Experiment on Anomalous Phase Synchronization

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Abstract. We report the first experimental evidence of anomalous phase synchronization in two coupled Chua's oscillators. The anomalous onset of phase synchronization shows an initial increase in frequency difference with coupling before following a usual transition to phase synchronization as monotonic decrease in frequency difference with a maximum at an intermediate coupling. The role of coupling asymmetry on anomalous effect is revealed for both diffusive and unidirectional coupling.

INTRODUCTION

The studies on synchronization of interacting nonlinear oscillators are of fundamental importance in physics, biology and engineering [1-2]. Of this phase synchronization (PS) [3-4] is found to play crucial roles in many weakly interacting living systems. PS is observed in electronics experiment [5], cardio-respiratory interaction [6] and ecology [7]. In case of PS, two or many interacting oscillators with varying natural frequencies develop a phase locking relation ($m:n$, m and n are integers) for weak coupling although the amplitudes remain almost uncorrelated.

The common notion [8] of transition to PS is that the frequency difference between interacting oscillators decreases monotonically with coupling and disappears above a critical value. Recently, a mark departure from this common notion has been observed [9] in a Foodweb model. A large population of nonidentical oscillators shows an increasing frequency disorder with coupling and then follows the usual monotonic decrease with further increase in coupling. Such an unusual enlargement of frequency disorder with an intermediate maximum is denoted as *anomalous phase synchronization* (APS). APS is a universal phenomenon [9], which may be observed in any coupled system and even only in two interacting systems. The exact condition for APS in any system can be derived with an appropriate choice of system parameters when two or more parameters of a system are functionally dependent. APS originates in the nonisochronicity of oscillations and arises when nonisochronicity increases with the natural frequency of oscillation. Nonisochronicity as a shear of phase flow [10] induces an amplitude dependence of natural frequency of an oscillator. Synchronization can also be enhanced, in contrast, by this mechanism when nonisochronicity and natural frequency have negative covariance.

In this paper we report the first experimental evidence of APS in two coupled Chua's oscillators. Two diffusively coupled Chua's oscillators show the anomalous transition to

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in-phase synchronization for comparatively large frequency mismatch while a transition to antiphase is observed at much weaker coupling by reversing the sign of frequency mismatch. However, we also find anomalous onset to antiphase by controlling frequency mismatch in this mode. The antiphase regime includes both out-of-phase ($0<\Delta\phi<\pi$) and antiphase ($\Delta\phi=\pi$) where $\Delta\phi$ is the phase difference of interacting oscillators. Further a desynchronization phase [11] of coexisting in-phase and antiphase has been observed in our experiment for intermediate coupling when a transition from antiphase to in-phase is found for large coupling. In case of APS, smooth variation in the attractor topology is observed without any coupling threshold. The coupling asymmetry is found to play an important role in APS. We also show evidences of APS for the extreme case of coupling asymmetry in unidirectional coupling.

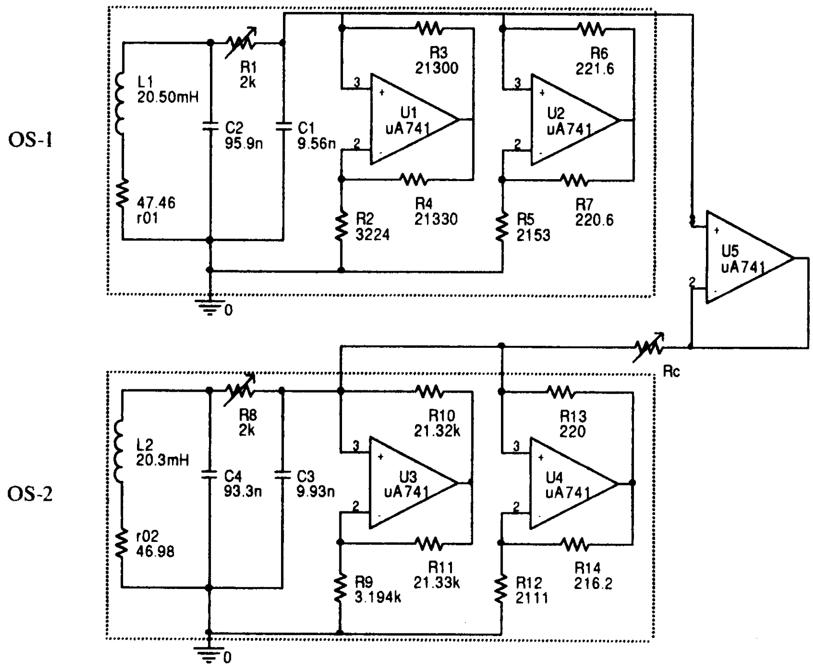

FIGURE 1. Coupled Chua's oscillator: oscillators OS-1 and OS-2 within dotted boxes.

COUPLED CHUA'S CIRCUIT

Two coupled nonidentical Chua's oscillators (OS-1, OS-2) are shown in Fig.1. Each oscillator consists of resistor $R_{1,8}$, inductor $L_{1,2}$, capacitors $C_{1,3}$ and $C_{2,4}$ and two Op-amp (U1-U2 or U3-U4) representing a piecewise linear function. The unity gain amplifier (U5) is used for undirectional coupling. The coupling strength increases with decrease in resistance $R_C(=1/\varepsilon)$. The state variables are the voltages $V_{C1,C3}$ and $V_{C2,C4}$ at corresponding capacitor nodes, and the inductor current $I_{1,2}$. The natural frequency $\omega_{1,2}$ of an uncoupled Chua's oscillator [12] is given by

$$\omega_{1,2} = \frac{1}{L_{1,2}C_{2,4}}\sqrt{1+\frac{L_{1,2}M_i}{R_{1,8}C_{1,3}}} \qquad \text{where } M_i = a_{1,2}, b_{1,2} \qquad (1)$$

where the slopes of piecewise linear function, $a_{1,2}$ and $b_{1,2}$, are given by

$$a_{1,2} = (-\frac{1}{R_{2,9}} - \frac{1}{R_{5,12}}), \quad b_{1,2} = (\frac{1}{R_{3,10}} - \frac{1}{R_{5,12}}) \tag{2}$$

Equation (1) clearly shows that the natural frequency of a Chua's oscillator depends on the reactive components and the slopes of the piecewise linear function while the slopes depend upon several resistive components establishing the functional dependence of several parameters. All circuit components are different, which introduce the mismatch in the natural frequency of the oscillators. All components are fixed throughout this paper except the resistance $R_{1,8}$, which is varied to obtain different dynamical features, period to chaos. Two similar state variables $V_{C1}(t)$ and $V_{C3}(t)$ at capacitor nodes C_1 and C_3 respectively are monitored using a 2-channel digital oscilloscope for varying coupling resistance R_C. Data acquisition is made by 8-bit memory (100MHz) of the oscilloscope for 2500 data points at each snapshot. The instantaneous phases $\phi_{1,2}(t)$ of measured signals are determined using the Hilbert transform [3,4] and mean frequencies $\Omega_{1,2}(\varepsilon)$ of the coupled oscillators are estimated as the mean rate of change of $\phi_{1,2}(t)$. An index of relative phase difference in terms of mean frequencies, $\Delta\Omega(\varepsilon)=2(\Omega_1-\Omega_2)/(\Omega_1+\Omega_2)$, is taken as a measure [7] of synchronization, which is the frequency difference as percentage of mean frequencies of the oscillators $\Omega_i(\varepsilon)$ (i=1,2). Phase synchronization is established when the relative phase difference disappears ($\Delta\Omega\approx0$).

EXPERIMENT ON ANOMALOUS PHASE SYNCHRONIZATION

We controlled the natural frequency mismatch of the oscillators (uncoupled state) by tuning the resistance $R_{1,8}$ to induce onset of APS thereby either to enhance or to inhibit synchronization. In our experiment for diffusive coupling (U5 removed), R_1 and R_8 are so selected that both the oscillators (OS-1, OS-2) are kept in limit cycle state and the natural frequency $\Omega_1(\varepsilon=0)=\omega_1$ of OS-1 is larger than $\Omega_2(\varepsilon=0)=\omega_2$, the frequency of OS-2. For the selected parameters, the frequency difference $\Delta\Omega(\varepsilon)$ with coupling as shown in Fig.2(a) first increases and then decreases before disappearing ($\Delta\Omega=0$) above a critical coupling $\varepsilon\geq\varepsilon_{C2}$. Evidently, the frequency difference increases without any threshold as may be observed for two different frequency mismatch (solid and dotted traces), $\Delta\omega=\omega_1-\omega_2=\Delta\Omega(0)$. As usual a shift in critical coupling $\varepsilon\geq\varepsilon_{C2}$ is observed, which increases with mismatch. A sufficient condition for APS is defined [9] as $d\kappa/d\omega>0$ where κ is the slope of individual frequency with coupling. This condition is clearly satisfied in our experiment as evident from the plot of individual frequency $\Omega_{1,2}(\varepsilon)$ with coupling below $\varepsilon<\varepsilon_{C1}$ shown in Fig.2(b). The slope of Ω_1 (solid line) defined by $d\Omega_1/d\varepsilon$ is larger than $d\Omega_2/d\varepsilon$ (dotted line) as observed during the increasing trend of $\Delta\Omega(\varepsilon)$ for coupling $\varepsilon<\varepsilon_{C1}$ indicated by the dotted vertical line. The larger the individual frequency, the higher is the slope. This confirms the existence of anomalous transition to PS. The $\Delta\Omega(\varepsilon)$ decreases for $\varepsilon>\varepsilon_{C1}$ after an intermediate maximum and finally disappears for large coupling $\varepsilon>\varepsilon_{C2}$, when in-phase synchrony sets in. It may be noted here that, in uncoupled state, the individual frequency $\Omega_1(0)>\Omega_2(0)$, when $\Delta\omega$ is assumed positive here.

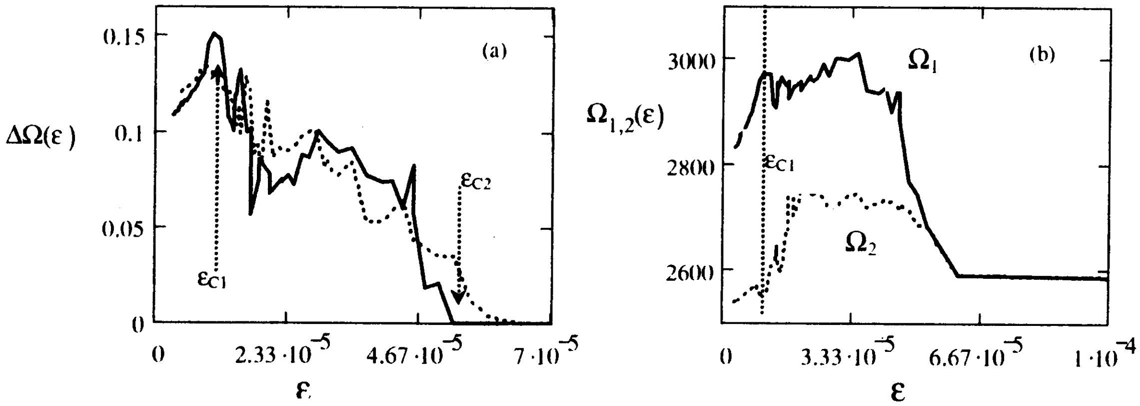

FIGURE 2. Anomalous phase synchronization: oscillators are both period-1, (a) $\Delta\Omega(\varepsilon)$ with coupling (ε) for $\Delta\omega$=279Hz (solid trace, R_1=1570Ω, R_8=1449Ω) and 289Hz (dotted trace, R_1=1570Ω, R_8=1447Ω), (b) individual frequencies $\Omega_{1,2}(\varepsilon)$ with coupling for $\Delta\omega$=279Hz corresponding to dotted curve in (a). ε_{C1}=1E-5 for solid trace.

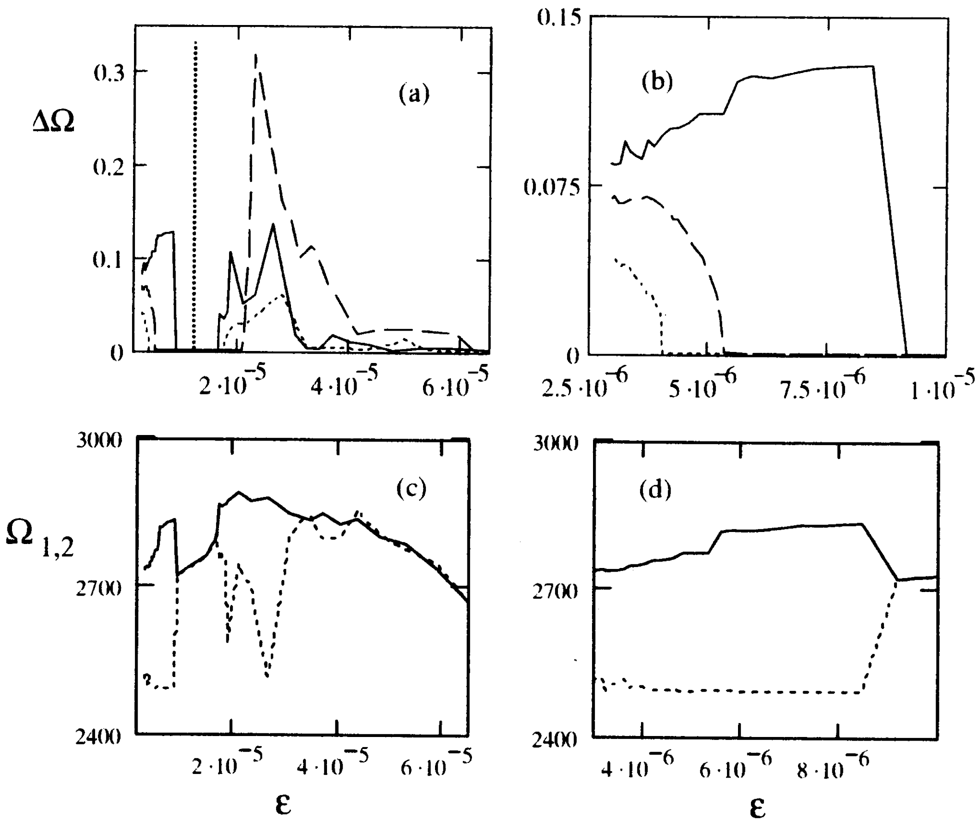

FIGURE 3. Anomalous transition to antiphase: (a) APS for $\Delta\omega$=-219Hz (solid line) when R_1=1452 (OS-1:chaotic), R_8=1540Ω (OS-2:period-1), and usual transition for $\Delta\omega$=-180Hz (dashed line) when R_1=1445Ω (OS-1:chaotic), R_8=1530Ω (OS-2:period-1), enhanced synchronization for $\Delta\omega$=-124Hz (dotted line) with R_1=1468Ω (OS-1:chaotic), R_8=1516Ω (OS-2: period-1), (b) magnified version of (a) in the coupling range 2.5E-6<ε<1E-5, (c) individual frequencies $\Omega_{1,2}$ for $\Delta\omega$ =219Hz, and (d) magnified versions of (c) in the coupling range 3.5E-6<ε<1E-5 confirms the necessary condition $d\kappa/d\omega>0$ for APS as explained in the text.

We observed anomalous onset to antiphase (Fig.3) for the first time in any coupled system by reversing the sign of $\Delta\omega$ when $\Omega_1(0)<\Omega_2(0)$. For $\Delta\omega$= -124Hz (dotted line) enhanced transition to antiphase is observed while for a larger mismatch, $\Delta\omega$= -180Hz, usual transition to antiphase is seen. For further increase in $\Delta\omega$=-219Hz, APS is clearly evident (solid line) in Fig.3(b). The frequency difference for all three mismatches disappears first at out-of-phase ($0<\Delta\phi<\pi$) and it continues until a desynchronization regime appears as shown in Fig.3(a). The coupled oscillators attain antiphase ($\Delta\phi=\pi$) just before the onset of desynchronization when the frequency difference shows large fluctuations. The desynchonization appears for the intermediate coupling range during the transition from antiphase to in-phase. The coupling threshold for out-of-phase increases with mismatch as shown in the enlarged version (Fig.3b) of the regime left to the vertical line in Fig.3(a). The time series for out-of-phase and antiphase are shown in Fig.4. A coupling threshold is also seen for the onset of desynchronization. To investigate the nature of shift in coupling threshold for the onset of desynchronization and associated complex dynamics is our future interest. We restrict this report to anomalous transition to PS only. During transition from desynchronization to in-phase shown in Fig.3(a), the coupling threshold increases with mismatch as usual. The individual frequencies $\Omega_{1,2}(\varepsilon)$ with coupling (ε) are shown in Fig.3(c)-(d).

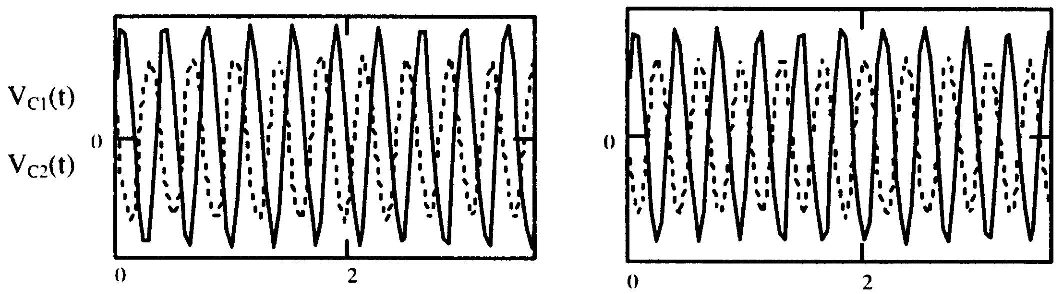

FIGURE 4. Time series of antiphase regime: $V_{C1}(t)$ and $V_{C3}(t)$ with time in solid and dotted lines respectively are out-of-phase in for coupling $\varepsilon\geq5.917E-6$ on the left and antiphase on the right for $\varepsilon\geq1.282E-5$ for mismatch $\Delta\omega$=-180Hz shown in dashed trace in Fig.3(a).

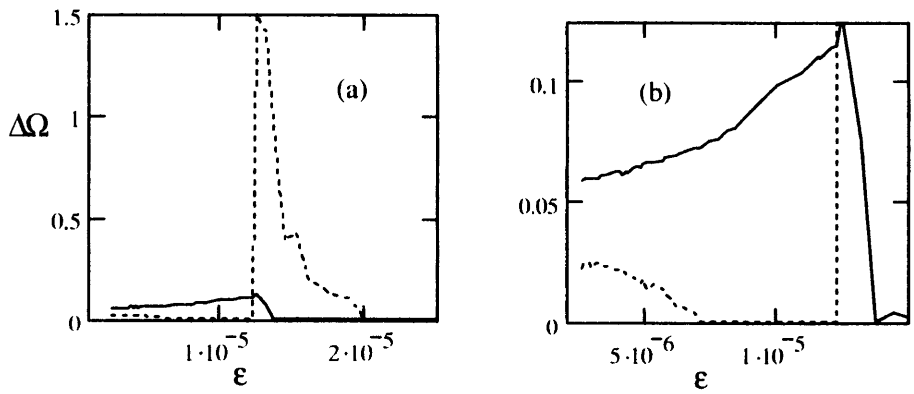

FIGURE 5. Anomalous synchronization for unidirectional coupling: (a) for chaotic driver (2533Hz), and period-2 response (2660Hz), $\Delta\omega$=127Hz, APS to in-phase (solid line); for periodic driver (2563Hz) and chaotic response (2458Hz), usual transition to antiphase and then to in-phase via desynchronization (dotted line) for $\Delta\omega$= -105Hz, $R_1=1446\Omega$, $R_8=1484\Omega$, (b) magnified version of (a) in the coupling range 2.5E-6<ε>1.5E-5. A part of desynchronization trace (dotted line) is seen in (b) for ε>1.5E-5.

The coupling asymmetry clearly plays important roles in the onset of APS. The effect of coupling asymmetry is further elaborated in the extreme case of asymmetry for unidirectional coupling. The driver oscillator OS-1 is chaotic and the response OS-2 is a limit cycle oscillator for appropriate choice of R_1 and R_8 respectively, when the natural frequency of response (OS-2) oscillator (ω_2) is larger than the driver (OS-1) frequency (ω_2). Anomalous transition to in-phase synchronization (solid line) is seen in Fig.5(a) for this positive mismatch ($\Delta\omega$=127Hz). For a negative mismatch ($\Delta\omega$= -105Hz), when the response frequency is lower than the driver frequency, enhanced transition to antiphase regime (dotted line) is observed for very weak coupling. Both out-of-phase and antiphase are also observed here before the onset of desynchronization similar to what is observed for diffusive coupling but no details are given here.

CONCLUSION

The first experimental evidence of APS is reported here using two coupled nonidentical Chua's oscillators. Moreover, anomalous onset to antiphase is first observed in any coupled system. The role of coupling asymmetry on APS is clearly evident from the experiment. The desynchronization regime is our interest of future investigation.

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