PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Preface

Elbert E.N Macau, Celso Grebogi and Jürgen Kurths

Phil. Trans. R. Soc. A 2008 **366**, 489-491
doi: 10.1098/rsta.2007.2120

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

Phil. Trans. R. Soc. A (2008) **366**, 489–491
doi:10.1098/rsta.2007.2120
Published online 13 August 2007

Preface

Mathematical models can provide a powerful description of the real world. Usually, there is a phenomenon of interest that one wants to describe or understand. By using fundamental laws and observations, a simplified representation of the phenomenon can be obtained and consolidated in mathematical equations. It is expected that these mathematical equations capture the essential and relevant aspects of the phenomenon in the context of a specific desired purpose. Furthermore, these equations can be analytically and computationally explored to lead to predictions for the behaviour of the phenomenon under a set of changes and operational conditions.

In 1961, Lorenz (1963) was working on a computer model for the atmosphere dynamics. It was a nonlinear model based on the evolution of the Rayleigh–Bérnard instability (Saltzman 1962), which results when a fluid layer subjected to gravity is heated from below. At one time, he repeated a computation simulation by initializing from a previously obtained value. His computer operated using six precision digits, but the results were printed out with just three digits. Lorenz did input these three digits back into the computer and restarted his program. After some time, the trajectory of the weather pattern diverged from the original result. This small and tiny truncation error in the fourth decimal was somehow amplified by his numerically implemented model so that, after a short time, the new solution was completely uncorrelated from the previously obtained one. By a careful analysis of this phenomenon, combined with an innovative way of data analysis, he established that this phenomenon of 'sensitive dependence on initial conditions' was a fundamental characteristic of his mathematical model and it should be present in nature. With this computational experiment and ensuing mathematical analysis, he captured for the first time the essence of the chaotic behaviour in a physically relevant model.

The pioneering work of Lorenz triggered much research work on deterministic physically relevant models to properly understand the behaviour he had uncovered and its consequence in nature. As such, the same deterministic aperiodic behaviour that presents a sensitive dependence on initial conditions and whose time evolution is eventually trapped in a specific region of the phase space, exhibiting a very intricate geometric structure (fractal), was found in a large class of nonlinear systems. The methods introduced by Lorenz to detect this deterministic aperiodic behaviour in the context of data time-series were improved and extended to allow for a proper recognition and characterization of the phenomenon based on a formal and coherent framework. Chaotic dynamics is recognized and characterized using quantifiers that are not particular to a

One contribution of 14 to a Theme Issue 'Experimental chaos II'.

specific trajectory, but they capture the invariant characteristics of the chaotic system. The development of these quantifiers for data series analysis continues to be a very active area of research.

The use of these quantifiers in computational experiments is facilitated because one has all the available state space variables of the simulated system. However, this is not the case in real experiments. Typically, the result of an experiment is a time-series that captures the time variation of one or at most a few specific variables of the system. It means that, in an experiment, the complete state space variables of the system are in general not available. Hence, the main problem is how to reconstruct the system dynamics from a limited dataset, which is essential to allow for the characterization of a possible dynamical behaviour. The major breakthrough on this problem was the introduction of the technique of time-delay embedding (Takens 1981). The basic idea behind this method is that the evolution of any single variable of the system has information about the evolution of the other variables. The dynamics of the system is thus implicitly contained in the time history of any single variable. Consequently, a topological equivalent state space can be reconstructed by looking at a single variable and its measurements at given time delays. The measurement and its delayed values are viewed as new coordinates, defining a single point in a multidimensional state space. This procedure is repeated on the available data series and so the system dynamics can be reconstructed and properly characterized. Over the years, other techniques have been proposed, but this method continues to be the canonical one.

This second issue of the *Phil. Trans. R. Soc. A* on Experimental Chaos focuses on data analysis theory, models and applications of chaotic dynamical systems. They represent the up-to-date techniques related to the proper handling of data that comes from real experiments or observation of natural process and were originally selected from the lectures that were presented at the IX Experimental Chaos Conference, held at the National Institute for Space Research—INPE in São José dos Campos, Brazil, from 29 May to 1 June 2006. This is the world's main conference on experimental chaos and is the proper stage on which experimental breakthroughs are usually presented. Therefore, we are confident that the most relevant developments related to the techniques of characterization of chaotic dynamics in experiments are properly reported here.

As guest editors for this issue, we wish to thank all those who accepted our invitation to submit manuscripts for consideration. We would also like to thank the many individuals who served as referees. We hope that these articles can motivate and even foster the development of techniques for the analysis of experiments, thus allowing for a continuous and better understanding of the role of chaotic dynamics in our world.

Elbert E. N. Macau[1], Celso Grebogi[2] and Jürgen Kurths[3]
[1]*Laboratory for Computation and Applied Mathematics—LAC, National Institute for Space Research—INPE, CEP 12245-970, São José dos Campos, São Paulo, Brazil*
E-mail address: elbert@lit.inpe.br
[2]*School of Engineering and Physical Sciences, Kings College, University of Aberdeen, Aberdeen AB24 3FX, UK*
[3]*Institute of Physics, University of Potsdam, 14469 Potsdam, Germany*

References

Lorenz, E. 1963 Deterministic nonperiodic flow. *J. Atmos. Sci.* **20**, 130–141. (doi:10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2)

Saltzman, B. 1962 Finite amplitude free convection as an initial value problem. *J. Atmos. Sci.* **19**, 329–341. (doi:10.1175/1520-0469(1962)019<0329:FAFCAA>2.0.CO;2)

Takens, F. 1981 Detecting strange attractors in turbulence. *Spring. Lect. Notes Math.* **898**, 366–381.