PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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Preface

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Preface

The Chaotic Dynamics was discovered in the scenario of a mathematical model. Around the end of the nineteenth century, Poincaré (1908) was working to find out whether our Solar System is stable or not. To make this problem more mathematically tractable, he considered a special case in which two massive particles, the primaries, move in circular orbits around their centre of mass, while another particle with a smaller mass moves in the plane of the primaries so that it suffers their gravitational influence but its presence does not disturb the motion of the primaries. This is the so-called circular restricted three-body problem. Even this simplified model represents a huge mathematical challenge to be analysed. To overcome this challenge, Poincaré introduced qualitative techniques of geometry and topology to understand the global properties of the solutions. By using these techniques, he was able to glimpse the chaotic motion that appears on complicated and apparently unpredictable trajectories that were close to periodic orbits, but spread out in bounded regions of the phase space. Analysing this motion, Poincaré (1908) concluded that '... it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon'. Thus, Poincaré identified in his mathematical model the property of sensitive dependence on initial conditions, which is the hallmark of chaos.

In the twentieth century, the studies initiated by Poincaré were further developed by Birkhoff, Cartwright and Littlewood, Levinson, Anosov, Kolmogorov, Moser, Arnold, Peixoto and Smale, and Shilnikov, among many others. They made important contributions to understand mathematically the chaotic behaviour. Now, it is mathematically known that any nonlinear, continuous in-time dynamical system with more than one degree of freedom can display chaos, and this dynamical behaviour can be present in attractive as well as non-attractive invariant sets. The chaotic behaviour can be explained by topological operations of stretching and folding that take place in the state space (Smale 1967).

Despite being studied in mathematics for over a century, only in the last couple of decades of the twentieth century has the profound impact of chaos in science, engineering and medicine been recognized. And this recognition happened in part as a consequence of the widespread use of powerful computers that allow not only for extensive simulations of nonlinear models of natural process, but also for careful analysis of the results by using properly developed tools that require intensive graphical support. As such, there emerged a scenario in which the chaotic behaviour was completely understood both mathematically and in computer simulations, and also in the corresponding experimental systems

One contribution of 15 to a Theme Issue 'Experimental chaos I'.

that the simulations were trying to model. This issue of the *Phil. Trans. R. Soc. A* on Experimental Chaos presents a unique collection of experimental works whose processes are of scientific and technological importance.

The first fundamental experiment concerning the presence of chaotic dynamics in the real world was conducted in 1975 by Gollub and Swinney in a Couette flow cell (Gollub & Swinney 1975). The space between two concentric cylinders is filled with a fluid, and one or both of them are rotated with a fixed angular velocity. As the angular velocity increases, the fluid showed progressively more complex flow patterns, with an involved time dependence. In this experiment, they measured the velocity of the fluid at a given point. Following the experiment, as the rotation rate is increased, they observed transitions from a velocity that is constant in time at the beginning to a periodically varying velocity and finally to an aperiodically varying velocity. They were interested in characterizing how this aperiodically varying velocity, which may suggest the presence of a chaotic dynamics, develops from the periodical one as the angular velocity of the cylinders increases.

The mechanisms that take place in such transitions are called bifurcation scenarios. For this experiment, the scenario is related to the presence of a geometric structure called torus. This shape describes the motion in the phase space of two or more independent oscillations with well-defined frequencies. Whenever the movement of a system can be reduced in the phase space to a torus, its dynamics is a non-chaotic one because it does not have sensitive dependence on the initial condition, meaning that trajectories which start close to each other remain close on average.

The experiment was conceived to determine which of the two proposed transition scenarios may explain the onset of turbulence. In the Landau scenario (Landau & Lifshitz 1959), proposed in 1944, turbulence appears as a consequence of an ever higher number of oscillations that are excited as the rotation rates of the cylinders are increased. As such, the dynamics would not be chaotic. This scenario was challenged by Ruelle and Takens, who gave, in 1971, a mathematical argument (Ruelle & Takens 1971) that the motion in a three- or higher-dimensional torus is generically chaotic. The result of the experiment showed that the Landau scenario was incomplete and that the behaviour 'seems to be of the general type described by Ruelle and Takens'. This remarkable experiment opened up the way for an extensive investigation regarding all the aspects of the phenomenon of chaotic motion in nature.

The articles that appear in this issue were selected from the lectures presented at the IX Experimental Chaos Conference, held at the National Institute for Space Research—INPE in São José dos Campos, Brazil, from 29 May to 1 June 2006. This is the World's main conference on experimental chaos and is the proper stage on which breakthrough experimental results are usually presented for the first time. We are confident that most relevant experimental works on chaotic dynamics are properly reported here.

As guest editors for this issue, we wish to thank all those who accepted our invitation to submit manuscripts for consideration. Also, we would like to thank the many individuals who served as referees. We hope that these articles can motivate and foster further experimental work that will allow for a continuous and better understanding of the role of chaotic dynamics in our world.

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