies that at the time step t_n the incremental coupling χ_n should not be large. (iii) The incremental coupling χ_n is a nonincreasing function on the time step n, and the ultimate intensity $\bar{S}=\sum_{i=1}^{\infty}\chi_i$ exists. This requirement means that after the time step t_n, the total intensity $\sum_{i=1}^{n}\chi_i$ is convergent and χ_n approaches zero as the time step n tends to infinity. (iv) The ultimate intensity $\bar{S}$ can be adjusted. This is consistent with realistic cases where the intensities (or couplings) for synchronization are in a certain range (such as networks of Rössler oscillators coupled through partial states). We can further discuss the relationship between network synchronization and network topology by adjusting the ultimate intensity $\bar{S}$.

Summing up the above analysis, the term $\varepsilon e^{-n/n_0}$ is one suitable choice of the incremental coupling χ_n. Hence, we choose $\chi_n=\varepsilon e^{-n/n_0}$. We then adjust the coupling strength by

$$\begin{aligned} W^{n+1}_{ij^n_{\max}} &= W^n_{ij^n_{\max}} + \varepsilon e^{-n/n_0}, \\ W^{n+1}_{ij} &= W^n_{ij}, j\neq j^n_{\max}, \end{aligned} \tag{5}$$

where n_0 is a suitable positive integer. Here we call this mechanism [namely, Eqs. (2) and (5)] the modified DO mechanism.

In this paper, the initial coupling strengths in networks are assumed to be zero.[23] Hence, the intensities are uniform

at the time step t_n, since the intensity of each oscillator increases by the same amount $\varepsilon e^{-n/n_0}$ at the time step t_n. Further, the intensity S_i for oscillator i is bounded by the limit $\bar{S}=\lim_{n\to\infty}S_i$, where

$$\bar{S}=\varepsilon e^{-1/n_0}/(1-e^{-1/n_0}). \tag{6}$$

We can adjust the ultimate intensity by the suitable parameter n_0. For a fixed ε, when n_0 is larger (smaller), the intensity $\bar{S}$ is larger (smaller).

III. ENHANCED SYNCHRONIZABILITY IN SCALE-FREE NETWORKS

We briefly review the stability of networks

$$\dot{\mathbf{x}}_i=\mathbf{F}(\mathbf{x}_i)+\sigma\sum_{j\neq i,j=1}^{N}G_{ij}^0(\mathbf{H}(\mathbf{x}_j)-\mathbf{H}(\mathbf{x}_i)),\quad 1\leqslant i\leqslant N, \tag{7}$$

where σ is the overall strength. For a generally asymmetric matrix $G^0=(G_{ij}^0)$, the variational equation on the synchronous state $\{\mathbf{x}_i=\mathbf{s},\ \forall i\}$ is $\dot{\mathbf{u}}_i=[D\mathbf{F}(\mathbf{s})-\sigma\lambda_l D\mathbf{H}(\mathbf{s})]\mathbf{u}_i$, where D is the Jacobian operator, and λ_l is the complex eigenvalue of the Laplacian matrix L $(=-G^0)$, satisfying $\mathrm{Re}(\lambda_1)\leqslant\mathrm{Re}(\lambda_2)\leqslant\cdots\leqslant\mathrm{Re}(\lambda_N)$. The largest Lyapunov exponent (LLE), i.e., $\Lambda(\epsilon,\eta)$, of the master stability equation $\dot{\mathbf{v}}=[D\mathbf{F}(\mathbf{s})-(\epsilon+i\eta)D\mathbf{H}(\mathbf{s})]\mathbf{v}$ is a function of ϵ and η, which is the master stability function (MSF).[22] Let $\mathcal{R}$ be the region in the complex plane where the MSF provides a negative LLE. The synchronization condition is that the set $\{\sigma\lambda_l:\lambda_l\neq 0\}$ is entirely contained in $\mathcal{R}$.[22] Here we only consider the case where the region $\mathcal{R}$ is bounded, which is shown by the dashed line in Figs. 4(a) and 4(c). A better synchronizability is achieved if simultaneously the ratio $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ and $\max|\mathrm{Im}(\lambda_l)|$ are minimized.[10,12]

In this paper, we have two aims based on networks (1) and (7). One is to realize the synchronization in network (1), in which all the oscillators have uniform intensities during the transition to synchronization. The other is to examine the synchronizability in network (7) when the coupling matrix G^0 is assigned by the coupling matrix from the synchronization in network (1), during or after the adaptation. Our analysis and simulation are based on BA networks.[4] Initially, M oscillators with labels $i=1,\ldots,M$ are fully connected. At every time step a new oscillator is introduced to be connected to M existing oscillators. The probability that the new oscillator is connected to oscillator i depends on degree k_i, i.e., $\Pi_i=k_i/\sum_j k_j$. Here we choose Rössler networks to illustrate the effectiveness of our mechanism: $\mathbf{x}_i=(x_i,y_i,z_i)$, $\mathbf{F}(\mathbf{x}_i)=(-0.97y_i-z_i,0.97x_i+0.15y_i,z_i(x_i-8.5)+0.4)$, $\mathbf{H}(\mathbf{x}_i)=(x_i,0,0)$, and $\phi(\mathbf{x}_i,\mathbf{x}_j)=|x_i-x_j|+|y_i-y_j|+|z_i-z_j|$. In order to measure the synchronization, we define the average error as $E=(1/N)\sum_{i=1}^N|\mathbf{x}_i-\bar{\mathbf{x}}|$, where $\bar{\mathbf{x}}=(1/N)\sum_{i=1}^N\mathbf{x}_i$ is the global mean field.

In our simulations the initial conditions for oscillators are randomly chosen from Rössler attractor (here, $t_0=0$). The parameter n_0 in Eq. (5) is $n_0=1200$. Hence, the limit $\bar{S}$ is about 1.2 if the value $\varepsilon=0.001$. From Fig. 1, the synchronization in network (1) is realized effectively. From Eqs. (2) and (5), all the oscillators have uniform intensities during the transition to synchronization, regardless of heterogeneous degrees and initial conditions. It is consistent with the necessary condition for the optimal synchronizability in the static mechanisms.[11,13] But this is totally different from the dynamical mechanisms.[18,19] The average intensity $S(k)$ over oscillators with degree k increases as $S(k)\sim k^{\beta}$ with $\beta\sim 0.5$.[18]

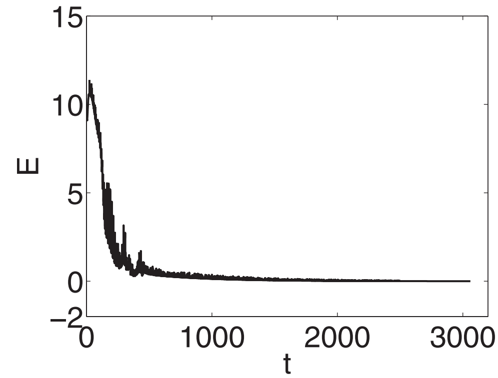

FIG. 1. The average error E as a function of time t. The parameters are $N=1000$, $M=5$, $\tau=1$, $\varepsilon=0.001$, and $n_0=1200$.

During the transition to synchronization, the ratio $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ in network (7) with $G^0=G$ decreases towards the optimal synchronizability $R_{\mathrm{opt}}\approx 3.8$ (Fig. 2). The value R_{opt} is decided by the eigenratio of the Laplacian matrix of $G'(0)$, where $G'(\alpha)=(G'_{ij}(\alpha))$ with $G'_{ij}(\alpha)=(k_ik_j)^{\alpha}/\sum_{j\in K_i}(k_ik_j)^{\alpha}$ and $G'_{ii}(\alpha)=-1$.[11] From Eqs. (2) and (5), the incoming link to be adjusted for each oscillator is always chosen to be the pair of oscillators with the maximal synchronization difference in the previous time interval, which greatly decrease the ratio $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$. However, there still exists the discrepancy between the ultimate value of $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ and R_{opt}. Now we explain the reason for the discrepancy. Due to the "winner-take-all" strategy inherent in the DO mechanism, the coupling strengths W_{ij} for oscillator i are almost uniform statistically as the time step n approaches infinity; namely, $W_{ij}\sim k_i^{-1}$. Unfortunately, the exact uniform coupling strength $W_{ij}=k_i^{-1}$ cannot be realized by dynamical mechanisms. In order to show it, we define the average standard deviation $E_{\mathrm{sd}}(k)=(1/l_k)\sum E_l^0$ between G^0

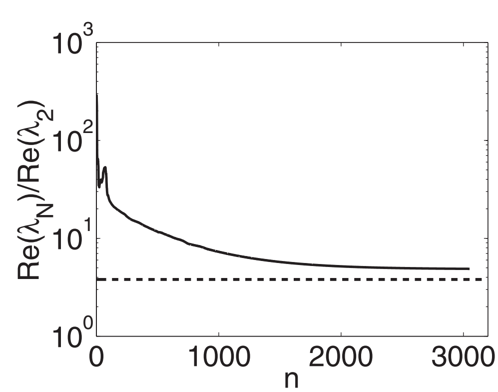

FIG. 2. The ratio $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ as a function of the adjustment step n. Solid line: the ratio by the modified DO mechanism; dashed line: R_{opt}. The parameters are the same as those in Fig. 1.

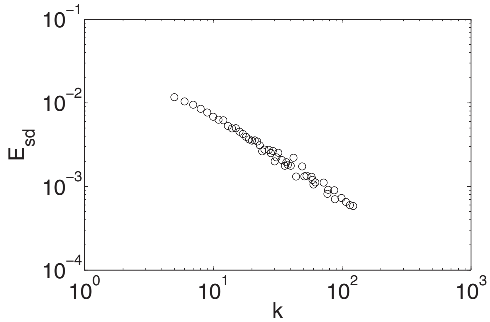

FIG. 3. Standard deviation $E_{sd}(k)$ as a function of degree k. The parameters are the same as those in Fig. 1.

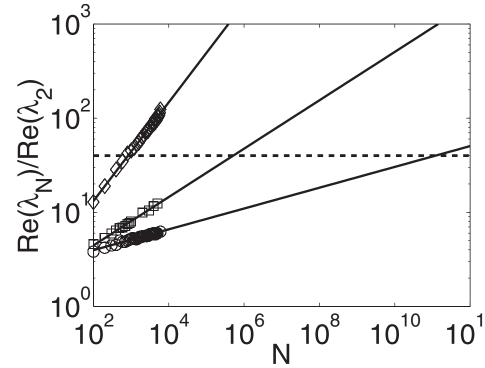

FIG. 5. The ratio $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ for different size of network (7). Diamond: unweighted networks; square: networks with adaptive coupling (see Ref. 18); circle: the ratio by the modified DO mechanism; solid line: fitting; dashed line: $R_\rho=\delta_2/\delta_1\approx 40$. The parameters M, τ, ε, n_0 are the same as those in Fig. 1. All the estimates are averaged over 20 realizations of networks.

given by the following Eq. (8) and $G'(0)$, where l_k is the number of oscillators with degree k and $E_l^0=(1/k)\sqrt{\Sigma_{j\neq i}(G_{ij}^0-1/k)^2}$ (Fig. 3). From this figure, the exact uniform coupling strength $W_{ij}=k_i^{-1}$ cannot be realized by dynamical mechanisms. This may be the reason for the discrepancy between the ultimate value of $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ and R_{opt}.

We assign the coupling matrix G^0 in network (7) by

$$G^0 = G_{\mathrm{norm}} = G_{\mathrm{end}}/\bar{S}, \tag{8}$$

where G_{end} is the coupling matrix of network (1) after the adaptation. Since all the oscillators have uniform intensities, the Laplacian matrices of G_{norm} and G_{end} have equal ratios $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$. When $\sigma=1.5$, all the nonzero eigenvalues of the Laplacian matrix of σG_{norm} are located in a very narrow region around the real axes in the region $\mathcal{R}$, and the absolute values of imaginary parts are sufficiently small [Figs. 4(b) and 4(c)].

The ratio $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ in network (7) with $G^0=G_{\mathrm{norm}}$ increases slightly with increasing the network size N, and this can be well fitted by a power-law dependence, which means the synchronizability decreases slightly (Fig. 5). From the fitting and the value R_ρ, we find that the network (7) is still synchronizable until $N\approx 10.$[11] The size of the network (7) that can be synchronizable exceeds by several orders of magnitude the size of unweighted networks ($\approx 10^3$) and networks with adaptive coupling ($\approx 8\times 10^5$).[18] Obviously, this is a great enhancement of the synchronizability in networks, compared with unweighted networks and networks with adaptive coupling.[18] It should be pointed out that for different size of networks, $\max|\mathrm{Im}(\lambda_l)|$ is sufficiently small (the maximal value is less than 0.1).

For the coupling matrix $G^0=G_{\mathrm{norm}}$, all the eigenvalues are fully contained within the unit circle centered at 1.[24] Thus, $0\leqslant \mathrm{Re}(\lambda_l)\leqslant 2$, $|\mathrm{Im}(\lambda_l)|\leqslant 1$, and the largest $\mathrm{Re}(\lambda_N)$ never diverges, independently of the network size N.[10] During the transition to synchronization in network (1), $S_{\max}/S_{\min}$ is always equals to 1. But in Refs. 14 and 18, the synchronizability decreases with the increasing of $S_{\max}/S_{\min}$, and $S_{\max}/S_{\min}$ increases with the increasing of the size N. Hence, the synchronizability here is better than Ref. 18, whose main aim is to reduce the heterogeneity of the intensities adaptively.

For the fixed n_0 and N, we discuss the effect of parameters τ and ε on the synchronizability in network (7) with $G^0=G_{\mathrm{norm}}$ [Figs. 6(a) and 6(b)]. The value ε can be chosen in a wide range, and the length τ can be arbitrary large. In our simulations, the value ε belongs to $[0.0001, 0.005]$ From Figs. 6(a) and 6(b), the ratio $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ is almost independent of the values of τ and ε.

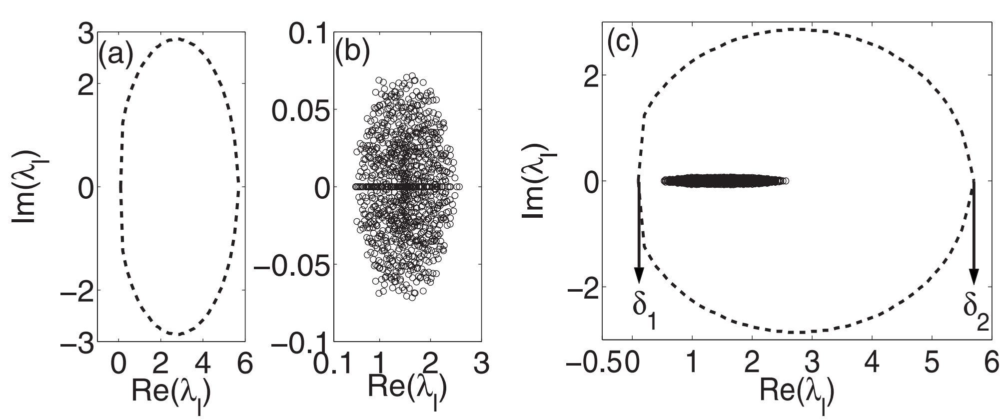

FIG. 4. (a) The stability region $\mathcal{R}$ bounded by the dashed line. (b) Distribution of nonzero eigenvalues λ_l of the Laplacian matrix of σG_{norm}. (c) The location of nonzero eigenvalues λ_l in the region $\mathcal{R}$. Circles: nonzero eigenvalues by the modified DO mechanism; $\delta_1\approx 0.144$ and $\delta_2\approx 5.76$ are the minimum and maximum of real parts in the region $\mathcal{R}$, respectively. The parameters N, M, τ, ε are the same as those in Fig. 1, and $\sigma=1.5$.

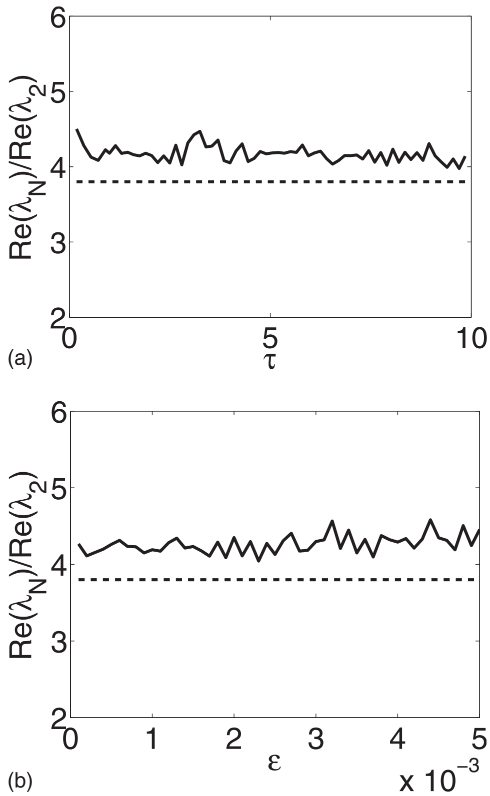

FIG. 6. The ratio $\mathrm{Re}(\lambda_N)/\mathrm{Re}(\lambda_2)$ for different τ (a) and ε (b). Solid line: the ratio by the modified DO mechanism. Dashed line: R_{opt}. The parameters N, M, n_0 are the same as those in Fig. 1. All the estimates are averaged over 20 realizations of networks.

IV. CONCLUSION

In this paper, we introduce a modified dynamical optimization coupling scheme to enhance the synchronizability in the scale-free networks as well as to keep uniform and converging intensities during the transition to synchronization. Moreover, the size of networks that can be synchronizable exceeds by several orders of magnitude the size of unweighted networks.

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