Rent taxation, inequality and the cost of housing

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Abstract

Housing costs are an important potential driver of rising inequality. There is a broad literature suggesting that land rent taxation can be used to ameliorate this inequality due to its non-distortionary properties. In practice, only a portion of total residential land is subject to rent taxation, since most countries abstain from taxing homeowners imputed rents. We built a dynamic macroeconomic model that disentangles land and structures from housing and study the welfare implications and distortionary properties of land rent taxation. We find that the lack of imputed rent taxation causes land rent taxation to be distorionary. Opposite to the thought that land rent taxation can be beneficial for tenants, we find that the lack of imputed rent taxation causes land rent taxation to be detrimental to tenants' welfare. This results since increases in the land rent tax rate decrease the implicit cost of land for self-use of homeowners, which increase their demand for land. This in turn raises the housing cost of tenants. To have an idea about the magnitude of these effects we apply the model to the Germany economy.

Keywords: Housing, imputed rent, land rent taxation, home value
JEL: R38 (real estate market, government policy), R31 (housing supply), R21 (housing demand), D31 (personal income, wealth, and their distribution), H31 (fiscal policies, households)

1. Introduction

The wealth gap between homeowners and tenants in Western economies is striking. In the euro area, the median net wealth of homeowners was 23 times larger than the median net wealth of tenants during 2010, 2014 and 2017 (ECB, 2020a-c). In Germany and the US, the wealth gap increased further over time, as the net wealth

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of homeowners increased, while the net wealth of tenants declined. For the US, the gap increased from 37 fold to 44, in Germany the gap went from 22 fold in the year 2014 to 26.6 fold in the year 2017 (Bricker, et al. 2017 and ECB, 2020a-c).

For long, the wealth disparity between landlords and tenants has received the attention of philosophers and scholars alike, often investigating ways to improve the lot of the tenants and aiming for a more equal distribution of wealth. For example, in The Land Question Henry George (1881) proposes a single land rent tax as a means to improve the wellbeing of tenants. George's proposal, backed by the thought that a rent tax on a fixed asset is non-distortionary (refer to Bryson (2011), and Stiglitz (2015)) has led to an extensive line of research studying the use of a single land rent tax to finance the optimal provision of public goods.[1] The premise that a land tax is non-distortionary was pointed out before George by Adam Smith and later confirmed by Ricardo (1821). Feldstein (1977) was the first to challenge the premise that a land rent tax is non-distortionary. Employing a model with imperfect altruism, he showed that land rent taxation fosters capital accumulation and is, therefore, distortionary. Thus, land tax reform following the ideas of George (1881) needs to be considerate of trading off tenants' welfare against distortionary economic effects.

Inspired by policies supporting home ownership a large body of literature has analyzed the effects of housing tax breaks on homeownership rates and tenants and homeowners welfare[2]. A prominent example of these policies is the lack of owner-occupier imputed housing rent taxation in many countries in Europe and the US. Imputed housing rent taxation would treat owners-occupiers as tenants in their own house, making artificial rent payments to themselves subject to income taxation (see Floetotto et al. (2016)).[3] In practice and at large, the tax on housing rental income taxes both the land lot and the structure. Separating the taxation of land and structures is relevant from the point of view that rent taxation is distortionary, as it hinders structure investments. Whereas the distortionary properties of land rent taxation deserve further investigation under current tax systems. To the best of our knowledge, the literature considering the lack of owner-occupier imputed rent taxation ignores that land and structures could be taxed separately. Consequently, it has not disentangled the distributional (and efficiency) effects of land taxation on tenants and homeowners yet. This paper adds to the literature by differentiating the taxation of rented land and the taxation of structures. Accordingly, it analyses the

[1]See for example, Arnott and Stiglitz (1979), Schweizer (1986), Arnott (2004), Fu (2005), Kawano (2014) and Mattauch et al. (2018).

[2]A renascence of housing literature started in 2007 after the sub-prime crisis.

[3]As opposed to what is done in practice George was interested in the potential rent and tax revenues from all land (Andelson (2003)).

	Germany (Thousands of 2017 euros)			Euro Area (Thousands of 2017 euros)			US (Thousands of 2016 dollars)	
Homeowners	236.1	225.9	277.0	233.1	199.4	203.0	201.5	231.4
Tenants	11.3	10.2	10.4	10.1	8.7	9.0	5.5	5.2

Table 1: Median net wealth . Own calculation using data from Bricker, et al. (2017) and ECB (2020a-c)

following questions: what is the effect of the lack of taxing imputed land rents on the non-distortionary properties of land rent taxation? In such a setting, is land rent taxation beneficial for tenants? Most importantly, what are the wealth distributional implications for landlords, owner occupiers, and tenants resulting from an increase in land rent taxes?

To analyze these questions we develop a dynamic model that includes heterogeneous consumers, namely landlords, owner-occupiers and tenants.[4] The economy produces the standard final good that is used for consumption and capital investments. In addition, a sector produces new residential structures, which add up to the undepreciated stock of structures. In the economy there is also a fixed land endowment. Landlords and owner-occupiers invest in new structures and optimally trade land and structures at prices that clear the respective markets. Consumers derive utility from the standard final good and also from the residential services that owned or rented land and structures provide. The treatment of land in the model closely resembles the treatment of land in the model developed by Kiyotaki et al (2011). Landlords use portions of their land and structure holdings for own use, and the rest is rented out to tenants at rental prices that clear rental markets. As we focus on analyzing the distributional effects of land rent taxation we consider the case when the sole purpose of the government is to finance a recurrent fixed level of government expenditure.

We find that when the imputed rents of homeowners are not taxed, a tax on income from renting out land is distortionary, in the sense that changes in this tax affect all the aggregate variables of the economy. Within the context of the model, we analytically show that in the absence of imputed rent taxation, the homeowners' per-unit opportunity cost of land for self-use is lower than the land rental price tenants pay. Moreover, the larger the land rent tax rate is, the larger the gap between the homeowners' per-unit opportunity cost of land for self-use and the land rental price (similar arguments apply for the case of structures). An increase in the

[4]With some abuse of language we call owner-occupiers those that own land and structures for self-use only, while we call landlords those that live in owned land and structures but that also rent out land and structures to tenants.

land rent tax rate, thus creates different incentives not only in terms of land use but also in the incentives to increase (or decrease) the stock of structures due to the substitution possibilities between land and structures. Moreover, the increase in the gap of the cost of housing of homeowners and tenants, resulting from an increase in the land rent tax has the potential of having large welfare consequences.

Our numerical simulations for the case of Germany confirm our analytical results. In addition our numerical results indicate that in the absence of imputed rent taxation an increase in the land rent tax rate has a negative effect in tenants welfare. This is the case even when the extra revenues of the increased tax rate are equally distributed across consumers.

The paper is organized as follows. In Section 2 we present the model. In Section 3 we present analytical results regarding the distortionary properties of land rent taxation. In Section 4 we present our numerical results after applying the model to the German economy. Section 5 concludes.

2. The model

We consider a dynamic economy initially endowed with capital, residential structures and land. There are two producing sectors. The first produces the typical final good (identified with subscript F) that is used for consumption and investment in the capital stock of the economy. The second sector constructs residential structures, henceforth only called structures (subscript S). Structures are the man-made part of housing such as walls, roofs and the like, but households combine structures with land (which they either rent or own) to obtain housing services. The stock of structures changes with production and depreciation; in contrast, the aggregate amount of land L is fixed through time. The sum of structures' values and land values equal the value of residential real estate. The separation of structures and land allows to attribute changes in residential real estate value to changes in land and structures values.

Consumers can take three distinct roles: landlord, owner-occupier or tenant.[5] Landlords and owner-occupiers own structures and land for self use, only landlords rent out access real estate to tenants. All consumers are endowed with capital, for tenants, this is their only asset. There is a government that finances a fixed and exogenously given public expenditure by levying taxes on income streams and assets,

[5] For the characterization of the implications of policies on income and wealth of different consumer types, exogenous roles are sufficient. Models with endogenous consumer classes are found in studies on home-ownership rates (cf. Jeske et al (2013), Floetotto et al. (2016), and Sommer (2018))

and possibly lump-sum taxes.

2.1. Firms

The production of final good Y_F and new residential structures Y_S employ capital K_i and labor N_i for $i = F, S$ and use a constant returns to scale Cobb-Douglas technology as follows (for brevity and when there is no room for confusion we omit writing variables as a function of time)

$$Y_i = A_i K_i^{\alpha_i} N_i^{1-\alpha_i}, \tag{1}$$

where $0 < \alpha_i < 1$ and $A_i > 0$. For given prices p_i, q_K and w, the representative firm of sector i maximizes profits $p_i Y_i - q_K K_i - w N_i$ subject to (1). q_K and w are the rental price of capital and the labor wage rate. The final good is presumed numéraire ($p_F = 1$), and p_S denotes the price of structures. The firms' first order conditions reduce to a set of zero profit conditions such that the price of each output equals its marginal cost. Moreover, since the production functions are Cobb-Douglas each input-expenditure/output-value ratio equals the respective input share parameter, e.g. $q_k K_F/(p_F Y_F) = \alpha_F$. Let $mc_S(q_k, w)$ denote the marginal cost of producing residential structures, zero profits imply that $mc_S(q_k, w)$ equals the price of structures p_S and likewise for good F ($mc_F(q_k, w) = 1$).

As we abstract from capital installations costs the firms' problem is static. All intertemporal decisions rest with the consumers, such that consumers' investments in structures determine the scale of production Y_S.

2.2. Consumers

Landlords, homeowners, and tenants are respectively denoted with subscripts LL, O and T. There are z_{LL} landlords, z_O owner-occupiers and z_T tenants. Consumers differ in their type, but, all consumers within one type are presumed to be identical.

Consumer i's discounted utility from final good consumption $c_{F,i}$ and housing services $c_{H,i}$ equals

$$\int_0^\infty \frac{c_i^{1-\theta}}{1-\theta} e^{-\rho t} dt \tag{2}$$

where

$$c_i \equiv A_C c_{F,i}^{\beta_F} (c_{H,i} - \gamma)^{\beta_H} \tag{3}$$

is a Stone Geary composite with A_C, β_F, β_H, $\gamma > 0$, $\beta_F + \beta_H = 1$ and the regularity condition $c_{H,i} > \gamma$ depicting a minimum housing service requirement.[6] $\theta > 0$ is the inverse of the elasticity of intertemporal substitution and $\rho > 0$ is the rate of time preference. In turn, housing services $c_{H,i}$ are obtained by combining rented or owned land $l_{c,i}$ and structures $s_{c,i}$ according to[7]

$$c_{H,i} = A_H l_{c,i}^{\varphi} s_{c,i}^{1-\varphi} \tag{4}$$

where $A_H > 0$ and $0 < \varphi < 1$. The representation of housing as a composite of land and structure allows us to capture that housing is typically bought or rented without differentiation of its components while maintaining flexibility to track the value of land and structure separately, particularly when subject to different kinds of taxation.

At time t the value of the assets of consumer i equal

$$a_i(t) \equiv k_i(t) + \sigma_i \left[p_L(t) l_i(t) + p_S(t) s_i(t) + b_i(t)\right] \quad \text{for} \quad i = LL, O, T \tag{5}$$

where $\sigma_i = 1$ for $i = LL, O$ and $\sigma_T = 0$. We presume that tenants are fully financially constrained such that they cannot borrow to finance their own land and structure, and exclusively save by holding capital $k_T(t)$. The capital stock of consumer i for $i = LL, O, T$ at time t is denoted $k_i(t)$. The land and structures holdings of landlords and owner-occupiers are denoted by $l_i(t)$ and $s_i(t)$; and $p_L(t)$, and $p_S(t)$ are their respective prices. Landlords and owner-occupiers lend $b_i(t)$ to each other at an endogenously determined rate of interest (if $b_i(t) < 0$ then borrowing). The budget constraint of a tenant equals (for brevity we omit writing variables as a

[6]Using US data Davis and Ortalo-Magné (2011) find that, over time and across metro areas, median housing (rental and utilities) expenditure shares are roughly constant. Despite that Davis and Ortalo-Magné (2011) findings "are not without controversy" as Davis and Nieuwerburgh (2015) point out many researchers justify employing a standard Cobb-Douglas utility function based on their findings. Using US data for the year 2015, Larrimore and Schuetz (2017) find that the median renter in the lowest quintile of the income distribution pays 56% of their income on rent, whereas the second quintile pays 28%. They also find that the median renter in the higher quintiles spends less than 20% of their income on rent. Albouy et al. (2016) have similar findings to those of Larrimore and Schuetz (2017). Using German data Dustmann et al. (2018) find that in the year 1993 people in the bottom quintile of the income distribution spent 27% of their income in housing, and 39% in 2013. They also find that for the top quintile this share went from 16% in 1993 to 14% in 2013. We introduce a Stone Geary utility function to account for these diverse findings and to account for the minimum housing requirement established by German regulation.

[7]Following the findings of Epple et al. (2010) who use Pennsylvanian data; and Ahlfeldt and McMillen (2014) who use Berlin, Chicago, and Pittsburgh data

we use a Cobb-Douglas representation for the production of housing services.

	asset flow	expression	T	LL	O
(1)	(taxed) labor income	$w\,(1-\tau_N)\,n_i$	$+$	$+$	$+$
(2)	interest income	$r_B b_i$	Ø	$+$	$+$
(3)	(taxed) capital property value	$(1-\tau_K)\,(q_K-\delta_K)\,k_i$	$+$	$+$	$+$
(4)	(taxed) land value	$(\dot{p}_L-\xi_L p_L)\,l_i$	Ø	$+$	$+$
(5)	receipts from land rented out	$q_L l_r$	Ø	$+$	Ø
(6)	tax on land rent	$\tau_L\,(q_L-\xi_L p_L)\,l_r$	Ø	$-$	Ø
(7)	tax on imputed land rent	$\tau_L\,(q_L-\xi_L p_L)\,l_{c,i}$	Ø	$-$	$-$
(8)	(taxed) structure value	$(\dot{p}_S-(\xi_S+\delta_S)\,p_S)\,s_i$	Ø	$+$	$+$
(9)	receipts from structures rented out	$q_S s_r$	Ø	$+$	Ø
(11)	tax on structure rent	$\tau_S\,(q_S-(\xi_S+\delta_S)\,p_S)\,s_r$	Ø	$-$	Ø
(12)	tax on imputed structure rent	$\tau_S\,(q_S-(\xi_S+\delta_S)\,p_S)\,s_{c,i}$	Ø	$-$	$-$
(13)	demand for final good	$c_{F,i}$	$-$	$-$	$-$
(14)	expenditure on rented land	$q_L l_{c,i}$	$-$	Ø	Ø
(15)	expenditure on rented structures	$q_S s_{c,i}$	$-$	Ø	Ø
(16)	lump-sum taxation	τ_i	$-$	$-$	$-$

Table 2: Income and wealth accumulation. Columns T , LL and O show with which sign ($+/-$) the expression enters the budget equation of tenant (T), landlord (LL) and owner-occupier (O), or not (Ø).

function of time)

$$\begin{aligned}\dot{k}_T =& w\,(1-\tau_N)\,n_T \\ &+(1-\tau_K)\,(q_K-\delta_K)\,k_T \\ &-c_{F,T}-q_L l_{c,T}-q_S s_{c,T}-\tau_T\end{aligned} \tag{6}$$

A dot over a variable denotes its time derivative, i.e. $\dot{k}=\partial k/\partial t$. τ_N is a labor income tax rate and n_T is the labor endowment of a tenant. q_K is the rental price of capital, δ_K is the constant rate of capital depreciation and τ_K is a capital income tax rate. q_L and q_S are the rental prices of land and structures. Thus, the assets of a tenant increase due to labor and capital income net of taxes and capital depreciation. A tenant's assets decrease due to final good consumption expenditure and due to rent expenses on land and structures. Finally, her assets also decrease due to the lump-sum tax τ_T.

Table 2 provides an overview financial flows in the economy, tracking income and expenditure as well as asset valuation and taxation. The tenant's wage income (net of labor tax), capital holding and expenditures for consumption, land and structures rent and lump sum tax (cf. 6) are found in rows 1, 3, 13-16, respectively.

For the budget equation of the landlord, let $l_r\,(t)$ and $s_r\,(t)$, respectively, denote the land and structures that each landlord rents out to tenants at time t. The amount of land and structures that a landlord employs for self use, respectively, equal

$$l_{c,LL}(t) = l_{LL}(t) - l_r(t) \quad \text{and} \quad s_{c,LL}(t) = s_{LL}(t) - s_r(t) \tag{7}$$

and for the case of an owner-occupier

$$l_{c,O}(t) = l_O(t) \quad \text{and} \quad s_{c,O}(t) = s_O(t) \tag{8}$$

so that the amount of land and structures she uses for own housing services equals her holding of land and structures.

Landlord and owner-occupier share the same equations for labor and capital income with the tenant, as well as the expenditure on final good consumption and lump-sum tax. In row 2 of Table 2 the interest rate on lending and borrowing is denoted by r_B. Tax rates on income from renting out land and structures in rows 6 and 7 are, respectively, denoted by τ_L and τ_S, and ξ_L and ξ_S in rows 4 and 6-8 denote property value tax rates. As structures depreciate slower than capital we set a specific rate of depreciation for structures and denote it δ_S. We explain equation (9) for the case of a landlord so that $i = LL$, and $\mu_{LL} = 1$. The term $(\dot{p}_L - \xi_L p_L) l_{LL}$ in row 4 indicates that the assets of a landlord increase when the price of land increases ($\dot{p}_L > 0$), but decrease due to property value taxes ($\xi_L p_L l_{LL}$). In row 5, $q_L l_r$ is a landlord's income from renting out land l_r. Rows 6 and 7 describe the land rent tax on land rented out after property tax deductions for land rented out (row 6) and – when rents are imputed – for the self used land of landlord and owner-occupier (row 7). Very similar explanations apply for the case of structures, except that one needs to account for the effects of structures' depreciation (rows 11 and 12).

Equation (9) collects all terms from Table (2) for $i \in \{LL, O\}$. We introduce two dummy variables to track the taxation of imputed rents (π) and receiver of rental payments (μ): In the absence of imputed land rent taxation π_L equals one, and when imputed land rents are taxed π_L equals zero. The same holds for π_S in the case of structures. We define parameter μ_i as $\mu_{LL} = 1$ and $\mu_O = 0$ to capture that only landlords rent out land and structures.

$$\begin{aligned}\dot{a}_i =& w\left(1-\tau_N\right) n_i + r_B b_i \\ &+ \left(1-\tau_K\right)\left(q_K-\delta_K\right) k_i \\ &+ \left\{\left(\dot{p}_L-\xi_L p_L\right) l_i + \mu_i q_L l_r - \tau_L\left(q_L-\xi_L p_L\right)\left[\mu_i l_r + \left(1-\pi_L\right) l_{c,i}\right]\right\} \\ &+ \left\{\left(\dot{p}_S-\left(\xi_S+\delta_S\right) p_S\right) s_i + \mu_i q_S s_r \right. \\ &\left. - \tau_S\left(q_S-\left(\xi_S+\delta_S\right) p_S\right)\left[\mu_i s_r + \left(1-\pi_S\right) s_{c,i}\right]\right\} \\ &- c_{F,i} - \tau_i\end{aligned} \tag{9}$$

Let $\bar{k}_i$ denote the initial endowment of consumer i for $i = LL, O, T$. Taking prices, tax rates and lump-sum tax as given, and given $\bar{k}_T$, each tenant maximizes (2) subject to her respective budget constraint. Also taking as given prices, tax rates and lump-sum tax and given initial non-real-estate wealth $\bar{k}_j + \bar{b}_j$, and initial endowments of land $\bar{l}_j$, and structures $\bar{s}_j$ (for $j = LL, O$) each landlord and each owner-occupier maximizes (2) subject to her respective budget constraint and subject to the restriction

$$\lim_{t\to\infty} a\left(t\right) e^{-\int_0^t r_B(v)dv} \geq 0 \tag{10}$$

to rule out the possibility of Ponzi games. All first order conditions are provided in Appendix A.

2.3. Consumer choices

Before we close the model by stating the government and the market clearing conditions, we characterize the main trade-offs that motivate consumer choices. To this end, we indicate how the owner-occupiers' and tenants' first order conditions depart from those of the landlords.

The landlord's first order conditions lead to non-arbitrate conditions between the returns on lending, on holding capital, land and structures. Since landlords hold land to rent it out, but, also for own use, the landlord's first order conditions imply relations that make a landlord indifferent between renting out land or holding it for self use (a similar arguments applies for structures). For brevity we focus on the case of land and explain how these relations are derived and what they imply. The first order conditions with regard to lending b_{LL} and land rented out l_r (please refer to Appendix A) lead to

$$r_B = \frac{\dot{p}_L + \left(1-\tau_L\right)\left(q_L-\xi_L p_L\right)}{p_L} \tag{11}$$

This is a non-arbitrage condition indicating that the return on lending should equal the land price increase plus the rental price of land−net of taxation and applicable tax deductions−to land price ratio. The first order conditions of b_{LL}, land for self use $l_{c,LL}$, and final good consumption $c_{F,LL}$ imply

$$r_B = \frac{\frac{\partial c_{LL}}{\partial c_{H,LL}} \frac{\partial c_{H,LL}}{\partial l_{c,LL}}}{\frac{\partial c_{LL}}{\partial c_{F,LL}}} \frac{1}{p_L} + \frac{\dot{p}_L - \xi_L p_L - (1-\pi_L)\,\tau_L\,(q_L - \xi_L p_L)}{p_L} \tag{12}$$

Equating equations (11) and (12) implies

$$\frac{\frac{\partial c_{LL}}{\partial c_{H,LL}} \frac{\partial c_{H,LL}}{\partial l_{c,LL}}}{\frac{\partial c_{LL}}{\partial c_{F,LL}}} = (1-\pi_L \tau_L)\, q_L + \pi_L \tau_L p_L \xi_L \tag{13}$$

This indicates that when imputed land rents are not taxed ($\pi_L = 1$) the marginal rate of substitution between $l_{c,LL}$ and $c_{F,LL}$ equals the rental price of land net of rent taxation $(1-\tau_L)q_L$ plus the tax deductions associated with renting out land $\tau_L \xi_L p_L$. This is nothing else than the cost of giving up renting out a unit of land and instead employing it for self-use (or its per-unit *opportunity cost*). Similar equations to (11), (12), and (13) are also found for structures.

Denote the right-side of (13) as

$$Q_L \equiv (1-\pi_L \tau_L) q_L + \pi_L \tau_L \xi_L p_L, \tag{14}$$

and also let

$$Q_S \equiv (1-\pi_S \tau_S) q_S + \pi_S \tau_S (\delta_S + \xi_S) p_S \tag{15}$$

Using the first order conditions with regard to $c_{F,LL}$, $l_{c,LL}$, l_r, $s_{c,LL}$, and s_r and the Stone Geary composite $c_{LL} = A_C c_{F,LL}^{\beta_F} (c_{H,LL} - \gamma)^{\beta_H}$, one can readily verify that a landlord's expenditure in final goods plus the opportunity cost of land and structures for self use equal

$$c_{F,LL} + Q_L l_{c,LL} + Q_S s_{c,LL} = p_{c,LL} c_{LL} + Q_H \gamma \tag{16}$$

with

$$p_{c,LL} = \frac{Q_H^{\beta_H}}{\beta_F^{\beta_F} \beta_H^{\beta_H} A_C} \quad \text{and} \quad Q_H \equiv \frac{Q_L^{\varphi} Q_S^{1-\varphi}}{\varphi^{\varphi}(1-\varphi)^{1-\varphi} A_H}, \tag{17}$$

The cost associated with the minimum housing requirement γ equals $Q_H \gamma$. A landlord's Euler equation is given by

$$\frac{\dot{c}_{LL}}{c_{LL}} = \frac{(1-\tau_K)\,(q_K - \delta_K) - \rho - \frac{\dot{p}_{c,LL}}{p_{c,LL}}}{\theta} \tag{18}$$

A similar equation to that of (12) is also found for owner-occupiers. In the case of tenants their expenditure in final goods and housing is given by

$$c_{F,T} + q_L l_{c,T} + q_S s_{c,T} = p_{c,T} c_T + q_H \gamma \tag{19}$$

with

$$p_{c,T} = \frac{q_H^{\beta_H}}{\beta_F^{\beta_F} \beta_H^{\beta_H} A_C} \quad \text{and} \quad q_H \equiv \frac{q_L^{\varphi} q_S^{1-\varphi}}{\varphi^{\varphi}(1-\varphi)^{1-\varphi} A_H}, \tag{20}$$

For detailed derivations please refer to Appendix A.

2.4. The government

Let K, L, and S denote the aggregate stock of capital, land and residential structures and let $N = \sum_{i\in\{LL,T,O\}} z_i n_i$ denote the aggregate labor endowment. The government finances a fix government consumption G via taxes on capital, labor, land and structures and lump-sum taxes and its budget constraint is given by

$$\begin{aligned} G = & \sum_{i\in\{LL,O,T\}} z_i \tau_i + w\tau_N N + \tau_K (q_K - \delta_K) K \\ & + \tau_L (q_L - p_L \xi_L) [z_T l_{c,T} + (1-\pi_L)(z_{LL} l_{c,LL} + z_O l_{c,O})] \\ & + \tau_S (q_S - (\xi_S + \delta_S) p_S) [z_T s_{c,T} + (1-\pi_S)(z_{LL} s_{c,LL} + z_O s_{c,O})] \\ & + \xi_L p_L L + \xi_S p_S S \end{aligned} \tag{21}$$

The government budget constraint (21), thus, indicates that once imputed rents are taxed ($\pi = 0$) the entire stocks of land and structures are subject to rent taxation.

2.5. Equilibrium and steady state

Definition 1. An equilibrium is the sequence of quantities $\{c_{F,i}(t), l_{c,i}(t), s_{c,i}(t), l_r(t), s_r(t), b_i(t), k_i(t), l_j(t), s_j(t), Y_x(t), K_x(t), N_x(t), \tau_i(t)\}_{t=0}^{\infty}$ for $i = LL, O, T$, $j = LL, O$, and $x = F, S$, and prices, and tax rates $\{w(t), q_h(t), p_L(t), p_S(t), r_B(t), \tau_L(t), \tau_S(t), \xi_L(t), \xi_S(t)\}_{t=0}^{\infty}$ for $h = K, L, S$ such that given prices and tax rates: i) $\{c_{F,i}(t), l_{c,i}(t), s_{c,i}(t), k_i(t), l_j(t), s_j(t), b_j(t)\}_{t=0}^{\infty}$ for $i = LL, O, T$ and $j = LL, O$ solve the landlords, owner-occupiers and tenants respective utility maximization problem; ii) $\{Y_x(t), K_x(t), N_x(t)\}_{t=0}^{\infty}$ for $x = F, S$ satisfy that the value marginal product of each input factor equals its respective rental price/wage and that the zero profit conditions hold; iii) lump-sum taxes $\tau_i(t)$ for $i = LL, O, T$ and tax rates $\tau_L(t)$, $\tau_S(t)$, $\xi_L(t)$, and $\xi(t)$ satisfy the government's budget constraint; iv) the following conditions hold:

– the labor market clears $\sum_{i\in\{LL,O,T\}} z_i n_i = N_F(t) + N_S(t)$;

– the capital market clears $\sum_{i\in\{LL,O,T\}} z_i k_i = K_F(t) + K_S(t)$;

– the land market clears $L = \sum_{j\in\{LL,O\}} z_j l_j$;

– the residential structures market clears $S = \sum_{j\in\{LL,O\}} z_j s_j$;

– the land rental market clears $z_{LL} l_r = z_T l_{c,T}$;

– the structures rental market clears $z_{LL} s_r = z_T s_{c,T}$;

– borrowing satisfies $\sum_{j\in\{LL,O\}} z_j b_j = 0$;

– the aggregate stock of residential structures evolves according to $\dot{S} = Y_S - \delta_S S$; and

– the aggregate stock of capital evolves according to $\dot{K} = Y_F - \delta_K K - \sum_{i\in\{LL,O,T\}} z_i c_{F,i} - G$.

We know provide the definition of steady-state

Definition 2. A steady-state is an equilibrium such that for some initial endowments $\bar{k}_i$, $\bar{l}_j$, and $\bar{s}_j$ for $i = LL, O, T$, and $j = LL, O$ the quantities, prices and tax rates $c^*_{F,i}(t)$, $l^*_{c,i}(t)$, $s^*_{c,i}(t)$, $l^*_r(t)$, $s^*_r(t)$, $b^*_i(t)$, $k^*_i(t)$, $l^*_j(t)$, $s^*_j(t)$, $Y^*_x(t)$, $K^*_x(t)$, $N^*_x(t)$, $\tau^*_i(t)$, $w(t)$, $q_h(t)$, $p_L(t)$, $p_S(t)$, $r_B(t)$, $\tau_L(t)$, $\tau_S(t)$, $\xi_L(t)$, and $\xi_S(t)$ for $i = LL, O, T$; $j = LL, O$; $x = F, S$; and $h = K, L, S$ are constant for all t.

2.6. *Analytical findings on land rent income taxation distortionary properties.*

We now provide some analytical results regarding the gap between the per-unit cost of land and structures for self use of landlord and owner-occupiers and the rental price of land that tenants pay. We then investigate the distortionary properties of land rent taxation on the aggregate level of capital and structures in the economy.

Proposition 1 (Equal housing costs for homeowners)**.** *The landlords' and owner-occupiers' per-unit opportunity costs of land for self use are equal. The same holds for the per-unit opportunity costs of structures.*

Proof. See Appendix B.1. □

Proposition 2 (Housing cost gap)**.** *If the imputed rents of landlords and owner-occupiers are not taxed ($\pi = 1$); and the land value tax rate (ξ_L) is sufficiently small, then, the landlords and owner-occupiers per-unit opportunity costs of land for self use is lower than the tenant's per-unit land rental cost. The same argument applies for structures and for the per-unit cost of housing services.*

Proof. See Appendix B.2. □

If imputed rents are not taxed and if tenants are relatively poorer than homeowners, then they do not only pay a larger share of their income in housing due to the minimum consumption requirement γ, but, also because the lack of imputed rent taxation imposes an implicit lower cost of housing for homeowners. This further implies that the lack of imputed rent taxation can have distinct effects on the savings possibilities of tenants and homeowners, and therefore, it can also effect the distribution of wealth. To have a better understanding of the magnitude of such effects in Section 4 we numerically simulate the model by applying it to the German economy.

Since we focus on analyzing the distortionary properties of land rent taxation, before further proceeding we know clearly define what we mean by this.

Definition 3. Given a fixed government expenditure land rent taxation is said to be distortionary if an increase in the land rent tax rate changes the aggregate levels of output, capital and structures stocks.

Proposition 3 (Distortionary land taxation)**.** *Assume $\tau_N = \tau_K = \xi_L = \xi_S = 0$, and that tenants neither contribute to lump-sum taxation ($\tau_T = 0$) nor save; but, instead simply use their labor income to pay their consumption of final goods and housing services. If landlords' and owner-occupiers' land imputed rents are not taxed, then at the steady state an increase in the land rent tax rate is distortionary for the accumulation of structures and capital.*

Proof. See Appendix B.3. □

As long as imputed rents are not taxed, an increase in the land rent tax rate, by decreasing the opportunity cost of land for self use, creates incentives for landlords and owner-occupiers to increase their land for self-use. In turn, the supply of land for rent decreases and the rental price of land increases. As tenants substitute land for structures this leads to a potential increase in the rental price of structures, and, thus the stock of structures may increase. Our numerical simulations indicates that this is the case, but, the increase in the stock of structures is at the expense of capital accumulation.

3. Numerical analysis

3.1. Calibration

We calibrate the model to the German economy. Using the Household Finance and Consumption Survey Statistical Tables (ECBa-c) we derive information about

Parameter		value
Capital share parameter of firm F	α_F	0.35
Shift parameter F	A_F	0.96
Capital share parameter of firm S	α_S	0.20
Shift parameter S	A_S	1.11
Capital depreciation rate	δ_K	0.10
Structures depreciation rate	δ_S	0.02
Land share parameter in housing	φ	0.14
Housing shift parameter	A_H	1.38
Rate of time preference	ρ	0.02
Elasticity of interteporal substitution	$1/\theta$	1.00
Consumption housing share parameter	β_H	030
Consumption shift parameter	A_C	1.00
Minimum housing consumption requirement	γ	5.8
Share of landlords in population		10%
Share of owner-occupiers in population		35%
Share of tenants in population		55%
Income tax rate (for land only for benchmark)	τ_N=τ_S=τ_L	24%
Capital income tax rate (corporate+individual)	τ_K	49%
Property tax rates for land and structures	$\xi_L = \xi_L$	0.15%

Table 3: Model Parameters.

the share of tenants and homeowners net wealth in total wealth. We use this information to assess the initial endowments of net wealth of consumers in the economy. Parameter values we employ are summarized in Table 3

We presume that there are 100 consumers in the economy, with the share of each type of costumer as indicated in Table 3.

The net wealth of the top 90-100 percentile in Germany has a median wealth of 861 thousand euros, we think of them as landlords. The 60-90 percentile has a median net wealth of 216 thousand euros, we think of them as owner-occupiers. For tenants we use the net wealth of the 10-60 percentile of 12 thousand euros. We compute the steady-state wealth of the economy and distribute the total value of assets according the wealth shares. All consumers are presumed to have an equal labor endowment which we set to unity.

First, we derive a steady-state benchmark scenario (first scenario) by considering current median tax rates for labor, capital, rental housing income, and housing value (property).[8] The government's expenditure, which we keep fixed throughout all the

[8]Recently housing value (property) taxes were eliminated in Germany, we include these taxes since our calibration relating wealth distribution uses is for 2017 data.

scenarios, is set equal to the revenue collection from benchmark taxes. To facilitate the analysis, we assume that the economy is first in the steady-state benchmark. We then introduce land rent tax changes and/or imputed land and/or structures rents taxation and compute transition paths. In all the scenarios extra government revenues are equally transferred across consumers.

3.2. Scenarios

The aim of our numerical analysis is to investigate the distributional effect of taxing land rental income. To explore this question, we run scenarios where we double the currently prevailing tax rate of 24% to 48%. This experiment is complemented by two variations, where we extend land rental income taxation (a) to imputed rents for land in self use and (b) to real estate as the composite of land and structure. We summarize the scenario design in Table 3.2.

In the benchmark scenario therefore, the land rental income tax is set to $\tau_L = 0.24$, and neither, imputed rents of land nor structures are taxed ($\pi_L = \pi_S = 1$) as it is the case in Germany. In the second scenario, we double the benchmark tax rate (τ_L) in income from renting out land, raising from 24% to 48%. In the third scenario, imputed land rents are taxed ($\pi_L = 0$) under the benchmark land rent tax rate. The forth scenario is like the third scenario, but with a tax rate τ_L raising from 24% to 48%. In our fifth scenario, *both* imputed land and structures rents are taxed ($\pi_L = \pi_S = 0$) under benchmark tax rates. The sixth scenario is similar to the fifth scenario, but again with a raise in tax rate τ_L from 24% to 48%. Table 3.2 provides an overview on the settings in each scenario.

Scenario	1 (benchmark)	2	3	4	5	6
Land rental income tax rate τ_L	24%	48%	24%	48%	24%	48%
Land imputed rent taxation	No	No	Yes	Yes	Yes	Yes
Structures imputed rent taxation	No	No	No	No	Yes	Yes

Table 4: Scenario design.

3.2.1. Benchmark housing cost markup for tenants

In all the plots, the black line depicts the steady-state benchmark scenario. Henceforth, when referring to both landlords and owner-occupiers, we simply write homeowners. Our theoretical analysis revealed that exempting imputed rents from taxation drives a wedge between the cost of housing of homeowners and tenants.

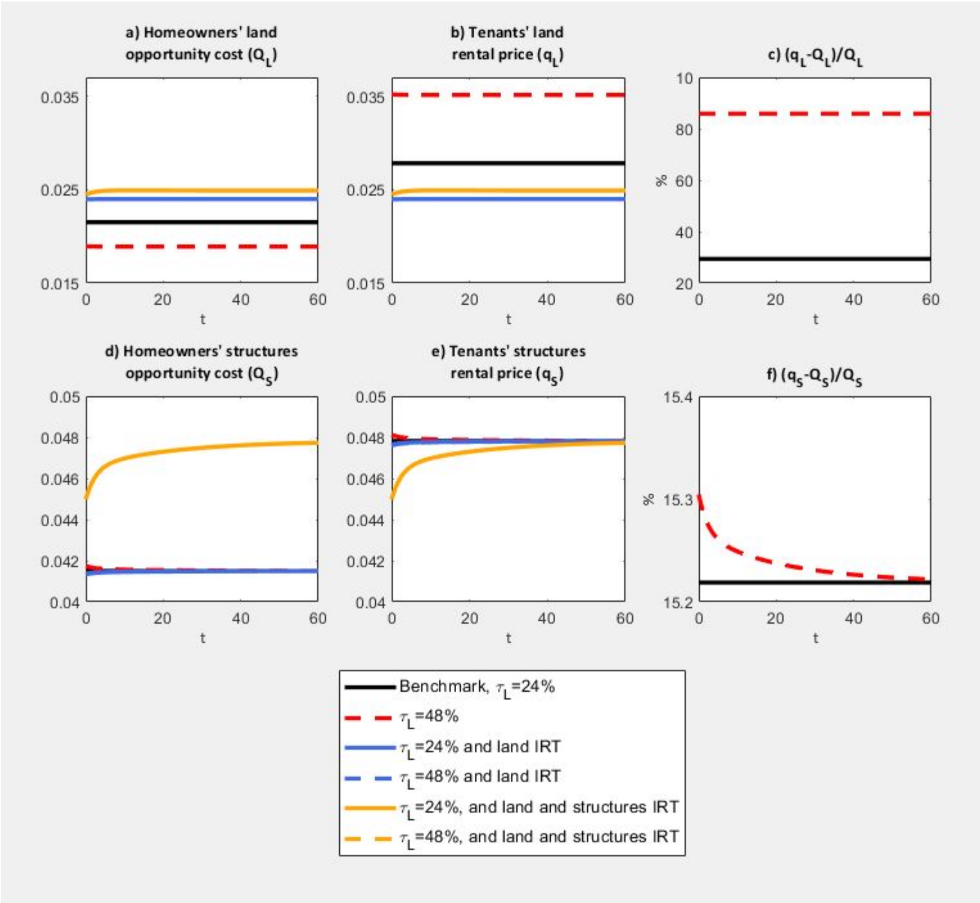

Figure 1: Rental prices

Cost of housing are shown in Figure 1 which opportunity costs for self use of land and structures (panels a and d), rental prices for land and structures (panels b and e) as well as their difference. Consistent with our derivations, when homeowners' imputed rents are not taxed, the per-unit opportunity cost of land (Q_L) for homeowners' self use is lower than the rental price of land q_L, (refer to the black lines in the plots of Figure 1). For the benchmark scenario, we estimate that q_L is 30% larger than Q_L to the lack of imputed rent taxation (top row of the panels). Thus, tenants face substantially a larger cost for employing land than homeowners. The same arguments applies for the case of structures except that benchmark lines in the plots of Figure 1 are hidden "below" the blue line of another simulation (bottom row in Figure 1). The rental price of structures (q_S) tenants pay is about 15% larger than the per-unit opportunity cost of homeowners for using structures (Q_S). Considering equations (17) and (20) the combined effect of these gaps implies a composite price of housing of tenants (q_H) that is 17% larger than the composite per-unit opportunity cost of housing of homeowners (Q_H).

3.2.2. Increase in land rent tax rate, absent of imputed rent taxation (scenario 2)

Scenario 2 doubles the tax on land rental payments τ_L relative to the benchmark to $\tau_L = 0.48$ but leaves imputed land rents exempt from taxation ($\pi_L = 1$). In all the plots, scenario 2 corresponds to the red-dashed line. At this tax rate, the wedge between homeowner and tenant costs of renting land raises from 30 to 85 percent (Figure 1c). But beyond the effects on costs, this kind of a tax reform has implication for the valuation and accumulation of structures and capital.

In Figure 2 we plot aggregate structures and capital stocks levels and their percentage increase from the benchmark scenario. Figure 2 confirms Proposition 3 about the distortionary effects of land rent taxation on assets accumulation when imputed land rents are not taxed ($\pi_L = 1$). Numerically, the stock of structures increases while the capital stock decreases.

Figure 3 shows the welfare of each type of consumer achieved in each of the scenarios with the benchmark scenario shown in black (or first column). The red (or the second) column is the result of the second simulation. Despite the increased stock of structures compared to the benchmark, tenant's welfare decreases[9] under the second scenarios when compared to the benchmark (cf. Figure 3). The reason for this is that as τ_L increases, the opportunity cost of using land for self use (Q_L) for landlords (and owner-occupiers) decreases (refer to Figure 1). In simple words, if a landlord has to pay more taxes when she rents out land, she rather holds more land for self-use. Therefore, homeowners increase their demand of land for self-use compared to the benchmark (refer to Figure 4). Landlords supply less land to the land rental market, and the land rental price (q_L) increases compared to the benchmark scenario (refer to Figure 1). Tenants make up for the lower supply of land and increased q_L by increasing their demand for structures, especially since they need to satisfy their minimum housing consumption requirement. As Figure 1 indicates, this put pressures in the rental price of structures q_S which increases in the short run; accordingly, the opportunity cost of holding structures for self-use also increases Q_S. The increase in q_S increases the price and construction of structures and, thus, the stock of structures increases at the expense of capital accumulation (see Figures 5 and 2). Finally, the increase in τ_L induces a decline in the price of land compared to the benchmark (see Figure 5). Taken all together, the increase in τ_L leads to a welfare loss for tenants which observe a rise in housing costs. Landlords also experience a decrease in welfare due the loss of income induced by increased

[9]Since the instantaneous utility of consumption composite c_i is logarithmic, and since tenants' composite consumption $c_T(t)$ is less than one in our simulations, aggregate discounted utility is negative, but this shall not be a problem.

taxation. Owner-occupiers benefit from the increase in τ_L as it decreases their cost of using land and therefore their welfare increase.

3.2.3. Land imputed rent taxation (scenario 3)

The third scenario introduces imputed land rent taxation with benchmark tax rates and is depicted with a blue solid line in the plots. Once the imputed land rents of homeowners are taxed, the homeowners per-unit opportunity cost of using land (Q_L) increases compared to the benchmark (refer to Figure 1). Consistent with our derivations Q_L equals the land rental price (q_L). A lager Q_L compared to the benchmark, induces homeowners to reduce their use of land for self use, owner-occupiers sell land to the landlords, and the amount of rented land increases. In turn the rental price of land q_L decreases compared to the benchmark (see Figure 1). The decline in q_L leads tenants to increase their demand for rented land and substitute structures for rented land so that their demand for structures declines. The reverse effects happens for owner-occupiers. Landlords decrease their land and structures holdings for self use. The overall effect of these changes leads to a decline in the stock of structures. Instead, the stock of capital increases. Due to land imputed rent taxation, the price of land decreases which makes landlords buy land from owners-occupiers, despite the decline in q_L compared to the benchmark. Land imputed rent taxation generates extra revenues for the government which are equally lump-sump transferred to all consumers. The decline in the rental price of land and lump-sump transfer lead to an increase in tenants' welfare compared to the benchmark (refer to Figure 3, dark blue column). Since land imputed rents are taxed on the land for self-use of homeowners, this implies and extra cost for homeowners. With extra government revenues equally redistributed across the population, they do make up for the extra expenditure of homeowners due to imputed rent taxation. Thus, homeowners' welfare declines (refer to Figure 3, dark blue column).

3.2.4. Land imputed rent taxation and increased in land rent tax rate (scenario 4)

Refer to the blue-dashed line for the results of scenario 4. Since many times the results of scenario 4 coincide with the results of scenario 3, when a blue-dashed line is not present in the plot, then please refer to the blue-solid line. In the forth scenario where land imputed rents are taxed, an increase in the land rent tax compared to scenario 3 leads to distributional changes, but leave aggregate variables unchanged (refer to Figures 2 and 4). Due to the increase in the land rent tax the price of land decreases even further than the price in scenario 3. As homeowners make larger payments due to a larger land rent tax, both in land rented out and land for self use, than in scenario 3, this lead to a decline in their consumption of

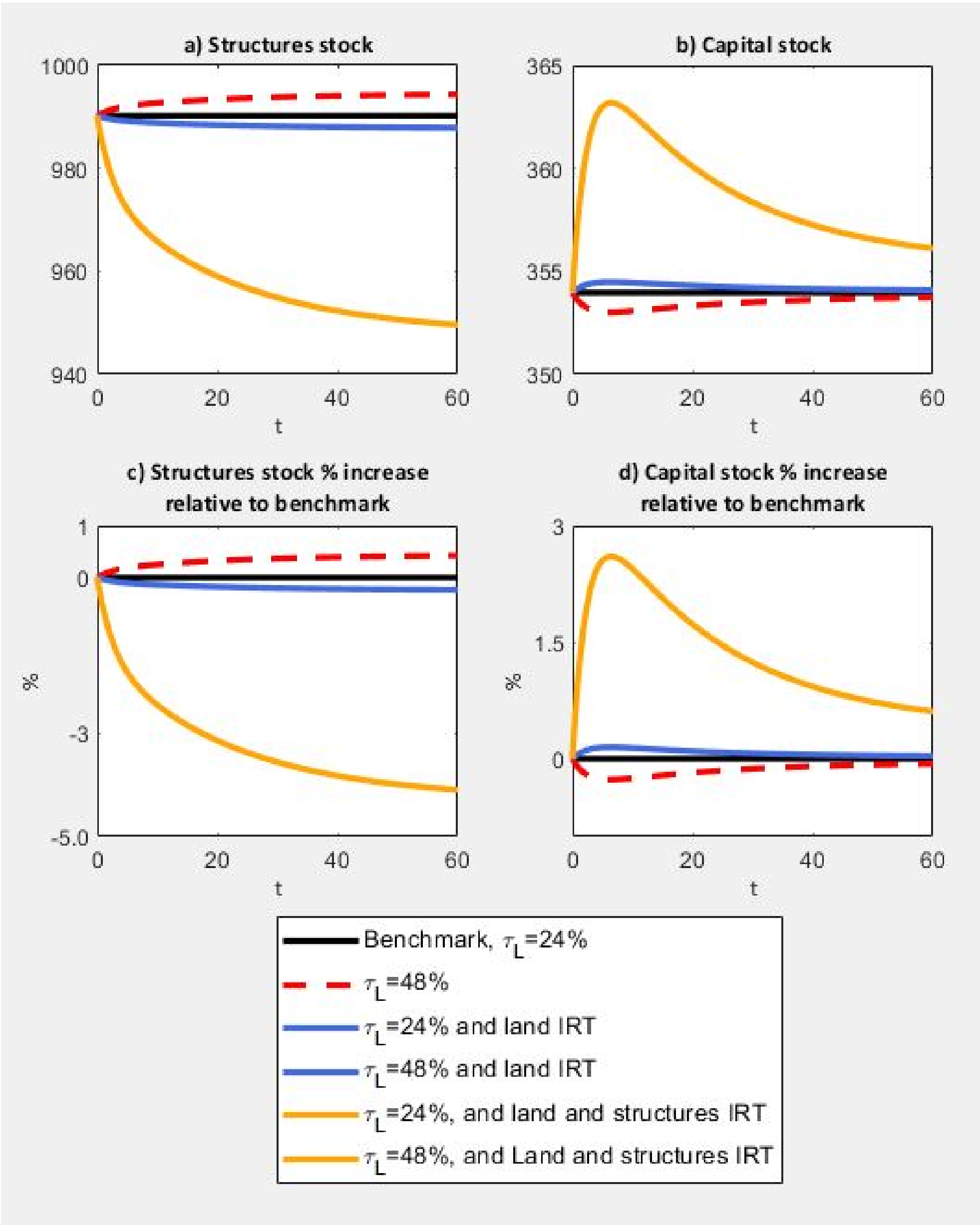

Figure 2: Structures and capital stocks

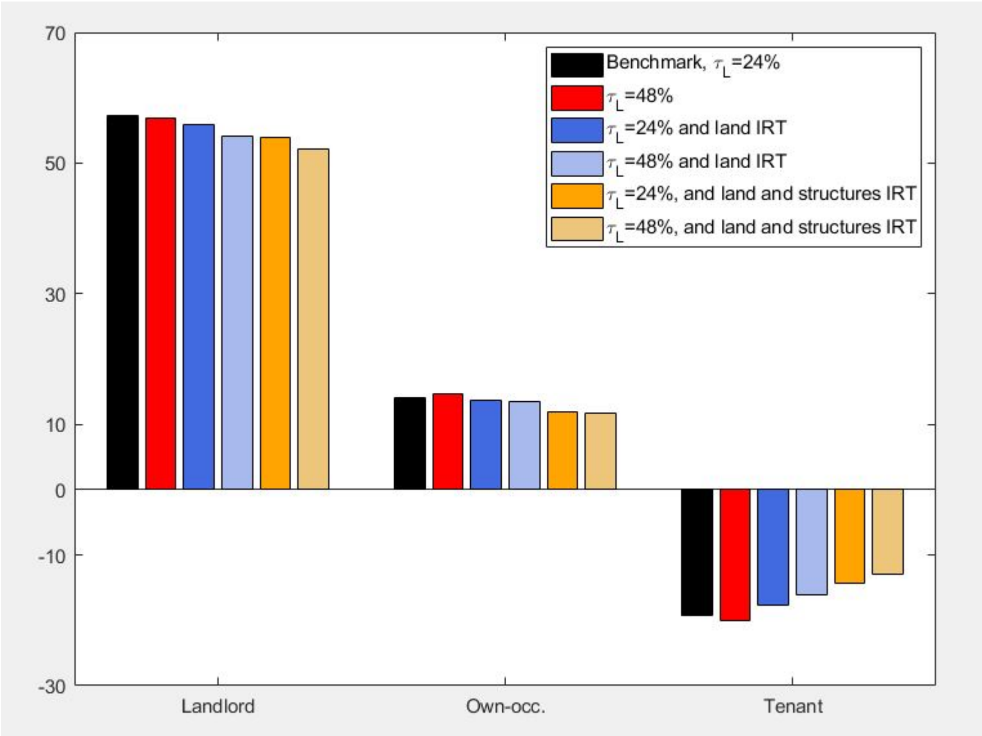

Figure 3: Welfare

housing services and also leads to an increase in government revenues. The increase in government revenues leads to larger transfers towards all consumers. Tenants increase the amounts of land and structures they rent, compared to scenario 3. These effects lead to an increase in the welfare of tenants, but the welfare of homeowners declines (refer to light-blue columns in Figure 3).

3.2.5. Land and structures imputed rent taxation (scenario 5)

The solid orange line is used to depict the results of scenario 5. When the structures imputed rents are taxed, this leads to a decline in the price of structures, and a large decrease in the stock of structures compared to the previous cases (refer to Figures 2 and 5). In this case not only the per-unit opportunity cost of land of homeowners equals the rental price of land, but also the opportunity cost of structures equals the rental price of structures. The taxation of structures imputed rents of homeowners leads to a decline in the demand of structures of homeowners, and thus the rental price of structures declines compared to the previous cases (refer to Figure 1) . Since government transfers are now larger than in the previous cases, and since the rental price of structures declines, this leads to a welfare gain for tenants. Instead homeowners suffer a welfare loss (refer to dark-orange columns in Figure 3).

land

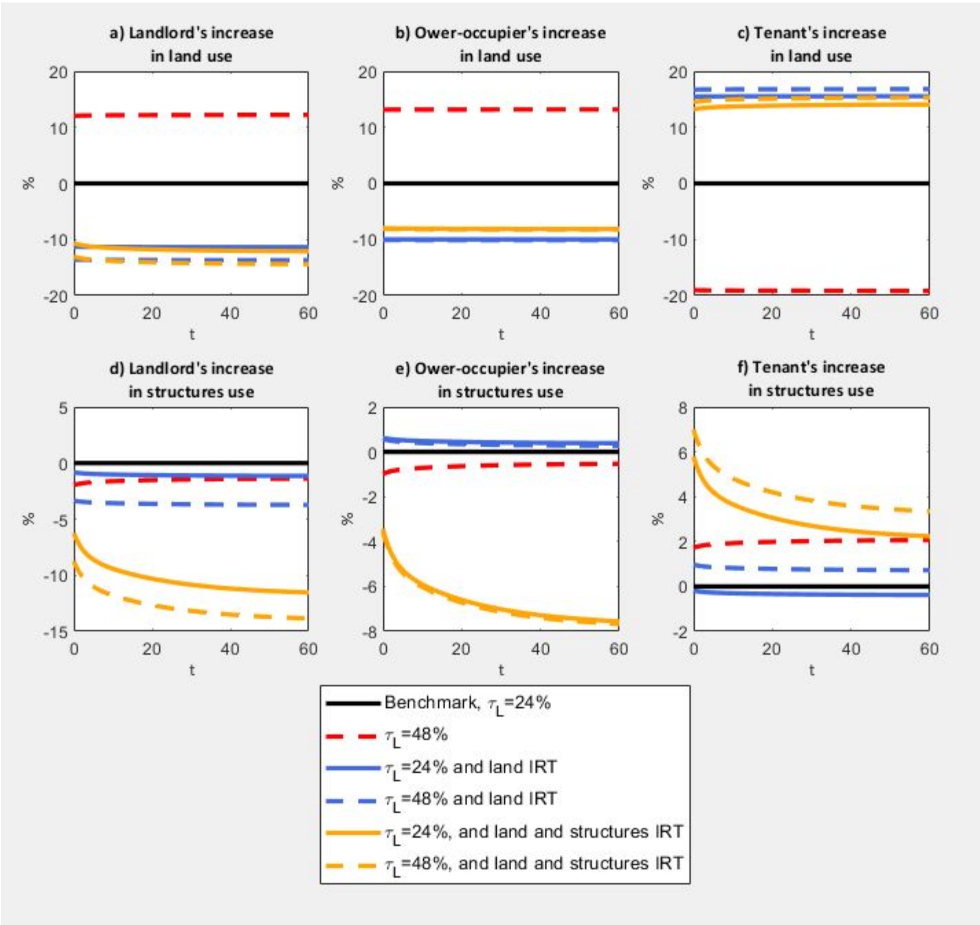

Figure 4: Increase in land and structures use from benchmark

3.2.6. Land and structures imputed rent taxation and increased in land rent tax rate (scenario 6)

Scenario 6 is depicted with a dashed-orange line in the figures. Many of the results of this scenario coincide with the results of scenario 5, so in the absence of a dashed-orange line refer to the solid orange line. Since land and structures imputed rents are taxed, the land rent tax is non-distortionary. Since we only increase the land rent tax rate, but leave the structures rent tax rate unchanged, the stock of structures and capital remain equal to those of scenario 5. The increase in the land rent tax rate has mostly distributional effects, so that the welfare of tenants increases while the welfare of homeowners declines (refer to light-orange columns in Figure 3).

4. Conclusions

Income inequality is thought to set incentives to work harder and take entrepreneurial risks, and hence to spur economic growth. Yet high levels of inequality are a risk to social cohesion and social polarization, and governments are keenly aware of these risks. Housing costs are an important potential driver of rising inequality as highlighted by recent reports of the European Central Bank (ECB, 2020a-c). There is a broad literature that suggests policy reforms to reduce these inequalities (e.g. Causa et al. 2019). We contribute by analyzing the distributive role of land value and taxation aimed specifically at the rental income of land in real estate tax reform, and separating role of land and the structures upon it that make up the real estate composite.

Exempting imputed rents from taxation is a known source of distortion and inequality (see, for example, Fatica and Prammer 2018), and we find that when imputed rents are exempt, the (opportunity) costs of housing for homeowners are substantially below the rent paid by tenants. Our modeling confirms this insight and sheds light on the underlying drivers. We find that the gap between housing costs of homeowners and tenants is driven by the land rent tax and the land value (property) tax. If the property land tax rate is sufficient small then the rent paid by tenants is larger than the cost of housing for homeowners. Intuitively, a landlords when deciding whether to self-use or rent land compares the marginal utility of land for self use against what he would get by renting it out (that is, the rental price of land net of taxes). When calibrated to the case of Germany and its current taxation of real estate, the cost markup for tenants as estimated by the model is substantial at 15% for structures and 30% for land.

Against this backdrop, this study explored two options for tax reform: (a) dou-

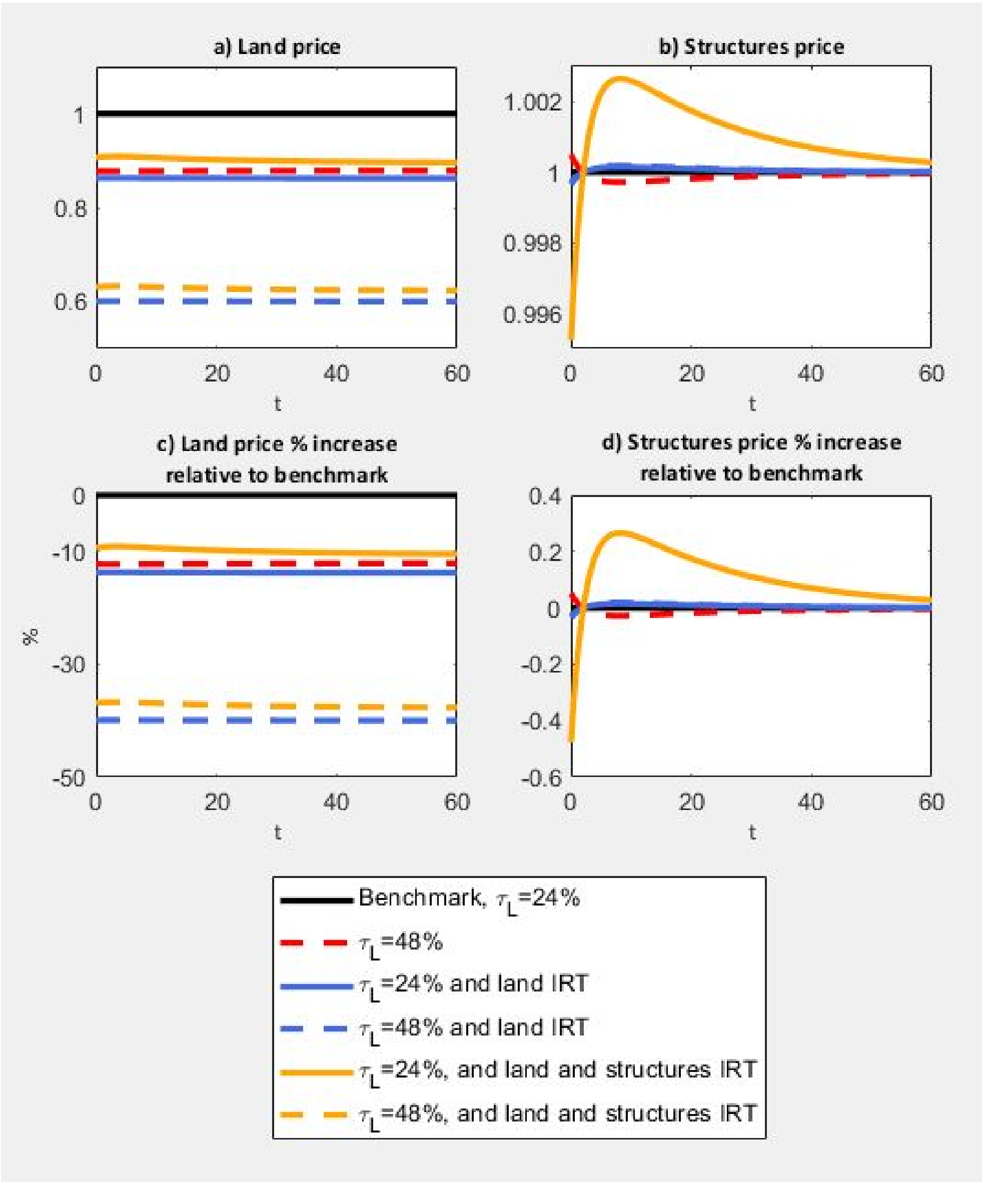

Figure 5: Land and structures prices

bling the tax on land rental income, and (b) expanding taxation of land rental income to imputed rents. Higher taxes on land rental income not only exacerbates the housing cost gap. The increased land rent tax rate furthermore distorts asset accumulation away from capital and towards higher stocks of structures as long as imputed rents are exempt. By creating incentives for homeowners to increase their land for self-use, the supply of land for rent is decreased and the rental price of land increases. When tenants in response substitute land for structures they drive up both rental price and stock of structures. The numerical simulations indicates that indeed the stock of structures increases at the expense of capital accumulation.

The root cause for the housing cost gap and the distortionary effects of the land rental income tax is the lack of taxing imputed rents. Indeed in numerical simulations where tax reform extends to imputed rents the housing cost gap vanishes and the effect on accumulation of structure and capital is reversed.

The welfare analysis reveals that policy makers face a trade-off and there is no one tax reform that would make all consumers better off. If tenants are to be made better of by reducing inequality, our analysis suggests that it has to happen at the expense of both landlords and owner-occupiers. It also echoes the note of caution of above: simply raising the tax on land rental income of landlords will backfire. The larger housing cost gap also translates to a welfare loss for tenants. The policy implication of this finding seems clear: if the purpose of tax reform is to improve the lot of tenants and reduce inequality then an effort to tax the land rental incomes cannot exempt imputed rents.

An extension of the model could analyze the recent Berlin policy that puts a price ceiling on rental housing. Such ceiling should have detrimental implications for investments in housing. The model presented here can help shed light on the distributional implication of such policy, which we leave for future research.

Appendix A. Consumers' first order conditions

We first derive the first order conditions associated with a landlord's optimization problem. Substituting $l_{LL}=l_{c,LL}+l_r$ and $s_{LL}=s_{c,LL}+s_r$ from (7) into (5) and (9), setting the time derivative of (5) equal to (9), and rearranging, the budget constraint (5) for $i=LL$ can be rewritten as

$$\begin{aligned}\dot{k}_{LL}=&w\left(1-\tau_N\right)n_{LL}+r_B b_{LL}\\&+\left(1-\tau_K\right)\left(q_K-\delta_K\right)k_{LL}\\&+\left\{q_L l_r-\xi_L p_L\left(l_{c,LL}+l_r\right)-\tau_L(q_L-\xi_L p_L)\left[l_r+\left(1-\pi_L\right)l_{c,LL}\right]\right\}\\&+\left\{q_S s_r-\left(\xi_S+\delta_S\right)p_S\left(s_{c,LL}+s_r\right)\right.\\&\left.-\tau_S\left(q_S-\left(\xi_S+\delta_S\right)p_S\right)\left[s_r+\left(1-\pi_S\right)s_{c,LL}\right]\right\}\\&-c_{F,LL}-\tau_{LL}-p_L\left(\dot{l}_{c,LL}+\dot{l}_r\right)-p_S\left(\dot{s}_{c,LL}+\dot{s}_r\right)-\dot{b}_{LL}\end{aligned}\tag{A.1}$$

Let $u\left(c\right)=\frac{c^{1-\theta}}{1-\theta}$. The current-value Hamiltonian associated with the maximization problem of a landlord equal

$$H_{LL}=u\left(c_{LL}\right)+\lambda_{kLL}\dot{k}_{LL}+\lambda_{lcLL}\dot{l}_{c,LL}+\lambda_{lr}\dot{l}_r+\lambda_{scLL}\dot{s}_{c,LL}+\lambda_{sr}\dot{s}_r+\lambda_{bLL}\dot{b}_{LL}\tag{A.2}$$

where λ_{kLL}, λ_{lcLL}, λ_{lr}, λ_{scLL}, λ_{sr}, and λ_{bLL} are, respectively, the co-state variables of k_{LL}, $l_{c,LL}$, l_r, $s_{c,LL}$, s_r, and b_{LL} (for a similar setting refer to Strulik, (2003)). Using $c_{LL}\equiv A_C c_{F,LL}^{\beta_F}(c_{H,LL}-\gamma)^{\beta_H}$ and $c_{H,LL}=A_H l_{c,LL}^{\varphi}s_{c,LL}^{1-\varphi}$ from (3) and (4), the first order conditions associated with a landlord's maximization problem and transversality condition are given by

$$c_{F,LL} : u'(c_{LL}) \frac{\partial c_{LL}}{\partial c_{F,LL}} = \lambda_{kLL} \tag{A.3a}$$

$$\dot{l}_{c,LL} : \lambda_{kLL} p_L = \lambda_{lcLL} \tag{A.3b}$$

$$\dot{l}_r : \lambda_{kLL} p_L = \lambda_{lr} \tag{A.3c}$$

$$\dot{s}_{c,LL} : \lambda_{kLL} p_S = \lambda_{scLL} \tag{A.3d}$$

$$\dot{s}_r : \lambda_{kLL} p_S = \lambda_{sr} \tag{A.3e}$$

$$\dot{b}_{LL} : \lambda_{kLL} = \lambda_{bLL} \tag{A.3f}$$

$$k_{LL} : \lambda_{kLL} (1 - \tau_K)(q_K - \delta_K p_F) = \rho \lambda_{kLL} - \dot{\lambda}_{kLL} \tag{A.3g}$$

$$l_{c,LL} : u'(c_{LL}) \frac{\partial c_{LL}}{\partial c_{H,LL}} \frac{\partial c_{H,LL}}{\partial l_{c,LL}} - \lambda_{kLL} [\xi_L p_L + (1 - \pi_L) \tau_L (q_L - \xi_L p_L)] \tag{A.3h}$$

$$= \rho \lambda_{lcLL} - \dot{\lambda}_{lcLL}$$

$$l_r : \lambda_{kLL} (1 - \tau_L)(q_L - \xi_L p_L) = \rho \lambda_{lr} - \dot{\lambda}_{lr} \tag{A.3i}$$

$$s_{c,LL} : u'(c_{LL}) \frac{\partial c_{LL}}{\partial c_{H,LL}} \frac{\partial c_{H,LL}}{\partial s_{c,LL}} \tag{A.3j}$$

$$- \lambda_{kLL} [(\xi_S + \delta_S) p_S + (1 - \pi_S) \tau_S (q_S - (\xi_S + \delta_S) p_S)]$$

$$= \rho \lambda_{lcLL} - \dot{\lambda}_{lcLL}$$

$$s_r : \lambda_{kLL} (1 - \tau_S)(q_S - (\xi_S + \delta_S) p_S) = \rho \lambda_{sr} - \dot{\lambda}_{sr} \tag{A.3k}$$

$$b_{LL} : \lambda_{kLL} r_B = \rho \lambda_{bLL} - \dot{\lambda}_{bLL} \tag{A.3l}$$

$$\lim_{t \to \infty} \lambda_{kLL}(t) \, a_{LL}(t) \, e^{-\rho t} = 0 \tag{A.3m}$$

Taking the time derivative of equations (A.3b)-(A.3f), making appropriate substitutions into equations (A.3g), (A.3i), (A.3k), and (A.3l), and simplifying leads to the following non-arbitrage conditions

$$r_B = (1 - \tau_K)(q_K - \delta_K), \tag{A.4a}$$

$$r_B = \frac{\dot{p}_L + (1 - \tau_L)(q_L - \xi_L p_L)}{p_L}, \tag{A.4b}$$

$$r_B = \frac{\dot{p}_S + (1 - \tau_S)(q_S - (\xi_S + \delta_S) p_S)}{p_S}. \tag{A.4c}$$

Similar manipulations into equations (A.3h) and (A.3j), and also employing

(A.3a) lead to

$$r_B = \frac{\frac{\partial c_{LL}}{\partial c_{H,LL}} \frac{\partial c_{H,LL}}{\partial l_{c,LL}}}{\frac{\partial c_{LL}}{\partial c_{F,LL}}} \frac{1}{p_L} + \frac{\dot{p}_L - \xi_L p_L - (1 - \pi_L) \tau_L (q_L - \xi_L p_L)}{p_L} \tag{A.5a}$$

$$r_B = \frac{\frac{\partial c_{LL}}{\partial c_{H,LL}} \frac{\partial c_{H,LL}}{\partial s_{c,LL}}}{\frac{\partial c_{LL}}{\partial c_{F,LL}}} \frac{1}{p_S} + \frac{\dot{p}_S - (\xi_S + \delta_S) p_S - (1 - \pi_S) \tau_S (q_S - (\xi_S + \delta_S) p_S)}{p_S} \tag{A.5b}$$

Similar first order conditions also apply for owner-occupiers except those related to renting out land and structures (equations (A.3c), (A.3e), (A.3i), and (A.3k)).

The current-value Hamiltonian associated with the maximization problem of a tenant equals

$$\begin{aligned} H_T = & \frac{\left(\overbrace{A_C c_{F,T}^{\beta_F} \left(\overbrace{A_H l_{c,T}^{\varphi} s_{c,T}^{1-\varphi}}^{c_{H,T}} - \gamma \right)^{\beta_H}}^{c_T} \right)^{1-\theta}}{1-\theta} + \lambda_{kT} [w (1 - \tau_N) n_T \\ & + (1 - \tau_K) (q_K - \delta_K) k_T - c_{F,T} - q_L l_{c,T} - q_S s_{c,T} - \tau_T] \end{aligned} \tag{A.6}$$

The first order conditions associated with a tenat's maximization problem and transversality condition are given by

$$c_{F,T} : u'(c_T) \frac{\partial c_T}{\partial c_{F,T}} = \lambda_{kT} \tag{A.7a}$$

$$l_{c,T} : u'(c_T) \frac{\partial c_T}{\partial c_{H,T}} \frac{\partial c_{H,T}}{\partial l_{c,T}} = \lambda_{kT} q_L \tag{A.7b}$$

$$s_{c,T} : u'(c_T) \frac{\partial c_T}{\partial c_{H,T}} \frac{\partial c_{H,T}}{\partial s_{c,T}} = \lambda_{kT} q_S \tag{A.7c}$$

$$k_T : \lambda_{kT} (1 - \tau_K) (q_K - \delta_K p_F) = \rho \lambda_{kT} - \dot{\lambda}_{kT} \tag{A.7d}$$

$$\lim_{t \to \infty} \lambda_{kT}(t) k_T(t) e^{-\rho t} = 0 \tag{A.7e}$$

Using (A.7b) and (A.7c) we get $s_{c,T} = \frac{1-\varphi}{\varphi} \frac{q_L}{q_S} l_{c,T}$. Substituting this last expression into $c_{H,T} = A_H l_{c,T}^{\varphi} s_{c,T}^{1-\varphi}$ and solving for $l_{c,T}$ leads to

$$l_{c,T} = \phi \frac{q_H}{q_L} c_{H,T} \quad \text{with} \quad q_H \equiv \frac{q_L^{\varphi} q_S^{1-\varphi}}{\phi^{\varphi} (1 - \varphi)^{1-\varphi} A_H} \tag{A.8}$$

Using (A.7a), (A.7b) and (A.8) implies

$$c_{F,T} = \frac{\beta_F}{\beta_H} q_H \left(c_{H,T} - \gamma\right) \tag{A.9}$$

Substituting into $c_T = A_C c_{F,T}^{\beta_F} \left(c_{H,T} - \gamma\right)^{\beta_H}$ and solving for $c_{H,T}$ we get

$$c_{H,T} = \frac{\beta_H}{q_H} p_{c,T} c_T + \gamma \quad \text{with} \quad p_{c,T} = \frac{q_H^{\beta_H}}{\beta_F^{\beta_F} \beta_H^{\beta_H} A_C} \tag{A.10}$$

combining these equations one can verify that the tenants expenditure in final goods and rented land and structures equal

$$c_{F,T} + q_L l_{c,T} + q_S s_{c,T} = \underbrace{\beta_F p_{c,T} c_T}_{c_{F,T}} + \underbrace{\beta_H p_{c,T} c_T + q_H \gamma}_{q_L l_{c,T} + q_S s_{c,T}} = p_{c,T} c_T + q_H \gamma. \tag{A.11}$$

Substituting (A.10) into (A.9) implies $c_{F,T} = \beta_F p_{c,T} c_T$. Substituting $c_{F,T} = \beta_F p_{c,T} c_T$ into (A.3a) leads to $c_T^{-\theta} = p_{c,T} \lambda_{kT}$. Time differentiating this last expression leads to the Euler equation of a tenant which equals

$$\frac{\dot{c}_T}{c_T} = \frac{\left(1 - \tau_K\right)\left(q_K - \delta_K p_F\right) - \rho - \frac{\dot{p}_{c,T}}{p_{c,T}}}{\theta} \tag{A.12}$$

To obtain the landlord's expenditure in final goods plus opportunity cost of land and structures (equation (16)) and Euler equation (18) simply use (13), derive a similar expression for the case of structures, and follow the same steps we used to derive equations (A.11) and (A.12).

Appendix B. Proofs

Appendix B.1. Proof of Proposition 1.

The first order conditions of owner-occupiers imply

$$r_B = \frac{\frac{\partial c_O}{\partial c_{H,O}} \frac{\partial c_{H,O}}{\partial l_{c,O}}}{\frac{\partial c_O}{\partial c_{F,O}}} \frac{1}{p_L} + \frac{\dot{p}_L - \xi_L p_L - \left(1 - \pi_L\right) \tau_L \left(q_L - \xi_L p_L\right)}{p_L} \tag{B.1}$$

Equating (12) to equation (B.1) implies

$$\frac{\frac{\partial c_O}{\partial c_{H,O}} \frac{\partial c_{H,O}}{\partial l_{c,O}}}{\frac{\partial c_O}{\partial c_{F,O}}} = \left(1 - \pi_L \tau_L\right) q_L + \pi_L \tau_L p_L \xi_L \tag{B.2}$$

A similar argument follows for the case of structures.

Appendix B.2. Proof of Proposition 2.

The poof immediately follows by comparing $Q_L$from equation (14) to q_L. Similarly comparing Q_S from equation (15) to q_S, and Q_H to q_H (from equations (17) and (20), respectively).

Appendix B.3. Proof of Proposition 3.

Let $\Omega_F \equiv \alpha_F^{\alpha_F}(1-\alpha_F)^{1-\alpha_F} A_F$ and $\Omega_S \equiv \alpha_S^{\alpha_S}(1-\alpha_S)^{1-\alpha_S} A_S$

Using firms' zero profit conditions and non-arbitrage conditions at the steady state one can verify that

$$q_K^* = \frac{\rho}{1-\tau_K} + \delta_K \tag{B.3a}$$

$$mc_F = \frac{q_K^{\alpha_F} w^{1-\alpha_F}}{\Omega_F} = 1 \Rightarrow w^* = \left(\frac{\Omega_F}{(q_K^*)^{\alpha_F}}\right)^{\frac{1}{1-\alpha_F}} \tag{B.3b}$$

$$p_S^* = \frac{q_K^{\alpha_S}(w^*)^{1-\alpha_S}}{\Omega_F} \tag{B.3c}$$

$$q_S^* = \frac{\rho + (1-\tau_S)\delta_S}{1-\tau_S} p_S^* \tag{B.3d}$$

$$Q_S^* = (1-\pi_S\tau_S)q_S^* + \pi_S\tau_S\delta_S p_S^* \tag{B.3e}$$

$$Q_H^* = \frac{(Q_L^*)^{\varphi}(Q_S^*)^{1-\varphi}}{\varphi^{\varphi}(1-\varphi)^{1-\varphi}A_H} \tag{B.3f}$$

$$q_H^* = \frac{(q_L)^{\varphi}(q_S^*)^{1-\varphi}}{\varphi^{\varphi}(1-\varphi)^{1-\varphi}A_H} \tag{B.3g}$$

With $Q_L^* = (1-\pi_L\tau_L)q_L$.

Using market clearing conditions for labor, capital and structures one can verify that the capital stock of the economy equals

$$\begin{aligned} K^* =& \frac{1}{fac}\{\left(\frac{1}{\delta_S}\frac{\alpha_F}{p_S^*} + \alpha_S\frac{1-\varphi}{Q_S}\frac{\beta_H}{\beta_F}\right)\frac{w^* N}{\alpha_F - \alpha_S} \\ &+ \frac{1-\varphi}{Q_S^*}\frac{\beta_H}{\beta_F}G - (1-\varphi)\frac{Q_H^*}{Q_S^*}(z_1+z_2)\gamma \\ &- \left[\frac{\beta_H}{Q_S^*}(q_H^*\gamma - w^* n_3) + \frac{1}{q_S^*}(\beta_H w^* n_3 + \beta_F q_H^* \gamma)\right](1-\varphi)z_3\} \end{aligned} \tag{B.4}$$

where

$$fac = \left((1-\varphi)\frac{\beta_H}{\beta_F}\frac{1-\alpha_S}{Q_S^*} + \frac{1}{\delta_S}\frac{1-\alpha_F}{p_S^*}\right)\frac{q_K^{ss}}{\alpha_F-\alpha_S} - \frac{1-\varphi}{Q_S^*}\frac{\beta_H}{\beta_F}\delta_K$$

The partial derivatives of K^* with regard to $q_L$and τ_L, respectively equal

$$\frac{\partial K^*}{\partial q_L} = -\frac{\gamma(1-\varphi)\varphi}{q_L}\frac{\frac{Q_H^*}{Q_S^*}(z_1+z_2)+\left(\frac{\beta_H}{Q_S^*}+\frac{\beta_F}{q_S^*}\right)q_H^* z_3}{fac} \tag{B.5a}$$

$$\frac{\partial K^*}{\partial \tau_L} = -\pi_L\frac{\gamma(1-\varphi)\varphi}{1-\pi_L\tau_L}\frac{\frac{z_1+z_2}{Q_S^*}}{fac} \tag{B.5b}$$

Aggregating the budget constraints of landlords and owner-occupiers, the expenditure in final goods of homeowners minus the cost of the minimum housing requirement equals

$$\begin{aligned} p_{c,1}^*(z_1c_1^*+z_2c_2^*) =& \frac{1}{\beta_F}[\left(\frac{1-\alpha_S}{\alpha_F-\alpha_S}q_K^*-\delta_K\right)K^* \\ &-\frac{\alpha_S w^*}{\alpha_F-\alpha_S}N+z_3\beta_F\left(q_H^*\gamma-wn_3\right)-G] \end{aligned} \tag{B.6}$$

The partial derivatives of $p_{c,1}^*(z_1c_1^*+z_2c_2^*)$ with regard to $q_L$and τ_L, respectively equal

$$\frac{\partial p_{c,1}^*(z_1c_1^*+z_2c_2^*)}{\partial q_L} = \frac{\frac{1-\alpha_S}{\alpha_F-\alpha_S}q_K^*-\delta_K}{\beta_F}\frac{\partial K^*}{\partial q_L}+z_3\varphi\gamma\frac{q_H^*}{q_L^*} \tag{B.7a}$$

$$\frac{\partial p_{c,1}^*(z_1c_1^*+z_2c_2^*)}{\partial \tau_L} = \frac{\frac{1-\alpha_S}{\alpha_F-\alpha_S}q_K^*-\delta_K}{\beta_F}\frac{\partial K^*}{\partial \tau_L} \tag{B.7b}$$

The market clearing condition for land implies

$$L = \frac{\varphi}{Q_L^*}[\beta_H p_{c,1}^*(z_1c_1^*+z_1c_1^*)+(z_1+z_2)Q_H^*\gamma] \tag{B.8}$$

$$+z_3\frac{\varphi}{q_L}(\beta_H w^* n_3+q_H\beta_F\gamma) \tag{B.9}$$

Applying the implicit function theorem to this last equation and using the derivatives above, leads to an expression for $\frac{dq_L}{d\tau_L}$ that is trivially different than zero except when π_L equals zero. Using the expression for $\frac{dq_L}{d\tau_L}$ and applying the the chain rule for total differentiation to K^* implies $\frac{dK^*}{d\tau_L}\neq 0$ as long as $\pi_L=1$.

References

Ahlfeldt, G. M., McMillen, D. P. (2014) New Estimates of the Elasticity of Substitution between Land and Capital. Lincoln Institute of Land Policy.

Albouy, D. Ehrlich, G. Liu, Y. (2016) Housing Demand, Cost-of-Living Inequality, and the Affordability Crisis. NBER Working Paper 22816

Arnott, R. J., Stiglitz, J. E. (1979) Aggregate land rents, expenditure on public goods, and optimal city size. Quarterly Journal of Economics, 93(4), 471–500.

Arnott, R. J. (2004) Does the Henry George Theorem Provide a Practical Guide to Optimal City Size?, American Journal of Economics and Sociology 63, 1057–1090

Bricker, J., Dettling, L. J., Henriques, A., Hsu, J.W., Jacobs, L., Moore, K. B., Pack, S., Sabelhaus, J., Thompson, J., Windle, R. A. (2017) Changes in U.S. Family Finances from 2013 to 2016: Evidence from the Survey of Consumer Finances. Federal Reserve Bulletin, 103(3).

Bryson, P. J. (2011) The economics of Henry George: History's Rehabilitation of America's Greatest Early Economist. New York: Palgrave Macmillan.

Causa, O., Woloszko, N., & Leite, D. (2019). Housing, wealth accumulation and wealth distribution: Evidence and stylized facts. OECD Working Papers. https://dx.doi.org/10.1787/86954c10-en

Davis, M. A. and Ortalo-Magné, F. (2011) Household expenditures, wages, rents. Review of Economic Dynamics, 14(2): 248-261.

Davis M. A., Van Nieuwerburgh, S. (2015) Housing, Finance, and the Macroeconomy. In Handbook of Regional and Urban Economics, Eds. G. Duranton, J. V. Henderson, W. C. Strange. 5: 753-811

ECBa (2020) European Central Bank's Household Finance and Consumption, Survey Statistical Tables, Wave 2010.

https://www.ecb.europa.eu/pub/economic-research/research-networks/html/researcher_hfcn.en.

ECBb (2020) European Central Bank's Household Finance and Consumption, Survey Statistical Tables, Wave 2014.

https://www.ecb.europa.eu/pub/economic-research/research-networks/html/researcher_hfcn.en.

ECBc (2020) European Central Bank's Household Finance and Consumption, Survey Statistical Tables, Wave 2017.

https://www.ecb.europa.eu/pub/economic-research/research-networks/html/researcher_hfcn.en.

Fatica, S., & Prammer, D. (2018). Housing and the Tax System: How Large Are the Distortions in the Euro Area?. Fiscal Studies, 39(2), 299–342.

Feldstein, M. S. (1977) The surprising incidence of a tax on pure rent: A new answer to an old question. Journal of Political Economy 85(2): 349-360.

Fu, S. (2005) Dynamic Henry George Theorem and Optimal City Sizes in Essays on urban agglomeration economies Ph.D. thesis, Boston College.

George H. (1881) The Land Question.

Epple, D., Gordon, B., and Sieg, H. (2010) A New Approach to Estimating the Production Function for Housing. American Economic Review 100: 905–924

Kawano, M. (2014) Optimal process of urbanization in a developing country dynamic Henry George theorem. Letters in Spatial Resource Sciences 7, 195–204 https://doi.org/10.1007/s12076-013-0111-x

Kiyotaki, N., Michaelides A., Nikolov, K. (2011) Winners and Losers in Housing Markets. Journal of Money, Credit and Banking, 43: 256-296.

Jeske, K., Krueger, D., Mitman K. (2013) Housing, Mortgage Bailout Guarantees and the Macro Economy. Journal of Monetary Economics, 60 (8).

Larrimore, J., Schuetz, J. (2017) Assessing the Severity of Rent Burden on Low-Income Families. Federal Reserve bank Notes. Board of Governors of the Federal Reserve System. https://doi.org/10.17016/2380-7172.2111.

Mattauch, L., Siegmeier, J., Edenhofer, O., Creutzig, F. (2018) Financing Public Capital through Land Rent Taxation: A Macroeconomic Henry George Theorem. Public Finance Analysis. 74(3), 340-360.

Ricardo, D. (1821) On the Principles of Political Economy and Taxation.

Schweizer, U. (1986) General Equilibrium in Space and Agglomeration. Fundamentals of Pure and Applied Economics 5: 151–185.

Stiglitz, J. E. (2015) In praise of Frank Ramsey's contribution to the theory of taxation. The Economic Journal