A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

February 2007

EPL, **77** (2007) 40006
doi: 10.1209/0295-5075/77/40006

www.epljournal.org

Detecting phase synchronization by localized maps: Application to neural networks

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received 21 September 2006; accepted in final form 20 December 2006
published online 9 February 2007

PACS 05.45.Xt – Synchronization; coupled oscillators
PACS 05.45.-a – Nonlinear dynamics and chaos
PACS 84.35.+i – Neural networks

Abstract – We present an approach which enables to state about the existence of phase synchronization in coupled chaotic oscillators without having to measure the phase. This is done by observing the oscillators at special times, and analyzing whether this set of points is localized. In particular, we show that this approach is fruitful to analyze the onset of phase synchronization in chaotic attractors whose phases are not well defined, as well as, in networks of non-identical spiking/bursting neurons connected by chemical synapses.

Many neural networks rely on a synchronous behavior for a proper functioning, *e.g.* information transmission [1,2], pattern recognition [3], and learning [4]. Nevertheless, the onset of synchronization in a network may also lead to some diseases as Parkinson disease [5] and epilepsy [6]. Studies on this topic have been concentrated on synchronization of networks of identical chaotic oscillators, in which the onset of complete synchronization takes place [7]. However, they do not treat the onset of synchronization in networks of non-identical chaotic oscillators, case typically found in nature [8], where complete synchronization is much harder to be achieved. Indeed, in such networks a weaker kind of synchronization may take place, the phase synchronization (PS). The condition for PS between two subsystems k and j can be written as

$$|\phi_k(t) - r\phi_j(t)| < c, \tag{1}$$

where $\phi_{k,j}$ is the phase of the subsystem $\Sigma_{k,j}$, $c \in \mathbb{R}$ is a constant, and r is a rational number. PS is a common phenomenon in interacting chaotic oscillators [8], and plays a major role in physical processes linked to communication [9], and communication processes in the human brain [6,10].

In order to state about the existence of PS, one has to introduce a phase $\phi(t)$ for the chaotic oscillator, which is not straightforward. Indeed, there is no general definition of phase to chaotic attractors, and depending on the situation one has to decide which phase is more suitable. In fact, in some oscillators it is rather unclear which phase one should use, especially in non-coherent oscillators with more than one time scale, typically found in neuronal dynamics with bursting/spiking behavior.

In this letter, we present a general and easy way to identify PS without having to access explicitly the phase. The approach consists in defining maps, which are a natural extension of the stroboscopic map, to coupled chaotic oscillators, in which the oscillators are observed at special times. PS implies the existence of maps of the attractor that appear as localized structures in the accessible phase space. The fact that PS produces subsets of the attractor that are localized structures, by particular observations was previously used as a way to detect PS in chaotic oscillators [8,11,12]. Here, we extend these results by demonstrating that localized sets can be constructed while in PS by means of *any typical physical* observation, which has a strong impact in the field of experimental physics, since in the laboratory measurements are restricted to the limitations of the experiment. Note that since this approach does not require any further calculation, but just the analysis of whether the sets are localized, it can be used in real-time experiments for PS detection. We illustrate the power of this approach by analyzing PS in a network of non-identical Hindmarsh-Rose (HR) spiking/bursting neurons connected via chemical synapses.

The classic stroboscopic map is defined in periodically driven chaotic oscillators. It consists in sampling the chaotic trajectory at times nT_0, where n is an integer and T_0 is the period of the driver. The stroboscopic map was used to detect PS [8,12]. The basic idea is

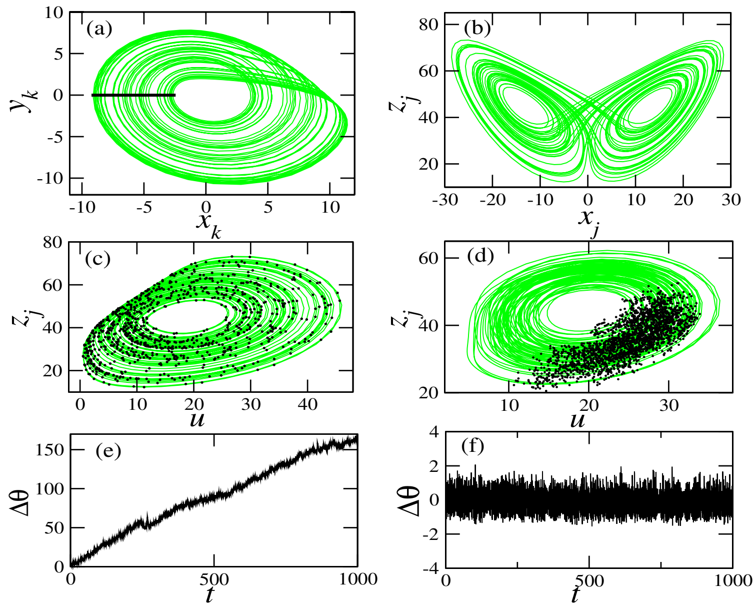

Fig. 1: PS onset between a Lorenz oscillator driven by a Rössler oscillator. The attractor of the oscillators are depicted in gray (a-d). In (a) we show the projection of the Rössler attractor into the subspace (x_k, y_k). The black line shows the Poincaré section at $y_k = 0$ with the constrain $\dot{y}_k < 0$. The projection of the Lorenz attractor in the subspace (x_j, z_j) is depicted in (b), and in (c,d) into the subspace (u, z_j). For $\epsilon = 0.0$, the set $\mathcal{D}_j$ (in black) spreads over the attractor of the oscillator Σ_j (c), and there is no PS; the phase difference diverges (e). For $\epsilon = 13.0$ the oscillators present PS. The set $\mathcal{D}_j$ is localized, it does not fulfill the attractor of Σ_j (d); the phase difference is bounded (f).

that if the stroboscopic map is localized in the attractor, PS is present. To generalize the stroboscopic map to coupled chaotic oscillators, we do the following: Given two subsystems Σ_k and Σ_j, we observe Σ_k at times when some event in the oscillator Σ_j happens. As a consequence of these observations, we get a discrete set $\mathcal{D}_k$. Then, we demonstrate that if there is PS, then the set $\mathcal{D}_k$ is localized.

In order to introduce our ideas, we analyze PS in two coupled chaotic oscillators, namely the Lorenz oscillator driven by the Rössler one. Furthermore, we extend this result to general compact oscillators. The subsystem Σ_k corresponds to the Rössler oscillator and Σ_j to the Lorenz one. They are coupled unidirectionally in the driver response scheme. An event in the Σ_k is considered to happen when its trajectory crosses a Poincaré section $y_k = 0$. As a result, we get the series of times $(\tau_k^i)_{i \in \mathbb{N}}$, where τ_k^i is the time at which the i-th crossing of the trajectory of Σ_k occurs in a Poincaré plane. The two coupled oscillators are given by

$$\begin{aligned}
\dot{x}_k &= -\alpha(y_k + z_k), \\
\dot{x}_j &= \sigma(y_j - x_j) + \epsilon(x_k - x_j), \\
\dot{y}_k &= \alpha(x_k + 0.2 y_k), \\
\dot{y}_j &= r x_j - y_j - x_j z_j, \\
\dot{z}_k &= \alpha[0.2 + z_k(x_k - 5.7)], \\
\dot{z}_j &= x_j y_j - \beta z_j,
\end{aligned}$$

with $\alpha = 13$, $\sigma = 16$, $r = 45.92$, and $\beta = 4$. Since the trajectory of the Rössler oscillator rotates around a fixed point (fig. 1(a)), we can define a phase $\theta_k = \tan^{-1}(y_k/x_k)$ which gives: $\theta_k(t) = \int_0^t (\dot{y}_k x_k - \dot{x}_k y_k)/(x_k^2 + y_k^2) \mathrm{d}t$. The trajectory of the Lorenz does not have a unique center of rotation, see fig. 1(b). However, if we consider the projection (u, z_j) with $u = \sqrt{x_j^2 + y_j}$, the trajectory projected into this subspace presents a unique center of rotation. Thus we also define a phase $\theta_j = \tan^{-1}[(z_j - z_{j0})/(u - u_0)]$, where $(u_0, z_{j0}) = (19, 45)$ is the center of rotation in the subspace (u, z_j), which gives $\theta_j(t) = \int_0^t [\dot{z}_j u - \dot{u}(z_k - z_{j0})]/[(u - u_0)^2 + (z_j - z_{j0})^2] \mathrm{d}t$.

For $\epsilon = 0.0$, we construct the set $\mathcal{D}_j$ by sampling the trajectory of Σ_j at times τ_k^i. The set $\mathcal{D}_j$ spreads over the trajectory of Σ_j; there is no PS, the phase difference $\Delta\theta = \theta_k - \theta_j$ diverges (fig. 1 (c,e)). Indeed, a calculation of the frequencies shows that $\langle \dot{\theta}_k \rangle \approx 13.94$ and $\langle \dot{\theta}_j \rangle \approx 13.75$. As we increase the coupling, PS appears. In particular, for $\epsilon = 13.0$ the set $\mathcal{D}_j$ is localized, and the phase difference is bounded (fig. 1(d,f)). The average frequencies are $\langle \dot{\theta}_k \rangle = \langle \dot{\theta}_j \rangle \approx 13.95$.

Next, we demonstrate that the sets $\mathcal{D}_j$ of the attractor that appear as localized structures imply PS, and vice versa. We first show for a Poincaré section for a better understanding of the ideas, and then we generalize these results to any possible event. Σ_j is given by the dynamical system $\dot{\mathbf{x}}_j = \mathbf{G}_j(\mathbf{x}_j)$, let F_j^t be the flow, Γ_j the Poincaré section, and Π_j the Poincaré map associated to the section

Γ_j, such that given a point $\mathbf{x}_j^i \in \Gamma_j$, so $\mathbf{x}_j^{i+1} = \Pi_j(\mathbf{x}_j^i) = F_j^{\Delta\tau_j^{i+1}}(\mathbf{x}_j^i)$, where $\Delta\tau_j^i = \tau_j^i - \tau_j^{i-1}$. From now on, we use a rescaled time $t' = t/\langle T_j \rangle$. For a slight abuse of notation we omit the "$\,'\,$". The average return time is given by $\langle T_k \rangle = \sum_{i=0}^{N} \Delta\tau_k^i / N = \tau_k^N / N$, and the time is rescaled, such that $\langle T_k \rangle = 1$. From our hypothesis that both oscillators present phase-coherent oscillations, there is a number κ_k such that $|\tau_k^i - i\langle T_k \rangle| \leqslant \kappa_k$, where $\kappa_k \ll 1$. If both oscillators are in PS, then $\langle T_k \rangle = \langle T_j \rangle$, and so

$$|\tau_k^i - \tau_j^i| \leqslant \tilde{\kappa}, \tag{2}$$

with $\tilde{\kappa} \leqslant \kappa_k + \kappa_j \ll 1$. Now, we analyze one typical oscillation, using the basic concept of recurrence. Given the following starting points $\mathbf{x}_k^0 \in \Gamma_k$ and $\mathbf{x}_j^0 \in \Gamma_j$, we evolve both until $\mathbf{x}_j^0$ returns to Γ_j. Let us introduce $\Delta\tau^i = \Delta\tau_k^i - \Delta\tau_j^i$. So, $\mathbf{F}_j^{\Delta\tau_j^1}(\mathbf{x}_j^0) = \Pi_j(\mathbf{x}_j^0) = \mathbf{x}_j^1 \in \Gamma_j$. Analogously, $\mathbf{F}_k^{\Delta\tau_j^1}(\mathbf{x}_k^0) = \mathbf{F}_k^{\Delta\tau_k^1 + \Delta\tau^1}(\mathbf{x}_k^0) = \mathbf{F}_k^{\Delta\tau^1} \circ \mathbf{F}_k^{\Delta\tau_k^1}(\mathbf{x}_k^0) = \mathbf{F}_k^{\Delta\tau^1}(\Pi_k(\mathbf{x}_k^0)) = \mathbf{F}_k^{\Delta\tau^1}(\mathbf{x}_k^1)$. Now, by using the fact that $|\Delta\tau^i| < \tilde{\kappa}$, we can write: $\mathbf{F}_k^{\Delta\tau^1}(\mathbf{x}_k^1) \approx \mathbf{x}_k^1 + \mathbf{G}(\mathbf{x}_k^1)\tilde{\kappa} + O(\tilde{\kappa}^2)$. So, given a point $\mathbf{x}_k \in \Gamma_k$ evaluated by the time when the trajectory of Σ_j returns to the section Γ_j, the point $\mathbf{x}_k$ returns near the section Γ_k, and vice versa. For a general case, we have to show that a point, in the section Γ_k, evolved by the flow for an arbitrary number of events in the oscillator Σ_j, still remains close to Γ_k. But, this is straightforward, since $|\sum_{i=0}^{N} \Delta\tau^i| = |\tau_k^N - \tau_j^N| < \tilde{\kappa}$. So, we demonstrated that the PS regime implies the localization of the set $\mathcal{D}_k$. Now, we show that the localization of the set $\mathcal{D}_k$ implies PS. Supposing that we have a localized set $\mathcal{D}_k$, so, eq. (2) is valid, by the above arguments. Therefore, we just have to show that eq. (2) implies PS. To do so, we note that at every crossing of the trajectory with the Poincaré section the phase increases 2π, as a consequence $\phi_k(\tau_k^i) = 2i \times \pi$. Then, $|\phi_k(\tau_k^i) - \phi_j(\tau_k^i)| = |\phi_k(\tau_k^i) - \phi_j(\tau_j^i + \zeta)|$, where $\zeta = \tau_k^i - \tau_j^i$. Now, expanding the phase ϕ_j in Taylor series around τ_j^i, we have $\phi_j(\tau_j^i + \zeta) \approx 2i\pi + \dot{\phi}_j(\tau_j^i) \times \zeta + \mathcal{O}(\zeta^2)$, as a result, the phase difference can be written as

$$|\phi_k(\tau_k^i) - \phi_j(\tau_k^i)| \leqslant \Lambda \times |\tau_k^i - \tau_j^i| \leqslant \Lambda \times \tilde{\kappa}, \tag{3}$$

where, $\Lambda = \max_{t,j}\{\dot{\phi}_j(t)\}$. Therefore, we showed that boundness in eq. (2) implies a bound in the phase difference at times τ_k^i. However, since the phase depends smoothly on time, and the Poincaré section can be smoothly changed, the boundness in eq. (3) also holds at the continuous time. Thus, we conclude our result.

An important point to stress is that it is not always possible to define a Poincaré Section on the attractor in such a way that a phase increases 2π every crossing. As an example we quote the non-coherent attractors with no proper rotations, where the definition of such section is not possible. Moreover, even if the oscillators are coherent, it might happen that the accessible data is not suitable to define a section, but rather to measure the entrance of the trajectory in some small region of the phase space. That does not constitute a problem, because PS implies localization of the set $\mathcal{D}_k$, independently on the event definition.

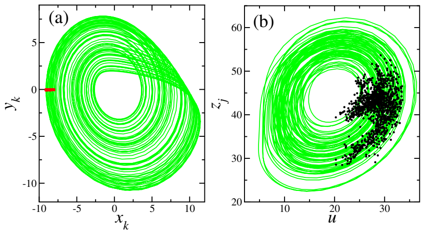

Fig. 2: PS implies the existence of localized sets, which is generated by the observation of an arbitrary typical event. The attractors of the oscillators are depicted in gray. For $\epsilon = 13.0$, we show the projections of the Rössler Lorenz attractors (a,b). The black bold line shows the segment $\mathcal{S}_k$ (a). $\mathcal{D}_j$ is constructed by observing the oscillator Σ_j whenever the trajectory of Σ_k crosses the segment $\mathcal{S}_k$. In (b) we show the set $\mathcal{D}_j$ in black. Since there is PS, $\mathcal{D}_j$ is localized.

Let us first discuss the idea of localization. If the set $\mathcal{D}$ is a subset of Φ, we say that $\mathcal{D}$ is localized (with respect to Φ) if there is a cross-section Ψ and a neighborhood Λ of Ψ, such that $\mathcal{D} \cup \Lambda = \emptyset$. In particular, for practical detections, one may check whether $\mathcal{D}$ is localized, by the following technique. If there is PS, for $\mathbf{y} \in \mathcal{D}$ there exist infinitely many $\mathbf{x} \in \Lambda$ such that $\mathbf{y} \cap B_\ell(\mathbf{x}) = \emptyset$, where $B_\ell(\mathbf{x})$ is an open ball of radius ℓ centered at the point $\mathbf{x}$, and ℓ is small. Then, we may vary $\mathbf{y}, \mathbf{x}$ (one may take $\mathbf{x}$ to be an arbitrary point of the attractor) and ℓ, to determine whether $\mathcal{D}$ is localized.

The event definition that generates the time series $\{\tau_k^i\}_{i \in \mathbb{N}}$ can be arbitrary. Therefore, the event could be a local maximum/minimum, the crossing of a dynamical variable with a threshold, the entrance in an ε-ball, and so on. The only constraint is the event must be typical. We also suppose that there is a function phase ϕ_k, in such a way that $\dot{\phi}_k = \Omega_k$, where Ω_k is continuous and $\Omega_k \leqslant \Upsilon$. Under such hypotheses, we can state that: *Given any typical event, with positive measure, in the oscillator* $\Sigma_{k,j}$, *generating the times*$(t_{k,j}^i)_{i \in \mathbb{N}}$, *if there is PS the observation of* Σ_k *at* $(t_j^i)_{i \in \mathbb{N}}$ *generates a localized set* $\mathcal{D}_k$.

Next, we demonstrate this result, doing so we extent these ideas to non-coherent oscillators. The strategy to demonstrate the previous results to an arbitrary event is the following: i) Note that the phase $\phi_j(t)$ naturally defines a section, namely $\tilde{\Gamma}_j$, in the attractor such that at the N-th crossing of the trajectory of Σ_j with $\tilde{\Gamma}_j$ the phase is equal to $N \times 2\pi$. Obviously, this section depends on the initial conditions. ii) Suppose that we construct the set $\mathcal{D}_k$ by observing the trajectory of Σ_k at every crossing of the trajectory of Σ_j with $\tilde{\Gamma}_j$. Then, following the previous results, PS implies the localization of $\mathcal{D}_k$, and vice versa. iii) Suppose that we have a small piece $P_{\tilde{\Gamma}_j}$ of the section

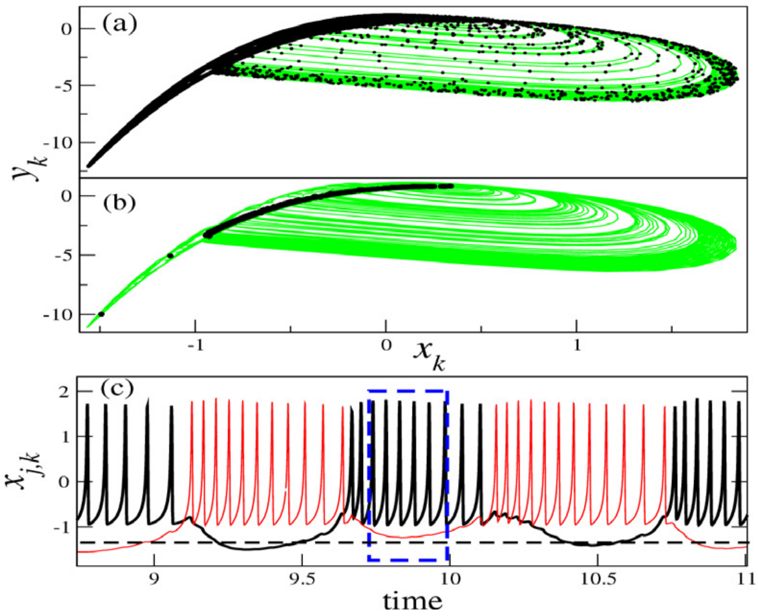

Fig. 3: Onset of PS in two HR neurons coupled via inhibitory synapses. In (a-b), we plot the attractor projection (x_k, y_k) in gray, and the set $\mathcal{D}_k$ in black, constructed by observing the subsystem Σ_k whenever Σ_j crosses the threshold $x_j = -1.3$ represented by the dashed line in (c). In (a) the set $\mathcal{D}_k$ spreads over the attractor which shows that there is no PS. In (b) the set $\mathcal{D}_k$ is localized which shows the presence of PS. In (c) we present the time series of the membrane potential of (b). The threshold (dashed line) can mislead the burst occurrence (see the box) leading to the wrong statement that there is no PS. For (a) the parameters are $I_k = 3.12$ and $g_{syn} = 0.75$, for (b) and (c) $I_k = 3.12$ and $g_{syn} = 0.85$.

$\tilde{\Gamma}_j$, such that the crossings of the trajectory of Σ_j with $P_{\tilde{\Gamma}_j}$ produces a subsequence $(\tau_j^{n_i})_{n_i \in \mathbb{N}}$ of the sequence $(\tau_j^i)_{i\in\mathbb{N}}$. Thus, we just note that if the observation of the trajectory of Σ_k at times $(\tau_j^i)_{i\in\mathbb{N}}$ gives place to a localized set $\mathcal{D}_k$, the observation at times $(\tau_j^{n_i})_{n_i\in\mathbb{N}}$ also gives place to a localized set $\tilde{\mathcal{D}}_k$ which is a subset of $\mathcal{D}_k$. Therefore, we showed that the observation of the trajectory of Σ_k, when the trajectory of Σ_j returns to $P_{\tilde{\Gamma}_j}$, also leads to a localized set in Σ_k. iv) Next, we show that an event does not have to be a piece of the section $\tilde{\Gamma}_j$ in order to obtain a localized set in Σ_k. Indeed, given an ε-ball event that produces the time series $\tilde{\tau}_j^i$, in Σ_j, there is, at least, one intersection of this ball with the section $\tilde{\Gamma}_j$. Since $\tilde{\Gamma}_j$ depends on the initial conditions, we can choose an initial condition right at the ε-ball event. Next, we choose $P_{\tilde{\Gamma}_j}$ such that it is completely covered by the ε-ball. Since the measure of the ε-ball is small, $\varepsilon \ll 1$, the time difference between crossings of the trajectory with $P_{\tilde{\Gamma}_j}$ and the ε-ball, namely $\tilde{\tau}_j^i - \tau_j^{n_i}$, is also small. Therefore, if we observe the trajectory of Σ_k at times $(\tilde{\tau}_j^i)_{i\in\mathbb{N}}$, we get a localized set in Σ_k close to the set $\tilde{\mathcal{D}}_k$. Thus, we conclude our result.

In order to illustrate these ideas, we consider again the Lorenz oscillator driven by the Rössler oscillator. As we showed before, for $\epsilon = 13.0$ there is PS. Thus the sets $\mathcal{D}$ must be localized independently of the event chosen. We define the event in the oscillator Σ_k to be the crossing of the trajectory $\mathcal{S}_k = \{x_k, y_k, z_k \in \mathbb{R} | x_j < -8, y_k = 0, \dot{y_k} < 0\}$. These crossings generate the times $(t_k^i)_{i\in\mathbb{N}}$. $\mathcal{S}_k$ is depicted in black bold line in fig. 2(a) together with the attractor of the Rössler oscillator depicted in gray. The observation of the trajectory of Σ_j at the times $(t_k^i)_{i\in\mathbb{N}}$ generates a localized set $\mathcal{D}_j$ (fig. 2(b)).

Keeping these results in mind, we analyze the onset of PS between two non-coherent neurons of the HR type coupled by chemical synapses. The neurons are described by a 4-dimensional HR model [13] which consists of four coupled differential equations: $\dot{x}_k = ay_k + bx_k^2 - cx_k^3 - dz_k + I_k + g_{syn}\sum_j \gamma_{kj} I_{syn}(x_j)$, $\dot{y}_k = e - y_k + fx_k^2 - gw_k$, $\dot{z}_k = \mu(-z_k + R(x_k + H))$, $\dot{w}_k = \nu(-kw_k + r(y_k + l))$, x_k represents the membrane potential, y_k is associated with fast current dynamics, and (z_k, w_k) are associated with slow currents, I_{syn} is the synaptic input, and γ_{kj} is the connectivity matrix: $\gamma_{kj} = 1$ if neuron j is connected to neuron k, and $\gamma_{kj} = 0$, otherwise, with $j \neq k$.

We set the parameters of the model in order to obtain a spiking/bursting behavior [13]. Then, we couple the neurons by means of chemical synapses. The current I_{syn} injected in the postsynaptic cell is given by [14]: $I_{syn}(x_j) = S_j[x_j - V_{rev}]$, $\tau \frac{dS_j}{dt} = \frac{S_{\infty j}(x_i) - S_j}{S_0 - S_{\infty j}(x_i)}$, where V_{rev} is the synaptic potential, and τ is the timescale governing receptor binding. S_∞ is given by $S_\infty(V) = \tanh[(V - V_{th})/V_{slope}]$, if $V > V_{th}$ and 0, otherwise. The synapse parameters are chosen in order to have an inhibitory effect, so, we set: $V_{th} = -0.80$, $V_{slope} = 1.00$, $V_{rev} = -1.58$, and $S_0 \geqslant 1$.

Now, the time series of events τ_j^i is the time at which the i-th crossing of membrane potential x_j reaches a threshold, namely $x=-1.3$. We fix $I_k=3.1200$ and $I_j=3.1205$, then for $g_{syn}=0.75$ the set $\mathcal{D}_k$ spreads over the attractor (fig. 3(a)); there is no PS. As we increase g_{syn}, the coupled neurons undergo a transition to PS, *i.e.* the set $\mathcal{D}_k$ is localized, fig. 3(b). The neurons are highly non-coherent, due to the existence of two time scales, and the inhibitory synapse which causes one neuron that is in a spiking behavior to inhibit the other neuron, which hyperpolarizes, but still tries to spike. This competition generates even more non-coherence in the phase space. As a consequence, it is rather unclear how one can calculate the phase for such dynamics. What has been currently done is to estimate the phase of the chaotic neuron by assuming that in every crossing in a given direction of the membrane potential with a threshold, the phase increases 2π [8]. The main problem with this approach is that the phase is threshold dependent, so, it can lead to the false statement that PS does not exist. We illustrate this problem in fig. 3(c), for the same parameter as in fig. 3(b); there is, indeed, PS. For a threshold $x=-1.3$ (dashed line), one burst is missed, what makes the phase difference to be no longer bounded as the time goes to infinity, leading to the wrong statement that there is no PS. Our approach, on the other hand, is not event dependent. Indeed, as we showed, a localized set $\mathcal{D}_k$ exists for this threshold.

Next, we analyze a network of 16 non-identical HR neurons, connected with excitatory chemical synapses. In order to simulate a mismatch in the intrinsic current, we set $I_i=3.12+\eta_i$, where η_i are uniformly distributed within the interval $[-0.05, 0.05]$. To simulate the excitatory synapses, we use the same I_{syn}, but changing the value of V_{rev}. Note that if $V_{rev}\geqslant x_i(t)$ the neuron presynaptic always injects a positive current in the postsynaptic neuron. In the following, we set $V_{rev}=2$. Our network is a homogeneous random network, *i.e.* all neurons receive the same number k of connections, namely $k=4$. We constrain g_{syn} to be equal to all neurons. We identify the amount of phase synchronous neurons by analyzing whether the sets $\mathcal{D}_j$ are localized, occupying no more than 80% of the attractor of Σ_j.

The onset of PS in the whole network takes place at $g^*_{syn}\approx 0.47$, so all neurons become phase synchronized. As the synapse strength crosses another threshold, $\tilde{g}_{syn}\approx 0.52$, the neurons undergo a transition to the rest state, and they no longer present oscillatory behavior. Clusters of PS appear even for g_{syn} far smaller than g^*_{syn}. In fact, right at $g_{syn}=0.04$, some clusters of neurons exhibit PS among themselves. These clusters seem to be robust under small perturbations. Clusters of PS inside the network may offer a suitable environment for information exchanging. Each one can be regarded as a channel of communication, since they possess different frequencies, and therefore each channel of communication operates in different bandwidths. This scenario of cluster formation is neither restricted to this HR model nor to the synapse model. It can also be found in square-wave and parabolic bursters.

In conclusion, we have proposed an extension of the stroboscopic map, as a general way to detect PS in coherent/non-coherent oscillators. The idea consists in constraining the observation of the trajectory of an oscillator at the times in which an event occurs in the other oscillator. We have shown that if PS is present, the maps of the attractor appear as a localized set in the phase-space, and vice-versa. The ideas herein provide a reliable and easy way of detecting PS, without having to explicitly measure the phase. This method can be applied in experiments in real time and networks.

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We thank M. Thiel, M. Romano and C. Zhou for useful discussions, and the Helmholtz Center for Brain and Mind Dynamics (TP and JK) and Alexander von Humboldt (MSB) for the financial support.

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