Managing the climate rent: How can regulators implement intertemporally efficient mitigation policies?*

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Abstract

This paper provides a formal framework to analyse informational and commitment requirements of several intertemporal price and quantity instruments for mitigating global warming. We ask under what conditions and to what extend the regulator can shift the complex and daunting intertemporal optimization of fossil resource use to markets. Mitigation always generates an intertemporal *climate rent* which reflects the stock-dependent damages and emerging scarcities of the atmospheric carbon deposit. In order to calculate and to manage this climate rent appropriately, common policy instruments like Pigouvian taxes or emissions trading presume perfect information about resource demand, extraction costs, reserve sizes and damages for the entire planning horizon. To reduce these informational requirements we develop an alternative policy approach – a state dependent tax rule – that relies only on current observations of cumulative extraction (or atmospheric carbon concentration). Within a cost-benefit analysis, this instrument is capable to shift the complex intertemporal optimization problem completely to the resource sector when resource owners are homogeneous. Under a cost-effective carbon budget approach, emissions trading with banking and borrowing can also unburden the regulator from solving the intertemporal social planner optimization problem. Additionally, we discuss which instruments can obtain an optimal allocation even if resource owners employ discount rate mark-ups (i.e. due to imperfect commitment or insecure property rights). While an emissions trading scheme without banking and borrowing is robust against discount rate mark-ups, resource taxes have to be modified in order to achieve an optimal allocation.

Abbreviations: BAU – business-as-usual; CBA – cost-benefit analysis; ETS – emissions trading scheme; ITR – intertemporal trading rate

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1 Introduction

Global warming is a stock-pollutant externality caused by the accumulation of greenhouse gases in the atmosphere. A main component of a successful climate policy consists of pricing global emissions – primarily from burning fossil fuels as they provide the main contribution to global warming [IPCC 2007, p. 28]. This price signal can be obtained by taxes or quantity instruments like emission trading schemes (ETS). Many of the successfully regulated environmental problems are more or less static problems. Sulphur emissions and particulate matters, for example, are easily measurable and citizens experience the impacts on environmental quality and health within short time horizons. The same applies to river and ground water quality. Environmental regulation is usually an iterative process where environmental policies are introduced and evaluated within few years. Evaluation of these problems is made by indicators describing the environmental quality and by the improvement directly perceived by citizens. At the same time, the impact of economic costs of environmental measures can be assessed. These short evaluation periods allow for a tightening or alleviating of environmental regulation – depending on the experienced costs and benefits.

With respect to global warming, however, this proceeding is not appropriate: The response of the climate system takes several decades and a fast "correction" of temperature levels is impossible due to this high inertia [Solomon et al. 2009]. Additionally, the stakes are high: The impacts of temperature increases are probably severe and long-lasting, as well as the impacts on the economy due to ambitious mitigation [Stern 2007]. Above all it is interfered in the fossil resource economy: regulating emissions directly affects fossil resource extraction. If fossil resource owners intertemporally maximize profits, they sensitively respond to announced and implemented climate policies [Sinn 2008, Edenhofer and Kalkuhl 2011]. As the expectations about future regulation strongly influence extraction and investment decisions, the "success" of already implemented policies with respect to achieved emission reductions can hardly be assessed. Hence, an iterative "muddling-through" policy based on a static externality concept is hardly feasible. Policies have to explicitly take into account the intertemporal dimension of the stock-pollutant problem.

The aim of this paper is to explore the institutional and informational requirements for achieving efficient allocation of resource extraction (and, hence, emissions). In contrast to existing works on resource extraction and global warming that focus on a social planner perspective [eg. Hoel and Kverndokk 1996, Farzin 1996] we explicitly consider the incentive, information and rent structure of this optimization problem as motivated by the green paradox [Sinn 2008]. We go beyond Sinn's analysis by providing a systematic comparison of optimal intertemporal price and quantity instruments. In particular, we draw on the literature on the intertemporal management of exhaustible resources [eg. Hotelling 1931, Dasgupta and Heal 1979, Dasgupta et al. 1981] and intertemporal emissions trading [eg. Kling and Rubin 1997, Leiby and Rubin 2001] when exploring designs of efficient and effective climate policy instruments in presence of profit maximizing fossil resource suppliers. We discuss several Hotelling-like models from a social planner and decentralized market perspective. The social planner model serves as a benchmark for the socially optimal solution. In the decentralized model, we study the incentive effect and the rent incidence on the resource sector that anticipates the policy instrument of the regulator. Furthermore, we perform our analysis within a cost-benefit (CBA) framework as well as in a cost-effectiveness (carbon budget) framework where the cumulative amount of emissions has been set as environmental target exogenously. Maximizing expected welfare under a cost-benefit approach is not possible if catastrophic impacts have a sufficiently high probability (i.e. if they are described by a fat-tailed probability distribution) [Weitzman 2009]. Such fat-tails evolve due to highly convex damages or due to the uncertainty about the parameters describing the probability distribution of the climate sensitivity (deep uncertainty). If the expected value of climate damages does not converge, greenhouse gas concentration targets could be derived by using an exogenous value-of-statistical-life parameter. In this case, concentration targets can be interpreted as an insurance against catastrophic climate change [Weitzman 2010].

Informational problems for implementing climate policy are usually associated with uncertainties about the costs and benefits of mitigation. Reducing uncertainties about damages of global warming requires better knowledge on the climate system and on the impacts of climate and weather on production and well-being. As the states of the climate are not traded on markets, we cannot rely on markets to find the correct valuation of climate damages – it falls into the domain of public scientific and economic research. The costs of mitigation – which are the benefits of using fossil resources – , however, can be estimated using market information as fossil resources are traded on markets. The lowest informational requirement for an efficient mitigation policy is therefore the knowledge of the damage function: Without knowing the damage function, the regulator cannot hope to implement optimal policies.

Existing approaches to internalize the damages of carbon emissions, however, rely always on both: information about damages *and* information about costs. By developing a general and flexible framework, we will illustrate formally the high informational requirements for a broad set of policies that are typically considered: Pigouvian taxes and emissions trading schemes. In both cases the regulator needs ex ante perfect information about size and quality (extraction costs) of fossil resources, technological progress in extraction and backstop technologies, fossil resource demand as well as climate damages for the entire time horizon (e.g. for the entire 21st century). Additionally, she needs to credibly commit to the announced policy. As the efficient level of carbon taxes or emission caps is not constant in time the regulator has to continuously revise tax levels or emission caps to remain on the optimal trajectory. In practice, however, such revisions are costly as they might provoke wasteful rent-seeking behavior which may dilute the commitment and credibility of climate policy. Hence, implementing the efficient Pigouvian tax may be very difficult for the regulator.

We illustrate the problems of commitment by studying the impact of discount-rate mark-ups reflecting policy risks (these discount rate mark-ups could also be motivated by insecure property rights or imperfect futures markets). In any case, optimal carbon taxes have to be modified to consider this additional distortion. In contrast, emissions trading schemes (without banking and borrowing) are fairly robust against discount rates; only the permit price increases if resource owners use higher discount rates. Hence, within our Hotelling model, quantity instruments can achieve an optimal allocation even if no commitment for future caps exists.

After illustrating and discussing informational and commitment requirements of standard approaches, we develop an alternative approach that achieves optimality at least for the case of homogeneous resource owners. The idea is to feed the climate damages through a resource stock dependent *tax rule* back to the resource sector who has to consider them within its intertemporal optimization problem. In contrast to Pigouvian taxes which are a previously announced time-dependent *tax path* for the entire planning period, our *tax rule* adjusts instantaneously to the current state of the world. The government does only need to measure carbon concentration in the air and to know the general functional form of damages – which is the lowest imaginable informational requirement.

Recent works of Golosov et al. [2011] and Gerlagh and Liski [2012] develop optimal Pigouvian taxes that require only the knowledge of specific parameters of the damage function and the endogenous saving rates. Golosov et al. [2011] argue in particular, that Pigouvian taxes are proportional to consumption in the steady state as savings form a constant fraction of output. The limitation of these findings is, however, that they assume a specific functional form for damages (exponential damages multiplicative to output), neglect extraction costs and assume a Cobb-Douglas production form (to justify a steady state solution with exhaustible resources). Our framework, however, is less specific on production technologies or damage functions, does not assume a steady state and allows for non-constant extraction costs.

The remainder of the paper is structured as follows: Section 2 starts with an analysis of optimal instruments within a cost-benefit analysis: We derive optimal Pigouvian taxes and optimal emissions trading schemes with and without intertemporal flexibility. Although these instruments have been studied elsewhere (in modified model settings and, in the case of emissions trading, without considering fossil resources explicitly), we provide a formal synthesis of all instruments within one single framework. The formal analysis of the common approaches emphasizes the high informational and commitment problems. We therefore present one new approach of an adaptive tax rule that demands less information for the regulator but is only efficient if resource owners are homogeneous. Finally, we supplement our formal analysis with a consideration on distorted discount rates and to what extent modified policies can correct them. Section 3 provides a similar analysis of cost-effective instruments within carbon budget framework. In this framework a stock-dependent tax rule is not feasible any more. However, an emissions trading scheme with banking and borrowing leads to similarly low informational requirements as the tax rule in the cost-benefit framework. The implications of the theoretical findings are finally discussed in the Conclusions.

2 The cost-benefit-approach

The analysis in this section is based on the modified Hotelling model presented in Sinn [2008]. We focus only on that part of the economy that generates intermediate or final goods (or services) from fossil energy use. Hence, $f(R)$ describes a partial production function from fossil resources R (or, how fossil resources contribute to overall economic output). These reseources are extracted from a (finite) resource stock S at marginal extraction costs $c(S)$.[1] We use the common assumption that production is increasing and concave in R, i.e. $f_R > 0$ and $f_{RR} < 0$.[2] As easily accessible resource sites are exploited first, we assume that extraction costs rise with depletion and are convex, thus $c_S < 0, c_{SS} \geq 0$. Focusing on the supply side, we neglect decay rates of carbon dioxide in the atmosphere and carbon dioxide storage technologies. We assume that by burning fossil resources a proportional amount of carbon dioxide is emitted into the atmosphere which remains there for a long time; thereby we describe (expected) damages $d(S)$ as function increasing in cumulative extraction of fossil fuels, implying $d_S < 0$.[3]

We consider a finite time horizon T which might be considered as relevant for designing and implementing instruments by policy makers. We abstract from terminal scrap values of the resource stock in the final period T in a world without climate damages. Instead, we consider a social scrap value function $F(S(T)), F_S \geq 0$ which reflects irreversible and persistent damages of global warming that are not valued by individual resource owners.[4] However, all of our findings translate to an infinite time model (without scrap value). In most cases, the T just becomes an ∞ and the scrap value term diminishes. We give some expressions for optimal policies under an infinite time horizon in the footnotes to the respective propositions.

The policy instruments we consider are taxes on fossil resource extraction or a quantity rationing system that limits extraction by issuing a limited amount of permits that are needed for

[1]To improve the readability of this paper, we will usually suppress the time-dependency of flow and stock variables like $R(t)$, $S(t)$ and so forth.

[2]In the following, we use the notation g_x for the partial derivative of g with respect to x, thus: $g_x := \frac{\partial g(x)}{\partial x}$. Likewise, $\dot{g} := \frac{dg}{dt}$ denotes the derivative of g with respect to time.

[3]Archer [2005] estimates that 17-33% of the emitted carbon dioxide remains in the atmosphere within approximately 1,000 years. Solomon et al. [2009] reports even higher numbers: After stopping carbon emissions immediately, atmospheric carbon concentration will fall to 40% after 1,000 years. These numbers suggest that carbon uptake rates should lie between 0.04% and 0.18%. Thus, for a policy analysis within a time-horizon of one century, uptake rates might be negligible.

[4]Indeed, we could extend our analysis by considering one scrap value function $F^R(S(T))$ for resource owners and another scrap value function $F^S(S(T))$ describing the valuation of the final stock by the society. As it turns out, the policy implications do only depend on the differences between the scrap value functions $F(S(T)) := F^S(S(T)) - F^R(S(T))$. Hence, assuming $F^R(S(T)) \equiv 0$ simplifies the formal analysis without constraining the generality of our results.

extraction. These instruments presuppose a strong institution that is able to monitor and control all relevant flows of carbon. As most of the fossil resources are consumed by industrialized countries, monitoring of fossil resource use and implementation of taxes or cap-and-trade systems are feasible although complete coverage over all resource exporting countries is difficult.[5] Besides monitoring resource flows and implementing taxes, calculating the optimal amount of taxes requires different kind of information. Optimal taxes (as well as optimal emission caps) are the outcome of an optimization problem that relies on further information on the structure and the parameters that describe the economy as well as the climate system. We therefore explore which of this information is necessary for the regulator and which information can be processed by profit optimizing agents.

In this paper, we always assume that fossil reserves are abundant in the sense that they are not fully extracted within the planning horizon (i.e. $S(T) > 0$) even if no damages exist. This can be justified by convex marginal extraction costs [Farzin 1992, Hoel and Kverndokk 1996], the existence of backstop technologies providing substitutes for R, or by the relative abundance of subterranean fossil carbon [eg. BGR 2009] in comparison to the limited demand of carbon within the planning horizon. The Assumption 1 gives a more restrictive formal condition that always leads to abundant resource stocks:

Assumption 1. *There exists $\tilde{S} \geq 0$ such that $c(\tilde{S}) > f_R(0)$. In words, if a certain amount of resources has been extracted, marginal extraction costs of resources $c(S)$ exceed marginal productivity f_R. This implies that extraction is ceased and the stock of fossil resources is not fully extracted within the planning horizon.*

This set of simplifying assumptions helps to clarify and highlight the supply-side dynamics by pointing out the intertemporal dimension of the control problem.

2.1 The social planner economy

The social planner maximizes the net present value of output $f(R)$ minus extraction costs $c(S)R$ and damages $d(S)$ with respect to the discount rate r. The optimization problem constrained by the scrap value function $F(S(T))$ and the initial resource stock size $S(0) = S_0$ reads:

$$\max_R \int_0^T \left(f(R) - c(S)R - d(S)\right) e^{-rt} \, dt + F(S(T))e^{-rT} \tag{1}$$

subject to:

$$\dot{S} = -R \tag{2}$$
$$S(0) = S_0 \tag{3}$$

The solution of the intertemporal optimization problem is characterized by:

Proposition 1. *(Socially optimal resource extraction) If a social planner maximizes intertemporal output according to (1–3), then: (a) the optimal solution (R^*, S^*) is determined by the following system of equations:*

$$r = \frac{\dot{f}_R(R^*) - d_S(S^*)}{f_R(R^*) - c(S^*)} \tag{4}$$
$$\dot{S}^* = -R^* \tag{5}$$
$$F_S(S^*(T)) = f_R(R^*(T)) - c(S^*(T)) = \lambda^*(T) \tag{6}$$
$$S(0) = S_0 \tag{7}$$

[5] Incomplete coverage provokes 'leakage' which can be exacerbated due to the inelastic supply of fossil resources [Sinn 2008].

(b) the shadow price λ^* for the stock S^* is given by:

$$\lambda^*(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T (c_S(S^*)R^* + d_S(S^*))e^{r(t-\xi)}\ d\xi \tag{8}$$

Proof. (a) We set up the corresponding Hamiltonian function $H = f(R) - c(S)R - d(S) - \lambda R$. Application of the maximum principle leads to the first-order condition with respect to R, the equation of motion for the shadow price λ, and the transversality condition:

$$\lambda = f_R(R) - c(S) \tag{9}$$

$$\dot{\lambda} = r\lambda - H_S = r\lambda + c_S(S)R + d_S(S) \tag{10}$$

$$0 = (\lambda(T) - F_S(S(T)))S(T) \tag{11}$$

By substituting (9) and its derivative with respect to time into (10) we obtain the social Hotelling rule (4). Furthermore, the transversality condition (11) together with Assumption 1 implies that $\lambda(T) = F_S(S(T))$. (b) Solving the differential equation (10) for given $\lambda(T)$ yields:

$$\lambda(t) = \lambda(T)e^{-r(T-t)} - \int_t^T (c_S(S)R + d_S(S))e^{r(t-\xi)}\ d\xi \tag{12}$$

□

For a zero scrap value function ($F(S(T)) \equiv 0$), Proposition 1 implies that marginal extraction costs increase up to marginal resource productivity. If the marginal scrap value is positive ($F_S(S(T)) > 0$), however, resources in the ground are assigned this additional value in the final period. This may be the case if society considers persistent and irreversible damages due to resource extraction after the planning period T. Equation (8) resembles the well-known rent dynamics for exhaustible resources with stock-dependent extraction costs [eg. Farzin 1992]. However, the familiar formula is extended by the term $d_S(S)$ under the integral reflecting the stock-pollutant dynamics of resource extraction and the marginal scrap value term $F_S(S(T))$. Hoel and Kverndokk [1996] derive a similar result for an infinite time horizon.[6] As we will show below, the rent dynamics in Equation (8) has to be induced by policy instruments in order to achieve an optimal decentralized solution.

2.2 Resource taxes in a decentralized market economy

Ordering fossil resources according to their extraction costs allows to consider resource heterogeneity by extraction costs that increase in cumulative extraction [Solow and Wan 1976, Swierzbinski and Mendelsohn 1989]. Therefore, aggregate industry behavior can be studied by focusing on a representative firm that takes resource prices $p(t) = f_R(R(t))$ and resource taxes $\tau(t)$ as given and maximizes intertemporal profit:

$$\max_R \int_0^T (p - c(S) - \tau)R)\, e^{-rt}\ dt \tag{13}$$

subject to:

$$\dot{S} = -R \tag{14}$$

$$S(0) = S_0 \tag{15}$$

[6] Hoel and Kverndokk [1996] assume extraction costs that rise without any bound implying that the optimal tax converges to zero in the long run: $\lim_{T\to\infty} \tau(T) = 0$. Considering an infinite time horizon in our framework would imply the transversality condition $\lim_{t\to\infty} \lambda^*(t)S^*(t)e^{-rt} = 0$. The appropriate shadow price λ^* in (8) would change to $-\int_t^\infty (c_S(S^*)R^* + d_S(S^*))e^{r(t-\xi)}\ d\xi$

In contrast to the social objective function (1), the resource sector does not consider social damages due to extraction during and after the planning horizon. By applying the maximum principle with λ as shadow price for the resource stock, we obtain (just along the lines of the proof of Proposition 1):

$$0 = p - c(S) - \tau - \lambda \tag{16}$$
$$\dot{\lambda} = r\lambda + c_S(S)R \tag{17}$$
$$0 = \lambda(T)S(T) \tag{18}$$

which leads to the private Hotelling rule and terminal condition:

$$r = \frac{\dot{p} - \dot{\tau} + r\tau}{p - c(S)} \tag{19}$$
$$\tau(T) = p(T) - c(S(T)) \tag{20}$$

because $S(T) > 0$. Without climate damages (i.e. $d(s) \equiv 0$), the decentralized market economy leads to the same extraction path as the social planner economy. In the presence of damages, however, the Hotelling rules in social planner and decentralized market economy (4) and (19) diverge for a zero tax level. In the decentralized market economy, the social optimum can be achieved by introducing a resource tax according to:

Proposition 2. *(Optimal resource tax) If the regulator knows the socially optimal extraction path S^* according to Proposition 1 and if she can commit at $t = 0$ to the tax path $\tau(t)$ over the entire planning horizon, then (a) the resource tax*[7]

$$\tau(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S(S^*)e^{r(t-\xi)}\, d\xi \tag{21}$$

achieves the optimal extraction path and (b) the rent in the resource sector is given by:

$$\lambda(t) = -\int_t^T c_S(S^*)R^* e^{r(t-\xi)}\, d\xi \tag{22}$$

Proof. (a) Differentiating (21) with respect to time, we obtain $\dot{\tau} = r\tau - d_S(S^*)$. Substituting this into the private Hotelling rule (19) and considering the fact that in the market equilibrium prices equal marginal productivities, i.e. $p = f_R(R)$, we obtain the socially optimal Hotelling rule (4). Furthermore, $\tau(T) = F_S(S^*(T))$ ensures that the private transversality condition (20) equals the social transversality condition (6). (b) The equation for λ follows from the solution of the differential equation (17) with $\lambda(T) = 0$ due to $S(T) > 0$.

□

Note that the sum of the resource owners' scarcity rent λ and the resource tax τ describes the entire rent dynamics and is expressed by:

$$\tau(t) + \lambda(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T (c_S(S^*)R^* + d_S(S^*))e^{r(t-\xi)}\, d\xi \tag{23}$$

which is exactly the resource shadow price in the social planner model as expressed in Eq. (8). The first summand denotes the (cumulative) scarcity of resources due to high stock externalities ($F_S(S^*(T)) > 0$). The second summand describes the dynamics of extraction costs and climate damages. In the following, we will denominate the rent component associated with τ *climate rent* as it evolves due to the stock-pollutant dynamics $d_S(S)$ and the cumulative scarcity by future damages expressed in $\tau(T) = F_S(S(T))$. Proposition 2 confirms that τ is indeed

[7] The infinite time model would yield $\tau(t) = -\int_t^\infty d_S(S^*)e^{r(t-\xi)}\, d\xi$

incentive-compatible in a decentralized economy as suggested by the social planner model of Hoel and Kverndokk [1996]: The tax achieves that intertemporally maximizing resource owners adjust their extraction path to the social optimum. Proposition 2 also makes the *informational requirement* for the government explicit: Implementing the optimal tax requires to solve the social planner problem as stated by Proposition 1. In an economy without climate damages, governments can delegate the task of determining an optimal allocation of resources completely to private resource owners: Proposition 2 confirms that without a climate externality no additional tax nor other government intervention is necessary. In contrast, under climate damages, implementing an optimal resource tax requires extensive amounts of information as well as a great ability to commit on a time-dependent tax, both of which are difficult to achieve:

- Calculating the optimal tax requires a full assessment of the resource stock size, extraction costs, resource demand, climate damages and the discount rate.
- Additionally, the regulator will have to commit to this tax for now and forever to incentivize the resource sector correctly. As the tax changes in time, it will provoke continuous public debates about the appropriate level as well as wasteful rent-seeking behavior to change the tax in favor of organized lobby groups.

Thus, the informational and commitment requirements for the regulator are quite high which makes deviations from the social optimum likely. Technological progress on extraction technologies (extraction costs) and backstop technologies (resource demand) is difficult to forecast and knowledge about the size and accessibility of reserves is often used strategically by oil exporting countries. A suboptimal tax can lead to an acceleration of extraction if the tax growth rate is high and the initial tax level is too low [Sinn 2008, Edenhofer and Kalkuhl 2011]. The conventional Pigouvian tax to internalize stock-pollutant damages is therefore difficult to implement in reality.

2.3 Emissions trading schemes

Proposition 2 proves that the informational requirements to implement a socially optimal resource tax are daunting. As a suboptimal tax can lead to an accelerated resource extraction worsening global warming, Sinn [2008] suggests a global emissions trading scheme as a foolproof alternative. Below, we elaborate the informational requirements and possible designs of efficient emissions trading schemes (ETS). As to our knowledge cap-and-trade systems have been studied without the link to the resource sector, we explicitly consider the interplay of quantity instruments with the resource sector in order to study the extraction and rent dynamics.

2.3.1 Emissions trading without banking and borrowing

We first focus on a conventional ETS where permits for resource extraction $C(t)$ which are only valid for one time period are issued. If a resource owner wants to sell a unit of resource, he has to use one permit. Thus, the regulator can effectively limit the resource use to C. This does, however, not imply that resource extraction always equals the permit path (it could be profitable for resource owners to extract less than the cap allows). We do not study the conditions under which such an undersupply of resources can occur as it requires rather tedious calculations. Instead, we assume that optimal extraction under climate policy is always lower than the business-as-usual extraction:

Assumption 2. *(Scarcity of permits) In each period, there are fewer permits issued than resources extracted in the zero-damage (BAU) case, i.e.*

$$C(t) < R^B(t) := R^*(t)_{|d(S)\equiv 0} \tag{24}$$

As we will show, this assumption guarantees that all permits are used at each point in time and no undersupply of resources occurs. The optimal ETS is characterized by the following proposition:

Proposition 3. *(Optimal ETS without banking) If the regulator issues permits $C(t) = R^*(t)$ along the socially optimal extraction path of Proposition 1, then (a) the optimal extraction is achieved, (b) the resource rent is given by $\lambda + \theta$ according to:*

$$\lambda(t) = -\int_t^T c_S(S^*)R^* e^{r(t-\xi)}\, d\xi \tag{25}$$

$$\theta(t) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S(S^*)e^{r(t-\xi)}\, d\xi \tag{26}$$

Proof. (a) We have to show that all permits are used, i.e. that $R(t) = C(t) = R^*(t)$. The optimization problem of the resource sector is given by $\max_R \int_0^T (p - c(S))Re^{-rt}\, dt$ subject to the constraints $\dot{S} = -R$, $S(0) = S_0$, $R(t) \leq C(t)$. The Hamiltonian function then reads $H = (p-c(S))R - \lambda R - \theta(C-R)$, where θ denotes the shadow price for the binding constraint $R \leq C$. Applying the maximum principle leads to the following first-order condition, equation of motion, transversality and Kuhn-Tucker condition, respectively:

$$0 = p - c(S) - \lambda - \theta \tag{27}$$

$$\dot{\lambda} = r\lambda + c_S(S)R \tag{28}$$

$$0 = \lambda(T)S(T) \tag{29}$$

$$0 = \theta(C - R) \tag{30}$$

Assumption 1 and Eq. (29) imply that $\lambda(T) = 0$. Solving the differential equation (28) with $\lambda(T) = 0$ we obtain

$$\lambda(t) = -\int_t^T c_S(S)Re^{r(t-\xi)}\, d\xi \tag{31}$$

From assumption 2 follows that $R \leq R^* < R^B$ and therefore $S > S^B$ and $c_S(S) > c_S(S^B)$ as $c_{SS}(S) > 0$ (the superscript B denotes the BAU case where damages are neglected). This implies that

$$\lambda(t) = -\int_t^T c_S(S)Re^{r(t-\xi)}\, d\xi < -\int_t^T c_S(S^B)R^B e^{r(t-\xi)}\, d\xi = \lambda^B(t) \tag{32}$$

With (27) we obtain $\lambda = p - c(S) - \theta$ and with (16) and $\tau = 0$ (in BAU) we have $\lambda^B = p^B - c(S^B)$. The inequality (32) therefore reads:

$$p - c(S) - \theta < p^B - c(S^B) \tag{33}$$

which can be rearranged to

$$(p - p^B) + c(S^B) - c(S) < \theta \tag{34}$$

As p decreases with higher R (because $p = f_R$ and $f_{RR} < 0$) and $R < R^B$ it follows $p > p^B$. Likewise, $S^B < S$ and $c_S < 0$ imply $c(S^B) > c(S)$. Therefore, (34) leads to $\theta > 0$ and due to the Kuhn-Tucker condition (30), we have $R(t) = C(t)$. (b) As R follows the socially optimal path R^*, (25) directly follows from (31). From (27) follows that the rent in the resource sector is given by $p - c(S) = \lambda + \theta$. In particular, $p(T) = c(S(T)) + \theta(T)$. As $R(t) = R^*(t)$ and $p = f_R$, the difference $p - c(S)$ is the same as in the social Hotelling model (9) which implies together with (8):

$$\lambda + \theta = p^* - c(S^*) = F_S(S^*(T))e^{-r(T-t)} - \int_t^T (c_S(S^*)R^* + d_S(S^*))e^{r(t-\xi)}\, d\xi \tag{35}$$

Substituting λ from (25) into (35), we finally obtain (26).

□

The shadow price θ for permits exactly equals the optimal resource tax (21) and thus reflects the climate rent. It is worthwhile to note that it has not been specified which party profits from the new climate rent – the resource sector or the regulator. If the regulator issues permits for free to the resource sector, the resource sector receives the extraction rent λ and adds the user cost θ to the resource price. His rent is then given by $\lambda + \theta$. Alternatively, the regulator can sell (or auction) the permits with a price up to θ and absorb the climate rent completely. In accordance with conventional wisdom this rent can be captured by the regulator without any intertemporal efficiency losses.

2.3.2 Emissions trading with banking and borrowing

Instead of controlling the time path of permits in each period, banking and borrowing of permits gives markets the flexibility to decide when to use the issued permits. On the first view, this could be a possibility to reduce informational requirements for the regulator as the market can now determine the intertemporal allocation. A free intertemporal permit trade, however, would result in a Hotelling path. Within this market, permits are treated like an exhaustible resource – one permit used now is not available in the future. This Hotelling-path is not socially optimal because the intertemporal allocation of marginal damages is not taken into account properly [Kling and Rubin 1997]. This problem could be resolved by introducing intertemporal trading rates. Leiby and Rubin [2001] have calculated intertemporal trading rates (ITR) which change the effective size of the pollution allowance held by a permit owner from one period to the next and lead to an optimal intertemporal reallocation of permits. We apply this approach to our model with a fossil resource sector to study whether the regulator can shirk the information and commitment problems as raised under the previous ETS without banking and borrowing. In order to analyze banking and borrowing within our framework, only small modifications are required. The objective function and equation of motion for the resource stock remain unchanged. However, we add an equation of motion for the permit stock b. The permit stock decreases by one unit for one unit of resource use and grows at $\gamma(t)$ – the intertemporal trading rate (ITR).

$$\dot{b} = -R + \gamma b \tag{36}$$

To keep our analysis simple, we restrict it to the case where the regulator issues b_0 permits once in the initial period for the entire time horizon.

Proposition 4. *(Optimal ETS with banking) If the regulator knows the optimal extraction path S^* of Proposition 1, then (a) she can achieve the socially optimal extraction path by issuing b_0 permits in the beginning and allowing for banking of permits with the intertemporal trading rate γ according to:*

$$b_0 = S_0 + \frac{\int_0^T e^{-r\xi} d_S^* S^*\, d\xi - S^*(T)F_S(S^*(T))e^{-rT}}{-\int_0^T e^{-r\xi} d_S^*\, d\xi + F_S(S^*(T))e^{-rT}} \tag{37}$$

$$\gamma = \frac{-d_S^*}{F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)} d\xi} \tag{38}$$

(b) the rent in the resource sector is given by $\lambda + \mu$ where:

$$\lambda = -\int_t^T c_S^* R^* e^{r(t-\xi)}\, d\xi \tag{39}$$

$$\mu = F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S^* e^{r(t-\xi)}\, d\xi \tag{40}$$

Proof. See Appendix A. □

As it turns out, the formula for the ITR γ is in accordance with the formula given by Leiby and Rubin [2001] who abstract from the fossil resource sector. We further extend their analysis by giving a formula for the optimal size of the initial permit stock b_0. In principle, optimal intertemporal permit trading requires two regulating screws. Besides the ITR, the regulator has to issue the optimal number of permits in the first period which can be traded over the entire time horizon. While the ITR γ enforces the optimal timing of extraction, b_0 enforces the optimal cumulative resource consumption in accordance with the transversality condition of the social planner problem. As the regulator has to calculate *ex ante* the damages and the extraction along the social optimum $d_S(S^*(t))$ and $S^*(t)$, respectively, the informational requirements of Propositions 2 remain unchanged. Introducing banking and borrowing cannot discharge the regulator from difficult intertemporal optimization decisions by using market mechanisms. The ETS with banking resembles the resource rent dynamics with a stock externality as given by (8). It becomes apparent that the regulator could capture the rent associated with the shadow price of permits μ. Applying an auctioning mechanism, she could sell permits in the first period at maximum price μ_0, which equals the discounted value of the cumulative tax income from the optimal resource tax (21).

2.4 A new approach to internalize damages: stock-dependent resource taxes

Usually, regulators cannot and do not commit *ex ante* to a time-dependent tax path $\tau(t)$ for long time horizons. Instead, regulation is more an iterative process where the resource tax is dependent on the estimation of marginal damages from the cumulative resource extraction. When concentrations rise, the regulator increases the tax to account for higher social damages. In this section we ask whether the regulator can achieve the optimal extraction path by implementing a resource tax $\tau(S)$ which is adjusted to the current concentration of carbon in the atmosphere. The regulator announces explicitly how she modulates the tax and the resource sector responds to this tax adjustment rule. Under rather restrictive assumptions, such a tax rule can indeed achieve the optimal allocation path:

Proposition 5. *(Stock dependent tax) If the regulator imposes a resource tax $\tau(S)$ which increases with cumulative emissions, thus $\tau_S(S) \leq 0$, then:*
(a) if there are $n > 1$ resource owners, the tax induces a flatter (steeper) extraction path R^i of the i-th resource owner than in the social optimum if

$$\tau_S(S) \underset{(<)}{>} \frac{-d_S(S) - r\tau(S)}{\sum_{j=1, j\neq i}^{n} R^j} \tag{41}$$

(b) if there are $n > 1$ resource owners, the tax rule

$$\tau(S) = \frac{-d_S(S)}{r} \tag{42}$$

leads to a steeper (flatter) resource price path compared to the optimal extraction if damages are strictly convex (concave).

(c) Implementing the tax rule (42) leads to the socially optimal Hotelling rule (4) if (i) damages are linear or (ii) if there is only one (competitive) resource owner. In order to meet the socially optimal transversality condition, the regulator has furthermore to commit to the terminal-period payment rule $\varsigma(S(T))$:[8]

$$\varsigma(S(T)) = \frac{d(S(T))}{r} - F(S(T)) \tag{43}$$

The combined rent and tax dynamics is as follows:

$$\lambda(t) + \tau(t) = F_S(S(T))e^{-r(T-t)} - \int_t^T (c_S(S^*)R^* + d_S(S^*))e^{r(t-\xi)}\, d\xi \tag{44}$$

Proof. (a) Setting up the Hamiltonian for the i-th resource owner reads $H^i = (p - c^i(S^i) - \tau(S))R^i - \lambda^i R^i$. The first order condition $\lambda^i = p - c^i(S^i) - \tau(S)$ and the equation of motion $\dot{\lambda}^i = r\lambda^i + c^i_s(S^i)R^i + \tau_S(S)R^i$ lead to the Hotelling rule:

$$r = \frac{\dot{p} + \tau_S \sum_{j=1,j\neq i}^n R^j}{p - c^i(S^i) - \tau(S)} = \frac{\dot{p} + r\tau(S) + \tau_S \sum_{j=1,j\neq i}^n R^j}{p - c^i(S^i)} \tag{45}$$

The socially optimal extraction for n heterogeneous resource stocks $S^1, ...S^n$ is derived by solving

$$\max_{R^i} \int_0^T \left(f(R) - \sum_{i=1}^n c^i(S^i)R^i - d(S) \right) e^{-rt}\, dt + F(S(T))e^{-rT} \tag{46}$$

subject to $R = \sum_{i=1}^n R^i, S = \sum_{i=1}^n S^i, \dot{S}^i = -R^i$. The resulting Hotelling rule reads:

$$r = \frac{\dot{f}_R(R) - d_S(S)}{f_R(R) - c^i(S^i)} \tag{47}$$

By comparing (45) with (47), (a) follows.

(b) Substituting (42) into (41) implies that the right-hand-side of (41) equals zero and (b) follows directly from (a).

(c) Substituting (42) into (45) leads to the socially optimal Hotelling rule (47) if damages are linear (i.e. $\tau_S(S) \equiv 0$) or if only one resource owner exists (i.e. $\sum_{j=1,j\neq i}^n R^j = 0$). In the case of one (competetive) resource owner, the transversality condition under a payment rule $\varsigma(S)$ reads $\lambda(T) = \varsigma_S(S(T))$. Together with the first-order condition, this leads to $\varsigma_S(S(T)) + \tau(S) = p(T) - c(S(T))$. Substituting (43), we obtain the socially optimal terminal condition (6).

Solving the equation of motion for λ , we obtain:

$$\lambda(t) = \lambda(T)e^{-r(T-t)} - \int_t^T c_S(S)Re^{-r(\xi-t)}d\xi - \int_t^T \tau_S(S)Re^{-r(\xi-t)}d\xi \tag{48}$$

Using partial integration for $\int_t^T \tau_S(S)Re^{-r(\xi-t)}d\xi$ yields:

$$\lambda(t) + \tau(S) = (\lambda(T) + \tau(S(T)))e^{-r(T-t)} - \int_t^T c_S(S)Re^{-r(\xi-t)}d\xi + r\int_t^T \tau(S)e^{-r(\xi-t)}d\xi$$

Substituting the tax rule $\tau(S) = -d_S(S)/r$ and using $F_S(S(T)) = \lambda(T) + \tau(T)$ due to the final payment rule, we finally obtain (44)

□

[8] For the infinite time horizon there is no additional final period payment rule necessary as long as marginal damages are not increasing without bound (i.e. $-d_S(S) < \infty$ for $S \geq 0$; as fossil resources are finite, maximal marginal damages are $-d_S(0)$). The transversality condition in the decentralized economy reads $\lim_{t\to\infty} \lambda(t)S(t)e^{-rt} = 0$. If marginal damages are bounded, $\lim_{t\to\infty} S(t)\tau(t)e^{-rt} = 0$ and therefore $\lim_{t\to\infty}(\lambda(t) + \tau(t))S(t)e^{-rt} = 0$. As $\lambda + \tau = \lambda^*$, the socially optimal transversality holds.

The insight from Proposition 5a is that regulators who adjust the carbon tax with the actually observed cumulative emissions can induce an accelerated or postponed resource extraction, depending on the tax rule $\tau(S)$. If the tax rule reacts strongly on S, i.e. if $|\tau_S(S)|$ is large, an accelerated extraction becomes more likely. Proposition 5b transfers this to a specific tax rule (42) which turns out to be optimal under certain (restrictive) conditions in the last part of Proposition 5. Proposition 5c describes how the intertemporal climate externality is successfully shifted to the resource owner who internalizes the stock-pollutant dynamics and calculates the optimal intertemporal allocation. In this case, the informational requirements for the regulator are relatively low: The regulator neither needs to know the optimal stock size $S^*(t)$ nor marginal productivity or extraction costs of resources along the optimum in advance. She only has to know the functional form of the damage function $d(S)$ and to commit to the *tax rule* and terminal-period *payment rule*. The resulting tax depends entirely on the cumulative extraction chosen by the resource owner – which is easily observable by the regulator by measuring the atmospheric carbon concentration. Hence, this tax rule allows unburdening the regulator completely from the task of finding an intertemporally optimal extraction path.

In contrast to the previous analysis, heterogeneity of resource owners becomes crucial under feedback policy rules.[9] If there are several resource owners, such a tax rule suffers from an additional externality between different resource owners. If damages are convex, a high aggregated stock S leads to a low resource tax which benefits all resource owners in the same way. Thus, if the i-th resource owner postpones extraction, all resource owners will benefit from lower resource taxes. At the same time, he has to carry an elevated tax burden for these resources caused by all resource owners together. Hence, he has an incentive to extract as fast as possible (as long as taxes are low). Proposition 5 gives an explanation, how the anticipation of an announced tax rule can lead to inefficient extraction paths due to a coordination problem in the resource sector. If damages are convex and the tax increases with cumulative extraction (i.e. $\tau_S < 0$), resource owners will react with an accelerated extraction which can lead to higher climate damages if the cumulative amount of extracted resources is not reduced appropriately. In order to overcome the coordination problem of the resource sector responding to a tax which is adjusted to the aggregate stock size S, the regulator has to link the tax rule to the *individual* resource stock of each resource owner:

Proposition 6. *(Individually adjusted optimal stock-dependent taxes) If there are n identical resource owners (i.e. with the same extraction cost function and initial resource stock) and the regulator announces to the i-th resource owner the resource tax rule $\tau^i(S^i)$ and the terminal-period payment rule $\varsigma^i(S^i)$*

$$\tau^i(S^i) = \frac{-d_S(nS^i)}{r} \tag{49}$$

$$\varsigma^i(S^i(T)) = \frac{1}{n}\left(\frac{d(nS^i(T))}{r} - F(nS^i(T))\right) \tag{50}$$

which depends explicitly on the i-th resource owners' cumulative extraction S^i, resource owners extract along the socially optimal extraction path.

Proof. The proof follows directly from the proof of Proposition 5. The individual tax rule leads for each resource owner to the Hotelling rule (cf. Eq. 45)

$$r = \frac{\dot{p} + r\tau^i(S^i)}{p - c(S^i)} = \frac{\dot{p} - d_S(nS^i)}{p - c(S^i)} \tag{51}$$

As all resource owners are identical, $S = nS^i$ and thereby the socially optimal Hotelling rule (47) applies. The terminal-period payment guarantees the socially optimal transversality condition. □

[9] Appendix B provides a short proof that the Pigouvian tax does also hold for heterogeneous resource owners.

The tax rule extrapolates the stock-damage caused by each resource owner's extraction behaviour through multiplication with factor n. Although each resource owner only causes the fraction $1/n$ of social damage, he internalizes the entire stock-pollutant dynamic as if timing and extend of the externality would solely depend on himself. The underlying assumption of identical resource owners is still very restrictive. In the more realistic case of heterogeneous resource owners, there is no simple tax rule that internalizes the stock externality appropriately (and without knowing already the socially optimal allocation (R^*, S^*)). The reason is that the share of each resource owner's cumulative extraction S^i on total cumulative extraction S is in general not constant. This makes it impossible to determine the contribution of individual resource owners to global damages (as in (49)) without using information about other resource owners' extraction paths.

2.5 Further aspects: Performance of tax and quantity instruments under suboptimal discount rates

Due to the intertemporal dynamics of the problem, however, discount rates of agents and of the society play a crucial role. In particular, when property rights for resources are insecure, capital or futures markets are incomplete, or regulatory uncertainty to imperfect commitment of governments, agents' effective discount rate could be higher than in the representative-household economy [eg. Sinn 2008]. The choice of discount rates from the social planners perspective can also to be discussed in a normative framework [eg. Stern 2007, Heal 2009]: while individuals discount utility during their life-time due to impatience or uncertainty over the individual's income stream, the society might use a different rate to discount the utility of generations in the far-distant future. Hence, resource owners might use an additional discount markup υ which leads to the discount rate $r + \upsilon$. There is, however, only little work on correcting suboptimal discount rates (despite from the approach to influence interest rates as proposed by Sinn [2008]). In this subsection we therefore study to what extent the previously described policy instruments are capable to simultaneously correct discount rate mark-ups and the climate externality.

If suboptimal discount rates distort the intertemporal allocation, the optimal tax and ETS policy has to be adjusted according to:

Proposition 7. *(Suboptimal discount rates) If the resource sector discounts profits with rate $r+\upsilon$ which differs from the discount rate r of the social planner's problem, then: (a) the optimal resource tax from Proposition 2 has to be modified according to*

$$\tau(t) = F_S(S^*(T))e^{-(r+\upsilon)(T-t)} - \int_t^T d_S(S^*)e^{(r+\upsilon)(t-\xi)}\, d\xi + \upsilon \int_t^T (p^* - c(S^*))e^{(r+\upsilon)(t-\xi)}\, d\xi \tag{52}$$

and (b) the efficiency of the ETS without banking is not affected. The shadow price for permits, however, changes according to:

$$\theta = F_S(S^*(T))e^{-r(T-t)} - \int_t^T d_S(S^*)e^{r(t-\xi)}\, d\xi + \int_t^T c_S(S^*)R^* \left(e^{(r+\upsilon)(t-\xi)} - e^{r(t-\xi)}\right)\, d\xi \tag{53}$$

In particular, θ increases in υ for $0 \le t < T$.

Proof. (a) See Appendix C.1. (b) The permit path $C(t)$ enforces the resource extraction path $R(t) = C(t)$ as permits are scarce (Assumption 2). Thus, the final price for resources $p^* = p(R^*)$

and the marginal extraction costs $c(S^*)$ follow the socially optimal path. The shadow price for resources λ, however, changes to:

$$\lambda(t) = -\int_t^T c_S(S^*)R^* e^{(r+\upsilon)(t-\xi)}\ d\xi \tag{54}$$

With (27), it follows that $\theta = p^* - c(S^*) - \lambda$. For $p^* - c(S^*)$ we can substitute the right-hand-side of Eq. (35) which gives us together with (54) the shadow price for θ under the discount rate $r + \upsilon$ (53). Finally, for $0 \leq t < T$:

$$\frac{\partial \theta}{\partial \upsilon} = \upsilon \underbrace{\int_t^T (t-\xi) c_S(S^*) R^* e^{(r+\upsilon)(t-\xi)}\ d\xi}_{>0} \tag{55}$$

as $c_S(S^*) < 0$.

□

Proposition 7 states that the resource tax has to change in order to achieve the optimal allocation while the permit path of the ETS without banking and borrowing remains unaffected. As long as the permit constraint is binding only the user cost for permit scarcity is affected. The higher the private discount rate markup $\upsilon > 0$, the higher is the valuation of the user cost θ. If permits are grandfathered, suboptimal discount rates make no difference in final resource prices. If permits are auctioned, the resource sector's willingness to pay for permits changes due to the modified user costs. Although suboptimal discount rates do not change the efficient extraction path, they lead to a different climate rent level. Suboptimal discount rates in an ETS with banking and borrowing, however, are hard to correct as they affect both intertemporal arbitrage conditions for the permit as well as the resource path. The solution of Leiby and Rubin [2001] to simply add υ to the ITR γ does not lead to an optimal allocation if the fossil resource supply is integrated. Nevertheless, a higher ITR should always give an incentive to postpone permit and resource use. For the stock-dependent tax rule derived in Proposition 6 we could not find a modification that eliminates the impact of higher discount rates. Although both instruments, the resource tax and the ETS without banking and borrowing, can achieve the optimal allocation even under distorted discount rates, the informational requirements differ: For the tax policy, the regulator has additionally to know the discount rate markup υ that resource owners actually use.

3 The carbon-budget-approach

The cost-benefit-approach requires a balancing of the damages from the use of fossil resources against the opportunity costs of postponed resource extraction. Quantifying the damages of climate change, however, is a difficult and controversial task. There are deep uncertainties in the climate system, in regional market and non-market impacts and in normative parameters like discount rates, risk aversion or assumed substitution possibilities between physical capital and ecosystem services. Furthermore, tipping points in the earth system can lead to irreversible, abrupt and catastrophic impacts when certain thresholds of the temperature increase are crossed [Lenton et al. 2008]. The controversy in the choice of many normative and uncertain parameters and the complexity of the damage dynamics might explain why many policy makers rather define concentration or temperature targets like the two-degree target. Although there is no scientific agreement on the two-degree target, there is an ongoing discussion within economics whether the possibility of catastrophic risks allows for applying cost-benefit analysis to climate change [Posner 2004, Weitzman 2009; 2010]. Therefore, some researchers argue that defining emission targets or carbon budgets might be at least a pragmatic approach to reduce the likelihood of catastrophic risks substantially. As Meinshausen et al. [2009] show, achieving such a temperature target like the two-degree target depends mainly on the cumulative emissions until 2050. Hence, a more practical way of communicating and negotiating climate targets could be based on (global or national) caps for cumulative emissions – a so-called *carbon budget* [WBGU 2009]. The carbon budget approach, however, does not directly imply a choice between different policy instruments in order to achieve the temperature limit in a cost-effective way. The purpose of this part is to clarify the precise requirements for the design of policy instruments.

3.1 The social planner economy

Implementing a carbon budget CB for cumulative extraction is only meaningful, if it enforces a binding constraint. We formulate a similar, albeit more general assumption than Assumption 2:

Assumption 3. *(Scarcity of the carbon budget) Cumulative extraction in the absence of the carbon budget (BAU) exceeds the carbon budget:*

$$CB < \int_0^T R^B \, dt < S_0 \tag{56}$$

A socially optimal allocation under the carbon budget approach is described by a small modification of the social planner economy formulated in Sec. 2.1. We remove the damage and scrap value terms and add instead the carbon budget constraint to the intertemporal optimization problem:

$$\max_R \int_0^T (f(R) - c(S)R)\, e^{-rt} \, dt \tag{57}$$

subject to:

$$\dot{S} = -R \tag{58}$$
$$\dot{C} = -R \tag{59}$$
$$S(0) = S_0 \tag{60}$$
$$C(0) = CB \tag{61}$$

The optimal allocation is described by:

Proposition 8. *(Socially optimal resource extraction) If a social planner maximizes intertemporal output according to (57–61), then: (a) The optimal solution $(R^\star, S^\star)$ is determined by the following system of equations:*

$$r = \frac{\dot{f}_R(R^\star)}{f_R(R^\star) - c(S^\star)} \tag{62}$$
$$\dot{S}^\star = -R^\star \tag{63}$$
$$S(0) = S_0 \tag{64}$$
$$S(T) = S_0 - CB \tag{65}$$

(b) The shadow prices $\lambda^\star$ and $\mu^\star$ for S and C, respectively, are given by:

$$\lambda^\star(t) = -\int_t^T c_S(S^\star) R^\star e^{r(t-\xi)} \, d\xi \tag{66}$$
$$\mu^\star(t) = \mu_T^\star e^{-r(T-t)} \tag{67}$$

where $\mu_T^\star = f_R(R^\star(T)) - c(S^\star(T)) = f_R(R^\star(T)) - c(S_0 - CB)$.

Proof. (a) We set up the corresponding Hamiltonian function $H = f(R) - c(S)R - \lambda R - \mu R$. Applying the maximum principle leads to the following first-order and transversality conditions:

$$\lambda + \mu = f_R(R) - c(S) \tag{68}$$
$$\dot{\lambda} = r\lambda + c_S(S)R \tag{69}$$
$$\dot{\mu} = r\mu \tag{70}$$
$$0 = \lambda(T)S(T) \tag{71}$$
$$0 = \mu(T)C(T) \tag{72}$$

Differentiating (68) with respect to time and rearranging with (69) and (70), we obtain the social Hotelling rule (62). Assumption 1 and (71) imply that $\lambda(T) = 0$. As shown in Appendix D, Assumption 3 implies that the entire budget is consumed, i.e. $C(T) = 0, \mu(T) > 0$ and, hence, $S(T) = S_0 - CB$. (b) Solving (69) with $\lambda(T) = 0$, we obtain (66). From $\lambda(T) = 0$ and (68) follows $\mu(T) = f_R(T) - c(S(T))$ – and with (70) we get (67).

□

Hence, the optimal allocation under a carbon budget simply follows the Hotelling rule. The transversality condition is determined by the size of the carbon budget. The rent can be decomposed into a term reflecting increasing extraction costs λ and a term reflecting the scarcity of the carbon budget μ which we also denote as *climate rent.* Resembling the results of the cost-benefit framework, it will turn out that policies have to generate this climate rent term in order to achieve the optimal allocation.

3.2 Resource taxes in a decentralized market economy

As the decentralized market dynamics equals the one described in the CBA Sec. 2.2, we merely restate the private Hotelling rule and the terminal condition:

$$r = \frac{\dot{p} - \dot{\tau} + r\tau}{p - c(S)} \tag{73}$$
$$\tau(T) = p(T) - c(S(T)) \tag{74}$$

Proposition 9. *(Optimal resource tax) If the regulator knows $\mu_T^\star$ (according to Proposition 8) and if she can commit at $t = 0$ to the tax path $\tau(t)$ over the entire planning horizon, then (a) the resource tax*

$$\tau(t) = \mu_T^\star e^{-r(T-t)} \tag{75}$$
$$\mu_T^\star = f_R(R^\star(T)) - c(S_0 - CB), \tag{76}$$

where $R^\star(T)$ denotes the final resource extraction from the social planner optimum (Proposition 8), achieves the optimal extraction path. (b) The rent in the resource sector is given by:

$$\lambda(t) = -\int_t^T c_S(S)Se^{r(t-\xi)}\, d\xi \tag{77}$$

Proof. (a) Plugging τ from (75) and its derivative into the private Hotelling rule (73) and utilizing the fact that in the market equilibrium $p = f_R$, we obtain the social Hotelling rule (62). The transversality condition of the decentralized resource sector (74) implies that $p(T) - c(S(T)) = \mu_T^{CB}$ which equals the social transversality condition derived in Proposition 8. Hence, $S(T) = S_0 - CB$. (b) Same proof as in Proposition 2 (b).

□

Similar to the cost-benefit framework, the regulator faces the same informational requirements in the social planner as well as in the decentralized market economy. The optimal resource tax is a pure budget scarcity price that reflects the scarcity of the (exhaustible) carbon budget according to the Hotelling rule. There is only a rent for reserves with low extraction costs (which diminishes if extraction costs are constant). Hence, the climate rent term μ from the social planner economy equals exactly the resource tax τ.

3.3 Emissions trading scheme

3.3.1 Emissions trading without banking and borrowing

An optimal emissions trading scheme where permits cannot be banked or borrowed is described by:

Proposition 10. *(Optimal ETS without banking) If the regulator issues permits $C(t) = R^\star(t)$ along the socially optimal extraction path of Proposition 8, then (a) the optimal extraction is achieved, (b) the resource rent is given by $\lambda + \theta$ according to:*

$$\lambda(t) = -\int_t^T c_S(S)Re^{r(t-\xi)}\, d\xi \tag{78}$$

$$\theta(t) = \mu_T^\star e^{-r(T-t)} \tag{79}$$

$$\mu_T^\star = f_R(R^\star(T)) - c(S_0 - CB) \tag{80}$$

Proof. The proof is along the lines of the proof of Proposition 3. □

Again, Proposition 10 requires that the regulator can calculate the socially optimal resource extraction path for the entire time horizon. The informational requirements are not lower than in the social planner economy or under a resource tax policy. The shadow price for permits θ (which would be observed on a market for tradable permits) equals the optimal tax in each period. Similar to the previous section where we studied CBA compatible instruments, we denote the scarcity price for carbon θ as *climate rent.* The regulator could absorb this rent by auctioning permits or she could shift this rent to resource owners by a grandfathering scheme.

3.3.2 Emissions trading with banking and borrowing

Alternatively, the regulator can allocate the permits from the carbon budget in the first period to the resource owners and allow for intertemporal flexibility when to use the permits. As objective function and constraints equal those of the social planner problem, the market reproduces the socially optimal solution:

Proposition 11. *(Optimal ETS with banking) If the regulator issues CB permits in the initial period which can be banked by resource owners, then (a) the optimal extraction is achieved, (b) the resource rent is given by $\lambda + \theta$ according to:*

$$\lambda(t) = -\int_t^T c_S(S)Re^{r(t-\xi)}\, d\xi \tag{81}$$

$$\theta(t) = \mu_T^\star e^{-r(T-t)} \tag{82}$$

$$\mu_T^\star = f_R(R^\star(T)) - c(S_0 - CB) \tag{83}$$

Proof. (a) and (b) follow directly from Proposition 8 with resource rent $p - (c(S)) = \lambda + \theta$ and $\theta = \mu$. □

The initial permit price θ_0 has to be set at the level which equals cumulative permit (or resource) demand under the carbon budget CB. As it turns out, the problem is equivalent to the emission tax problem (75) and $\theta_0 = \tau_0$. But in contrast to the taxation scheme, the market has to determine $\theta_0 = \mu_T^\star e^{-rT}$ by estimating the demand function and the extraction cost curve. This, however, requires a complete set of future markets to achieve an intertemporal market equilibrium [Dasgupta and Heal 1979, pp. 100–110]. The regulator could issue permits for free (e.g. in a grandfathering mode to resource owners) or sell them at maximum price $\theta(t)$ – thus she can divide the scarcity rent in a non-distortionary way between several economic actors. As the regulator may not estimate $\theta(t)$ correctly, she could auction the entire permit stock in the first period. The rent left to the resource owner then reduces to λ.

3.4 Further aspects: Performance of tax and quantity instruments under suboptimal discount rates

Equal to the analysis in the cost-benefit section, we briefly study how suboptimal discount rates influence the performance of the previously studied policy instruments.

Proposition 12. *(Suboptimal discount rates) If the resource sector discounts profits with rate $r + \upsilon$ which differs from the discount rate r of the social planner's problem and if the regulator furthermore knows the socially optimal extraction and price paths $S^\star, R^\star, p^\star$ and $\mu_T^\star$ of Proposition 8, then: (a) The optimal allocation can be achieved if the resource tax from Proposition 9 is modified according to*

$$\tau(t) = \mu_T^\star e^{-(r+\upsilon)(T-t)} + \upsilon \int_t^T (p^\star - c(S^\star)) e^{(r+\upsilon)(t-\xi)} \, d\xi \tag{84}$$

(b) The efficiency of the ETS without banking is not affected; the shadow price for permits, however, changes according to:

$$\theta(t) = \mu_T^\star e^{-r(T-t)} + \int_t^T c_S(S^\star) R^\star \left(e^{(r+\upsilon)(t-\xi)} - e^{r(t-\xi)} \right) \, d\xi \tag{85}$$

In particular, θ increases in υ. (c) Under the ETS with banking and borrowing, the optimal allocation can be achieved if the the regulator introduces an additional resource tax according to:

$$\tau(t) = \upsilon \int_t^T (p^\star - c^\star(S)) e^{(r+\upsilon)(t-\xi)} \, d\xi \tag{86}$$

Proof. For (a) and (c) see Appendix C.2 and C.3; (b) follows basically along the lines of the proof of Proposition 7 (b). □

If the discount rate in the resource sector exceeds the social discount rate ($\upsilon > 0$), the resource tax has to increase at a lower rate compared to the case where $\upsilon = 0$ in order to provide an incentive for future extraction. In line with the findings obtained in the CBA framework, the ETS without banking and borrowing is the most robust instrument – as long as the regulatory institution uses the 'right' discount rate. In the latter case, suboptimal discount rates only affect the shadow price for permits and, thus, the distribution of the permit rent in case permits are auctioned by the regulator. In particular, the optimal permit price does not increase at a constant rate and is therefore not consistent with intertemporal maximization of the permit rent. This is the reason why an ETS with banking and borrowing is suboptimal: High discount rates of permit owners lead to a steeper permit price path and, thus, to an accelerated extraction. Within the banking-and-borrowing ETS, the regulator additionally has to tax resource extraction. This, however, requires the regulator to have all the necessary information on optimal timing and demand for resources for the entire time horizon.

	Informational requirement for government	Commitment requirement for government	Robustness against suboptimal discounting
Cost-benefit approach:			
Tax path	high	high	low
Tax rule	low	high	low
ETS w/o banking	high	low	high
ETS with banking	high	high	low
Carbon budget approach:			
Tax path	high	high	low
ETS w/o banking	high	low	high
ETS with banking	low	high	low

Table 1: Comparison of policy instruments.

4 Conclusion

Table 1 summarizes the main findings of our paper. Our analysis has emphasized that in a deterministic world price and quantity instruments can differ with respect to the distribution of informational requirements between market and regulator and their robustness against discount rate mark-ups. In particular, the cost-benefit approach has to deal with more complex intertemporal rent dynamics as the carbon budget approach due to its aim to allocate climate damages efficiently in time.

Due to the complexity of the stock-pollutant problem markets are hardly able to manage the climate rent intertemporally in an efficient way without substantial government intervention. It seems to be unavoidable to entrust a regulatory institution with the challenging task to find an extraction path that is close to the social optimum. This requires perfect information about the availability of fossil resource, extraction costs, climate damages, fossil resource demand as well as the availability of backstop technologies. In contrast to static environmental problems an iterative adjustment process for the carbon tax will hardly be able to approach the optimal emission level. Adapting carbon taxes to observed cumulative emissions can indeed induce an accelerated extraction (green paradox). In the case of rather homogeneous resource owners or linear stock-dependent damages, such a stock-dependent tax rule (together with a final-period payment rule) leads to an optimal emission level within the cost-benefit framework. In this case, the regulator is completely discharged from the task to calculate and to control optimal emission pathways. She does only need to know the functional form of climate damages and a credible commitment to the tax and final-period payment rule.

In the carbon-budget approach, only an intertemporally flexible permit trade could dispense the regulator from finding the intertemporally efficient extraction path. All other instruments rely crucially on the performance of the regulatory institution to implement an intertemporally efficient allocation plan. However, delegating the task of intertemporal optimization to decentralized agents in a market environment requires a complete set of future markets. Until now, future markets for commodities or resources have not been established for planning horizons of many decades or even an entire century. Existing future markets are often thin and suffer from volatile prices due to high uncertainties and speculations. Hence, it is questionable whether intertemporally flexible permit markets are really capable to find an optimal allocation.

Intertemporal optimality crucially depends on the discount rate that economic agents use for their investment and saving decisions. Insecure property rights, incomplete future markets and uncertainty about climate policies lead to high risk premiums which are added to the discount rate of resource owners. Furthermore, the discount rate itself (as derived from the Ramsey rule) may be too high from a normative point of view. Theoretically, resource taxes could cure this additional market failure but depend on the precise evaluation of the extraction dynamics which becomes even more complex when the regulators discount rate differs from the market discount rate. In contrast, an emissions trading schemes without intertemporal flexibility for resource owners always ensures an optimal extraction path regardless of the discount rate used by resource owners. If the supply of permits is lower than the resource supply under the absence of climate policy, resource owners will always extract resources up to the emission cap: No under-supply can occur. In this case, suboptimally high discount rates do only increase the permit price and reduce the scarcity rent associated with the in-situ resources.

These considerations indicate the need for an institution enabling a reasonable intertemporal management of the climate rent which takes into account the behavior of the owners of exhaustible resources. As all efficient policy instruments are in general non-constant in time, a continuous adjustment of taxes or emissions caps is necessary. Without a strong and credible commitment, such revisions provoke high transaction costs due to ongoing rent-seeking activities of affected actors. Hence, a carbon bank or atmospheric trust as proposed by [Barnes et al. 2008] could – similar to central banks – improve the commitment to a long-term climate policy. If such an institution is capable to solve the intertemporal optimization problem, it could implement optimal emission caps or carbon taxes. If such an institution is not able to solve this problem, a pragmatic second-best approach would be to commit only to a fixed cumulative emissions budget. In this case, finding the intertermporal efficient outcome is delegated to the market. Even if the carbon budget cannot be derived by a cost-benefit analysis, there are reasons to apply emission targets when catastrophic risks can occur with a very low likelihood. Then, an intertemporally efficient allocation of permits within the budget is obtained if complete futures markets can be developed. The choice between quantity and tax instruments depends on empirical assumptions about the capability of governments to long-term commit compared to the capability of markets to ensure intertemporal efficiency.

Appendix

A CBA-ETS with banking and borrowing

The quantity trading ratio changes the effective volume of emissions through banked permits b by rate $\gamma(t)$. The optimization problem for the resource sector with initial conditions $S_0 = S(0)$ and $b_0 = b(0)$ reads:

$$\max_R \int_0^T (p - c(S))Re^{-rt}dt \tag{87}$$

$$\dot{S} = -R \tag{88}$$

$$\dot{b} = -R + \gamma b \tag{89}$$

From the the Hamiltonian $H = (p-c(S))R-\lambda R-\mu(R-\gamma b)$ we obtain the first order-conditions

$$\lambda = p - c(S) - \mu \tag{90}$$

$$\dot{\lambda} = r\lambda + c_S(S)R \tag{91}$$

$$\dot{\mu} = r\mu - \gamma\mu \tag{92}$$

and the transversality conditions:

$$S(T)\lambda(T) = 0 \tag{93}$$

$$b(T)\mu(T) = 0 \tag{94}$$

In the following, we derive the optimal value for b_0 and the optimal *policy trajectory* for $\gamma(t)$ that guarantees a socially optimal solution as characterized in Proposition 1.

A.1 The optimal intertemporal trading rate $\gamma(t)$

Differentiating (90) and substituting (92), we obtain:

$$\dot{\lambda} = \dot{p} + c_S(S)R - (r - \gamma)\mu \tag{95}$$

Equating (95) with (91) and using (90) yields:

$$\dot{p} = r(p - c(S)) - \gamma\mu \tag{96}$$

The socially optimal price path, however, from (4) is given by:

$$\dot{p} = r(p - c(S)) + d_S(S) \tag{97}$$

By equating (97) with (96) and using (92), we obtain:

$$-d_S(S) = \gamma\mu = r\mu - \dot{\mu} \tag{98}$$

Solving for μ, we get $\mu(t) = e^{rt}\int_0^t d_S(S)e^{-r\xi}d\xi + \mu_0 e^{rt}$. For known $\mu(T)$ we can calculate $\mu_0 := \mu(0) = -\int_0^T e^{-r\xi}d_S(S)\ d\xi + \mu(T)e^{-rT}$ and obtain for μ:

$$\mu = \mu(T)e^{-r(T-t)} - \int_t^T d_S(S)e^{r(t-\xi)}d\xi \tag{99}$$

Now, we can calculate γ by using (98) and (99):

$$\gamma = \frac{-d_S(S)}{\mu} = \frac{-d_S(S)}{\mu(T)e^{-r(T-t)} - \int_t^T d_S(S)e^{r(t-\xi)}d\xi} \tag{100}$$

For $S(T) > 0$, the transversality condition (93) implies $\lambda(T) = 0$ and with (90) $\mu(T) = p(T) - c(S(T))$. As in the optimum $f_R(R(T)) - c(S(T)) = F_S(S(T))$ (see Proposition 1), it follows with $p = f_R$ that $\mu(T) = F_S(S(T))$ for the social optimum.

A.2 The optimal initial permit stock b_0

Solving (89) yields

$$b(t) = e^{\int_0^t \gamma d\xi} \int_0^t \left(-Re^{-\int_0^\xi \gamma du}\right) d\xi + b_0 e^{\int_0^t \gamma d\xi} \tag{101}$$

By using the substitution $\phi := -\int_0^t \mu_0^{-1} e^{-rs} d_s(S)ds - 1$ we can re-write γ as $\gamma = \frac{\partial}{\partial t}(-\ln(-\phi)) = -\frac{\dot{\phi}}{\phi}$ and obtain for (101): $b(t) = \frac{-1}{\phi(t)}\left(\int_0^t \phi(\xi)R(\xi)\ d\xi + b_0\right)$. For given terminal value $b(T)$, we can calculate b_0 as follows:

$$b_0 = -b(T)\phi(T) - \int_0^T \phi(\xi)R(\xi)\ d\xi = -b(T)\phi(T) + \int_0^T \phi(\xi)\dot{S}(\xi)\ d\xi \tag{102}$$

For an optimal solution, $\mu(T) > 0$ for as otherwise the trading ratio $\gamma(T)$ in (100) is not defined. From the transversality condition (94) then follows that $b(T) = 0$. Using integration by substitution and the definition of ϕ, we get:

$$b_0 = \phi(T)S(T) - \phi(0)S(0) - \int_0^T \dot{\phi}(\xi)S(\xi)\ d\xi \tag{103}$$

$$= \phi(T)S(T) + S_0 + \frac{\int_0^T e^{-r\xi} d_S(S)S\ d\xi}{\mu_0} \tag{104}$$

By plugging in $\phi(T)$ and μ_0 from above, the initial permit stock is finally described by:

$$b_0 = S_0 + S(T)\frac{-\mu(T)e^{-rT}}{-\int_0^T e^{-r\xi} d_S\ d\xi + \mu(T)e^{-rT}} + \frac{\int_0^T e^{-r\xi} d_S(S)S\ d\xi}{-\int_0^T e^{-r\xi} d_S(S)\ d\xi + \mu(T)e^{-rT}} \tag{105}$$

$$= S_0 + \frac{\int_0^T e^{-r\xi} d_S(S)S\ d\xi - S(T)\mu(T)e^{-rT}}{-\int_0^T e^{-r\xi} d_S(S)\ d\xi + \mu(T)e^{-rT}} \tag{106}$$

A.3 The resource rent

The rent π in the resource sector is determined by resource prices minus extraction costs, i.e. $p - c(S)$. From (90) follows that $\pi = \lambda + \mu$. With the solution of the differential equation for λ (91) and the equation for μ (99),we obtain $\pi = (\mu(T) + \lambda(T))e^{-r(T-t)} - \int_t^T (d_S + c_S R)e^{r(t-\xi)}\ d\xi$

B Optimality of Pigouvian taxes for heterogeneous resource owners

The Hotelling rule for a given tax *path* $\tau(t)$ in a heterogeneous resource model (see Eq. (45)) reads

$$r = \frac{\dot{p} - \dot{\tau} + r\tau}{p - c^i(S_i)}$$

Substituting the Pigouvian tax path (21) gives

$$r = \frac{\dot{p} - d_S(S)}{p - c^i(S_i)}$$

which is identical to the socially optimal Hotelling rule for heterogeneous resource owners (47).

C Suboptimal discount rates

C.1 Optimal resource tax in the cost-benefit approach

If the resource sector uses the discount rate ρ instead of the socially optimal discount rate r, the re-arranged private Hotelling rule (19) reads:

$$\rho(p - c(S)) = \dot{p} - \dot{\tau} + \rho\tau \tag{107}$$

The re-arranged socially optimal Hotelling rule (4) with $p = f_R$ in the market equilibrium is:

$$r(p^* - c(S^*)) = \dot{p}^* - d_S^* \tag{108}$$

Substituting $\dot{p}$ from (108) into (107), we obtain for the optimal solution $\dot{\tau} = \rho\tau + d_S^* + (r - \rho)(p^* - c(S^*))$. Solving the ODE for given $\tau(T)$ yields:

$$\tau(t) = \tau(T)e^{-\rho(T-t)} - \int_t^T d_S^* e^{\rho(t-\xi)}\ d\xi - (r - \rho)\int_t^T (p^* - c(S^*))e^{\rho(t-\xi)}\ d\xi \tag{109}$$

In order to achieve the social transversality condition (11), we set $\tau(T) = F_S(S^*(T))$.

C.2 Optimal resource tax under a carbon budget without ETS

Under the budget approach applies the private Hotelling rule from (107). The re-arranged socially optimal Hotelling rule (62), however, does not contain a damage term and reads with $p = f_R$:

$$r(p - c(S)) = \dot{p} \tag{110}$$

Substituting $\dot{p}$ from (110) into (107) and solving the ODE for given $\tau(T)$, we obtain:

$$\tau(t) = \tau(T)e^{-\rho(T-t)} - (r - \rho)\int_t^T (p^* - c(S^*))e^{\rho(t-\xi)}\, d\xi \tag{111}$$

In order to achieve the social transversality condition within the budget approach, we set $\tau(T) = \mu_T^{CB}$.

C.3 Optimal resource tax under a carbon budget with ETS

Under the ETS with banking and borrowing, we have to consider the Hotelling rules (110) and (107) which yields to the same formula for the optimal tax as (111) without ETS. The social transversality condition, however, is already achieved by the limited permit stock, implying $\tau(T) = 0$, and thus:

$$\tau(t) = -(r - \rho)\int_t^T (p^* - c(S^*))e^{\rho(t-\xi)}\, d\xi \tag{112}$$

D Exhaustion of the entire carbon budget

Proof for $C(T) = 0$:

Let us assume, that the permit stock is not exhausted, i.e. $C(T) > 0$. From (72) follows that $\mu(T) = 0$ which implies that (with $\lambda(T) = 0$ and (68)) $f_R(R(T)) = c(S(T))$. As in the BAU case $S^B(T) > 0$ and thus, $\lambda^B(T) = 0$, it follows that $f_R(R^B(T)) = c(S^B(T))$ (where x^B denotes the corresponding variable in the BAU-scenario without binding carbon budget constraint, i.e. with $CB = S_0$). Thus, we have:

$$f_R(R(T)) = c(S(T)) \tag{113}$$

$$f_R(R^B(T)) = c(S^B(T)) \tag{114}$$

From Assumption 3 follows that

$$S(T) > S^B(T) \tag{115}$$

Equations (113–115) imply together with $f_{RR} < 0$ and $c_S < 0$ that $R(T) > R^B(T)$. As $\int_0^T R\, dt < \int_0^T R^B\, dt$ and $R(T) > R^B(T)$ there must exist a $t^* : 0 < t^* < T$ with:

$$R(t^*) = R^B(t^*) \tag{116}$$

$$R(t) \geq R^B(t) \quad \text{for } t^* \leq t \leq T \tag{117}$$

In particular, this implies $\int_0^{t^*} R\, dt < \int_0^{t^*} R^B\, dt$ and thus (considering $c_S < 0$)

$$c(S(t^*)) < c(S^B(t^*)) \tag{118}$$

The Hotelling rules for the budget and BAU probem read:

$$r = \frac{\dot{f}_R}{f_R - c(S)} = \frac{\dot{f}_R^B}{f_R^B - c(S^B)} \tag{119}$$

Using $\dot{f}_R = f_{RR}\dot{R}$, we get by rearranging (119) in $t = t^*$:

$$\underbrace{\dot{R}(t^*) - \dot{R}^B(t^*)}_{\geq 0 \quad \text{from}(117)} = \underbrace{\frac{r}{f_{RR}}}_{<0} \underbrace{[c(S^B(t^*)) - c(S^B(t^*))]}_{>0 \quad \text{from}(118)} \tag{120}$$

which leads to a contradiction as the right hand side is strictly negative while the left hand side is positive (or zero). Thus, the initial assumption $C(T) > 0$ is not valid and it follows that $C(T) = 0$.

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