Asset pricing and the carbon beta of externalities

Ottmar Edenhofer[a,b,c], Kai Lessmann[a,b,*], Ibrahim Tahri[a]

[a]*Potsdam Institute for Climate Impact, P.O. Box 60 12 03, 14412 Potsdam, Germany*
[b]*Mercator Research Institute on Global Commons and Climate Change, Berlin, Germany*
[c]*Technische Unversity Berlin, Berlin, Germany*

Abstract

Climate policy needs to set incentives for actors that face imperfect, distorted markets and large uncertainties about the costs and benefits of abatement. Investors price uncertain assets according to their return and risk. Carbon prices, derived within this asset pricing framework, reflect not only uncertainty about the returns but their correlations, captured by the carbon beta of abatement. We study carbon pricing and financial incentives in a consumption based asset pricing model (CCAPM) distorted by regulatory failure, technology spillover and time-inconsistency. We find that investment in abatement is delayed under all three distortions as they reduce asset return and decrease the equilibrium rate of return. However, their direct (all else the same) effect on the carbon beta and risk premium of abatement can be decreasing (technology externality), increasing (time-inconsistency) or neutral (regulatory failure). Numerical simulations suggest that the effect on the risk premium is around 30 basis points. For time-inconsistency, the associated welfare costs are significant. The efficient equilibrium can be restored by carbon pricing and financial incentives, implemented in our model by a regulatory authority and by a long-term investment. The regulator commands carbon pricing and the fund provides subsidies to reduce technology costs or to boost investment returns. The investment subsidy creates a financial incentive that acts as an additional carbon price such that the investment fund can support climate policy when the actions of the regulatory falls short. All these instruments must also consider the investment risk and the sequence of their implementation. Then, the investment fund can pave the path for carbon pricing in latter periods, when financial incentives are used to increase abatement in earlier periods and can increase the feasibility of ambitious carbon pricing.

Keywords: Carbon budget, CCAPM, policy instruments, external effect
JEL: Q54, D81, G12

1. Introduction

Carbon pricing, such as the Emission Trading System of the European Union (EU ETS), addresses the external effects of climate change and the associated market failure. It is consensus

*Corresponding author
Email address: lessmann@pik-potsdam.de (Kai Lessmann)

within climate economics that additional market failures merit further policy intervention and additional policy instruments. The design of the necessary instruments depends on the specific market distortion, including asymmetric information, lack of commitment or incomplete contracts to name but a few. Furthermore, when large uncertainties about climate impacts and climate policy turn abatement decisions into risky investment, the climate policies need to take these uncertainties into account, as Cai et al. (2016) and Barnett (2019) demonstrate for 'physical risk' and 'transition risk', respectively.

Financial economics emphasizes the importance of the correlation of investment returns: investments that pay off in bad states of the world are worth more than investments that pay off in good states. Hence for private sector investors the *beta* of an investment is decisive, which measures the correlation of its return with the market return (Sharpe, 1964; Lintner, 1965) or consumption (Lucas, 1978). The decision to invest in emission abatement as an asset is thus informed by the *carbon beta* (Dietz et al., 2018; Gollier, 2020). In our paper, climate economics and financial economics meet to improve our understanding of how additional distortions affect the pricing of risks, the associated risk premium demanded, and the optimal policy response. We study the asset pricing problem for emission abatement projects subject to three additional distortions: in the growth rate of the carbon price, the carbon budget as indicator for the overall ambition of the climate policy, and due to a technology externality. We find that the distortions are distinct in their effect on carbon betas and risk premium. The risk premium has to be derived endogenously dependent on the type of externalities and the choice of policy instruments to correct them.

It turns out that sub-optimal growth rate of the carbon price (for a given carbon budget) does not directly change the risk premium (carbon beta), although in the distorted equilibrium the risk premium is lower. In contrast, the technology externality reduces the risk premium while non-credibility of the emission budget raises the risk premium, all else equal. Risk in terms of co-variance instead of variance is one of the most important insights of finance to the design of policy instruments: The more policy makers want to push abatement technologies and the less credible the carbon budget is the more investors demand for a higher risk premium.

The paper compares, within a consumption based asset pricing model (CCAPM), the socially optimal risk premium with a risk premium determined by decentralized markets. It argues that policy instruments should be chosen to achieve the socially optimal risk premium.

We derive from a calibrated version of our analytical model two main results: First, we confirm a key insight of Gollier (2020) in a decentralized economy with distortions: the risk premium on abatement is substantial, putting the socially optimal rate of return well above the risk-free rate. This rate is decisive for discounting return and the timing of abatement activities. Hence ignoring risks has welfare costs and leads to a mis-allocation between consumption and investment projects. The primarily ethical debate on social discounting needs to be complemented by quantifying the macro-economic risks for investors. Otherwise, climate economics would focus on the quantitatively less important component of the social discount rate. Second, the impact of market failure and ill-designed policy on the risk premium deserves more attention as they distort the risk premium way from the social optimal level. Similarly, we quantify the welfare losses and risk premium of time-inconsistency when regulators cannot commit to their policies. We highlight the sequencing of policies in which a long-term fund paves the way to ambitious carbon pricing when the regulator might fail to implement creditable long-term carbon price path.

The paper is organized as follows. In Section 2 we discuss the literature and clarify our contribution. Section 3 presents the social planner model for the normative benchmark and the decentralized economy for the market equilibrium with a technological externality and the climate externality. Carbon prices are used to achieve a social optimum. The respective carbon betas of these externalities are derived. In Section 4 we introduce a long-term fund with two additional policy instruments: subsidies on market interest rates for loans and up-front capital costs. Sections 5 and 6 are dedicated to the calibration of the model and its numerical results, in particular the quantification of the risk premiums, and welfare costs of carbon pricing policies which exhibit a lack of commitment of the regulator. The final section offers conclusion and outlook.

2. Motivation and literature

Mainstream models in climate economics have been mostly relying on deterministic cost-benefit approaches (Nordhaus, 2007, 2014) or on cost-effectiveness analysis of Integrated Assessment Models (IAMs) often used in the IPCC assessment reports (IPCC, 2018). Both approaches share the use of risk-free social discount rate. The social discount rate might then be used either as a normative benchmark (Stern et al., 2006) or to replicate observed market behavior (Nordhaus, 2007). Both approaches ignore the specific macro-economic risks arising from the uncertain behavior of the key elements making-up the climate-economic model. Yet, real-world investors demand for a risk premium is based on the covariance of uncertainties.

More recently, researchers have begun to investigate the risk premium of physical and transitional risks. One important strand of literature explores primarily the physical risks of climate change including the damage on productive assets. The second strand of literature focusing on transitional risk reflect costs arising from transformation to a low carbon economy (Giglio et al., 2020). The third strand of literature traces government policies to the carbon beta or the risk premium. Additionally real world climate policies reflect at least partially the existence of multiple externalities which deserve a deeper analysis within the context of financial economics.

Physical risks of climate change: A majority of studies on the role and implications of uncertainty in climate-economic models has so far focused on climate change impacts. A wide literature on this topic was developed using stochastic general equilibrium models with recursive preferences (a la Epstein and Zin, 1989, 1991). Within this modeling framework, studies have mostly focused on two sources of uncertainty namely damage uncertainty and growth uncertainty. Several papers report on the effect of damage uncertainty and its implication for climate policy, for example Crost and Traeger (2014), Rudik (2020) and more recently Hambel et al. (2021), who find the social cost of carbon to be heavily driven by the assumptions about the damage specification. Other papers studied the impact of growth uncertainty (Jensen and Traeger, 2014; Cai et al., 2013; Cai and Lontzek, 2019). These stochastic equilibrium models with recursive preferences have the advantage of capturing how policy incorporates anticipated learning and the ability to calculate the optimal tax on carbon emissions. Most of these models treating uncertainty are solved numerically, with little attention to analytical insights (but see Golosov et al. 2014, Van der Ploeg and van den Bremer 2018 and Hambel et al. 2021 for examples of closed-formed solutions of stochastic IAMs).

A more recent approach to studying the question of physical risks looks into climate change as an asset pricing problem, where CO_2 in the atmosphere or inversely abatement levels are considered assets. Financial-economics models of decision under risk can provide interesting insights on the implications of uncertainty in the estimation of the carbon price. Bansal et al. (2016) explores the impact of climate change and long-run risk on the social cost of carbon and asset prices; Dietz et al. (2018) look into the elasticity of climate damages, where they obtained a positive and close to one climate-beta value for investment maturities of up to about one hundred years. A result that is mostly due to the overwhelming positive effect of uncertainty about emissions-neutral technological progress.; Daniel et al. (2019) estimate the optimal carbon price today as opposed to further delaying its implementation in the future and the social cost of doing so.

Transitional risks: This strand of literature focuses primarily on transitional risks. Gollier (2020) provides an innovative analytical framework, where he analyses the effects of abatement technologies and economic growth uncertainties on the dynamics of efficient carbon prices, interest rates and risk-premiums. His results highlight the positive correlation between aggregate consumption and marginal abatement costs along the optimal abatement path, thus implying a positive carbon beta and an efficient growth rate of expected carbon prices larger than the risk-free rate. Lemoine (2021) derives an analytical model to portray the different channels through which uncertainty affects the social cost of carbon, and goes on to quantitatively estimate the impact of different sources of uncertainties on the the marginal value of emission reductions.

Government policies, asset prices and multiple externalities: In addition to the above cited literature, exploring the link between climate change and asset prices, our paper contributes also to the work on the interaction between government actions and asset prices. Pastor and Veronesi (2012), Baker et al. (2016) and Kelly et al. (2016) are some examples in this area that focus on the impact of policy uncertainty on asset prices that face different degrees of exposures to these risks. Our paper looks more precisely into what implications a regulators' lack of commitment in implementing a credible long-term carbon price trajectory has for the beta of abatement investments and therefore their risk premium. On top of the above mentioned areas of research, our modeling framework takes into consideration the role of multiple externalities explored extensively in climate economics. In fact, it has been shown extensively in the recent literature that additional externalities have far reaching consequences for the design of policy instruments: R&D investments, learning-by-doing investments (Jaffe et al., 2005; Kalkuhl et al., 2012), interaction with the fiscal system (Goulder, 2013; Franks et al., 2015), lack of commitment (Kalkuhl et al., 2020) all require well-designed policy packages to achieve second-best or even first-best outcomes.

Drawing on these strands of literature we build a decentralized market equilibrium version of the CCAPM model from Gollier (2020), where we include a technology externality as well as political economy constraints on carbon pricing in order to investigate their effect on the carbon beta.

3. The model

The focus of our research is to explore some policy instruments that could address additional distortions beyond the climate change externality. This necessitates a market equilibrium representation of the economy. Prior to introducing the decentralized problems for all agents, we characterize the socially efficient solution from a social planner perspective. Our presentation follows the social planner model of Gollier (2020) but extends the model to include a technological externality.

3.1. Social planner benchmark

Assume a social planner who considers utility of consumption $u(C_t)$ in two periods $t = 0, 1$. Consumption C_t is the residual of endowed income Y_t and abatement expenditures A_t. The abatement level K_t reduces emissions in period t; furthermore, abatement K_0 in first period has a spillover effect on future abatement cost, such that abatement at $t = 0$ affects abatement cost at $t = 1$: $A_1(K_0, K_1)$. The planner's objective is to limit emissions to a carbon budget of T. The exogenous carbon budget (rather than a Pigouvian carbon price that reflects the social cost of carbon) is in accordance with current EU climate policy, which relies on a carbon budget consistent with carbon neutrality by 2050 in its *Green Deal*. Moreover, uncertainties in quantifying climate damages might still be too large for social cost of carbon to be a useful point of reference for policy makers. The problem of the planner is thus:

$$\max_{K_0,K_1} u(C_0) + e^{-\rho}\mathbb{E}[u(C_1)] \tag{1}$$

$$\text{such that } C_0 = Y_0 - A_0(K_0) \tag{2}$$

$$C_1 = Y_1 - A_1(K_0, K_1) \tag{3}$$

$$T = (Q_0 Y_0 - K_0)e^{-\delta} + Q_1 Y_1 - K_1 \tag{4}$$

Assuming an interior solution, the first order conditions of the planner yield the following asset pricing equation.

$$u'(C_0)A_0' = e^{-(\rho+\delta)}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right] \tag{5}$$

To derive the risk premium in a beta form representation, we view abated emissions in equation (5) as an asset with cost A_0' and expected gross return $R_1^A = \left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)/A_0'$. We use the following Lemma to rewrite (5) using a beta form representation of the risk premium.

Lemma 1. *Consider a representative agent with time-additive expected utility, with a subjective discount rate* ρ *and a constant relative risk aversion* ξ*, in a discrete-time setting with a risk-free asset traded each period. Assuming the relative growth rate of consumption* $g_\tau^c = c_\tau/c_0 - 1$ *and gross return* $R_\tau = \frac{e^{-\delta\tau}A_\tau'}{A_0'} = e^{-\delta\tau}R_\tau^A$ *to be jointly lognormally distributed, then*

$$\frac{1}{\tau}\ln\left(\mathbb{E}\left[R_\tau^A\right]\right) = \delta + \frac{1}{\tau}\ln R^f + \frac{1}{\tau}\xi\sigma_{g^c}Corr\left[\ln R_\tau^A, \ln\frac{c_\tau}{c_0}\right]\sigma[\ln R_\tau^A]$$

and in beta-form

$$\mathbb{E}\left[\frac{A_\tau'}{A_0'}\right]^{\frac{1}{\tau}} = e^{\delta + r^f + \phi\eta}$$

with

$$\phi = \frac{Cov\left[r_\tau, \tilde{g}_\tau^c\right]}{Var\left[\tilde{g}_\tau^c\right]} \quad \text{and} \quad \eta = \frac{1}{\tau}\xi Var\left[\tilde{g}_\tau^c\right]$$

and r^f*,* r_τ*, and* $\tilde{g}_\tau^c$ *represent respectively* $\ln R^f$, $\ln R_\tau$, *and* $\ln\frac{c_\tau}{c_0}$.

The proof is provided in Appendix A. Based on Lemma 1, we have the expression for the two-period risk premium of abatement investments:

$$A'_0 = e^{-(r_f+\phi\eta)}\mathbb{E}\left[e^{-\delta}\frac{\partial A_1}{\partial K_1} - \frac{\partial A_1}{\partial K_0}\right] \tag{6}$$

That is, the growth rate of social marginal abatement costs should exceed the risk-free rate by a risk-premium $\phi_t\eta_t$ where η_t and ϕ_t are the systematic risk premium and "carbon beta" respectively, as defined in Lemma 1.

3.2. The market economy

The decentralized economy is populated by three agents: a firm-owning household, a regulatory authority and a long-term investment fund. The firm-owning household (henceforth simply household) is endowed with the (stochastic) economic product and chooses consumption to maximize (expected) welfare but must constrain total emissions to keep an emission permit budget. To this end, the household controls emissions by investing in emission abatement projects.

The regulatory authority is in charge of the carbon pricing policy. Its policies, however, may be distorted due to political economy considerations leading it to implement a carbon price with a non-optimal growth rate or because of lacking credibility in the announcement of the emission budget.

The investment fund is investigated as a potential remedy to overcome distortions. The fund anticipates household and government actions and may chose to support abatement projects via a subsidy paid on project benefits or via an upfront technology subsidy. The next sections introduce these agents in turn.

3.3. Household

The problem of the household is similar to the problem of the social planner. However, the household only partially anticipates the technology externality (ψ), does not anticipate emissions decay (δ), and is subject to policy instruments: the government issues T_0 emission permits and sets an intertemporal trading ration (γ), a long-term investment fund offers a technology subsidy (κ) and a bonus on long-term abatement project (σ).

Carbon pricing

The household is subject to regulation via an emission permit budget T_0 specified by the regulator. To give the regulator control over the growth rate of the carbon price, emission permits are discounted at an intertemporal trading ratio by the regulator when banked. At $t = 1$ a banked permit covers e^{γ} emissions (instead of 1) where γ is the intertemporal trading ratio.[1] The necessary abatement at $t = 1$ can be expressed in terms of abatement at $t = 0$ and the emission permit budget:

$$\begin{aligned} T_0 =& (Q_0 Y_0 - K_0) + e^{-\gamma}(Q_1 Y_1 - K_1) \\ \Leftrightarrow \; K_1(K_0) =& Q_1 Y_1 + e^{\gamma}(Q_0 Y_0 - K_0 - T_0) \end{aligned} \tag{7}$$

Note that the initial emission permit allocation T_0 needs to be adjusted in case of $\gamma \neq 0$ for overall emission to remain in (and exhaust) the budget T according to the following rule.

$$T_0 = T + \tau_1(e^{-\gamma} - 1) \tag{8}$$

Technology & investment subsidies

The long-term investment fund can play two roles. It can help internalize the technology externality by paying a subsidy rate of κ on abatement expenditures $A_0(K_0)$.[2] The long-term investment fund can also pay a premium of $(1 + \sigma)$ on the benefits of abatement projects at $t = 0$ that reduce emissions by Δ_0 beyond a benchmark abatement level, say $\bar{K}_0$, such that $K_0 = \bar{K}_0 + \Delta_0$. We will see that the level of the benchmark only affects the total of revenues paid by the fund but is irrelevant for the incentive set by the policy. We can think of $(\bar{K}_0, \bar{K}_1)$ as the household's abatement choices in absence of fund policy. In general, an additional abatement project has two components. The cost component with $A_0(K_0) - A_0(\bar{K}_0)$ representing the additional abatement costs at $t = 0$, and the benefit component, where $A_1(\bar{K}_0, \bar{K}_1) - A_1(K_0, K_1(K_0))$, represents cost savings relative to the baseline as additional K_0 allows to reduce K_1 while still complying with the carbon budget T.

The budget equations of the household read

$$\begin{aligned} C_0 &= Y_0 - (1 - \kappa)[A_0(\bar{K}_0) + A_0(K_0) - A_0(\bar{K}_0)] \\ &= Y_0 - (1 - \kappa)A_0(K_0) \end{aligned} \tag{9}$$

$$\begin{aligned} C_1 &= Y_0 - A_1(\bar{K}_0, \bar{K}_1) + (1 + \sigma)[A_1(\bar{K}_0, \bar{K}_1) - A_1(K_0, K_1(K_0))] \\ &= Y_1 - (1 + \sigma)A_1(K_0, K_1(K_0)) + \sigma A_1(\bar{K}_0, \bar{K}_1) \end{aligned} \tag{10}$$

Households' optimization problem

Together with the objective to maximize welfare, household's problems hence becomes

[1] *Intertemporal trading ratios* are due to Leiby and Rubin (2001). We assume that the regulator adjusts T_0 in anticipation of the allocation such that in the end emissions do not exceed the carbon budget T.

[2] Like the investment subsidy below, the fund could also pay the technology subsidy for *additional* projects only. In this case the cost of the project would become $(1 - \kappa)A_0(K_0) + \kappa A_0(\bar{K}_0)$, i.e. abatement costs would be reduced for all of A_0 by $(1 - \kappa)$ except for abatement up to $\bar{K}_0$.

$$\max_{\{K_0\}} u(C_0) + e^{-\rho} u(C_1) \tag{11}$$

$$\begin{aligned} \text{subject to (9), (10)}: &\ \text{budget equations} \\ (7): &\ \text{emission permit budget} \\ \text{and given } (T_0, \gamma): &\ \text{the regulator's instruments} \\ (\sigma, \kappa): &\ \text{the fund's instruments} \end{aligned}$$

In (10), the premium paid by the investment fund, $(1+\sigma)$, amplifies the reduction in abatement costs but the term $\sigma A_1(\bar{K}_0, \bar{K}_1)$ corrects for the fact that the subsidy should not apply to benchmark abatement.

The fund subsidy σ could be implemented as a *financial contract* at time $t = 0$ that guarantees that emission permits from any (certified) additional emission reductions in $t = 0$ (defined as Δ_0 above) can be sold to the fund at $t = 1$ at a markup of $(1 + \sigma)$ above the market price. The fund would, in turn, resell the emission permits to the market (and hence to the household) at the market price, which should be $A_1'(K_0, K_1) \equiv \partial A_1 / \partial K_1$.[3]

The first-order condition reads

$$u'(C_0)\,(1-\kappa)\,A_0' = e^{-\rho} e^{\gamma} \mathbb{E}\left[u'\,(C_1)\,(1+\sigma)\left(A_1' - \frac{\partial A_1}{\partial K_0} \right) \right] \tag{12}$$

If households could not appropriate (or did not anticipate) any of the technology learning $\frac{\partial A_1}{\partial K_0}$ then they would act as if $-\frac{\partial A_1}{\partial K_0} = 0$. We model partial appropriation by introducing a scaling parameter ψ as a measure of the market failure into (12) like this:

$$u'\,(C_0)\,(1-\kappa)\,A_0' = e^{-\rho} \mathbb{E}\left[u'\,(C_1)\,e^{\gamma}\,(1+\sigma)\left(A_1' - (1-\psi)\,\frac{\partial A_1}{\partial K_0} \right) \right] \tag{13}$$

For $\psi = 1$ technological progress in $t = 1$ from abatement in $t = 0$ is a pure externality. That is, households do not take into consideration the feedback effect of abatement learning. For any $\psi < 1$, part of the externality is anticipated and thus internalized. For $\psi = 0$ there is no technology externality. Equation 13 shows how the household prices abatement investment.

[3] Strictly speaking, the only participants in the permit market of the model economy are the (representative) entrepreneur and the fund. The gross abatement cost of the entrepreneur at $t = 1$ (in the absence of the investment subsidy σ which is only offered for early abatement at $t = 0$) is $A_1'(K_1)$ and hence the highest price that the fund (as the only seller in this market) can set.

We can re-express asset pricing equation (13) using the risk free rate $r_f = \rho - \log\left(\frac{\mathrm{E}[u'(C_1)]}{u'(C_0)}\right)$ $\Leftrightarrow e^{\rho - r_f} = \frac{\mathrm{E}[u'(C_1)]}{u'(C_0)}$

$$\begin{aligned}(1-\kappa)\,A_0' =& e^{-r_f}\mathbb{E}\left[e^{\gamma}\,(1+\sigma)\left(A_1' - (1-\psi)\,\frac{\partial A_1}{\partial K_0}\right)\right] \\ &+ \mathrm{Cov}(u'(C_1), e^{\gamma}\,(1+\sigma)\,(A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}))/u'\,(C_0)\end{aligned} \tag{14}$$

And, with assumption of (log)normality as in Lemma 1

$$(1-\kappa)\,A_0' = e^{-(r_f+\phi\eta)}\mathbb{E}\left[e^{\gamma}\,(1+\sigma)\left(A_1' - (1-\psi)\,\frac{\partial A_1}{\partial K_0}\right)\right] \tag{15}$$

Equation (14) underlines the specific role of covariance in pricing abatement. It is precisely the covariance term in (14) which translates into the risk premium of (15). When asset return and marginal utility of consumption are uncorrelated and hence the covariance term in (14) vanish, so will the risk premium. Intuitively, while the asset return remained uncertain, it would then have no systematic effect on the marginal utility.

In this asset pricing equation (15), we consider $(1-\kappa)\,A_0'$ the price (or cost) of the asset, $e^{\gamma}\,(1+\sigma)\cdot\left(A_1' - (1-\psi)\,\frac{\partial A_1}{\partial K_0}\right)$ the asset benefit, and $r_A = r_f + \phi\eta$ the equilibrium return of the asset. The asset pricing equation (6) of the social planner has a similar in structure. It differs in that it takes into consideration the decay rate δ, while this is not part of the household's pricing equation since it is not anticipated by her as mentioned earlier. Instead, the marginal abatement in period 1, in the household's derived pricing equation (15), is amplified by the intertemporal trading ratio γ. In case there is no technology learning, the intertemporal trading ratio is optimal when it is equal in absolute terms to the decay rate. When γ is greater than optimal, it implies an underinvestment in the initial period 0.

Both policy instruments (κ and σ) play in essence a similar role in attempting to correct the distortions arising from the externalities or factors of political economy nature. Yet, the timing of their implementation is different. κ as a technology subsidy is paid upfront and is reduces abatement costs in period 0. σ is a contractual agreed 'reward' for investing in abatement in period 0 that is received in the next period.

Equation (15) suggests that mispricing of the asset, e.g. from ill-specified intertemporal trading ratio γ, that distorts the ratio of asset benefit to its cost can be rectified by σ or κ because both instruments directly affect the benefit-cost ratio.

3.4. Regulator and regulatory failures

A number of reasons can lead to inefficient carbon pricing; lobbying from interest groups (i.e. fossil fuel industrialists), and the government inability to commit to intended policies are some of the examples. Hence, even if benevolent regulators follow the same objective function as the one of the social planner and households, they may find themselves unable to implement efficient carbon pricing policies. Sub-optimal growth rate of the carbon price, as a result of political economy considerations, can be reflected for example in our model by an inefficient intertemporal trading ratio γ. More concretely, we can think of the case where the regulator relies on integrated assessment results as summarized in (IPCC WG3, 2014). As we know from Gollier (2020), the resulting growth rates of the carbon price would diverge from the optimum of this model. Another distortionary mechanism of market equilibrium can result from the external effect of technology learning introduced via the parameter ψ. Finally, another distortion can result from the lack of a commitment device for the carbon budget T by the regulator; making the announced emission permit budget T_0 in period 0 not credible.

4. The investment fund: optimal instruments

Parameters γ and ψ distort the asset pricing equation of the household compared to the social optimum. The next section derives instruments σ and κ to address the distortions analytically by comparing these asset pricing conditions. To this end, we equate the asset returns expressions of household and social planner, the right-hand-sides of equations (5) and (13).

$$e^{-(\rho+\delta)}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right] = e^{-\rho+\gamma}\mathbb{E}\left[u'\left(C_1\right)\frac{(1+\sigma)}{(1-\kappa)}\left(A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)\right] \tag{16}$$

$$\frac{1+\sigma}{1-\kappa} = e^{-(\gamma+\delta)}\frac{\mathbb{E}\left[u'(C_1)\left(A_1' - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[u'(C_1)\left(A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)\right]} \tag{17}$$

For single policies, i.e. $\sigma|_{\kappa=0}$ or $\kappa|_{\sigma=0}$, we have

$$\sigma = e^{-(\gamma+\delta)}\frac{\mathbb{E}\left[u'(C_1)\left(A_1' - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[u'(C_1)\left(A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)\right]} - 1 \tag{18}$$

$$\kappa = 1 - e^{\gamma+\delta}\frac{\mathbb{E}\left[u'(C_1)\left(A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[u'(C_1)\left(A_1' - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right]} \tag{19}$$

Applying Lemma 1, we rewrite the asset pricing equation

$$u'(C_0)A'_0 = e^{-\rho+\gamma}\mathbb{E}\left[u'(C_1)\frac{1+\sigma}{1-\kappa}\left(A'_1 - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)\right] \tag{20}$$

$$1 = \mathbb{E}\left[e^{-\rho}\frac{u'(C_1)}{u'(C_0)}e^{\gamma}\left(\frac{\left(A'_1 - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)}{A'_0}\frac{1+\sigma}{1-\kappa}\right)\right] \tag{21}$$

We define $A_1'^n$ such that

$$A_1'^n = A'_1 - (1-\psi)\frac{\partial A_1}{\partial K_0}$$

Now we need that $g_c \equiv \frac{C_1}{C_0} - 1$ and R_1 with

$$R_1 = e^{\gamma}\frac{A_1'^n(1+\sigma)}{A'_0(1-\kappa)} \tag{22}$$

to be jointly lognormally distributed.

Then

$$\frac{\mathbb{E}\left[A_1'^n \frac{1+\sigma}{1-\kappa}\right]}{A'_0} = e^{-\gamma+r_f+\tilde{\phi}\eta} \tag{23}$$

Here $\tilde{\phi}$ reflects the covariance of the asset return (with partial spillover) and subject to the various instruments.

Remember from (6) that it is optimal to price abatement according to

$$\frac{\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]}{A'_0} = e^{\delta+r_f+\phi\eta} \tag{24}$$

With some algebraic manipulation equation (24) is transformed into an analogous form

$$\mathbb{E}\left[e^{\phi\eta-\tilde{\phi}\eta}\frac{A_1'^n}{A'_0}\frac{(1+\sigma)}{(1-\kappa)}e^{(\gamma+\delta)}\right] = e^{\delta+r_f+\phi\eta} \tag{25}$$

We solve for optimal fund instruments by comparing expectations in (24) and (25).

4.1. Fund investment support σ

For an optimum σ that brings household behavior and planner in line we have the following based on (17):

$$\frac{1+\sigma}{1-\kappa} = e^{-(\delta+\gamma)} \frac{\mathbb{E}\left[u'(C_1)\left(A_1' - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[u'(C_1)\left(A_1'^n\right)\right]} \tag{26}$$

with some algebraic manipulations[4], we get the following equation:

$$\frac{1+\sigma}{1-\kappa} = e^{-(\delta+\gamma)}\left(\frac{\mathbb{E}\left[A_1'\right]}{\mathbb{E}\left[A_1'^n\right]} - \frac{\mathbb{E}\left[e^{\delta}\frac{\partial A_1}{\partial K_0}\right]}{\mathbb{E}\left[A_1'^n\right]}\right) + e^{-\gamma}\frac{\text{Cov}\left[u'(C_1), Z_1\right] - \text{Cov}\left[u'(C_1), Z_2\right]}{\mathbb{E}\left[u'(C_1)\right]\mathbb{E}\left[A_1'^n\right]} \tag{27}$$

where $Z_1 := e^{-\delta}\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)$ and $Z_2 := e^{\gamma}\frac{1+\sigma}{1-\kappa}A_1'^n$

Through equation (27), we have an asset price representation emphasizing the role of the policy instruments (left-hand-side, i.e. σ and κ). As briefly referred to earlier, we consider three distortion cases that policy instruments need to address. Namely, the case when the intertemporal trading ratio γ is not the optimal one, i.e. $\gamma \neq -\delta$; the case when technology learning spillover, $\frac{\partial A_1}{\partial K_0} \neq 0$, is not fully anticipated by households; the third case also related to technology spillover is when market participants have different perceptions on how the return of mitigation investment (co-)varies with consumption. The two instruments σ and κ are perfect substitutes. Both can be equally used to correct the distortions. We do not necessarily need to use both instrument to remedy the market distortions; σ and κ work in complementary fashion, i.e. we can use $\sigma = \kappa/(1-\kappa)$ instead of κ, and $\kappa = \sigma/(1+\sigma)$ instead of σ.

Alternatively,

$$\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right] = \mathbb{E}\left[e^{\phi\eta - \tilde{\phi}\eta}A_1'^n\frac{1+\sigma}{1-\kappa}e^{(\gamma+\delta)}\right]$$

$$\frac{1+\sigma}{1-\kappa} = e^{-(\gamma+\delta+(\phi-\tilde{\phi})\eta)}\frac{\mathbb{E}\left[A_1' - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]}{\mathbb{E}[A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}]} \tag{28}$$

Without learning spillover $\left(\frac{\partial A_1}{\partial K_0} = 0\right)$, or if the internalization ψ balances with the rate of decay (which is ignored by the household) such that $(1-\psi) = e^{\delta}$ then the expression in parentheses reduces to 1 and with ($\kappa = 0$), (28) simplifies to

$$(1+\sigma) = e^{-(\gamma+\delta+(\phi-\tilde{\phi})\eta)} \tag{29}$$

[4]See Appendix A for more detailed derivations

Hence, when there are no differences in risk perception (i.e. $\phi = \tilde{\phi}$), we get the same result as in (27). Equation (29) shows that in addition to correcting policy failures in the growth rate of the carbon price, σ can correct any deviation in risk premium as perceived by planner (ϕ) and household ($\tilde{\phi}$).

4.2. Fund technology subsidy κ

Solving for technology subsidy κ in equation (28), we get:

$$\kappa = 1 - e^{(\phi-\tilde{\phi})\eta} e^{(\gamma+\delta)}(1+\sigma) \frac{\mathbb{E}\left[A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}\right]}{\mathbb{E}\left[A_1' - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]} \tag{30}$$

Equation (30) reveals that in the absence of a fund investment support, the technology subsidy can also be used to address carbon pricing policy failure.

In summary, the instruments of the investment fund can be used to affect asset returns and thus steer the economy towards the socially optimal equilibrium. We assume that the fund imposes lump-sum taxes on the economy to generate the means for the required financial incentives. But the role of the risk premium (carbon beta) remained unclear. In the next section we turn to numerical simulations to shed light on the role of the carbon beta as the key determinant of the risk premium, and to estimate order-of-magnitudes of the distortionary effects. But first, we discuss the calibration.

5. Calibration

In most parameters choices, we follow Gollier (2020), which we summarize in Table 1. New parameters enter the model in the extension to technology learning.

Abatement cost function with learning.

Continued use of a given technology builds experience which translates into an improved efficiency of the technology. A prominent approach that captures such technology learning *by doing* is to make its marginal cost dependent on the past cumulative investment (see Guo and Fan, 2017, for a recent example). Samadi (2018) reports technological *learning rates* for electricity generation, where a learning rate of *lr* indicates a decrease in costs for each doubling of cumulative installed capacity. The study includes estimates for future learning rates for renewable energy technologies, reporting 3-5% for wind turbines and 12-20% for solar photovoltaics.

Parameter Descriptions	Notations	Values
annual rate of pure preference for the present	ρ	0.5%
parameter of relative risk aversion	γ	3
annual probability of a macroeconomic catastrophe	p	1.7%
mean growth rate of production in a business-as-usual year	μ_{bau}	2%
volatility of the growth rate of production in a business-as-usual year	σ_{bau}	2%
mean growth rate of production in a catastrophic year	μ_{cat}	-35%
volatility of the growth rate of production in a catastrophic year	σ_{cat}	25%
production in the first period (in GUS$)	Y_0	315,000
annual rate of natural decay of CO_2 in the atmosphere	δ	0.5%
carbon intensity of production in period 0 (in $GtCO_2e/GUS\$$)	Q_0	2.10×10^{-4}
carbon intensity of production in period 1 (in $GtCO_2e/GUS\$$)	Q_1	1.85×10^{-4}
expected carbon budget (in $GtCO_2e$)	μ_T	40
standard deviation of the carbon budget (in $GtCO_2e$)	σ_T	10
slope of the marginal abatement cost functions (in $GUS\$/GtCO_2e^2$)	b	1.67
slope of marginal abatement cost with learning (in $GUS\$/GtCO_2e^2$)	c_0	5.04
technology learning rate (in percent)	lr	20.0
marginal cost of abatement in the BAU, first period (in $GUS\$/GtCO_2e$)	a_0	23
expected future log marginal abatement cost in BAU	μ_θ	2.31
standard deviation of future log marginal abatement cost in BAU	σ_θ	1.21

Table 1: Benchmark calibration of the two-period model.

We build on the abatement cost function of Gollier (2020) at $t = 1$ ($A_1(K_1) = \theta K_1 + \frac{1}{2} b K_1^2$). In line with Guo and Fan (2017), we include the learning dynamics in the non-linear term. Here, past experience is simply given by first period investment K_0.

$$A_1(K_1, K_0) = \theta K_1 + \frac{1}{2} c_0 K_1^2 K_0^{-\alpha}$$

We explore an optimistic scenario with learning rate of $lr = 20\%$ at the upper end of the empirically observed learning rates (Samadi, 2018). The learning rate lr translates to the learning elasticity parameter as $lr = 1 - 2^\alpha$. For consistency with the original calibration, we adjust $c_0 < b$ such that $A_1(K_1) \approx A_1(K_1, K_0)$, taking (K_0, K_1) from the equilibrium without technology learning.

6. Numerical results

In this section we consider three distortions that rationalize an underinvestment: ill-adjusted carbon prices set by the regulator, technology learning as an externality to the household and non-credibility of the emission budget such that households expect issuance of additional emission permits. For all three distortions, we analyze their impact on asset pricing and risk premium,

scenario	K_0	$E[K_1]$	p_0	$E[p_1]$	r_f	ϕ	η	$r_f + \phi\eta$	welfare
$\gamma = 0.0$	31.0	66.0	74.7	131.0	1.23	0.99	2.27	3.48	0.754
$\gamma = -4.5\%$	17.0	78.9	51.4	152.6	1.06	0.77	2.28	2.83	0.807
$\psi = 0$	35.0	62.2	81.5	120.6	1.28	1.30	2.27	4.22	0.747
$\psi = 1$	32.5	64.5	77.3	126.8	1.25	1.02	2.27	3.57	0.750
$T = T^{gov}$	31.0	66.0	74.7	131.0	1.23	0.99	2.27	3.48	0.754
$T = T^{hh}$	15.3	80.5	48.5	155.3	1.04	0.75	2.28	2.76	0.820

Table 2: Overview of distortions. We report the effects of distorting the growth rate of the carbon price via the intertemporal trading ration γ, the effects of an externality in technology learning (parameter ψ), and the effect of (wrongly) expecting a doubling of the carbon budget T by issuing additional permits in the second period. Welfare is given as the difference to a no-policy scenario in percent of constant welfare-equivalent consumption levels.

and suggest investment fund policies to overcome the resulting inefficiencies. As above, fund instruments consists of subsidies on up-front capital costs and premium on investment returns.

Table 2 gives a preview of key insights: we summarize the impact of the three distortions on key variables of the model. All distortions delay abatement (note though, that with technology learning, the efficient reference case (row 3) differs from the reference cases for the other distortions in rows 1 and 5), and hence produce a steeper carbon price path. In the distorted equilibria, the 'carbon beta' ϕ lies below the reference case by a similar amount. As a measure of the distortion when the regulator's announcement of the carbon budget is non-credible, we show a scenario where the household invests in anticipation of a higher T but has to face the originally announced low T in the second period. The welfare costs of the distortion are largest for the this scenario (about 7 basis points) and second largest for distorted carbon price path (5 basis points, row 2). The effect of the technology externality on ϕ is similar but results in smaller underinvestment, the associated welfare costs are hence lower.

6.1. Ill-specified carbon price path

Due to various political economy reasons, regulator might impose a sub-optimal carbon price trajectory on the economy. Therefore, we discuss first the failure of the regulator to specify the optimal intertemporal trading ratio $\gamma = \gamma^*$. For simplicity, we consider one distortion at a time, i.e. this section is without technology learning (i.e. $lr = 0$). Hence, the optimal intertemporal trading ratio only reflects emissions decay $\gamma^* = -\delta$.

A central finding in Gollier (2020) is the divergence of the optimal growth rate of the carbon price derived from asset pricing approach (3.76%) and the carbon price growth rates found in the literature. As a point of reference, Gollier cites the average growth rate model scenarios in the

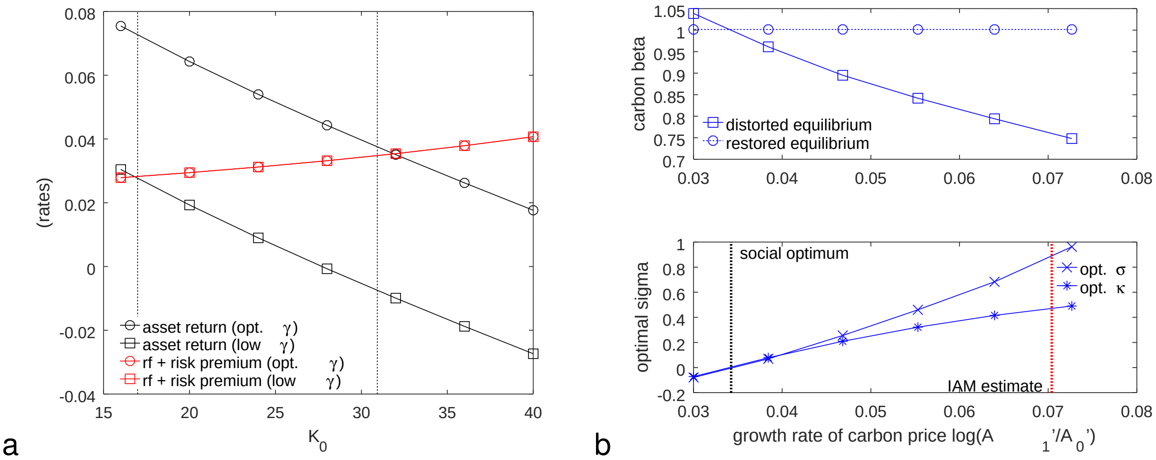

Figure 1: Ill-specified carbon pricing. Panel a shows the actual asset return as a function of K_0 around the equilibrium (in black) as well as the return demanded by investors (in red). Intersection curves of the same color indicate an equilibrium – at higher levels of investment, the actual return falls short of the demanded return and vice versa when K_0 is below equilibrium levels. Vertical lines indicate the precise equilibria; as the computation of risk premiums presumed normal distributions intersections and equilibria may diverge by small measures. Panel b documents the implementation of the optimal σ-policy a variation of γ over the range $\gamma \in [-0.05, 0.0]$. On top, we show the carbon beta in the distorted equilibrium and the equilibrium restored to the social optimum by the optimal fund subsidy σ. On the bottom, we show the optimal instruments (σ and κ).

IPCC database that is consistent with 2 degree warming (7.04%). We translate this into a policy failure scenario, where a regulator specifies an emission permit system with an intertemporal trading ratio γ such that the growth rate of the carbon price equals about 7% ($\gamma = -0.045$).

The ill-specified intertemporal trading ratio γ is below the optimal level, thus penalizing future use of permits more. This reduces the asset benefit of abatement, which arises from saving a permit for future use and creates the incentive to reduce current abatement K_0.[5] Figure 1a shows the asset return based on the cost of investment (marginal abatement costs A'_0) and its benefit (taking A'_1 and γ into account) consistent with the ill-adjusted carbon price. Unsurprisingly, the return is declining in K_0.[6]

Figure 1a also shows the required rate of return (the sum of risk-free rate and risk premium, $r_f + \phi\eta$). As the risk-free rate r_f and the systematic risk premium η barely move (cf. Table 2), the upward slope is mainly driven by the carbon beta (ϕ). Round points represent the socially optimum equilibrium (for $\gamma^* = -\delta = -0.5\%$). Square points show the distorted equilibrium for

[5]If the overall carbon budget was reduced by discounting permits, we would also see an incentive to use permits earlier. Remember though, that by assumption the regulator corrects for the discounting to implement the intertemporal trading ratio budget neutral (equation (8)).

[6]More K_0 implies less K_1; as $A''_t(K_t) > 0$ more K_0 implies lower benefits at higher costs, in other words a lower growth rate of the carbon price.

($\gamma = -4.5\%$). An intertemporal trading ratio $\gamma < \gamma^*$ reduces the benefit of any investment K_0. The return curve thus shifts downward. The risk premium is unaffected. The ill-specified γ-policy distorts household abatement choice and the equilibrium allocation but has no fundamental effect on the risk premium.

Why is γ 'neutral' with respect to the risk premium? The carbon beta is determined by the co-variance of marginal abatement costs and consumption growth. The growth rate of carbon prices changes the timing of marginal abatement costs but also the timing of consumption. Both effects do not change the co-variance between consumption and abatement investments (as the residual $C_1 = Y_1 - A_1(K_1)$). Therefore, the intertemporal trading ratio γ has no effect on the underlying relationship of abatement and consumption. This will be different in our later experiments with technology externality and with diverging emission budget, as both technology learning and relaxed emission budget affect the overall costs of the emission budget rather than its allocation over time.

In the distorted equilibrium, the carbon beta is below the social optimum value. This is because the carbon beta is declining in K_0.

Investment fund policies

The fund can use either instrument (σ or κ) to restore the social optimum as the market equilibrium. The top of Figure 1b shows the equilibrium carbon beta for a range of distorted equilibria where we vary γ over the interval $[-0.05, 0.0]$ to produce the carbon price growth rates shown on the x-axis. Below, we report the necessary instruments, either σ or κ, to restore the economy to the efficient equilibrium.

To understand the numerical difference of the two instruments, recall that for an equivalent effect from either σ or κ we need $(1 + \sigma) = (1 - \kappa)^{-1}$, or $\sigma = \kappa/(1 - \kappa)$. To have the same effect of a κ close to 100%, the regulator would need to use an infinitely large σ.

6.2. Technology externality

The technology externality ψ is a measure of how much technological learning (learning by doing) at a fixed learning rate (lr) is internalized by the household. The asset pricing equation of the household (15) reads

$$(1 - \kappa) A_0' = e^{-(r_f + \phi\eta)} \mathbb{E}\left[e^{\gamma} (1 + \sigma) \left(A_1' - (1 - \psi) \frac{\partial A_1}{\partial K_0} \right) \right] \tag{31}$$

In analogy to Figure 1 for the ill-specified carbon price path, Figure 2 shows asset return and the required rate of return by investors for the economy extended by technological learning. Notice

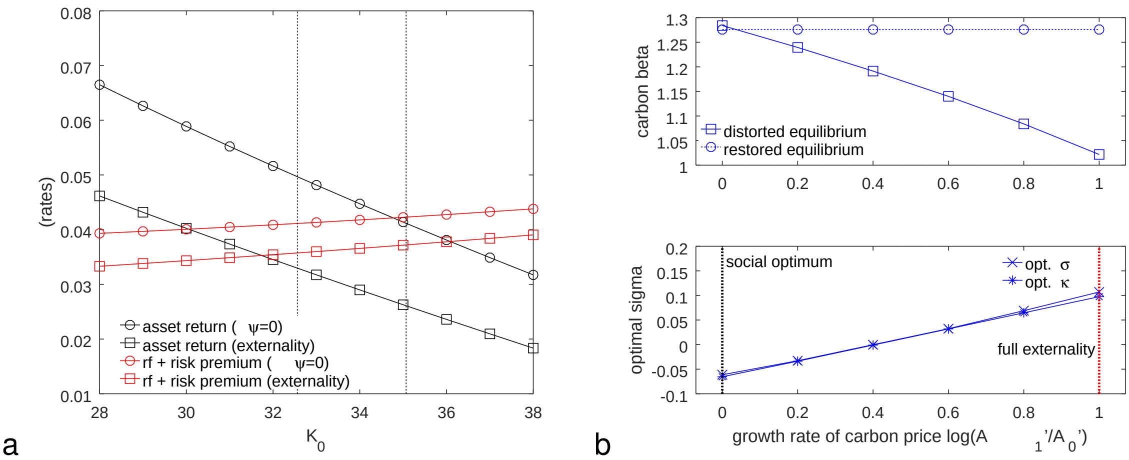

Figure 2: Technology externality. Panel a shows the actual asset return as a function of K_0 around the equilibrium (in black) as well as the return demanded by investors (in red). Intersection curves of the same color indicate an equilibrium (cf. Figure 1). Panel b documents the implementation of the optimal σ-policy a variation of γ over the range $\gamma \in [-0.05, 0.0]$. On top, we show the carbon beta in the distorted equilibrium and the equilibrium restored to the social optimum by the optimal fund subsidy σ. On the bottom, we show the optimal instruments (σ and κ).

that the optimum K_0 is higher with learning ($K_0 \approx 34.5$ instead of $K_0 \approx 30.5$), even though we calibrate the abatement cost function such that the social optimum of Section 6.1 still has the same marginal costs. With technological learning the optimal K_0 is higher because the learning triggered by K_0 raises the return. The asset return curve of the undistorted equilibrium (for $\psi = 0$, round bullet in Figure 2a) has thus shifted upwards (compared to Figure 1).

When technology learning is an externality ($\psi = 1$, square points in Figure 2a), this reduces the asset return. Ceteris paribus, this shifts the equilibrium to the left. That is, the household will underinvest in K_0. But ($\psi = 1$) has a second effect: it reduces the risk premium. The carbon beta captures how the asset return covaries with consumption C_1 (cf. Gollier, 2020, Fig. 3), linked by abatement costs A_1 in the household's budget equation ($C_1 = Y_1 - A_1(K_0, K_1)$). With technology learning, A_1 depends on K_0. When this is ignored by households, this translates to their estimate of the carbon beta and hence the risk premium.

Technology learning reduces abatement costs, hence reducing the expected return on investment. This increases investment risk because it reduces the chance that future economic output Y_1 is high enough (and emissions likewise) to drive up marginal abatement costs to levels such that the investment become profitable. The carbon beta in the undistorted equilibrium is therefore higher in the presence of technology learning ceteris paribus.

The technology *externality* is caused by the households' ignorance of technological learning, hence its effect on the carbon beta is opposite: the larger the extent to which technology learning is external (larger ψ), the lower the carbon beta from the perspective of the household. The reduced carbon beta and hence lower required rate of return would raise investment. The implications of ψ on actual asset return effect and the required rate of return including the risk premium thus work in opposite directions. In this calibration, the asset pricing effect is dampened by the risk premium effect but the former outweighs the latter: the overall effect of the technology externality is an underinvestment in K_0.

Investment fund subsidies

Again the investment fund can use either instrument to restore the efficient equilibrium (cf. Figure 2b). Additional numbers are shown in the appendix in Table B.5, which reports the financial variables for variations of the externality parameter ψ and both policy instruments σ and κ.

Both κ and σ ultimately increase the rate of return r_A. But the κ-policy works by reducing the asset price I_0 (net costs) while the σ-policy works by increasing asset benefit B_1.[7]

Climate policy uphill battle

With this endogenous change of the carbon beta climate policy becomes an uphill battle. As successful climate policy produces more abatement, in our case via the investment premium σ of the fund, resulting in a higher (carbon) beta of abatement and thus the risk premium demanded for subsequent investment in abatement projects. Figure 3 illustrates the effect for the distorted carbon price of Section 6.1. The investment subsidy σ increases investment in the first period but with decreasing marginal effectiveness as it increases the carbon beta with higher abatement levels. The asset return is reduced more strongly when consumption is low and thus the co-variance of asset return and consumption is strengthened. An investment that pays off in bad states of the world is worth more than an investment that pays off in good states. Paradoxically, successful climate policy leads to a higher risk premium for additional abatement. Risk in terms of co-variance instead of variance is one of the most important insights of financial economics to the design of policy instruments: The more policy makers want to push green abatement technologies the higher the risk premium demanded by investors. The social discount rate and the carbon price trajectory has to be adjusted accordingly, otherwise the risk an of overinvestment in green assets is increasing.

6.3. Policy failure: non-credible climate policy

In this section we explore scenarios where the regulator is unable to commit to the low emission budget $T_0 = T^{gov}$ announced at $t = 0$. A regulator that is not bound to carry out the announced

[7]The asset price (benefit) moves in the same direction when the asset benefit (price) is affected by σ (κ) but it both cases this effect is smaller such that in total, the rate of return r_A increases.

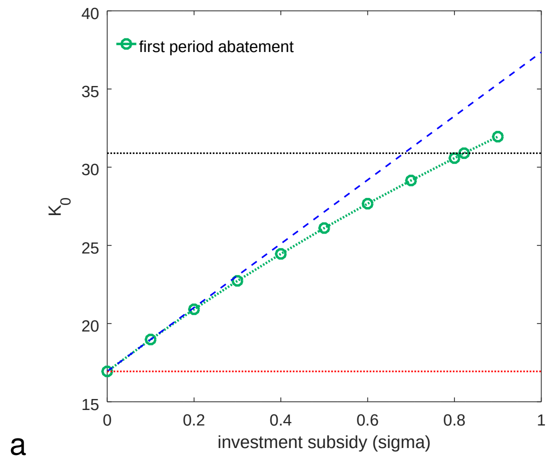

Figure 3: Fund subsidy. Effect on capital accumulation

climate policy may reconsider the emission permit budget at $t = 1$ and opt for a more lenient policy by issuing additional emission permits. The regulator may me prompted to reconsider the original policy if the investor does not invest as expected when announcing the policy and as would have been optimal. Gersbach and Glazer (1999) argue that investors may trigger the reconsideration by the regulator by strategically choosing not to invest. Below, we will suggest a discrete game of regulator and investor that creates a similar incentive problem. When we use the numerical model to compute the payoff structure for plausible assumptions, we find that it is indeed rational for the investor to hold up on investment.

Figure 4a visualizes the impact of shifting from a stringent budget of $T^{low} = 40$ GtCO2e to a lenient budget of $T^{high} = 80$ GtCO2e. The rate of return is substantially lower for the inflated budget, which reflects the deteriorated carbon price for this budget. The risk premium is considerably higher for the larger budget driven by a more than two-fold increase of the carbon beta, reflecting the lower chance of an abatement investment paying off at this large budget. When the household expects a larger emission budget, then the asset return is lower because – ceteris paribus – the carbon price in $t = 1$ will be lower. This implies underinvestment in K_0. But the investor will also demand a higher risk premium. The two effects work in the same direction thus amplifying the underinvestment.

Non-credible regulator

To introduce the commitment problem into the model, we consider the case where the regulator and household consider two discrete possibilities for the ultimate emission budget: when the regulator remains steadfast, no further emissions are issued and the ultimate budget will be

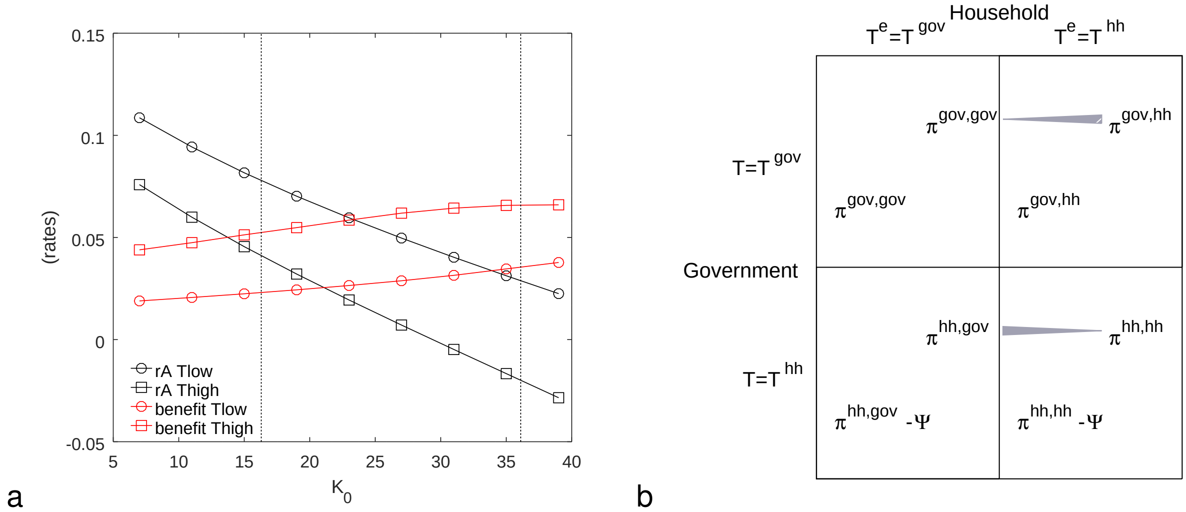

Figure 4: Commitment failure. Panel a shows the actual asset return as a function of K_0 around the equilibrium (in black) as well as the return demanded by investors (in red). Intersection curves of the same color indicate an equilibrium (cf. Figure 1).

equal to the announced budget. Else a regulator who topples in their resolve will issue additional emission permits ultimately imposing a high emission budget of $T_0 = T^{hh}$ with $T^{hh} < T^{gov}$.

When the regulator is able to commit to the emission budget $T_0 = T^{gov}$ at $t = 0$, the expectation of the household regarding the emission budget at $t = 1$ will be in line with the announcement, i.e. $T^e = T^{gov}$. When the announcement is non-credible, the household will expect $T^e = T^{hh}$.

This setup gives rise to a simple two-stage game where the household can take the investment decision K_0 either based on $T^e \in \{T^{gov}, T^{hh}\}$ at $t = 0$, whereupon the regulator faces the decision at $t = 1$ to stick with the announced emissions budget $T = T^{gov}$ or issue more permits to a total of $T = T^{hh}$. Figure 4b presents the payoffs in a 2×2 matrix, π^{ij} is the payoff of the household when expecting j while the regulator implements i. For a given choice of the regulator, the household will always prefer to act in accordance with it, i.e. $\pi^{gov,gov} > \pi^{gov,hh}$ and $\pi_{hh,hh} > \pi^{hh,gov}$, as this allows an efficient and hence welfare maximizing abatement choice (K_0, K_1). We indicate this preference by arrows in Figure 4b. Our modeling assumptions also imply that $\pi^{hh,hh}$ maximizes household welfare and in particular $\pi^{hh,hh} > \pi^{gov,gov}$ because the smaller budget implies higher abatement costs – and we did not include the benefits of avoided climate impacts in the model.

For the regulator, we assume a benevolent objective function, such that the regulator, too, maximizes household payoff π^{ij}. The regulator, though, incurs cost of failing to meet the announced emission budget, which we capture in a penalty term Ψ. The penalty may include the anticipated climate change damages to the economy, loss of reputation and cost of non-compliance with international climate treaties (cf. Kalkuhl et al., 2020, for a similar approach). Obviously the level

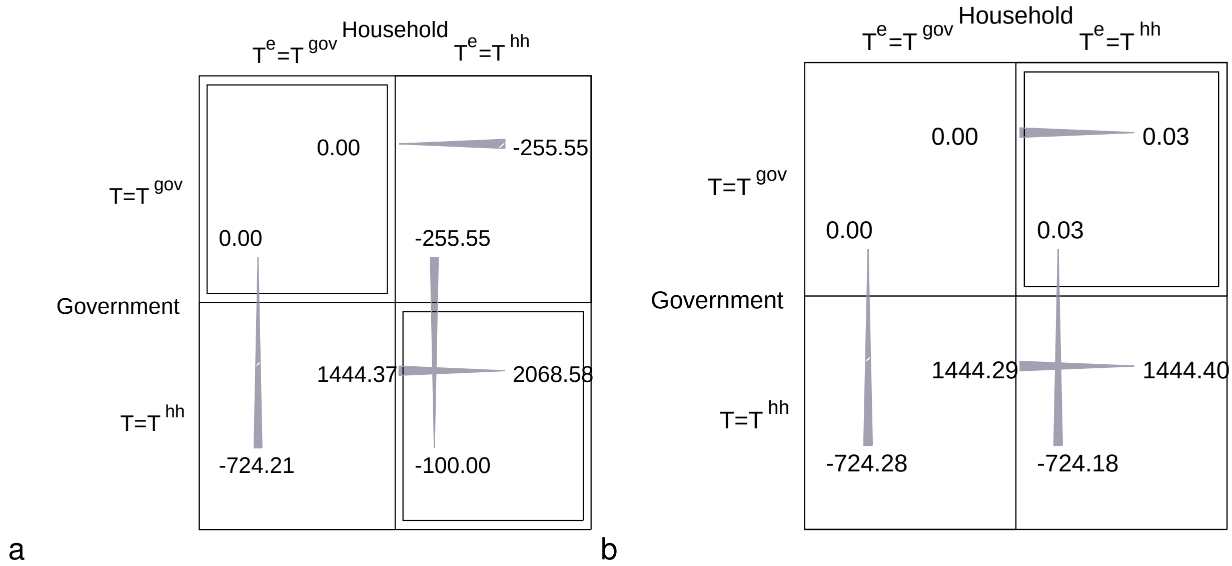

Figure 5: Payoff structure for the commitment game. Gray arrows indicate preferences, gray boxes indicate Nash equilibria without interference by the investment fund (panel a) and with σ policy by the fund (panel b).

of Ψ will be decisive for the preference of regulator. It is plausible to set $\Psi > \pi^{hh,hh} - \pi^{gov,gov}$ making the penalty severe enough to give the regulator a preference for the low budget T^{gov} given that the household goes along and also expects T^{gov} – or else regulator and household would agree that (T^{hh}, T^{hh}) was optimal. For excessively high penalties $\Psi > \pi^{hh,hh} - \pi^{gov,hh}$, $T = T^{gov}$ becomes a dominant strategy for the regulator. The household would anticipate this and hence expect $T^e = T^{gov}$. We therefore select $\Psi \in (\pi^{hh,hh} - \pi^{gov,gov}, \pi^{hh,hh} - \pi^{gov,hh})$.

Figure 5a shows the payoff matrix constant equivalent consumption levels of the household welfare, we normalize $\pi^{gov,gov}$ to zero (from US\$ 344 314) to ease comparison. The payoff structure supports two outcomes as Nash equilibria when regulator and household decide simultaneously, $(T^{gov,gov}, T^{gov,gov})$ and (T^{hh}, T^{hh}). In such a *Chicken Game*, where either one of the two equilibria is preferred by one actor, actors can pick the game outcome if they can credibly commit to a strategy. In our case, the household moves first (deciding on investment K_0 at $t = 0$) and can therefore select (T^{hh}, T^{hh}).

Intuitively, by delaying investment as if the household knew that the regulator will issue more permits up to T^{hh} at $t = 1$, the household creates a situation where the inefficiency of rushing enough abatement last minute for T^{gov} at $t = 1$ is so costly that at this point the regulator prefers to revise the policy and to issue additional permits.

Commitment by investment fund subsidies

The investment fund can support the regulator by subsidizing investment in abatement at $t = 0$ to raise investment K_0. As K_0 gets close to its optimal level for a low emission budget T^{gov}, the inefficiency of imposing T^{gov} even on a household that expected T^{hh} shrinks. Any $\sigma > 0$ thus reduces the incentive for the regulator to revise the announced policy of $T = T^{gov}$.[8]

Ideally, the fund subsidizes K_0 up to its optimal level for the T^{gov} budget. In this case, (T^{gov}, T^{hh}) is identical to (T^{gov}, T^{gov}), and becomes a Nash equilibrium instead of formerly (T^h, T^{hh}). This game-theoretic setting provides compelling insights into the sequencing of climate policy: The investment fund can pave the path for carbon pricing in latter periods, when subsidies are used to increase abatement in earlier periods. The risk premium can then be reduced to is socially optimal level. The intertemporal allocation of investment is not distorted. Additionally, due to investment funds strategy, the social optimum is a Nash equilibrium. It should also be noted that carbon pricing remains an essential part of the policy package, otherwise welfare costs will be increased and the budget constraint will be violated.

Table 3 presents additional numbers from the numerical simulations. Row 1 is the reference case with credible commitment, or (T^{gov}, T^{gov}). Rows 2-5 are (T^{hh}, T^{hh}) and (T^{gov}, T^{hh}), without and with optimal fund interventions, respectively. Rows 6-7 show analogous results for κ instead of σ. To compute rows 3, 5 and 7 we ran the model for $T = T^{hh}$ to compute the first stage decision of K_0. Then, taking K_0 as given we ran the model for the second stage of K_1 for $T = T^{gov}$. The investment decision in these row was taken with the expectation of a rate of return $E[r_A]$ reported in the row above, and therefore we omit values that related to the investment decision.

Rows 1 and 2 mirror the equilibria in Figure 4a for low and high T, respectively. As discussed above, moving to a larger emission budget lowers the asset return while increasing the risk premium, and investment in equilibrium K_1 is reduced for both of these reasons. In equilibrium, the asset return is higher for the larger budget (row 2 versus row 1) due to a higher growth rate of the carbon price – but at a price level at less than half of its row 1 value (cf. p_0, $E[p_1]$). The carbon beta and the risk premium more than double for the higher budget.

In the policy scenario (row 4) the investment subsidy σ is chosen such that K_0 (and hence p_0) match the reference case. Notice that the investment benefit B_1 includes the fund investment subsidy, such that the benefits (and subsequently the asset return) almost match the reference case benefits (row 1) despite a much lower carbon price at $t = 1$.

The risk premium remains high because the fund subsidy is a bonus paid in proportion to the

[8]This is exacerbated by the increasing inefficiency of the (T^{hh}, T^{hh}) outcome: as σ increases, the choice of K_0 will be inefficiently high.

Baseline with no credible carbon budget $T^{hh} = T^{gov}$															
T^{hh}	T^{gov}	σ	p_0	E[p_1]	I_0	E[B_1]	E[r_A]	K_0	E[K_1]	r_f	ϕ	η	$r_f + \phi\eta$	ΔW	ΔA_1
40	40	0.00	80.7	130.6	80.7	130.6	3.22	34.5	65.2	1.42	0.98	2.11	3.48	0.000	0.0
Noncredible carbon budget with $T^{hh} > T^{gov}$															
T^{hh}	T^{gov}	σ	p_0	E[p_1]	I_0	E[B_1]	E[r_A]	K_0	E[K_1]	r_f	ϕ	η	$r_f + \phi\eta$	ΔW	ΔA_1
100	100	0.00	38.6	71.6	38.6	71.6	4.12	9.3	30.8	1.46	2.15	2.17	6.13	0.000	0.0
100	40	0.00	38.6	172.7	38.6	172.7		9.3	90.4					-0.185	73.4
100	100	2.65	80.7	36.2	80.7	132.1	3.29	34.5	9.6	1.59	1.72	2.19	5.35	-0.183	-77.9
100	40	2.65	80.7	130.6	80.7	477.4		34.5	65.2					0.000	0.0
T^{hh}	T^{gov}	κ	p_0	E[p_1]	I_0	E[B_1]	E[r_A]	K_0	E[K_1]	r_f	ϕ	η	$r_f + \phi\eta$	ΔW	ΔA_1
100	100	0.73	22.1	36.2	22.1	36.2	3.29	34.5	9.6	1.59	1.72	2.19	5.35	-0.183	-77.9
100	40	0.73	22.1	130.7	22.1	130.7		34.5	65.2					0.000	0.0

Table 3: Policy failures fixed by adjusted σ. T^{hh} denotes the carbon budget expected by the household. In the baseline, the household accepts the regulators announced carbon budget as credible, hence $T^{hh} = T^{gov}$. In the following, the household expects the regulator's budget to be exceeded ($T^{hh} > T^{gov}$) and chooses K_0 based on this expectation. We explore scenarios where the regulator topples in his policy ($T^{gov} = T^{hh}$) or remains steadfast ($T^{gov} < T^{hh}$). When the long-term investment fund acts ($\sigma > 0$), its subsidy aims to lift K_0 to its optimum level. The last columns show efficiency costs of inaction by the regulator. We show the differences in welfare (ΔW in percent balanced growth equivalent change) and abatement costs (ΔA_1 in percent change) relative to the optimal solution for the carbon budget that is *implemented*.

asset return. The effect of the instrument is therefore on returns but not on risk, and any uncertainty about the asset return translates into an uncertainty about the corresponding subsidy payments.

The two cost metrics ΔW and ΔA_1 in the last columns measure gains in welfare (in percent balanced growth equivalents) and abatement costs (percent), respectively, relative to planner solution with the same carbon budget. Abatement costs are substantially higher in $t = 1$ if the household expected additional emission permits but the regulator does not supply any (row 3). The subsequent welfare loss underlines the temptation for the regulator to revise the announced policy. The scenario with intervention by the investment fund (row 4) shows similar welfare losses: it distorts the economy unnecessarily because the fund policy is undercut by the regulator revising the budget. But the substantially reduced abatement costs show that the fund policy has prepared the economy for a more ambitious carbon budget.

Rows 6-7 show analogous computations for a technology subsidy (κ) instead of the investment subsidy (σ). The result is almost identical. We highlight two differences:

- The technology subsidy reduces investment costs rather than boosting their benefit, which is reflected in much lower I_0 and B_1. The resulting $r_A = \log(B_1/I_0)$ is the same.
- Technology subsidy and investment subsidy differ in the time period when they are paid out. This lends an advantage to the technology subsidy which therefore has no commitment problem by definition. The investment subsidy is paid on returns that accrue at $t = 1$, whether this instrument is time-consistent therefore depends on the credibility of the funds commitment.

Arguably, the financial contract made out at $t = 0$ that guarantees the investment subsidy σ to the household at $t = 1$ is much harder to revert than it is to revise climate policy. Renegotiating the contract would in essence expropriate the household from the contractual benefits, and constitutional states often impose high barriers to protect their citizens from state arbitrariness. Nevertheless, it remains an advantage of the technology subsidy that such considerations are not necessary at all.

We summarize the results of the numerical analysis in Table 4, which reports the changes for asset prices and risk premium caused by the policy instruments under ceteris-paribus conditions and in a general equilibrium setting. In the first case, abatement investments are fixed, in the second-setting abatement investments are adjusted to the optimal level. All else the same, the suboptimal growth rate of the carbon price for a given carbon budget (row 1) does not change the risk premium (carbon beta), although in the distorted equilibrium the risks premium is lower (columns 3 and 4). In contrast, the technology externality reduces the risk premium while non-credibility of the emission budget raises the risk premium, all else the same (row 2).

	asset return		risk premium	
	ceteris paribus	equilibrium	ceteris paribus	equilibrium
ill-addjusted carbon price (γ)	$\ominus$	$\ominus$	$\odot$	$\ominus$
technology externality (ψ)	$\ominus$	$\ominus$	$\ominus$	$\ominus$
non-credibility (T^{gov})	$\ominus$	$\oplus$	$\oplus$	$\oplus$

Table 4: Summary of distortion effects on asset return and risk premium. We distinguish the direct effect, keeping all else the same (*ceteris paribus*), i.e. without adjustment of the investment decision K_0, and the entirety of equilibrium effects, i.e. after K_0 is adjusted to the new (distorted) equilibrium. We show reductions and increases relative to the undistorted equilibrium using $\ominus$ and $\oplus$, and denote neutrality with $\odot$.

7. Conclusions

This paper studies transition risk induced by climate policy in a CCAPM model, i.e. we focus primarily on risks to cash flows arising from a transition to a low-carbon economy induced by a carbon budget. The carbon budget captures physical risks of climate change such as the damage on productive assets indirectly: these risks are not modeled explicitly but are reflected by the budget in as much as they determine the choice of the carbon budget by the regulator. This modeling approach allows us to trace the carbon beta of abatement investments to climate policy instruments, e.g. carbon pricing, subsidies on up-front capital costs and investment premium. Additionally, we considered the risk premium induced by the lack of commitment of the regulator.

Studying climate policy through the lens of financial economics provide several crucial insights.

First, investments that pays off in a bad states of the world (with low economic growth) is worth more than an investment that pays off in good states (high economic growth). This basic truth from financial economics carries over to abatement investment and hence climate policy analysis: the co-variance, and therefore, the risk premium are key for the design of climate policy instruments. Policy instruments change the investment pathway and therefore, the climate beta. We find that the effect of climate policy instruments on the risk premium of abatement investments may be neutral, increasing or decreasing depending on the nature of the distortion that is addressed by the instrument.

Second, financial market actors such as an investment fund can in principle address the distortions by setting financial incentives for green investment but need to take into account the distorted risk perception of investors. As we have shown, financial incentives for investors can complement a carbon pricing policy and cure its dynamic inefficiency or pave the path towards more ambitious carbon pricing. The policy failure experiments have also emphasized the importance of carbon pricing. Carbon prices which reflect a lack of commitment exhibit a huge potential for increasing the risk premium, which then acts as a brake on abatement and climate policy.

Third, the nascent literature on applying asset pricing theory to climate change mitigation has focused its analysis on how risk and uncertainty affect first-best mitigation policies and the associated social costs of carbon or carbon price trajectories which are consistent with the carbon budget. The avenue which we have taken in this paper intends to connect the second-best analysis with this financial economics approach. We have shown that the risk premium is a fundamental endogenous variable determined by the regulator and agents on the financial markets, e.g. a long-term investment fund. The welfare losses of ill-adjusted risk premium might be significant given the mis-allocation of capital.

This study takes a first step to discuss climate policy as an asset pricing problem in a second-best setting. While the simple framework illustrates the key role of correlated risks our analysis remained stylized in many aspects with room for improvements and extensions. Not all of the distortions that we considered are modeled endogenously, and integrating a micro-foundation for the distortion (as with the technology externality) could produce further insights. Of course, extensions could add additional distortions to the model, short- and long-termism of investors or the introduction of incomplete markets into the model may be particularly interesting to shed light on the role of institutional investors.

The pioneering work of Nordhaus and of the IAMs have studied the interaction between climate change and the economy in deterministic setting. The design of first-best and second-best policy instruments have been carried out in a static and deterministic setting. The assessment of climate policy can benefit enormously when climate economics meets financial economics. The large uncertainties that are ubiquitous in the assessment of climate policy will enter investor decisions as substantial risk premiums. Getting a better understanding of level and the structure of risk premiums will help avoid the mis-allocation of scarce resources. That is what an important part of climate economics is all about.

Acknowledgments. The manuscript benefited from discussion in research seminars at the Potsdam Institute for Climate Impact Research and the Mercator Research Institute on Global Commons and Climate Change. We would in particular like to thanks Christian Gollier, Michael Jakob, Matthias Kalkuhl and Michael Pahle for their insightful comments.

References

Baker, S. R., Bloom, N., and Davis, S. J. (2016). Measuring Economic Policy Uncertainty. *The Quarterly Journal of Economics*, 131(4):1593–1636.

Bansal, R., Kiku, D., and Ochoa, M. (2016). Price of long-run temperature shifts in capital markets. *Capital Markets: Market Efficiency eJournal*.

Barnett, M. D. (2019). *A run on oil: Climate policy, stranded assets, and asset prices*. PhD thesis, The University of Chicago. Available at https://knowledge.uchicago.edu/record/1908.

Cai, Y., Judd, K. L., and Lontzek, T. S. (2013). The Social Cost of Stochastic and Irreversible Climate Change. *Working Paper 18704*, (January):1–38.

Cai, Y., Lenton, T. M., and Lontzek, T. S. (2016). Risk of multiple interacting tipping points should encourage rapid CO2 emission reduction. *Nature Climate Change*, 6(5):520–525.

Cai, Y. and Lontzek, T. S. (2019). The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734.

Cochrane, J. (2001). *Asset pricing*. Princeton Univ. Press, Princeton [u.a.].

Crost, B. and Traeger, C. (2014). Optimal co2 mitigation under damage risk valuation. *nature climate change*, (4):631—-636.

Daniel, K. D., Litterman, R. B., and Wagner, G. (2019). Declining CO2 price paths. *Proceedings of the National Academy of Sciences*, 116(42):20886–20891.

Dietz, S., Gollier, C., and Kessler, L. (2018). The climate beta. *Journal of Environmental Economics and Management*, 87:258–274.

Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–969.

Epstein, L. G. and Zin, S. E. (1991). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. *Journal of Political Economy*, 99(2):263–286.

Franks, M., Edenhofer, O., and Lessmann, K. (2015). Why Finance Ministers Favor Carbon Taxes, Even if They Do Not Take Climate Change into Account. *Environmental and Resource Economics*.

Gersbach, H. and Glazer, A. (1999). Markets and regulatory hold-up problems. *Journal of Environmental Economics and Management*, 37(2):151–164.

Giglio, S., Kelly, B., and Stroebel, J. (2020). Climate Finance.

Gollier, C. (2020). The cost-efficiency carbon pricing puzzle. Toulouse School of Economics, University of Toulouse-Capitole.

Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88.

Goulder, L. H. (2013). Climate change policy's interactions with the tax system. *Energy Economics*, 40:S3–S11.

Guo, J. X. and Fan, Y. (2017). Optimal abatement technology adoption based upon learning-by-doing with spillover effect. *Journal of Cleaner Production*, 143:539–548.

Hambel, C., Kraft, H., and Schwartz, E. (2021). Optimal carbon abatement in a stochastic equilibrium model with climate change. *European Economic Review*, 132:103642.

IPCC (2018). *Global Warming of 1.5 C*.

IPCC WG3 (2014). *Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press.

Jaffe, A. B., Newell, R. G., and Stavins, R. N. (2005). A tale of two market failures: Technology and environmental policy. *Ecological Economics*, 54(2-3):164–174.

Jensen, S. and Traeger, C. P. (2014). Optimal climate change mitigation under long-term growth uncertainty: Stochastic integrated assessment and analytic findings. *European Economic Review*, 69:104–125. Sustainability and Climate Change: From Theory to Pragmatic Policy.

Kalkuhl, M., Edenhofer, O., and Lessmann, K. (2012). Learning or lock-in: Optimal technology policies to support mitigation. *Resource and Energy Economics*, 34(1):1–23.

Kalkuhl, M., Steckel, J. C., and Edenhofer, O. (2020). All or nothing: Climate policy when assets can become stranded. *Journal of Environmental Economics and Management*, 100:102214.

Kelly, B., Pastor, L., and Veronesi, P. (2016). The price of political uncertainty: Theory and evidence from the option market. *The Journal of Finance*, 71(5):2417–2480.

Leiby, P. and Rubin, J. (2001). Intertemporal permit trading for the control of greenhouse gas emissions. *Environmental and Resource Economics*, 19(3):229–256.

Lemoine, D. (2021). The climate risk premium: how uncertainty affects the social cost of carbon. *Journal of the Association of Environmental and Resource Economists*, 8(1):27–57.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The Review of Economics and Statistics*, 47(1):13–37.

Lucas, R. E. (1978). Asset prices in an exchange economy. *Econometrica: Journal of the Econometric Society*, pages 1429–1445.

Nordhaus, W. (2007). Critical assumptions in the stern review on climate change. *Science*, 317(5835):201–202.

Nordhaus, W. (2014). Estimates of the social cost of carbon: Concepts and results from the dice-2013r model and alternative approaches. *Journal of the Association of Environmental and Resource Economists*, 1(1/2):273–312.

Pastor, L. and Veronesi, P. (2012). Uncertainty about government policy and stock prices. *The Journal of Finance*, 67(4):1219–1264.

Rudik, I. (2020). Optimal climate policy when damages are unknown. *American Economic Journal: Economic Policy*, 12(2):340–73.

Samadi, S. (2018). The experience curve theory and its application in the field of electricity generation technologies–a literature review. *Renewable and Sustainable Energy Reviews*, 82:2346–2364.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, 19(3):425–442.

Stern, N. H., Peters, S., Bakhshi, V., Bowen, A., Cameron, C., Catovsky, S., Crane, D., Cruickshank, S., Dietz, S., Edmonson, N., et al. (2006). *Stern Review: The economics of climate change*, volume 30. Cambridge University Press Cambridge.

Van der Ploeg, R. and van den Bremer, T. (2018). The risk-adjusted carbon price.

Appendix A. Proof of Lemma 1

Lemma 1. *Consider a representative agent with time-additive expected utility, with a subjective discount rate ρ and a constant relative risk aversion γ, in a discrete-time setting with a risk-free asset traded each period. Assuming the relative growth rate of consumption $g^c_\tau = c_\tau/c_0 - 1$ and gross return $R_\tau = \frac{e^{-\delta\tau}A'_\tau}{A'_0} = e^{-\delta\tau}R^A_\tau$ to be jointly lognormally distributed, then*

$$\frac{1}{\tau}\ln\left(\mathbb{E}\left[R^A_\tau\right]\right) = \delta + \frac{1}{\tau}\ln R^f + \frac{1}{\tau}\xi\sigma_{g^c}Corr\left[\ln R^A_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma[\ln R^A_\tau]$$

and in beta-form

$$\frac{1}{\tau}\ln\left(\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]\right) = \delta + r^f + \phi\eta$$

or

$$\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]^{\frac{1}{\tau}} = e^{\delta + r^f + \phi\eta}$$

with

$$\phi = \frac{Cov\left[r_\tau, \tilde{g}^c_\tau\right]}{Var\left[\tilde{g}^c_\tau\right]} \qquad \text{(Appendix A.1)}$$

$$\eta = \frac{1}{\tau}\gamma Var\left[\tilde{g}^c_\tau\right] \qquad \text{(Appendix A.2)}$$

and r^f, r_τ, *and* $\tilde{g}^c_\tau$ *represent respectively* $\ln R^f$, $\ln R_\tau$, *and* $\ln\frac{c_\tau}{c_0}$.

Proof. Given the form of the utility function assumed, $u(c) = c^{1-\xi}/(1-\xi)$ then

$$\frac{u'(c_\tau)}{u'(c_0)} = \left(\frac{c_\tau}{c_0}\right)^{-\xi} = \exp\left\{-\xi\ln\left(\frac{c_\tau}{c_0}\right)\right\},$$

which is lognormally distributed, $\ln\left(\frac{c_\tau}{c_0}\right) \sim N\left(\bar{g}, \sigma^2_{g^c}\right)$

$$\mathbb{E}\left[\frac{u'(c_\tau)}{u'(c_0)}\right] = \mathbb{E}\left[\exp\left\{-\xi\ln\left(\frac{c_\tau}{c_0}\right)\right\}\right] = \exp\left\{-\xi\bar{g} + \frac{1}{2}\xi^2\sigma^2_{gc}\right\}$$

and

$$\frac{\sigma\left[u'(c_\tau)/u'(c_0)\right]}{\mathbb{E}\left[u'(c_\tau)/u'(c_0)\right]} = \sqrt{e^{\xi^2\sigma^2_{gc}} - 1} \approx \xi\sigma_{gc}$$

The price of a risky asset can be expressed as[9]:

$$P_{i,\tau} = \mathbb{E}\left[m_\tau B_{i,\tau}\right]$$

with $B_{i,\tau}$representing the payoff of the risky asset and m_τ the stochastic discount factor (also known as the state-price deflator) which is defined as $m_\tau \equiv \beta u'(c_\tau)/u'(c_0)$ with β the discount factor

$$\mathbb{E}\left[m_\tau B_{i,\tau}\right] = \mathbb{E}\left[m_\tau\right]\mathbb{E}\left[B_{i,\tau}\right] + \text{Cov}\left[B_{i,\tau}, m_\tau\right]$$

$$P_{i,\tau} = \mathbb{E}\left[m_\tau\right]\mathbb{E}\left[B_{i,\tau}\right] + \text{Cov}\left[B_{i,\tau}, m_\tau\right]$$

[9]See Cochrane (2001) for more general asset pricing models

According to the asset pricing model, even though expected returns can vary across assets and time, expected discounted returns should be the same equal to 1. Then,

$$1 = \mathbb{E}\left[m_\tau\right]\mathbb{E}\left[R_{i,\tau}\right] + \text{Cov}\left[R_{i,\tau}, m_\tau\right]$$

$$\mathbb{E}[R_{i,\tau}] = \frac{1}{\mathbb{E}\left[m_\tau\right]} - \frac{1}{\mathbb{E}\left[m_\tau\right]}\text{Cov}\left[R_{i,\tau}, m_\tau\right]$$

$$\mathbb{E}[R_{i,\tau}] = R_t^f - \frac{\text{Cov}\left[R_{i,\tau}, u'\left(c_\tau\right)/u'\left(c_0\right)\right]}{\mathbb{E}\left[u'\left(c_\tau\right)/u'\left(c_0\right)\right]}$$

$$= R_t^f - \frac{\sigma\left[u'\left(c_\tau\right)/u'\left(c_0\right)\right]}{\mathbb{E}\left[u'\left(c_\tau\right)/u'\left(c_0\right)\right]}\sigma_t\left[R_{i,\tau}\right] Corr\left[R_{i,\tau}, u'\left(c_\tau\right)/u'\left(c_0\right)\right]$$

Hence,

$$\mathbb{E}[R_{i,\tau}] = R_t^f - \xi\sigma_g\sigma_t\left[R_{i,\tau}\right] Corr\left[R_{i,\tau}, u'\left(c_\tau\right)/u'\left(c_0\right)\right]$$

□

Instead of using marginal rate of substitution, the relation between *expected returns* and *relative consumption growth* can be approximated via $g_\tau^c = \frac{c_\tau}{c_0} - 1$. This is obtained by applying a first-order Taylor approximation of $u'(c_\tau)$ around c_0.

$$\frac{u'(c_\tau)}{u'(c_0)} \approx \frac{u'(c_0) + u''(c_0)(c_\tau - c_0)}{u'(c_0)} = 1 - \xi(c_0)g_\tau$$

where $\xi(c_0) = -c_0u''(c_0)/u'(c_0)$ is the relative risk aversion of the individual evaluated at time 0 consumption level, and $g = c_\tau/c_0 - 1$ is the relative growth rate of consumption over the period. Replacing the above equation, we get

$$\mathbb{E}[R_\tau] - R^f \approx \xi(c_0)\text{Cov}\left[R_\tau, g_\tau^c\right] \tag{Appendix A.3}$$

Assuming a time-additive utility with constant relative risk aversion parameter and the consumption growth to be lognormally distributed

$$\ln(1 + g_\tau^c) \equiv \ln\left(\frac{c_\tau}{c_0}\right) \sim N(\bar{g}^c, \sigma_g^2) \tag{Appendix A.4}$$

then

$$\mathbb{E}[R_\tau] - R^f = \sigma[R_\tau]\sqrt{e^{\xi^2\sigma_g^2} - 1}Corr\left[R_\tau, \left(\frac{c_\tau}{c_0}\right)^{-\xi}\right] \quad \text{(Appendix A.5)}$$

$$\mathbb{E}[R_\tau] - R^f \approx \xi\sigma_{g^c}\sigma[R_\tau]Corr\left[R_\tau, \frac{c_\tau}{c_0}\right] \quad \text{(Appendix A.6)}$$

One can make a stronger assumption, and assume that both gross rate of return R_i of the asset and the consumption growth g^c are jointly lognormally distributed. In that case,

$$\mathbb{E}[\ln(R_\tau)] - \ln R^f + \frac{1}{2}\text{Var}[\ln R_\tau] = \xi\sigma_{g^c}Corr\left[\ln R_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma[\ln R_\tau] \quad \text{(Appendix A.7)}$$

equivalently,

$$\ln(\mathbb{E}[R_\tau]) - \ln R^f = \xi\sigma_{g^c}Corr\left[\ln R_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma[\ln R_\tau] \quad \text{(Appendix A.8)}$$

Following Gollier, P_0 is equivalent to A'_0 and $B_\tau = e^{-\delta\tau}A'_\tau$, and β the discount factor can be expressed as $\beta = e^{-\rho\tau}$. Hence,

$$R_\tau = \frac{B_\tau}{P_0} = \frac{e^{-\delta}A'_\tau}{A'_0} = e^{-\delta}R^A_\tau \quad \text{(Appendix A.9)}$$

Then we can re-write equ. (43) as

$$\mathbb{E}\left[\ln\left(\frac{e^{-\delta\tau}A'_\tau}{A'_0}\right)\right] - \ln R^f + \frac{1}{2}\text{Var}\left[\ln\left(\frac{e^{-\delta\tau}A'_\tau}{A'_0}\right)\right] = \xi\sigma_{g^c}Corr\left[\ln R_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln(R_\tau)\right]$$

$$\ln\left(\mathbb{E}\left[\frac{e^{-\delta\tau}A'_\tau}{A'_0}\right]\right) = \ln R^f + \xi\sigma_{g^c}Corr\left[\ln R_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln(R_\tau)\right]$$

$$-\delta\tau + \ln\left(\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]\right) = \ln\left(\frac{1}{e^{-\rho\tau}\mathbb{E}\left[u'(c_\tau)/u'(c_0)\right]}\right) + \xi\sigma_{g^c}Corr\left[\ln R^A_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln\left(R^A_\tau\right)\right]$$

$$\ln\left(\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]\right) = \delta\tau + \rho\tau - \ln\left(\mathbb{E}\left[\frac{u'(c_\tau)}{u'(c_0)}\right]\right) + \xi\sigma_{g^c}Corr\left[\ln R^A_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln\left(R^A_\tau\right)\right]$$

$$\underbrace{\frac{1}{\tau}\ln\left(\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]\right)}_{g} = \delta + \underbrace{\rho - \frac{1}{\tau}\ln\left(\mathbb{E}\left[\frac{u'(c_\tau)}{u'(c_0)}\right]\right)}_{r^f} + \frac{1}{\tau}\xi\sigma_{g^c}Corr\left[\ln R^A_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln\left(R^A_\tau\right)\right]$$

It can be expressed in ”beta-form”

$$g = \delta + r^f + \phi\eta \qquad \text{(Appendix A.10)}$$

with,

$$\phi\left[r_\tau, \tilde{g}^c_\tau\right] = \frac{\text{Cov}\left[r_\tau, \tilde{g}^c_\tau\right]}{\text{Var}\left[\tilde{g}^c_\tau\right]}$$

$$\eta = \frac{1}{\tau}\xi\text{Var}\left[\tilde{g}^c_\tau\right]$$

Note:
A random variable X has log-normal distribution, if the random variable $Y = \ln X$ is normally distributed. Let μ be the mean of Y and σ^2 be the variance of Y so that $Y = \ln X \sim N(\mu, \sigma^2)$.
For $Y \sim N(\mu, \sigma^2)$ and $\xi \in \mathbb{R}$ we have

$$\mathbb{E}_t\left[e^{-\xi Y}\right] = \exp\left\{-\xi\mu + \frac{1}{2}\xi^2\sigma^2\right\}$$

$$var_t\left[e^{-\xi Y}\right] = \left(\mathbb{E}_t\left[e^{-\xi Y}\right]\right)^2\left[e^{-\xi^2\sigma^2} - 1\right]$$

Deriving expressions for optimal instruments

$$e^{-(\rho+\delta)}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right] = e^{-\rho+\gamma}\mathbb{E}\left[u'(C_1)\frac{1+\sigma}{1-\kappa}A_1'^n\right]$$

$$\mathbb{E}\left[u'(C_1)e^{-\delta}\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right] = \mathbb{E}\left[u'(C_1)e^{\gamma}\frac{1+\sigma}{1-\kappa}A_1'^n\right]$$

With $Z_1 := e^{-\delta}\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)$ and $Z_2 := e^{\gamma}\frac{1+\sigma}{1-\kappa}A_1'^n$ we have

$$\mathbb{E}\left[u'(C_1)Z_1\right] = \mathbb{E}\left[u'(C_1)Z_2\right]$$
$$\mathbb{E}\left[u'(C_1)\right]\mathbb{E}\left[Z_1\right] + \text{Cov}\left[u'(C_1), Z_1\right] = \mathbb{E}\left[u'(C_1)\right]\mathbb{E}\left[Z_2\right] + \text{Cov}\left[u'(C_1), Z_2\right]$$

$$\mathbb{E}\left[Z_1\right] = \mathbb{E}\left[Z_2\right] + \mathbb{E}\left[u'(C_1)\right]^{-1}\left(\text{Cov}\left[u'(C_1), Z_2\right] - \text{Cov}\left[u'(C_1), Z_1\right]\right)$$

$$e^{-\delta}\left(\mathbb{E}\left[A_1'\right]-\mathbb{E}\left[e^{\delta}\frac{\partial A_1}{\partial K_0}\right]\right)=e^{\gamma}\frac{1+\sigma}{1-\kappa}\mathbb{E}\left[A_1'^{n}\right] + \frac{1}{\mathbb{E}\left[u'\left(C_1\right)\right]}\left(\mathrm{Cov}\left[u'\left(C_1\right),Z_2\right]-\mathrm{Cov}\left[u'\left(C_1\right),Z_1\right]\right)$$

Appendix B. Additional numerical results

(a) Externality (for $\psi = 1$ learning is fully external)

ψ	I_0	B_1	r_A	r_f	ϕ	η	$\phi\eta$	$r_f + \phi\eta$
0.000	83.32	146.70	3.77	1.28	1.10	2.30148	2.54	3.81
0.250	82.00	142.56	3.69	1.27	1.08	2.30111	2.50	3.76
0.500	80.56	138.00	3.59	1.26	1.07	2.30072	2.45	3.71
0.750	78.95	132.94	3.47	1.25	1.05	2.30031	2.41	3.65
1.000	77.10	127.26	3.34	1.23	1.03	2.29987	2.36	3.59

(b) Technology subsidy

κ	I_0	B_1	r_A	r_f	ϕ	η	$\phi\eta$	$r_f + \phi\eta$
0.000	77.17	127.12	3.33	1.22	1.02	2.30000	2.36	3.57
0.100	72.88	121.42	3.40	1.24	1.07	2.30079	2.46	3.70
0.111	72.39	120.79	3.41	1.25	1.08	2.30089	2.47	3.72
0.200	68.25	115.32	3.50	1.27	1.13	2.30188	2.59	3.87
0.300	63.30	108.61	3.60	1.31	1.20	2.30341	2.77	4.07
0.400	57.90	101.20	3.72	1.35	1.30	2.30552	2.99	4.34
0.500	51.97	92.82	3.87	1.40	1.42	2.30849	3.29	4.69

(c) Investment subsidy

σ	I_0	B_1	r_A	r_f	ϕ	η	$\phi\eta$	$r_f + \phi\eta$
0.000	77.17	127.12	3.33	1.22	1.02	2.30000	2.36	3.57
0.125	81.41	135.85	3.41	1.25	1.08	2.30089	2.47	3.72
0.200	83.80	140.89	3.46	1.26	1.11	2.30147	2.55	3.81
0.400	89.62	153.50	3.59	1.30	1.19	2.30315	2.74	4.04
0.600	94.86	165.04	3.69	1.34	1.27	2.30492	2.93	4.26
0.800	99.60	175.73	3.79	1.37	1.35	2.30671	3.11	4.48

Table B.5: Technology externality. *Note: Optimal policies are merged into the parameter variations (in-between the equidistant steps of the parameter variation).*