Communication

When do increasing carbon taxes accelerate global warming? A note on the green paradox

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A B S T R A C T

The “green paradox” by Hans-Werner Sinn suggests that increasing resource taxes accelerate global warming because resource owners increase near-term extraction in fear of higher future taxation. In this note, we show that this effect does only occur for the specific set of carbon taxes that increase at a rate higher than the effective discount rate of the resource owners. We calculate a critical initial value for the carbon tax that leads to a decreased cumulative consumption over the entire (infinite) time horizon. Applying our formal findings to carbon taxes for several mitigation targets, we conclude that there is a low risk of a green paradox in case the regulator implements and commits to a permanently mal-adjusted tax. This remaining risk can be avoided by emissions trading schemes as suggested by Sinn—as long as the emission caps are set appropriately and the intertemporal permit market works correctly.

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1 The tax is ad-hoc because it is not derived from an optimality or efficiency criterion.

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1. Introduction

The green paradox of Sinn (2008) analyzes possible responses of intertemporally profit maximizing fossil resource owners to climate policy, which are omitted by many existing studies on climate policy instruments. The focus on the dynamics of fossil fuel supply leads to important implications regarding the effectiveness and robustness of policy instruments. By examining several tax schemes, Sinn concludes that “measures to reduce carbon demand, ranging from taxes on fossil fuel consumption to the development of alternative energy sources […] will not mitigate the problem of global warming” (Sinn, 2008, p. 388). The analysis of Sinn is in particular based on a formal examination of increasing ad-hoc\textsuperscript{1} cash-flow taxes for resource owners within a Hotelling model. Increasing cash-flow taxes raise the value of resources extracted in the present relative to the resources extracted in the far-distant future. As such, taxes, for Sinn’s model assumptions, always exhaust the entire stock within infinite time, such a relative up-valuation of early extracted fossil resources leads to higher near-term extraction compared to the zero-tax case. Thus, increasing cash-flow taxes accelerate extraction and worsen global warming (the “green paradox”). Therefore, Sinn proposes several other policies like decreasing cash-flow taxes, constant unit taxes, \textit{in-situ} subsidies, emissions trading, or capital source taxes which slow down and postpone extraction.

In this note, we focus on the impact of increasing unit taxes on resource extraction instead of cash-flow taxes because the economic and political debate mainly centers on CO\textsubscript{2} or carbon taxes as unit taxes which are not linked to actual prices of fossil resources (e.g. IPCC, 2007, pp. 755–756; Nordhaus, 2008; Stern, 2008; Edenhofer et al., 2010). As it turns out, the denomination ‘increasing taxes’ has a very different meaning and incentive effect for unit and cash-flow taxes. This comes from the intertemporal dynamics of the resource extraction model which generates increasing resource prices due to increasing scarcities. We use basically the same model as Sinn (2008) and extend the original model by a formal analysis of unit taxes. While Sinn focuses only on a pure timing effect of policies (i.e. a pure intertemporal reallocation of resource extraction without affecting the cumulative amount of extraction), we will also take into account a volume effect (i.e. a lower cumulative amount of extraction within an infinite time horizon).

2. The effect of increasing resource taxes

In order to keep the analysis simple, we assume constant extraction costs $c \geq 0$ and focus on the tax and price dynamics of the standard Hotelling problem for a competitive resource industry as presented in Sinn (2008). There, resource owners maximize profit according to:

\begin{align*}
& \max \int_0^\infty (1-\gamma(t))(p(t)-c)q(t)e^{-\rho t}dt \\
& S(t) = \frac{dS(t)}{dt} = -q(t) \\
& S(0) = S_0
\end{align*}

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where \( p(t) \) is the resource price, \( q(t) \) the resource extraction, \( r \) the interest rate, \( S(t) \) the resource stock in the ground and \( 0 < S_0 < \infty \) the initial stock size. The resource price is determined by the demand function \( q(p) \) with \( q(p) = (\theta q(p))/C_0 < 0 \) and \( q(p) > 0 \) for all \( p > 0 \). In contrast to Sinn’s increasing cash-flow tax \( v(t) = 1 - \theta q(p) \sigma \) with \( \sigma \leq 0 \) we will in the following consider the unit tax \( \tau(t) = \tau_0 e^{rt} \) which increases at a constant rate \( \theta \). The maximization problem, hence, reads:

\[
\max \int_0^\infty (p(t) - \tau(t) - c)q(t)e^{-r t} dt
\]

\[
\dot{S}(t) = -q(t)
\]

\[
S(0) = S_0
\]

By setting up the Hamiltonian (in the following, we suppress the explicit time-dependency of the variables \( p, q, \tau, \lambda, S \))

\[
H = (p - \tau - c)q + \lambda \dot{S}
\]

we obtain as first-order conditions:

\[
\lambda = p - \tau - c
\]

\[
\dot{\lambda} = r \lambda.
\]

These lead to the Hotelling rule

\[
r = \frac{p - \tau}{p - \tau - c} = \frac{\dot{p} - \tau}{p - c} = \frac{\dot{p} + (r - \theta)\tau}{p - c}
\]

and to the transversality condition

\[
0 = \lim_{t \to \infty} \frac{\lambda e^{-r t}}{e^{-r t}} = \lim_{t \to \infty} (p - \tau - c)Se^{-r t}
\]

The resource price cannot be lower than the sum of the tax and the extraction costs, \( p \geq \tau + c \). We define \( \tau^*_\infty \) as the initial tax level where a pure tax-and-extraction-cost-price would equalize cumulative demand with the entire resource stock, i.e.

\[
\int_0^\infty q(\tau_0 e^{rt} + c) dt = S_0
\]

(2)

**Proposition 1.** (a) If \( \tau_0 \leq \tau^*_\infty \), the entire resource stock will be depleted in infinite time, i.e. \( S_\infty := \lim_{t \to \infty} S = 0 \).

(b) If \( \tau_0 > \tau^*_\infty \), the stock will not be exhausted in infinite time, i.e. \( S_\infty > 0 \); we say the tax provokes a volume effect.

**Proof.** Solving the inhomogeneous linear differential equation (1) for the resource price \( p \) with \( p_0 := p(0) \), we obtain:

\[
p = \tau_0 e^{rt} + (p_0 - \tau_0 - c)e^{rt} + c
\]

The transversality condition reads:

\[
(p_0 - \tau_0 - c)S_\infty = 0
\]

(4)

The final size of the resource stock \( S_\infty \) is characterized by the cumulative demand \( q(p) \) with the consumer price \( p \) given by (3):

\[
S_\infty = S_0 - \int_0^\infty q(\tau_0 e^{rt} + (p_0 - \tau_0 - c)e^{rt} + c) dt
\]

(5)

(a): Suppose that \( S_\infty > 0 \). Then the transversality condition (4) implies that \( (p_0 - \tau_0 - c) = 0 \). Hence, final resource stock

\[
eq \max \int_0^\infty (p(t) - \tau(t) - c)q(t)e^{-r t} dt
\]

\[
\dot{S}(t) = -q(t)
\]

\[
S(0) = S_0
\]

Hence, if the initial resource tax level is set higher than \( \tau^*_\infty \), the cumulative extraction will always be reduced and the stock will be prevented from exhaustion, i.e. \( S_\infty > 0 \).

From (3) and (4) follows immediately:

**Corollary 1.** If \( \tau_0 > \tau^*_\infty \), the resource price will be completely determined by the tax and the extraction costs: \( p = \tau_0 e^{rt} + c \). The resource owners reap zero profits.

In case the tax provokes a volume effect, the carbon tax reflects the scarcity rent for the de facto resource stock \( S_0 := S_0 - S_\infty \). In contrast, if the initial resource tax level was set equal or below \( \tau^*_\infty \), there would be no volume effect of the tax and the entire resource stock would be exhausted despite an increasing resource tax, i.e. \( S_\infty = 0 \).

With respect to the time path of the resource extraction, we can now distinguish three cases concerning the term \( r - \theta \) in (1):

(1) the carbon tax grows at the discount rate;
(2) the carbon tax grows at a rate lower than the discount rate; and
(3) the carbon tax grows at a rate higher than the discount rate.

Table 1 summarizes the different cases and their implications which are discussed in detail in the following.

Case 1: The carbon tax grows at the discount rate. If \( r = r_0 \), the increasing tax will not influence the relative time path of the resource price and, hence, resource extraction. If the initial tax level \( \tau_0 \) is equal or below \( \tau^*_\infty \) as defined by (2), the resource tax will simply absorb the scarcity rent without any distortions (Dasgupta and Heal, 1979, p. 364). The initial consumer price \( p_0 \) is at the level that equals the total resource price with the cumulative demand over an infinite time horizon. If, in contrast, the initial tax level \( \tau_0 \) is above \( \tau^*_\infty \), the consumer price for resources will equal the tax and is at each point in time strictly higher than in the no-tax case. Thus, a unit tax that increases with the discount rate has no timing effect—here it can have a substantial volume effect in decreasing demand and conserving the resource stock. With such a unit tax, a green paradox cannot occur.

Case 2: The carbon tax grows at a rate lower than the discount rate. If \( r < r_0 \), the resource tax will have a clear timing effect. The resource tax reflects the scarcity rent for the de facto resource stock, i.e. \( r \). In contrast, if the initial resource tax level was set equal or below \( \tau^*_\infty \), there would be no volume effect of the tax and the entire resource stock would be exhausted despite an increasing resource tax, i.e. \( S_\infty = 0 \).

With respect to the time path of the resource extraction, we can now distinguish three cases concerning the term \( r - \theta \) in (1):

(1) the carbon tax grows at the discount rate;
extraction and therefore reduces climate damages by the timing effect and – if \( t_0 > r^* \) additionally by the volume effect.

Case 3: The carbon tax grows with a rate higher than the discount rate. If \( \theta > r \), the resource tax will have a clear timing effect. The price path is steepened according to (1) and, thus, extraction is accelerated. The volume effect, however, depends once more on the initial tax level. An initial level below \( r^* \) does not decrease the cumulative extraction. Climate damages increase compared to the zero-tax case because the resources are extracted too early. This is the case for the classical green paradox as described in Sinn (2008). If, in contrast, the tax level is above \( r^* \), the cumulative extraction will be lowered and the stock will be prevented from exhaustion. In this case, we have two driving forces on climate damages with an antithetic impact. While the volume effect leads to lower long-term extraction, near-term extraction could actually increase due to the timing effect. The higher the initial tax level, the stronger is the volume effect and, thus, the timing effect diminishes. In principle, every value of \( S^w \) could be achieved if the initial tax level was set appropriately high.

In a related study, Hoel (2010) argues that a carbon tax of 179 $/tC0\(_2\) (656 $/tC) will definitely reduce carbon emissions from the beginning and that such immediate emissions reduction is likely to occur for carbon taxes higher than 367 $/tC. Due to the heterogeneity of fossil resources in extraction costs and demand, it is difficult to calculate exactly the critical initial tax level leading to sufficient lower (cumulative) extraction. In the following, we will therefore focus on the timing effect of carbon tax proposals and their impact on resource extraction.

### Table 1
Assessment of resource unit taxes with respect to the zero-tax case. \( r \) — effective discount rate of the resource owners; \( \theta \) — rate of the tax increase, \( t_0 \) — initial tax level. Impact on damages: ‘–’ denotes a reduction of damages; ‘+’ an increase of damages.

<table>
<thead>
<tr>
<th>Tax increases at discount rate ( \theta = r )</th>
<th>Slowly increasing tax ( \theta &lt; r )</th>
<th>Fast increasing tax ( \theta &gt; r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_0 ) Small ((T_0 \leq T^*))</td>
<td>( t_0 ) small ((T_0 &gt; T^*))</td>
<td>( t_0 ) large ((T_0 &gt; T^*))</td>
</tr>
<tr>
<td>Timing effect</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Volume effect</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Green paradox</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Impact on damages</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>compared to zero-tax case</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Volume effect</td>
<td>Timing effect</td>
<td>Timing and volume effect</td>
</tr>
<tr>
<td>Timing effect</td>
<td>Timing effect</td>
<td>Timing effect</td>
</tr>
</tbody>
</table>

### Fig. 1
Carbon taxes in 2005-USD per ton of carbon as calculated for several temperature scenarios by the integrated assessment models DICE-2007 (Nordhaus, 2008, pp. 92-93), REMIND, MERGE, TIMER, E3MG and POLES (Edenhofer et al., 2010).

What carbon taxes do integrated assessment models suggest? In general, carbon taxes are very sensitive to many parameters concerning the climate system, damages and technological progress (Edenhofer et al., 2006). Fig. 1 shows carbon taxes calculated by several models. Table 2 gives also the initial tax level and growth rate of the respective exponential carbon taxes which are approximated to the models’ taxes. Even for ambitious mitigation targets (450 ppm) most of the taxes have moderate growth rates between zero and three percent and are lower than the risk-free interest rates within these models. Only in the MERGE model the tax growth rate is with 5.8% on a very high level.

Resource owners discount their resource rent usually at the market interest rate which differs with respect to region and risk of financial assets. While the long-run rate of return of UK or US government bonds is about 1.5%, long-run rates of private equity are around 6–7% (Stern, 2008). Sinn (2008) argues that resource owners may add an additional risk premium if the ownership of their resources in the ground is insecure due to (geo)political instability—an analogous argument will hold if futures markets

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4 Although social planer models can – according to the welfare theorems – mimic market dynamics if no externalities and market failures exist, both models lead to different results in second-best worlds. This is, for example, relevant if the discount rate of a social planer differs from the rate of private households (see Heal (2009) for the debate on normative vs. positive discounting).
Table 2
Initial tax level $t_0$ and tax growth rate $\theta$ for the approximated exponential carbon tax $t = t_0e^{\theta t}$. The approximation of Fig. 1 models’ ten-year taxes (from 2015 to 2095) is calculated by linear regression of the log-values with the least square method. The $R^2$ values measure how good the approximated tax fits to the models’ tax. The last column shows − when available − the average discount rate $r$ (return on capital) which applies in the models.

<table>
<thead>
<tr>
<th></th>
<th>$t_0$ (t/1000 ppm CO$_2$)</th>
<th>$\theta$ (%)</th>
<th>$R^2$</th>
<th>$r$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DICE-2007</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal</td>
<td>586</td>
<td>37.2</td>
<td>1.9</td>
<td>0.994</td>
</tr>
<tr>
<td>1.5 C</td>
<td>(420 ppm CO$_2$)</td>
<td>209.2</td>
<td>2.1</td>
<td>0.774</td>
</tr>
<tr>
<td>2 C</td>
<td>(465 ppm CO$_2$)</td>
<td>54.2</td>
<td>3.3</td>
<td>0.992</td>
</tr>
<tr>
<td>2.5 C</td>
<td>(544 ppm CO$_2$)</td>
<td>36.0</td>
<td>2.8</td>
<td>0.999</td>
</tr>
<tr>
<td>Sterilization (404 ppm CO$_2$)</td>
<td>316.0</td>
<td>1.3</td>
<td>0.985</td>
<td>5.5</td>
</tr>
<tr>
<td>Low-cost backstop (340 ppm CO$_2$)</td>
<td>5.0</td>
<td>0.2</td>
<td>0.999</td>
<td>5.5</td>
</tr>
<tr>
<td>REMIND 2 C</td>
<td>(450 ppm CO$_2$-eq.)</td>
<td>72.1</td>
<td>2.6</td>
<td>0.802</td>
</tr>
<tr>
<td>MERGE 2 C</td>
<td>(450 ppm CO$_2$-eq.)</td>
<td>23.9</td>
<td>5.8</td>
<td>0.999</td>
</tr>
<tr>
<td>TIMER 2 C</td>
<td>(450 ppm CO$_2$-eq.)</td>
<td>375.6</td>
<td>1.4</td>
<td>0.530</td>
</tr>
<tr>
<td>E3MG 2 C</td>
<td>(450 ppm CO$_2$-eq.)</td>
<td>43.5</td>
<td>0.0</td>
<td>0.525</td>
</tr>
<tr>
<td>POLES 2 C</td>
<td>(450 ppm CO$_2$-eq.)</td>
<td>121.6</td>
<td>3.7</td>
<td>0.790</td>
</tr>
</tbody>
</table>

for fossil resources are incomplete. Adelman (1986) estimated that effective discount rates of OPEC countries exceed 25% partly due to poor diversification of OPEC’s economies and other political economy aspects—while in industrialized countries discount rates are estimated to be around 10%. Even if these numbers differ from current discount rates, it is very likely that discount rates of resource owners exceed market interest rates significantly.

4. Policy implications

So far, we focused on the incentive effect of an arbitrarily set carbon tax. An optimal carbon tax as calculated by Kalkuhl and Edenhofer (2010) follows a complex dynamics and requires a precise understanding of the damages of global warming. While such an optimal tax does not provoke an accelerated extraction, the consideration of second-best taxes as done by Sinn (2008) and by this paper gives important hints on the robustness of carbon taxes. As the mere possibility of an accelerated resource extraction exists in case the carbon tax is (and permanently remains) mal-adjusted, this instrument could be perceived too risky to prevent dangerous climate change. An emissions trading scheme as suggested by Sinn (2008) can avoid this risk— as long as the emission caps are set appropriately and the intertemporal permit market works correctly.

As an alternative to a global cap-and-trade scheme, Sinn (2008) proposes a capital income tax harmonization within OECD countries as a robust fool-proof instrument. Extraction is always slowed down because such a tax reform lowers the effective discount rate of resource owners. This instrument, however, is in practice not capable to achieve ambitious mitigation targets: Firstly, capital tax rates cannot be set very high as they lead to distortions in investment decisions implying lower welfare. Second, capital taxes cannot reduce cumulative extraction, i.e. they cannot provoke a volume effect (see Appendix for proof). By the specific choice of resource and capital tax instruments, Sinn (2008) completely rules out the volume effect. This is a strong limitation: The volume effect could become relevant if policy makers commit to concentration targets or cumulative carbon emissions in order to prevent the crossing of tipping points in the climate system (see WBGU, 2009 for the carbon budget proposal). An optimal carbon tax under such a carbon budget grows at the discount rate and the initial tax level is set such that cumulative extraction equals the carbon budget (see Kalkuhl and Edenhofer (2010) for a formal analysis). However, a mal-adjusted tax can again provoke an accelerated extraction and an emissions trading scheme may be the superior alternative.

5. Conclusion

By implementing carbon (unit) taxes in Sinn’s (2008) model, we have shown that an accelerated resource extraction due to increasing carbon taxes (green paradox) is limited to specific conditions: The initial tax level has to be lower than a certain threshold and the tax has to grow permanently at a rate higher than the discount rate of resource owners. We showed that a prominent set of carbon taxes for several mitigation targets is not at high risk to provoke a green paradox. However, in order to avoid the small risk of a green paradox, quantity instruments might be preferable if they are implemented appropriately and markets work correctly. The capital income tax proposed by Sinn can be useful to slow down extraction, but it is not capable to achieve low stabilization targets. If regulators nevertheless rely on carbon taxes (i.e. due to political constraints) the initial tax level should be high enough and the long-run tax-growth rate equal or below market interest rates.

Acknowledgments

We wish to thank Kai Lessmann, Brigitte Knopf and two anonymous reviewers for their constructive support.

Appendix

Introducing a constant capital tax $k < 1$ changes the effective discount rate of resource owners’ maximization problem to $\tilde{r} = r(1-k)$. The resulting Hotelling rule is then:

$$r(1-k) = \frac{p}{p-c}$$  \hspace{1cm} (A1)

Solving the differential equation (A1) for $p(t)$ gives:

$$p = (p_0-c)e^{(1-k)t} + c$$  \hspace{1cm} (A2)

Putting (A2) into the transversality condition $0 = \lim_{t \to \infty} (p-c) Se^{-(1-k)t}$ yields:

$$0 = \lim_{t \to \infty} (p_0-c)S$$

Hence, either the entire resource stock has to be exhausted (i.e. $\lim_{t \to \infty} S(t) = 0$) or the resource price equals always the extraction costs, i.e. $p = c$ due to (A2). The latter condition, however, contradicts the stock clearing condition because with positive demand, the cumulative demand exceeds the initial resource stock:

$$\int_0^\infty q(p)dt = \int_0^\infty q(c)dt = \infty > S_0.$$  

References


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