PIK Report

No. 96

THE TIME SCALES OF THE CLIMATE-ECONOMY FEEDBACK AND THE CLIMATIC COST OF GROWTH

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PIK

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ISSN 1436-0179 POTSDAM, APRIL 2005

Abstract

This paper is based on the perception that the inertia of climate and socio-economic systems are key parameters in the climate change issue. In a first part, it develops and implements a new approach based on a simple integrated model with a particular focus on an innovative transient impact and adaptation modelling. In a second part, a *climate-economy feedback* is defined and characterized. It is found that: (i) it has a 70-year characteristic time, which is long when compared to the system's other time-scales, and it cannot act as a natural damping process of climate change; (ii) mitigation has to be anticipated since the feedback of an emission reduction on the economy is significant only after a 20-year delay and really efficient after a one-century delay; (iii) the IPCC methodology, that neglects the feedback from impacts to emissions, is acceptable up to 2100, *whatever is the level of impacts*. This analysis allows also to define a *climatic cost of growth* as the additional climate change damages due to the additional emissions linked to economic growth. Usefully, this metric for climate change damages is particularly independent of the baseline scenario.

1. Introduction

One major challenge in the modeling of Climate Change is the taking into account of the various characteristic times involved: the climate inertia, which may be responsible for quasi-irreversibility and implies anticipated decisions, and the socio-economic inertia, which precludes an instantaneous large reduction in anthropogenic emissions and makes human societies more or less vulnerable to brutal changes in climate patterns. To address this challenge, an analysis of the dynamic behaviour of each sub-system involved is necessary, together with an understanding of how their own behaviours interact in the coupled system.

Such a dynamic approach is a necessary complement to the enumerative approach, which focuses on how climate change affects welfare at a particular point in time (see e.g. Nordhaus (1991), Cline (1992) or Mendelsohn and Neumann (1999)). This complementarity has already been discussed by Fankhauser and Tol (2005) and Tol (2002b), but faces many difficulties: (i) the fundamental differences in the nature of the objects under scrutiny, and the corresponding differences between socio-economic and physical science models; (ii) the variety of temporal and spatial scales involved; (iii) the multiplicity of the influence channels between environment and society; and (iv) the controversies surrounding both the value judgments at stake and the confidence into scientific results.

This paper aims at demonstrating how the TEF/ZOOM approach, which has been applied in a diversity of other fields, can be used to tackle these very difficulties[1]. Its interest lies in that it allows for precise analyses of the characteristic times, for dynamic characterizations of feedbacks, and hence for an understanding of the roles of dynamics and inertia in the evolution of the climate-economy system.

This paper focus on the dynamics of climate change impacts and on the characteristic times of the coupled climate-economy system. It does not try to provide an assessment of the climate change economic damages, but rather aims at providing robust information on the coupling process time scales, which may help to understand the climate-economy system behaviour. The understanding of the climate-economy feedback is also necessary to justify the IPCC approach, which is currently based on a ”one-direction” coupling: from socio-economic and emission scenarios, climate scenarios are built, and climate change impacts are evaluated. No feedback from economic impacts to the economic and emission scenario is taken into account. This paper proposes an assessment of the validity of this methodology.

The low complexity model whose results are used as a basis for the analysis is

[1]More information on the TEF/ZOOM approach and its applications can be found on http://www.lmd.jussieu.fr/ZOOM.

K	Productive capital	trillions U.S.$
Y	Production	trillions U.S.$
I	Investment	trillions U.S.$
τ_d	Depreciation characteristic time	years
L	population (proportional to labour)	millions of inhabitants
γ_L	population growth	% per year
A	technical progress	No unit
E	Greenhouse gas emissions to atmosphere	GtC/year
D	Emission intensity	No unit
T_s	Surface air temperature	K
T_{ada}	"Economic" temperature	K
X	Climate change impacts on productivity	No unit

Table 1: Model common variables.

described in part **2.**. It is implemented following a precise formalism, presented in part **3.**. It is then used to study the climate-economy feedback through temporal simulations and a rigorous feedback analysis (in part **4.**).

2. Model

The simple model providing the basis for our analyses is composed of five modules: a climate module; a macroeconomic module; a demographic module; an emission module and an impact module. These models have in common the variables reproduced in Tab. 1.

a. Climate module

A single column of atmosphere, containing only water vapour, CO_2, and 3 layers of clouds, is considered. The atmospheric column is divided into 2 layers (troposphere and stratosphere) and caps an oceanic mixed layer 50 m thick. The lapse rate (*i.e.* the temperature change with respect to altitude) is fixed in the troposphere and stratosphere and is null in the ocean. The temperature in each object is determined by: the sea surface temperature(SST) for the ocean; the mid-troposphere temperature (T_{trp}) for the troposphere; and the tropopause temperature (T_{str}) for the stratosphere. These 3 objects exchange water fluxes, sensible heat fluxes, latent heat fluxes, and long wave (LW) radiative fluxes. These fluxes are modelled by a 1D radiative model using a Malkmus narrow-band model with a water vapour continuum. The principles behind this module were explored by Green (1967) and developed by Cherkaoui et al. (1996). The 3 objects also receive short wave (SW) fluxes from space. The complete description of the model is provided in Hallegatte et al. (2005).

b. Macroeconomic module

The macroeconomic module is a classical Solow (1956) growth model, of comparable complexity than other compact integrated climate-economy models (*e.g.* the DICE model developed by Nordhaus (1994)). The model is written in a simulation formalism, without optimisation, and the saving ratio is fixed at 20%. The model accounts for exogenous technical progress (impacting productivity and CO_2 emissions per unit of production).

The primary equations of the growth model are the following:

$$
\begin{aligned}
\frac{dK}{dt} &= I - \frac{1}{\tau_d} \cdot K && (1)\\
Y &= X \cdot A \cdot \lambda \cdot K^{1/3} \cdot L^{2/3} && (2)\\
I &= \alpha_I \cdot Y && (3)\\
\frac{dA}{dt} &= \gamma_A \cdot A && (4)
\end{aligned}
$$

Where K is the productive capital; Y is production; I is investment; A is the total productivity; λ is a production calibration parameter set so that $A = 1$ at $t = 0$; α_I is the saving ratio; γ_A is the productivity growth; X represents the climate impacts on productivity. To facilitate analyses in terms of characteristic times, the depreciation is represented by a capital life-time (τ_d) rather than by the classical depreciation rate.

To better understand the long-term behaviour of the model, it is useful to separate the effect of population and productivity growth from the other effects: for given productivity and labour supply, and if no impacts are considered, the equilibrium values of K, Y and I are proportional to $(L \cdot A^{3/2})$ (this property is derived from the previous equations where all derivatives are set to zero). Consequently, the following *"normalized"* variables are defined:

$$
\begin{aligned}
K^* &= K \cdot \frac{L_0}{L} \cdot A^{-3/2} && (5)\\
Y^* &= Y \cdot \frac{L_0}{L} \cdot A^{-3/2} && (6)\\
I^* &= I \cdot \frac{L_0}{L} \cdot A^{-3/2} && (7)
\end{aligned}
$$

Where L_0 is the initial population. Note that the normalized variables equal the original variables at $t = 0$ since $A(0) = 0$. The corresponding equations read:

$$\frac{dK^*}{dt} = \gamma_K \cdot K^* \tag{8}$$

$$\gamma_K = \frac{I^*}{K^*} - \frac{1}{\tau_d} - \gamma_L - \frac{3}{2}\gamma_A \tag{9}$$

$$Y^* = X \cdot \lambda \cdot L_0^{2/3} \cdot K^{*\,1/3} \tag{10}$$

$$I^* = \alpha_I \cdot Y^* \tag{11}$$

$$\frac{dA}{dt} = \gamma_A \cdot A \tag{12}$$

The variable Y^* is the production, normalized by the steady-state production of a model without climate change impacts and with constant population and productivity.

c. Demographic module

The demographic module is the same as DICE's. It reproduces a demographic scenario leading to a stabilisation of the world population around 11.5 billions inhabitants in 2200, an intermediate scenario between the SRES/A2 and the SRES/B2 (see IPCC (2000)).

The equations are the following:

$$\frac{dL}{dt} = \gamma_L \cdot L \tag{13}$$

$$\gamma_L = \gamma_L^0 \cdot e^{-\frac{t}{\tau_L}} \tag{14}$$

To focus on economic dynamics, no impacts of economic dynamics or climate change impacts on population growth are accounted, even though they could constitute a significant channel of climate -economy interaction (see IPCC (2001a), Chp 9). The climate change impacts on labour productivity (as the malaria impact discussed by Gallup et al. (1999)) are also neglected.

d. Emission module

All greenhouse gases are modelled by an equivalent CO_2 concentration. Emissions are assumed to be proportional to production through a unique factor, modelling both energy intensity and carbon intensity. An exogenous emission-intensity decrease compensates the growth in emissions caused by a technical-progress-driven production growth.

$$E = \frac{1}{A^{3/2}} \cdot \beta \cdot Y = \beta \cdot \frac{L}{L_0} \cdot Y^* \quad (15)$$

For the sake of simplicity, no carbon cycle module is implemented into the model. Only a natural carbon sink of 40% of the emissions is considered (it corresponds to the value observed at present). Note that baseline emission growth without any abatement policy leads as in many other studies to a doubling of the CO_2-equivalent concentration in the end of the XXIth century. Obviously, this rather crude modelling of emissions would welcome many improvements, but it should be sufficient for our purpose.

e. Impact and Adaptation module

Climate change impacts on the socio-economic system have two components: an absolute component, which measures the productivity change associated with a stabilized climate; a transient component, which measures the costs associated to the adaptation of the socio-economic system to a changing climate. In the following, it is assumed that there is no absolute impact of climate change on productivity. This hypothesis is a very optimistic one since it assumes that society is able to adapt to any climate and that no climate is better than the others. It focuses on the transition period in which the socio-economic system is not adapted to climate and assumes that if climate is stabilized for a long enough period, impacts disappear. Moreover, no direct climate change impact on welfare is taken into account[2].

The transient component cannot but involve an adaptation process: an endogenous adaptation process is modelled by an "adaptive temperature" (T_{ada}). This temperature is equal to the "climate temperature" at the equilibrium, but it diverges from it whenever climate changes faster than the adaptation characteristic time of the socio-economic system (τ_{ada}).

$$\frac{dT_{ada}}{dt} = \frac{1}{\tau_{ada}}(T_s - T_{ada}) \quad (16)$$

When T_{ada} and T_s differ, the socio-economic system is not adapted and it faces impacts (i) through productivity losses (modelled by X) and (ii) through a shortening of the life-time of productive capital, because ill-adapted capital is more likely to be damaged by new climate conditions.

[2] It has already been mentioned that this article does not aim at providing an assessment of the climate change damages to feed a decision-making process but aims at improving our understanding of the coupling processes between climate and economy.

Both impacts are assumed proportional to the maladjustment of T_{ada} and hence proportional to the adaptation effort.

$$X = 1 - \alpha_T \cdot |T_{ada} - T_s| \quad (17)$$

$$\tau_d = \tau_d^0 \cdot (1 - \alpha_\tau \cdot |T_{ada} - T_s|) \quad (18)$$

Of course, the characteristic time of the adaptation τ_{ada} is strongly related to the capital depreciation time: the more frequently the productive capital is replaced, the easier and less costly the adaptation process. In the following, τ_{ada} is fixed by: $\tau_{ada} = 5 \cdot \tau_d^0$. It means that if the real surface temperature is constant, the maladjustment and the impacts converge to zero with an e-folding time of $(5 \cdot \tau_d)$: there is no permanent impacts, impacts are only temporary, due to ill-adaptation to new climate conditions. Because the model has only one sector, it is impossible to take into account the productive capital heterogeneity and the differences in adaptation pace in different sectors (*e.g.* housing and infrastructure), justifying the fact that the adaptation characteristic time τ_{ada} is much larger than the mean depreciation time τ_d. An extension of the model with two sectors will be presented in a following paper.

No distinction is made between autonomous adaptation and planned adaptation in this modeling, in spite of their essential differences. All the complexity of the adaptation process is here reduced to a single characteristic time, representing the time needed by the economy to adapt in reaction to temperature changes.

This formalism is very crude, as is the damage function formalism, but it takes into account the transition period in which the socio-economic system is not adapted to climate. The advantages of this formulation are: (i) climate change intensity and rate are both taken into account; (ii) present climate is not used as an absolute reference; (iii) a characteristic pace of adaptation is introduced; (iv) any temperature change (increase or decrease) has negative impacts. We argue this specification is more realistic than the classical damage function (already criticized by Tol (1996)), which assumes that a temperature decrease is always beneficial for the economic system and that damage intensity depends on the initial temperature.

f. Parameter values

The parameters used by the model are a ”best guess scenario”. Their values are given in Tab. 2. Most of them are the DICE parameters (see Nordhaus (1994)), the others are assumptions. Particularly, the initial productive capital is set so that the production function gives an initial production consistent with the observed one.

L_0	Initial population	5632.7 millions (DICE)
K_0	Initial productive capital	21 trillions U.S.$
Y_0	Initial production	14.6 trillions U.S.$
T_0	Initial surface temperature	287.0 K
T_{ada}^0	Initial adaptive temperature	287.0 K
τ_d^0	Initial depreciation time	20 years
γ_L^0	Initial population growth	1.57% per year (DICE)
τ_L	Time of population growth decrease	4.5 years (DICE)
β	Initial emission intensity	0.5 GtC / trillion U.S.$
λ	Production factor	0.01685 (DICE)
α_I	Saving ratio	20% (DICE)
γ_A	Productivity growth	1.5% per year
τ_{ada}	Adaptation charateristic time	100 years
α_T	Productivity loss due to a 1 K maladjustment	2% in the ”moderate impact” run
α_τ	τ_d change due to a 1 K maladjustment	5% in the ”moderate impact” run

Table 2: Model parameters.

In the ”moderate impacts” run, a 1 K maladjustment reduces production by 2%. This is slightly higher than usual assumptions (see Fankhauser et al. (1999), Tol (2002a) and IPCC (2001a), Chp.19), but is mostly compensated by the fact that adaptation is explicitely modelled and by the fact that climate change impacts on population and labor productivity are neglected. To assess the influence of this parameter, a simulation with a 4% productivity decrease for a 1 K maladjustment is also carried out and is refered to as ”strong impacts”.

The climate change impact on depreciation time is less documented (although its existence has been pointed out by Fankhauser et al. (1999)). The ”moderate impacts” scenario assumes that a 1 K maladjustment decreases the depreciation time by 5%; The ”strong impacts” simulation assumes a 10% decrease of the depreciation time for a 1 K maladjustement.

This parameter set is one of the possible sets of parameters. The aim of the next section is to provide some robust information, that does not depend too strongly on the parameter choice.

3. Model implementation

Because of the high degree of uncertainty characterising the climate change issue, the range of plausible values is large for most of the model's parameters. As a consequence, almost any result can be demonstrated by selecting a particular set of parameter values. This suggests the need of a new approach, able to produce robust information and to rigorously quantify the robustness of each produced information. To progress in this direction, the model is built according to Transfer Evolution Formalism (TEF) prescriptions.

a. The Transfer Evolution Formalism Prescriptions

The TEF is a tool for system analysis and simulation (see Appendix A for a more detailed description). The model presented in the previous section is mathematically represented by a set of equations, belonging to two kinds:

1. A set of *cells,* which are elementary models and correspond to state equations such as:

$$\begin{array}{rcl} \frac{\partial \boldsymbol{\eta}_\alpha}{\partial t} & = & \mathbf{G}_\alpha(\boldsymbol{\eta}_\alpha, \boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, ...) \\ \frac{\partial \boldsymbol{\eta}_\beta}{\partial t} & = & \mathbf{G}_\beta(\boldsymbol{\eta}_\beta, \boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, ...) \\ \cdots & & \end{array} \tag{19}$$

   The $\boldsymbol{\eta}_\alpha$ are the state variables of each cell and $\boldsymbol{\varphi}$ represents the dependent boundary conditions, *i.e.* the variables considered as boundary conditions by a cell, but that depend on the complete model state. This dependent boundary conditions are required to make the cells correspond to well-posed problems.

2. A set of *transfers,* which are associated to the dependent boundary conditions and correspond to equations such as:

$$\begin{array}{rcl} \boldsymbol{\varphi}_1 & = & \mathrm{f}_1(\boldsymbol{\eta}_\alpha, \boldsymbol{\eta}_\beta, ..., \boldsymbol{\varphi}) \\ \boldsymbol{\varphi}_2 & = & \mathrm{f}_2(\boldsymbol{\eta}_\alpha, \boldsymbol{\eta}_\beta, ..., \boldsymbol{\varphi}) \\ \cdots & & \end{array} \tag{20}$$

Let also $\boldsymbol{\eta}$ be the state vector of the complete system and $\boldsymbol{\varphi}$ be the vector of the dependent boundary conditions. When initial conditions are given at time t_0, the system is a well-posed problem.

The TEF solution for solving the system consists in building, for each time step, the differential of the dynamical system around its current state ($\eta(t_n)$). It is proved in Appendix A that the Borel transform of the obtained Tangent Linear System (TLS) can be written as:

$$\left\{ \begin{array}{rcl} \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}](\tau) & = & \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{dec}](\tau) + \underline{\underline{\mathcal{F}}}(\tau)\, \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}](\tau) \\ \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}](\tau) & = & \left[1 + \underline{\underline{\mathcal{C}}}(\tau)\right]^{-1} \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}_{ins}](\tau) \end{array} \right. \tag{21}$$

where $\mathcal{B}[f]$ is the Borel transform of f(t); τ is the Borel variable; $\mathring{\delta}\eta(t)$ and $\mathring{\delta}\varphi(t)$ are the solutions of the TLS; and where the quantities $\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{dec}$, $\underline{\underline{\mathcal{F}}}$, $\underline{\underline{\mathcal{C}}}$, $\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}_{ins}$ can be calculated from the elementary Jacobian matrices and vectors at time t_n.

The first equation of (21) describes the evolution of the state variables. The state variables evolve because: i) of their internal inertial evolutions $\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{dec}$ (which would be obtained if transfer models were changed to constant transfer model with $\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi} = \mathbf{0}$) ; ii) of the evolution of their boundary conditions ($\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi} \neq \mathbf{0}$). The matrix $\underline{\underline{\mathcal{F}}}$ describes the influence of transfer variables on state variables, and independently of the type of model used for these transfers ($\underline{\underline{\mathcal{F}}}$ is independent of the model of $\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}$).

In the second equation, $\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}_{ins}$ represents the variation of transfer variables if $\mathring{\boldsymbol{\delta}}\boldsymbol{\eta} = \mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{dec}$ (*i.e.* if the cell models were changed to decoupled models with $\underline{\underline{\mathcal{F}}} = 0$). Consequently, $\underline{\underline{\mathcal{C}}}$ represents the effect of cell and transfer coupling.

All numerical results presented in the paper use a software developed by the author and others to implement models expressed with the TEF. Thanks to its use of the Crank-Nicolson scheme, it is capable of computing numerically the Borel transform of the TLS matrix coefficients and solutions on the real axis $\tau > 0$. An approximation of the step-by-step evolution of the complete system is obtained by solving the system (21) and through the relationships:

$$\begin{array}{lcl} \boldsymbol{\delta\eta} & \approx & 2\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}](\frac{\delta t}{2}) \\ \boldsymbol{\delta\varphi} & \approx & 2\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}](\frac{\delta t}{2}) \end{array} \qquad (22)$$

where $\boldsymbol{\delta\eta}$ and $\boldsymbol{\delta\varphi}$ are the state and transfer variable variations in the complete model during a time step δt.

From these formulas, in addition to the time evolution of the model, one may assess separately the decoupled and the coupled evolution of each subsystem (through the matrix $\mathcal{F}$), and get access to the subsystem interactions (through the matrix $\mathcal{C}$).

4. The Climate-Economy Feedback

The existence of a climate-economy feedback, coming from the emission variation due to climate change impacts at a given emissions to GDP ratio, is one of the key issues in the building of consistent climate change scenarios: a quantification of the involved time scales and of the magnitude of this effect is necessary to achieve a rigorous design of the simulations with high complexity models. The following part aims at providing some ideas on these essential figures.

a. Model Simulations

An extensive study of the climate module is available in Hallegatte et al. (2005). The version used in this study is the 'Increasing cloud cover' version, which assumes an increase in the high level cloud cover with respect to temperature. The climate

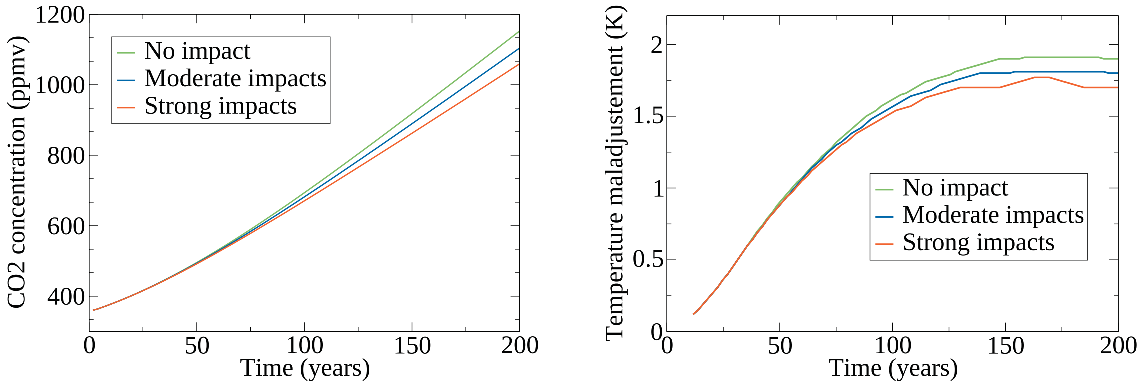

Figure 1: CO_2 concentration evolution (left panel) and difference between surface temperature and adaptive temperature, *i.e.* the maladjustment (right panel) over 200 years for the 3 runs.

sensitivity of the model to doubling CO_2 concentration is found to be +2.8 K, which is within the GCM's sensitivity spectrum (see IPCC (2001b), Chp 9).

A set of simulations is carried out to assess the validity of the complete model: a simulation without climate change impacts ("no impact"), a simulation with "moderate impacts" and a simulation with "strong impacts". The left panel of Fig. 1 shows that the doubling CO_2 concentration (660 ppmv) is reached about 2100 for the 3 runs. The concentration in 2200 lies between 1050 ppmv and 1150 ppmv. The right panel of Fig. 1 shows that, because of how adaptation is modelled, the level of impact is stabilized from 2150, when the climate change slows down.

The production evolution is reproduced in the left panel of Fig. 2. Apparently, the production is not impacted so much: in 2100 the production growth is reduced by about 6% over one century in the "moderate impacts" simulation, which is negligible when compared to the economic growth during the same period (a 2000% rise).

To understand the underlying processes, it is necessary to focus on another variable: the growth of the normalized production, $\gamma_Y = (dY^*/dt)/Y^*$, *i.e.* the growth rate of the production normalized by the long-term production without climate change impacts and with constant population and productivity. The right panel of Fig. 2 shows the evolution of γ_Y on 200 years. In the case without impact, the increase in normalized production ($\gamma_Y > 0$) comes from the decrease in the population growth rate. Because adaptation takes time to change the economic system, the growth reduction due to climate change impacts is significant over the medium term: between 0.05% and 0.2% between 2025 and 2075 depending on the impact

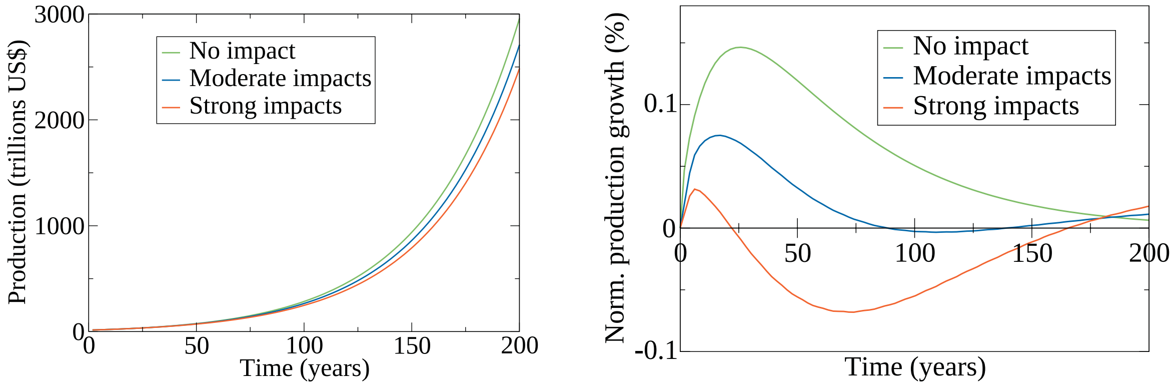

Figure 2: Production evolution (left panel) and γ_Y evolution (right panel) over 200 years for the 3 runs.

level. After 2175, the climate change is slow enough to allow adaptation to compensate and prevent damages. Because the absolute impacts are assumed to be null, the climate change damages are null over the long-term and the normalized production growth pathways converge whatever is the level of impacts. However, this does not prevent damages from being significant over more than one century.

This figure allows to understand that, in the production evolutions, the final difference comes mainly from invisible production losses in the first century, which are amplified by the economic growth and become visible in the second century. The production figures hide the real damages: the invisible shocks over the medium term are serious although the visible long-term difference between scenarios does not really matter.

These results emphasize the fact that it is not trivial to analyse a model trajectory and to characterize and quantify a selected process in the simulation. To address this issue, a new tool is proposed in order to separate the different effects. It is the aim of the feedback analysis that is carried out in the next section on this simple example, which is interesting *per se* but also demonstrates the interest of the method.

b. Feedback Definition

In our formalism, we define a feedback loop as a set of processes interfaced by transfer variables $\{\varphi_i$,i=1,..,n$\}$ in which the evolution of each variable, $\delta\varphi_j$, depends only on $\delta\varphi_{j-1}$, and $\delta\varphi_1$ depends only on $\delta\varphi_n$.

Using the formalism proposed by Bode (1945) in electric circuit theory, a feedback is usually characterized by its gain (g) or its factor (f), defined by:

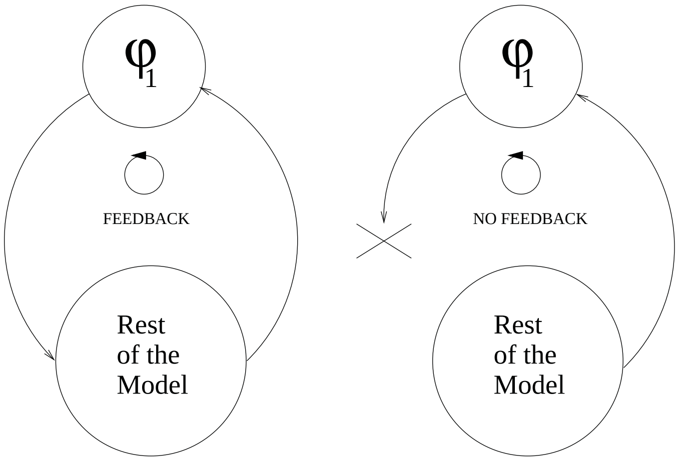

Figure 3: Scheme of a feedback (left) and illustration of the open-loop model (right).

$$(1-g)\cdot\delta\varphi_1^\infty \quad = \quad \frac{1}{f}\cdot\delta\varphi_1^\infty = \delta\varphi_1^0 \tag{23}$$

where $\delta\varphi_1^\infty$ is the equilibrium change in φ_1, when a perturbation in the forcing is applied; $\delta\varphi_1^0$ is the equilibrium change in φ_1 for the same perturbation but in the absence of the feedback (*i.e.* when at least one link between two variables of the loop is cut). The feedback gain is thus defined by a difference between two equilibrium, and will be hereafter called the *static gain* .

However, feedbacks are dynamic processes and transient effects can be essential. In our case, since the equilibrium costs are null, the transients are alone of interest. Hence, a feedback characterization which describes the whole dynamics of the response is needed. The proposed methodology aims at generalizing the feedback static gain to take into account the feedback dynamics.

c. Feedback dynamic study

In order to analyze the dynamics of the feedback, the model Tangent Linear System (TLS) is studied. Since the system is not linear, the TLS evolves with time. Leaving aside transient states, the study will be limited to the equilibrium state, where the TLS is autonomous. The use of the normalized variables (*e.g.* Y^*; see section b.) allows to consider an equilibrium state of the model, even though the real variables (*e.g.* Y) are growing over time.

As is well known, poles of Laplace transform of TLS solutions are eigenmodes of the system. The same holds for Borel transform: determining the poles of the

Borel transform yields the complete dynamics of the system. Since the Borel transform of TLS matrix coefficients and solutions are numerically computed on the real axis $\tau > 0$, the problem of describing the dynamics of a system is thus reduced to that of determining the poles of the Borel transform of the TLS solution from its numerical values on the positive real axis.

The method to study one feedback loop is very elementary: the TEF elimination process is based on the fact that if one is pursuing the elimination procedure of all variables but one, say $\mathring{\delta}\varphi_1$, from the second row of system (21), then the remaining scalar equation reads :

$$(1 + \mathcal{C}'_{11}(\tau)) \cdot \mathcal{B}[\mathring{\delta}\varphi_1](\tau) \quad = \quad \mathcal{B}[\mathring{\delta}{\varphi_1}'_{ins}](\tau) \tag{24}$$

where $\mathring{\delta}\varphi'_{1,ins}$ is the φ_1 change predicted by the TLS when the rest of the system (that takes into account all of the eliminated variables) is insensitive to φ_1 variation (in other terms: when the loop is cut just after φ_1 in Fig. 3). The reduced matrix $\mathcal{C}'_{11}(\tau)$, or rather $g_1(\tau) = -\mathcal{C}'_{11}(\tau)$, represents the effect of closing the feedback loop: φ_1 perturbation $\rightarrow$ perturbation impact on the rest of the system $\rightarrow$ further φ_1 perturbation. Contrary to the feedback static gain, the *feedback dynamic gain* g_1 is a function of τ. Equation (24) may be rewritten as:

$$\mathcal{B}[\mathring{\delta}\varphi_1](\tau) \quad = \quad (1 - g_1(\tau))^{-1} \cdot \mathcal{B}[\mathring{\delta}{\varphi_1}'_{ins}](\tau) \tag{25}$$

Hence, the poles of $\mathring{\delta}\varphi_1(\tau)$ are i) the poles of $\mathring{\delta}{\varphi_1}'_{ins}$, *i.e.* the poles of the open-loop model; ii) the poles of $(1 - g_1(\tau))^{-1}$, *i.e.* the poles corresponding to the feedback. The inverse Borel transform of Eq. (25) provides the full dynamics of the feedback, *i.e.* the temporal response of the perturbed variable, and reads:

$$\mathring{\delta}\varphi_1(t) \quad = \quad \mathcal{B}^{-1}\left[\frac{1}{1 - g_1(\tau)}\right] * \frac{d}{dt}\mathring{\delta}\varphi_{1_{ins}}(t) \tag{26}$$

Note that the function $g_1(\tau)$ generalizes the feedback static gain, since:

$$\lim_{t\rightarrow+\infty} \mathcal{B}^{-1}[g_1(\tau)](t) \quad = \quad \lim_{\tau\rightarrow+\infty}[g_1(\tau)] \quad = \quad g \qquad \text{(static gain)} \tag{27}$$

But where the feedback static gain only describes the response corresponding to an asymptotic behaviour (the equilibrium value), Eq. (26) describes the whole response dynamic of $\mathring{\delta}\varphi_1$ and thus the whole dynamics of the feedback. Moreover, this approach explicitly shows that feedbacks are indeed a linear concept.

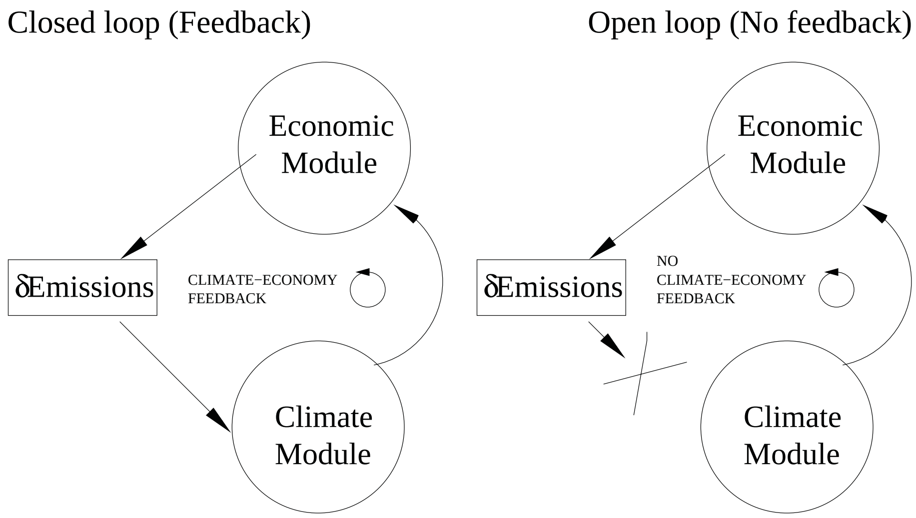

Figure 4: Scheme of the climate-economy feedback (left); and illustration of the open-loop model (right).

d. The Climate-Economy Feedback

Choosing the emissions E as the last retained variable, Eq. (26) becomes:

$$\mathring{\delta}E(t) \;=\; \mathcal{B}^{-1}\left[\frac{1}{1-g_E(\tau)}\right] * \frac{d}{dt}\mathring{\delta}E_{ins}(t) \qquad (28)$$

where $\mathring{\delta}E$ is the E change predicted by the TLS, in the closed loop case, after an exogenous perturbation; $\mathring{\delta}E_{ins}(t)$ is the E variation obtained in the open loop case, after the same perturbation; and $g_E(\tau)$ is the feedback dynamic gain of the ”climate-economy feedback”.

In other terms, $\mathring{\delta}E(t)$ is the complete model response after an exogenous perturbation; and $\mathring{\delta}E_{ins}(t)$ is the response of a model based on the IPCC methodology, neglecting the feedback from the climate impacts to the emissions.

We can then define the *feedback factor function* as:

$$FF_E(t) \;=\; \mathcal{B}^{-1}\left[\frac{1}{1-g_E(\tau)}\right] \qquad (29)$$

The feedback factor function corresponding to the climate-economy feedback, $FF_E(t)$, is the response of E to a perturbation that would have lead to a 1 GtC step[3] of E at $t = 0$ in the open-loop model, that follows the IPCC methodology.

[3]Since we use the TLS in this analysis, the response is proportional to the perturbation.

Hypothesis	λ_1	τ_1	λ_2	τ_2	static gain g
Moderate imp.	$7.5 \cdot 10^{-2}$	30.5 yrs	$-1.9 \cdot 10^{-1}$	77.5 yrs	-11.5%
Strong imp.	$2.3 \cdot 10^{-1}$	33.6 yrs	$-4.5 \cdot 10^{-1}$	62.9 yrs	-20.7%

Table 3: Climate-Economy feedback poles and static gains for 2 hypotheses on impact levels, as calculated by numerical computation of the TLS of the model.

As illustrated in Fig. 4, this methodology gives the difference between the responses of the closed-loop model (in which climate impacts influence emissions) and the IPCC-like model. It thus provides an assessment of the validity of the IPCC methodology.

Note that if a step function is here used as the perturbation of the model, the relationship (28) allows to build the response to the model to any other kind of perturbations.

Thanks to the Jacobian matrix of the model, that are explicitely calculated from the model equations, the system (21) can be numerically solved, and the inverse Borel transform (see Appendix A) yields then the climate-economy feedback factor function:

$$FF_E(t) = 1 + \lambda_1 \cdot \left(1 - e^{-\frac{t}{\tau_1}}\right) + \lambda_2 \cdot \left(1 - e^{-\frac{t}{\tau_2}}\right) \qquad (30)$$

The numerical values in the two hypotheses on the impact levels are reproduced in Tab. 3, and the feedback factor functions are shown in Fig. 5.

The complexity of the model is here reduced to two poles: the response of the model to a step in emissions has two components, each of them characterized by its intensity and its characteristic time. Such a description of the model response is very rigorous and separates the intensity of the phenomena and their characteristic time. This allows to produce more robust information than single simulations and to question explicitly the problem of inertia.

The feedback factor function shows the race between climate change impacts and adaptation processes. One should mention its very long characteristic times. It denotes the time needed by the whole system to react to a perturbation and is due to the fact that climate change is a problem of stock: variables that matter are the CO_2 concentration and the stock of capital, which are cumulative variables. In this case, an additional emission enhances the climate change, which will impact on the economy and then reduce the emissions. This process needs more than 60 years to act, and an emission reduction does not change the impacts for about 20 years. Such length of time compared with other characteristic times of the climate and of the socio-economic system shows that this feedback is not capable to act as a natural ”damping process” which might automatically adapt the anthropogenic emissions to the climate sensitivity. In other words, if impacts are found to be serious, the emission reductions corresponding to economic damages will arrive too late to control the climate change and avoid stronger damages over a time-scale of a few centuries: if climate change is dangerous, a strongly anticipated abatement policy is the only way to avoid it.

It is noteworthy that, while the feedback intensity varies with the impact level, the characteristic times do not change much. The conclusion concerning the absence of a natural damping process that control climate change is thus independent of the level of the damages.

The length of this time scale, and its independence to the impact level, demonstrates also that the IPCC methodology provides an acceptable estimate of the damages over the common time horizon of one century, even if the climate impacts are strong. Indeed, the fact that most of the impacts are expected for the second part of the XXIth century and the characteristic time of the feedback show that, *whatever is the level of impacts*, only a limited part of these impacts can be reduced by the taking into account of the feedback from impacts to emissions up to 2100.

Finally, this emission feedback function can be equivalently expressed in terms of production (cf Eq. (15)): if the production is permanently increased by 1$, the emissions are increased, and $FF_E(t)$ shows how the additional 1$ of production is reduced in time t by the corresponding additional climate change. For example, one century after the production step, the additional production due to this step is reduced by approximately 7% in the moderate impact case. Finally, after three centuries, about 12% of any additional production would be lost because of the corresponding additional climate change (still in the moderate impacts case). This allows to define a *climatic cost of growth* as the additional cost of the impacts due to the additional emissions due to economic growth. This metric is an original and rigorous way of quantifying climate change damages, that is less dependent on the emission scenario than other quantification methods.

5. Conclusive discussion

Three types of conclusions can be derived from this exercise. First, in pure methodological terms, it couples a conventional economic model to a simple climate module in such a way that the characteristic times can be rigorously scrutinized. In particular, a new impact and adaptation modelling is proposed: the absolute impacts linked to a stabilized climate state, and the transient impacts caused by a changing climate, are explicitly differentiated. Transient impacts involve an *adaptive temperature*, *i.e.* the temperature to which the socio-economic system is adapted.

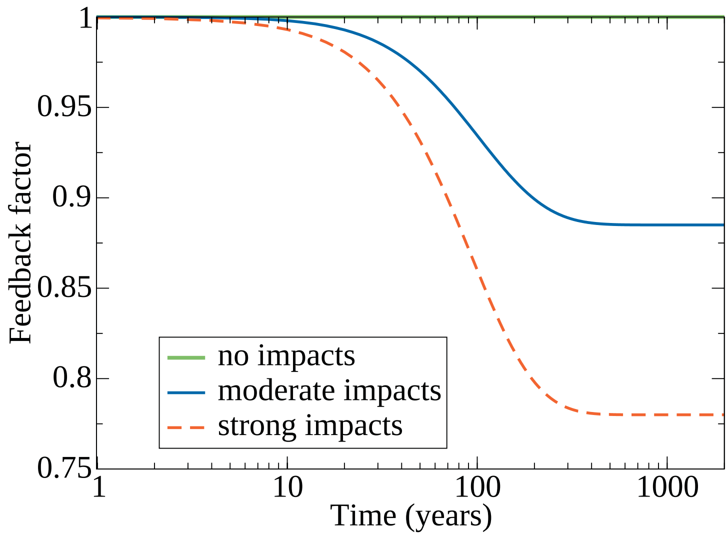

Figure 5: Feedback factor function of the climate-economy feedback. It provides the full model response to a perturbation, that would have lead to a 1 GtC step at $t = 0$ in the open-loop model. It also represents the difference between an IPCC-like scenario and a full-model scenario.

Whenever the adaptive temperature differs from the real surface temperature, the socio-economic system faces impacts (through a lower productivity and a shorter lifetime of the capital). Moreover, an adaptation process, summarizing autonomous and planned adaptation, drives the adaptive temperature toward the real temperature with a given characteristic time.

Second, this paper demonstrates, based on a simple exercise, the interest of the TEF/ZOOM methodology as a tool to overcome some limitations of classical simulations, which are often difficult to analyse rigorously: the TEF/ZOOM methodology allows for a precise definition of the feedback function characterizing the dynamics of a feedback loop (through the additional change of a variable perturbed by a step). Applied to the climate-economy feedback, this method leads to the conclusion that the climate-economy feedback has a feedback static gain of -10%, with a 70-year characteristic time. The feedback gain can also be interpreted as the elasticity of the final emissions (or, equivalently, of the final production) with respect to a permanent increase of the emissions (respectively of the production): if a constant additional amount of goods is produced each year, about 12% of this amount will finally be lost each year because of climate change impacts. In other words, a 1% growth rate results only in a 0.93% growth after one century and in a

0.88% growth over the (very) long term. This can be interpreted as a *climatic cost of growth*. This is an original way of measuring the climate change damages, which is less dependent on the emission scenario than usual damage metrics.

This model brings out three insights that deserve further investigation: (i) the absence of impact over the long-term (thanks to adaptation) does not preclude significant mid-term impacts, making it essential to take into account the time profile of the climate change impacts; (ii) The time scale of the climate-economy feedback indicates that, because of the inertia of the climate and economic systems, the damages cannot act as a natural damping process controlling climate change: if strong impacts happen, their influence on concentration occurs too late to control climate change and avoid stronger impacts. The weak sensitivity of the feedback characteristic times to changes in the impact level demonstrates the robustness of this conclusion. (iii) This time scale shows that climate change management requires a large anticipation, since the first effects of a mitigation effort influence back the economy only 20 years later.

Last, the length of the climate-economy feedback time scale demonstrates that following the IPCC methodology, and neglecting the feedback from climate impacts to emissions in the economic assessment of climate change, should not influence much the results up to 2100, even if the impacts are strong. Here, the point is not to know if the climate-economy feedback is weak enough to be neglected, the point is that this feedback is too slow to act significantly over a few decades, independently of its magnitude.

6. Acknowledgments

The author wishes to acknowledge METEO-FRANCE for financial support and to thank Alain Lahellec, Jean-Yves Grandpeix and Robert Franchisseur for their support, together with Patrice Dumas, Frédéric Ghersi, Jean-Charles Hourcade and two anonymous referees for helpful comments and remarks. The remaining errors are entirely the author's.

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A Appendix: the Transfer Evolution Formalism

a. Tangent Linear System Analysis

As explained in the article, the model is mathematically represented by a set of equations of two kinds:

1. cells:

$$\begin{array}{rcl} \frac{\partial \boldsymbol{\eta}_\alpha}{\partial t} & = & \mathbf{G}_\alpha(\boldsymbol{\eta}_\alpha, \boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, ...) \\ \frac{\partial \boldsymbol{\eta}_\beta}{\partial t} & = & \mathbf{G}_\beta(\boldsymbol{\eta}_\beta, \boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, ...) \\ \dots & & \end{array} \tag{A-1}$$

2. transfers:

$$\begin{array}{rcl} \boldsymbol{\varphi}_1 & = & \mathrm{f}_1(\boldsymbol{\eta}_\alpha, \boldsymbol{\eta}_\beta, ..., \boldsymbol{\varphi}) \\ \boldsymbol{\varphi}_2 & = & \mathrm{f}_2(\boldsymbol{\eta}_\alpha, \boldsymbol{\eta}_\beta, ..., \boldsymbol{\varphi}) \\ \dots & & \end{array} \tag{A-2}$$

Let $\boldsymbol{\eta}$ be the state vector of the complete system and $\boldsymbol{\varphi}$ be the vector of the dependent boundary conditions. With initial conditions at time t_0, the system is a well-posed problem.

The method consists in building the first order development of the dynamical system around its current state ($\eta(t_n)$). For each cell α, it reads:

$$\begin{array}{r} \frac{\partial \boldsymbol{\eta}_\alpha(t)}{\partial t} = \frac{\partial(\boldsymbol{\eta}_\alpha(t_n)+\boldsymbol{\delta\eta}_\alpha(t))}{\partial t} = G_\alpha(\boldsymbol{\eta}_\alpha(t_n), \varphi(t_n)) + (\underline{\underline{\frac{\partial G_\alpha}{\partial \eta_\alpha}}})(\boldsymbol{\eta}_\alpha(t_n), \varphi(t_n)) \cdot \boldsymbol{\delta\eta}_\alpha(t) \\ +(\underline{\underline{\frac{\partial G_\alpha}{\partial \varphi}}})(\boldsymbol{\eta}_\alpha(t_n), \varphi(t_n)) \cdot \boldsymbol{\delta\varphi}(t) + \mathcal{O}((t-t_n)^2) \end{array} \tag{A-3}$$

where $\boldsymbol{\delta\eta}_\alpha(t) = \boldsymbol{\eta}_\alpha(t) - \boldsymbol{\eta}_\alpha(t_n)$, and $\boldsymbol{\delta\varphi}(t) = \boldsymbol{\varphi}(t) - \boldsymbol{\varphi}(t_n)$.

The Tangent Linear System (TLS) corresponding to system (A-3) is, for each cell α:

$$\left\{ \begin{array}{c} \frac{\partial \mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_\alpha(t)}{\partial t} = \mathbf{G}_\alpha|_{t_n} + \underline{\underline{\frac{\partial G_\alpha}{\partial \eta_\alpha}}}\Big|_{t_n} \mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_\alpha(t) + \underline{\underline{\frac{\partial G_\alpha}{\partial \varphi}}}\Big|_{t_n} \mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}(t) \\ \mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}(t) = \sum_\beta \underline{\underline{\frac{\partial f}{\partial \eta_\beta}}}\Big|_{t_n} \mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_\beta(t) + \underline{\underline{\frac{\partial f}{\partial \varphi}}}\Big|_{t_n} \mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}(t) \end{array} \right. \tag{A-4}$$

where the suffix β sweeps the list of sub-domains.

We approximate the true time evolution of the model ($\boldsymbol{\delta\eta}_\alpha(t)$ and $\boldsymbol{\delta\varphi}(t)$) by $\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_\alpha(t)$ and $\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}(t)$, the TLS solutions, since they differ only by $\mathcal{O}((t-t_n)^2)$.

In formulation (A-4), the Jacobian matrices appear contain critical information for the analysis of the interactions between variables. The TLS can be solved by various methods, including Laplace transforms. Rather than Laplace transformation, we shall use the more convenient Borel transformation defined by:

$$f(t) \xrightarrow{\mathcal{B}} \mathcal{B}[f](\tau) = \frac{1}{\tau}\int_0^\infty e^{-t/\tau} f(t)dt = \frac{1}{\tau}\widetilde{f}(\frac{1}{\tau}) \tag{A-5}$$

where $\widetilde{f}(p)$ stands for the Laplace transform of $f(t)$. Contrary to the Laplace variable, the Borel variable τ is real and homogeneous with time.

Because $\mathcal{B}[\partial f/\partial t] = (1/\tau)\mathcal{B}[f]$, the Borel transform of Eq. (A-4) reads:

$$\left\{\begin{array}{c} \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_\alpha] = \overbrace{\left[1-\tau \underline{\underline{\left.\frac{\partial G_\alpha}{\partial \eta_\alpha}\right|_{t_n}}}\right]^{-1} \tau\, \mathbf{G}_\alpha|_{t_n}}^{\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{\alpha,dec}]} + \tau \overbrace{\left[1-\tau \underline{\underline{\left.\frac{\partial G_\alpha}{\partial \eta_\alpha}\right|_{t_n}}}\right]^{-1} \underline{\underline{\left.\frac{\partial G_\alpha}{\partial \varphi}\right|_{t_n}}}}^{\underline{\underline{\mathcal{F}}}} \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}] \\ \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}] = \sum_\beta \underline{\underline{\left.\frac{\partial f}{\partial \eta_\beta}\right|_{t_n}}} \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_\beta] + \underline{\underline{\left.\frac{\partial f}{\partial \varphi}\right|_{t_n}}} \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}] \end{array}\right. \tag{A-6}$$

If the cell variables $\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}$ are eliminated from the second equation, the complete system of equations (which includes cells) becomes:

$$\left\{\begin{array}{c} \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}] = \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{dec}] + \underline{\underline{\mathcal{F}}}\,\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}] \\ \left[1+\underline{\underline{\mathcal{C}}}\right] \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}] = \mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}_{ins}] \end{array}\right. \tag{A-7}$$

where the quantities $\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{dec}]$, $\underline{\underline{\mathcal{F}}}$, $\underline{\underline{\mathcal{C}}}$, $\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}_{ins}]$ depend on τ and can be calculated from the elementary Jacobian matrices and vectors at time t_n.

The first equation of (A-7) describes the evolution of the state variables. The state variables evolve because: i) of their internal inertial evolutions $\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{dec}$ (which would be obtained if transfer models were changed to constant transfer model with $\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi} = \mathbf{0}$) ; ii) of the evolution of their boundary conditions ($\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi} \neq \mathbf{0}$). The matrix $\underline{\underline{\mathcal{F}}}$ describes the influence of transfer variables on state variables, and independently of the type of model used for these transfers ($\underline{\underline{\mathcal{F}}}$ is independent of the model of $\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}$).

In the second equation, $\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}_{ins}$ represents the variation of transfer variables if $\mathring{\boldsymbol{\delta}}\boldsymbol{\eta} = \mathring{\boldsymbol{\delta}}\boldsymbol{\eta}_{dec}$ (*i.e.* if the cell models were changed to decoupled models with $\underline{\underline{\mathcal{F}}} = 0$). Consequently, $\underline{\underline{\mathcal{C}}}$ represents the effect of cell and transfer coupling.

The developed expression of the matrix $\underline{\underline{\mathcal{C}}}$ shows how the partial derivatives defined at the cell and transfer level combine. The coefficients of the coupling matrix are rational fractions of the variable τ. This is the way the full dynamic of the system bounds the remaining variables after an elimination process.

b. Numerical solution of the Transfer Evolution Formalism

For large systems, the above matrices are huge and sparse, and exhibit an internal structure that depends upon the connections between cells and transfers. The full algorithm of the ZOOM[4] solver follows a technique called "relaxed super-nodes hyper multi-frontal method" (cf. Liu (1992)). We focus here on the principles of the resolution that explain how the system dynamics is described by the coupling coefficients.

Equivalence between Borel transform and the Crank-Nicolson scheme

It is easily shown that the Crank-Nicolson resolution of the system (A-4) with a time step δt, is identical to its Borel transform (A-7), with the correspondence $\tau \longleftrightarrow \frac{\delta t}{2}$.

To demonstrate this equivalence, let $\hat{\delta} X$ be the time evolution of variable X approximated by a Crank-Nicolson scheme, and consider the linear system:

$$\frac{\partial \eta(t)}{\partial t} = A \cdot \eta(t) \tag{A-8}$$

If $\eta(t) = \eta_0 + \delta\eta(t)$, with $\delta\eta(0) = 0$, it may be rewritten as:

$$\frac{\partial(\eta_0 + \delta\eta(t))}{\partial t} = A \cdot (\eta_0 + \delta\eta(t)) \tag{A-9}$$

If a Crank-Nicolson scheme is applied to the system (A-9), with a time step δt, the discretized equation reads:

$$\frac{\hat{\delta}\eta(\delta t)}{\delta t} = A\frac{1}{2}(2\eta_0 + \hat{\delta}\eta(\delta t)) \tag{A-10}$$

which gives the time evolution of η, since $\hat{\delta}\eta(\delta t) \approx \delta\eta(\delta t)$ for small δt.

For any $t > 0$, $\hat{\delta}\eta(t)$ is given by:

$$\hat{\delta}\eta(t) = \left(1 - \frac{t}{2}A\right)^{-1} A\eta_0 \cdot t \tag{A-11}$$

Now, the Borel transform of the system (A-9) reads:

$$\mathcal{B}\left(\frac{\partial\delta\eta(t)}{\partial t}\right) = \frac{1}{\tau}\mathcal{B}(\delta\eta)(\tau) = \mathcal{B}(A \cdot (\eta_0 + \delta\eta(t))) = A\mathcal{B}(\eta_0) + A\mathcal{B}(\delta\eta)(\tau) \tag{A-12}$$

[4] ZOOM is a TEF dedicated solver developed by authors and colleagues.

which can be rewritten (because $\mathcal{B}(k) = k$) as:

$$\mathcal{B}(\delta\eta)(\tau) \;\; = \;\; (1 - \tau A)^{-1} A \eta_0 \tau \qquad \text{(A-13)}$$

Equations (A-11) and (A-13) show that the Crank-Nicolson integration of a linear system is equivalent to the Borel transform of the system, through the relationship:

$$\hat{\delta}\eta(t) \;\; = \;\; 2 \cdot \mathcal{B}(\delta\eta)(\frac{t}{2}) \qquad \text{(A-14)}$$

Time evolution of the model

For each time step, the ZOOM solver solves the second matrix equation of (A-7) for $\mathcal{B}[\mathring{\delta}\varphi]$. The first equation is then solved for $\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}]$. Thanks to the property (A-14), this gives an approximation of the temporal evolution of the model variables between t_n and $t_n + \delta t$.

TLS Analysis

As is well known, poles of Laplace transform of TLS solutions are eigenmodes of the system. The same holds for Borel transform: determining the poles of the Borel transform yields the complete dynamic of the system.

ZOOM is able of computing numerically the Borel transform of the TLS solution ($\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\eta}](\tau)$ and $\mathcal{B}[\mathring{\boldsymbol{\delta}}\boldsymbol{\varphi}](\tau)$) on the real axis $\tau > 0$. The problem of describing the dynamics of a system is thus reduced to that of determining the poles of the Borel transform of the TLS solution from its numerical values on the positive real axis.

In particular, in Eq. (A-7), the poles of $\mathcal{B}[\mathring{\delta}\varphi](\tau)$ are i) the poles of $\mathcal{B}[\mathring{\delta}\varphi_{ins}]$, i.e. the poles of the model without taking into account the interactions between sub-systems; ii) the poles of $(1 + \underline{\underline{\mathcal{C}}})^{-1}$, i.e. the poles corresponding to the sub-system interaction. The inverse Borel transform of Eq. (A-7), obtained by an identification of simple elements, provides the full dynamics of the model. The methodology consists here in fitting the Borel transform with a linear combination of sigmoid and bump functions, which are the only possible Borel transforms of linear differential equation solutions. From the characteristic times of the corresponding poles and their residue, the original function can easily be reconstructed without inverse Borel transform.

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