Using dependent types in models of climate change impacts

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PIK: Potsdam Institute for Climate Impact Research

"At PIK researchers in the natural and social sciences work together to study global change and its impacts on ecological, economic and social systems. They examine the Earth system's capacity for withstanding human interventions and devise strategies for a sustainable development of humankind and nature.

PIK research projects are interdisciplinary and undertaken by scientists from the following Research Domains: Earth System Analysis, Climate Impacts and Vulnerabilities, Sustainable Solutions and Transdisciplinary Concepts and Methods."

"Who sent you hither? Wherefore do you come?" R3 I.iv.174

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"What's in a name?" RJ II.ii.43

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"It cannot be thus long; the sides of nature / Will not sustain it." AC I.iii.17-18

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"Confusion's near." Cor III.i.189

Different concerns: scientific, economic, political, ethical . . .

Different methodologies: empirical, simulations, Gedankenexperimente, stakeholder dialogues, participatory games . . .

Different specialized languages

Common ground: English, Mathematics, and “the classics” (Western cannon)

“You must translate. ’Tis fit we understand them.” Ham IV.i.2

Example: “Vulnerability”

“. . . a human condition or process resulting from physical, social and environmental factors which determine the likelihood and damage from the impact of a given hazard” (UNDP Annual Report, 2004)

“Vulnerability [. . .] is a way of conceptualizing what may happen to an identifiable population under conditions of particular risk and hazards.” (Cannon et al. 2004)

“. . . the degree to which a system is susceptible to and unable to cope with, adverse effects of climate change, including climate variability and extremes. ” (The Intergovernmental Panel on Climate Change, 2007)

“We first survey the plot, then draw the model” 2H4 I.iii.42

A formalization of vulnerability as measure of possible future harm.

$$
\begin{array}{lcl}
\textbf{data}\ \mathit{State} & = & \ldots \\
\textbf{data}\ \mathit{Evolution} & = & [\mathit{State}] \quad \text{-- or } T \to \mathit{State} \\
\mathit{possible} & :: & \mathit{Functor}\ f \Rightarrow \mathit{State} \to f\ \mathit{Evolution} \\
 & & \quad\quad \text{-- E.g. } [\mathit{Evolution}] \text{ or } [(\mathit{Evolution}, \mathit{Float})] \\
\mathit{harm} & :: & \mathit{Preorder}\ v \Rightarrow \mathit{Evolution} \to v \\
\mathit{measure} & :: & \mathit{Preorder}\ w \Rightarrow f\ v \to w \\
\mathit{vulnerability} & :: & \mathit{State} \to w \\
\mathit{vulnerability} & = & \mathit{measure} \circ \mathit{fmap}\ \mathit{harm} \circ \mathit{possible}
\end{array}
$$

“. . . formal ostentation” Ham IV.v.215

Monotonicity condition:

$$
\begin{aligned}
harm &:: Preorder\ v \Rightarrow Evolution \rightarrow v \\
measure &:: Preorder\ w \Rightarrow f\ v \rightarrow w
\end{aligned}
$$

Increasing harm should lead to increased vulnerability.

Formally: for every non-decreasing function $nd :: V \rightarrow V$ we have, for every $s :: F\ V$

$$measure\ (fmap\ nd\ s) \geqslant measure\ s$$

"...her woes the more increasing" Ven.254

The type of a vulnerability measure is:

$$
\begin{array}{l}
\exists\, m : F\ V \rightarrow W \\
\quad (\forall < nd, \_ > : Nondec\ V \\
\qquad (\forall\ s : F\ V \\
\qquad\quad (m\ (fmap\ nd\ s) \geqslant m\ s)))
\end{array}
$$

For all vulnerability measures we've encountered so far, it's very easy to prove they satisfy the monotonicity condition or to show they don't.

"... 'tis a common proof" JC II.i.21

Multi-agent economic models: agents are discrete systems with input, e.g.:

$$\mathit{firm} : \mathit{State} \times \mathit{Command} \rightarrow \mathit{State}$$

but not every command is valid in every state (e.g. you cannot invest more than you have):

$$\mathit{firm} : (s : \mathit{State})\ (c : \mathit{Command})\ \{\mathit{check} : c\ \mathit{AllowedInState}\ s\} \rightarrow \mathit{State}$$

“Lend less than thou owest” KL I.iv.119

Usualy, there are a number of kinds of states: normal, bankrupt, etc.

$$
\begin{aligned}
&\textbf{data}\ \mathit{StateKind} : \mathit{Set}\ \textbf{where}\\
&\quad \mathit{normal}\ \ : \mathit{StateKind}\\
&\quad \mathit{bankrupt} : \mathit{StateKind}\\
&\qquad \ldots
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{State} : \mathit{StateKind} \to \mathit{Set}\\
&\mathit{State}\ \mathit{normal} \quad = \mathit{Capital}\ \times\ \mathit{Stocks}\ \times \ldots\\
&\mathit{State}\ \mathit{bankrupt} = \mathit{Trustees}\ \times\ \mathit{ProtectedAssets}\ \times \ldots
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{firm} : (\mathit{sk} : \mathit{StateKind})\ (s : \mathit{State}\ \mathit{sk})\ (c : \mathit{Command})\\
&\qquad \{\mathit{check} : c\ \mathit{AllowedInState}\ s\ \mathit{OfKind}\ \mathit{sk}\}\\
&\qquad \to (\mathit{sk}' : \mathit{StateKind})\ \times\ (\mathit{State}\ \mathit{sk}')
\end{aligned}
$$

"Such a dependency of thing on thing" MM V.i.62

Similar to multi-agent models: model coupling.

E.g.: economic + climate (PIAM), ocean + atmosphere + sea-ice + vegetation (Climber3), trade + energy sector + climate (REMIND), . . .

Compatibility conditions: boundary conditions have to match, types of systems have to be taken into account (stochastic + deterministic = ?), temporal and spatial scales . . .

"And all combined..." RJ II.iii.56

The information needed to ensure compatibility is mostly not available.

Poor documentation of models: research papers give too high-level a view, user guides are incomplete, comments in the code are too low-level (and sometimes misleading).

A lot of the effort in coupling is spent in overcoming software issues: different programming languages, operating systems, etc. This, and not the specification of the models, is perceived by scientists to be the most important part of model coupling.

Efforts to construct a typology of models and couplings have been largely unsuccessful: too many concerns addressed at the same time.

"And shall I couple hell?" Ham I.v.93

A proposal: start with very simple models which can be implemented in e.g. Agda, and use the type system to obtain specifications of these models, with a view to formulating adequate compatibility conditions.

Develop a library or a DSL for specifying this kind of models and their interactions.

Add more detail, moving in the direction of the models actually in use, until you can reimplement them in the new framework, or use the framework to drive them.

"and to your audit comes / Their distract parcels, in combined sums." LC.231

A different kind of problem: many scientists use an environment and code around it.

Example: choose $x \in D$ to maximize $u(x)$ such that $g(x) \leqslant 0$.

"choose production to maximize profit while meeting the 2°C target"

GAMS is very good at this, as long as g is given explicitely. But climate models are given as ODEs or PDEs. So GAMS users write ad-hoc integration methods.

"Why old men, fools, and children calculate," JC I.iii.65

Numerical methods are easy to get wrong:

inappropriate discretization, choice of inappropriate method, incorrect checks of convergence, no testing for singularity conditions, no validation of results, etc.

Again, the gap between the mathematical description and a good implementation is wider than commonly acknowledged.

"No, we detest such vile base practices." *TG IV.i.73*

A possible solution: diminish the distance from the mathematical description to implementation.

Use constructive mathematics a la Bishop.

Constructive mathematics as DSL for scientific programming.

CTT as foundation for modeling.

"Well, we leave that to the proof." 1H4 II.ii.67