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Infrastructure and Inequality: Insights from Incorporating Key Economic Facts about Household Heterogeneity

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Abstract

We study the trade-off between equity and growth in the context of tax-financed investment in public capital. Taking into account stylized facts on wealth accumulation, we model agent heterogeneity through differences in saving behavior, income source and time preference. In contrast to the results of studies that introduce heterogeneity through different initial endowments only, we find that under our heterogeneity assumptions an equity-efficiency trade-off does not necessarily occur. We show that a consumption tax or a capital tax, levied to finance public capital, does not increase inequality. In our model capital tax-financed public investment has even an inequality-reducing effect - thus allowing for Pareto-improving public investment that decreases inequality. Additionally we find that agents differ in their preferred tax rates. These results are valid for both, the case of endogenous growth and the case of steady state convergence and do not require the assumption of an identical rate of pure time preference across all households.

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1 Introduction

Public investment has recently received much attention as an attractive option for accomplishing two main objectives of economic policy: Promoting efficiency and growth as well as reducing inequality in wealth. For instance, public spending on education has been recommended as a means to both reduce inequality and boost growth (OECD, 2012). Moreover, increased public spending on education or infrastructure, could counteract regional inequality while enhancing long run growth (The Economist, 2011). The empirical literature on the relationship between public spending and inequality is, however, sparse and ambiguous (see Calderón and Chong 2004 and the review in Chatterjee and Turnovsky 2012).

Previous theoretical work has found that when public investment can increase growth or average welfare, it will inevitably increase inequality in the long run, regardless of the financing mechanism (Chatterjee and Turnovsky, 2012) and that inequality reducing public investment always decreases the efficiency (Alesina and Rodrik, 1994). In this article we argue the opposite case, i.e. that an equity-efficiency trade-off concerning public investment does not necessarily exist: Capital taxation can both promote inequality reduction and efficiency gains when its revenue is spent on public investment. Consumption taxation enhances the efficiency but leaves the level of inequality virtually unchanged. Moreover, up to a certain level of all taxes, efficiency is always enhanced, which makes all agents better off and thus constitutes a Pareto-improvement. Our results rest on an alternative approach of modeling household heterogeneity that is based on stylized facts about income sources, real-word saving behavior and time preferences.

In Chatterjee and Turnovsky (2012) heterogeneity is introduced only in initial endowments and differences in saving motives or time preference are not accounted for; Alesina and Rodrik (1994) account for differences in income sources but ignore other types of heterogeneity. A related study by Chiroleu-Assouline and Fodha (2014) employs an overlapping generations model, in which agents differ in their time preferences and skill levels, but also arrive at the conclusion that capital tax-financed public investment increases inequality. [1] A model with these types of heterogeneity assumptions however is not able to reproduce the observed wealth distribution (De Nardi, 2004; De Nardi and Yang, 2014). "[F]or many purposes, the representative-

[1]Their analysis is in fact a contribution to the double dividend literature, since the capital tax is interpreted as a carbon tax on polluting capital. The authors find that the incidence of a capital tax that mitigates the effects of a negative production externality, can be inequality-enhancing if the tax revenues are not used to relieve pre-existing labor taxes. Since the avoided damages of the negative production externality can be interpreted as a capital tax-financed public good, their study also confirms the results by (Chatterjee and Turnovsky, 2012).

consumer model should be abandoned in favor of a model that matches key microeconomic facts" Carroll (2000). Here we comply with this request for the case of the distributional effects of public investment: By departing from the standard assumptions on heterogeneity we obtain results strikingly at odds with previous work on the subject.
In our model we account for the following important economic facts:[2] Rich households have been shown to save in a dynastic fashion, while households in the middle-income cohorts exhibit more of a life-cycle saving behavior (Attanasio, 1994; Dynan et al., 2004; Browning and Lusardi, 1996), neither motive in isolation can reproduce the observed wealth distribution (Carroll, 1998). The wealthier a household is, the more his income sources shift away from wage income towards business and capital income[3] and the more likely that he is self-employed (Quadrini, 1997; Diaz-Gimenez et al., 2011; Wolff, 1998). Lawrance (1991) find that wealthier households have lower rates of time preference. Krusell and Smith (1998) show that allowing for different time preference rates is a key factor in reproducing the wealth distribution.

We develop a general equilibrium model in which high-income households are characterized by having a dynastic saving behavior and capital income as their only income source. Middle-income households are life-cycle savers, which split their labor income between current consumption and savings for retirement. Low-income households do not save or even dissave and are thus omitted. High-income households are modeled as a representative infinitely-lived agent, and the middle-income households are modeled as a representative overlapping-generations agent.[4] The model also allows for agents to differ in their time preference rate. We then calibrate the model to closely match stylized facts of the U.S. economy (see Section 2.7 for details).

In our companion paper (Mattauch et al., 2014) we show analytically, for a basic version of the model, that under those assumptions of heterogeneity, capital tax-financed public investment can enhance productivity while reducing inequality. Here we generalize the basic model, in order to assess more channels through which public investment affects the distribution of wealth, welfare and income and to allow for a close comparison of our results to the results described in Chatterjee and Turnovsky (2012). Assessing the effect of labor income taxation requires introducing a labor-leisure choice,

[2]There are other factors which also influence the wealth distribution but go beyond the scope of this work: Most importantly the transmission of human capital within families and the existence of public insurance systems (De Nardi and Yang, 2014) as well as differences in rates of return (Guvenen, 2006) and others.

[3]This might not be true for the lowest income decile. Since the focus of our study is on the middle and upper class households we can ignore this fact.

[4]In order to highlight the underlying mechanisms we choose to only look at two extreme cases of saving behavior: completely altruistic in the case of the infinitely-lived agent, and pure life-cycle in the case of the overlapping generations agent.

which also leads to differences between labor and consumption taxation. Public capital is modeled as productivity and leisure enhancing; it thus affects the economy through two different channels [5].

In our model tax-financed public capital acts on the distribution in three different ways: First, through a change in the policy, the aggregate level of capital changes. Second, agents are affected differently by different tax instruments and finally, agents react to policies by changing their leisure level and their saving behavior.

The aim of the present article is to study the effects of our assumption of household heterogeneity on the performance of different tax instruments used to finance public investment in infrastructure. The tax instruments are examined in terms of their efficiency and their distributional implications.

This article makes two main contributions. First, we compare several financing mechanisms for public investment (labor, capital and consumption taxation) and assess similarities and differences to previous work which arise through the different way heterogeneity is modeled. Our second contribution is that the results of our modeling are more general than those in previous work, since they hold for endogenous growth as well as for steady state convergence and they do not depend on the assumption of homogeneous time preference rates across all agents.
Concerning our first contribution, our main findings regarding the comparison of a capital, labor or consumption tax for financing public investment can be summarized as follows (see Section 3.1 for details on the steady state analysis and Section 4.1 for details on the endogenous growth analysis).

Our two main results differ from Alesina and Rodrik (1994) and Chatterjee and Turnovsky (2012), due to our choice of modeling agent heterogeneity: (i) Higher levels of wealth, welfare and income and a reduced dispersion of these economic variables can be achieved by a policy of capital tax-financed public investment. (ii) Financing public capital through a consumption tax has virtually no effect on the distribution of these variables (as opposed to a strong negative effect in Chatterjee and Turnovsky 2012).
On the other hand we confirm three of their findings: (i) Tax-financed public capital enhances overall productivity up to a certain tax level independent of the financing mechanism. (ii) Labor tax-financed public capital increases welfare, wealth and income up to a certain tax rate, but leads to an increase in their dispersion (only Chatterjee and Turnovsky 2012). (iii) High-income households prefer the output maximizing tax rate (only Alesina and Rodrik 1994).

[5] Since most public goods such as for example infrastructure and health care affect productivity and utility at the same time, it is crucial to account for both channels to avoid incorrect conclusions. See Chatterjee and Ghosh (2011) for more details.

Additionally, we determine the optimal tax level for each tax mechanism. We are thus able to determine systematically whether the government's instrument choice is Pareto-optimal. We find that agents differ in their preferred levels of taxation. This result extends the well-established outcome of studying public investment (Barro, 1990) that there is a single optimal tax rate, which is determined by the trade-off between the productivity enhancing effect of public capital and the distortionary effect of its financing.

Concerning our second contribution, we show that it is not necessary to make the restrictive parameter assumption of constant returns in accumulable factors (and hence) of endogenous growth to obtain meaningful results about the equity-efficiency trade-off in an intertemporal context: We find that the general trends in the steady state results for a capital and a labor tax are preserved in the endogenous growth case, except for small variations for low tax rates, but that this does only partially apply to the results of consumption taxation. The reasons for this model behavior are presented in Section 4.1. Additionally, our model does not require the knife-edge assumption that all households have the same rate of pure time-preference.[6]

The remainder of this article is organized as follows: Section 2 outlines the model and its calibration. In Section 3 we characterize the model results for convergence to the steady state, while in Section 4.1 the results for endogenous growth are described. In Section 4.2 we verify the robustness of our results by varying critical parameters and Section 5 concludes the article.

2 Model

The three most important features of the model are that (i) household heterogeneity is modeled through different saving behavior and different income sources: high-income households whose bequest motive is perfectly altruistic and who rely only on capital income are modeled as a representative infinitely-lived agent. Middle-income households who save according to a life-cycle motive are modeled as a representative overlapping-generations agent with labor and capital income. (ii) Public and private capital are combined in a weighted product, the composite externality. By varying the weight parameter we can vary the role capital plays in production: When the weight parameter of private capital equals 1 the role of private capital is analogue to the case examined by Romer (1986). For a weight parameter of

[6]In a neoclassical growth model this assumption is necessary to avoid that the agent with the lowest time preference rate ends up owning all capital in steady state (Becker, 1980).

private capital equal to 0, public capital plays the same role as in the model by Barro (1990). (iii) Public capital plays a dual role in our model, enhancing both the value of leisure in the utility function, and total productivity. Since it would not provide us with additional insights we neglect population growth and assume that the size of the representative households does not change. Still we account for the fact that the households are different in size in the calibration of the model (see Section 2.7).

2.1 The firm

The production sector is modeled as a single representative firm. Labor is provided by the middle-income household only, while both households supply capital. Production occurs with a Cobb-Douglas production function:

$$F(K_t, h_t) = \hat{A} K_t^{\alpha} h_t^{1-\alpha}, \qquad \hat{A} = A X_t^{\beta}, \qquad 0 < \alpha, \beta < 1 \tag{1}$$

with $h_t = L - l_t$ being the portion of the total time L that middle-income households dedicate to work. The remainder of their time is used for leisure l_t. $X_t = K_t^{\epsilon} K_{G,t}^{1-\epsilon}$, with $0 < \epsilon < 1$, represents a composite production externality, modeled as a weighted product of private and public capital. The capital entering the production function is the sum of the middle-income households' savings from the last period S_{t-1} and the high-income households' capital $K_{h,t}$:

$$K_t = S_{t-1} + K_{h,t}. \tag{2}$$

Note, that for $\alpha + \beta < 1$ the economy converges to a steady state. But if $\alpha + \beta = 1$ and if the ratio of public to private capital remains constant, the model will display endogenous growth behavior. This can be deduced by an equivalent of Equation (1):

$$F(K_t, h_t) = A K_t^{\alpha+\beta} (h_t)^{1-\alpha} \left(\frac{K_{G,t}}{K_t} \right)^{(1-\epsilon)\beta}.$$

A representative firm maximizes its profit:

$$\Pi_t = F(K_t, h_t) - (r_t + \delta_K) K_t - w_t h_t$$

where r_t and w_t represent the rental rates the firms have to pay to the households for capital and labor and δ_k is the depreciation rate of private capital. The following first-order conditions are obtained:

$$r_t + \delta_K = \frac{\partial F(K_t, h_t)}{\partial K_t} = \alpha A \left(\frac{h_t}{K_t} \right)^{1-\alpha} X_t^{\beta} \tag{3}$$

$$w_t = \frac{\partial F(K_t, h_t)}{\partial h_t} = (1-\alpha) A \left(\frac{K_t}{h_t} \right)^{\alpha} X_t^{\beta}. \tag{4}$$

2.2 The high-income households

The high-income households are modeled as a representative infinitely-lived agent, to which we will from time to time also refer as "ILA". She derives utility from either consumption C_t or leisure l_h, which is fixed for this agent. We later show in chapter 4.2 that the results of this paper are independent from the level of leisure the high-income households receive as long as it remains in a plausible range (see Table 5). Future utility is discounted by the time preference rate ρ_h. Her lifetime utility is given by

$$U = \sum_{t=0}^{t_{\text{final}}} u_t^{\text{ILA}} \cdot \frac{1}{(1+\rho_h)^t}, \tag{5}$$

with

$$u_t^{\text{ILA}} = \left(\frac{1}{b}\right) \left(C_t^a + \theta (X_t l_h)^a\right)^{\left(\frac{b}{a}\right)},$$

where $a = 1 - \frac{1}{\sigma_{\text{Intra}}}$, with σ_{Intra} being the intratemporal elasticity of substitution between consumption and leisure and $b = 1 - \frac{1}{\sigma_{\text{Inter}}}$, with σ_{Inter} being the intertemporal elasticity of substitution. θ is a weight factor for the leisure term.

The ILA chooses her levels of consumption C_t and capital accumulation $K_{h,t}$ to maximize Equation (5) according to her budget constraint:

$$K_{h,t+1} - K_{h,t} = (1-\tau_K) r_t K_{h,t} - (1+\tau_c) C_t, \tag{6}$$

where τ_c represents a consumption and τ_k a capital income tax. The agent takes the returns to capital, r_t, as well as all taxes as given by the firm and the government, respectively.

Solving the optimization problem yields the following Euler equation:

$$\frac{\left(\frac{\partial u^{\text{ILA}}}{\partial C_{t-1}}\right)}{\left(\frac{\partial u^{\text{ILA}}}{\partial C_t}\right)} = \frac{1 + (1-\tau_K) r_t}{1+\rho_h}. \tag{7}$$

2.3 The middle-income households

The middle-income households are modeled as a representative Diamond-type overlapping-generations agent, to whom we will refer from time to time as an "OLG" agent and who lives for just two periods. The duration of each period is thirty years. In the first period the agent decides how to divide her fixed time endowment L between work ($h_t = L - l_{y,t}$) and leisure ($l_{y,t}$) and how much of her labor income (w_t) she saves for the second period (Equation 9). In the second period, the savings plus the interests are consumed (see Equation 10). We use the subscript "y" to denote the young agent, and "o" to denote the old agent.

The lifetime utility of the OLG agent is given by:

$$u_t^{\mathrm{OLG}} = \frac{1}{b}\left(C_{y,t}^a + \theta(X_t l_{y,t})^a\right)^{\frac{b}{a}} + \frac{1}{(1+\rho_m)}\frac{1}{b}\left(C_{o,t+1}^a + \theta(X_{t+1} l_o)^a\right)^{\frac{b}{a}}, \quad (8)$$

where l_o is the fixed leisure endowment of the old agent. We show in Section 4.2 that the level of this parameter does not change the character of the results as long as it remains in a plausible range. The young agent discounts her own old age by a factor ρ_m. The agent chooses $l_{y,t}$ and S_t to maximize her lifetime utility subject to the two budget constraints:

$$(1+\tau_c)C_{y,t} = (1-\tau_w)w_t(L-l_t) - S_t \quad (9)$$

$$(1+\tau_c)C_{o,t+1} = (1+(1-\tau_K)r_{t+1})S_t, \quad (10)$$

where τ_w is a tax on labor. Solving the optimization problem yields the Euler equations:

$$\frac{\left(\frac{\partial u_t^{\mathrm{OLG}}}{\partial C_{y,t}}\right)}{\left(\frac{\partial u_t^{\mathrm{OLG}}}{\partial C_{o,t+1}}\right)} = (1+(1-\tau_K)r_{t+1}) \quad (11)$$

$$\frac{\left(\frac{\partial u_t^{\mathrm{OLG}}}{\partial C_{y,t}}\right)}{\left(\frac{\partial u_t^{\mathrm{OLG}}}{\partial l_{y,t}}\right)} = \frac{(1+\tau_c)}{(1-\tau_w)\cdot w_t}. \quad (12)$$

2.4 The government

The government levies taxes to finance investment in a public capital stock K_G. Public capital is depreciates at the rate δ_G. The tax level is set exogenously, which means that the government does not optimize. We nevertheless can find the preferred tax rates of each agent by comparing their utilities in different steady states.

$$K_{G,t+1} - K_{G,t} = \tau_K \cdot r_t \cdot K_t + \tau_w \cdot h_t \cdot w_t + \tau_c \cdot (C_t + C_{y,t} + C_{o,t}) - \delta_G K_{G,t} \quad (13)$$

Subsequently, the relative merit of financing public investment by the three distinct taxes will be compared.

2.5 Equilibrium and the Pasinetti Paradox

For $\alpha + \beta < 1$ the system converges to a steady state for all parameter combinations evaluated numerically (see Section 4.2). In the following variables at their steady state levels are denoted by a tilde. We see from Equation (7) that at this steady state, the high-income households' rate of pure time preference determines the steady state interest rate of the aggregate economy $\tilde{r}$:

$$\frac{1+(1-\tau_K)\tilde{r}}{1+\rho_h}=1 \Rightarrow \tilde{r}=\frac{\rho_h}{(1-\tau_K)}. \tag{14}$$

This entails that in our model a form of the Pasinetti Paradox occurs. In a model with two types of households, one of them only receiving income through capital interests – the "capitalists" – the Pasinetti Paradox states that, in the steady state, the interest rate is solely determined by the "capitalists"' pure rate of time preference and the rate of capital taxation τ_k. The paradox implies that the high-income households, in steady state, can react to an increase in the middle-income households' saving only by decreasing their saving. For more details on the Pasinetti Paradox in the context of a simpler version of this model see Mattauch et al. (2014). Note that the Pasinetti Paradox does not occur for endogenous growth.

2.6 Measures of distribution

We take the coefficient of variation σ as a measure of dispersion in wealth, welfare and income (see e.g. Ray 1997 for details on inequality measures). The cohorts represented by the two agents are of unequal size (see chapter 2.7 on calibration), which has to be reflected in the calculation of the coefficient. In the following N is the total size of the households, while N_{m} and N_{h} stand for the size of the middle and high-income household. The index "pc" marks a per capita variable:

$$\sigma_K=\frac{\sqrt{\frac{1}{N}\left(N_{\mathrm{m}}(S_{\mathrm{pc}}-\mu_K)^2+N_{\mathrm{h}}(K_{h,\mathrm{pc}}-\mu_K)^2\right)}}{\mu_K},$$

with μ_K being the mean:

$$\mu_K=\frac{N_{\mathrm{h}}K_{h,\mathrm{pc}}+N_{\mathrm{m}}S_{\mathrm{pc}}}{N}.$$

2.7 Calibration

We calibrate the model such that in the baseline scenario the high-income households make up five percent of the population, while owning 62 % of total wealth and the middle-income households make up the next 55 % of the population while owning the remaining 38 % of total wealth. These numbers are chosen to match a study on wealth inequality in the U.S. (Wolff, 2010). The model also roughly complies with the fact that 50 – 60 % of U.S. net worth accumulation is due to wealth transfers from one generation to another (Gale and Scholz, 1994). In the baseline scenario a minimal stock of public capital is already provided through a consumption tax, which is the least distorting of the three types of taxes.

The above results use the parameterization displayed in Table 1. All values are chosen for timesteps of thirty years.

Symbol	Parameter	Value	Value (yearly)
α	Elasticity of capital in production	0.4	–
β	Exponent of public capital in production	0.2	–
δ_G	Depreciation of public capital	0.7	4%
δ_k	Depreciation of private capital	0.7	4%
ϵ	Exponent of private capital in composite externality	0.6	–
L	Time endowment of middle-income household	1	–
l_h, l_o	Leisure of agents with only capital income	0.71	–
ρ_h	High-income households' time preference rate	0.45	1.2%
ρ_m	Middle-income households' time preference rate	6	6.7%
σ_{Inter}	Intertemporal elasticity of substitution	0.4	–
σ_{Intra}	Intratemporal elasticity of substitution	0.76	–
θ	Share of leisure in utility function	1.75	–

Table 1: Standard calibration of the model

We choose these values to match the calibration used by Chatterjee and Turnovsky (2012) as closely as possible. Whenever we deviate from their calibration the reason lies in the different type of household heterogeneity used in our model: We model the households such that high-income households have a lower time preference rate than middle-income households, in accordance with findings by Lawrance (1991) and Dynan et al. (2004). Leisure is constant for agents receiving only capital income, which is true for the old middle-income household and the high-income household.[7]

3 Results

The purpose of the present article is to assess the impact of our assumption of household heterogeneity on the performance of three policy instruments in terms of their efficiency and their distributional consequences: capital income taxation, labor income taxation and consumption taxation. We analyze the performance of policy instruments by considering their impact on wealth, welfare, income, their distribution and on aggregate output. The performance of the various taxes is evaluated relative to a scenario in which a basic level of public capital is supplied by a 2 % consumption tax.

We find that in the long run the trade-off between equality and efficiency can be avoided in the following sense: Capital taxation as a financing-option reduces inequality, while promoting efficiency up to a certain tax level. This finding is opposed to Chatterjee and Turnovsky (2012) who use the same

[7] We show in Section 4.2 that our results are robust for any leisure value in the range of 50-100 %.

setup except for modeling agent heterogeneity exclusively through different initial endowments.
The disadvantage of a capital tax is, naturally, that it harms aggregate efficiency more than a labor or a consumption tax, due to its disincentive to accumulate capital. A labor tax increases inequality, but a consumption tax leaves the degree of inequality almost unchanged again differing from Chatterjee and Turnovsky (2012) where a consumption tax has a strong inequality increasing effect. For low tax rates, public investment is Pareto-improving for all tax mechanisms (Section 3.1).
The short run effects can, for some financing mechanisms, be adverse: e.g. a labor tax can decrease short run wealth inequality. A consumption tax is almost distribution-neutral in the long run, but has strong distributional impacts in the short run (Section 3.2).
In Section 4.1 we also determine the model's behavior along the growth path and find that most of the results obtained in the steady state analysis also hold for endogenous growth.
This section is divided into two parts: In the first part, Section 3.1, we describe the effect of each financing mechanism for public capital for the case of convergence to a steady state. We discuss the effects of the policy on welfare, capital and income of each agent as well as on aggregate output and their effects on the dispersion of wealth, welfare and income as measures of inequality. In Section 3.2, we describe the effects of the policies on the transitional dynamics.

3.1 Steady state analysis

In this section we investigate the long term effects of increased public investment for a broad range of exogenously given capital, labor and consumption tax rates. We write dX to denote the percentage change of the variable X with respect to the baseline scenario of a 2 % consumption tax. We use the term welfare for the level of utility at the steady state, not taking into account the utility values in the transition to this steady state.

3.1.1 Capital tax

When financing an increase in public capital with a tax on capital income, we find the following four effects:

1. Dispersion in wealth, welfare and income decreases for rising τ_k (see Figure 1 on the left)

2. Output is maximized for a 30 % capital tax

3. For tax rates up to 64 % the policy is Pareto-improving (see Figure 1 on the right)

4. Middle-income households prefer a higher capital tax rate (40%) than high-income households (30%) (see Figure 1 on the right)

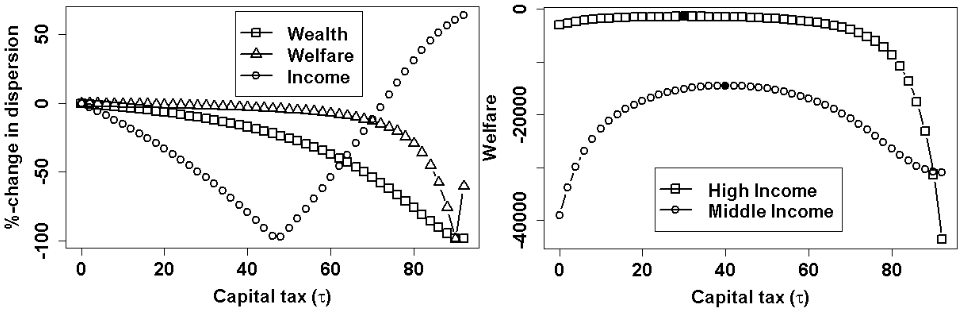

Figure 1: Effects of capital tax-financed public investment on the dispersion of wealth, welfare and income (left side) and on the welfare of both agents (right side). The downward spikes in the left figure reflect the points where middle-income households are equal in a certain variable to high-income households. For even higher tax rates the dispersion increases again, but this time the middle-income households are better off.

	dY(%)	du_{ILA}(%)	du_{OLG} (%)	$d\sigma_K$ (%)	$d\sigma_u$ (%)	$d\sigma_{\text{Inc}}$ (%)
$\tau_Y^{\max} = 0.3$	+31.0	+53.3	+61.2	-11.0	-1.8	-54.0
$\tau_{u,\text{ILA}}^{\max} = 0.3$	+31.0	+53.3	+61.2	-11.0	-1.8	-54.0
$\tau_{u,\text{OLG}}^{\max} = 0.4$	+28.6	+51.0	+62.9	-17.1	-2.9	-79.1

Table 2: Steady state effects of a capital tax-financed increase in public spending. In the column on the left, the levels of capital tax rates which maximize output and utility of the different agents are given. In the remaining columns the changes in output, welfare and dispersion are given in percent, as compared to the baseline.

These results are explained as follows: Since the model has Pasinetti properties (see also Section 2.5), a capital tax increases the interest rate in the long run (see Equation 14), high-income households reduce their savings and thus the income and wealth dispersion decreases. For low capital taxes the public capital stock and with it the composite externality increases, which increases the returns to labor (see Equation 4) and thus further decreases the dispersion in income.[8] These effects combined lead to a larger reduction in consumption and thus in welfare for high-income households than for middle-income households. Thus dispersion in all three variables

[8]Some parts of the tax incidence also fall on the middle-income households through the depressing effect a capital tax can have on the wage rate. In our model this effect is offset by the positive effect of public investment on both factors.

decreases.[9]

A Pareto-improvement exists because of the positive effect of the composite externality on utility and production. Whenever the positive effect of public investment outweighs the negative effect of taxation Pareto-improvements are possible.

3.1.2 Labor tax

A labor tax affects only the middle-income households, since the high-income households don't receive any labor income. The effects of labor tax-financed public capital are displayed in Figure 2. Our main findings are:

1. Dispersion in all three variables increases (see Figure 2 on the left)
2. Output is maximized for a labor tax bigger than 92 %
3. The policy is Pareto-improving up to more than 92% (see Figure 2 on the right)
4. Middle-income households prefer a lower income tax rate (64%) than high-income households (> 92 %) (see Figure 2 on the right)

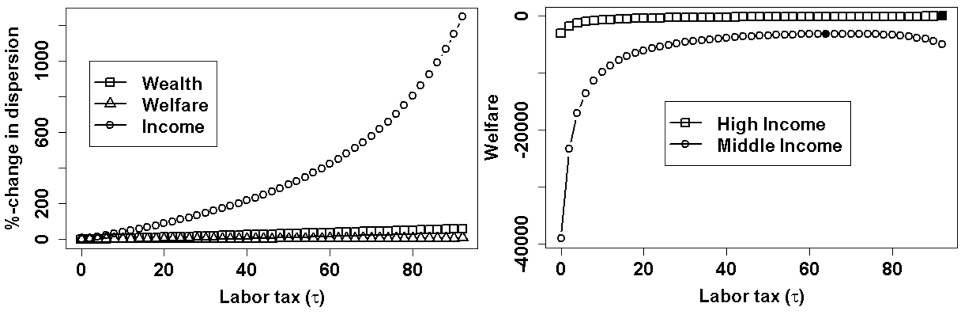

Figure 2: Effects of labor tax-financed public capital on the dispersion of wealth, welfare and income (left graph) and on the welfare of both agents (right graph). The high-income households prefer the maximum wage tax rate, since they do not receive wage income but benefit from public investment. The tax rate preferred by the middle-income households is quite high, which is a consequence of the current calibration, where the benefits of public investment outweigh the negative effects of a labor tax up to a tax rate of 64%.

[9] In the case of a capital tax the labor-leisure decision plays only a minor role: Total leisure for the middle-income households is slightly decreased since the value of leisure increases due to an increase in the composite externality. The composite externality increases as long as the increase in public capital offsets the decrease in private capital.

	dY(%)	du_{ILA}(%)	du_{OLG} (%)	$d\sigma_K$ (%)	$d\sigma_u$ (%)	$d\sigma_{Inc}$ (%)
$\tau_Y^{max} > 0.92$	> 411	> 98	> 89	> 58	> 8	> 1153
$\tau_{u,ILA}^{max} > 0.92$	> 411	> 98	> 89	> 58	> 8	> 1153
$\tau_{u,OLG}^{max} = 0.64$	+255	+96	+92	+41	+5	+480

Table 3: Steady state effects of a labor tax-financed increase in public spending. In the column on the left, the levels of labor tax rates which maximize output and utility of the different agents are given. In the remaining columns the changes in output, welfare and dispersion are given in percent, as compared to the baseline. Some values are outside the feasible range of taxes in our model and are thus marked with a ">" sign.

These results are explained as follows: A labor tax solely affects the middle-income households' income, which increases income dispersion strongly. Since the middle-income households' saving decision depends on the level of the wage income, their savings decrease, which causes wealth dispersion to increase. Labor taxation increases the leisure consumption ratio, which can be seen in Equation (C.5). The increasing composite externality has an opposing effect on the leisure consumption ratio (since $a < 0$, for $\sigma_{Intra} < 1$), which dominates with the current parameterization (specified in Section 2.7), so leisure decreases. The high-income household experiences a stronger increase in welfare due to its non-taxed income and the leisure-enhancing effect of the composite externality, while the middle-income household has reduced consumption through labor income taxation and reduced leisure, which causes inequality in welfare also to increase. The mechanism for the Pareto-improvement described for the capital tax also applies here, it is even stronger since labor taxation decreases the private capital stock less than direct capital taxation.

3.1.3 Consumption tax

The consumption tax has the broadest tax base of the three taxes, since all agents, the infinitely-lived and the young and the old overlapping-generations agents are taxed. Financing public capital with a consumption tax has the following effects:

1. Output is maximized for a tax rate of > 90 %
2. The policy is Pareto-improving for consumption taxes up to more than 90 % (see Figure 3 on the right)
3. Both households prefer a consumption tax > 90 % (see Figure 3 on the right)
4. Dispersion in all three variables changes only slightly (see Figure 3 on

the left)[10]

	dY(%)	du_{ILA}(%)	du_{OLG} (%)	$d\sigma_K$ (%)	$d\sigma_u$ (%)	$d\sigma_{Inc}$ (%)
$\tau_Y^{max} > 0.9$	> 217.2	> 91.3	> 92.4	-0.5	-1.4	1.4
$\tau_{u,ILA}^{max} > 0.9$	> 217.2	> 91.3	> 92.4	-0.5	-1.4	1.4
$\tau_{u,OLG}^{max} > 0.9$	> 217.2	> 91.3	> 92.4	-0.5	-1.4	1.4

Table 4: Steady state effects of a consumption tax-financed increase in public spending. In the column on the left, the levels of consumption tax rates which maximize output and utility of the different agents are given. In the remaining columns the changes in output, welfare and dispersion are given in percent, as compared to the baseline. Some values are outside the feasible range of taxes in our model and are thus marked with a ">" sign.

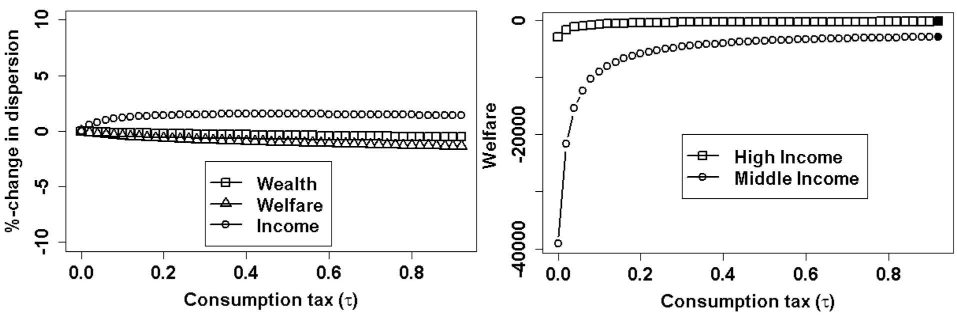

Figure 3: Effects of consumption tax-financed public capital on the dispersion of wealth, welfare and income (left side) and on the welfare of both agents (right side).

The results in the case of consumption taxation can be explained as follows: Since the tax itself affects both agents in the same way, it does not have an effect on the dispersion. There are slight effects on the dispersion of all three variables, which are orders of magnitude smaller than the effects of a capital or a labor tax. As in the case of labor taxation, a higher consumption tax increases the leisure consumption ratio, but an increasing composite externality decreases it. The value-enhancing effect of the composite externality on leisure again dominates, since the level of the composite externality is highest with consumption taxation, so the total effect is leisure-decreasing. To summarize: the value of leisure increases strongly with increased public investment, so the agent works more to be able to consume more. Public investment in turn raises wages. Both effects combined lead to very high preferred consumption tax rates for both agents.

[10]A change we attribute to incomplete convergence of our model (see Figure 4)

3.1.4 Summary: Comparing the different taxes

When comparing the results from Tables 2, 3 and 4, two main differences become apparent:

(i) The dispersion in all variables is strongly reduced by capital tax-financing of public spending, while labor tax-financing increases it. A consumption tax hardly changes the dispersion in all variables.

(ii) A consumption tax enhances the economy's output the most up to 20 %. Above that threshold a labor tax outperforms the consumption tax, while a capital tax performs worst. We attribute this to the disincentive to accumulate capital caused by the capital tax. For higher tax levels aggregate efficiency is highest for labor taxation, since labor taxation in this setup reduces leisure time thus causing middle-income households to work more and thereby increasing the public capital stock.

(i) and (ii) together suggest an equity-efficiency trade-off between capital tax-financing and consumption tax-financing.

By contrast all taxes constitute a Pareto-improvement up to a certain tax rate. This result depends crucially on the base level of public capital. When the public capital stock is already at its optimal level, further investment does not enhance both agents' welfare and thus will not lead to a Pareto improvement.

3.2 Transitional effects

In addition to the steady state analysis we also analyze the transitory dynamics of the system, since short run distributional effects can go into opposite directions compared to long run effects, as is the case in Chatterjee and Turnovsky (2012). We examine the impact of an unanticipated policy shock: When the system is in a steady state, public spending is increased from the baseline level to a level which increases output by 30 %.

We find two main results: (i) Short term effects opposite to the long run outcome are found only in the case of labor taxation: Wealth inequality is decreased in the short term, but then converges to a steady state with increased wealth inequality (see Figure 7 in the Appendix). (ii) A consumption tax has almost no long run effects on the distribution, but strong short run effects: Wealth inequality is decreased while income inequality is strongly increased in the short term (see Figure 4). The dynamics for a capital tax are displayed in Figure 8, which can be found in the Appendix.

Short term effects for both labor and consumption can be explained as follows. The slight initial decrease in the wealth distribution can be attributed to the Pasinetti property of the system: A sudden increase in public spending increases both factor prices thus saving of both agents increases. Since

the high-income household wants to force the interest rate back to her time preference rate, she decreases her saving, thus wealth inequality is reduced until the interest rate converges to the high-income household's time preference rate. The strong reaction of the income distribution for a labor and a consumption tax can also be explained by the Pasinetti property of the system: A sudden increase in public spending increases both factor prices. Since the interest rate before the shock is already at its Pasinetti level, the productivity enhancing shock causes the interest rate to converge to its steady state levels from above, while the wage rate converges to its steady state levels from below. This leads to higher capital income than wage income in the short run and thus to increased income inequality. This effect is not visible at the moment of the shock, $t = 0$, since the savings level of the middle-income household has already been determined in the time step preceding the shock.
In the case of capital taxation (see Figure 8) dispersions in all variables converge to their steady state values without noteworthy short run effects except for the strong initial decrease in the income dispersion which accrues to the the fact that middle-income households determine their savings in the period before the shock.

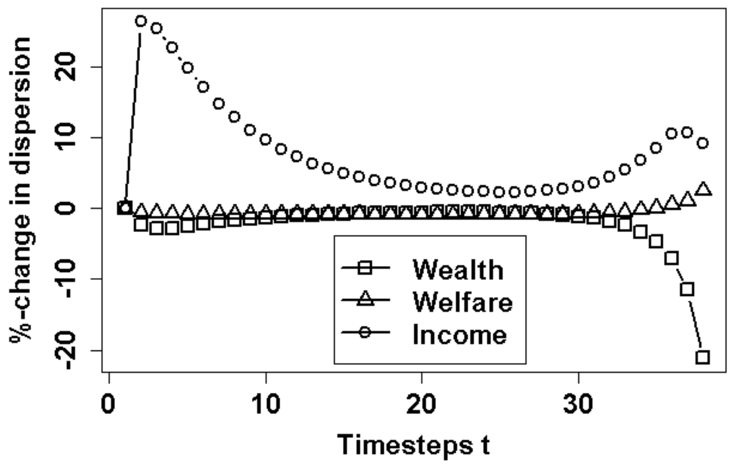

Figure 4: Transitory effects of an unanticipated increase of the consumption tax from the baseline steady state to a new steady state. The new steady state has a 30 % higher output level than the baseline. Even though the long run effect of consumption tax-financed public investment are almost distribution-neutral, there are strong short run effects.

4 Endogenous growth and sensitivity analysis

This section is split into two parts: In Subsection 4.1 we analyze the case of endogenous growth. In Subsection 4.2 the drivers of the model results are characterized by disabling some of the model features and the robustness of the results is analyzed by varying important parameters within their plausible ranges.

4.1 Endogenous growth analysis

In this section we present our findings for the case of endogenous growth, which requires constant instead of diminishing returns in accumulable factors, hence we set $\beta = 1 - \alpha$ (see Section 2.1). For this parameter choice the economy converges to a steady growth path on which consumption and capital for both agents, as well as public capital and the composite externality grow at the same rate g.
The purpose of analyzing this case is twofold: It permits a closer comparison of our results to the results obtained by Chatterjee and Turnovsky (2012) and Alesina and Rodrik (1994) and examines the robustness of the findings for steady state convergence presented in Section 3.1. The differences to the steady state analysis are mainly driven by the fact, that the Pasinetti Paradox does not occur in the case of endogenous growth (for more details on the Pasinetti Paradox see Section 2.5). Along the growth path we consider changes in the growth rate rather than in output as an indicator of efficiency.

We obtain three main results: (i) Similar to the steady state analysis, a consumption tax is the most efficient instrument, followed by a labor tax. A capital tax is least growth-enhancing (see Figure 5). (ii) Capital and labor taxation yield results very similar to the steady state analysis, except for slight variations in the case of low tax rates, which are explained below. (iii) The results for a consumption tax deviate from the steady state results (Figure 6 c). This behavior is analyzed in detail below.

For labor tax rates up to 20 %, income and welfare dispersion increase as in the steady state analysis (see Figure 6 b). But wealth dispersion slightly decreases, an effect which we only obtain because public capital is very productive in the case of endogenous growth. This outcome can be explained by examining the effects of an increase in labor tax-financed infrastructure spending:

(1) Leisure (l_t) decreases because the quality of public capital is enhanced, while total capital increases. This leads to a decreased interest rate and an increased wage rate (see Equations 3 and 4).

(2) Public capital and thus the composite externality is increased, which

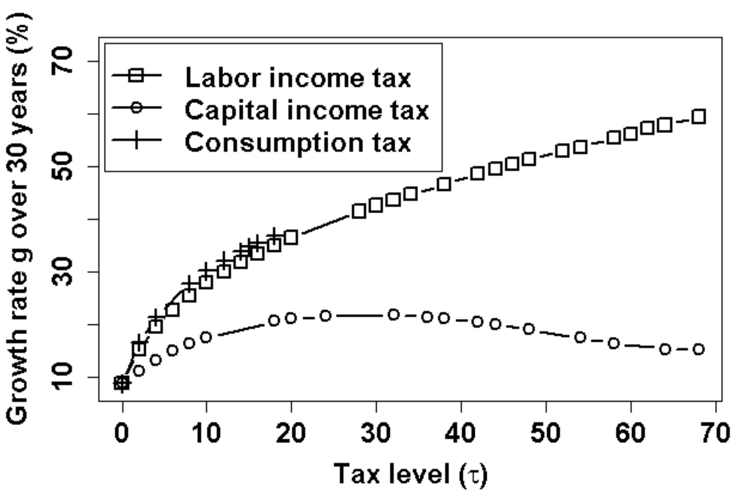

Figure 5: Effects of public investment on the steady state growth rate: A consumption tax is the most growth-enhancing way to finance infrastructure investment, directly followed by a labor tax. A capital-tax is the least growth enhancing policy instrument. Growth reaches its maximum already at a 30 % capital tax, while the other taxes enhance growth up to their maximum levels.

enhances both the interest and the wage rate.[11]

(3) Combining both (1) and (2) leads to increases in both factor prices since effect (2) outweighs effect (1) for the interest rate. However it also leads to an increased ratio of wage rate to interest rate due to effect (1).

For small tax rates the productivity enhancing effect of public capital more than offsets the negative effect of taxation and due to (3), the middle-income households' savings are affected more strongly by labor tax-financed public spending than the high-income households' savings.[12]
For tax rates below 8 %, effect (2) is also at work in the case of a capital tax, which leads to a small increase in income dispersion (see Figure 6 a). From 8 % on the negative effect of capital taxations outweighs this

[11] An effect unobserved in the case without endogenous growth, in which the interest rate always stays at the level determined by the high-income households' time preference rate due to Pasinetti's Paradox.

[12] This effect is not visible in the income dispersion since the labor component of the middle-income households' income benefits less from labor tax-financed infrastructure spending than the capital component due to the negative impact of the labor tax. The overall effect is that the middle-income households' income benefits less from infrastructure spending than the high-income households' income.

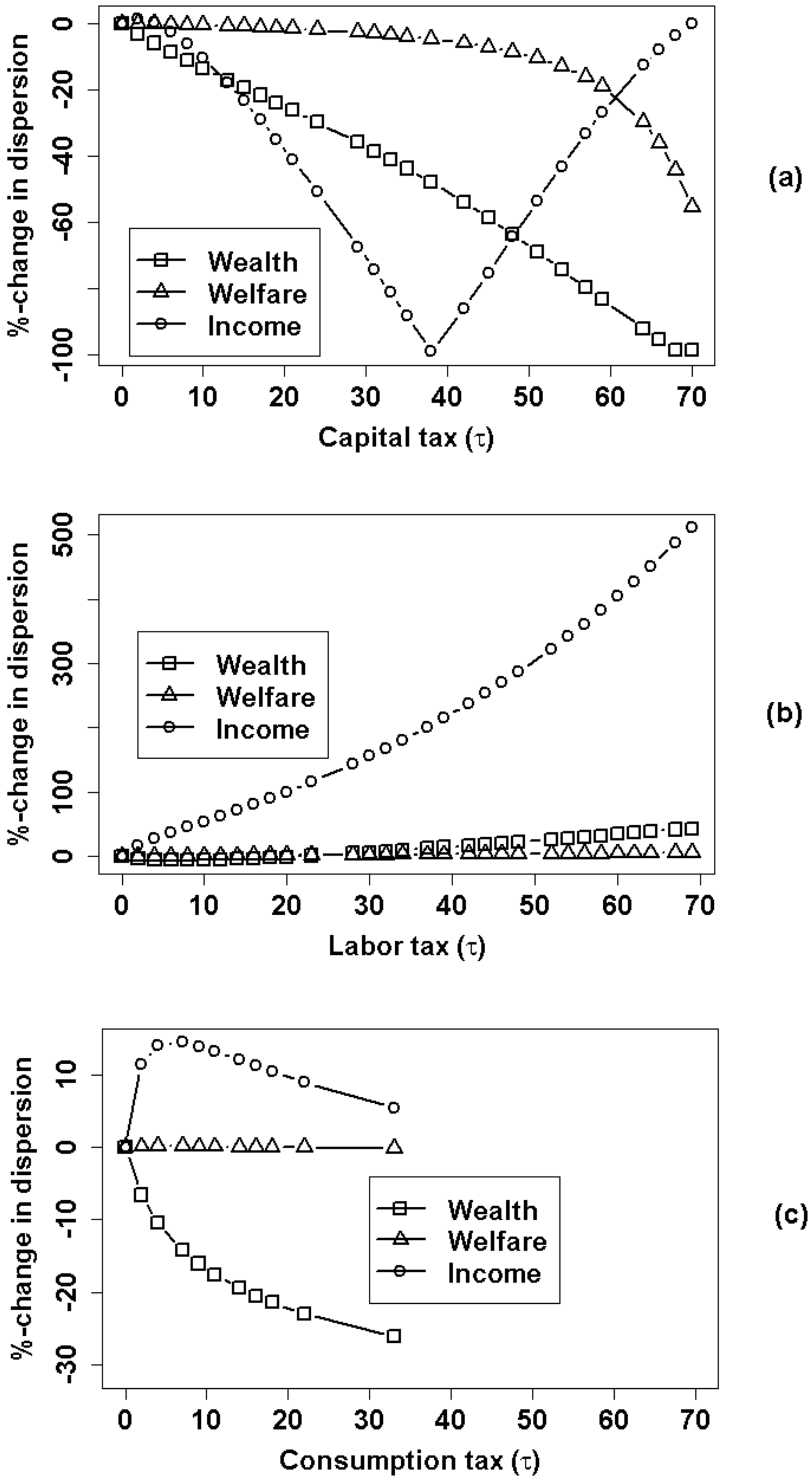

Figure 6: Effects of infrastructure financing through (a) capital, (b) labor and (c) consumption taxation on the dispersion of wealth, welfare and income for the case of endogenous growth. The downward spikes in Figure (a) reflect the points where middle-income households are equal in a certain variable to high-income households. For even higher tax rates the dispersion increases again, but this time the middle-income households are better off.

effect. Both of these effects are quite small for our current parameterization. Consumption tax-financed infrastructure investment leads to decreased wealth dispersion, but to an increase in income dispersion for taxes up to 10 %. For higher consumption taxes the income dispersion declines as well. There is hardly any effect on welfare dispersion (Figure 6 c).
The mechanisms of an increase in the consumption tax are the same as for a labor tax, so (1) to (3) still hold. But the negative effect of labor taxation on the middle-income households' income is missing, so for tax rates above 10 %, the middle-income households' income is affected more strongly by infrastructure spending than the high-income households' income and thus income dispersion declines from this point on. For tax values below 10 % the strong productivity-enhancing effect of infrastructure investment causes the capital component of the income to increase more strongly than the labor component, which leads to an increase in the income dispersion.

4.2 Sensitivity analysis

We find that the character of our results from Section 3.1 does not change in all of the scenarios described in Table 5. Discussing the full sensitivity analysis would go beyond the scope of this paper, but the three most important results should briefly be noted:
(i) For $\beta = 0$ a Pareto-improvement is still possible even though the composite externality is only utility-enhancing. (ii) In the case of $\theta = 0$, in which the composite externality is only production enhancing we also have the possibility of Pareto-improving policies. This means that our results do not depend on whether the composite externality affects production or welfare, as long as it affects one of them positively. (iii) The results are robust in ϵ which means that they also hold with a Romer (1986) and a Barro (1990) type of representation of the roles of public and private capital. Thus our assumption about household heterogeneity is the main driver of all observed effects.

Symbol	Parameter	Range
β	Exponent of public capital in production	$[0, 0.3]$
ϵ	Exponent of private capital in composite externality	0.0, 1.0
l_h, l_o	Leisure of agents with only capital income	$[0.5, 1]$
$\rho_h = \rho_m$	Time preference rate (yearly)	4%
θ	Share of leisure in utility function	$[0.0, 2.0]$
CES utility of the type $u_t = \left(\frac{1}{b}\right)\left((1-\theta)C_t^a + \theta(X_t l_t)^a\right)^{\left(\frac{b}{a}\right)}$, with $\theta = 0.5$		

Table 5: Sensitivity analysis of the model. The character of the results does not change for these parameter variations.

5 Conclusion

The present article studies the trade-off between equity and efficiency for public investment. We introduce a concept of household heterogeneity that is based on stylized facts about empirical saving behavior and differences in income sources and time preference. We show that these assumptions about household heterogeneity lead to results about the impact of public investment that starkly differ from the standard case in which heterogeneity stems from different initial endowments only, as recently examined by Chatterjee and Turnovsky (2012). We make two main contributions:

First, we find that in the long run, capital tax-financed public investment can be productivity-enhancing up to a certain tax level and reduce inequality in wealth, welfare and income at the same time. Consumption tax-financing also enhances productivity but leaves inequality in wealth, welfare and income virtually unchanged. This means that for these two financing mechanisms there is a tax range where no equity-efficiency trade-off exists for the financing of public capital. On the contrary, financing public capital with labor taxes increases inequality in all three variables, although it still enhances productivity up to a certain level.

Second, the type of household heterogeneity examined in this article leads to more robust results than heterogeneity that consists solely of different initial endowments: Our findings regarding the performances of policy instruments do neither require the assumption of endogenous growth, nor of identical rates of pure time preference of all agents. The results in the case of endogenous growth remain largely the same for the case of capital and labor taxation.The results for a consumption tax in the case of endogenous growth differ from the steady state outcome: after an increase in income inequality for low tax rates, inequality decreases in income and wealth, while remaining the same in welfare.
In summary we can say that also in the case of endogenous growth we observe many possible scenarios where efficiency and equity can be enhanced at the same time.

We conclude that the equity-efficiency trade-off is highly sensitive to the way heterogeneity is modeled. In the light of these findings, the standard assumption of heterogeneity in initial asset endowments seems questionable. A proper analysis of public policies thus should take into account differences in households which are beyond initial endowments.

This model framework could be extended to assess additional questions of public policy, for example climate policy, health spending or pension systems. A further refinement of the model structure could be to include mobility between income classes, the transmission of human capital within families and the existence of public insurance schemes. Optimal policies could be derived by introducing an optimizing government with different welfare functions, in order to assess the implications of different welfare norms on equity and efficiency of the economy.

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Appendices

A Transitional dynamics for a labor and a consumption tax

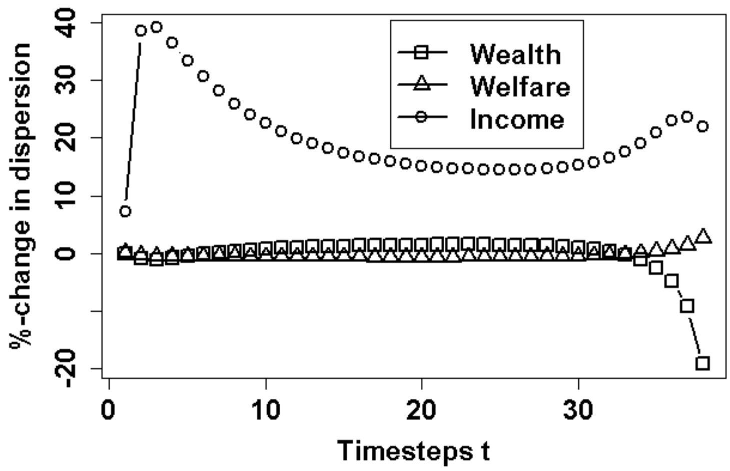

Figure 7: Transitory effects of an unanticipated increase of the labor tax from the baseline steady state to a new steady state. The new steady state has a 30 % higher output level than the baseline. Even though the long term effects of labor tax-financed public investment is inequality-increasing, it decreases short term wealth inequality.

B First-order conditions high-income households

The Lagrangian of the optimization problem of the high-income household can be written as:

$$\mathcal{L} = \sum_{t=0}^{t_{\text{final}}} u_t^{\text{ILA}} \cdot \frac{1}{(1+\rho_h)^t} + \lambda_t \left((1 + (1-\tau_K)r_t)K_{h,t} + (1+\tau_c)C_t - K_{h,t+1}\right).$$

The first-order conditions of the high-income household then are:

$$(1 + (1-\tau_K)r_t)\lambda_t = \lambda_{t-1}, \tag{B.1}$$

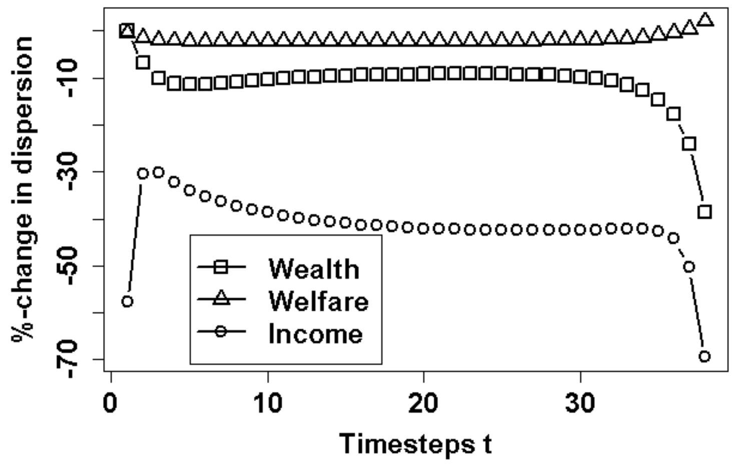

Figure 8: Transitory effects of an unanticipated increase of the capital tax from the baseline steady state to a new steady state. The new steady state has a 30 % higher output level than the baseline. For the case of capital tax financing, the model approximates the steady state monotonically, except for a strong first period decrease in income inequality, which can be attributed to the fact that middle-income households choose their saving level in the period before the shock, whilst the high-income households choose their level of saving already anticipating the shock.

$$\frac{\partial u_t^{\mathrm{ILA}}}{\partial C_t} \cdot \frac{1}{(1+\rho_h)^t} = \lambda_t(1+\tau_c). \tag{B.2}$$

Calculating the derivations in Equation (7) yields the explicit Euler equation:

$$\frac{\left(C_{t-1}^a + \theta(X_{t-1}l_h)^a\right)^{\left(\frac{b}{a}-1\right)}}{\left(C_t^a + \theta(X_t l_h)^a\right)^{\left(\frac{b}{a}-1\right)}} \left(\frac{C_t}{C_{t-1}}\right)^{(1-a)} = \frac{1+(1-\tau_K)r_t}{1+\rho_h} \tag{B.3}$$

C First-order conditions middle-income households

The Lagrangian of the optimization problem of the middle-income household can be written as:

$$\mathcal{L} = u_t^{\mathrm{OLG}} - \vartheta_t \cdot \left(C_{y,t}(1+\tau_c) + \frac{(1+\tau_c)C_{o,t+1}}{(1+(1-\tau_K)r_{t+1})} - (1-\tau_w)w_t(L-l_t) \right).$$

The first-order conditions are calculated as:

$$\vartheta_t = \frac{\partial u_t^{\text{OLG}}}{\partial C_{y,t}} \frac{1}{(1+\tau_c)} \tag{C.1}$$

$$\vartheta_t = \frac{\partial u_t^{\text{OLG}}}{\partial C_{o,t+1}} \frac{(1+(1-\tau_K)r_{t+1})}{(1+\tau_c)} \tag{C.2}$$

$$(1-\tau_w)w\vartheta_t = \frac{\partial u_t^{\text{OLG}}}{\partial l_{y,t}} \tag{C.3}$$

Combining Equations (C.1) and (C.2) we get the Euler equation (11); By combining (C.1) and (C.3) we get Equation 12. By calculating the partial derivatives of u_t^{OLG} and inserting them into Equation (11) and Equation (12) we get the explicit expressions:

$$\frac{C_{o,t+1}}{C_{y,t}} = \left(\left(\frac{1+(1-\tau_k)r_{t+1}}{(1+\rho_m)} \right) \cdot \left(\frac{C_{o,t+1}^a + \theta(X_{t+1}l_{o,t+1})^a}{C_{y,t}^a + \theta(X_t l_{y,t})^a} \right)^{((\frac{b}{a})-1)} \right)^{\frac{1}{(1-a)}} \tag{C.4}$$

Here we can see that the intertemporal decision is only directly influenced by capital taxation, as this expression only depends on τ_k.

$$\frac{l_{y,t}}{C_{y,t}} = X_t^{\frac{a}{(1-a)}} \left(\theta \cdot \frac{(1+\tau_c)}{(1-\tau_w)w_t} \right)^{\frac{1}{(1-a)}} \tag{C.5}$$

By contrast, we infer from the second Euler Equation that the intragenerational labor leisure decision is only directly influenced by consumption and labor taxation: the higher the labor or consumption tax, the higher the chosen levels of leisure.

D Steady state equations of the economy

By formulating the equations for the system's steady state we can gain important insights about the main drivers of steady state behavior. Additionally we can verify if the dynamic model which is solved numerically, is solving correctly.
In the following all steady state variables are denoted by a tilde. From Equations (6) and (14) it is easy to obtain an expression for the ILA's steady state consumption $\tilde{C}$:

$$\tilde{C} = \rho_h \tilde{K}_h. \tag{D.1}$$

The middle-income household's first-order conditions (Eqs. 9, 10, C.4 and C.5) and the first-order conditions of the firm (Equations 3 and 4) remain the same in the steady state.

The steady state level of public capital $\tilde{K}_G$ is given by:

$$\delta_G \tilde{K}_G = \tau_K \cdot \tilde{r}\tilde{K} + \tau_w \cdot \tilde{h}\tilde{w} + \tau_c \cdot (\tilde{C} + \tilde{C}_y + \tilde{C}_o). \tag{D.2}$$

Together with the Equation (2) we have a system of partially nonlinear equations.

By combining the steady state Equations (14), (D.1), (D.2) with the first-order conditions of the OLG agent (9, 10, C.4, C.5) and the firm (3 and 4), we can eliminate $\tilde{r}, \tilde{w}$ and $\tilde{C}$:

$$(1+\tau_c)\tilde{C}_y = (1-\tau_w)\left((1-\alpha)A\tilde{K}^{\alpha}\right)\tilde{X}^{\beta}(L-\tilde{l}_y)^{(1-\alpha)} - \tilde{S},$$

$$\tilde{C}_o = \frac{(1+\rho_h)}{(1+\tau_c)}\tilde{S},$$

$$\frac{\tilde{C}_o}{\tilde{C}_y} = \left(\left(\frac{1+\rho_h}{(1+\rho_m)}\right) \cdot \left(\frac{\tilde{C}_o^a + \theta(\tilde{X}l_o)^a}{\tilde{C}_y^a + \theta(\tilde{X}\tilde{l}_y)^a}\right)^{((\frac{b}{a})-1)}\right)^{\frac{1}{(1-a)}},$$

$$\frac{\tilde{l}_y}{\tilde{C}_y} = \tilde{X}^{\frac{2a-1}{(1-a)}}\left(\theta \cdot \frac{(1+\tau_c)}{(1-\tau_w)\left((1-\alpha)A\left(\frac{\tilde{K}}{(L-\tilde{l}_y)}\right)^{\alpha}\right)}\right)^{\frac{1}{(1-a)}},$$

$$\frac{\rho_h}{(1-\tau_K)} + \delta_K = \alpha A\left(\frac{(L-\tilde{l}_y)}{\tilde{K}}\right)^{1-\alpha}\tilde{X}^{\beta},$$

and

$$\delta_G \tilde{K}_G = \frac{\tau_K}{1-\tau_K} \cdot \rho_h \tilde{K} + \tau_w \cdot (L-\tilde{l}_y)(1-\alpha)A\left(\frac{\tilde{K}}{(L-\tilde{l}_y)}\right)^{\alpha}\tilde{X}^{\beta} + \tau_c \cdot (\rho_h \tilde{K}_h + \tilde{C}_y + \tilde{C}_o).$$

For the sake of readability we did not insert the expressions for $\tilde{K} = \tilde{K}_h + \tilde{S}$ and for $\tilde{X} = \tilde{K}^{\epsilon}\tilde{K}_G^{1-\epsilon}$. Now we only have a set of six partially non-linear equations in $\tilde{K}_h, \tilde{S}, \tilde{K}_G, \tilde{C}_y, \tilde{C}_o$ and $\tilde{l}_y$.

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