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POTSDAM - A SET OF ATMOSPHERE STATISTICAL-DYNAMICAL MODELS: THEORETICAL BACKGROUND

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Abstract

The concept and theoretical background are presented of the hierarchy of Potsdam Statistical-Dynamical Atmospheric Models (POTSDAM). In this, the procedure of the derivation of the generalized statistical-dynamical equations (SDEs) is described at length which are the result of the spatial and temporal averaging of the primitive Reynolds-type hydrothermodynamical equations over the horizontal and time scales of the order of $\Delta L \approx 1000 - -3000\ km$ and $\Delta\tau \approx 10 - -30\ days$, respectively. Applying a scale/magnitude analysis of the order of terms the generalized SDEs are reduced to the basic SDEs of POTSDAM. By the vertical integration of these latter the governing SDEs are developed then, with the use of the universal vertical structure of the climate variables on the $(\Delta L, \Delta\tau)$ scale. Under some additional simplifications the governing SDEs of POTSDAM-2 model are obtained which is used as the atmospheric module of CLIMBER-2 Earth system Model of Intermediate Complexity.

Contents

Introduction

Current studies in the field of climate modelling are carried out using models of different complexity, starting from the simplest energy–balance models (EBMs) up to highly sophisticated three—dimensional general circulation models (GCMs).

Atmospheric modules of GCMs (AGCMs) use primitive hydrothermodynamical equations (see, e.g., [8, 9, 20, 187, 106]). These are state-of-the-art modules with high spatial and temporal resolutions: in the best of them the latitudinal and longitudinal resolution is up to 2°, the number of vertical levels is more than 20, and the time step is up to 10 minutes. AGCMs are widely used for simulating present climate atmospheric characteristics in terms of the annual cycle and intraseasonal and interannual variability, as well as for evaluating natural and anthropogenic climate impacts [139, 151, 187]. However, due to their high degree of complexity, AGCMs have a relatively long turnaround time, even on the most advanced computer systems. This fact places certain limitations on the application of AGCMs in the cases of multivariant analysis of probable future climate change scenarios or long–term (e.g., paleoclimate) integrations. Moreover, different parts of AGCMs are not usually equally developed (e.g., cloudiness, which is an important component of the climate system, is one of the relatively weak points of these models [146]).

Atmospheric modules of EBMs (AEBMs) are highly efficient in terms of the running time and find a great many applications in the climate change studies (see, e.g., [25, 200, 213, 154, 163, 153, 89, 3]). It should be noticed, however, that these models are much less sophisticated than AGCMs and include only a small number of climatic variables and feedbacks.

In [90] a basic concept of the stochastic climate models was first formulated. Such models are widely used for the evaluation of the variability of the climate characteristics, as well as for the estimation of the probability of large climate deviations (see, e.g., [121, 215, 46, 10]). A relatively weak point of such models is that the module describing the evolution of slow variables in these models is mainly based on a simplified EBM approach.

Another tool for exploring climate change impacts are 1-dimensional (only the z-coordinate is resolved) atmospheric radiative–convective and radiative–turbulence models (see, e.g., [140, 181, 107, 135]). These models are predominantly used for the assessment of the influence of the radiatively active atmospheric constituents on the Earth's thermal regime.

Atmospheric statistical–dynamical models (ASDCMs) occupy an intermediate position between AGCMs and AEBMs (as regards the number of the explicitly described climatic processes and variables) in the hierarchy of atmospheric models. These models are widely employed in the analysis of differerent climate-forming mechanisms and feedbacks, as well as in explorations of future climate projections and paleoclimate integrations (see, e.g., [1, 192, 209, 165, 191, 201, 167, 212, 15]). The crucial points of the conventional ASDCMs are the zonal averaging applied in the majority of these models to the climate variables when deriving the model's governing equations, and the basically heuristic character of the parameterisations of the subgrid (e.g., synoptic) processes.

There exists, so far, an obvious gap between simple and comprehensive models. This paper describes an attempt to develop an atmospheric climate module of intermediate complexity. The model can be treated as a multilevel nonzonal grid-cell ASDCM. It is concerned with a large set of climatic processes and feedbacks, comparable with that used in the comprehensive models. Due to its rather low temporal resolution and simplified governing equations, our model has a fast turnaround time. This makes it suitable for long-term simulations of climate system dynamics, particularly for studies of the paleoclimate and long-term future climate scenarios.

In the paper, we present the theoretical background for the hierarchy of Potsdam Statistical-Dynamical Atmospheric Models (POTSDAM) which are under development at PIK. The first generation, POTSDAM-2, is already implemented in the Earth system model of intermediate comlexity CLIMBER-2. The paper is organized as follows. In Chapter 1 the basic concept of POTSDAM is stated. In Chapter 2 the so-called primitive equations of the atmosphere are set out which are used as the background for POTSDAM. Chapter 3 describes the procedure of the derivation of the generalized statistical-dynamical equations (SDEs) from the primitive ones. In Chapters 4 and 5 a set of simplified basic SDEs of POTSDAM are shown which result from the application of the scale/amplitude analyses to the generalized SDEs derived in Chapter 3. In Chapter 6, corresponding SDEs for the diabatic processes are given. Chapter 7 outlines the governing SDEs of the POTSDAM-3 model which are the result of vertical integration of the mentioned basic SDEs within the troposphere and stratosphere layers, with the use of the universal vertical structure (UVS) of climate variables in these layers. Finally, in Chapter 8 a simplified version (POTSDAM-2) of POTSDAM-3 is described. Appendices A and B contain the description of some details of the model equation derivation.

Chapter 1

The concept of Potsdam Statistical-Dynamical Atmosphere Models

The main concept of POTSDAM is that the basic features of the long–term evolution of the atmosphere can be expressed in terms of nonstationary, nonlinear interactions among large–scale, long–term components (LSLTC) of the main atmospheric variables with characteristic temporal and horizontal spatial scales equal to and more than $\Delta\tau \approx 15 - 30\ days$ and $\Delta L \approx 1000 - 3000\ km$, and the ensembles of the transient synoptic–scale eddies/waves represented by their $(\Delta L, \Delta\tau)$ averaged ensemble statistical characteristics (ESC), e.g., second moments (SM). More fast small-scale energy loading atmospheric processes are diagnostically parameterised in the model. Namely, atmospheric small/mesoscale eddies/waves are treated in terms of the vertical and horizontal coefficients of "turbulence", and the contribution of the ensembles of dry/moist convection and fast gravitational waves is described by diagnostic formulas in terms of LSLTC and ESC.

The phenomenological premises for this approach are

1. The existence in the Earth's atmosphere of a limited set of basic energy-loading, slowly-trending quasi-stationary atmospheric objects (patterns) that are topologically (structurally) stable on $(\Delta L, \Delta\tau)$ scales. Based on the results of a large body of theoretical, modelling and empirical research (see, e.g., [182, 221, 19, 30, 155, 33, 142, 132, 141, 152, 159, 18, 216, 156, 180, 24, 164, 53, 111, 32, 160, 233, 72, 119, 183, 203, 231, 232, 239, 101, 48, 4, 162, 211, 173, 212, 139, 187, 85, 136, 106, 240]) we attribute to those objects the following atmospheric structures: quasi-zonal flows (including subtropical and polar jet streams), mean meridional circulations (Hadley, Ferrel and polar cells), thermally and orographically induced quasi-stationary planetary-scale waves, Walker circulations (including the Southern Oscillation), large-scale quasi-stationary equatorial Kelvin waves, Madden-Julian circulation cells, quasi-stationary atmospheric centres of action, depressions and highs (e.g., Siberian and Azores Highs, Iceland and Aleutian Lows), monsoon circulation patterns, large-scale ultra-long (persistent) blockings. The characteristic linear scales of listed large-scale quasi-stationary objects are of the order of or larger than ΔL [205, 30, 33, 18, 159, 156, 180, 193, 197, 62, 176, 93, 98, 183, 48, 4, 161], and the characteristic temporal scales are of the order of or larger than $\Delta\tau$ [30, 33, 180, 62, 176, 183, 239, 101, 161, 136]

2. The existence in the Earth's atmosphere, on $(\Delta L, \Delta\tau)$ scales, of topologically stable, quasi-stationary slowly-trending fields (patterns) of energy-loading ensemble characteristics

of the transient synoptic-scale eddies/waves and fronts. The specific linear and temporal scales of the mentioned quasi-stationary ensemble patterns are of the order of or larger than, respectively, ΔL and $\Delta\tau$ [159, 156, 62, 98, 105, 183, 161]

3. The existence of characteristic spatial/temporal correlation radii for the above-mentioned faster small-scale energy-loading atmospheric processes, which (radii) are well below the corresponding scales of the objects/patterns listed in points 1. and 2.[133, 246, 157, 43, 16, 244, 110, 6, 7, 95, 55, 17, 225, 224]

We assume that the individual objects entering the sets of atmospheric structures mentioned in points 1., 2. and 3., as well as the sets themselves, are universal, i.e., topologically (structurally) stable under a broad range of climatic states, even state somewhat distant from the present-day climate conditions. We also anticipate that the phenomenological features of the mentioned individual objects are also universal, in the mentioned sense.

The second conceptual point of the model approach is based on the assumption supported by the empirical and theoretical studies, that the vertical structure of the main atmospheric variables at $(\Delta L, \Delta\tau)$ scale can be adequately represented by a low number of layers. The vertical distributions of the main LSLTC within these layers, as well as the layers themselves, are considered universal in the above-mentioned sense. For example, the temperature vertical structure $\hat{T}(z)$ (hereinafter the circumflex over the variable denotes the $(\Delta L, \Delta\tau)$ averaging) at a variety of the climate conditions is assumed to be represented by a surface layer (SL), planetary boundary layer (PBL), free troposphere, stratosphere and mesosphere, while $\hat{T}(z)$ inside these layers is described by a limited set of the universal functions. E.g., $\hat{T}(z)$ in the free troposphere and stratosphere is presumed to be a piecewise linear function of height, with the tropospheric lapse rate $\hat{,}$ dependent mainly on the sea surface air temperature. The vertical distribution of the specific humidity $\widehat{q_v}(z)$ within SL is surmised to be adequately depicted by the universal functions depending on the SL thermal and dynamic stratification. Above the surface layer, the vertical profile of $\widehat{q_v}(z)$ is supposed to be universally regulated by the processes of the small/meso and macroturbulent vertical mixing and large-scale vertical advection, on the one hand, and precipitation, on the other hand. Tropospheric cloudiness at this scale is assumed to be closely approximated by a three-layer model. The $(\Delta L, \Delta\tau)$ ensemble-averaged vertical structure of the main variables in the "representative" synoptic object is anticipated to be adequately described by a few vertical modes attributed mainly to the processes of the baroclinic and barotropic instabilitis of the large-scale atmospheric flows as the basic source of the synoptic eddy/wave generation. This assumption on the universal vertical structure (UVS) of the main LSLTC and the representative synoptic objects at $(\Delta L, \Delta\tau)$ scale is supported by the results of a large number of theoretical and modelling studies, as well as the empirical explorations in a variety of geographical regions and periods of time (see, e.g., [221, 155, 97, 42, 216, 38, 156, 198, 199, 234, 111, 160, 21, 217, 239, 236, 116, 237, 4, 211, 212, 84, 161]). The use of UVS allows one to reduce the 3-dimensional description of the atmosphere to a set of 2-dimensional, vertically integrated prognostic equations, and at the same time, provide the possibility to reconstruct the 3-dimensional atmospheric fields for any purpose (e.g., for the computation of the radiative fluxes) at any time step.

The third basic point of the model approach is the assumption that the set of the main physical (generally nonlinear) mechanisms and feedbacks standing behind the long-term large-scale evolution and interaction among the above-mentioned sets of climate-forming objects (structures) is also universal [205, 33, 32, 203, 232, 238].

It is important to note that the chosen interval $\Delta\tau$ of time averaging is considerably longer than the characteristic duration of the lifecycles of the individual entities (eddies/waves and fronts) which constitute the ensembles of transient synoptic objects (ESC), and also $\Delta\tau$ is much longer than the residence time of the mentioned individual synoptic objects over the $(\Delta L)^2$ area, due to the high phase velocity of these latter [62, 98]. Meantime, $\Delta\tau$ is of the order of (or less than) the characteristic duration of the lifecycles of the individual entities which enter the above-mentioned set of the LSLTC, as well as of the residence time over the $(\Delta L)^2$ area of the individual objects from the set of LSLTC (due to the low phase velocity of these latter) [62, 98, 183, 239, 101]. So, the $(\Delta\tau, \Delta L)$ averaging, on the one hand, permits to obtain rather stable energy-loading ensemble characteristics of transient synoptic objects [105, 183, 239], even in the absence of the pronounced (if at all) minimum in the time spectra of atmospheric variables in the $\Delta\tau$ range of periods [227, 230, 62, 98]; on the other hand, it allows one to realistically describe the individual LSLTC objects. In this, the seasonal cycle of ESC and LSLTC is not significantly distorted. Furthermore, the ΔL averaging scale is optimal to capture the basic spatial structures of LSLTC and ESC. In other words, LSLTC and transient synoptic objects appear to be "well separated" in three-dimensional (specific wavelength, period, phase velocity, see, e.g., [62]) phase space, that enables one to obtain stable ensemble characteristics of ESC and individual characteristics of LSLTC objects on the $(\Delta\tau, \Delta L)$ scale.

Aforesaid taken together justifies the application of the $(\Delta L, \Delta\tau)$ spatial/temporal averaging procedure which we apply to a set of the atmosphere primitive hydrothermodynamical equations to obtain the atmosphere climate equations, and makes it possible then to develop parameterisations and to perform a scale/magnitude analysis of the developed equations which could be applicable to a broad range of climate states.

A starting point for the derivation of POTSDAM equations is a set of the original (primitive) hydrothermodynamical equations for the atmosphere in the inelastic and Reynolds approximations (see Chapter 2). Any atmospheric variable entering a set of primitive equations is represented then by a large-scale long-term component (LSLTC) with spatial/temporal scales equal to or more than $\Delta L/\Delta\tau$, and the deviation from it. This latter is considered to mainly attribute to the atmospheric synoptic processes.

Following this idea, the mentioned primitive equations are subjected to a procedure of $(\Delta L, \Delta\tau)$ spatial/temporal averaging. In this, the hydrostatic, quasi–solenoidal and quasi–geostrophic approximation for the synoptic component of motion is used, and a close-to-Gaussian distribution, with respect to time and space, of the synoptic eddy/wave ensembles is supposed [121, 160, 45, 238]. As a consequence of the averaging procedure the resultant statistical-dynamical equations (SDE) for LSLTC include, besides LSLTC, the ensemble statistical characteristics (ESC) of the synoptic component, e.g., second moments, SM (auto– and cross–correlation functions). An additional set of SDE for ESC is derived from a set of the above-mentioned primitive equations, by applying a procedure which is conventionally used in statistical fluid dynamics and in the theory of turbulence [114], [150]. The supposed close-to-Gaussian distribution for the ensembles of the synoptic eddies/waves provides a basis for a rigorous procedure of reducing an infinite set of SDE for ESC to a limited number of SDE for SM. The outlined procedures of the derivation of SDE for LSLTC and SM are described at length in Chapter 3.

A set of the mentioned generalized SDE then undergo spatial–temporal scale–magnitude analysis, to obtain the basic SDE of POTSDAM for LSLTC and SM presented, respectively, in Chapters 4 and 5. The diabatic terms which enter the right-hand sides of correspondent basic equations are described in Chapter 6.

Finally, a set of governing SDE is derived implementing a procedure of vertical integration of the basic SDE within the aforementioned atmospheric layers, with the use of the assumption on the universal vertical structure (UVS) of LSLTC and ESC (see Chapter 7 where the most general setup of the POTSDAM governing SDE, POTSDAM-3–which it is envisaged will be applied in a future generation of the CLIMBER climate model–is outlined, and Chapter 8 in which a simplified version of POTSDAM-3, POTSDAM-2, is described in detail).

The $(\Delta L, \Delta\tau)$ averaging (which allows one to avoid an explicit description of the fast, e.g., synoptic-scale, components of the atmospheric motions), as well as the vertical integration, make it possible to run the model with the time step of the order of a day. As a result, the model is much less time-consuming than AGCMs.

Without violation of the basic concept and structure of the model, the Gaussian-type distribution of the synoptic ensembles can, in principle, be replaced in POTSDAM by a more refined one, with more a complex, in the general case, closure procedure. Let us notice here that by virtue of the Central Limit Theorem (see, e.g., [90]), the description of the synoptic ensembles at the level of SM is the more precise, the closer the real atmosphere is to an ideal one in which the above–mentioned phenomenological premises 1.–3. hold with the absolute accuracy, i.e. $\Delta\tau_{sy}$ and ΔL_{SY} are infinitesimal.

A zonal prototype of POTSDAM was earlier developed in [165] and improved in [166]. A number of the investigations on the relative role of different climate-forming factors were pursued using that option of the model (see, e.g., [171]). A nonzonal prototype–described in detail in [172]–was derived in [167], [168] and used then for the study of the decadal climate variability in [86]. Currently POTSDAM-2–the first generation of POTSDAM–is utilized as the atmospheric module of the CLIMBER-2 Earth system model of intermediate complexity [170] which is employed for a large body of climate research (see, e.g., [64, 63, 23, 65]).

Chapter 2

Background primitive equations

In this Chapter, we present a complete set of the so-called primitive equations of the atmosphere which are utilized as such in AGCMs. In POTSDAM, these equations are exploited as a starting point for the derivation of the basic SDE described below in Chapters 4 and 5.

The original energy balance equation used as one of the background equations for POTSDAM is written in the inelastic approximation (see, e.g., [228, 30])

$$\frac{\partial \rho_0 T}{\partial t} + \nabla_H \cdot \rho_0 \vec{v}_H T + \frac{\partial \rho w T}{\partial z} = \frac{1}{c_v}(-p\nabla \cdot \vec{v} + Q^T). \tag{2.1}$$

Here t – time; c_v – the specific heat of air at constant volume; T – air temperature; ρ – air density; z — height; $\vec{v} = (u, v, w)$ – p — air pressure; and ρ_0 is given by

$$\rho_0 = \rho_{00} \exp(-z/H_{00}), \tag{2.2}$$

where $H_{00} = RT_{00}/g$, R –the gas constant for the air, ρ_{00} and T_{00} – (constant) reference air density and temperature, respectively.

The Q^T term in (2.1) stands for sources (sinks) of energy, i.e., the net heating rate (per unit volume)

$$\frac{1}{c_v}Q^T = q_R^T + q_{LSC}^T + q_{MC}^T + q_{SMT}^{T,c} + q_{SMT}^T \equiv q^T \tag{2.3}$$

Here q_R^T, q_{LSC}^T, q_{MC}^T, $q_{SMT}^{T,c}$ and q_{SMT}^T denote influxes of heat due to radiation, large–scale condensation, moist convection condensation, moist convection (mainly vertical) mixing and small/mesoscale turbulent horizontal/vertical mixing, respectively, each divided by c_v.

The q_R^T and q_{SMT}^T terms can be written in the form

$$q_R^T = -div\vec{F}_R^T, \quad q_{SMT}^T = -div\vec{F}_{SMT}^T \tag{2.4}$$

where $\vec{F}_R^T$ is thw (divided by c_v) sum of the shortwave (solar) and longwave (terrestrial) radiative fluxes, and $\vec{F}_{SMT}^T$ is the (divided by c_v) small/mesoscale turbulent heat flux

$$\vec{F}_{SMT}^T = \left\{-\overline{\overline{K}}_H \rho_0 \frac{\partial T}{a\cos\phi\partial\lambda}, -\overline{\overline{K}}_H \rho_0 \frac{\partial T}{a\partial\phi}, -\overline{\overline{K}}_z \rho \frac{\partial \Theta^*}{\partial z}\right\}, \tag{2.5}$$

$$\frac{\partial \Theta^*}{\partial z} = \frac{\partial T}{\partial z} + ,\ _a - ,\ _c \tag{2.6}$$

Here λ – longitude, ϕ – latitude, a – the Earth's radius, $\overline{\overline{K}}_H = \frac{c_p}{c_v}K_H$, $\overline{\overline{K}}_z = \frac{c_p}{c_v}K_z$, where K_H and K_z are horizontal and vertical kinematic small/mesoscale "eddy" diffusion coefficients for heat, c_p – the specific heat of air at constant pressure, , $_a$ is the adiabatic lapse rate, and , $_c$ is the so-called counter–gradient factor [26, 179, 44, 225, 99, 247]. Hereafter, for brevity, $\overline{\overline{K}}_H$ and $\overline{\overline{K}}_z$ are referred to as the horizontal and vertical diffusion coefficients for heat.

The background primitive dynamical equations are the Reynolds equations in the inelastic approximation (see, e.g., [228])

$$\frac{\partial \rho_0 u}{\partial t} + \nabla_H \cdot \rho_0 \vec{v}_H u + \frac{\partial \rho w u}{\partial z} = -\nabla_\lambda p + f \rho_0 v + \frac{\rho_0 u v}{a} \tan\phi + F_\lambda, \qquad (2.7)$$

$$\frac{\partial \rho_0 v}{\partial t} + \nabla_H \cdot \rho_0 \vec{v}_H v + \frac{\partial \rho w v}{\partial z} = -\nabla_\phi p - f \rho_0 u - \frac{\rho_0 u^2}{a} \tan\phi + F_\phi \qquad (2.8)$$

Here $f = 2\omega \sin\phi$ – the Coriolis parameter, where ω is the Earth's rotation angular velocity; $\nabla_\lambda p = \partial p / a \cos\phi \partial\lambda$; $\nabla_\phi p = \partial p / a \partial\phi$; F_λ and F_ϕ are, correspondingly, λ and ϕ components of the small/mesoscale + cumulus "turbulent" friction force $\vec{F}$

$$\vec{F} = div \overset{\leftrightarrow}{\tau}, \qquad (2.9)$$

where $\overset{\leftrightarrow}{\tau}$ is a stress tensor (see, e.g., [132, 160, 161]). A simplified form of F_λ and F_ϕ description is involved

$$F_\lambda = \rho_0 \overline{\overline{K}}_{DH} \Delta_{\lambda\phi} u + \nabla_z \left(\overline{\overline{K}}_{Dz} \rho \nabla_z u \right) + F_\lambda^c, \qquad (2.10)$$

$$F_\phi = \rho_0 \overline{\overline{K}}_{DH} \Delta_{\lambda\phi} v + \nabla_z \left(\overline{\overline{K}}_{Dz} \rho \nabla_z v \right) + F_\phi^c, \qquad (2.11)$$

where $\overline{\overline{K}}_{DH}$ and $\overline{\overline{K}}_{Dz}$ are, correspondingly, horizontal and vertical coefficients of viscosity, $\Delta_{\lambda\phi}$ – horizontal Laplace operator on a sphere of radius a, $\nabla_z = \partial/\partial z$, and F_λ^c and F_ϕ^c terms describe the zonal and meridional components of the "friction" force due to cumulus convection. Eqs. (2.10) and (2.11) are written in an approximation of generalized (with cumulus contribution accounted for) Reynolds friction, neglecting the non–diagonal terms in $\overset{\leftrightarrow}{\tau}$.

The background primitive water vapour balance equation is written in terms of the specific humidity q_v

$$\frac{\partial \rho_0 q_v}{\partial t} + \nabla_H \cdot \rho_0 \vec{v}_H q_v + \frac{\partial \rho w q_v}{\partial z} = q^v, \qquad (2.12)$$

where the q^v term describes sources (sinks) of water vapour (per unit volume)

$$q^v = q_{LSC}^v + q_{MC}^v + q_{SMT}^{v,c} + q_{SMT}^v, \qquad (2.13)$$

Here q_{LSC}^v, q_{MC}^v and $q_{SMT}^{v,c}$ designate water vapour influx due to large-scale condensation, moist convection condensation and mixing, respectively, and q_{SMT}^v denotes small/mesoscale "turbulent" flux $\vec{F}_{SMT}^v$ of this substance

$$q_{SMT}^v = -div \vec{F}_{SMT}^v, \qquad (2.14)$$

where

$$\vec{F}_{SMT}^v = \left\{ -\overline{\overline{K}}_{vH} \rho_0 \frac{\partial q_v}{a \cos\phi \partial\lambda}, -\overline{\overline{K}}_{vH} \rho_0 \frac{\partial q_v}{a \partial\phi}, -\overline{\overline{K}}_{vz} \rho \frac{\partial q_v}{\partial z} \right\}. \qquad (2.15)$$

Here $\overline{\overline{K}}_{vH}$ and $\overline{\overline{K}}_{vz}$ are the horizontal and vertical diffusion coefficients for humidity.

The continuity equation among a set of the background primitive equations is that one conventionally used for the description of long-term large-scale atmospheric processes [33, 223, 132]

$$\partial_t \rho = -(\nabla_H \cdot \rho_0 \vec{v}_H + \frac{\partial \rho w}{\partial z}) \tag{2.16}$$

The last two equations closing the set of background primitive equations for POTSDAM are the hydrostatic balance equation

$$\frac{\partial p}{\partial z} = -\rho g, \tag{2.17}$$

where g is the acceleration due to gravity, and the equation of state

$$\rho = \frac{p}{RT} \tag{2.18}$$

Chapter 3

Procedure of the derivation of the generalized SDE

I. Large-scale long-term atmospheric variables

To obtain the SDE for large-scale long-term components of the atmospheric variables (LSLTC) the $(\Delta L, \Delta\tau)$ spatial/temporal averaging defined above in Chapter 1 is applied to the set of primitive equations (2.1)—(2.18). In this, any variable in (2.1)—(2.18) is written in the form [167, 168, 169]

$$y_i = \hat{y}_i + y_i', \quad y_i = T, q_v, u, v, \ldots \tag{3.1}$$

where $\hat{y}_i$ is the $(\Delta L, \Delta\tau)$ average of y_i and y_i' is the deviation of y_i from $\hat{y}_i$. Synoptic–scale processes are supposed to be mainly responsible for y_i'. The y_i' component is assumed to be quasi-solenoidal (nondivergent), see, e.g., [30, 149, 160]

$$\nabla \cdot \vec{v'} = 0, \tag{3.2}$$

and of a Gaussian type [121, 45, 238]. The latter gives

$$\hat{y}_i' = \widehat{y_i' y_j' y_k'} = \widehat{y_i' y_j' y_k' y_l' y_m'} = \ldots = 0 \tag{3.3}$$

for all odd moments of y_i' and

$$\widehat{y_i' y_j'} = \widehat{y_j' y_i'}, \quad \widehat{y_i' y_j' y_k' y_l'} = \widehat{y_i' y_j'}\,\widehat{y_k' y_l'} = \widehat{y_i' y_l'}\,\widehat{y_j' y_k'} = \widehat{y_i' y_k'}\,\widehat{y_l' y_j'}, \ldots \tag{3.4}$$

for all even moments of y_i' (see, e.g., [114, 150]). The equations (3.2)—(3.4) are the essence of the $(\Delta L, \Delta\tau)$ averaging. Applying this averaging procedure to the equations (2.1)—(2.18), and using (3.2)— (3.4) a set of the SDE for LSLTC can be written as follows [167, 168, 169]

$$\partial_t \rho_0 \hat{y}_i + \nabla_H \cdot \left(\rho_0 \hat{y}_i \hat{\vec{v}}_H + \rho_0 \widehat{y_i' \vec{v}_H'} \right) + \nabla_z \left(\hat{\rho} \hat{y}_i \hat{w} + \hat{\rho} \widehat{y_i' w'} + \hat{y}_i \widehat{\rho' w'} + \hat{w} \widehat{\rho' y_i'} \right) = \hat{F}_i, \tag{3.5}$$

where $i = 1, \ldots, 4$; $y_1 = T$; $y_2 = q_v$; $y_3 = u$; $y_4 = v$;

$$\hat{F}_1 = -\frac{1}{c_v} \hat{p} \nabla \cdot \hat{\vec{v}} + \hat{q}^T, \tag{3.6}$$

$$\hat{F}_2 = \hat{q}^v, \tag{3.7}$$

$$\hat{F}_3 = -\nabla_\lambda \hat{p} + f \rho_0 \hat{v} + \frac{\rho_0}{a} \tan\phi \left(\hat{u}\hat{v} + \widehat{u'v'} \right) + \hat{F}_\lambda, \tag{3.8}$$

$$\hat{F}_4 = -\nabla_\phi \hat{p} - f\rho_0 \hat{u} - \frac{\rho_0}{a} \tan\phi \left(\hat{u}^2 + \widehat{u'^2}\right) + \hat{F}_\phi \quad (3.9)$$

$$0 = -(\nabla_H \cdot \rho_0 \hat{\vec{v}}_H + \nabla_z \left(\hat{\rho}\hat{w} + \widehat{\rho' w'}\right)) \quad (3.10)$$

$$\hat{\rho} = \frac{\hat{p}}{R\hat{T}}(1 + \frac{\widehat{T'^2}}{\hat{T}^2}) - \frac{\widehat{p'T'}}{R\hat{T}^2} \quad (3.11)$$

$$\nabla_z \hat{p} = -\hat{\rho} g \quad (3.12)$$

II. Synoptic moments

The SDE (3.5)— (3.12) for LSLTC contain, besides $\hat{y}_i$, the SM (second moments) of the synoptic component $\widehat{y'_i y'_j}$. To deduce the corresponding model initial equations for $\widehat{y'_i y'_j}$ in the free troposphere from the primitive equations given in Chapter 2, a special procedure is applied [167, 168] similar to that one conventionally used in statistical fluid dynamics and the theory of turbulence (see, e.g., [114, 150]). Namely, to derive the SDE, as an example, for $\widehat{u'T'}$ the left-hand sides of the primitive equations (2.1) and (2.7), with the usage of the continuity equation (2.16), are transformed to

$$\partial_t \rho_0 y_i + \nabla_H \cdot \rho_0 \vec{v}_H y_i + \nabla_z \rho w y_i = \rho_0 \partial_t y_i + \rho_0 \vec{v}_H \cdot \nabla_H y_i + \rho w \nabla_z y_i, \quad (3.13)$$

where in that case $i = 1, 2$; $y_1 = T$; $y_2 = u$. The equations (2.1) and (2.7) are multiplied then, respectively, by u' and T', summarized and subjected to $(\Delta L, \Delta\tau)$ averaging.

The same procedure is applied to all pairs of the primitive equations (2.1), (2.7), (2.8), (2.12), to obtain the corresponding model initial equations for the cross–correlation functions $\widehat{v'T'}$, $\widehat{u'q'_v}$, $\widehat{v'q'_v}$, $\widehat{T'q'_v}$, $\widehat{u'v'}$.

The model initial equations for the auto–correlation functions $\widehat{T'^2}$, $\widehat{q'^2_v}$, $\widehat{u'^2}$, $\widehat{v'^2}$ are obtained by multiplying any y'_i (where, in this case, $y'_i = T', q', u', v'$) into the transformed primitive equation for y_i followed by $(\Delta L, \Delta\tau)$ averaging.

Multiplying the transformed primitive equations (2.1), (2.7), (2.8) and (2.12) by w' and applying the $(\Delta L, \Delta\tau)$ averaging one can obtain the equations for the vertical cross–correlation functions $\widehat{T'w'}$, $\widehat{u'w'}$, $\widehat{v'w'}$ and $\widehat{q'_v w'}$.

The above–mentioned (see (3.2) — (3.4)) features of the synoptic–scale components are accounted for in the procedure, and the additional assumptions are taken:

1. the "representative" individual synoptic object is supposed to be quasi–geostrophic in the free troposphere, with the characteristic horizontal $\widetilde{L_{sy}}$ and vertical $\widetilde{H_{sy}}$ spatial scales close to $L_{Ro,sy}$ and $H_g Ro_{sy}$, respectively (hereinafter the tilde over the variable stands for its characteristic value). Here $H_g \approx H_{00} \approx 10^4$ m is the atmospheric scale height, $Ro_{sy} = \frac{\widetilde{U_{sy}}}{f\widetilde{L_{sy}}} \approx (0.1 \div 0.3)$ and $L_{Ro,sy} \approx 10^6$ m are, respectively, the Rossby number and the internal Rossby deformation radius inherent in the individual synoptic objects, $\widetilde{U_{sy}} \approx 10$ m/s is the characteristic value of the horizontal velocity in these objects [149, 160]. $L_{Ro,sy}$ is assumed to be much smaller than the scale of the Coriolis parameter variation with the latitude $L_f \approx a$. Near the equator, $L_{Ro,sy}$, Ro_{sy} and f transform , respectively, to the synoptic scale equatorial Rossby deformation radius $L_{Ro,sy,e}$, the equatorial Rossby number $Ro_{sy,e}$ and $f_e = \beta_0 L_{Ro,sy,e}$ ([160]), where $\beta_0 =$

$\frac{2\omega}{a}$, with a smooth matching for these variables at the (specified) northern and southern boundaries of the equatorial belt. Based on the premise of quasi–geostrophicity the UVS (universal vertical structure) of u' and v' in the "representative" individual synoptic object in the free troposphere is presumed to be adequately approximated by the barotropic and the first baroclinic modes [160, 217] correspondent to the synoptic disturbance with the largest growth rate. The UVS of the synotic-scale temperature component T' in the individual object is also anticipated to be basically described by these barotropic and first baroclinic modes (see e.g., [155, 97]).

All the ensemble characteristics of synoptic eddies/waves (e.g., second moments, i.e., auto- and cross-correlation functions) in the free troposphere are assumed to have the specific horizontal spatial scale of the order of the Obukhov deformation radius $L_O \equiv L_{RO,bt} = \frac{(gH_g)^{\frac{1}{2}}}{f} \approx 3 \cdot 10^6\ m$, the latter being a so called "screening" radius for the baroclinic eddy-eddy ensemble interaction ([160, 82, 83]. The specific vertical spatial scale of the synoptic eddies/waves ensemble characteristics in the free troposphere (except those ones including vertical velocity w'), in accordance with the Charney-Drazin [33] theory of vertical propagation of wave disturbances, is assigned the tropopause height (approximated, to a first order, by H_g). The "vertical" synoptic-scale ensemble characteristics (i.e., those ones including synoptic vertical velocity w') are assumed to have the characteristic vertical scale of the order of $H_g Ro_{sy}$ imposed by the specific vertical scale of synoptic vertical velocity w' [149, 160] in that part of the atmosphere;

2. taking into account the ratio of the characteristic synoptic time scale $\tau_S \approx 3$ days to that of the radiative processes $\tau_R \approx 10 - 20$ days (see, e.g., [101]) and accounting for the characteristic values of the penetrative cumulus cloud amount (see, e.g., [183]) the terms $\widehat{y_i' q_R^{T'}}$, $\widehat{y_i' {q_{SMT}^{T,c}}'}$, $\widehat{y_i' {q_{SMT}^{v,c}}'}$, $\widehat{y_i' F_\lambda^{c'}}$ and $\widehat{y_i' F_\phi^{c'}}$ are neglected in the model initial equations;

3. the estimation
$$\rho' \approx -\frac{\rho}{T} T' \tag{3.14}$$
(incompressibility approximation for the synoptic-scale objects) is used [30] for the description of ρ' in "horizontal" $\widehat{\rho' u'}$ and $\widehat{\rho' v'}$ second moments as well as in $\widehat{\rho' T'}$, $\widehat{\rho' q_v'}$, $\widehat{\rho' \rho'}$ and $\widehat{\rho' p'}$ cross-correlation terms in the free troposphere

4. instantaneous adaptation of the vertical (i.e., including w') synoptic-scale second moments to LSLTC and all other SM is assumed

Notice that the assumptions 3. and 4. are the direct consequences of the hydrostatic (see eq. (2.17)) and quasi-solenoidal (see eq. (3.2)) character of the synoptic component of the atmospheric motions.

Using eqs. (3.2)–(3.4) and assumptions 1.–4. the SDE for the synoptic second moments $u'T'$, $v'T'$, $u'q_v'$, $v'q_v'$ in the free troposphere can be written as follows [167, 168] (hereinafter the circumflex over variables in the text and all formulas below, except (3.23), (3.24) is dropped, so that y_i and $y_i' y_j'$ stand, respectively, for $\widehat{y_i}$ and $\widehat{y_i' y_j'}$)

$$\rho_0 \partial_t \phi_i' \psi_j' + \rho_0 \left\{A_H\right\}_{i,j} + \rho_0 \vec{v}_H \cdot \nabla_H \phi_i' \psi_j' + \rho \left\{A_W\right\}_{i,j} + \rho w \nabla_z \phi_i' \psi_j' -$$
$$-\nabla_z (\frac{\rho}{T}\ T' w'\ \phi_i' \psi_j') - \phi_i' \psi_j' \nabla_z \rho' w' - w \frac{\rho}{T} [\phi_i' (\delta_{1j} \psi_j' + \delta_{2j} \psi_{j-1}') \nabla_z \psi_j +$$

$$+(\delta_{1j}{\psi'_j}^2+\delta_{2j}\psi'_j\psi'_{j-1})\nabla_z\phi_i]=(-1)^{i-1}\frac{\rho_0}{a}\tan\phi(\delta_{1i}F_{1j}+\delta_{2i}F_{2j})+$$
$$+\delta_{1j}F_{1i}+\delta_{2j}F_{2i}+\Delta(\phi'_i\psi'_j) \qquad (3.15)$$

Here $i,j=1,2$, $\phi_i=u$ at $i=1$, $\phi_i=v$ at $i=2$, $\psi_j=T$ at $j=1$, $\psi_j=q_v$ at $j=2$,

$$\{A_H\}_{i,j}=\phi'_i\vec{v'}_H\cdot\nabla_H\psi_j+\psi'_j\vec{v'}_H\cdot\nabla_H\phi_i,$$
$$\{A_W\}_{i,j}=\phi'_iw'\nabla_z\psi_j+\psi'_jw'\nabla_z\phi_i,$$
$$F_{1i}=\phi'_iq'_1-(1/c_v)\phi'_ip'\nabla\cdot\vec{v},$$
$$F_{2i}=\phi'_iq'_2,$$
$$q'_1={q^T_{LSC}}'+{q^T_{MC}}',$$
$$q'_2={q^v_{LSC}}'+{q^v_{MC}}',$$
$$F_{1j}=\phi_1\phi'_2\psi'_j+\phi_2\phi'_1\psi'_j,$$
$$F_{2j}=2\phi_1\phi'_1\psi'_j$$
$$\Delta(\phi'_i\psi'_j)=\overline{\overline{K}}_{DH}\rho_0\Delta_{\lambda\phi}\phi'_i\psi'_j+\nabla_z(\overline{\overline{K}}_{Dz}\rho\nabla_z\phi'_i\psi'_j),$$

δ_{mn} —the Kronecker delta.

The SDE for $u'v'$ in the same domain of the atmosphere is [167], [168]

$$\rho_0\partial_tu'v'+\rho_0u\nabla_\lambda u'v'+\rho_0v\nabla_\phi u'v'+\rho_0v'\vec{v'}_H\cdot\nabla_Hu+\rho_0u'\vec{v'}_H\cdot\nabla_Hv-$$
$$-\frac{\rho}{T}w(u'T'\nabla_zv+v'T'\nabla_zu)+\rho w\nabla_zu'v'+u'w'\rho\nabla_zv+v'w'\rho\nabla_zu+$$
$$+\nabla_z(\frac{\rho}{T}\ \ T'w'\ \ u'v')-u'v'\nabla_z\rho'w'=\frac{\rho_0}{a}\tan\phi\left(uv'^2+vu'v'-2uu'^2\right)+\Delta(u'v') \qquad (3.16)$$

The SDE for the auto–correlation functions T'^2, ${q'_v}^2$, u'^2, v'^2 in the free troposphere are as follows [167], [168]

$$\frac{1}{2}\rho_0\partial_ty'^2_i+\rho_0y'_i\vec{v'}_H\cdot\nabla_Hy_i+\frac{1}{2}\rho_0\vec{v}_H\cdot\nabla_Hy'^2_i+\frac{1}{2}\rho w\nabla_zy'^2_i+$$
$$+\rho y'_iw'\nabla_zy_i-\frac{\rho}{T}wy'_iT'\nabla_zy_i-\frac{1}{2}\nabla_z(\frac{\rho}{T}\ y'_iT'\ y'_iw')-\frac{1}{2}y'^2_i\nabla_z\rho'w'=$$
$$=\delta_{1i}F_{y1}+\delta_{2i}F_{y2}+\delta_{3i}F_{y3}+\delta_{4i}F_{y4}+\frac{1}{2}\Delta\left(y'^2_i\right) \qquad (3.17)$$

where $i=1,\ldots,4$, $y_1=T$, $y_2=q_v$, $y_3=u$, $y_4=v$,

$$F_{y1}=y'_1q'_1-\frac{1}{c_v}y'_1p'\nabla\cdot\vec{v},$$
$$F_{y2}=y'_2q'_2,$$
$$F_{y3}=\frac{\rho_0}{a}\tan\phi\left(y_4y'^2_3+y_3y'_3y'_4\right),$$
$$F_{y4}=-2\frac{\rho_0}{a}\tan\phi y_3y'_3y'_4,$$

The SDE for $u'w'$ "vertical" SM is obtained by multiplying w' into the transformed primitive equation for u followed by $(\Delta L, \Delta\tau)$ averaging with eqs.(3.2)— (3.4) and assumptions 1.–4. taken into account [167]

$$\rho_0 w'\vec{v'}_H \cdot \nabla_H u + \rho_0 \vec{v}_H \cdot \nabla_H u'w' - \rho_0 \vec{v}_H \cdot (u'\nabla_H w') + \rho w \nabla_z u'w' - \\ -\rho w u' \nabla_z w' - \frac{\rho}{T} T'w'w\nabla_z u + \rho w'^2 \nabla_z u - \nabla_z (\frac{\rho}{T}\ T'w'\ u'w') - u'w'\nabla_z \rho' w' + \\ \frac{1}{2}\frac{\rho}{T} u'T'\nabla_z w'^2 = \frac{\rho_0}{a} \tan\phi (uv'w' + vu'w') + \Delta\,(u'w') \tag{3.18}$$

Using the analogous procedure the SDE for $v'w'$ second moment is as follows [167]

$$\rho_0 w'\vec{v'}_H \cdot \nabla_H v + \rho_0 \vec{v}_H \cdot \nabla_H v'w' - \rho_0 \vec{v}_H \cdot (v'\nabla_H w') + \rho w \nabla_z v'w' - \\ -\rho w v' \nabla_z w' - \frac{\rho}{T} T'w'w\nabla_z v + \rho w'^2 \nabla_z v - \nabla_z (\frac{\rho}{T}\ T'w'\ v'w') - v'w'\nabla_z \rho' w' \\ +\frac{1}{2}\frac{\rho}{T} v'T'\nabla_z w'^2 = -2\frac{\rho_0}{a} \tan\phi\ u\ u'w' + \Delta\,(v'w') \tag{3.19}$$

Within the same approximation the SDE for $T'w'$ and $q_v'w'$ "vertical" SM in the free troposphere can be written in the form [167]

$$\rho_0 w'\vec{v'}_H \cdot \nabla_H T + \rho_0 \vec{v}_H \cdot (w'\nabla_H T') + \\ +\rho w w' \nabla_z T' - \frac{\rho}{T} T'w'w\nabla_z T + \rho w'^2 \nabla_z T - \nabla_z (\frac{\rho}{T}\ T'^2\ w'^2) - T'w'\nabla_z \rho' w' \\ +\frac{1}{2}\frac{\rho}{T} T'^2\nabla_z w'^2 = -\frac{w'p'}{c_v} \nabla \cdot \vec{v} + w'q_1' + \Delta\,(T'w') \tag{3.20}$$

$$\rho_0 w'\vec{v'}_H \cdot \nabla_H q_v + \rho_0 \vec{v}_H \cdot (w'\nabla_H q_v') + \\ +\rho w w' \nabla_z q_v' - \frac{\rho}{T} T'w'w\nabla_z q_v + \rho w'^2 \nabla_z q_v - \nabla_z (\frac{\rho}{T}\ T'w'\ q_v'w') - q_v'w'\nabla_z \rho' w' + \\ +\frac{1}{2}\frac{\rho}{T} T'q_v'\nabla_z w'^2 = w'q_2' + \Delta\,(q_v'w') \tag{3.21}$$

The SDE for $T'q_v'$ SM (which, in particular, enters the expression for $q_v'w'$ "vertical" SM) in the same atmospheric compartment is obtained by mutiplying q_v' and T', respectively, into transformed primitive energy and water vapour balance equations followed by summation and $(\Delta L, \Delta\tau)$ averaging of resultant equations with the formulas (3.2)—(3.4) and the assumptions 1.–4. taken into account, which gives [167]

$$\rho_0 \partial_t T'q_v' + \rho_0 \vec{v}_H \cdot \nabla_H T'q_v' + \rho_0 T'\vec{v}_H' \cdot \nabla_H q_v + \rho_0 q_v'\vec{v}_H' \cdot \nabla_H T - \frac{\rho}{T} T'^2 w \nabla_z q_v + \\ +T'w'\rho\nabla_z q_v + \rho w \nabla_z T'q_v' - \frac{\rho}{T} T'q_v' w\nabla_z T + q_v'w'\rho\nabla_z T - \nabla_z (\frac{\rho}{T}\ T'w'\ T'q_v') - \\ -T'q_v'\nabla_z \rho' w' = T'q_2' + q_v'q_1' - \frac{q_v'p'}{c_v} \nabla \cdot \vec{v} + \Delta\,(T'q_v') \tag{3.22}$$

To evaluate the $\widehat{p'y_i'}$ cross–correlation functions the expression for p' can be obtained by subtracting $\hat{p} = R(\hat{\rho}\hat{T} + \widehat{\rho'T'})$ from $p = R\rho T$ which gives

$$p' = R\left(\rho T' + T\rho' + \rho' T' - \widehat{\rho' T'}\right) \tag{3.23}$$

Multiplying (3.23) by any y' and applying the $(\Delta L, \Delta\tau)$ average results in the estimate

$$p'y' = R\left[\rho T'y' + T\rho'y' + y'(\rho'T' - \widehat{\rho'T'})\right] \tag{3.24}$$

Using eq.(3.14) the $\rho'T'$ term in the SDE can be written in the form

$$\rho'T' = -\frac{\rho}{T}T'^2 \tag{3.25}$$

To the same approximation the $\rho'u'$, $\rho'v'$ and $\rho'q_v'$ moments are

$$\rho'u' = -\frac{\rho}{T}u'T' \tag{3.26}$$

$$\rho'v' = -\frac{\rho}{T}v'T' \tag{3.27}$$

$$\rho'q_v' = -\frac{\rho}{T}q_v'T' \tag{3.28}$$

The SDE for w'^2, $\rho'w'$ and $\nabla_z\rho'w'$ SM in the free troposphere are derived below in Chapter 5 (see eqs. (5.17)– (5.22),(5.23)–(5.25) and (5.26)).

Let us notice here that in the above-listed formulas of this Chapter it is assumed that the horizontal $\overline{\overline{K}}_H$, $\overline{\overline{K}}_{vH}$ and vertical $\overline{\overline{K}}_z$, $\overline{\overline{K}}_{vz}$ diffusion coefficients for humidity and heat are equal to the horizontal $\overline{\overline{K}}_{DH}$ and vertical $\overline{\overline{K}}_{Dz}$ coefficients of viscosity, respectively.

The corresponding SDE for the synoptic–scale diabatic terms q_1', q_2' can be, in the general case, written in the form

$$q_{1,2}' = \frac{\partial q_{1,2}}{\partial T}T' + \frac{\partial q_{1,2}}{\partial q_v}q_v' + \ldots + \frac{1}{2}\frac{\partial^2 q_{1,2}}{\partial T^2}T'^2 + \ldots + \frac{\partial^2 q_{1,2}}{\partial T \partial q_v}T'q_v' + \ldots, \tag{3.29}$$

where $\frac{\partial q_{1,2}}{\partial y_i}$, $\frac{\partial^2 q_{1,2}}{\partial y_i^2}$, etc. are the first, second, etc. derivatives of $q_{1,2}$ with respect to their arguments taken at $y_i = \hat{y}_i$. Multiplying (3.29) by any y_i' and applying the $(\Delta L, \Delta\tau)$ average the $y'q_{1,2}'$ moments can be expressed in terms of LSLTC and SM.

The vertical profiles of the above-listed SM in the stratosphere are assumed to be adequately represented by the Charney–Drazin formulas [33] for the vertical distribution of the trapped wave, with the wave vector squared $K_w^2 = (2\pi/L_{Ro,sy})^2$, which gives the following SDE for SM in any resolved stratospheric layer ($z \geq H_{tr}$)

$$y_i'y_j' = {y_i'y_j'}_l \exp\left\{-\left[\frac{2N}{f}\left(K_w^2 + \gamma_w^2 - \frac{\beta}{[\overline{u}]}\right)^{1/2} - \frac{1}{H_{0,l}}\right](z - H_l)\right\} \tag{3.30}$$

Here ${y_i'y_j'}_l$ is the value of $y_i'y_j'$ at $z = H_l$ where H_l equals the tropopause height H_{tr} (for the model lowest stratospheric layer) or the height of the upper boundary of the adjacent lower stratospheric layer (for all other model stratospheric layers) ; $[\overline{u}]$ is the zonally averaged and mass-weighted zonal component of $\vec{v}$ in a given stratospheric layer; N is the Brunt–Vaisala frequency ($= \sqrt{\frac{g}{\Theta}\frac{\partial\Theta}{\partial z}}$, where Θ is the potential temperature) for stratospheric layer; $\gamma_w = f/2NH_{0,l}$, $H_{0,l} = RT_l/g$ — the scale height for air density at the temperature $T_l \equiv T(H_l)$, $\beta = \frac{\partial f}{a\partial\phi}$. In the equatorial stratosphere the $L_{Ro,sy}$, f and β parameters are replaced by their equatorial values (see assumption 1.), with a smooth matching of these quantities at the (prescribed) northern and southern latitudinal boundaries of the equatorial belt.

The description of the SDE for SM in the planetary boundary layer (PBL) will be given below in Chapter 4.

Let us notice here that the assumption on the quasi-geostrophic character of the synoptic objects retains its validity even for the free atmosphere regions rather close to the equator [160], with $L_{Ro,sy}$, Ro_{sy} and f replaced, respectively, by $L_{Ro,sy,e}$, $Ro_{sy,e}$ and f_e (see assumption 1.).

The generalized SDE (3.5), (3.10)— (3.12), (3.15)–(3.22), (3.24)–(3.30) describe the large–scale long–term evolution of the atmosphere and can be applied to any domain (surface layer, planetary boundary layer, free troposphere and stratosphere of high and middle latitudes and in the tropics). However, these equations are too complicated and give no advantage in computational time, compared to the primitive ones. Applying scale-magnitude analysis and some additional assumptions the generalized SDE will be reduced in Chapters 4 and 5 to a set of much more simple and computationally much less expensive basic SDE which adequately describe the major large–scale long–term atmosphere processes and hence can be used for the development of climate models of intermediate complexity.

Chapter 4

Basic SDE for long-term large-scale atmosphere variables

In this Chapter, by applying a scale/magnitude analysis the generalized SDE for LSLTC obtained in Chapter 3 are reduced to the model basic SDE for LSLTC in the free atmosphere and PBL of high/middle latitudes and the tropics (see [168, 169]). We anticipate that the basic SDE for LSLTC and SM have to adequately describe a totality of the main energy loading atmospheric climate structures in their interaction. As already mentioned in Chapter 1, based on the results of a large body of theoretical, model and empirical research we attribute to those structures the following atmospheric objects: quasi-zonal flows (including subtropical and polar jet streams), mean meridional circulations (Hadley, Ferrel and polar cells), thermally and orographically induced quasi-stationary planetary-scale waves, Walker circulations (including Southern Oscillation), large-scale quasi-stationary equatorial Kelvin waves, Madden-Julian circulation cells, quasi-stationary atmospheric centres of action, depressions and highs (e.g., Siberian and Azores Highs, Iceland and Aleutian Lows), monsoon circulations, large-scale ultra-long (persistent) blockings, ensembles of the transient synoptic-scale eddies/waves and fronts. We suggest this set of the atmospheric structures to be universal, i.e., structurally stable under a broad range of climatic states, even somewhat away from the present climate conditions. (Notice here once again that the ensembles of the dry/moist covection and small/mesoscale turbulence which can also be placed in the main energy loading atmospheric structures are described in the model by diagnostic parameterizations.)

This forms the basis for a scale/magnitude analysis of the generalized SDE to reduce them to the basic SDE. First we evaluate the characteristic magnitudes and spatial/temporal scales of the variables in the generalized SDE for LSLTC by using the correspondent characteristic values for these variables inherent in the listed main energy loading atmospheric structures. Passing to the nondimensional variables, the dimensionless SDE are then derivedfrom the dimensional ones. The members in the dimensionless equations are revealed which contribute to a zero, first, second, etc. approximation (i.e., terms of the order of 10^0, 10^{-1}, 10^{-2}, etc.). Retaining the zero and the first order terms and coming back to the dimensional equations we obtain the basic SDE for LSLTC which are discussed below in this Chapter.

4.1 Extratropics

4.1.1 Free atmosphere

In this Section, we illustrate in detail the technique of the applied scale/magnitude analysis and the derivation of the basic SDE for LSLTC. As the first example we consider the equations for u and v in the extratropical free troposphere. A starting point are the generalized SDE (3.5) for these two variables. First u and v in these equations are represented by a sum of geostrophic u_g, v_g and ageostrophic u_a, v_a components. Taking into account the magnitudes and spatial/temporal scales of the fields u_g, v_g, u_a, v_a attributed to the above-listed main energy loading atmospheric structures [19, 33, 97, 132, 159, 216, 156, 192, 160, 49, 93, 100, 231, 232, 48, 4, 240] we specify the following relations among the characteristic values of the parameters of these fields in the extratropical free atmosphere

$$\widetilde{u_g} \geq \widetilde{v_g} \approx U_g \approx 10\ m/s \qquad (4.1)$$

$$\widetilde{u_a} \approx \widetilde{v_a} \approx U_a \qquad (4.2)$$

$$U_a \approx \varepsilon_{U_a} U_g, \qquad (4.3)$$

where $\varepsilon_{U_a} \approx (0.1 \div 0.3)$. As a typical example, a monsoon component of ageostrophic motion in the planetary boundary layer (PBL) has a specific magnitude of the order of $U_g \widetilde{\sin\alpha}$, where $\widetilde{\sin\alpha} \approx 0.1 \div 0.3$ is the characteristic value of the cross-isobar angle (see Subsection 4.1.2). This estimate can be taken as the upper limit for the free atmosphere reverse monsoon the latter being normally weaker than the direct monsoon in the PBL. Spatial scales for the geostrophic and ageostrophic components are

$$L(\widetilde{u_g}) \approx L(\widetilde{v_g}) \approx L_g \qquad (4.4)$$

$$L(\widetilde{u_a}) \approx L(\widetilde{v_a}) \approx L_a \qquad (4.5)$$

$$L_g \approx L_a \approx L_O \qquad (4.6)$$

$$H(\widetilde{u_g}) \approx H(\widetilde{v_g}) \approx H_{00} \approx 10^4\ m \qquad (4.7)$$

$$H(\widetilde{u_a}) \approx H(\widetilde{v_a}) \approx H_a = \varepsilon_{z,U_a} H_{00} \qquad (4.8)$$

where $\varepsilon_{z,U_a} \approx (0.3 \div 0.5)$. Again, as an example, a direct monsoon component of ageostrophic motion has a specific vertical scale of the order of $\widetilde{Z_B}$, where $\widetilde{Z_B} \approx (1.5 \div 3) \cdot 10^3\ m$ is the characteristic value of the height of the planetary boundary layer. The vertical scale of the reverse monsoon usually developed in the free atmosphere is of the order of $\sqrt{H_{00} (\frac{2\widetilde{K_z}}{\widetilde{f}})^{\frac{1}{2}} \cdot e}$ where $\widetilde{K_z}$ is the characteristic value of the effective kinematic vertical "diffusion" coefficient due to small/mesoscale and synoptic eddies. This vertical scale [$\approx (2 \div 3) \cdot 10^3\ m$] is close in magnitude to that for the lower direct monsoon branch.

In (4.1)—(4.8) $\widetilde{L(y)}$ and $\widetilde{H(y)}$ denote, respectively, the characteristic horizontal and vertical spatial scales for the variable y, and $L_O = L_{RO,bt} = \frac{(gH_g)^{\frac{1}{2}}}{f} \approx 3 \cdot 10^6\ m$ designates the Obukhov deformation radius (the Rossby external (barotropic) deformation radius). The synoptic Rossby deformation radius $L_{Ro,sy}$ and the Rossby barotropic deformation radius $L_{RO,bt}$ are related by the following formula (see, e.g., [149, 160])

$$L_{Ro,sy} = L_{RO,bt} Ro_{sy} \tag{4.9}$$

On the other hand, the characteristic spatial scale $\widetilde{L_{sy}}$ of the individual tropospheric synoptic eddy/wave (which is closely approximated by $L_{Ro,sy}$, by virtue of the basically baroclinic nature of the synoptic eddy/wave generation) is approaching the following specific scale [149]

$$\widetilde{L_{sy}} \approx L_{Ro,sy} \approx L_{RO,bt}\ \alpha_0, \tag{4.10}$$

where $\alpha_0 = (\frac{N^2 H_g}{g})^{\frac{1}{2}}$ is the dimensionless parameter of static stability and N is the Brunt–Vaisala frequency [149, 160].

As was already mentioned above, the characteristic time scales of the processes under consideration are of the order of (or longer than) $\Delta\tau$. We will use as the characteristic one the following value of $\Delta\tau$

$$\Delta\tau \approx (15 \div 30) \text{ days}, \tag{4.11}$$

so that the specific magnitudes of the time variations $\widetilde{\partial_t u_g}$, $\widetilde{\partial_t v_g}$ and $\widetilde{\partial_t u_a}$, $\widetilde{\partial_t v_a}$ of u_g, v_g and u_a, v_a fields can be estimated as follows

$$\widetilde{\partial_t u_g} \approx \widetilde{\partial_t v_g} \approx \frac{U_g}{\Delta\tau} \tag{4.12}$$

$$\widetilde{\partial_t u_a} \approx \widetilde{\partial_t v_a} \approx \frac{U_a}{\Delta\tau} \tag{4.13}$$

The characteristic magnitudes of the vertical motions $\widetilde{w_g}$ and $\widetilde{w_a}$ attributed, correspondingly, to the geostrophic and ageostrophic winds can be evaluated using the continuity equation (3.10), under the assumption that the characteristic vertical scales $\widetilde{H(w_g)}$ and $\widetilde{H(w_a)}$ are of the order of H_{00} and H_a, respectively, which gives

$$\widetilde{w_g} \approx U_g \frac{H_{00}}{L_g} \tag{4.14}$$

$$\widetilde{w_a} \approx U_a \frac{H_a}{L_a} \tag{4.15}$$

The generalized SDE (3.5) for u and v include, besides LSLTC, the terms describing the synoptic second moments u'^2, v'^2, $u'v'$, $u'\rho'$, $v'\rho'$, $u'w'$, $v'w'$, $\rho'w'$. Based on the results obtained in [30, 155, 97, 132, 159, 216, 156, 209, 192, 160, 4, 211, 212, 238] and using (3.14) we admit the following scale/magnitude relations among u'^2, v'^2, $u'v'$, $u'\rho'$, $v'\rho'$ ensemble characteristics of the synoptic component and those of LSLTC in the extratropical free atmosphere

$$(\widetilde{u'^2})^{\frac{1}{2}} \approx (\widetilde{v'^2})^{\frac{1}{2}} \approx \widetilde{U'}, \tag{4.16}$$

where

$$\widetilde{U'} \approx U_g \tag{4.17}$$

$$\widetilde{u'v'} \approx \varepsilon_{u'v'} (\widetilde{U'})^2, \tag{4.18}$$

where

$$\varepsilon_{u'v'} \approx (0.1 \div 0.3) \tag{4.19}$$

$$\widetilde{u'\rho'} \approx \frac{\rho_0}{T_{00}} \widetilde{u'T'} \tag{4.20}$$

$$\widetilde{v'\rho'} \approx \frac{\rho_0}{T_{00}} \widetilde{v'T'}, \tag{4.21}$$

where

$$\widetilde{u'T'} \approx \varepsilon_{u'T'} (\widetilde{u'^2})^{\frac{1}{2}} (\widetilde{T'^2})^{\frac{1}{2}}, \tag{4.22}$$

$$\widetilde{v'T'} \approx \varepsilon_{v'T'} (\widetilde{v'^2})^{\frac{1}{2}} (\widetilde{T'^2})^{\frac{1}{2}}. \tag{4.23}$$

Here $\varepsilon_{u'T'} \approx \varepsilon_{v'T'} \approx \varepsilon_{u'v'}$, and $(\widetilde{T'^2})^{\frac{1}{2}} \approx (2 \div 5)\ K$ is given by the eq. (4.49).

$$L(\widetilde{u'^2}) \approx L(\widetilde{v'^2}) \approx L(\widetilde{u'v'}) \approx L(\widetilde{u'T'}) \approx L(\widetilde{v'T'}) \approx \widetilde{L'}, \tag{4.24}$$

where

$$\widetilde{L'} \approx L_{RO,bt} \tag{4.25}$$

Hence, we assume that the horizontal spatial scale of the ensembles of synoptic-scale eddies/waves is of the order of the Obukhov deformation radius L_O (see assumption 1. in Chapter 3 and accompanying references).

$$H(\widetilde{u'^2}) \approx H(\widetilde{v'^2}) \approx H(\widetilde{u'v'}) \approx H(\widetilde{u'T'}) \approx H(\widetilde{v'T'}) \approx \widetilde{H'}, \tag{4.26}$$

where

$$\widetilde{H'} \approx H_{00}. \tag{4.27}$$

Based on the results obtained in [30, 155, 97, 132, 159, 216, 156, 209, 192, 160, 4, 211, 212, 238] we adopt the following estimations for typical magnitudes and vertical spatial scales of $u'w'$ and $v'w'$ terms entering the equations (3.5) for u and v

$$\widetilde{u'w'} \approx \varepsilon_{u'w'} \widetilde{U'} (\widetilde{w'^2})^{\frac{1}{2}} \tag{4.28}$$

$$\widetilde{v'w'} \approx \varepsilon_{v'w'} \widetilde{U'} (\widetilde{w'^2})^{\frac{1}{2}}, \tag{4.29}$$

where $\varepsilon_{u'w'} \approx \varepsilon_{v'w'} \approx \varepsilon_{u'v'}$

$$H(\widetilde{u'w'}) \approx H(\widetilde{v'w'}) \approx H_{00} Ro_{sy} \tag{4.30}$$

As it was already mentioned we suggest $L_O = L_{Ro,bt}$ to be the characteristic horizontal spatial scale for these second moments as well as for all other SM [232].

From eqs. (4.28), (4.29) it is seen that the characteristic value of synoptic-scale vertical velocity $(\widetilde{w'^2})^{\frac{1}{2}}$ enters the formulas for the typical magnitudes of the terms $u'w'$ and $v'w'$. To obtain an estimation for the characteristic value of $(\widetilde{w'^2})^{\frac{1}{2}}$ we use the formula for the square of the "adiabatic" synoptic-scale vertical velocity ${w'^2}_{ad}$ which is derived in Chapter 5 (see eqs. (5.18)– (5.20))

$$\widetilde{w'^2} \approx {w'^2}_{ad}(\frac{H_g}{2}) = C_w(\frac{\beta}{f})^2(\frac{g}{fT_{00}})^2(\nabla^2_\lambda\ T^*(0) + \nabla^2_\phi\ T^*(0) + \nabla^2_\phi\ \overline{T(0)})\ {z'_g}^4(\frac{1}{2} - ,_T \frac{z'_g}{3})^2\ \alpha_0^{-2} \tag{4.31}$$

Here C_w is a constant of the order of unity, $\overline{y}$ and y^*, respectively, denote the zonal average and deviation from it, $\beta = \nabla_\phi f$, $z'_g \approx \frac{H_g}{2}$, and $,_T = \frac{\partial \Gamma}{\partial T_0}$, where , is the free troposphere temperature lapse rate and T_0 is the temperature obtained by a linear extrapolation of the free troposphere temperature to a sea surface, with the same lapse rate. Eq.(4.31) provides the estimation for $(\widetilde{w'^2})^{\frac{1}{2}}$ term.

Using eqs. (5.23)–(5.25) derived in Chapter 5, the characteristic magnitude of the term $\rho' w'$ in the model initial equations (3.5) for u and v in free atmosphere can be estimated as follows [168]

$$\widetilde{\rho' w'} \approx \frac{\tilde{\rho}}{H_{00}} \widetilde{U'} \frac{H(\widetilde{\rho' w'})}{\widetilde{L_{sy}}} H(\widetilde{\rho' w'}) \tag{4.32}$$

where the specific vertical scale $H(\widetilde{\rho' w'})$ is given by

$$H(\widetilde{\rho' w'}) \approx H_{00} Ro_{sy} \tag{4.33}$$

Combining estimations (4.32), (4.33) the characteristic magnitude of $\nabla_z \rho' w'$ term can be evaluated by the formula

$$\nabla_z \widetilde{\rho' w'} \approx \frac{\tilde{\rho}}{H_{00}} \widetilde{U'} \frac{H(\widetilde{\rho' w'})}{\widetilde{L_{sy}}} \tag{4.34}$$

Taking into consideration (4.1)—(4.33) and accounting for the characteristic values of the horizontal $\widetilde{\overline{\overline{K}}}_{DH}$ and vertical $\widetilde{\overline{\overline{K}}}_{Dz}$ coefficients of viscosity (see, e.g., [222, 111, 101]) one can derive analogous estimations for F_λ and F_ϕ terms in the eqs. (3.5) for u and v in free atmosphere.

We make a further conventional assumption that the pressure and the Coriolis terms in eqs. (3.5) for u and v in the extratropical free atmosphere are of the same order [30, 160]. We then extract the terms $\nabla_\lambda p$, $\rho_0 f v_g$ and $\nabla_\phi p$, $\rho_0 f u_g$ from the eqs. (3.5) for u and v. These terms give the basic SDE for the geostrophic wind components u_g and v_g (see eqs. (4.38)–(4.43), (4.44), (4.45)). The remaining parts of the eqs.(3.5) for u and v give the equations for u_a and v_a. To pass on to the dimensionless equations we divide the mentioned equations for u_a and v_a by $\rho_0 \tilde{f} \widetilde{U_a}$ (where $\tilde{f} \approx 2^{\frac{1}{2}}\omega$ is the characteristic value of the Coriolis parameter for the extratropical latitudes) and enter the nondimensional variables $y_{nd} = \frac{y}{\tilde{y}}$. Applying the scale/magnitude estimates (4.1)–(4.34) (see eqs. [A1]–[A47] and the accompanying text in the Appendix A) and retaining the zero and first order terms, the dimensionless equations for u_a and v_a are derived. Coming back to the dimensional variables we arrive at the basic SDE for u_a and v_a [168, 169]

$$\frac{1}{a\cos\phi}\frac{\partial({u_g}^2 + u'^2)}{\partial\lambda} + \frac{1}{a\cos\phi}\frac{\partial(u_g v_g + u'v')\cos\phi}{\partial\phi} - \frac{1}{a}\tan\phi(u_g v_g + u'v') + \frac{\partial(u_g w_g + u'w')}{\partial z} - f v_a = \frac{1}{\rho_0}F_\lambda \tag{4.35}$$

$$\frac{1}{a\cos\phi}\frac{\partial(u_g v_g + u'v')}{\partial\lambda} + \frac{1}{a\cos\phi}\frac{\partial({v_g}^2 + v'^2)\cos\phi}{\partial\phi} + \frac{1}{a}\tan\phi({u_g}^2 + u'^2) + \frac{\partial(v_g w_g + v'w')}{\partial z} + f u_a = \frac{1}{\rho_0}F_\phi, \tag{4.36}$$

where the terms F_λ and F_ϕ in the case of the absence of cumulus convection in the free troposphere and in the disregard of the gravity-wave drag are written in terms of the horizontal and vertical coefficients of viscosity (see eqs. (2.10) and (2.11)). The vertical coefficient of viscosity $\overline{\overline{K}}_{Dz}$ in the free troposphere in this case is given by the eq. (6.65). In the presence of cumulus convection in the free troposphere the $\overline{\overline{K}}_{Dz}$ term is described by the eq. (6.69), see Section 6.4. $\overline{\overline{K}}_{Dz}$ in the free troposphere is also affected by the gravity-wave drag. The corresponding formulas accounting for this effect are listed in Section 6.4. In the model, the horizontal coefficient of viscosity $\overline{\overline{K}}_{DH}$ is assigned a constant value in the PBL and free troposphere and is not affected by cumulus covection and gravity-wave drag.

Notice that the 3rd member in the left-hand sides of the eqs. (4.35) and (4.36) is not negligible, with a given accuracy only in the polar regions.

Taking into account that the zonal average of v_g in the model is equal to zero (see eqs.(4.39), (4.41), (4.43)) the zonal averaging of the eq.(4.35) results in the following equation for the zonally averaged meridional velocity (the horizontal component of the mean meridional circulation) $\overline{v} \equiv \overline{v}_a$ in the extratropical free troposphere [168]

$$f\overline{v} - \frac{1}{a\cos\phi}\frac{\partial(\overline{u_g v_g} + \overline{u'v'})\cos\phi}{\partial\phi} + \frac{1}{a}\tan\phi(\overline{u_g v_g} + \overline{u'v'}) - \nabla_z(\overline{u_g w_g} + \overline{u'w'}) = -\frac{1}{\rho_0}\overline{F_\lambda} \quad (4.37)$$

For futher consideration it is convenient to represent the long-term large-scale geostrophic wind components u_g and v_g in the form

$$u_g = u_{g,0} + u_T, \quad (4.38)$$

$$v_g = v_{g,0} + v_T. \quad (4.39)$$

Here

$$u_{g,0} = -\frac{1}{\rho_{00} f}\frac{\partial p_0}{a\partial\phi}, \quad (4.40)$$

$$v_{g,0} = \frac{1}{\rho_{00} f}\frac{\partial p_0}{a\cos\phi\partial\lambda}, \quad (4.41)$$

where p_0 is the sea surface pressure, and

$$u_T = -\int_0^z \frac{g}{T_{00} f}\frac{\partial T}{a\partial\phi}\,dz, \quad (4.42)$$

$$v_T = \int_0^z \frac{g}{T_{00} f}\frac{\partial T}{a\cos\phi\partial\lambda}\,dz \quad (4.43)$$

are the geostrophic wind components associated with the so called thermal wind. For brevity, in the text below u_T and v_T are referred to as the thermal wind components.

Accounting for the quasi–linear UVS of the temperature vertical profile in the extratropical free troposphere (see, e.g., [156, 111, 84, 161]) the eqs. (4.42) and (4.43) for the thermal wind constituents in this atmospheric domain can be approximated by

$$u_T = -\frac{g}{fT_{00}}\frac{z - \frac{1}{2}\frac{\partial\Gamma}{\partial T_0}z^2}{1 - \frac{\partial\Gamma}{\partial T_0}z}\nabla_\phi T \quad (4.44)$$

$$v_T = \frac{g}{fT_{00}}\frac{z - \frac{1}{2}\frac{\partial\Gamma}{\partial T_0}z^2}{1 - \frac{\partial\Gamma}{\partial T_0}z}\nabla_\lambda T \quad (4.45)$$

Here $\,_T = \frac{\partial \Gamma}{\partial T_0}$, where , is the free troposphere temperature lapse rate which is mainly a function of T_0 (see eq.(7.8)), and T_0 is the temperature resultant from a linear extrapolation of the free troposphere temperature to a sea surface, with the same lapse rate.

Equations (4.35)–(4.37), (4.40), (4.41), (4.44), (4.45) are a set of basic SDE for the LSLTC of the horizontal winds in the extratropical free atmosphere.

Applying the same scale/magnitude estimates (4.1) – (4.33) to the continuity equation (3.10) we obtain the following equation

$$\nabla_H \cdot \rho_0 \vec{v}_T + \nabla_z(\rho w_T) + \nabla_H \cdot \rho_0 \vec{v}_n + \nabla_z \rho w_n + \nabla_z \rho' w' = 0, \tag{4.46}$$

In (4.46) $\vec{v}_T = \{u_T, v_T\}$, $\vec{v}_n = \{u_{g,0} + u_a, v_{g,0} + v_a\}$, and the large-scale long-term (i.e., $(\Delta L, \Delta \tau)$ averaged) vertical velocity is subdivided into the term w_T associated with the thermal wind $\vec{v}_T$, and the deviation from it w_n attributed to the deviation $\vec{v}_n$ of the horizontal velocity from the thermal wind component. By the definition, w_T obeys the following basic SDE

$$\nabla_H \cdot \rho_0 \vec{v}_T + \nabla_z(\rho w_T) = 0 \tag{4.47}$$

in the total column of the troposphere, with the zero lower boundary condition at the sea surface. Notice here that the total geostrophic vertical velocity w_g entering eqs. (4.35) – (4.37) yields the SDE analogous to (4.47) (with the same zero lower boundary condition) but with $\vec{v}_T$ and w_T replaced with $\vec{v}_g$ and w_g, respectively. Then, by virtue of (4.46), (4.47), the basic SDE for w_n reads

$$\nabla_H \cdot \rho_0 \vec{v}_n + \nabla_z \rho w_n + \nabla_z \rho' w' = 0, \tag{4.48}$$

The lower boundary condition $(\rho w_n|_{Z_B})$ for the free troposphere w_n at the top of the PBL is discussed below in the Subsection 4.1.2.

To obtain the basic SDE for the temperature in the extratropical free troposphere the scale/magnitude estimates (4.1) – (4.33) are applied to the equation (3.5) for the temperature ($y_1 = T$), and the eq.(3.25) is invoked. Additionally, the characteristic value $\widetilde{T'}$ of the temperature perturbations inherent in the synoptic-scale objects is evaluated from the thermal wind equation, taking into account the dominant contribution of the thermal wind component to the dynamic fields of the synoptic objects in the free atmosphere, which gives [168]

$$\widetilde{T'} \approx \frac{L_{Ro,sy}}{\widetilde{H'}} \frac{\tilde{U'} \tilde{f}}{g} T_{00}, \tag{4.49}$$

The analogous evaluation of the characteristic magnitudes of the temperature disturbances $\widetilde{T^*}$ (hereinafter the star designates the azonal component of the correspondent variable) in the above-listed (nonzonal) energy loading atmospheric structures in the free atmosphere of high and middle latitudes leads to a following estimate [168]

$$\widetilde{T^*} \approx \frac{\widetilde{L^*}}{H_{00}} \frac{\widetilde{U^*} \tilde{f}}{g} T_{00}, \tag{4.50}$$

where $\widetilde{L^*}$ and $\widetilde{U^*}$ are approximated by L_a and U_a.

The order of the characteristic latitudinal gradient of the zonally averaged temperature $\overline{T}$ in the above-mentioned (quasi-zonal) atmospheric energy-loading structures can be assessed as follows

$$\widetilde{\nabla_\phi \overline{T}} \approx \frac{\delta \widetilde{\overline{T}}_{ep}}{L_{\overline{T}}}, \tag{4.51}$$

where $L_{\overline{T}} \approx \frac{2\pi a}{4}$ and $\widetilde{\delta \overline{T}_{ep}} \approx 45\ K$ is the characteristic scale of the equator-to-pole zonal temperature difference.

The specific magnitude $\widetilde{\partial_t T^*}$ of the time variations of T^* is estimated as follows

$$\widetilde{\partial_t T} \approx \frac{T^*}{\Delta\tau} \tag{4.52}$$

We assume that the specific magnitude $\widetilde{\partial_t \overline{T}}$ of $\overline{T}$ time variations is of the same order as that of T^*.

The characteristic vertical changes of temperature at the synoptic and LSLTC scales are derived from the above-mentioned UVS of these fields. The eq. (3.25) provides the estimation for the $\rho' T'$ term which enters the equation (3.5) for the temperature, and the characteristic vertical scale of the term T'^2 in the right side of (3.25) is derived from the UVS of synoptic-scale component T' of temperature

$$H(\widetilde{T'^2}) \approx H_{00} \tag{4.53}$$

Based on the results obtained in [159, 216, 209, 192, 4, 211, 212, 238] we adopt the following estimation for typical magnitude and vertical spatial scale of the term $T'w'$ entering the model initial equations (3.5) for T

$$\widetilde{T'w'} \approx \varepsilon_{T'w'} \widetilde{T'}, (\widetilde{w'^2})^{\frac{1}{2}} \tag{4.54}$$

where $\varepsilon_{T'w'} \approx \varepsilon_{u'w'} \approx \varepsilon_{v'w'} \approx \varepsilon_{u'v'}$, and

$$H(\widetilde{T'w'}) \approx H_{00} Ro_{sy} \tag{4.55}$$

In the eq.(3.5) for the temperature the heat source/sink terms and the term describing the work done by the atmosphere are anticipated to be of one and the same order. We take into account equations (4.44), (4.45), (4.47) and divide the eq.(3.5) for $y_1 = T$ by $\rho_0 \widetilde{\nabla_\phi \overline{T}} U_a$ to pass on to a nondimensional equation for T, and we introduce under this procedure the nondimensional variables $y_{nd} = \frac{y}{\tilde{y}}$. As a result, the dimensionless equation for the free troposphere temperature is obtained. The scale/magnitude estimations (4.1) – (4.34), (4.49)– (4.51) are then applyed to estimate the order of terms in that dimensionless equation (see eqs. [A48]–[A57] and the accompanying text in the Appendix A). In this, we retain in the equation the terms of zero and first order which brings one to the basic dimensionless equation for T. Finally, coming back to the dimensional variables we obtain the basic SDE for the temperature in the extratropical free troposphere [168, 169]

$$\begin{aligned}\partial_t \rho_0 T = -\nabla_\lambda (\rho_0 T u_{T,eff}) - \nabla_H \cdot (\rho_0 T \vec{v}_n) - \nabla_z (\rho T w_n) - \nabla_H \cdot (\rho_0 T' \vec{v}'_H) - \\ \nabla_z (\rho T' w') - \frac{1}{c_v} p\, div \vec{v} + q^T,\end{aligned} \tag{4.56}$$

where

$$u_{T,eff} = \frac{1}{\rho T} \frac{g\rho_0}{a} H_{00}^2 F_0(z) \frac{T_0}{T_{00}} \left(, \ -\frac{1}{2} \frac{\partial,}{\partial T_0} T_0 \right) \frac{\partial}{\partial \phi} \left(\frac{1}{f} \right), \tag{4.57}$$

$$\begin{aligned}F_0(z) = 1 - \frac{\partial,}{\partial T_0} H_{00} \left\{ 1 - \left[\frac{1}{2} \left(\frac{z}{H_{00}} \right)^2 + \frac{z}{H_{00}} + 1 \right] \exp\left[-\frac{z}{H_{00}} \right] \right\} - \\ \left(\frac{z}{H_{00}} + 1 \right) \exp\left[-\frac{z}{H_{00}} \right].\end{aligned}$$

As is seen from the eqs. (4.56) and(4.57), the fast thermal wind component contributes to the advection only through the beta-term. This allows one to use in the model governing SDE for tropospheric temperature–which is the result of the vertical integration of the eq. (4.56) over the total troposphere column (see Chapters 7 and 8)–a time step of integration up to one day, without the violation of the CFL criterion.

The basic SDE for q_v in the same domain of the atmosphere is derived from the equation (3.5) for the specific humidity ($y_2 = q_v$). Under this procedure, the saturated specific humidity at the given temperature and pressure is taken as the characteristic value of q_v

$$\widetilde{q_v} \approx {q_v}^{sat}(T, p), \tag{4.58}$$

while the characteristic value of q_v' is assigned the difference between the saturated specific humidities at temperatures T and $T + \widetilde{T'}$, under a given pressure

$$\widetilde{q_v'} \approx {q_v}^{sat}(T + \widetilde{T'}, p) - {q_v}^{sat}(T, p) \tag{4.59}$$

Analogously, the characteristic value of q_v^* is assigned the difference between the saturated specific humidities at temperatures T and $T + \widetilde{T^*}$, under a given pressure

$$\widetilde{q_v^*} \approx {q_v}^{sat}(T + \widetilde{T^*}, p) - {q_v}^{sat}(T, p) \tag{4.60}$$

The specific horizontal scale $\widetilde{L_q}$ of latitudinal/longitudinal changes of q_v^* is assumed to be of the order of

$$\widetilde{L_q} \approx L_{Ro,bt}, \tag{4.61}$$

and the specific latitudinal gradient $\widetilde{\nabla_\phi \overline{q_v}}$ of zonally averaged q_v is assigned the value

$$\widetilde{\nabla_\phi \overline{q_v}} \approx \frac{\partial {q_v}^{sat}(\overline{T}, \overline{p})}{\partial \overline{T}} \widetilde{\nabla_\phi \overline{T}} \tag{4.62}$$

The characteristic magnitude of the vertical gradient $\widetilde{\nabla_z q_v}$ of water vapour content is specified as follows

$$\widetilde{\nabla_z q_v} \approx \frac{{q_v}^{sat}(T, p)}{\widetilde{Z_B} \cdot \ e}, \tag{4.63}$$

where the characteristic magnitude of the time variations $\widetilde{\partial_t q_v}$ is assumed to be of the order of

$$\widetilde{\partial_t q_v} \approx \frac{\widetilde{q_v^*}}{\Delta\tau} \tag{4.64}$$

The specific values of $u'q_v'$ and $v'q_v'$ fluxes are estimated by

$$\widetilde{u'q_v'} \approx \varepsilon_{u'q_v'} (\widetilde{u'^2})^{\frac{1}{2}} \widetilde{q_v'}, \tag{4.65}$$

$$\widetilde{v'q_v'} \approx \varepsilon_{v'q_v'} (\widetilde{v'^2})^{\frac{1}{2}} \widetilde{q_v'}. \tag{4.66}$$

Here $\varepsilon_{u'q_v'} \approx \varepsilon_{v'q_v'} \approx \varepsilon_{u'T'} \approx \varepsilon_{v'T'}$. As was already mentioned above, the characteristic horizontal spatial scale $L_{q'}$ for $u'q_v'$ and $v'q_v'$ SM is presumed to be of the order of $L_{Ro,bt}$.

We adopt the following estimation for typical magnitude and vertical spatial scale of the term $q_v'w'$ entering the equation (3.5) for q_v

$$\widetilde{q_v'w'} \approx \varepsilon_{q_v'w'} \widetilde{q_v'} (\widetilde{w'^2})^{\frac{1}{2}}, \tag{4.67}$$

where $\varepsilon_{q'_v w'} \approx \varepsilon_{T'w'}$, and

$$H(\widetilde{q'_v w'}) \approx (H_{00} Ro_{sy} \widetilde{h_B})^{\frac{1}{2}} \tag{4.68}$$

To pass on to a nondimensional equation the eq.(3.5) for q_v is divided by $\nabla_z \widetilde{\rho q_v} w_T$ and the nondimensional variables $y_{nd} = \frac{y}{\tilde{y}}$ are introduced. As a result, the dimensionless equation for the free troposphere specific humidity is obtained. The scale/magnitude estimates (4.1) – (4.34), (4.58)–(4.68), (4.49)– (4.55), (3.25) are then applied to the nondimesional equation (see eqs. [A58]–[A66] and the accompanying text in the Appendix A) and the members of the zero and first order are retained. Under this procedure, the source/sink term in the right side of the equation is assumed to be of the zero order. As a result, the nondimensional equation for q_v in the extratropical free troposphere is obtained [168, 169]. Coming back to the dimensional equation one arrives at the basic SDE for q_v

$$\partial_t \rho_0 q_v + \nabla_\lambda \rho_0 q_v \left(u_T + u_n\right) + \nabla_\phi \rho_0 q_v \left(v_T + v_n\right) +$$
$$\nabla_H \cdot (\rho_0 q'_v \vec{v}'_H) + \nabla_z [\rho q_v (w_T + w_n) + \rho q'_v w'] = q^v \tag{4.69}$$

In contrast to the eq. (4.56), the total wind velocity enters the eq. (4.69). However, due to the fact that the major part of the atmospheric water vapour is contained in the lower troposphere, the high values of thermal wind in the upper troposphere do not affect the stability of the numerical integration of the SDE governing water vapour in the model. This latter is just the equation (4.69) integrated over the total troposphere column (see Chapters 7 and 8). So the time step of the order of one day can be used in the governing SDE for water vapour without violation of the CFL criterion.

The last two equations closing the set of model basic SDE for LSLTC in the free troposphere of high and middle latidudes are the eqs. (3.11) and (3.12) which are subjected to the same scale/magnitude analysis, where the zero- and first-order terms

$$\rho = \rho(z)(1 - (1/T(0))(T - T(0))), \tag{4.70}$$

where $\rho(z) = \rho_0(1 - (1/T_{00})(T(0) - T_{00}))$, and

$$\nabla_z p = -\rho g \tag{4.71}$$

For what follows, it is convenient to rewrite these two equations to the following approximation using the quasi-linear UVS of temperature in the free troposphere

$$\rho = \rho'(0)(\frac{T_0 - , z'}{T_0})^{\frac{g}{R\Gamma} - 1} \tag{4.72}$$

$$p_2 - p_1 = -g \int_{z_1}^{z_2} \rho dz, \tag{4.73}$$

where $\rho'(0) = \rho_0(1 - (1/T_{00})(T_0 - T_{00}))$, z' is the absolute elevation above a sea level, and p_1 and p_2 stand for the LSLTC of pressure at two arbitrary levels $z = z_1$ and $z = z_2$ in the free atmosphere.

As is seen from the eqs.(4.40), (4.41), (4.35)– (4.48) the u, v and w fields in the model extratropical free troposphere are determined by the temperature T and the synoptic moments u'^2, v'^2, $u'v'$, $u'w'$, $v'w'$, $\rho'w'$ in this part of the atmosphere, as well as by the sea level pressure p_0 and "frictional" terms. The correspondent SDE for the mentioned synoptic moments are derived in the next Chapter, while the formulas for the zonally averaged $\overline{p_0}$ and azonal ${p_0}^*$ components of p_0 are deduced in Chapter 7. Under this procedure, the equations (4.72) and (4.73) are accounted for. In Chapter 7 the expressions for the lapse rate , , the tropopause height H_{tr} and the vertical profile of the specific humidity in the free troposphere are also obtained. The "frictional" terms are described in detail in Sections 6.3 and 6.4. The equations for the temperature (4.56) and specific humidity (4.69) in the model extratropical free troposphere include the temperature T, specific humidity q_v as well as the fields u, v, w, p_0, sources/sinks of sensible and latent heats, and $T'u'$, $T'v'$, $T'w'$, $q_v'u'$, $q_v'v'$, $q_v'w'$ SM in this part of the atmosphere. The correspondent model basic equation for the terms $T'u'$, $T'v'$, $T'w'$, $q_v'u'$, $q_v'v'$, $q_v'w'$ are derived in the next Chapter, while the expressions for the diabatic (source/sink) members for free atmosphere are determined above in Chapter 2 and described in detail in Chapter 6.

In the stratosphere of extratropical latitudes the geostrophic approximation is used for the zonally averaged zonal component of wind $\overline{u}$. The vertical profile of $\overline{u}$ in each stratospheric layer is represented in this case by the formula for the zonally averaged thermal wind. To describe the nonzonal components of zonal u^* and meridional v^* winds in this region of the atmosphere, the reduced nonzonal parts of the equations (3.15) for $y_3 = u$ and $y_4 = v$ are used in the Charney–Drazin approximation for vertically trapped and propagating waves [33] modified by introducing gravity–wave drag, as well as Rayleigh friction and planetary–wave breaking mechanisms based on [66, 67, 68, 69, 70] findings (see Section 6.4). The partition of u^* and v^* into thermal and nonthermal wind components is performed and the expansions of the tropospheric fields of u_T^*, v_T^* and u_n^*, v_n^* at $z = H_{tr}$ into the spherical harmonic series are used as the lower boundary conditions for u^* and v^* in the stratosphere.

The estimation of the magnitudes and spatial/temporal scales of terms in the zonally averaged equation (3.15) for $y_4 = v$ in the stratosphere of high and middle latitudes leads to the following basic SDE for the zonally averaged meridional velocity $\overline{v} \equiv \overline{v}_a$ in this part of the atmosphere [168]

$$f\overline{v} - \nabla_\phi \left(\overline{u'v'} + \overline{u}\;\overline{v}\right) + \frac{2}{a}\tan\phi\left(\overline{u'v'} + \overline{u}\;\overline{v}\right) - \frac{\partial(\overline{u'w'} + \overline{u}\;\overline{w})}{\partial z} - \frac{1}{\rho_0}\frac{\partial\overline{u}\overline{\rho' w'}}{\partial z} = -\frac{1}{\rho_0}\overline{F_\lambda} \quad (4.74)$$

To match the solutions for $\overline{v}$ at the tropopause and at the interface of the stratospheric layers (in the version of POTSDAM with multi-layer stratosphere) $z = H_l$ the correspondent lower boundary condition at $z = H_l$ is implied on the zonal average of $\frac{\partial\overline{\rho' w'}}{\partial z}$ (see eq.(5.28)), to provide continuity of $\overline{v}$ at $z = H_l$.

The model basic SDE for the temperature T in the stratosphere is analogous to the eq.(4.56). Again, the thermal wind component enters the advection terms only through the beta-term. To obtain the governing SDE the mentioned basic SDE for the stratospheric temperature is integrated over the stratospheric layer(s), under the assumption of a quasi-linear UVS of the temperature. At the top $z = H$ of the atmosphere the condition of the zero vertical momentum, heat and moisture fluxes is imposed, by applying a procedure advocated in [87].

The governing SDE for water vapour in the stratosphere is obtained by the vertical integration of the eq.(4.69) in the limits of the stratospheric layer(s), with omitted nonstationary and large-scale horizontal advection terms.

4.1.2 PBL

Applying the scale/magnitude analysis described in Section 4.1.1 to the equations (3.5) for $y_3 = u$ and $y_4 = v$ in the extratropical planetary boundary layer (PBL) and surface layer (SL) brings about a modified Ekman formulation of Taylor. This formulation includes an Ekman layer above a surface layer of the constant momentum flux in this latter, with continuity of the wind direction and wind stress across the interface between PBL and SL (see, e.g., [220, 87]). In this case, the equations for u and v in the PBL are as follows

$$0 = -\nabla_\lambda p + f\rho_{00} v + \nabla_z \overline{\overline{K}}_{Dz} \rho_{00} \nabla_z u \tag{4.75}$$

$$0 = -\nabla_\phi p - f\rho_{00} u + \nabla_z \overline{\overline{K}}_{Dz} \rho_{00} \nabla_z v \tag{4.76}$$

The effects implied by the baroclinicity of the PBL are neglected in this approximation.

To derive the basic SDE for the horizontal components of velocity in the PBL we integrate the equations (4.75), (4.76) from the upper boundary of the SL to the upper boundary of the PBL and take into account the continuity of the wind stress at the top of the SL, with the constant momentum flux within this layer. Neglecting the wind stress at the top of the PBL as compared to that at its bottom (see, e.g., [124, 133, 246]) and using a bulk-formula (see, e.g., [41, 42]) for the description of the surface wind stress one can obtain

$$< u > = < u_g > - \frac{c_D V_s v_s}{f h_B}, \tag{4.77}$$

$$< v > = < v_g > + \frac{c_D V_s u_s}{f h_B}, \tag{4.78}$$

where $< y >$ denotes the vertical averaging of y over the part of the PBL above the SL, c_D is the drag coefficient, u_s and v_s are the values of u and v at the Stephenson screen level, V_s is the module of the surface wind and $h_B = Z_B - z_{sl} \approx Z_B$ is the PBL depth in the limits from the top z_{sl} of the SL to the top Z_B of the PBL. In view of the continuity of the wind direction and eddy stress across the interface between PBL and SL the u_s, v_s and the surface wind stress τ_s yield the relationships [220, 87]

$$u_s = < u_g > \ \varepsilon \cos\alpha - < v_g > \ \varepsilon \sin\alpha \tag{4.79}$$

$$v_s = < v_g > \ \varepsilon \cos\alpha + < u_g > \ \varepsilon \sin\alpha \tag{4.80}$$

$$\tau_s \equiv c_D \rho_{00} {V_s}^2 = \rho_{00} V_g \sin\alpha \ (2\ |f|\ \overline{\overline{K}}_{Dz})^{\frac{1}{2}} \tag{4.81}$$

Here α is the cross-isobar angle, V_g is the module of the geostrophic wind in the PBL, and $\varepsilon = (1 - \sin 2\alpha)^{\frac{1}{2}}$. Assuming that eqs. (4.79) and (4.80) are valid also for the instantaneous winds in the PBL involves the relationship between V_g and V_s

$$V_s = \varepsilon\ V_g, \tag{4.82}$$

where

$$V_s = ({u_s}^2 + {u'_s}^2 + {v_s}^2 + {v'_s}^2)^{\frac{1}{2}}, \tag{4.83}$$

$$V_g = ({u_g}^2 + {u'_h}^2 + {v_g}^2 + {v'_h}^2)^{\frac{1}{2}} \tag{4.84}$$

In (4.84) the synoptic scale components are represented by their values at the top of the PBL where they are assumed to be geostrophic, as was already mentioned above (hereinafter the lower index h denotes the value of the correspondent variable at the top of the PBL). On the strength of (4.79), (4.80) and assuming that, to a first approximation, ${u'_h}^2 \approx {v'_h}^2$, ${u'_s}^2 \approx {v'_s}^2$ and the cross-isobar angle is small in the extratropics (see, e.g., [80, 156, 87, 105]) the formulas for ${u'_s}^2$ and ${v'_s}^2$ are

$$ {u'_s}^2 = \varepsilon^2 \ {u'_h}^2 \ (\cos\alpha)^2 \tag{4.85} $$

$$ {v'_s}^2 = \varepsilon^2 \ {v'_h}^2 \ (\cos\alpha)^2 \tag{4.86} $$

Substituting (4.79), (4.80), (4.81), (4.82) in (4.77), (4.78) yields

$$ < u > = < u_g > - C_\alpha < v_g > \sin\alpha \tag{4.87} $$

$$ < v > = < v_g > + C_\alpha < u_g > \sin\alpha, \tag{4.88} $$

where

$$ C_\alpha = \frac{V_g}{f h_B} \frac{(2\ |f| \overline{\overline{K}}_{Dz})^{\frac{1}{2}}}{V_s} \ \varepsilon \cos\alpha \tag{4.89} $$

The depth of the PBL h_B at the spatial/temporal scales considered in the model can be estimated by the formula (see, e.g., [43, 246])

$$ h_B = C_{hb} \left(\frac{2\overline{\overline{K}}_{Dz}}{|f|}\right)^{\frac{1}{2}} \tag{4.90} $$

where C_{hb} is a dimensionless parameter of the one unit. Substitution of the eqs.(4.82), (4.90) in the eq.(4.89) gives

$$ C_\alpha = \frac{\cos\alpha}{C_{hb}}, \tag{4.91} $$

so that the eqs.(4.77),(4.78) can be rewritten as

$$ < u > = < u_g > - \frac{\cos\alpha}{C_{hb}} < v_g > \sin\alpha \tag{4.92} $$

$$ < v > = < v_g > + \frac{\cos\alpha}{C_{hb}} < u_g > \sin\alpha \tag{4.93} $$

The ageostrophic components of $< u >$ and $< v >$ are, hence, as follows

$$ < u_a > = - \frac{\cos\alpha}{C_{hb}} < v_g > \sin\alpha \tag{4.94} $$

$$ < v_a > = \frac{\cos\alpha}{C_{hb}} < u_g > \sin\alpha \tag{4.95} $$

Assuming that eqs.(4.92), (4.93) are valid for the instantaneuos winds, the equations for the $< u'^2 >$ and $< v'^2 >$ SM can be written as follows [168]

$$ < u'^2 > = {u'_h}^2 - \frac{\cos\alpha}{C_{hb}} u'_h v'_h \sin\alpha \tag{4.96} $$

$$ < v'^2 > = {v'_h}^2 + \frac{\cos\alpha}{C_{hb}} u'_h v'_h \sin\alpha, \tag{4.97} $$

while the expression for the $< u'v' >$ SM yields

$$ < u'v' > = u'_h v'_h \tag{4.98} $$

In [168] the vertical coefficient of viscosity $\overline{\overline{K}}_{Dz}$ in the PBL is governed by the turbulent kinetic energy balance equation (see, e.g., [133, 246]). In POTSDAM, $\overline{\overline{K}}_{Dz}$ is described by more simple formulas in which this variable depends on the bulk Richardson number Ri_B in the PBL and the temperature stratification of this layer [87]

$$\overline{\overline{K}}_{Dz} = \frac{c_1 + c_2 \Delta T/\Delta z}{1 + c_3 \Delta T/\Delta z}, \qquad \frac{\Delta T}{\Delta z} \le 0$$

$$\overline{\overline{K}}_{Dz} = \frac{c_1}{1 + c_4 \mathrm{Ri}_B}, \qquad \frac{\Delta T}{\Delta z} > 0, \tag{4.99}$$

where

$$\mathrm{Ri}_B = \frac{g \Delta T/\Delta z}{\Theta_h^* \left[\Delta u_s^2 + \Delta v_s^2\right]}, \tag{4.100}$$

$$\Delta T = \Theta_h^* - T_0,$$

$$\Delta z = h_B,$$

$$\Delta u_s^2 = (u_h - u_s)^2,$$

$$\Delta v_s^2 = (v_h - v_s)^2,$$

$$\Theta_h^* = \Theta^*(Z_B),$$

$$u_h =< u_g >,$$

$$v_h =< v_g >,$$

c_1, c_2, c_3, c_4 are empirical coefficients [220, 41, 87] and T_s is the temperature at the Stephenson screen level. In the presence of stratiform and/or cumulus cloudiness in the PBL the formula (4.99) is modified to account for the influence of cloudiness on $\overline{\overline{K}}_{Dz}$ (see eqs. (6.62)– (6.64) and accompanying text, and eq. (6.69)).

Notice that accounting for the cumulus convection and the gravity-wave drag invokes the additional "vertical" terms in the eqs. (4.75) and (4.76). Analogous terms describing the influence of the cumuli and gravity waves appear in the right-hand sides of the above-listed equations (4.35), (4.36) and (4.37) in the free troposphere. The corresponding formulas for these cumuli and gravity-wave drag terms in the PBL and the free troposphere are exhibited below in Sections 6.3 and 6.4. In Section 6.4 the influence is also discussed of the gravity-wave drag, and planetary-wave breaking and Rayleigh friction on the dynamical fields in the stratosphere.

The λ and ϕ components of the surface wind stress are

$$\tau_{s\lambda} = C_D \rho_{00} \left(u_s^2 + {u'_s}^2 + v_s^2 + {v'_s}^2\right) u_s \tag{4.101}$$

$$\tau_{s\phi} = C_D \rho_{00} \left(u_s^2 + {u'_s}^2 + v_s^2 + {v'_s}^2\right) v_s \tag{4.102}$$

Following [87] the drag coefficient C_D in the model is a function of z_{sl} (parameter of the model), aggregated surface roughness z_0 (which accounts for the contributions from the large-scale orography, standard deviation of the sub-grid orography and vegetation–land cover type), and the bulk Richardson number Ri_s for the SL at the neutral temperature stratification of this latter.

The lower boundary condition for the free troposphere w_n (see eq. (4.48)) is the continuity of this variable at the top of the PBL, which gives

$$\rho w_n|_{Z_B} = \rho_{00}\vec{v}_{Hs}\cdot\nabla_H z_0 - \int_{z_{sl}}^{Z_B} \left(\nabla_\lambda \rho_{00} < u > + \nabla_\phi \rho_{00} < v >\right) dz - \rho' w'|_{Z_B}, \qquad (4.103)$$

where $\vec{v}_{Hs} = \{u_s, v_s\}$.

In the same approximation, the model SDE for the temperature in the PBL outside the equatorial belt is as follows [168]

$$\rho_{00}\partial_t < T > + \rho_{00}\left(\nabla_H \cdot < \vec{v}_H >< T > + \nabla_H \cdot < \vec{v'}_H T' >\right) = \left(F_T^s - F_T^{p-}\right)/h_B + < q^T >_{res} \qquad (4.104)$$

where $< \vec{v}_H > = \{< u >, < v >\}$ and $\vec{v'}_H = \{u', v'\}$. The terms F_T^{p-} and F_T^s in the eq. (4.104) are the vertical fluxes of heat due to large-scale and synoptic-scale vertical advection and small-scale turbulence at the top of the PBL and at the SL/PBL interface, respectively

$$F_T^{p-} = +\left(\rho(w_n T + w'T') + F_T^{p+}\right)\Big|_{z=Z_B^+}, \qquad (4.105)$$

$$F_T^s = \rho_{00} c_h V_s (T_g - T_s), \qquad (4.106)$$

where F_T^{p+} is the vertical turbulent heat flux at the lower boundary of the free troposphere, c_h is the heat transfer coefficient which is a function of c_D and Ri_s, and T_g is the temperature of the underlying surface.

The $< q^T >_{res} = < q_R^T > + < q_{LSC}^T > + < q_{MC}^T > + < q_{SMT}^{T,c} >$ term in (4.104) describes the correspondent heat influx to PBL due to radiation, large–scale condensation, moist convection condensation and moist convection mixing (see eq. (2.3)).

The PBL baroclinic effects being neglected, one can write in the first approximation

$$T'(z) \approx T_h' \qquad (4.107)$$

where $T_h' \left(= T'\left(z = Z_B^+\right)\right)$ is the synoptic disturbance of T at the bottom of the free atmosphere. As a result, the $< u'T' >$ and $< v'T' >$ second moments entering eq.(4.104) are as follows

$$< u'T' > = < u_h' T_h' > - \frac{\cos\alpha}{C_{hb}} < v_h' T_h' > \sin\alpha \qquad (4.108)$$

$$< v'T' > = < v_h' T_h' > + \frac{\cos\alpha}{C_{hb}} < u_h' T_h' > \sin\alpha \qquad (4.109)$$

By virtue of (4.107) the T'^2 SM in the PBL is

$$T'^2 = {T_h'}^2 \qquad (4.110)$$

The vertical diffusion coefficient for heat in the PBL is assumed to be equal to the vertical coefficient of viscosity. The lower boundary condition analogous to (4.81) but for the heat flux is applied at the PBL/SL interface

$$\rho_{00} c_h V_s (T_s - T_g) = (c_v/c_p)\, \rho_{00} \overline{\overline{K}}_{Dz} \nabla_z \Theta^*\Big|_{z=z_{sl}} \qquad (4.111)$$

This equation is used to determine the temperature at the Stephenson screen level T_s. The $\nabla_z\Theta^*$ term entering (4.111) yields the eq.(2.6) at $z = z_{sl}$.

In the same approximation the model SDE for the specific humidity in the PBL outside the equatorial belt is as follows [168]

$$\rho_{00}\partial_t < q_v > +\rho_{00}\left(\nabla_H \cdot < \vec{v}_H >< q_v > +\nabla_H \cdot < \vec{v'}_H q'_v >\right) = \left(F_q^s - F_q^{p-}\right)/h_B + < q^v >_{res} \tag{4.112}$$

The terms F_q^{p-} and F_q^s in the eq. (4.112) stand for the vertical fluxes of moisture at the top of the PBL and at the SL/PBL interface, respectively

$$F_q^{p-} = +\left(\rho(wq_v + w'q'_v) + F_q^{p+}\right)\Big|_{z=Z_B^+}, \tag{4.113}$$

$$F_q^s = \alpha_q \rho_{00} c_q V_s (q_g - q_{vs}), \tag{4.114}$$

where F_q^{p+} is the vertical turbulent moisture flux at the lower boundary of the free troposphere, c_q is the humidity transfer coefficient which is assumed to equal c_h, q_{vs} is the specific humidity at the Stephenson screen level, q_g is the saturated specific humidity at the temperature of the underlying surface and surface air pressure, and α_q is the coefficient dependent on the hydrological properties of the underlying layer [47].

The $< q^v >_{res} = < q^v_{LSC} > + < q^v_{MC} > + < q^{v,c}_{SMT} >$ term in (4.112) describes the correspondent water vapour influx in PBL due to large–scale condensation, moist convection condensation and moist convection mixing (see eq. (2.13)).

The vertical profile of q'_v in the main part of the PBL above the SL is approximated in the model by

$$q'_v(z) \approx q'_{vs} \exp\left[-\frac{z}{H_q}\right] \tag{4.115}$$

The analogous equation is used in POTSDAM for the description of the vertical distribution of q_v in the PBL

$$q_v(z) \approx q_{vs} \exp\left[-\frac{z}{H_q}\right] \tag{4.116}$$

The scale height H_q for the the specific humidity is determined below in Chapter 8.

Taking (4.115) into account the expressions for the $u'q'_v$ and $v'q'_v$ second moments in the PBL are as follows

$$u'q'_v = u'_h q'_{vh} \exp\left(\frac{Z_B - z}{H_q}\right) - \frac{\cos\alpha}{C_{hb}} v'_h q'_{vh} \exp\left(\frac{Z_B - z}{H_q}\right) \sin\alpha \tag{4.117}$$

$$v'q'_v = v'_h q'_{vh} \exp\left(\frac{Z_B - z}{H_q}\right) + \frac{\cos\alpha}{C_{hb}} u'_h q'_{vh} \exp\left(\frac{Z_B - z}{H_q}\right) \sin\alpha, \tag{4.118}$$

The analogous equation for $T'q'_v$ in the PBL is

$$T'q'_v = T'_h q'_{vh} \exp\left(\frac{Z_B - z}{H_q}\right) \tag{4.119}$$

Averaging these three equations with respect to z over the total PBL depth gives formulas for $< u'q'_v >$, $< v'q'_v >$ and $< T'q'_v >$, the first two of which enter the eq. (4.112).

Accounting for (4.115) the ${q'_v}^2$ SM in the model PBL is described by

$${q'_v}^2 = {q'_{vh}}^2 \exp\left(\frac{2(Z_B - z)}{H_q}\right) \tag{4.120}$$

The vertical diffusion coefficient for humidity in the PBL is assumed to be equal to the vertical coefficient of viscosity. The lower boundary condition analogous to (4.81) and (4.111) but for the turbulent flux of humidity is applied at the PBL/SL interface

$$\alpha_q \rho_{00} c_q V_s (q_{vs} - q_g) = \rho_{00} \overline{\overline{K}}_{Dz} \nabla_z q_v \Big|_{z=z_{sl}} \tag{4.121}$$

This equation is used for the determination of q_{vs}. The factor $\nabla_z q_v$ entering the last equation is described in [168] using formula (4.116). The item F_q^{p+} in (4.113) is governed by taken with the negative sign expression in the right–hand side of (4.121) , with $\nabla_z q_v$ factor derived from the expression for the vertical profile of the specific humidity in the free troposphere (for further detail see Chapter 8).

Neglecting the baroclinic effects in the PBL and applying eq.(3.2) the "vertical" SM in this layer above the SL are represented in the model by the following formulas [168]

$$u'w' = u'_h w'_h - \frac{\cos\alpha}{C_{hb}} v'_h w'_h (Z_B - z) \sin\alpha \tag{4.122}$$

$$v'w' = v'_h w'_h + \frac{\cos\alpha}{C_{hb}} u'_h w'_h (Z_B - z) \sin\alpha \tag{4.123}$$

$$T'w' = T'_h w'_h \tag{4.124}$$

$$q'_v w' = q'_{vh} w'_h \exp\left(\frac{Z_B - z}{H_q}\right) \tag{4.125}$$

To the same approximation, the w'^2 and $\rho' w'$ SM in the PBL are

$$w'^2 = w_h'^2 \tag{4.126}$$

$$\rho' w' = \rho' w'_h \exp\left(\frac{Z_B - z}{H_{00}}\right) \tag{4.127}$$

Applying the zero lower boundary condition for w' at the underlying surface the "vertical" SM in the $z_0 < z < z_{sl}$ range of heights are described in the model as linear functions of z with zero values at $z = z_0$ and with values correspondent to (4.122)–(4.127) at $z = z_{sl}$.

4.2 Tropics

4.2.1 Free atmosphere

Using the same scaling analysis as in Subsection 4.1.1 the set of the generalized SDE (3.5), (3.8), (3.9) for $y_3 = u$ and $y_4 = v$ in the equatorial (tropical) free atmosphere can be reduced to the following basic SDE for u and v^* [168]

$$\partial_\lambda \rho_0 (u^2 + u'^2) + \partial_\phi \rho_0 u'v' + \partial_z \rho (w^* u + w'u') = -\partial_\lambda p + f_e \rho_0 v^* + F_\lambda(u) \tag{4.128}$$

$$\partial_\lambda \rho_0 (uv^* + u'v') + \partial_\phi \rho_0 v'^2 + \partial_z \rho (w^* v^* + w'v') = -\partial_\phi p + f_e \rho_0 u + F_\phi(v^*), \tag{4.129}$$

where $f_e = \beta_0 L_{Ro,e}$ is the above-mentioned "equatorial" Coriolis parameter, $\beta_0 = 2\omega/a$ and $L_{Ro,e}$ is the equatorial Rossby deformation radius given by [160]

$$L_{Ro,e} = \left(\frac{N H_{00}}{\beta_0}\right)^{1/2} \tag{4.130}$$

The Hadley cell component $\overline{v}$ in the model equatorial free atmosphere is described by the eqs. (4.37), (4.74) in which f is replaced by f_e.

The model basic SDE for T and q_v in this atmospheric domain are the same as in the extratropical region, and the governing SDE for these variables in the equatorial belt are obtained by the vertical integration of the basic SDE in the limits of the above-mentioned atmospheric layers.

The boundary conditions of the continuity of the solution and its first derivatives with respect to horizontal coordinates are set in the model at $\phi = \phi_e^0$, where ϕ_e^0 is the latitude (in both hemispheres) at which the following equality is met [168]

$$f \mid_{\phi=\phi_e^0} = f_e. \tag{4.131}$$

4.2.2 PBL

Exploiting the same magnitude and spatial/temporal analysis of terms the basic SDE for u and v^* in the equatorial boundary layer are [168]:

$$\partial_z\{\rho(wu + u'w')\} = \nabla_\lambda p + f_e\rho_0 v^* + F_\lambda(u), \tag{4.132}$$

$$\partial_z\{\rho(wv + v'w')\} = \nabla_\phi p + f_e\rho_0 u + F_\phi(v^*). \tag{4.133}$$

The in-PBL part of the Hadley cell $\overline{v}$ is assumed to close at any latitude the bulk mass flux due to zonally averaged meridional circulation integrated over the total column of the atmosphere (as in the extratropical latitudes, see eq. (7.15)).

The SDE for T and q_v in this region of the atmosphere are the same as those for the PBL of high and middle latitudes except that the terms describing the horizontal small/mesoscale eddy influxes of heat $-div_H \vec{F}^T_{SMT,H}$ and humidity $-div_H \vec{F}^v_{SMT,H}$ (see eqs. (2.4), (2.5) and (2.14), (2.15)) are taken into account in the left sides of eqs. (4.104) and (4.112), respectively.

The boundary conditions of the continuity of T, q_v and their first derivatives with respect to horizontal coordinates are set at $\phi = \phi_e^0$. The analogous boundary conditions with respect to the vertical coordinate are assigned at the top of the equatorial PBL.

Chapter 5

Basic SDE for synoptic moments

The basic SDE for the SM in the stratosphere and PBL are given above in Chapters 3 and 4 (see eqs. (3.30), (4.85),(4.86), (4.96), (4.97),(4.98), (4.108), (4.109), (4.110), (4.117)–(4.120), (4.122)–(4.127)). In those equations, the SM in the free troposphere at the tropopause and at the top of the PBL are used, respectively, as the lower and upper boundary conditions. In this Chapter, the basic SDE are derived for the synoptic second moments in the free troposphere.

We apply scale/magnitude estimates (3.25), (4.1) – (4.34), (4.49)– (4.55), (4.58)–(4.68) to the generalized SDE for the "horizontal" SM and auto-correlation functions described in Chapter 3 (see eqs. (3.15)– (3.17)). In this, each component of the dynamic fields u and v is subdivided additionally into a mean zonal component and a deviation from it, under the assumption that

$$\widetilde{\widetilde{v_i}} \le \widetilde{u_i^*} \approx \widetilde{v_i^*} \approx \varepsilon_* \widetilde{\widetilde{u_g}} \tag{5.1}$$

where in this case $i = g, a, T, n$ and $\varepsilon_* \approx (0.1 \div 0.3)$.

Retaining zero- and first-order terms in the mentioned generalized SDE (see estimations [B1]–[B128] and accompanying text in the Appendix B) results in the following basic SDE for the "horizontal" SM in the free extratropical troposphere [168]

$$\frac{\partial u'^2}{\partial t} + u\nabla_\lambda u'^2 + v\nabla_\phi u'^2 + 2u'w'\nabla_z u - \frac{1}{\rho_0} u'^2 \nabla_z \rho' w' = \frac{2}{a} v u'^2 \tan\phi + 2F_{u',u'}, \tag{5.2}$$

$$\frac{\partial v'^2}{\partial t} + u\nabla_\lambda v'^2 + v\nabla_\phi v'^2 + 2v'w'\nabla_z v - \frac{1}{\rho_0} v'^2 \nabla_z \rho' w' = -\frac{4}{a} u u'v' \tan\phi + 2F_{v',v'}, \tag{5.3}$$

$$\frac{\partial u'v'}{\partial t} + u\nabla_\lambda u'v' + v\nabla_\phi u'v' + v'^2\nabla_\phi u + u'^2 \nabla_\lambda v + v'w'\nabla_z u + u'w'\nabla_z v - \\ -\frac{1}{\rho_0} u'v'\nabla_z \rho' w' = \frac{1}{a}\tan\phi u \left(v'^2 - 2u'^2\right) + F_{u',v'}, \tag{5.4}$$

$$\frac{\partial u'T'}{\partial t} + u\nabla_\lambda u'T' + v\nabla_\phi u'T' + u'^2\nabla_\lambda T - \frac{1}{\rho_0} u'T'\nabla_z \rho' w' = \\ = \frac{1}{a}\tan\phi (uv'T' + vu'T') + F_{u',T'}, \tag{5.5}$$

$$\frac{\partial v'T'}{\partial t} + u\nabla_\lambda v'T' + v\nabla_\phi v'T' + v'^2\nabla_\phi T - \frac{1}{\rho_0} v'T'\nabla_z \rho' w' = -\frac{2}{a}\tan\phi \; u \; u'T' + F_{v',T'}, \tag{5.6}$$

$$\frac{\partial T'^2}{\partial t} + u\nabla_\lambda T'^2 + v\nabla_\phi T'^2 - 2T'w', \ -\frac{1}{\rho_0} T'^2 \nabla_z \rho' w' = 2F_{T',T'} + \frac{2\mathcal{L}}{\rho_0 c_v} T'm', \quad (5.7)$$

$$\frac{\partial T' q'_v}{\partial t} + \vec{v}_H \cdot \nabla_H T' q'_v + T' w' \nabla_z q_v + q'_v w' \nabla_z T - \frac{1}{\rho_0} T' q'_v \nabla_z \rho' w' = \frac{1}{\rho_0}(T' q'_2 + q'_v q'_1) + + F_{T',q'_v} \quad (5.8)$$

$$\frac{\partial u' q'_v}{\partial t} + u\nabla_\lambda u' q'_v + v\nabla_\phi u' q'_v + u'^2 \nabla_\lambda q_v - \frac{1}{\rho_0} u' q'_v \nabla_z \rho' w' = \frac{1}{a}\tan\phi (uv' q'_v + vu' q'_v) + F_{u',q'_v}, \quad (5.9)$$

$$\frac{\partial v' q'_v}{\partial t} + u\nabla_\lambda v' q'_v + v\nabla_\phi v' q'_v + v'^2 \nabla_\phi q_v - \frac{1}{\rho_0} v' q'_v \nabla_z \rho' w' = -\frac{2}{a}\tan\phi \ u \ u' q'_v + F_{v',q'_v}, \quad (5.10)$$

$$\frac{\partial {q'_v}^2}{\partial t} + u\nabla_\lambda {q'_v}^2 + v\nabla_\phi {q'_v}^2 + 2q'_v w' \nabla_z q_v - \frac{1}{\rho_0} {q'_v}^2 \nabla_z \rho' w' = 2F_{q'_v,q'_v} - \frac{2}{\rho_0} q'_v m'. \quad (5.11)$$

Here $F_{y'_j,y'_k}$ terms represent the small/mesoscale "turbulent" diffusion for the correspondent SM, m — condensation rate, c_p — the specific heat of the air at constant pressure, $\mathcal{L}$ — the latent heat of the phase transition and

$$T'm' = \beta_q q'_v T' + \beta_w T' w', \quad (5.12)$$

$$q'_v m' = \beta_q {q'_v}^2 + \beta_w q'_v w', \quad (5.13)$$

where

$$\beta_q = \frac{\rho_0 f_r}{\tau_0} \frac{\partial n_{st}}{\partial f_r}, \quad (5.14)$$

$$\beta_w = \frac{\rho_0 q_v}{\tau_0} \frac{\partial n}{\partial w} + C_u \left({w'^2}^{1/2} + |w| \right)^{-1} \quad (5.15)$$

In (5.14), (5.15) f_r — relative humidity, τ_0— characteristic time of the cloud generation (see eq. (8.82) and accompanying text), n_{st} — stratiform cloud amount in the layer below the considered level, and C_u is given by

$$C_u = C_{u,i} n_{cu}, \quad (5.16)$$

where $C_{u,i}$ and n_{cu} are, respectively, condensation rate per unit volume in cumuli interior and cumulus cloud amount in the layer below the considered level (see Section 6.3). Based on the results reported in [50] the dependence of the stratiform cloud amount on the large–scale fields of the relative humidity and vertical velocity is described by the formula (6.12). The correspondent formulas for C_u and n_{cu} are given in Section 6.3. Notice that the equations for $T'm'$ and $q'_v m'$ are the result of the application of the formula (3.29) to the diabatic terms (described in Section 6.3) associated with the condensation in stratus and cumuli, retaining the zero- and first-order terms [168, 172].

The w'^2 SM in the free troposphere is represented in the model by a sum of the "adiabatic" term ${w'^2}_{ad}$ and the additional "diabatic" term ${w'^2}_{cond}$ due to the condensation in stratus and cumuli

$$w'^2 = {w'^2}_{ad} + {w'^2}_{cond}. \quad (5.17)$$

To derive the equation for the synoptic-scale vertical velocity w'_{ad} we use eq. (3.2). We integrate that equation from the surface (at the zero lower boundary condition) to the arbitrary level in the free atmosphere, under the thermal wind approximation for u' and v' components and quasi–linear UVS of T' which gives the following formula for w'_{ad} [168]

$$w'_{ad} \approx \frac{\beta}{f}\ |\vec{v'}|\ z'\ (\frac{1}{2} - ,\ _T\frac{z'}{3}) \approx \frac{\beta}{f}\frac{g}{fT_{00}}|\nabla_H T'(0)|\ z'^2(\frac{1}{2} - ,\ _T\frac{z'}{3}) \tag{5.18}$$

Here $\beta = \nabla_\phi f$, $T'(0)$ is the synoptic-scale perturbation of temperature at the surface and z' is the height above the surface.

Square eq. (5.18) and apply $(\Delta L, \Delta\tau)$ averaging. By virtue of (4.9), (4.10), (4.49)–(4.51) one can write

$$[\nabla_H\ T'(0)]^2 \approx [\nabla_H\ T(0)]^2\ Ro_{sy}^{-2} \approx [\nabla_H\ T(0)]^2\ \alpha_0^{-2}, \tag{5.19}$$

which gives the estimation for $w'^2{}_{ad}$

$$w'^2{}_{ad} \approx C_w(\frac{\beta}{f})^2(\frac{g}{fT_{00}})^2([\nabla_\lambda\ T^*(0)]^2 + [\nabla_\phi\ T^*(0)]^2 + [\nabla_\phi\ \overline{T(0)}]^2)\ z'^4(\frac{1}{2} - ,\ _T\frac{z'}{3})^2\ \alpha_0^{-2}, \tag{5.20}$$

where C_w is a constant of the order of unity, and $\overline{y}$ and y^*, respectively, denote the zonal average and deviation from it.

The dimensionless static stability parameter α_0 in (5.20), as well as in all the formulas for the SM in the free troposphere is represented as follows

$$\alpha_0^2 = \frac{R}{g}(,\ _{a,*} - ,\ ), \tag{5.21}$$

where

$$,\ _{a,*} = ,\ _{wa}V_c + ,\ _a(1 - V_c),$$

$,\ _a$ — adiabatic lapse rate, $,\ _{wa}$ — moist adiabatic lapse rate, V_c - relative volume of the span of the cloudiness in the layer below a given level.

The term $w'^2{}_{cond}$ in the free troposphere is evaluated following [205] from the entropy equation which gives

$$w'^2{}_{cond} = \beta_0^2\frac{\beta_q^2 q_v'^2 + \beta_w^2 w'^2{}_{ad}}{1 - \beta_0^2\beta_w^2} \approx \beta_0^2\left(\beta_q^2 q_v'^2 + \beta_w^2 w'^2{}_{ad}\right) \tag{5.22}$$

Here

$$\beta_0 = \frac{\mathcal{L}R\Theta}{g\rho_0 c_p \alpha_0^2 T},$$

where $\Theta = T + ,\ _{a,*}z$.

The model basic SDE for the $\rho' w'$ vertical moment in the free troposphere is derived by applying the Prandtl concept of "mixing length" [178] but for macroturbulent synoptic-scale eddies which gives the following formula for the characteristic value of ρ' in the $\rho' w'$ term

$$\rho' \approx \frac{\partial\rho}{\partial z}H_g Ro_{sy} \tag{5.23}$$

Using eq.(3.2) the w' factor in $\rho' w'$ SM is approximated by

$$w' \approx \frac{\sqrt{u'^2 + v'^2}}{L_{Ro,sy}}H_g Ro_{sy} \tag{5.24}$$

Combining eqs. (5.23) and (5.24) gives

$$\overline{\rho' w'} = -\epsilon_{\rho' w'} \frac{\sqrt{\overline{u'^2} + \overline{v'^2}}}{L_{Ro,sy}} (H_g Ro_{sy})^2 \frac{\partial \rho}{\partial z} \tag{5.25}$$

where $\epsilon_{\rho' w'} \approx (0.1 \div 0.3)$. The negative sign in the right side of (5.25) designates the positive correlation between synoptic-scale perturbations of air density and vertical velocity (i.e., upward (positive) mass flux due to ensembles of synoptic-scale eddies). Taking into account eq. (5.25) the $\nabla_z \overline{\rho' w'}$ SM in the free troposphere can be approximated by [168]

$$\nabla_z \overline{\rho' w'} = -\epsilon_{\rho' w'} \frac{\sqrt{\overline{u'^2} + \overline{v'^2}}}{L_{Ro,sy}} (H_g Ro_{sy})^2 \frac{\partial^2 \rho}{\partial z^2} \tag{5.26}$$

The basic SDE for $\overline{\rho' w'}$ and $\nabla_z \overline{\rho' w'}$ vertical moments in the stratosphere are obtained from the eq. (3.30) which gives

$$\overline{\rho' w'} = \overline{\rho' w'}_l \exp\left\{ - \left[\frac{2N}{f} \left(K_w^2 + \gamma_w^2 - \frac{\beta}{[\overline{u}]} \right)^{1/2} - \frac{1}{H_{0,l}} \right] (z - H_l) \right\} \tag{5.27}$$

$$\nabla_z \overline{\rho' w'} = \nabla_z \overline{\rho' w'}_l \exp\left\{ - \left[\frac{2N}{f} \left(K_w^2 + \gamma_w^2 - \frac{\beta}{[\overline{u}]} \right)^{1/2} - \frac{1}{H_{0,l}} \right] (z - H_l) \right\} \tag{5.28}$$

The characteristic time τ_{ens} of the SM adjustment to the LSLTC can be evaluated from the eqs. (5.2)—(5.11) as

$$\tau_{ens} = |\rho_0 / \nabla_z \overline{\rho' w'}|$$

. Using the eq.(5.26) this yields the estimate $\tau_{ens} \approx (10 \div 40)$ days. So, the synoptic–scale ensembles can exhibit a remarkable deviation from the instantaneous adaptation to the large–scale long-term thermodynamical and dynamical patterns.

The small/mesoscale "turbulent" diffusion terms in the right–hand sides of eqs. (5.2)–(5.11) can be written as follows

$$F_{y'_j, y'_k} = \overline{\overline{K}}_{DH} \Delta_{\lambda\phi} \overline{y'_j y'_k} + \overline{\overline{K}}_{Dz} \frac{1}{\rho_0(z)} \frac{\partial}{\partial z} \left(\rho_0(z) \frac{\partial \overline{y'_j y'_k}}{\partial z} \right), \tag{5.29}$$

for the cross–correlations and

$$F_{y'_j, y'_j} = \frac{1}{2} \overline{\overline{K}}_{DH} \Delta_{\lambda\phi} \overline{y'^2_j} + \frac{1}{2} \overline{\overline{K}}_{Dz} \frac{1}{\rho_0(z)} \frac{\partial}{\partial z} \left(\rho_0(z) \frac{\partial \overline{y'^2_j}}{\partial z} \right) - \frac{1}{2} \overline{\overline{K}}_{Dz} \overline{\left(\frac{\partial y'_j}{\partial z} \right)^2}, \tag{5.30}$$

for the auto-correlations.

Notice that the equations (5.2)—(5.11) for the "horizontal" SM have a structure analogous to those for the LSLTC: they include the advection of the synoptic-scale ensemble characteristics by the large-scale long-term motions, small/mesoscale friction or diffusion and source/sink terms. The generation of the "horizontal" SM is mainly due to the "baroclinic" terms $\rho \overline{y'_i w'} \nabla_z y_i$ (in those regions where $\nabla_z y_i > 0$). These terms can be referred to as large-scale generation terms. The second source for the ensembles of the synoptic component is the stratus and cumuli condensation, as well as additional vertical motions (diabatic generation) attributed to these processes. The dissipation is basically caused by the synoptic-scale vertical mass transport $y'_i w' \nabla_z \rho' w'$ (in those cases when the latter is upward) which represents the work done by the ensembles of the synoptic-scale objects against the force of gravity (large-scale dissipation), and also by the small/mesoscale "turbulent" friction or diffusion (small/mesoscale dissipation).

To derive the basic SDE for the "vertical" SM we apply scale/magnitude estimates (3.14), (4.1)– (4.34), (4.49)–(4.55), (4.58)–(4.68) to the generalized SDE (3.18)–(3.21) for these SM retaining the zero- and first-order terms (see eqs. [B129]–[B170] and accompanying text in the Appendix B). Under this procedure, using eq. (5.18), thermal wind representation for u', v', and quasi–linear and quasi-exponential approximations, respectively, for the UVS of T' and q'_v we can estimate the terms $u'\nabla_\lambda w'$, $u'\nabla_\phi w'$, $u'\nabla_z w'$, $v'\nabla_\lambda w'$, $v'\nabla_\phi w'$, $v'\nabla_z w'$, $w'\nabla_\lambda T'$, $w'\nabla_\phi T'$, $w'\nabla_z T'$, $w'\nabla_\lambda q'_v$, $w'\nabla_\phi q'_v$ and $w'\nabla_z q'_v$ in eqs. (3.18)–(3.21) as follows [168]

$$u'\widetilde{\nabla_\lambda} w' \approx \frac{\beta}{f} z'_g (\frac{1}{2} - \,, _T \frac{z'_g}{3})[\nabla_\lambda u'v' + (1 - \,, _T z'_g)^2 \; (\frac{1}{2}\nabla_\phi + \frac{\beta}{f}) \; v'^2] \tag{5.31}$$

$$u'\widetilde{\nabla_\phi} w' \approx -\frac{\beta}{2f} \; z'_g \; (\frac{1}{2} - \,, _T \frac{z'_g}{3}) \; \nabla_\lambda \; u'^2 \tag{5.32}$$

$$u'\widetilde{\nabla_z} w' \approx \frac{\beta}{f} \; [(\frac{1}{2} - 2, _T \frac{z'_g}{3}) + (\frac{1}{2} - \,, _T \frac{z'_g}{3}) \frac{1 - 2, _T z'_g}{1 - \,, _T z'_g}] \tag{5.33}$$

$$v'\widetilde{\nabla_\lambda} w' \approx \frac{\beta}{2f} z'_g (\frac{1}{2} - \,, _T \frac{z'_g}{3}) \nabla_\lambda v'^2 \tag{5.34}$$

$$v'\widetilde{\nabla_\phi} w' \approx \frac{\beta}{f} \; z'_g \; (\frac{1}{2} - \,, _T \frac{z'_g}{3}) \; (\frac{1}{2} \; \nabla_\phi - \frac{\beta}{f}) \; v'^2 \tag{5.35}$$

$$v'\widetilde{\nabla_z} w' \approx \frac{\beta}{f} \; [\frac{z'_g}{2} \; (\frac{1}{2} - \,, _T \frac{z'_g}{3}) \; \nabla_z + (\frac{1}{2} - 2, _T \frac{z'_g}{3})] \; v'^2 \tag{5.36}$$

$$w'\widetilde{\nabla_\lambda} T' \approx \beta \; (\frac{1}{2} - \,, _T \frac{z'_g}{3}) \frac{T_{00}}{g} \; v'^2 \tag{5.37}$$

$$w'\widetilde{\nabla_\phi} T' \approx -\beta \; (\frac{1}{2} - \,, _T \frac{z'_g}{3}) \frac{T_{00}}{g} \; u'v' \tag{5.38}$$

$$w'\widetilde{\nabla_z} T' \approx -\frac{\beta}{2f} \; z'_g \; (\frac{1}{2} - \,, _T \frac{z'_g}{3}) \frac{g \; z'_g}{f T_{00}} \frac{1}{1 - \,, _T z'_g} \;, _T \; \nabla_\lambda T'^2 \tag{5.39}$$

$$w'\widetilde{\nabla_\lambda} q'_v \approx \beta \; (\frac{1}{2} - \,, _T \frac{z'_{vg}}{3}) \frac{T_{00}}{g} \; v'^2 \; \frac{\partial q_{v,sat}(0)}{\partial T(0)} \frac{q_v(0)}{q_{v,sat}(0)} \tag{5.40}$$

$$w'\widetilde{\nabla_\phi} q'_v \approx -\beta \; (\frac{1}{2} - \,, _T \frac{z'_{vg}}{3}) \frac{T_{00}}{g} \; u'v' \; \frac{\partial q_{v,sat}(0)}{\partial T(0)} \frac{q_v(0)}{q_{v,sat}(0)} \tag{5.41}$$

$$w'\widetilde{\nabla_z} q'_v \approx -\frac{\beta}{2f} \; z'_{vg} \; (\frac{1}{2} - \,, _T \frac{z'_{vg}}{3}) \frac{g \; z'_{vg}}{f T_{00}} \frac{1}{1 - \,, _T z'_{vg}} \;, _T \; \nabla_\lambda T'^2 \frac{\partial q_{v,sat}(0)}{\partial T(0)} \frac{q_v(0)}{q_{v,sat}(0)}, \tag{5.42}$$

where $z'_{vg} \approx \sqrt{z'_g H_q}$.

As a result, the model basic equations for "vertical" SM in the free troposphere are written in the form [168]

$$w'^2 \nabla_z T - \frac{1}{\rho_0} T' w' \nabla_z \rho' w' = \frac{1}{\rho_0} w' q'_1 + \frac{1}{\rho_0} \Delta \, (T'w') \tag{5.43}$$

$$w'^2\nabla_z q_v - \frac{1}{\rho_0}q'_v w'\nabla_z\rho' w' = \frac{1}{\rho_0}w' q'_2 + \frac{1}{\rho_0}\Delta\left(q'_v w'\right) \tag{5.44}$$

$$\vec{v}_H \cdot \nabla_H u' w' + w'^2\nabla_z u - \frac{1}{\rho_0}u' w'\nabla_z\rho' w' = \frac{1}{\rho_0}\Delta\left(u' w'\right) \tag{5.45}$$

$$\vec{v}_H \cdot \nabla_H v' w' + w'^2\nabla_z v - \frac{1}{\rho_0}v' w'\nabla_z\rho' w' = \frac{1}{\rho_0}\Delta\left(v' w'\right) \tag{5.46}$$

In the adiabatic approximation, neglecting the "spherical" (including $\tan\phi$ factor) terms the zero–order equations for the synoptic–scale second moments can be obtained from the eqs. (5.2)—(5.11), (5.43)—(5.46) by retaining only the zero-order terms. Those represent a balance among large-scale generation, large-scale dissipation and advection, which gives

$$u\nabla_\lambda u'^2 + v\nabla_\phi u'^2 + 2u' w'\nabla_z u - \frac{1}{\rho_0}u'^2\nabla_z\rho' w' = 0 \tag{5.47}$$

$$u\nabla_\lambda v'^2 + v\nabla_\phi v'^2 + 2v' w'\nabla_z v - \frac{1}{\rho_0}v'^2\nabla_z\rho' w' = 0 \tag{5.48}$$

$$u\nabla_\lambda u' v' + v\nabla_\phi u' v' + v'^2\nabla_\phi u + u'^2\nabla_\lambda v + v' w'\nabla_z u - \frac{1}{\rho_0}u' v'\nabla_z\rho' w' = 0 \tag{5.49}$$

$$u\nabla_\lambda u' T' + v\nabla_\phi u' T' + u'^2\nabla_\lambda T - \frac{1}{\rho_0}u' T'\nabla_z\rho' w' = 0 \tag{5.50}$$

$$u\nabla_\lambda v' T' + v\nabla_\phi v' T' + v'^2\nabla_\phi T - \frac{1}{\rho_0}v' T'\nabla_z\rho' w' = 0 \tag{5.51}$$

$$u\nabla_\lambda T'^2 + v\nabla_\phi T'^2 - 2T' w', \; - \frac{1}{\rho_0}T'^2\nabla_z\rho' w' = 0 \tag{5.52}$$

$$u\nabla_\lambda T' q'_v + v\nabla_\phi T' q'_v + T' w'\nabla_z q_v + q'_v w'\nabla_z T - \frac{1}{\rho_0}T' q'_v\nabla_z\rho' w' = 0 \tag{5.53}$$

$$u\nabla_\lambda u' q'_v + v\nabla_\phi u' q'_v + u'^2\nabla_\lambda q_v - \frac{1}{\rho_0}u' q'_v\nabla_z\rho' w' = 0 \tag{5.54}$$

$$u\nabla_\lambda v' q'_v + v\nabla_\phi v' q'_v + v'^2\nabla_\phi q_v - \frac{1}{\rho_0}v' q'_v\nabla_z\rho' w' = 0 \tag{5.55}$$

$$u\nabla_\lambda {q'_v}^2 + v\nabla_\phi {q'_v}^2 + 2q'_v w'\nabla_z q_v - \frac{1}{\rho_0}{q'_v}^2\nabla_z\rho' w' = 0 \tag{5.56}$$

$$T' w' = \frac{\rho_0 w'^2}{\nabla_z\rho' w'}\partial_z\ T, \tag{5.57}$$

$$q'_v w' = \frac{\rho_0 w'^2}{\nabla_z\rho' w'}\partial_z\ q_v, \tag{5.58}$$

$$w'^2\nabla_z u - \frac{1}{\rho_0}u' w'\nabla_z\rho' w' = 0 \tag{5.59}$$

$$w'^2\nabla_z v - \frac{1}{\rho_0}v' w'\nabla_z\rho' w' = 0 \tag{5.60}$$

The terms w'^2 and $\nabla_z \rho' w'$ are described in this case by the equations (5.20) and (5.26), with $\rho = \rho_0$ in (5.26).

To avoid the necessity of solving the three–dimensional equations (5.2)– (5.11) the shape of the vertical profiles of the horizontal SM in (5.29), (5.30) is specified in POTSDAM as it results from the zero-order approximation (5.47)–(5.56), with neglected advective terms. Under this procedure, to compute the vertical SM at any given time step it is convinient to use the LSLTC as the inputs at that very time step as well as the u'^2, v'^2 and $q_v'^2$ fields computed in the previous time step. The vertical SM are then utilized to calculate the horizontal SM accounting for the values of the LSLTC and SM obtained at the previous time step.

The basic SDE for the SM in the equations for the extratropical SM described above in this Chapter but with the replacement of the Coriolis parameter, the Rossby number, the Rossby deformation radius and β by the equatorial ones. Under this procedure, a smooth matching of the SM at the (prescribed) northern and southern latitudinal boundaries of the equatorial belt is applied.

Chapter 6

Basic SDE for diabatic processes

6.1 Radiation

In this Section, we describe the shortwave and longwave components of the radiative influx q_R^T which contributes in POTSDAM to item q^T in the right-hand sides of the eqs. (4.56) and (4.104) for the LSLTC of temperature in the free atmosphere and PBL.

6.1.1 Shortwave radiation

The computation of shortwave radiation (SWR) fluxes in the model [168] is based on the traditional two–stream approximation (see, e.g. [58, 109, 245]).

To calculate the upward and downward solar radiation fluxes a method proposed in [219, 229, 218]is applied. The method is based on a two–stream delta–Eddington approximation [195] of the transport equation in gas–aerosol atmosphere in UF ($0.2\mu m < \lambda < 0.4\mu m$) and VI ($0.4\mu m \leq \lambda < 0.75\mu m$) spectrum ranges outside the water vapour and carbon dioxide absorption bands (in Section 7.1 λ denotes a wavelength of radiation). The parameters of the SWR module in these spectrum ranges for clear sky conditions are optical thickness τ, an asymmetry factor of the phase function G, and a single scattering albedo ω_0. To obtain τ, G and ω_0 the vertical distribution of the effective scattering and absorption coefficients and asymmetry factors due to various aerosols (from [194]), the Rayleigh scattering (from [76]) and molecular absorption for the basic atmosphere gases (from [235]) are used as the input parameters in UF and VI bands.

For the cloudy atmosphere the input parameters of the SWR module in these spectral intervals are supplemented by the heights of the upper and lower boundaries of the cloud layers, as well as their optical thickness τ_c, single scattering albedo ω_c and asymmetry factor of phase function G_c for each cloud layer. These parameters of cloud water are assumed to be wavelength independent in UF and VI bands [195], and the absorption by cloud water is neglected [76]. The formulas for UF and VI values of τ_c, G_c and ω_c calculations are taken from [195], [208] and [190], respectively.

In the NI spectral band ($0.75\mu m \leq \lambda < 4\mu m$) the absorption by water vapour, carbon dioxide and water droplets and crystals (in clouds) is important. To calculate the upward and downward solar fluxes in the NI spectral range a combined delta-Eddington method is applied treating this absorption in terms of the integral transmission functions (ITFs) [229, 218]. The calculations of the fluxes for clear sky conditions are performed in the NI band in the approximation of the nonscattering atmosphere. The ITFs for water vapour are taken from [61], the ITFs for various aerosols are derived using the optical models proposed in [194]. The maximum number of three reflections among cloud layers and/or cloud layers and the underlying surface are allowed in the model based on the results obtained in [190]. The two–stage calculations of NI fluxes in the cloudy atmosphere are used. In the first stage the computations of the upward and downward fluxes at any level in the atmosphere are performed using the delta–Eddington method. In doing so both fluxes at any level are expanded into a sum of the partial fluxes which experience different numbers of reflections among the cloud layer(s) and the underlying surface. The geometric optics approximation is used in this procedure. The "diffusivity factor" $\beta_d = 1.66$ is taken into account to calculate the effective geometric pathlengths among clouds or cloud and underlying surface; the effective geometric pathlengths for transmission through and reflection from the cloud are calculated as functions of cloud optical thickness and the cosine of the zenith angle ξ [61], the latter being set equal to $1/\beta_d$ for all upward fluxes and for downward fluxes to the clouds (as well as to the underlying surface) located beneath the other one(s). For all other fluxes ξ is assigned the value of the solar zenith angle ξ_0. The formulas for optical thickness, albedo and transmission of clouds are taken from [208]. In the second stage each partial upward and downward flux at any level is multiplied by the ITF due to water vapour, cloud water/crystals, carbon dioxide and aerosols (from [61, 190, 194]) developed on the partial geometric pathlength corresponding to this flux. The SWR module contains 19 levels (12 in the troposphere and 7 in the stratosphere).

In POTSDAM a simplified version of the SWR module described above is implemented. Namely, the total SWR band is divided into two subintervals: UF+VI and NI. In the former, aerosols, ozone and nonabsorbing clouds are radiatively active substances, while in the latter water vapour, clouds and aerosols are taken into account. In both subintervals the geometric optics approximation mentioned above is employed. The ITF of ozone in the UF+VI band is represented by the parameterization taken from [118]. The albedo, transmission and absorption of the aerosol layer(s) in the clear sky (cloudy) atmosphere for the partial upward and downward fluxes of solar irradiance are treated in this spectral interval in terms of the zenith angle, the aerosol effective "opacities" along the partial pathlengths and the real and imaginary parts of the aerosol refractive indices following [243]. The albedo of a single effective cloud layer is described in the UF+VI band by the formulas derived in [166] as a function of the zenith angle, the β_d factor, the integral liquid content and of the cloud scattering coefficient. In the NI band the additional absorption (in terms of the corresponding ITFs) along each partial pathlength due to water vapour and liquid/crystal water in clouds is accounted for following [60].

In both the UF+VI and NI spectrum ranges the surface albedo is parameterized according to [47] for the land areas and according to [37] for the oceanic areas.

The effective optical pathlengths in the cloud layer for the transmission and reflection are taken to be equal to $l_T \approx 3\Delta h_c$ and $l_A \approx \Delta h_c$, respectively, where Δh_c is the geometric thickness of the cloud [61].

6.1.2 Longwave radiation

The computation of the longwave radiative (LWR) transfer in the model [168] is based on a traditional two–stream approximation (see, e.g., [58, 109, 245]). The total LWR range is divided into four bands $3.96\mu m < \lambda \leq 7.98\mu m$, $7.98\mu m < \lambda \leq 11.76\mu m$, $11.76\mu m < \lambda \leq 20.10\mu m$, and $20.10\mu m < \lambda \leq 100\mu m$. Water vapour and aerosols are taken into account in the first band; water vapour (with its dimers allowed for as is proposed in [185]), aerosols and ozone are considered to be radiatively significant constituents in the second one; water vapour, carbon dioxide, aerosols are accounted for in the third spectral region; and water vapour and aerosols are taken into account in the fourth band. Based on the Curtis–Godson approximation (see, e.g., [235]) the integral transmission function (ITF) of the i–th gas in the j–th spectral range for any (z_1, z_2) radiative atmospheric layer depends in the model on the effective absorber mass calculated by the following formula

$$M_{i,j}(z_1, z_2) = \int_{z_1}^{z_2} \rho_i(z) \left(\frac{p(z)}{p_{00}}\right)^{k_{ij}} \left(\frac{T(z)}{T_{00}}\right)^{l_{ij}} dz, \tag{6.1}$$

where T_{00} and p_{00} are the reference temperature and pressure, $\rho_i(z)$ is the partial density of the i–th gas mutiplied by β_d [186], and k_{ij} and l_{ij} are the specified parameters derived from [186, 58, 185, 54].

The ITFs for water vapour and carbon dioxide in the first, third and fourth bands (e.g., outside the $8 - 12 \mu m$ window) are taken from [147], while in the second spectral range the ITF treatment from [147] for water vapour is modified by the allowance for the water vapour dimers [79, 78, 177] based on the model of the dimer absorption coefficient described in [185]. The ITF for ozone is approximated in the model by a function of its effective absorber mass following [79].

The ITFs for various aerosols are derived in [77] using the models of spatial distributions and optical properties of aerosols given in [194]. To calculate the upward and downward longwave radiation fluxes at different levels as well as influxes to corresponding layers, the algorithm proposed in is used [78, 177]. The LWR module in [168] contains 19 atmospheric levels (12 in the troposphere and 7 in the stratosphere). The algorithm is developed in such a way that clouds can occupy an arbitrary number of radiative layers adjacent to each other or (widely) separated. The longwave radiative properties of water drops in each layer occupied by cloud are described in terms of the integral liquid water content in this layer and liquid water absorption coefficient dependent on the type of the clouds: low– and middle-level stratiform, cirrus, cumuli [36]. For the layers occupied by crystal or mixed liquid/crystal water (depending on the temperature of the layer) the LWR properties of liquid/crystal water mixture is described using the model proposed in [129].

When calculating the upward and downward LWR fluxes at any l–th radiative level the total ITF in each of four spectral intervals for an atmospheric layer confined between a given level and any other m–th level is represented by the product of the partial ITFs of radiatively active substances deposited between the l–th and m–th levels. Following [87] the surface LWR emissivity is treated in the model as a function of the vegetation/land–cover type (over land and sea ice) and surface air wind speed (over open water).

In POTSDAM a simplified algorithm of LWR calculations is employed*. LWR transfer is computed using two–stream approximation. Here we describe as an example the computation of the longwave radiative fluxes for the clear sky conditions. The upward LWR fluxes are computed in this case as follows

$$F_{\uparrow}(z) = B(z) + [B_s - B(0)]D(0, z) - \int_0^z D(z', z)\frac{dB(z')}{dz'}dz', \tag{6.2}$$

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* The algorithm is developed by A.V. Eliseev (A.M. Oboukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow)

where $B(z) = \sigma T^4(z), B_s = \epsilon_e \sigma T_g^4, \sigma$ — Stephan–Boltzmann constant, $T(z)$ — temperature at the level z, T_g — surface temperature, ϵ_e — surface LWR emissivity, $D(z, z')$ — ITF of the atmospheric layer confined between levels z and z'. Similarly, the downward LWR fluxes are computed according to

$$F_{\downarrow}(z) = B(z) - B(H)D(z, H) + \int_z^H D(z, z') \frac{dB(z')}{dz'} dz', \tag{6.3}$$

where H is the atmosphere upper boundary height.

The clear sky ITF is represented by

$$D = D_{vap} D_{CO_2} D_{O_3}, \tag{6.4}$$

where ITFs for water vapour, carbon dioxide and ozone are computed as follows

- Water vapour ITF [147]:

$$D_{vap} = \frac{1}{1 + A_{vap}(\beta_0 M_{vap})^{\beta_{vap}}}, \tag{6.5}$$

  where $M_{vap}[cm]$ — effective absorber mass for water vapour in a unit atmospheric column enclosed between levels z and z', $A_{vap} = 1.716, \beta_0 = 1.66, \beta_{vap} = 0.409$.

- Carbon dioxide ITF [168]:

$$D_{CO_2} = A_{0,CO_2} - A_{1,CO_2} \log M_{CO_2}, \tag{6.6}$$

  where $M_{CO_2}[cm]$ — effective absorber mass for carbon dioxide in a unit atmospheric column enclosed between levels z and z', $A_{0,CO_2} = 0.907$, $A_{1,CO_2} = 0.047$. This formula is in a good agreement (with an accuracy higher than $(3--5)\ -\%$) with observational data in a broad range of carbon dioxide content in the atmosphere, from $0.3cm$ up to 20 times as large as the present day one [168, 108]. Let us notice here that in the POTSDAM-2 version of the model an approximate formula is used for carbon dioxide ITF [147]:

$$D_{CO_2} = \frac{1 + A_{CO_2} A'_{CO_2} (\beta_0 M_{CO_2})^{\beta_{CO_2}}}{1 + A_{CO_2} (\beta_0 M_{CO_2})^{\beta_{CO_2}}}, \tag{6.7}$$

  where $A_{CO_2} = 0.247$, $A'_{CO_2} = 0.755$, $\beta_{CO_2} = 0.509$. This formula is valid within the mentioned accuracy in the $(0 \div 400)$ cm range of carbon dioxide effective absorber mass.

- Ozone ITF:

$$D_{O_3} = 1 - A_{O_3} M_{O_3}^{\beta_{O_3}}, \tag{6.8}$$

  where $M_{O_3}[g/cm^2]$ — effective absorber mass of ozone per unit area in an atmospheric column enclosed between levels z and z', $A_{O_3} = 13.8$, $\beta_{O_3} = 0.596$.

Water vapour and carbon dioxide effective absorber masses are computed in the LWR module of POTSDAM using (6.1), on the assumption that the vertical profiles are quasi-exponential for pressure and air density, as well as for water vapour. Ozone is assumed to occupy the stratosphere layer with prescribed vertical distribution taken from [22].

Both the oceanic and land surfraces, as well as the effective cloud layer are treated in POTSDAM as blackbody radiators in the thermal range of spectrum.

Total LWR fluxes in each model grid cell are computed as weighed sums of clear sky flux and the flux in the presence of clouds. To compute the LWR fluxes the total atmosphere column is subdivided in POTSDAM into 16 levels. The temperature and specific humidity at each level are computed using formulas (7.8) and (8.20), respectively.

Notice here that the incorporation in the SWR and LWR modules of the other radiatively active components of the atmosphere (methane, CFCs, etc) can be easily done in the framework of the developed approach. However, the atmosphere chemistry routine is still absent in POTSDAM. To this end, the inclusion of the most significant atmospheric chemistry processes in the troposphere and stratosphere modules is one of the necessary steps still to be performed to improve the model setup so as to account for the fundamental importance of chemistry reactions, e.g., in the lifecycle of ozone. This latter is recognized to be one of the basic atmospheric constituents which regulate the thermodynamical and dynamical processes in the Earth's climate system [39].

6.2 Large–scale condensation and precipitation

A model of the large-scale stratiform (stratus) cloudiness used in POTSDAM is proposed in [168]. This type of cloudiness is represented in [168] by three layers of clouds. The upper boundary h_{st}^l of the lower–layer stratus clouds is assumed to be located at the top of the PBL

$$h_{st}^l = Z_B \tag{6.9}$$

The lower boundary h_{st}^m of the middle–layer stratiform cloudiness is prescribed at the level in the free troposphere of an extremum for the total water vapour vertical flux F_{qz}

$$\partial_z F_{qz}|_{z=h_{st}^m} = 0 \tag{6.10}$$

The lower boundary h_{st}^h of cirrus clouds is specified in the model as the level in the free troposphere of an extremum of the total vertical mass flux F_{mz}

$$\partial_z F_{mz}|_{z=h_{st}^h} = 0 \tag{6.11}$$

These parameterizations of stratiform cloudiness boundary heights are based on corresponding empirical findings reviewed in [221, 130, 12, 58, 59] and are tested in [168] against the zonally averaged values of h_{st}^l, h_{st}^m and h_{st}^h reported in [221, 130, 58, 59].

Following the results obtained in [50] the lower (n_{st}^l), middle (n_{st}^m) and high (n_{st}^h) stratus cloud amounts are treated in the model in terms of the vertical velocities $w_{eff}\left(h_{st}^l\right)$, $w_{eff}\left(h_{st}^m\right)$ and $w_{eff}\left(h_{st}^h\right)$ developed at corresponding cloud bases, and the relative humidities $< f_r^l >$, $< f_r^m >$, $< f_r^h >$ which are the results of the vertical averaging of the relative humidity over the depths $\Delta p^l = \left(p\left(z_0\right), p\left(h_{st}^l + (\Delta z)_l\right)\right)$, $\Delta p^m = \left(p\left(h_{st}^l + (\Delta z)_l\right), p\left(h_{st}^m + (\Delta z)_m\right)\right)$ and $\Delta p^h = \left(p\left(h_{st}^m + (\Delta z)_m\right), p\left(H_{tr}\right)\right)$, respectively

$$n_{st}^x = \begin{cases} n_{st}^{x,0}, & \text{if } n_{st}^{x,0} \geq 0 \\ 0, & \text{if } n_{st}^{x,0} < 0 \end{cases} \tag{6.12}$$

where $n_{st}^{x,0} = \alpha_0^x + \beta_0^x < f_r^x >^2 [1 + \gamma_0^x \tanh\{\frac{w_{eff}(h_{st}^x)}{w_0}\}]$. In (6.12) $x = l, m, h$ and

$$w_{eff} = w_{nor} + K_w (w_{or,sd}^2)^{1/2} + K_w (w'^2)^{1/2}, \tag{6.13}$$

where $(w_{or,sd}^2)^{1/2}$ is that component of the total large-scale long-term vertical velocity w which attributes to standard deviation of explicitly unresolved random surface orography, while w_{nor} describes all other above-mentioned constituents of w (see Subsections 4.1.1 and 4.1.2). The factor $K_w \approx \frac{1}{4}$ in (6.13) takes into consideration that the predominant contribition to cloud formation from w' and $w_{or,sd}$ is provided within the periods of positive sign of these components, if the latter are thought of as sine-shaped in time processes with characteristic frequency and random amplitude and phase [168].

The terms $(\Delta z)_l$ and $(\Delta z)_m$ in (6.12) are height ranges of the span of the lower-level and mid-level stratus [206] specified in the model based on results obtained in [50, 206], w_0 is given by the eq. (6.17) and α_0^x, β_0^x, γ_0^x are numeric parameters for which the analytic formulas are derived in the model based on [51] findings, with additional allowing in POTSDAM for the effect exerted on these parameters by the number, composition and particle size distribution of CCN (cloud condensation nuclei) in correspondent layers [92, 202, 2, 226, 40, 112].

Based on the model of the hydrologic cycle in stratiform cloudiness developed in [166, 168] the geometric thickness of the lower (Δh_{st}^l), middle (Δh_{st}^m) and high (Δh_{st}^h) stratus clouds is described in POTSDAM by the following formula

$$\Delta h_{st}^x = C_0^x \Delta h_{st,0}^x \tanh\{\left(\frac{n_{st}^x}{n_{0,st}}\right)^{1/2}\} \tag{6.14}$$

Here $\Delta h_{st,0}^x$ is given by [168]

$$\Delta h_{st,0}^x = Ma_0 \sqrt{\frac{T_{00}}{T_0}} \frac{H_{00}}{\alpha_{0,d}} \left(\frac{h_{st}^x}{h_{st}^l}\right)^{c_{st}}, \tag{6.15}$$

where Ma_0 is a characteristic value of the Mach number for the atmosphere, $\alpha_{0,d} = [\frac{R}{g}(,\ _a - ,)]^{\frac{1}{2}}$, and $n_{0,st} \ll 1$ and $0 < c_{st} \ll 1$ are the dimensionless numeric parameters.

The dimensionless numeric parameters $C_0^x = O(1)$ in (6.14) depend on the number, composition and particle size distribution of CCN in correspondent layers.

Eq. (6.14) is deduced in [168] from the requirements (based on the results obtained, e.g., in [206, 12, 31, 214]) that the thickness of stratiform cloudiness correlates to a large degree with the characteristic vertical scales of small/mesoscale (at low levels) and synoptic scale (at higher levels) eddies/waves and that, at low cloud amounts, Δh_{st}^x is closely proportional to the square root of correspondent cloud amount n_{st}^x, while at moderate and high values of cloud amounts Δh_{st}^x depends (but only slightly) on n_{st}^x. The factor $(\frac{h_{st}^x}{h_{st}^l})^{c_{st}}$ in (6.15) describes the factor of the increase of the stratiform cloudiness characteristic lifetime with the increase of the cloud layer height.

In the framework of the same model of the hydrologic cycle in stratiform cloudiness proposed in [166, 168] the condensation rates (in mass per unut volume and unit time) in low (C_{st}^l), middle (C_{st}^m) and high (C_{st}^h) stratus clouds are described in POTSDAM as functions of relevant cloud amounts n_{st}^l, n_{st}^m and n_{st}^h, vertical velocities $w_{eff}\left(h_{st}^l\right)$, $w_{eff}(h_{st}^m)$ and $w_{eff}\left(h_{st}^h\right)$, geometric cloud thicknesses Δh_{st}^l, Δh_{st}^m and Δh_{st}^h, as well as the correspondent water vapour contents $Q_v^x = \rho q_v^x$

$$C_{st}^x = C_{cl}^x \tanh\{\frac{\Delta h_{st}^x}{(\Delta z)^x}\} \frac{w_{eff,SMT}\left(h_{st}^x\right)[1 + C_{cl,1}^x K_w^{\frac{1}{2}} \tanh\{\frac{w_{eff}\left(h_{st}^x\right)}{w_{eff,SMT}\left(h_{st}^x\right)}\}]}{H_{q,0}} n_{st}^x Q_v^x, \qquad (6.16)$$

where $C_{cl}^x = O(1)$ and $C_{cl,1}^x = O(1)$ are dimensionless numeric parameters which depend on the number, composition and particle size distribution of CCN in correspondent layers, $H_{q,0}$ is governed by the eq. (8.20) and $(\Delta z)^x$ are the geometric thicknesses of the Δp^x layers.

The term $w_{eff,SMT}\left(h_{st}^x\right)$ in (6.16) stands for the "effective" vertical velocity due to small/mesoscale turbulence

$$w_{eff,SMT} = w_0 = K_w \frac{K_z}{H_{q,0}} \qquad (6.17)$$

[166, 168], where K_z is given by the eq. (7.5).

With $w^{eff}\left(h_{st}^x\right) = C_{cl}^x \ \tanh\{\frac{\Delta h_{st}^x}{(\Delta z)^x}\} \ w_{eff,SMT}\left(h_{st}^x\right) \ [1 + C_{cl,1}^x K_w^{\frac{1}{2}} \tanh\{\frac{w_{eff}\left(h_{st}^x\right)}{w_{eff,SMT}\left(h_{st}^x\right)}\}]$ the eq. (6.16) can be rewritten as follows

$$C_{st}^x = n_{st}^x \frac{Q_v^x}{\tau_{st}^x}, \qquad (6.18)$$

where $\tau_{st}^x = \frac{H_{q,0}}{w^{eff}\left(h_{st}^x\right)}$ is the characteristic lifetime of water vapour in the stratiform cloudiness of the x-th layer. In this form the eq. (6.16) has a rather clear physical interpretation: the condensation rate in the x-th layer of stratiform cloudiness is determined by the rate of the "passage" of the water vapour total amount in the correspondent layer through the "cloud sink" the latter being regulated by the efficiency of the CCN in that layer.

The C_{st}^x terms give the expression for the q_{LSC}^v item (condensation rate in stratus clouds) in the water vapour balance equation in the limits of correspondent Δp^x layers. Multiplying C_{st}^x by $\frac{\mathcal{L}}{c_v}$ one can obtain the formula for the q_{LSC}^T term in the heat balance equation within the same layers. Integrations of C_{st}^x over the depths of Δp^x layers give the total condensation rates $C_{st,int}^x$ in these layers, and summation of the results of integrations provides the integral condensation rate in the atmosphere as a whole.

Eq. (6.16) produces also the explicit dependence of q_{LSC}^T and q_{LSC}^v on their arguments if all quantities in (6.16) are thought of as the instantaneous values of correspondent variables. This allows one to derive in an explicit form the expansions for q_{LSC}^T and q_{LSC}^v items in (3.29).

The total stratiform cloudiness precipitation rate P_r at the underlying surface is computed as a sum of $C_{st,int}^x$ which are modified by the dimensionless numeric factors $0 < C_{Pr,c}^x < 1$ dependent on CCN number, composition and particle-size distribution in correspondent layers [92, 202, 2, 226, 40, 112], as well as by rainfall evaporation below the cloud bases. The latter is parameterized in the model following the algorithm advocated in [196]. As a result, P_r in a given grid cell is written in the model as follows

$$P_r = (1 - C_{Pr,e}^l) C_{Pr,c}^l C_{st,int}^l + (1 - C_{Pr,e}^m) C_{Pr,c}^m C_{st,int}^m + (1 - C_{Pr,e}^h) C_{Pr,c}^h C_{st,int}^h, \qquad (6.19)$$

where the $C_{Pr,e}^x$ factor describes relative part of the precipitation developed at the cloud bases in correspondent cloud layers, which does not reach the underlying surface due to evaporation of rainfall. Under the procedure, the excess of the integral condesation rate over the primary precipitation (without allowing for the evaporation below the cloud bases) is removed from a given grid cell to adjacent ones, with the redestribution of the excess among the surrounding cells being dependent on the strengh and direction of wind over a given grid cell. The liquid water evaporated under the cloud bases is added to the given grid cell water vapour content [168].

Following [206, 12, 58, 11] the integral liquid water, crystal or mixed liquid water/crystal content in stratus clouds $M^x_{int,w}$ is governed in [166, 168] by

$$M^x_{int,w} = C_{mw}(C^x_0 \Delta h^x_{st,0})^2 \tanh\{(\frac{\Delta h^x_{st}}{C^x_0 \Delta h^x_{st,0}})^2\} \tanh\{\left(\frac{n^x_{st}}{n_{0,st}}\right)^{3/2}\} \frac{\exp\{-k_{wT}|T(h^x_{st}) - T_{M1}|\}}{T(h^x_{st})}, \tag{6.20}$$

where C_{mw}, k_{wT} and T_{M1} are dimensional numeric parameters.

Eq. (6.20) describes such features of the stratiform cloudiness UVS as high correlation of the integral liquid content with the geometrical thickness and cloud amount (raised to correspondent power) at low values of Δh^x_{st} and n^x_{st}, while at high values of Δh^x_{st} and n^x_{st} the correlation is weak [206, 12, 58]. In [11] it is argued that in different regions and periods of time $M^x_{int,w}$, as a function of temperature, has a maximum which is also taken into account in the eq. (6.20). This equation is used, in particular, in the radiative scheme to compute the radiative properties of stratus in the model. In doing so, the pure scattering albedo of stratiform clouds A^x_{st} in POTSDAM is a function of their optical thickness τ^x_{st} and of the cosine of the zenith angle ξ [166, 168]

$$A^x_{st} = A^x_{st}(\tau^x_{st}, \xi), \tag{6.21}$$

where τ^x_{st} is a function of $M^x_{int,w}$ and effective cloud droplet/crystal scattering coefficient σ^x_0 [166, 168]

$$\tau^x_{st} = \tau^x_{st}(\sigma^x_0, M^x_{int,w}), \tag{6.22}$$

and σ^x_0 depends, in particular, on the CCN number, composition and particle size distribution in the corresponding layer.

6.3 Cumulus convection

The early version of the cumulus convection scheme (CCS) is proposed in [168]*. The scheme treats ensembles of penetrative convection (PC), shallow convection (SC) and all large–scale climatic variables and synoptic moments for a given time step and given grid cell are the CCS inputs. The ensembles of cumuli are assumed to be in an equilibrium with the grid-cell LSLTC and SM and are described by diagnostic formulas. Clouds in cumuli ensembles are represented by a bulk cloud model. To compute the contribution from cumulus convection to the large–scale budgets of heat, moisture and momentum, the parcel method is applied. In the PC scheme the height of the base of the virtual cumulus clouds in any grid cell is defined as the condensation level z_c for the adiabatically ascending from $z = z_0$ parcel of the surface air. If z_c is located above the PBL, the CCS for PC does not switch on in a given grid cell in the model. If z_c level is reached inside the PBL, i.e.

$$z_c \leq Z_B, \tag{6.23}$$

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* The modified version of the CCS presented in the paper is developed by V.A. Semenov (A.M. Oboukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow)

the condition

$$w_{eff}(z_c) > 0, \tag{6.24}$$

is then checked. If (6.24) does not hold, CCS for PC does not switch on. Otherwise, the condition of large–scale low–level water vapour convergence in a given grid cell for the (z_0, Z_B) layer is checked [168]

$$C_{qv} \equiv -\int_{z_0}^{Z_B} \left[\vec{v}_H \cdot \nabla_H q_v + \nabla \cdot \vec{v}' q_v' + w \frac{\partial q_v}{\partial z} + + \frac{1}{\rho_0} q^v_{SMT} \right] \rho_0 \, dz > 0 \tag{6.25}$$

If the conditions (6.23)– (6.25) fulfil the level of buoyancy (free convection) z_{BU} for the surface air is defined at which the dry adiabatically (beneath the z_c level) and moist adiabatically (above z_c) ascending surface air parcel develops the temperature of the environmental air $T\,(z_{BU})$. Again, if $z_{BU} > Z_B$ the CCS for PC is not initialized. If the free convection level z_{BU} is lower than or equal to

$$z_{BU} \leq Z_B \tag{6.26}$$

the additional condition is applied

$$\left. \frac{\partial \Theta^*}{\partial z} \right|_{z_0 < z < z_{BU}} \equiv \left. \left(\frac{\partial T}{\partial z} + \Gamma_a - \Gamma_{c,l} \right) \right|_{z_0 < z < z_{BU}} < 0, \tag{6.27}$$

where $\Gamma_a = g/c_p$ and $\Gamma_{c,l}$ is the low-level counter–gradient factor. Two formulas are used in the model for $\Gamma_{c,l}$ description which bring one to close results [110, 166]

$$\Gamma_{c,l} = -0.61 T \partial q_v / \partial z \tag{6.28}$$

or

$$\Gamma_{c,l} = (\Gamma_a - \Gamma_{wa})\, n^l_{st} \tag{6.29}$$

Here Γ_{wa}–moist adiabatic lapse rate at $T = T_a = T_0 - \Gamma z_0$ and $p = p_0$.

The condition (6.27), (6.28) is checked in the surface layer assuming continuity of the vertical water vapour flux at the SL/PBL interface

$$\frac{\partial q_v}{\partial z} = \frac{E}{k \rho_{00} z_{sl} v_{fr}}, \tag{6.30}$$

where E is the evaporation/evapotranspiration from the surface, k - the von Karman constant, v_{fr} - the friction velocity ($v_{fr} = (\tau_s/\rho_{00})^{1/2}$).

The expressions (6.27),(6.28) and (6.27),(6.29) describe (in terms of different variables) the condition of the Bernard convection occurence, in the moist air. This convection is one of the probable mechanisms of the surface air parcels penetration through the "buffer layers" (z_0, z_c) and (z_c, z_{BU}) inside the PBL (see, e.g., [110]). If (6.28) and (6.29) do not hold, the condition of the Faller–Kaylor convection [57] is tested

$$Re > Re_c, \tag{6.31}$$

where Re and Re_c are, respectively, the Reynolds and critical Reynolds numbers for the PBL [57]

$$Re = \frac{V_g}{f Z_B},$$

$$Re_c = const$$

This dynamically caused convection can also exert the "buffer layer" penetration effect [57]. If the conditions (6.23)—(6.27) (or (6.31)) are met the CCS for PC is switched on in a given grid cell. In the case of superadiabatic or neutral stratification of the PBL in the model the buoyancy level coincides with z_0 and (6.26) is fulfilled. In this situation, the (z_0, z_{BU}) "buffer layer" for the development of PC does not exist and only (6.23)– (6.25) conditions are checked.

The CCS scheme for SC differs from the CCS for PC described above only by the second condition (see eq. (6.24)). Namely, in the CCS for SC the condition (6.24) is replaced by

$$w_{eff}(z_c) \leq 0, \tag{6.32}$$

If the sets of the above mentioned conditions for PC and SC do not hold in a given grid cell, the possibility of MC occurence is evaluated. Since this type of moist convection usually occupies levels above the PBL [24, 102, 95] the following procedure is applied in the model. The parcels of the environmental air located above z_0 are lifted (beginning with the lowest z_0 level) adiabatically (condensation heating is taken into account above the corresponding condensation level z_c), with checking for buoyancy. If for any parcel the buoyancy becomes positive at any z_{BU} level inside the troposphere

$$T_{u,v} \geq T_v \tag{6.33}$$

(where $T_{u,v}$ and T_v are correspondingly the virtual temperature of the ascending parcel and the environmental air), the respective z_c is defined as the cloud base of the virtual mid–level convection cumulus. The second condition for MC is

$$w_{eff}(z_c) > 0, \tag{6.34}$$

In the CCS for MC it is implicitly assumed that some mechanisms occur regarding the fast transport of the air parcels through the "buffer layer" (z_{in}, z_{BU}) (where z_{in} is the height of the initial position of the air parcel) which compensate negative buoyancy in the mentioned layer. The probable mechanisms are those of mesoscale circulations, e.g., wave–CISK [55] mechanism and/or conditionally symmetric instability [13, 14]. To take these two mechanisms into account in an explicit way the characteristic vertical mixing lengths l_{wc}^m, l_{si}^m (as functions of the large–scale environmental parameters) for these types of mesoscale circulation are specified in the model based on [55] and [13, 14], respectively. In this case the additional necessary condition for MC is implied

$$\Delta z_{ib} < max\left(l_{wc}^m, l_{si}^m\right), \tag{6.35}$$

where Δz_{ib} is the thickness of the (z_{in}, z_{BU}) layer. If (6.33)—(6.35) are obeyed the CCS for MC is initiated in a given grid cell.

Neglecting the downdrafts in cumuli interior and assuming that all the thermo- hydrodynamic characteristics of the environmental air are close to those averaged for the grid cell, the contributions from cumulus convection to the large–scale budgets of heat, moisture, zonal and meridional components of momentum are calculated in POTSDAM as follows [224, 168]

$$q_{MC}^T + q_{SMT}^{T,c} = -\frac{1}{c_v}\left\{\partial_z\left[M_u\left(S_u - S\right)\right] - \mathcal{L}C_u\right\}, \tag{6.36}$$

$$q^v_{MC} + q^{v,c}_{SMT} = -\partial_z \left[M_u \left(q_u - q_v \right) \right] - C_u \tag{6.37}$$

$$F^c_\lambda = -\partial_z \left[M_u \left(u_u - u \right) \right] \tag{6.38}$$

$$F^c_\phi = -\partial_z \left[M_u \left(v_u - v \right) \right] \tag{6.39}$$

Here $q^T_{MC} + q^{T,c}_{SMT}$, $q^v_{MC} + q^{v,c}_{SMT}$, F^c_λ and F^c_ϕ are, correspondingly, the partial influxes of heat (divided by c_v), moisture, zonal and meridional momentum per unit volume due to cumulus convection condensation and mixing, M_u – upward mass flux due to cumulus convection, $S = c_pT + gz$ —the grid–cell averaged dry static energy (per unit mass), $S_u = c_pT_u + gz$, T_u, q_u, u_u and v_u are, respectively, the dry static energy, temperature, specific humidity, zonal and meridional velocities inside the cumuli, C_u — condensation rate (per unit volume) due to cumuli.

The vertical profiles of the mass M_u, heat M_us_u and moisture M_uq_u fluxes are [244, 224]

$$\frac{\partial M_u}{\partial z} = E_u - D_u \tag{6.40}$$

$$\frac{\partial M_u S_u}{\partial z} = E_u S - D_u S_u + \mathcal{L}\rho c_u \tag{6.41}$$

$$\frac{\partial M_u q_u}{\partial z} = E_u q_v - D_u q_u - \rho c_u \tag{6.42}$$

Here E_u and D_u denote, respectively, mass entrainment to and detrainment from cumuli, c_u - condensation rate (per unit mass) in cumuli, so that $C_u = \rho c_u$. All water condensed in the cumulus clouds is assumed to fall out as cumuli precipitation, and the latter is then modified by taking into account the evaporation of the rainfall below the cloud base following [196]. Under this procedure the evaporated liquid water is added to the grid-cell water vapour q_v.

It is assumed that from the cloud base $z_c \equiv h^B_{cu}$ up to the zero-buoyancy level $z_{BU,0}$ which is considered to be a cloud top level, $z_{BU,0} \equiv h^u_{cu}$, the mass entrainment/detrainment exchange between cloud and enviroment occurs only as turbulent exchange through cloud edges and

$$E_u = D_u = \lambda_e M_u \tag{6.43}$$

[224], where λ_e is the entrainment coefficient assumed to depend inversely on the representative cumuli cloud radius r_{cu} [204]

$$\lambda_e = const/r_{cu} \tag{6.44}$$

In [168] a parameterisation for r_{cu} is proposed using the continuity equation for cumulus cloud interior. Herein the suppositions are applied of a linear relationship among w_u (where w_u is the vertical velocity inside the cumuli), r_{cu} and h^u_{cu} for low values of w_u (based on the results obtained in [6, 7]) and the existance (at high w_u) of maximum possible values for vertical $h^u_{cu,m}$ and horizontal $r_{cu,m}$ cumulus cloud sizes. The former is assumed to be equal to the tropopause height H_{tr}, due to inverse stratification of the low stratosphere which makes this layer impenetranable for cumulus convection. The latter is assigned the PBL depth h_B, based on [157, 158] and [188] models of the ascending air plumes involved into PC/SC and MC processes.

Notice that in POTSDAM r_{cu} is assigned a constant value, so that by virtue of (6.44) λ_e is a disposable numeric parameter in this model.

On the strengh of (6.40), (6.43) for $h_{cu}^B < z < h_{cu}^u$ one can write

$$\frac{\partial M_u}{\partial z} = E_u - D_u \equiv 0 \tag{6.45}$$

In the POTSDAM convection scheme an organized detrainment at the cloud top is neglected, and it is assumed that all moisture converged in the PBL C_{qv} is injected into the clouds through their base. Thus, imposing a moisture balance for the PBL and neglecting downdraft mass fluxes in cumuli one can obtain [224]

$$M_u q_u|_{h_{cu}^B} = C_{qv}, \tag{6.46}$$

which gives

$$M_u|_{h_{cu}^B} = \frac{C_{qv}}{q_{vs}}, \tag{6.47}$$

where q_{vs} is the surface air specific humidity.

The described scheme allows one to avoid computation of the organized entrainment into the cloud within $h_{cu}^B < z < h_{cu}^u$ layer [224] and gives the constant value of $M_u = M_u|_{h_{cu}^B}$ from the cloud base up to the cloud top.

Using (6.45) and the Clausius–Clapeyron equation for q_u the eqs. (6.41), (6.42) can be rewritten as follows

$$M_u(c_p \frac{\partial T_u}{\partial z} + g) = \lambda_e M_u c_p (T - T_u) + \mathcal{L}\rho c_u \tag{6.48}$$

$$M_u \frac{\partial q_u}{\partial z} = \lambda_e M_u (q_v - q_u) - \rho c_u, \tag{6.49}$$

where

$$\frac{\partial q_u}{\partial z} = q_u (\frac{\mathcal{L}}{R_v T_u^2} \frac{\partial T_u}{\partial z} + 1/H_{00}) \tag{6.50}$$

Here R_v - the gas constant for the water vapour. The system (6.48) — (6.50) allows one to determine three variables T_u, q_u and c_u provided the grid-cell air charachteristics are known. This system is integrated from h_{cu}^B to h_{cu}^u, with the following lower boundary conditions at $z = z_c \equiv h_{cu}^B$: mass flux given by (6.47), $q_u = q_{v,s}$, T_u equal to the temperature of the adiabatically ascending to the level z_c surface air parcel, and $c_u = 0$ below h_{cu}^B. Cloud top level is determined as the level where the virtual temperature in the cloud is lower than the virtual temperature of the enviromental air: $T_{u,v} \leq T_v$. From the system (6.48) — (6.50) one can obtain

$$\frac{\partial T_u}{\partial z} = \left[\lambda_e \mathcal{L}(q_v - q_u) + \lambda_e c_p (T - T_u) - g - \frac{\mathcal{L}}{H_{00}} q_u\right] / \left(c_p + \frac{\mathcal{L}^2 q_u}{R_v T_u^2}\right) \tag{6.51}$$

This equation is used for the computation of the temperature profile in the cloud. Then by the usage of the third equation of the system (6.48)–(6.50) one can compute q_u and, finally, determine c_u using any of the two first equations of this system.

As a result, the following expressions can be written for $q_{MC}^T + q_{SMT}^{T,c}$ and $q_{MC}^v + q_{SMT}^{v,c}$ (see eqs. (6.36), (6.37)) which are used in POTSDAM

$$q_{MC}^T + q_{SMT}^{T,c} = -\frac{M_u}{c_v} \{\lambda_e c_p (T - T_u) - c_p, \ - g\} \tag{6.52}$$

$$q_{MC}^v + q_{SMT}^{v,c} = -M_u \left(\lambda_e (q_v - q_u) - \frac{\partial q_v}{\partial z}\right) \tag{6.53}$$

The eqs. (6.52) and (6.53) give also the explicit dependence of the terms $q^T_{MC} + q^{T,c}_{SMT}$ and $q^v_{MC} + q^{v,c}_{SMT}$ on their arguments provided all quantities in (6.52), (6.53) are understood as the instantaneous values of the corresponding variables. This makes it possible to obtain the explicit expansions of these terms on their arguments in the eq. (3.29).

As mentioned above, all condensed water ($\int_{h^B_{cu}}^{h^u_{cu}} \rho c_u \, dz \equiv C_{t,cu}$) falls out as precipitation, the evaporation of the rainfall beneath the cloud base being taken into account.

With entrainment taken into account, the vertical distribution of the horizontal velocities inside cumuli is governed by [224, 168]

$$\partial_z(M_u u_u) = \lambda_e M_u(u - u_u) + (u - u_u)\partial_z M_u \tag{6.54}$$

$$\partial_z(M_u v_u) = \lambda_e M_u(v - v_u) + (v - v_u)\partial_z M_u \tag{6.55}$$

Combining eqs. (6.54), (6.55) with eqs. (6.38), (6.39) allows one to compute the contribution of cumuli to the zonal F^c_λ and meridional F^c_ϕ components of the large-scale momentum. Notice that in POTSDAM the F^c_λ and F^c_ϕ terms are computed using a simplified description of u_u and v_u in (6.38) and (6.39). Namely, u_u and v_u are assigned in (6.38) and (6.39) at any z level in the $h^B_{cu} < z < h^u_{cu}$ range of heights the values of the large-scale long-term u and v at the cloud base $z = h^B_{cu}$ (see, e.g., [184]). Taking into account eq. (6.45) the terms F^c_λ and F^c_ϕ can be written as follows

$$F^c_\lambda = M_u|_{z=h^B_{cu}} \, \partial_z u \tag{6.56}$$

$$F^c_\phi = M_u|_{z=h^B_{cu}} \, \partial_z v \tag{6.57}$$

In [168] an additional equation is used to describe the liquid/crystal water budget in cumulus clouds. In POTSDAM a simple assumption is adopted that the liquid/crystal water content $W(z)$ (in mass per unit volume) in cumuli is equal to

$$W(z) = C_{cu}\tau_{cu} \tag{6.58}$$

Here $C_{cu} = \frac{\rho c_u}{n_{cu}}$ is the condensation rate in unit volume of cumuli interior at a given z, and τ_{cu} is the cumuli characteristic life cycle time evaluated as

$$\tau_{cu} = C_{0,cu}\frac{h^u_{cu} - h^B_{cu}}{w_u|_{z=z_{cu}}}, \tag{6.59}$$

where $C_{0,cu} = O(1)$ and the representative vertical velocity in the cumulus interior $w_u|_{z=z_{cu}}$ is calculated based on the buoyancy equation for the vertical velocity in cumulus cloud derived in [6, 7]

$$w_u|_{z=z_{cu}} = C_{w,cu}\sqrt{g\frac{T_{u,v} - T_v}{T_v}|_{max}\frac{1}{1+\beta_{cu}}(h^B_{cu} + \frac{h^u_{cu} - h^B_{cu}}{3})} \tag{6.60}$$

where $\frac{T_{u,v}-T_v}{T_v}|_{max}$ is the value of $\frac{T_{u,v}-T_v}{T_v}$ at that level inside cumuli where it is a maximum, $C_{w,cu} \approx 0.01$ and $\beta_{cu} \approx 0.5$ [6, 7].

The cumulus amount n_{cu} is computed as

$$C_{qv}\tau_{cu} = n_{cu}\int_{h^B_{cu}}^{h^u_{cu}} \rho q_u \, dz \tag{6.61}$$

The eq. (6.61) is the necessary condition for the maintanence of the cumuli ensemble in the stationary state, with the lower and upper boundaries of clouds at h^B_{cu} and h^u_{cu} levels, respectively.

6.4 Small/mesoscale turbulence

Small/mesoscale vertical "turbulent" friction in the PBL for clear-sky conditions, in the absence of penetrating and shallow cumulus convection and disregarding of the gravity wave drag, is described in POTSDAM in terms of the vertical diffusion coefficient $\overline{\overline{K}}_{Dz}$ (see eq. (4.99)).

The stratiform cloudiness taken into account the effective vertical diffusion coefficient $\overline{\overline{K}}_{Dz,st}$ in the PBL yields

$$\overline{\overline{K}}_{Dz,st} = (1 - V_{st})\overline{\overline{K}}_{Dz} + V_{st}\overline{\overline{K}}_{Dz,cld}, \tag{6.62}$$

where $\overline{\overline{K}}_{Dz}$ is given by the eq. (4.99), the $\overline{\overline{K}}_{Dz,cld}$ term is the (prescribed) vertical diffusion coefficient in stratiform cloudiness, and V_{st} is the relative volume occupied by stratiform cloudiness in the PBL.

In the presence of cumulus clouds inside the PBL, the additional terms described by the eqs. (6.56) and (6.57) are introduced in the right-hand sides of (4.75) and (4.76), respectively. As a result, the terms $< u_g >$ and $< v_g >$ in the eqs.(4.77) and (4.78) (as well as in all other formulas in Section 4.1.2 including the $< u_g >$ and $< v_g >$ terms) are replaced in POTSDAM in the case of cumulus convection inside the PBL, respectively, by $< u_g >_c$ and $< v_g >_c$, where

$$< u_g >_c = < u_g > + \frac{M_u|_{z=h^B{cu}} < v_g >}{h_B f \rho_{00}} \tag{6.63}$$

$$< v_g >_c = < v_g > - \frac{M_u|_{z=h^B{cu}} < u_g >}{h_B f \rho_{00}} \tag{6.64}$$

In particular, the expressions for $\overline{\overline{K}}_{Dz}$ from [87] which are used in POTSDAM include $< u_g >_c$ and $< v_g >_c$ terms and hence, $\overline{\overline{K}}_{Dz}$ is also modified by the cumulus convection if the latter takes place in the PBL.

Free troposphere $\overline{\overline{K}}_{Dz,st}$ in the absence of cumulus convection is given in POTSDAM by the equation

$$\overline{\overline{K}}_{Dz,st} = (1 - V_{st})\overline{\overline{K}}_{Dz} + V_{st}\overline{\overline{K}}_{Dz,cld}, \tag{6.65}$$

in which $\overline{\overline{K}}_{Dz}$ is assumed to be constant with height, the characteristic value being estimated from [174, 175]. The V_{st} term in (6.65) denotes the relative volume occupied by the free troposphere stratiform cloudiness in the layer below a given level.

When penetrative, shallow or middle-level cumulus convection takes place in a grid cell in the free troposphere the terms F^c_λ and F^c_ϕ (described, correspondingly, by the eqs. (6.56) and (6.57)) appear in the right-hand sides of the equations (4.35) and (4.36), respectively, while the term $\overline{F^c_\lambda}$ is accounted for in the RHS of the eq. (4.37).

As a rough approximation, the F^c_λ and F^c_ϕ terms can be written in the free troposphere in terms of the vertical diffusion coefficient $\overline{\overline{K}}_{Dz,cu}$. Namely, representing F^c_λ and F^c_ϕ in the conventional form

$$F^c_\lambda = \nabla_z \overline{\overline{K}}_{Dz,cu} \rho \nabla_z u \tag{6.66}$$

$$F^c_\phi = \nabla_z \overline{\overline{K}}_{Dz,cu} \rho \nabla_z v, \tag{6.67}$$

assuming the quasi-linear profiles of u and v in this part of the atmosphere, neglecting a change of ρ in the $Z_B < z < h^u_{cu}$ range of heights and comparing the eqs. (6.56), (6.57) with the eqs. (6.66), (6.67) one can write

$$\nabla_z \overline{\overline{K}}_{Dz,cu} = \frac{M_u}{\rho}|_{z=h^B_{cu}} \tag{6.68}$$

Given n_{cu} in the PBL and the lower boundary condition for $\overline{\overline{K}}_{Dz,cu}$ obtained with the usage of the value of $\overline{\overline{K}}_{Dz}$ as modified by cumulus convection, the equation (6.68) completely determines $\overline{\overline{K}}_{Dz,cu}$ in the free troposphere.

As a result, the effective vertical diffusion coefficient $\overline{\overline{K}}_{Dz,eff}$ for the case of the occurence of the cumulus convection in a given grid cell can be written as follows

$$\overline{\overline{K}}_{Dz,eff} = (1 - n_{cu})\overline{\overline{K}}_{Dz,st} + n_{cu}\overline{\overline{K}}_{Dz,cu} \tag{6.69}$$

Hence, by virtue of the eqs. (5.29) and (5.30) the $F_{y'_j,y'_k}$ terms in the POTSDAM equations for the synoptic second moments in the free troposphere are also modified in the presence of cumuli convection. The cumulus convection also influences the SM in the free troposphere via LSLTC entering the equations for the SM, and via additional condensation and vertical velocity in cumuli (see eqs. (5.12), (5.13), (5.15)–(5.17), (5.20)). Finally, since the SM at the lower (upper) boundary of the free troposphere are used in POTSDAM as the upper (lower) boundary condition for the SM in the PBL (stratosphere) all the synoptic moments $y'_i y'_j$ in the total volume of the model grid cell are modified in the presence of the cumulus convection.

The effect of the sub–grid scale gravity waves on the momentum vertical transports in the PBL and free atmosphere is taken into consideration in the model using the parameterization scheme proposed in [143]. In this case additional items enter the RHS of the eqs. (4.35), (4.36), (4.37), (4.74), (4.75) – (4.81) which are attributed to gravity wave drag effect, namely, the terms

$$F^{gw}_\lambda = \frac{\partial \tau^{gw}_\lambda}{\partial z} \tag{6.70}$$

and

$$F^{gw}_\phi = \frac{\partial \tau^{gw}_\phi}{\partial z} \tag{6.71}$$

Here τ^{gw}_λ and τ^{gw}_ϕ are the λ– and ϕ–components of gravity wave stress (GWS) τ^{gw} which in turn can be represented by the sum

$$\tau^{gw} = \tau^w(z) + \tau^{Fr}(z) \tag{6.72}$$

The first term on the RHS of this equation describes a part of the GWS in the PBL and free atmosphere dependent on the pressure. This term is a function of $< u >$, $< v >$, V_s, as well as ρ, the PBL Brunt–Vaisala frequency N_s, the Froude number $Fr_s = N_s\ f^{1/2}_{1,s}/V_s$ in the PBL (where $f_{1,s}$ is a directionally dependent sub–grid scale orographic variance [187]), orographic anisotropy function $f_{2,s}$ measuring the two–dimensionality of the sub–grid scale orography [187], vertical distributions of a local wave Richardson number $Ri^w(z)$ [187] and pressure $p(z)$.

The term $\tau^{Fr}(z)$ represents the additional drag which occurs when the PBL flow is supercritical and the dynamical mechanism of resonant trapping of the gravity waves takes place leading to high-drag situations (see, e.g., [162]). This term depends on the same variables as the previous term, as well as on $Fr_{s,cr}$ — the PBL Froude number critical value, the Brunt–Vaisala frequency vertical profile $N(z)$, hydrostatic vertical gravity wave length L^{gw} and pressure p at $z = \frac{3}{4}L^{gw}$ [187]. In this, the vertical structure of a local gravity wave Richardson number $Ri^w(z)$ is calculated in [187] to describe the onset of "turbulence" due to the gravity waves which become convectively unstable [125] or encounter critical layers.

The horizontal small/mesoscale "turbulent" exchange in the model is described in terms of horizontal coefficients of diffusion for heat $\overline{\overline{K}}_H$ moisture $\overline{\overline{K}}_{vH}$, and horizontal of viscosity $\overline{\overline{K}}_{DH}$. In POTSDAM they are assigned constant values in the PBL and free atmosphere.

As already mentioned in Section 4.1.1, to describe the nonzonal components of zonal u^* and meridional v^* winds in the stratosphere the reduced nonzonal parts of the model initial equations (3.15) for $y_3 = u$ and $y_4 = v$ are used in the Charney–Drazin approximation [33] modified by introducing gravity–wave drag, as well as Rayleigh friction and planetary–wave breaking mechanisms based on [66, 67, 68, 69, 70] findings. Under this procedure, the Rayleigh friction is described by

$$F_\lambda^R = -K_R \; u^* \tag{6.73}$$

$$F_\phi^R = -K_R \; v^*, \tag{6.74}$$

where K_R is a constant Rayliegh friction coefficient. These terms are added to the RHS of the equations for u^* and v^* in the stratosphere written in the Charney–Drazin approximation [33].

The effect of planetary–wave breaking is parameterized in terms of a dissipation rate and is based on the suggestion that wave breaking can be viewed as an equilibrium between the flux of wave energy into the wave breaking region and its dissipation by nonlinear interactions [66]. The wave breaking region is identified by a breaking criterion [66, 67]

$$Br = \frac{|\partial_\phi q^*|}{|\partial_\phi \overline{q}|} \geq 1, \tag{6.75}$$

where q is the potential vorticity. Eq. (6.75) is a necessary condition for barotropic instability [35] which is advocated in [91] as the main mechanism of the development of the planetary Rossby-wave critical layers. In [168] the expansion of the u^* and v^* fields at the tropopause in spherical harmonic series is performed, with truncation at the scales of the spatial resolution of the model, and the condition (6.75) is then checked for all harmonics in each stratospheric layer resolved by the model.

Chapter 7

Towards POTSDAM-3

In this Chapter, with the use of the UVS of climatic variables discussed in the previous Chapters, the governing SDE of the POTSDAM model are derived in the most general setup by vertical integration of the above set of the basic SDE. These governing SDE are envisaged for implementation in the next generation of the model, POTSDAM-3. The next Chapter is concerned with the simplified version, POTSDAM-2, of the governing SDE developed below in this Chapter. POTSDAM-2 is already employed as the atmospheric module in the CLIMBER-2 climate model of intermediate complexity.

As already mentioned, the vertical integration of the basic SDE using UVS allows one, first, to significantly simplify the calculations in the model, and second, to have a fast turnaround time due to large time step that we may apply in the numerics without violation of the CFL criterion.

To obtain the POTSDAM-3 governing equations for the tropospheric temperature (specific humidity), the heat (water vapour) balance equations (4.56), ((4.69)) and (4.104), ((4.112)) are integrated in the model over z in the limits $z_B < z < H_{tr}$ and $z_0 < z < Z_B$, respectively, and then summarized, which gives the model's governing prognostic equations for temperature (water vapour content) in the total troposphere.

At the mentioned vertical integration, the temperature lapse rate , in the free troposphere is parameterized on the assumption that this atmospheric layer is well mixed in vertical direction analogous to SL, but with the synoptic scale eddies instead of small/mesoscale turbulence as the basic agent of the vertical thermal mixing in the free troposphere at the considered $(\Delta L, \Delta \tau)$ spatial/temporal scales [168]. Following this premise the vertical flux $T'w'$ governed by the eq.(5.57) is assigned a value in POTSDAM-3 nearly constant with z in the free troposphere

$$\partial_z \; T'w' \approx 0 \tag{7.1}$$

Then substitution of the eqs. (5.20), (5.26), (4.9), (4.10), (5.21), (5.47), (5.48), (5.59), (5.60), (4.38), (4.39), (4.42), (4.43), (4.35), (4.36) and (5.49) in (7.1) results in the term $\partial_z T$ which is nearly constant with z within the free troposphere at any latitude and longitude. This means that the condition (7.1) implies a constant with height value of the lapse rate , in the free troposphere. To parameterize , we employ such a feature of the UVS of the lower free atmosphere as the existence, in the vicinity of the PBL/free troposphere interface, of a range of heights

$$z_v = C_{zv} h_B \tag{7.2}$$

(where $C_{zv} = O(1)$) with very low values (nearing zero) of the total turbulent (small/mesoscale + macroturbulent) vertical heat flux, which is typical for a variety of regions and periods of time at $(\Delta L, \Delta\tau)$ spatial/temporal scale (see, e.g., [27, 28, 123, 29, 5, 222, 113, 122, 115, 103])

$$[c_v T' w' + \frac{1}{\rho} F^T_{SMT,z}]_{z_v} \approx 0, \tag{7.3}$$

where $F^T_{SMT,z}$ is the z-component of the vector of small/mesoscale turbulent heat flux that is equal to the (multiplied by c_v) $\vec{F}^T_{SMT}$ given by the eqs. (2.5) and (2.6). We next substitute

$$,_c|_{z_v} = [(,_a - ,_{wa})(n^l_{st} + n^l_{cu})]_{z_v} \tag{7.4}$$

and

$$K_z = K_{z,eff}|_{z_v} = [K_{z,d}(1 - n^l_{cu}) + K_{z,cu} n^l_{cu}]_{z_v} \tag{7.5}$$

in $F^T_{SMT,z}$, and account for (5.57) in (7.3). In (7.4), (7.5) n^l_{cu} is the penetrative cumulus convection cloud amount, and $K_{z,d}$ is the vertical small/mesoscale diffusion coefficient for heat in the absence of cumulus convection

$$K_{z,d} = \frac{c_v}{c_p} \overline{\overline{K}}_{Dz,st} \tag{7.6}$$

at $z = z_v$, where $\overline{\overline{K}}_{Dz,st}$ yields the eq.(6.65). The $K_{z,cu}$ term in (7.5) is equal to $\frac{c_v}{c_p}\overline{\overline{K}}_{Dz,cu}$, where $\overline{\overline{K}}_{Dz,cu}$ is described by the eq. (6.68) at $z = z_v$. As a result, the equation (7.3) can be rewritten as follows

$$c_v[\frac{\rho w'^2}{\nabla_z \rho' w'} \partial_z T]_{z_v} - c_p[K_{z,d}(1 - n^l_{cu}) + K_{z,cu} n^l_{cu}]_{z_v} [\partial_z T + ,_a - (,_a - ,_{wa})\,(n^l_{st} + n^l_{cu})]_{z_v} = 0 \tag{7.7}$$

and from (7.7) the temperature lapse rate $, \equiv -\partial_z T$ in the free troposphere is [166, 168]

$$, = \frac{1}{K_{sy}|_{z_v} \;/\; \overline{\overline{K_{z,eff}}}|_{z_v} + 1} \left[,_a(1 - n^l_{st} - n^l_{cu}) + ,_{wa}(n^l_{st} + n^l_{cu})\right]_{z_v}, \tag{7.8}$$

where

$$\overline{\overline{K_{z,eff}}} = (c_p/c_v) K_{z,eff}|_{z_v} \tag{7.9}$$

$$K_{sy}|_{z_v} = \frac{\rho w'^2}{\nabla_z \rho' w'}|_{z_v} \tag{7.10}$$

Eq. (7.8) gives values for $,$ which are in a good agreement with the observational data analysed in [145]. These latter can be represented with the formula

$$, = ,_{00} + \frac{\partial ,}{\partial T_0}|_{T_M}\,(T_0 - T_M)\,, \tag{7.11}$$

where $,_{00}$ and T_M are constants.

Under the above-mentioned vertical integration of the heat balance equation, the vertical distribution of temperature in the bulk of the PBL above the SL is assigned in POTSDAM-3 a linear profile, with the lapse rate equal to $(T(Z_B) - T_0)/h_B$.

In the procedure of derivation of the POTSDAM-3 governing SDE for temperature in the stratospheric layers, the vertical temperature profiles are assumed to be quasi–linear in these layers. The resultant governing SDE are written in terms of the vertical temperature gradients in these layers.

The water vapour vertical distribution in the PBL which is used in the procedure of the deriving POTSDAM-3 governing SDE for the specific humidity is parameterized following the algorithm developed in [166, 168]. In the mentioned papers, the characteristic time scales of the processes which regulate the horizontal and vertical distributions of water vapour (see eq. (2.12)) in the PBL are taken into account. Based on these estimates, the vertical small/meso- and synoptic scale "turbulent" exchange (correspondingly modified by stratiform cloudiness and moist convection), large-scale vertical advection and precipitation/mixing in stratus and cumulus clouds are argued to be the predominant processes in the formation of the water vapour vertical profile in that layer. The vertical distribution of q_v in the PBL can then be derived from the approximate balance among the mentioned vertical exchange processes and precipitation which gives (see eqs. (2.12)–(2.15), (3.5) for $y_2 = q_v$, (4.99), (4.116), (4.125), (6.16) at $x = l$, (6.45), (6.53), (6.62)–(6.64), (6.68), (6.69))

$$\frac{\partial}{\partial z} K^{z,eff} \rho \frac{\partial q_v}{\partial z} - \frac{\partial}{\partial z}(\rho q_v w) - \frac{1}{\tau^{eff}} \rho q_v = 0 \qquad (7.12)$$

Here $K^{z,eff}$ is the PBL effective vertical diffusion coefficient due to small/mesoscale and synoptic scale "turbulence" affected by stratiform cloudiness and cumulus convection. The term τ^{eff} in (7.12) is the effective (due to stratus and cumulus precipitation/mixing) life time of water vapour in clouds. Within the negligible lower-level cumulus entrainment approximation, the $K^{z,eff}$ and τ^{eff} terms are exact constants with height in the PBL (see eqs. (4.99), (4.116), (4.125), (6.16) at $x = l$, (6.45), (6.53), (6.62)–(6.64), (6.68), (6.69)). The second term in the left-hand side of (7.12) is dominated in the PBL by the first and the third terms. Allowing for a quasi-exponential character of ρ change with height in the layer the vertical profile of q_v governed by the eq.(7.12) is fairly well approximated with the exponential one. A correspondent exponent is referred to as $-\frac{1}{H_q}$ in the model. In this, H_q is totally determined by the eq. (7.12) provided the eqs. (4.99), (4.116), (4.125), (6.16) at $x = l$, (6.45), (6.53), (6.62)–(6.64), (6.68), (6.69) are allowed for.

In Δp^m and Δp^h layers (see Section 7.2) of the free troposphere the second member in the left-hand side of the eq. (7.12) is described by $\nabla_z \rho q_v (w_T + w_n)$ (see eq. (4.69)). The third term in the left-hand side of (7.12) is represnted in those layers by the condensation rates in stratiform clouds (see eq. (6.16)) and condensation/mixing rates in cumuli (see eq. (6.53)). The lower boundary conditions of the continuity of the specific humidity and total vertical flux of water vapour at the $\Delta p^l/\Delta p^m$ and $\Delta p^m/\Delta p^h$ interfaces are applied under the procedure of solving (7.12) in Δp^m and Δp^h layers. In the general case, the vertical profiles of specific humidity in Δp^m and Δp^h layers deviate from those which exponentially decay with height.

The POTSDAM-3 governing equations for water vapour in the model stratospheric layers are obtained by the vertical integration of the eq.(4.69) in the limits of these layers, with the omitted nonstationary and large-scale horizontal advection terms. Under the integration, quasi-exponential vertical profiles of relative humidities are assigned in each layer, and the "precipitation" from the stratospheric clouds is taken into account. The model's governing equations for water vapour in the stratospheric layers are written in terms of exponents for the vertical profiles of the relative humidity.

Thus the vertical distributions of the specific humidity in POTSDAM-3 are represented in different layers of the atmosphere by different formulas which guarantee that q_v does not exceed a saturation value at any level, and hence the Clausius-Clapeyron equation is observed. These formulas offer the continuity of q_v and total vertical moisture flux at the layer interfaces, and provide realistic values of the characteristics of three-layer cloudiness in POTSDAM-3.

To derive the POTSDAM-3 parameterization for the tropopause height H_{tr} it is assumed–following [147]–that the model's lowest stratospheric layer is, to a first approximation, in the radiative equilibrium and quasi-isothermal. In this, the absorption of solar radiation as well as the abssorption of longwave radiation from higher stratospheric layers are neglected in the mentioned layer, since they are small as compared with the absorption of longwave radiation from the underlying surface and the troposphere. Then the radiative balance equation for the layer, in the approximation R_1 from [58] can be written as

$$[B_a D_{tr} + B_{Htr}(1 - D_{tr})](1 - \sum_{i=1}^{3} n_i) + \sum_{i=1}^{3}[B_{c,i} D_{c,tr,i} + B_{Htr}(1 - D_{c,tr,i})]n_i = 2B_{Htr} \quad (7.13)$$

Here $B_a = \epsilon_e \sigma T_a^4$, $B_{Htr} = \sigma(T_a - , H_{tr})^4$, $i = 1, 2, 3$, $B_{c,i} = \sigma(T_a - , H_{c,i})^4$, where $H_{c,i}$ is the height of the upper boundary of the i-th cloud layer, n_i – the cloud amount of the i-th cloud layer as it is observed from the outer space in the longwave spectrum range, D_{tr} – the clear-sky ITF of the total tropospheric layer, $D_{c,tr,i}$ – the clear-sky ITF for the tropospheric layer above the i-th cloud layer. On the assumption that D_{st} is O(1), where D_{st} is the ITF of the total stratospheric layer, one can replace D_{tr}, $D_{c,tr,i}$, respectively, by D^{int}, $D^{int}_{c,i}$, where D^{int} and $D^{int}_{c,i}$ are the clear-sky ITF for the total atmosphere column and for the atmosphere column over the i-th cloud layer, correspondingly. As a result, the tropopause height H_{tr} developed from the eq. (7.13) is as follows

$$H_{tr} = \frac{T_a}{,}\{1 - [\frac{D^{int} + \sum_{i=1}^{3}(\frac{B_{c,i}}{B_a} D^{int}_{c,i} - D^{int})n_i}{1 + D^{int} + \sum_{i=1}^{3}(D^{int}_{c,i} - D^{int})n_i}]^{\frac{1}{4}}\} \quad (7.14)$$

At n_i, T_a and $H_{c,i}$ given, respectively, in [236, 237], [161] and [221], and for D^{int}, $D^{int}_{c,i}$ calculated as described above in Subsection 6.1.2, the H_{tr} represented by the eq.(7.14) is in agreement with the empirical data on the tropopause height from [137] with an accuracy of $1km$, both in seasonal course and in latitudinal/longitudinal distribution.

As is mentioned in Subsection 4.1.1, the u, v and w fields in the model extratropical free atmosphere are driven by the temperature T, the synoptic moments u'^2, v'^2, $u'v'$, $u'w'$, $v'w'$, $\rho'w'$ in this part of the atmosphere, as well as by the sea level pressure p_0 (see the eqs. (4.35)–(4.48), (4.74)). Here the derivation of correspondent formulas for the zonally averaged $\overline{p_0}$ and azonal $p_0{}^*$ components of p_0 is described.

Zonally averaged sea level pressure $\overline{p_0}$ is determined in the model from the condition [168] that the vertical integral over the total atmosphere column of the mass flux due to $\overline{v}$ is equal to zero, which provides (by virtue of the continuity equation) a zero value of the zonally averaged vertical mass flux at the upper boundary of the system. As far as the zonally averaged geostrophic component of the meridional component of wind in the model is equal to zero, the applied condition yields

$$\int_{z_0}^{H} \rho \overline{v}_a \, dz = 0 \quad (7.15)$$

Allowing for the eqs. (4.38), (4.40), (4.42), (4.37), (4.74) and (4.95) the eq. (7.15) is used in POTSDAM-3 as the equation for $\overline{p_0}$.

The azonal component of pressure $p^*_{0,T}$ attributed to the azonal component of temperature T^*_0 in the extratropical PBL is described in [168] leaning upon the linearized vorticity equation for the thermally induced components of the planetary-scale quasi-stationary Rossby waves at the equivalent-barotropic level $z = z_{eq}$ deduced in [19]. The equation relates the thermally induced LSLTC of the azonal pressure at this level $p^*_{e,T}$ to the LSLTC of the azonal temperature T^*_e at the same level. Namely, in [19] the following equation is derived for the correspondent component of the stream function $\Psi^*_{e,T}$ at $z = z_{eq}$ level

$$\alpha^e_{ic}\frac{\partial \Delta_H \Psi^*_{e,T}}{\partial \lambda} + 2\omega \frac{\partial \Psi^*_{e,T}}{\partial \lambda} - \frac{1}{a^2 \sin\vartheta}\frac{\partial \Delta_H \overline{\Psi_e}}{\partial \vartheta}\frac{\partial \Psi^*_{e,T}}{\partial \lambda} = \frac{K^e_H \Delta_H \Delta_H \Psi^*_{e,T}}{a^2} + \frac{a^2\omega}{T^e_0 \sin\vartheta}\frac{\partial \alpha^e_{ic}\sin^2\vartheta}{\partial \vartheta}\frac{\partial T^*_e}{\partial \lambda} \tag{7.16}$$

In (7.16) ϑ is co-latitude, Δ_H is the horizontal Laplace operator on a sphere of unit radius, $\overline{\Psi_e}$ and K^e_H are, respectively, the zonally averaged LSLTC of the stream function and the effective (due to small/meso- and synoptic-scale "turbulence") horizontal coefficient of viscosity at $z = z_{eq}$, and $T^e_0 = T_{00} -, {}_{00} z_{eq}$. The α^e_{ic} term in (7.16) is the so-called index of the atmosphere circulation which is given by

$$\alpha^e_{ic} = \frac{\overline{u_e}}{a \sin\vartheta}, \tag{7.17}$$

where $\overline{u_e}$ is the zonally averaged zonal wind at $z = z_{eq}$. The $\overline{\Psi_e}$ and $\overline{u_e}$ variables are related by the equation

$$\overline{u_e} = \frac{1}{a}\frac{\partial \overline{\Psi_e}}{\partial \vartheta} \tag{7.18}$$

The stream function $\Psi^*_{e,T}$ being known, the pressure $p^*_{e,T}$ is determined in [19] with the usage of the linearized equation of motion along the λ axis

$$\frac{1}{\rho^e_0 \sin\vartheta}\frac{\partial p^*_{e,T}}{\partial \lambda} = -\alpha^e_{ic}\frac{\partial}{\partial \lambda}\frac{\partial \Psi^*_{e,T}}{\partial \vartheta} + 2\alpha^e_{ic}\cot\vartheta \frac{\partial \Psi^*_{e,T}}{\partial \lambda} + 2\omega \cot\vartheta \frac{\partial \Psi^*_{e,T}}{\partial \lambda}, \tag{7.19}$$

where $\rho^e_0 \equiv \rho_0(z_{eq})$. Applying (7.16), (7.19) and exploring (4.72), (4.73), (7.8) the interrelation between $p^*_{0,T}$ and T^*_0 is derived in [168]. In POTSDAM-3, the same algorithm is used, with the important modification that the eqs. (7.16) and (7.19) are solved by expansions of the solution in terms of small parameters $\frac{\widetilde{\alpha^e_{ic}}}{\omega}$ and $\frac{\widetilde{K^e_H}}{a^2\omega}$ rather than by applying the spherical harmonic expansions for mentioned components of the temperature, pressure and stream-function which are employed in [19] and [168]. In doing so, the second terms in the left-hand and right-hand sides of (7.16), as well as the term in the left-hand side and the last term in the right-hand side of (7.19) are conventionally assumed to be of one and the same order in the corresponding equation. Let us notice here that the method applied in POTSDAM-3 of the expansion in terms of a small parameter allows one to make the computations less time-consuming, and on the top of this, more precise at regional scales. The application of the spherical harmonic expansion method implies the usage of a rather large number of harmonics to avoid an artificial teleconnections which could otherwise appear in the model if the orbital and toroidal indices of spherical harmonics are truncated at too low a value.

In a zero-order approximation (i.e., retaining only the second terms in both sides of (7.16), and the last term in the right-hand side and the term in the left-hand side of (7.19)) the solutions to the eqs. (7.16) and (7.19) are, respectively

$$\Psi^*_{e,T} = \frac{a^2}{2T^e_0 \sin\vartheta}\frac{\partial \alpha^e_{ic}\sin^2\vartheta}{\partial \vartheta}T^*_e \tag{7.20}$$

$$p^*_{e,T} = f\rho^e_0\Psi^*_{e,T} \tag{7.21}$$

Accounting for the terms of a first, second, etc. order in the expansions of the solutions to (7.16) and (7.19) in terms of small parameters $\frac{\widetilde{\alpha^e_{ic}}}{\omega}$ and $\frac{\widetilde{K^e_H}}{a^2\omega}$ brings about longitudinal ($\delta\lambda$) and latitudinal ($\delta\vartheta$) shifts between T^*_e, and $p^*_{e,T}$. Applying then (4.72), (4.73) and (7.8) gives correspondent relation between T^*_0 and $p^*_{0,T}$ fields.

The orographical azonal component of the sea surface pressure $p^*_{0,or}$ is described in POTSDAM-3 employing the linearized vorticity equation developed in [34] for the orographically induced components of the planetary-scale quasi-stationary Rossby waves at the equivalent-barotropic level $z = z_{eq}$ which relates the corresponding azonal component of pressure $p^*_{e,or}$ at this level to the LSLTC of the surface orography z_0

$$u(z_{eq})\nabla_\lambda(\Delta_{\lambda,\phi}p^*_{e,or}) + v(z_{eq})\nabla_\phi(\Delta_{\lambda,\phi}p^*_{e,or}) + \chi\ F_{fr}\Delta_{\lambda,\phi}p^*_{e,or} + \beta\nabla_\lambda p^*_{e,or} = -\frac{\rho^e_0 f^2}{H_g}w_{n,or} \tag{7.22}$$

Here $\chi = \frac{V(Z_B)}{V(z_{eq})}$, where $V(Z_B)$ and $V(z_{eq})$ are modules of the horizontal velocity, respectively, at $z = Z_B$ and $z = z_{eq}$, $F_{fr} = C_{fr}|\sin 2\alpha|\frac{(K_{Dz}|f|)^{\frac{1}{2}}}{H_g}$, where $C_{fr} = O(1)$, and $w_{n,or} = \vec{v}_{Hs}\cdot\nabla_H z_0$ [34]. Taylor's formula (4.81) is then employed. The last term in the left-hand side and the term in the right-hand side of (7.22) are conventionally anticipated to be of the same order [34]. The solution $p^*_{e,or}$ to (7.22) is found by applying a method of the expansion in terms of small parameters $\epsilon_1 = \frac{a}{L_O}Ro_{LO}$ and $\epsilon_2 = \widetilde{c_D}Ro_{LO}\frac{a}{H_g}(\frac{\widetilde{Z_B}}{z_{eq}})^2$, where $L_O = L_{Ro,bt}$, $Ro_{LO} = \frac{\tilde{U}}{fL_O}$. Under this scale/magnitude estimate, it is assumed that the characteristic value of the amplitude of the large-scale orography perturbations and the PBL height are the same order. Provided the value of $p^*_{e,or}$ is known, the sea surface pressure disturbance $p^*_{0,or}$ can be obtained using eq. (4.73), the pressure and density terms in this equation being thought of as the pressure $p^*_{or}(z)$ and density $\rho^*_{or}(z)$ disturbances due to LSLTC of the orographic forcing. Under the procedure, the quasi-barotropic character of these disturbances in the lower free troposphere is surmised [93, 231, 48], so that mentioned density preturbations can be approximated by

$$\rho^*_{or}(z) = \rho_0(z)\Delta z^*_{or}, \tag{7.23}$$

where the attributed perturbation of the geopotential height Δz^*_{or} is assumed to be constant over the lower free troposphere column. The Δz^*_{or} term is deduced from the expression

$$\Delta z^*_{or} = \frac{p^*_{e,or}}{g\rho_0(z_{eq})} \tag{7.24}$$

Substitution of (7.23), (7.24) in (4.73) and integration over (z_0, z_{eq}) range of heights yield the formula for $p^*_{0,or}$ provided the baroclinicity of the PBL is neglected.

The total azonal component of the sea surface pressure p^*_0 in POTSDAM-3 is the sum of $p^*_{0,T}$ and $p^*_{0,or}$.

Chapter 8

Governing SDE in POTSDAM-2

In this part of the paper, a simplified version, POTSDAM-2, of the POTSDAM-3 atmospheric module is presented in detail. As mentioned in the Introduction, POTSDAM-2 is already implemented in the Earth system model of intermediate comlexity CLIMBER-2.

A reduction of the eq. (7.8) for the temperature lapse rate , in the free troposphere to a more simple formula employed in POTSDAM-2 for this variable is as follows. The effective vertical diffusion coefficient for heat due to small/mesoscale and synoptic-scale "turbulence" is assumed to be continuous in the vicinity of the z_v level (discussed in Chapter 7) located in the transition range of heights from the PBL to the free atmosphere, i.e., an approximate equality

$$K_{sy}|_{z_v} \approx K_{z,eff}|_{z_v} \tag{8.1}$$

is introduced in (7.8).

For the description of n_{st}^l in (7.8) a reduced form of the eq. (6.12) at $x = l$ is invoked by omitting the term with vertical velocity and replacing the vertically averaged relative humidity $< f_r^l >$ by its value at $z = z_v$ in the right-hand side of (6.12). These assumptions are feasible taking into account that the vertical change of relative humidity in (z_0, z_v) layer at $(\Delta L, \Delta\tau)$ scale is relatively small (see, e.g., [221, 132]), and that the contribution of vertical velocity to n_{st}^l is also small compared to that of relative humidity [50].

In the regions where moist convection occurs, all three types of cumulus are supposed to be equiprobable at $(\Delta L, \Delta\tau)$ scale [236, 237], so that the term n_{cu}^l is put as $n_{cu}^l \approx \frac{1}{3} n_{cu}$ in (6.12), where n_{cu} is the total cumulus cloud amount.

With the usage of (4.116) the specific humidity at $z = z_v$ is expressed in terms of the surface air specific humidity q_{vs}, water vapour scale height H_q (see eq. (8.20)) and C_{zv} (see eq. (7.2)), under the assumption that $H_q \approx h_B$ and assigning $C_{zv} \approx 1.5$ [27, 28, 123, 29, 5, 222, 113, 122, 115, 103].

By virtue of the definition of T_0 (see eqs. (4.44), (4.45) and the text below) the temperature at $z = z_v$ is represented as $T(z_v) = T_0 + , z_v$. We then apply the Taylor-series expansion to any function $\Phi(T_0)$ entering (7.8)

$$\Phi(T_0) = \Phi(T_M) + \frac{\partial\Phi(T_0)}{\partial T_0}|_{T_M}(T_0 - T_M) + \frac{1}{2}\frac{\partial^2\Phi(T_0)}{\partial T_0^2}|_{T_M}(T_0 - T_M)^2 + \ldots, \tag{8.2}$$

where $T_M = const = 288K$, and rewrite $(T_0 - T_M)$ in the equivalent form $(T_0 - T_M + T_{00} - T_{00})$. As a result, dividing (7.8) by $,_a$, assuming $n_{cu}^l = O(10^{-1})$, retaining in the dimensionless (7.8) the zero- and first-order terms and coming back to the dimensional equation yield

$$, = ,_0 + ,_1(T_0 - T_{00})(1 - a_q q_{vs}^2) - ,_2 n_{cu}, \tag{8.3}$$

where

$$\Gamma_0 = \frac{c_v}{c_p K_0}\{\Gamma_a + [\Gamma_a - \Gamma_{wa}|_{p_{00},T_M} - |\frac{\partial \Gamma_{wa}}{\partial T}|_{p_{00},T_M}(T_M - T_{00})]\alpha_0^l\} \tag{8.4}$$

$$\Gamma_1 = \frac{1}{K_0}|\frac{\partial \Gamma_{wa}}{\partial T}|_{p_{00},T_M}\alpha_0^l \tag{8.5}$$

$$\Gamma_2 = \frac{1}{3K_0}(\Gamma_a - \Gamma_{wa}|_{p_{00},T_M}) \tag{8.6}$$

$$a_q = C_\Gamma \frac{\beta_0^l}{\alpha_0^l} \frac{1}{q_{v,sat}^2|_{p_{00},T_M}} \tag{8.7}$$

Here $C_\Gamma = \exp\ \{-3\}$, $K_0 = 1 + \frac{c_v}{c_p}$, and $\Gamma_{wa}|_{p_{00},T_M}$, $|\frac{\partial \Gamma_{wa}}{\partial T}|_{p_{00},T_M}$ and $q_{v,sat}|_{p_{00},T_M}$ are, respectively, the moist adiabatic lapse rate, the module of the partial derivative of the moist adiabatic lapse rate with respect to temperature and the saturated specific humidity at $p = p_{00}$, $T = T_M$, where $p_{00} = R\rho_{00}T_{00}$. Substitution of c_v, c_p, Γ_a, $\Gamma_{wa}|_{p_{00},T_M}$, $|\frac{\partial \Gamma_{wa}}{\partial T}|_{p_{00},T_M}$, and $\alpha_0^l \approx 1$ and $\beta_0^l \approx 2$ from [50, 206] to (8.4)–(8.7) brings about the following values of the parameters in (8.3): $\Gamma_0 = 5.5 * 10^{-3}\ Km^{-1}$, $\Gamma_1 = 5.7 * 10^{-5}\ m^{-1}$, $\Gamma_2 = 1.0 * 10^{-3}\ Km^{-1}$, $a_q = 1.0 * 10^3$. Being implemented in CLIMBER-2 eq. (8.3) gives values of Γ which are in good agreement with the empirical data on this variable for present climate conditions, as well as with the results of GCM equilibrium and transient CO_2 runs (see [170, 65]).

In the given later governing SDE of POTSDAM-2 (see eqs. (8.86), (8.88)), the vertical temperature structure under the procedure of the vertical integration of the basic SDE is assumed to be linear over the total troposphere, with the lapse rate Γ described by the eq. (8.3), while in a single stratospheric layer (see text below) a temperature is assigned constant with height, i.e.

$$T(z) = \begin{cases} T_a - \Gamma(z - z_0), & \text{if } z_o < z < H_{tr}, \\ T_a - \Gamma(H_{tr} - z_0), & \text{if } H_{tr} \leq z \leq H \end{cases} \tag{8.8}$$

Let us remind ourselves that T_a is a temperature which would occur near the surface if the lapse rate within the PBL were the same as in the free troposphere. However, in the lowest part of the PBL within the vicinity of the top of the SL the vertical profile of temperature can deviate significantly from that given by the eq. (8.8). Therefore, the Stephenson screen level temperature T_s that is employed to determine the surface sensible heat fluxes may not be set equal to T_a. To compute T_s a simplified version of the eq.(4.111) is applied in POTSDAM-2 in which $\overline{\overline{K}}_{Dz}$ is described by the eq. (4.99). In this, the $\nabla_z\Theta^*$ term in the right-hand side of (4.111) was approximated with $\nabla_z\Theta^* \approx (\Theta^*(Z_B) - T_s)/h_B$. Taking into account the counter-gradient factor [26, 179, 44] the term $\Theta^*(Z_B)$ can be written in the form

$$\Theta^*(Z_B) = T(Z_B) + \Gamma^* h_B, \tag{8.9}$$

where

$$\Gamma^* = \Gamma_a V_{dc} + \Gamma_{wa} V_{mc} + \Gamma V_{nc} \tag{8.10}$$

In (8.10) Γ_{wa} is the moist adiabatic lapse rate at $T = T_a$, $p = p_a$, and V_{dc}, V_{mc} and V_{nc} are, respectively, the relative volumes of the atmosphere at the uppermost part of the PBL in the vicinity of the free troposphere-PBL interface occupied by dry convection, moist convection and nonconvection regions, so that $V_{dc} + V_{mc} + V_{nc} = 1$. Neglecting, to a first approximation, V_{dc} and V_{mc} in the mentioned uppermost part of the PBL, applying eqs. (4.79)– (4.82), (4.90) and using the assumption on the neutral stratification ($Ri_s \approx 0$) of the SL eq.(4.111) yields

$$T_s - T_a \approx 2\frac{\sin\alpha}{(1-\sin 2\alpha)^{\frac{1}{3}}}(T_g - T_s) \tag{8.11}$$

At mean annual globally averaged α [87] the factor $2\frac{\sin\alpha}{(1-\sin 2\alpha)^{\frac{1}{3}}}$ in the right side of (8.11) is of the order of O(1), so that (8.11) reduces to

$$T_s \approx (T_g + T_a)/2 \tag{8.12}$$

Eq.(8.12) is used in POTSDAM-2 for the description of temperature T_s at the Stephenson screen level.

In Chapter 7 a procedure has been described that is applied in POTSDAM for the computation of the vertical profile of specific humidity in the PBL in terms of the exponent $-\frac{1}{H_q}$. In POTSDAM-2, more simple determination of H_q is utilized. To calculate H_q in this version of the model we use the eq. (7.12) in which $K^{z,eff}$ is represented based on findings in [96]. Namely, in [96] a formula is developed for the $K^{z,eff}$ term

$$K^{z,eff} = \frac{1}{2}(w^p_{eff})^2\tau_L \tag{8.13}$$

Here w^p_{eff} is the characteristic value of the effective (due to the combined action of the mentioned above vertical exchange and cloud processes) pulsation of the vertical velocity in the PBL, and τ_L is a specific time of conservation of w^p_{eff} sign in moving parcels (i.e., τ_L is the Lagrange time scale of the effective vertical pulsations). As shown in [134], τ_L can be written as follows

$$\tau_L = \tau_E\frac{V_m}{|V - V_m|}, \tag{8.14}$$

where τ_E is the Euler time scale of the effective vertical pulsations and $\frac{V_m}{|V-V_m|}$ is a characteristic value of a ratio of mean (long-term) horizontal velocity to its effective pulsation in the PBL. Representing $(w^p_{eff})^2$ by

$$(w^p_{eff})^2 = (\frac{K^{z,eff}}{H^p_{eff}})^2, \tag{8.15}$$

where H^p_{eff} is the effective vertical scale height of the vertical pulsations, and using (8.14) the $K^{z,eff}$ term described with (8.13) can be rewritten as follows

$$K^{z,eff} = 2\frac{(H^p_{eff})^2}{\tau_E}\frac{|V - V_m|}{V_m} \tag{8.16}$$

Based on the results obtained in [175] a characteristic value of the amplitude $A(|V - V_m|)$ of the effective horizontal velocity pulsations in the main part of the PBL above the SL can be approximated by a characteristic value of the mean (long-term) horizontal velocity V_m. Then, introducing an effective frequency ω^p_{eff} of the pulsations and averaging over the effective period of the pulsation, the $\frac{|V-V_m|}{V_m}$ term can be written as

$$\frac{|V - V_m|}{V_m} \approx C_K, \tag{8.17}$$

where $C_K = \sqrt{0.5}$.

We assume further that

$$H_q \approx H^p_{eff} \tag{8.18}$$

and

$$\tau^{eff} \approx \tau_E \tag{8.19}$$

The assumption (8.18) is quite reasonable taking into account that the vertical scale height for water vapour in the PBL is just formed by the above-mentioned effective vertical transfer and cloud processes, which effective vertical scale is represented by H^p_{eff}. Also, the relation (8.19) is feasible for the PBL, inasmuch as the cloud-forming processes in this layer (in particular, precipitation) are mainly determined with the mentioned effective vertical transfer due to the combined action of small/mesoscale and synoptic scale eddies, as well as moist convection (if this latter occurs in the PBL). Substitution of (8.16)–(8.19) in (7.12) with tuning factor $C_{Hq} = O(1)$ in the right-hand side of (8.18) and neglecting, to a zero approximation, the second term in the left side of (7.12) yields

$$H_q = H_{q,0} \equiv H_g \frac{1 - C_{q0}}{C_{q0}}, \tag{8.20}$$

where $H_g = RT_0/g$ and $C_{q0} = \frac{2C_K}{C^2_{Hq}}$. At $C_{Hq} = 1.33$ the formula (8.20) gives $H_q = 0.25 H_g \approx 0.25 H_{00} = 2.0 * 10^3 m$. The eq.(8.20) for H_q is used in POTSDAM-2 for the description of the exponential vertical distribution of specific humidity in the total column of the atmosphere, in particular, in the procedure of vertical integration in the governing SDE for this variable (see eq.(8.88)). The values of H_q term calculated by the eq. (8.20) are in a good agreement with the qualitative behaviour (weak dependence of H_q on horizontal coordinates and time) and quantitative values of this variable developed from the empirical data on vertical distributions of specific humidity, and relative humidity and temperature (see, e.g., [221, 132, 156]).

It should be noticed that eq. (8.16), with the eqs. (8.17), (8.18) and (8.20) taken into account, can be regarded as the determination of τ_E, provided that the computation of $K^{z,eff}$ entering the left-hand side of (8.16) is carried out applying above-mentioned complex procedure with the usage of the eqs. (4.99), (4.116), (4.125), (6.16) at $x = l$, (6.45), (6.53), (6.62)–(6.64), (6.68), (6.69). Then the eq. (8.19) can be met by tuning the empirical parameters which enter the formulas for the vertical small/meso- and synoptic-scale diffusion coefficients (see Subsection 4.1.2 and Section 6.4) and the equations for the condensation rates in lower-level stratus and cumuli (see Sections 6.2 and 6.3). So, the computation of H_q according to the simple algorithm described here by the eqs. (8.13)–(8.20) does not contradict the other model equations.

The second term in the left-hand side of (7.12) being taken into account, the expression for H_q is slightly modified by this small term and to a first approximation is given by

$$\frac{1}{H_q} = \frac{1}{H_{q,0}} - \frac{w}{K_z}, \tag{8.21}$$

where K_z is governed by the eq. (7.5) and $H_{q,0}$ is described by the eq. (8.20).

In POTSDAM-2, a stratiform cloudiness is described by a single effective cloud layer (see eqs. (8.56)–(8.85)). In accordance with the parameterization of the height of its lower boundary (see eq.(8.71)), the mentioned effective cloud layer is deposited in the lower troposphere where the vertical distribution of q_v is close to that developed in the (z_0, z_v) layer. The cloud amount of the effective cloud layer, condensation/precipitation rates and integral water content in it are described in POTSDAM-2 by somewhat modified formulas (6.12), (6.16) and (6.20), respectively (see eqs. (8.78)–(8.80), (8.83), (8.84)), which relate these terms to the low-troposphere relative humidity and vertical velocities. By virtue of employing such a model of cloudiness, the vertical profile of specific humidity in the total atmosphere column is governed in POTSDAM-2 by the eq. (4.116), i.e., the mentioned profile is expressed in terms of a single parameter — low-troposphere water vapour scale height H_q. The H_q scale height is highly representative for the lower troposphere in different seasons and various geographical regions. On the strengh of this, the basic characteristics of the atmosphere hydrologic cycle computed in POTSDAM-2 are in rather good agreement with the empirical ones, and with the analogous parameters computed using the original POTSDAM three-layer cloudiness scheme. The reason is that a majority of the water vapour mass in the real atmosphere is concentrated mainly in the lower troposphere, just where the water vapour vertical profile is pretty well approximated by $\exp(-\frac{z}{H_q})$.

For the same reason, the computation of the integral water vapour content, which is strictly speaking the only prognistic variable for water vapour in the model (due to above-mentioned vertical integration of the model equations), is not violated noticeably by such an approximation of the water vapour vertical profile, because of rather fast quasi-exponential decay (with very close to $-\frac{1}{H_q}$ exponent) with height of the specific humidity in the real atmosphere.

Water vapour as one of the radiatively active constituents also enters the radiation module of POTSDAM-2. For the same reason of predominantly lower-level deposition of this substance in the real atmosphere, with a vertical distribution being very close to $\exp(-\frac{1}{H_q})$, the description of the water vapour contribution to radiative fluxes at the upper and lower boundaries of the troposphere and at $z = H$ is rather precise in POTSDAM-2 as compared to the computations using the exact distribution of q_v in the atmosphere column. Just these radiation fluxes (due to vertical integration) enter the POTSDAM-2 prognostic governing equation for temperature.

So, despite the possibility to formally obtain unrealistic values of some variables which do not enter the model codes (e.g., values of relative humidity at high troposphere levels) the water vapour and cloudiness characteristics which actually enter POTSDAM-2 and regulate energy and water vapour balances are only slightly violated by the applied exponential parameterization of water vapour vertical profile. Moreover, anticipating the universal character of the above-mentioned processes which govern water vapour vertical distribution, one can assume the topological stability of this distribution at climate conditions even somewhat distant from present-day climate conditions.

The tropopause height H_{tr} is given in POTSDAM-2 by a simplified version of the eq. (7.14). Namely, based on the results obtained in [221, 236, 237], the evaluation of the order of terms in the right-hand side of this equation allows one to skip, to a zero-order approximation, the terms under the summation sign both in the numerator and denominator of the fraction in square brackets. This results in the following expression for H_{tr} used in POTSDAM-2

$$H_{tr} = \frac{T_0}{,}\{1 - [\frac{D^{int}}{1 + D^{int}}]^{\frac{1}{4}}\} \tag{8.22}$$

With a tuning factor $C_{HT} = 0.8$ in its right-hand side, the formula (8.22) governs the tropopause height in its seasonal course and the geographical distribution with an accuracy higher than 1 km, in comparision to the empirical data presented in [137].

To find the zonally averaged sea level pressure $\overline{p_0}$ we employ in POTSDAM-2 a special parameterization for the zonally averaged component of meridional velocity $\overline{v}$. The basic assumption is that the present-day structure of the mean meridional circulation with three pairs of cells (Hadley, Ferrel and polar) is robust and will exist under different climatic conditions. It is assumed that the average latitudinal width of every cell is approximately $\pi/6$ and that variations of the position of the borders between the cells ϕ_i follow the variations of the thermal equator, with amplitude decreasing toward the poles. The zonally averaged meridional velocity in the atmosphere is represented in the form

$$\overline{v}(\phi, z) = v_1(\phi)\, v_2(z) \tag{8.23}$$

The first term in the eq. (8.23) is the zonally averaged meridional velocity in the PBL which is computed as

$$v_1(\phi) = (-1)^i C_i \left(\frac{\tilde{T}_i}{T_{00}}\right)^3 \mid \Delta\tilde{T}_i \mid \sin\left(\pi \frac{\phi - \phi_i}{\phi_{i-1} - \phi_i}\right), \quad \phi_i < \phi \le \phi_{i-1}, \quad i = 1, 6. \tag{8.24}$$

where

$$\phi_i = \frac{(3-i)\pi}{6} + \frac{\phi_{te}}{(\mid 3 - i \mid\ +1)^2} \quad \text{for } i = 1, 5 \text{ and } \quad \phi_0 = \pi/2, \phi_6 = -\pi/2.$$

C_i are empirical constants prescribed for every cell. The average temperature for the i-th cell is defined as

$$\tilde{T}_i = (\sin\phi_{i-1} - \sin\phi_i)^{-1} \int_{\phi_{i-1}}^{\phi_i} \overline{T_0} \cos\phi \, d\phi, \tag{8.25}$$

and $\Delta\tilde{T}_i = \overline{T}_0(\phi_i) - \overline{T}_0(\phi_{i-1})$ represents the meridional temperature gradient within the i-th cell. The position of the thermal equator is defined as

$$\phi_{te} = 2 \frac{\tilde{T}_3 - \tilde{T}_4}{\tilde{T}_3 + \tilde{T}_4 - \tilde{T}_2 - \tilde{T}_5}\,. \tag{8.26}$$

Eq.(8.24) provides the description of the atmospheric mean meridional circulation in terms of linked heat engines of the first and second genus (see, e.g., [207, 131]) as described in [165]. This approach can be considered as a spatially aggregated version of the traditional description of meridional atmospheric circulation based on the solution of 2-D differential equations for the zonally averaged atmospheric velocity (e.g. [193, 197, 94, 126]). In particular, this approach implicitly accounts for the dependence of the mean meridional circulation on the mean meridional synoptic angular momentum flux $\overline{u'v'}$ (which dominates among all other terms governing $\overline{v}$ in the eq. (4.37)), since the latter, as a first approximation, can be expressed in terms of the meridional gradient of zonally averaged temperature [241].

The vertical structure of the zonally averaged meridional wind is specified in POTSDAM-2 in the form

$$v_2(z) = \begin{cases} 1 & \text{if } 0 < z < Z_B \\ C_v(z - Z_B)^2 & \text{if } Z_B \le z \le H_{tr} \\ 0 & \text{if } H_{tr} < z \le H \end{cases} \tag{8.27}$$

where the value of C_v is obtained from the condition $\int_0^{H_{tr}} v_2\, \rho\, dz = 0$.

The zonally averaged sea level pressure is computed by equating $v_1(\phi)$ to zonally averaged $< v_a >$ given by (4.95), using (4.40) and neglecting the influence of zonally averaged orography, which gives

$$\frac{\partial \overline{p_0}}{\partial \phi} = -\frac{v_1(\phi)\, a\, f\, \rho_{00}}{C_{hb}\, \overline{\sin \alpha}}\,. \tag{8.28}$$

The nonzonal component of sea level pressure $p^*_{0,T}$ attributed to thermally induced large-scale quasi-stationary planetary waves is governed in POTSDAM-2 by the simplified equations (7.20), (7.21), (4.72), (4.73) and (7.8) using which $p^*_{0,T}$ is expressed in terms of T^*_0. In this, the eq. (8.3), which is a simplified version of (7.8), is–to a zero approximation–rewritten in the form

$$, = ,_0 + ,_1(T_0 - T_{00}) \tag{8.29}$$

Eq.(4.72), within the same approximation, yields

$$\rho(z) = \rho_0(z)[1 - \frac{1}{T_{00}}(T_0 - T_{00})] \tag{8.30}$$

Substituting (8.30) in (4.73) and separating the nonzonal component gives, to the same approximation

$$p^*_{e,T} = p^*_{0,T} + R\rho(0)T^*_0\frac{z_{eq}}{H_g}(1 - ,_T z_{eq}), \tag{8.31}$$

where $,_T = ,_1$. Substituting (7.20) and (8.31), respectively, in the right- and left-hand sides of (7.21), allowing for (by virtue of (8.29)) $T^*_e \approx T^*_0(1 - ,_T z_{eq})$ and retaining a zero-order terms in (7.21) one can obtain

$$p^*_{0,T} = -\frac{g p_0 z_{eq}}{R T_0^2}T^*_0(1 - ,_T z_{eq}), \tag{8.32}$$

Replacing within the same approximation p_0 by $\overline{p}_0$ and T_0 by $\overline{T}_0$, and substituting $\overline{z}_{eq} \approx \frac{\overline{H}_{tr}}{2}$ [19] involve the following relation between T^*_0 and $p^*_{0,T}$, in a zero-order approximation

$$p^*_{0,T} = -\frac{g\overline{p}_0\overline{H}_{tr}}{2R\overline{T}_0^2}T^*_0, \tag{8.33}$$

which is exploited in POTSDAM-2.

Notice that accounting for the spatial resolution of POTSDAM-2 the $p^*_{0,or}$ term is omitted in this simplified version of POTSDAM, and allowing for the smallness of α the formulas (4.85), (4.86), (4.94)– (4.97), (4.108), (4.109), (4.117), (4.118), (4.122) and (4.123) are used in a simplified form, with the factor $\cos\alpha$ in these equations being set equal to unit.

For the description of the ageostrophic components of wind in the free troposphere, simplified versions of the eqs. (4.35), (4.36) are applied in POTSDAM-2. Namely, applying to these equations the scale/magnitude analysis described above in Chapter 4 and retaining only zero-order terms, the eqs. (4.35) and (4.36) are reduced to the following set of equations for u_a and v_a in the free troposphere

$$\frac{\partial u'w'}{\partial z} - f v_a = 0 \tag{8.34}$$

$$\frac{\partial v'w'}{\partial z} + f u_a = 0, \tag{8.35}$$

where $\overline{u'w'}$, $\overline{v'w'}$ SM are governed, respectively, by the eqs. (5.59), (5.60) and hence can be represented as follows

$$\overline{u'w'} = K\partial_z \, u \tag{8.36}$$

$$\overline{v'w'} = K\partial_z \, v, \tag{8.37}$$

where $K = \frac{\rho_0 \overline{w'^2}}{\nabla_z \overline{\rho' w'}}$. As discussed in Chapter 7, an assumption regarding the constancy with height of $\overline{T'w'}$ SM in the free troposphere implies only minor vertical variations of $\nabla_z T$, by virtue of (5.57). This is due to the only slight variations with height of K (see (5.57)). On the strength of it eqs. (8.34) and (8.35) can be rewritten as

$$K(\lambda, \phi)\frac{\partial^2(u_g + u_a)}{\partial z^2} - f v_a = 0 \tag{8.38}$$

$$K(\lambda, \phi)\frac{\partial^2(v_g + v_a)}{\partial z^2} + f u_a = 0, \tag{8.39}$$

where $K(\lambda, \phi)$ is assigned in POTSDAM-2 the value of $\frac{\rho_0 \overline{w'^2}}{\nabla_z \overline{\rho' w'}}$ at $z = \frac{H_g}{2}$. Taking into account that u_g and v_g are quasi-linear with height in the model's free troposphere (see eqs. (4.38)—(4.43) and (8.29)) the eqs. (8.38), (8.39) yield, to a zero order approximation

$$K(\lambda, \phi)\frac{\partial^2 u_a}{\partial z^2} - f v_a = 0 \tag{8.40}$$

$$K(\lambda, \phi)\frac{\partial^2 v_a}{\partial z^2} + f u_a = 0 \tag{8.41}$$

Imposing a zero upper boundary condition for w_a at $z = H_{tr}$ and requiring continuity of u_a and v_a at $z = Z_B$ a solution to (8.40), (8.41) is as follows

$$u_a = \frac{1}{4\epsilon_f}\left[(C_1 \cos \epsilon_f z - C_3 \sin \epsilon_f z)\, e^{\epsilon_f z} + (C_2 \cos \epsilon_f z + C_4 \sin \epsilon_f z)\, e^{-\epsilon_f z}\right] \tag{8.42}$$

$$v_a = \frac{1}{4\epsilon_f}\left[(C_3 \cos \epsilon_f z + C_1 \sin \epsilon_f z)\, e^{\epsilon_f z} + (C_4 \cos \epsilon_f z - C_2 \sin \epsilon_f z)\, e^{-\epsilon_f z}\right], \tag{8.43}$$

where

$$C_2 = U_a - C_1,$$

$$C_4 = V_a - C_3,$$

$$C_3 = \frac{U_a(I_3 + Z_B) + V_a I_4}{I_2 + I_4} + \frac{I_1 - I_3}{I_2 + I_4}C_1,$$

$$C_1 = \frac{U_a I_4 - V_a(I_3 + Z_B) + (I_3 - I_1)\frac{U_a(I_3+Z_B)+V_a I_4}{I_2+I_4}}{I_2 + I_4 + (I_1 - I_3)^2/(I_2 + I_4)} \tag{8.44}$$

$$\epsilon_f = (f/2K(\lambda, \phi))^{1/2} \tag{8.45}$$

$$U_a = \epsilon_f < u_a >, V_a = \epsilon_f < v_a >$$

$$I_1 = \frac{\exp(\alpha_1 z)}{\alpha_1^2 + \beta_1^2}(\alpha_1 \cos \beta_1 z + \beta_1 \sin \beta_1 z)\Big|_0^{H_{tr}-Z_B} \tag{8.46}$$

$$I_2 = \frac{\exp(\alpha_1 z)}{\alpha_1^2 + \beta_1^2}(\alpha_1 \sin \beta_1 z - \beta_1 \cos \beta_1 z)\Big|_0^{H_{tr}-Z_B} \tag{8.47}$$

$$I_3 = \frac{\exp(\alpha_2 z)}{\alpha_2^2 + \beta_1^2}(\alpha_2 \cos\beta_1 z + \beta_1 \sin\beta_1 z)\Big|_0^{H_{tr}-Z_B} \tag{8.48}$$

$$I_4 = \frac{\exp(\alpha_2 z)}{\alpha_2^2 + \beta_1^2}(\alpha_2 \sin\beta_1 z - \beta_1 \cos\beta_1 z)\Big|_0^{H_{tr}-Z_B} \tag{8.49}$$

$$\alpha_1 = \epsilon_f - 1/H_{00}, \alpha_2 = -\epsilon_f - 1/H_{00}, \beta_1 = \epsilon_f.$$

Notice that accounting for the cumulus convection and the gravity-wave drag would invoke the additional "vertical" terms in the eqs.(4.75) and (4.76). Analogous terms describing the influence of the cumuli and gravity waves would appear in the right-hand sides of the above-listed equations (4.35), (4.36) and (4.37) in the free troposphere. In POTSDAM-2, the mentioned terms are dropped for the sake of simplicity. Also, in this version of POTSDAM the last terms in the left sides of equations (4.46), (4.48) are omitted (as are the first-order terms).

The stratosphere is represented in POTSDAM-2 by a single isothermal layer which serves only to close the momentum, heat and moisture balance at the upper boundary of the system. The vertical flux of water vapour is negligible at the upper boundary H of the system taking into account that in POTSDAM-2 the vertical profile of specific humidity in the total atmosphere column is described by the eq. (4.116), with H_q governed by the eq. (8.20). So, it is necessary to find only the vertically averaged horizontal velocity in the stratosphere which has to satisfy the condition of zero vertical mass flux at the upper boundary of the atmosphere at $z = H$. The correspondent λ and ϕ components of this velocity which automatically match the mentioned condition are found from the equations

$$\int_{z_0}^{H} \rho u^* \, dz = \int_{z_0}^{H} \rho v^* \, dz = 0. \tag{8.50}$$

As mentioned above in Chapter 5, the second moments are described in POTSDAM-3 by the equations (5.2)—(5.11) and (5.43)–(5.46), with omitted "spherical" terms. In POTSDAM-2, simplification of the equations for the SM by retaining the zero-order members leads to the expressions (5.47)–(5.60), with neglected advective terms. This brings one to a "macroturbulent diffusion" approximation for the SM in POTSDAM-2. In this, equations (5.47)–(5.60) are then written in β-plane approximation, describing the f and β in all corresponding formulas by their values f_0 and β_0 at the representative latitude ϕ_0. The POTSDAM-2 governing equations for the temperature and specific humidity integrated with height include the terms $u'T'$, $v'T'$ and $u'q_v'$, $v'q_v'$ which stand for the horizontal heat and moisture transport due to synoptic-scale eddies/waves in the troposphere. These terms are given in POTSDAM-2 by the eqs. (5.50), (5.51) and (5.54) (5.55), respectively. Substituting (5.26) in (5.50), (5.51), (5.54), (5.55), assuming $u'^2 \approx v'^2$ and employing (4.9), (4.10) one can obtain, under the mentioned simplification as regards the SM

$$u'T' = -A_h \frac{\partial T}{a\cos\phi\partial\lambda}, \qquad v'T' = -A_h \frac{\partial T}{a\partial\phi}, \tag{8.51}$$

$$u'q_v' = -A_h \frac{\partial q_v}{a\cos\phi\partial\lambda}, \qquad v'q_v' = -A_h \frac{\partial q_v}{a\partial\phi}, \tag{8.52}$$

with the Austausch coefficient A_h given by

$$A_h = C_{Tq} \frac{(u'^2 + v'^2)^{1/2} L_{O,0}}{\alpha_{0,0}}, \tag{8.53}$$

where $C_{Tq} = O(1)$, $L_{O,0} = (RT_0)^{1/2}/(4f_0)$, $f_0 = f|_{\phi=\phi_0=\pi/4}$, $\alpha_{0,0} = \left(\frac{R}{g}(,_a - ,)\right)^{1/2}$. To a zero-order approximation, the relative volume occupied by clouds is assumed in POTSDAM-2 to be negligibly small in correspondent formulas, e.g., in the expression for α_0 (see eq. (5.21)). Eqs. (5.47), (5.48) are then utilized for the description of u'^2 and v'^2 in (8.51), (8.52), with the $u'w'$, $v'w'$ and $\nabla_z\rho'w'$ terms being expressed, respectively, by the eqs. (5.59), (5.60) and (5.26). In doing so, the term w'^2 described with the eq. (5.20) is substituted to the eqs. (5.59), (5.60), while the $\nabla_z u$ and $\nabla_z v$ terms in these equations are approximated with $\nabla_z u_g$ and $\nabla_z v_g$. As a result, the $u'^2 + v'^2$ term in (8.51), (8.52) is as follows

$$u'^2 + v'^2 = 2C^* \alpha_{0,0}^{-2}[(\nabla_\phi \overline{T_0})^2 + (\nabla_\phi T_0^*)^2 + (\nabla_\lambda T_0^*)^2] Z^2 (\frac{1}{2} - \frac{,_1 Z}{3}) \tag{8.54}$$

Here

$$C^* = C_U \frac{g^2 L_{O,0}\beta_0}{f_0^3 T_{00}^2}, \tag{8.55}$$

$Z = max(z, Z_B)$ and $C_U = O(1)$. Macroturbulent diffusion due to synoptic eddies/waves in the stratosphere is neglected in POTSDAM-2.

As already mentioned above, a stratiform cloudiness is represented in POTSDAM-2 by an effective stratiform cloud layer (ESCL). The evaporation of rain drops below cloud bases is neglected in this version of the model and precipitation in any grid cell is equal to the integral over the atmosphere column condensation rate. Also neglected in POTSDAM-2 are the effects exerted on cloud characteristics by changes in number, composition and particle-size distribution of CCN. The formula for the height of ESCL is developed in POTSDAM-2 based on some constraints which should be met in each grid cell. These constraints are as follows [168]

1. The ESCL has the cloud amount n_0 equal to the sum of partial cloud amounts n_i (where $i = 1, 2, 3$) of the three-layer stratiform cloudiness as they are observed from outer space in shortwave/longwave ranges of spectrum

2. The ESCL is located at such a level h_{st}^0 that the total radiative balance at the tropopause in the presence of the ESCL is equal to the analogous balance developed in the atmosphere with three-layer stratiform cloudiness, provided all other characteristics of the atmosphere are the same in both cases.

These constraints assure the identity of the radiative balance of the total troposphere-underlying layer system, as well as of the net radiative balance of the system as a whole in each grid cell in both cases, provided all other characteristics of the atmosphere are the same.

Applied to the longwave part of the total radiative balance at the tropopause, the constraint 2. in the approximation R_1 from [58] can be formalized by the relation

$$F_{R,net}^{cs}(1 - \sum_{i=1}^{3} n_i) + \sum_{i=1}^{3} n_i[B_{c,i}D_{c,tr,i} + B_{Htr}(1 - D_{c,tr,i})] =$$
$$F_{R,net}^{cs}(1 - n_0) + n_0[B_{c,0}D_{c,tr,0} + B_{Htr}(1 - D_{c,tr,0})] \tag{8.56}$$

Here $F_{R,net}^{cs}$ is the longwave radiation balance at the tropopause in the clear-sky part of the atmosphere, $B_{c,0}$ is the blackbody radiation at the temperature of the upper boundary of the ESCL, $D_{c,tr,0}$ –correspondent ITF of the tropospheric layer over the ESCL, n_3 is the cloud amount of the representative "blackbody" layer for cirrus clouds which relates to the "geometric" cirrus cloud amount n_3^g as follows

$$n_3 = \epsilon_{ci} n_3^g, \tag{8.57}$$

where $\epsilon_{ci} < 1$ is the cirrus cloud LWR emissivity. The stratus clouds of the lower and middle layers are assigned to be the blackbody radiators. All other notations in (8.56) are the same as in (7.13).

The constraint 1. implies

$$n_0 = \sum_{i=1}^{3} n_i \tag{8.58}$$

According to findings from [76] and [60], one can write within a first approximation for the Earth's free troposphere

$$\nabla_z[B(z)D(z)] \approx 0 \tag{8.59}$$

in any grid cell. Allowing for (8.58), (8.59) the eq. (8.56) is rewritten as

$$\sum_{i=1}^{3} n_i D_{c,tr,i} = \sum_{i=1}^{3} n_i D_{c,tr,0} \tag{8.60}$$

Expanding the integral transmission function $D(z)$ at any level in the free troposphere into Taylor series

$$D(z) = D_{c,tr,2} + \frac{\partial D(z)}{\partial z}|_{z=h^m_{st}}(z - h^m_{st}) + ..., \tag{8.61}$$

truncating the series at the second term and substituting the result in (8.60) yield

$$n_1 h^l_{st} + n_2 h^m_{st} + n_3 h^h_{st} = \sum_{i=1}^{3} n_i h^0_{st} \tag{8.62}$$

Eq. (8.62) can be rewritten as follows

$$h^0_{st} = h^l_{st} + \frac{n_3}{\sum_{i=1}^{3} n_i}(h^h_{st} - h^l_{st}) + + \frac{n_2}{\sum_{i=1}^{3} n_i}(h^m_{st} - h^l_{st}) \tag{8.63}$$

Assuming a statistically independent overlapping of clouds attributed to different layers and taking into account the characteristic values of the ratios among cloud amounts in different cloud layers [236, 237] we specify for n_1, n_2 and n^g_3 the following relation

$$n_1 \approx n_2 \approx n^g_3 \tag{8.64}$$

We assume further $\frac{h^h_{st} - h^l_{st}}{h^h_{st}} \approx 1$ and replace h^h_{st} by H_{tr}. The reason for this replacement is that h^h_{st} approximated by the eq. (6.11), with the eqs. (2.2), (4.47), (4.44), (4.45) and (5.20) taken into consideration, yields

$$h^h_{st} \approx 2(1 - \, , \, _1H_{00})(2 - \sqrt{2})H_g, \tag{8.65}$$

which gives for this variable the values in the $(9 - 11)km$ range of heights. The ITF of the troposphere layer above h^h_{st} at such values of this latter is of the order of O(1) which allows one to replace h^h_{st} by H_{tr} in (8.63). Accounting for (6.9) the eq. (8.63) then reads

$$h^0_{st} = Z_B + \frac{\epsilon_{ci}}{2 + \epsilon_{ci}} H_{tr} + \frac{1}{2 + \epsilon_{ci}}(h^m_{st} - h^l_{st}) \tag{8.66}$$

As discussed in Section 6.2, the height h^m_{st} of the middle-layer stratus clouds is assigned in POTSDAM the level of the extremum of the total vertical flux of water vapour F_{qz} (see eq. (6.10)). Taking into account that the synoptic–scale "macroturbulence" and large-scale vertical advection attributed to the thermal wind dominant in this part of the free troposphere h^m_{st} can be determined within a first approximation from the condition

$$\{\frac{\partial}{\partial z}\rho_0 q_v[w_T + (w'^2)^{\frac{1}{2}}]\}_{z=h^m_{st}} = 0 \tag{8.67}$$

Substituting eqs. (2.2), (4.47) (with (4.44), (4.45) allowed for), (4.116) and (5.20) to the eq. (8.67) brings about the following expression for h^m_{st}

$$h^m_{st} \approx \frac{H_q}{\frac{1}{2} + ,\, {}_1 H_q} \approx 2H_q \frac{H_g}{H_g + H_q} \approx 2H_q \frac{H_g}{H_g + H_{q,0}}, \tag{8.68}$$

where H_q and $H_{q,0}$ are given by the eqs. (8.21) and (8.20), respectively. Assuming that, to a first approximation, the PBL height closely correlates with the effective vertical scale height of the vertical pulsations H^p_{eff} and taking into account (8.18) one can write

$$h^l_{st} \approx H_q \frac{H_g}{H_g + H_{q,0}} \tag{8.69}$$

The term in round brackets of the eq. (8.66), hence, can be represented by

$$h^m_{st} - h^l_{st} \approx H_{q,0} \frac{H_g}{H_g + H_{q,0}} + H^2_{q,0} \frac{w}{K_z} \tag{8.70}$$

(see eq. (8.21) in which the smallness of the second term in the right side in comparison with the first one is allowed for)

Substitution of (8.70) to (8.66) finally gives, within the same approximation

$$h^0_{st} = [1 + \frac{1}{2 + \epsilon_{ci}} (\frac{H_g}{H_g + H_{q,0}})^2] Z_B + \frac{\epsilon_{ci}}{2 + \epsilon_{ci}} H_{tr} + \frac{1}{2 + \epsilon_{ci}} \frac{H^2_{q,0}}{K_z} w \tag{8.71}$$

For representative cirrus cloud LWR emissivity $\epsilon_{ci} \approx 0.5$ [127]–[129], [59] and the vertical small/mesoscale diffusion coefficient in the lower part of the free atmosphere $K_z \approx 10\ m^2 s^{-1}$ [123, 175], and with the value $H_{q,0}$ being calculated from (8.20), the coefficients at Z_B, H_{tr} and w in formula (8.71) are equal, respectively, to 1.25, 0.2 and $1.6 * 10^5 s$.

Applied to the shortwave part of the total radiative balance at the tropopause, the above-mentioned constraints 1. and 2. lead to the same expression (8.71) for h^0_{st}. Namely, neglecting to a first approximation the albedo and absorption of the over-cloud part of the troposphere, as well as the contribution from the upward shortwave fluxes which are formed at the lower layers and then transmitted through the lower- and middle-layer stratus, and assuming the cirrus clouds to be semi-transparant for those fluxes [60, 61, 127], the constraint 2. imposed on the shortwave radiative balance at the tropopause reads

$$F^{cs}_{S,net}(1 - \sum_{i=1}^{3} n_i) + \sum_{i=1}^{3} n_i [I'_0 \cos\xi(1 - A_{c,i})] = F^{cs}_{S,net}(1 - n_0) + n_0 [I'_0 \cos\xi(1 - A_{c,0})], \tag{8.72}$$

where $F^{cs}_{S,net}$ is the clear-sky shortwave radiative balance at the tropopause, I'_0 is the solar "constant" at the level of tropopause, $A_{c,1} = A^l_{st}$, $A_{c,2} = A^m_{st}$, $A_{c,3} = A^h_{st}$ and n_3 is related to n_3^g by the formula analogous to (8.57), with the coefficient ϵ_{ci} at n_3^g replaced by k_{n3} the latter being close to 0.5 value. Applying constraint 1. the eq. (8.72) can be rewritten as

$$\sum_{i=1}^{3} n_i A_{c,i} = \sum_{i=1}^{3} n_i A_{c,0} \tag{8.73}$$

Involving then eqs. (6.21), (6.22), (6.20), (6.14), (6.15) at $n_{st}^x \gg n_{0,st}$ and taking into account that $\left(\frac{h_{st}^x}{h_{st}^l}\right)^{c_{st}} \approx 1$ the albedo terms A_{st}^x in (8.73) are predominantly functions of correspondent temperatures $T(h_{st}^x)$. Expanding the albedo A_{st}^x of any x-th stratus cloud layer into the Taylor series

$$A_{st}^x = A_{st}^m + \frac{\partial A_{st}^x}{\partial T}|_{T=T(h_{st}^m)}[T(h_{st}^x) - T(h_{st}^m)] + ..., \tag{8.74}$$

(where, in this case, $x = 0, 1, 2, 3$) truncating the series at the second term, applying eq. (8.8) and substituting the result in (8.73) yield

$$n_1 h_{st}^l + n_2 h_{st}^m + n_3 h_{st}^h = \sum_{i=1}^{3} n_i h_{st}^0 \tag{8.75}$$

This equation is identical to the eq. (8.62). Sequential using equations (8.63)–(8.70), with ϵ_{ci} being replaced by k_{n3}, finally offers the same equation as (8.71) for the height of the effective cloud layer h_{st}^0.

For the description of the effective stratus layer cloud amount n_{st}^0 a simplified formula (6.12) is implemented in POTSDAM-2. Namely, to avoid usage of the logic operator in the n_{st}^0 program codes this term is approximated in POTSDAM-2 in the total range $0 < f_r^0 < 1$ of relative humidity f_r^0 at h_{st}^0 level by the following expression (cf. eq. (6.12))

$$n_{st}^0 = (f_r^0)^{k_n}(a'_1 + a'_2 F_c(w_{eff})) \tag{8.76}$$

Here w_{eff} is given by the eq. (6.13) at $z = h_{st}^0$, $F_c(w_{eff}) = 0.5\left(1 + tanh(\frac{w_{eff}}{w_0})\right)$ and w_0 obeys the eq. (6.17), where K_z (as already mentioned above) is assigned a constant value in POTSDAM-2. Numeric coefficients k_n, a'_1 and a'_2 in (8.76) are prescribed in POTSDAM-2 to provide the best fit of n_{st}^0 to cloud amount described by (6.12) at $z = h_{st}^0$ for $n_{st}^0 \geq n_{st,0}^0 = 0.2$. Notice that the values of n_{st}^0 in the dominant majority of cases exceed $n_{st,0}^0$ in any model grid cell. Assuming that changes of relative humidity with respect to z in the $Z_B < z < h_{st}^0$ range of heights are relatively small (see, e.g., [221]) we replace f_r^0 term in the eq. (8.76) by $C_{fr} f_r^{St}$ where $C_{fr} = const = O(1)$ and the relative humidity at the Stephenson screen level f_r^{St} is given by

$$f_r^{St} = q_{vs}/q_{sat}(T_a, p_a) \tag{8.77}$$

In (8.77) $q_{sat}(T_a, p_a)$ is the saturated specific humidity at the surface air temperature T_a and pressure p_a. As a result, the expression for the n_{st}^0 term is as follows

$$n_{st}^0 = (f_r^{St})^{k_n}(a_1 + a_2 F_c(w_{eff})), \tag{8.78}$$

where $a_1 = 0.8$, $a_2 = 0.1$ and $k_n = 1.5$ are constant numeric coefficients and w_0 in the $F_c(w_{eff})$ term is calculated using eqs. (6.17), (8.20) which gives $w_0 \approx 1.25 * 10^{-3} ms^{-1}$ at $K_z \approx 10\ m^2 s^{-1}$.

A significantly simplified version of the cumulus convection scheme (CCS) described in Section 6.3 is employed in POTSDAM-2. The eqs. (6.24), (6.25), (6.47) are used as the basic ones and the cumulus cloud amount n_{cu} is considered to be governed by water vapour flux $M_u q_u$ at the cumuli cloud base, if $M_u q_u > 0$. It is also taken into account that n_{cu} has a marked threshold (see, e.g., [56]) at the surface air temperature of about $T_{th} \approx 299K$ (+ 26 degree Celsius), which is persistently observed in different regions and periods of time. Based on [157, 158] and [188] models of the ascending air plumes involved into PC, SC and MC cumulus convection processes described in Section 6.3 the existence of maximum possible values of cumuli horizontal size and cloud amount n_{cm} is allowed for in POTSDAM-2. (Let us notice here that the value of the parameter n_{cm} strongly depends on the spatial resolution of the climate model [157, 158, 188]).

As a result, the term n_{cu} is described in POTSDAM-2 as a function of effective vertical velocity $w_{eff,b}$ and specific humidity $q_{vb} = q_{vs}\ \exp(-\frac{h^B_{cu}}{H_{q.0}})$ at the cumuli base, the threshold at $T \approx T_{th}$ being accounted for

$$n_{cu} = n_{cm} \tanh(\frac{q_{vs}\exp(-\frac{h^B_{cu}}{H_{q.0}})}{q_{sat}(T_{th}, p_a)}) \tanh(\frac{w_{eff,b}}{w_0}) \text{ if } w_{eff,b} > 0, \quad \text{and} \quad n_{cu} = 0 \text{ if } w_{eff,b} < 0, \tag{8.79}$$

where $q_{sat}(T_{th}, p_a)$ is the saturated specific humidity at $T = T_{th}$ and $p = p_a$. The effective height h^0_{cu} of cumuli is assigned close to that of stratus $h^0_{cu} \approx h^0_{st} = h^0$, so that $w_{eff,b}$ is assumed to be close to w_{eff}, and n_{cm} and h^B_{cu} are prescribed constant values 0.3 and $0.75 * 10^3 m$, respectively.

The total cloud amount n is calculated in POTSDAM-2 as follows

$$n = 1 - (1 - n_{cu})(1 - n^0_{st}). \tag{8.80}$$

The precipitation term P_r is computed in POTSDAM-2 by integration over the total atmosphere column of the eq. (6.16). Under this procedure, n^x_{st} is represented in (6.16) by n (see (8.80)), $w_{eff,SMT}(h^x_{st})$ is given by (6.17) at $K_z \approx 10\ m^2 s^{-1}$, and w_{eff} is described by (6.13) at $z = h^0$. The term Δh^x_{st} in (6.16) is accounted for by Δh^0, the latter being governed by (cf. (6.14), (6.15))

$$\Delta h^0 = C^0_0 M a_0 (\frac{T_{00}}{T_0})^{\frac{1}{2}} \frac{H_{00}}{\alpha_{0,d}}, \tag{8.81}$$

Taking into account that $K_w^{\frac{1}{2}} \ll 1$, the term $\frac{1}{\tau} \equiv \frac{1}{\tau^x_{st}}$ (cf. (6.16)–(6.18)), where in this case $x = 0$, is approximated by

$$\frac{1}{\tau} \equiv C^0_{cl} \tanh\{\frac{\Delta h^0}{(\Delta z)^0}\} \frac{w_0[1 + C^0_{cl,1} K_w^{\frac{1}{2}} \tanh\{\frac{w_{eff}(h^0)}{w_0}\}]}{H_{q,0}} \approx \frac{1}{\tau_0(1 - a_\tau F_c(w_e))}, \tag{8.82}$$

where $\tau_0 \approx \frac{H_{q,0}}{w_0 C^0_{cl} \tanh\{\frac{\Delta h^0}{(\Delta z)^0}\}}$ and $a_\tau \approx C^0_{cl,1} K_w^{\frac{1}{2}}$.

As a result, the term P_r is written in POTSDAM-2 as follows (cf. (6.18))

$$P_r = \frac{Q_q n}{\tau}, \tag{8.83}$$

where $Q_q = \int^H_{z_0} \rho q dz$ is the integral water vapour content in the atmosphere. Under the employed values of the parameters $\tau_0 \approx 5 * 10^5 s$ and $a_\tau \approx 0.5$.

Due to the strong dependence of precipitation on relative humidity, in a great majority of cases the surface air specific humidity does not reach saturation. However, to exclude rare cases when the surface air specific humidity becomes oversaturated, the relative humidity at the Stephenson screen level is controlled at each time step. If relative humidity exceeds 0.95, the water content above this threshold is removed from the atmospheric column and is added to the precipitation.

The fraction of precipitation in the form of snow is assumed to be equal to unit if $T_a < -5^0C$, and equal to zero if $T_a > 5^0C$. In between these values, a linear dependence on temperature of the fraction of precipitation in the form of snow is prescribed.

The integral liquid/crystal water content $M^0_{int,w}$ of cloud layer in POTSDAM-2 is described by the following formula (cf. eq. (6.20))

$$M^0_{int,w} = C_{mw}(\Delta h^0)^2 \frac{\exp\{-k_{wT}|T(h^0) - T_{M1}|\}}{T(h^0)}, \tag{8.84}$$

where h^0 and Δh^0 are given, respectively, by the eqs. (8.66) and (8.81).

The optical thickness τ_c of cloudiness in POTSDAM-2 obeys the equation (cf. eq. (6.22))

$$\tau_c = \sigma^0_0 M^0_{int,w}, \tag{8.85}$$

with constant cloud droplet/crystal scattering coefficient σ^0_0. The eq. (8.85) is used in POTSDAM-2 for computation of the albedo of cloudiness according to the eq. (6.21).

Finally, the energy balance equation, vertically integrated from the surface to the top of the atmosphere, which is one of POTSDAM-2 governing prognostic SDE, is written in a flux form in terms of potential temperature $\theta = T_a + ,_a(z - z_0)$ as follows (cf. (4.56), (4.57), (4.104))

$$\frac{\partial Q_T}{\partial t} = -\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}\int_{z_0}^{H} \rho\,(u_n\theta + u'\theta' + M^\theta_\lambda)dz + \frac{\partial}{\partial\phi}\int_{z_0}^{H} \cos\phi\rho\,(v_n\theta + v'\theta' + M^\theta_\phi)dz\right] - \int_{z_0}^{H} \rho w_T(,_a - ,)dz + {c_v}^{-1}(S_a + R_s - R_t + \mathcal{L}_e P_w + \mathcal{L}_s P_s + F_h) \tag{8.86}$$

Here $Q_T = \int_{z_0}^{H} \rho T dz$ is the internal energy (divided by c_v) of a total atmosphere column, $u'\theta' = u'T'$, $v'\theta' = v'T'$, and w_T is the LSLTC of the vertical velocity due to thermal wind computed in the troposphere layer by

$$w_T(z) = \frac{1}{\rho}\int_{z_0}^{z} \rho v_T(z)\frac{\partial f}{f\partial\phi}dz \tag{8.87}$$

In the stratosphere w_T is assumed in POTSDAM-2 to decrease linearly with pressure.

Taking into account that thermal wind contributes to the advection of temperature only through the beta-effect, the horizontal components of thermal wind are represented in the energy balance equation (8.86) only by the (8.87) term, which allows us to use in POTSDAM-2 a time step of integration of up to one day.

When developing the eq. (8.86) it is assumed additionally that $|1 - \frac{c_p}{c_v}(1 - \frac{R}{g},)|$ is of the order of $o(1)$. This assumption holds in the range of variations of , in its seasonal and latitudinal/longitudinal course for the present-day climate conditions, as well in the broad diapason of climate change (including LGM and mid-Holocene climates) explored in the CLIMBER-2 experiments.

The terms M_λ^θ and M_ϕ^θ in (8.86) stand for the mesoscale zonal and meridional horizontal diffusion of heat described by the formulas analogous to (8.51) but with the (constant) Austausch coefficient A_m (instead of A_h) correspondent to the mesoscale diffusion, S_a is the solar radiation absorbed in the atmosphere, R_s and R_t are the terrestrial radiation fluxes at the Earth's surface and at the top of the atmosphere, and $F_h \equiv F_T^s$ (see eq. (4.106)) is the sensible heat flux at the surface. The terms P_w and P_s denote precipitation in the liquid and snow forms, while $\mathcal{L}_e$ and $\mathcal{L}_s$ designate the corresponding latent heats.

The final governing prognostic SDE of POTSDAM-2 is the water vapour balance equation integrated from the underlying surface to the upper boundary of the atmosphere (cf. (4.69), (4.112))

$$\frac{\partial Q^v}{\partial t} = -\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}\int_{z_0}^{H}\rho\left(uq_v + u'q_v' + M_\lambda^q\right)dz + \frac{\partial}{\partial\phi}\int_{z_0}^{H}\cos\phi\rho\left(vq_v + v'q_v' + M_\phi^q\right)dz\right] + E - P_r, \tag{8.88}$$

where $Q^v = \int_{z_0}^{H}\rho q_v dz$ is the water vapour content in the total atmosphere column. The terms $E \equiv F_q^s$ (see eq. (4.114)) and P_r in the eq. (8.88) stand for evaporation/evapotranspiration and total precipitation respectively, M_λ^q and M_ϕ^q represent horizontal components of mesoscale diffusion of water vapour, given by the equations analogous to (8.52) but with A_m instead of A_h. In contrast to the eq. (8.86), the total wind velocity enters the eq. (8.88) but due to the fact that the major part of atmospheric water vapour is contained in the lower troposphere, the high values of thermal wind in the upper troposphere do not affect the stability of the numerical integration of the eq. (8.88) with a time step of about a day.

The main features of POTSDAM-2 are given in Table 8.1.

Table 8.1: Main features of POTSDAM-2

Spatial resolution	Horizontal: 51.4° lon × 10.° lat. Vertical: 3 layers – PBL (with SL), free troposphere, stratosphere (in the radiative scheme – 16 layers)
Temporal resolution	1–day time step
Basic equations	Prognostic equations for large–scale long–term temperature and specific humidity; diagnostic equations for large–scale long–term circulation variables and auto– and cross–correlation functions of synoptic components
Prescribed parameters	Optical parameters of atmospheric gases, water droplets and aerosols in solar and terrestrial radiation bands; aerosols and greenhouse gas concentrations
Main processes	
	Radiative transfer; large–scale circulation; macroscale eddy/wave horizontal and vertical transport of momentum, heat and moisture; small/mesoscale "turbulent diffusion"; large–scale condensation and cumulus convection; soil/vegetation/atmosphere and ocean/atmosphere momentum, heat and moisture exchange
Explicit temperature–related feedbacks	Water vapour; soil, vegetation and snow/ice albedo; cloudiness; lapse rate; horizontal and vertical transport processes
Radiatively active constituents	
Longwave radiation	H_2O, CO_2, O_3, cloudiness
Shortwave radiation	H_2O, O_3, aerosols, cloudiness
Running time	ca 10 s per model year on the IBM RISC 6000 Workstation

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Appendix A

Dividing by $\widetilde{\rho_0 \tilde{f} U_a}$ the eqs. (3.5) for u and v with the extracted geostrophic terms $\nabla_\lambda p$, $\rho_0 f v_g$, $\nabla_\phi p$, $\rho_0 f u_g$ and then applying the scale/magnitude estimates (4.1) – (4.34) provide the following evaluation of the order of terms in the equations for u_a and v_a

[A1] $\frac{\widetilde{\partial_t \rho_0 u_g}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 3. \cdot 10^{-2}$

[A2] $\frac{\widetilde{\partial_t \rho_0 v_g}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 3. \cdot 10^{-2}$

[A3] $\frac{\widetilde{\partial_t \rho_0 u_a}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 1. \cdot 10^{-2}$

[A4] $\frac{\widetilde{\partial_t \rho_0 v_a}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 1. \cdot 10^{-2}$

[A5] $\frac{\widetilde{\nabla_H \cdot (\rho_0 u_g \vec{v}_g)}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 2.5 \cdot 10^{-1}$

[A6] $\frac{\widetilde{\nabla_H \cdot (\rho_0 v_g \vec{v}_g)}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 2.5 \cdot 10^{-1}$

[A7] $\frac{\widetilde{\nabla_H \cdot (\rho_0 u_a \vec{v}_g)}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 7.5 \cdot 10^{-2}$

[A8] $\frac{\widetilde{\nabla_H \cdot (\rho_0 v_a \vec{v}_g)}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 7.5 \cdot 10^{-2}$

[A9] $\frac{\widetilde{\nabla_H \cdot (\rho_0 u_g \vec{v}_{H,a})}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 7.5 \cdot 10^{-2}$

[A10] $\frac{\widetilde{\nabla_H \cdot (\rho_0 v_g \vec{v}_{H,a})}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 7.5 \cdot 10^{-2}$

[A11] $\frac{\widetilde{\nabla_H \cdot (\rho_0 u_a \vec{v}_{H,a})}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 2.5 \cdot 10^{-2}$

[A12] $\frac{\widetilde{\nabla_H \cdot (\rho_0 v_a \vec{v}_{H,a})}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 2.5 \cdot 10^{-2}$

[A13] $\frac{\widetilde{\nabla_\lambda (\rho_0 u'^2)}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 5. \cdot 10^{-1}$

[A14] $\frac{\widetilde{\nabla_\phi (\rho_0 v'^2)}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 5. \cdot 10^{-1}$

[A15] $\frac{\widetilde{\nabla_\lambda (\rho_0 u' v')}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 1.5 \cdot 10^{-1}$

[A16] $\frac{\widetilde{\nabla_\phi (\rho_0 u' v')}}{\widetilde{\rho_0 \tilde{f} U_a}} \approx 1.5 \cdot 10^{-1}$

[A17] $\frac{\nabla_z \widetilde{(\rho u_g w_g)}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-1}$

[A18] $\frac{\nabla_z \widetilde{(\rho v_g w_g)}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-1}$

[A19] $\frac{\nabla_z \widetilde{(\rho u_a w_g)}}{\rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-2}$

[A20] $\frac{\nabla_z \widetilde{(\rho v_a w_g)}}{\rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-2}$

[A21] $\frac{\nabla_z \widetilde{(\rho u_g w_a)}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-2}$

[A22] $\frac{\nabla_z \widetilde{(\rho v_g w_a)}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-2}$

[A23] $\frac{\nabla_z \widetilde{(\rho u_a w_a)}}{\rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-3}$

[A24] $\frac{\nabla_z \widetilde{(\rho v_a w_a)}}{\rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-3}$

[A25] $\frac{\nabla_z \widetilde{(\rho u' w')}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-1}$

[A26] $\frac{\nabla_z \widetilde{(\rho v' w')}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-1}$

[A27] $\frac{\nabla_z \widetilde{(u_g \rho' w')}}{\rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-2}$

[A28] $\frac{\nabla_z \widetilde{(v_g \rho' w')}}{\rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-2}$

[A29] $\frac{\nabla_z \widetilde{(u_a \rho' w')}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-2}$

[A30] $\frac{\nabla_z \widetilde{(v_a \rho' w')}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-2}$

[A31] $\frac{\nabla_z \widetilde{(w_g \rho' u')}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-3}$

[A32] $\frac{\nabla_z \widetilde{(w_g \rho' v')}}{\rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-3}$

[A33] $\frac{\nabla_z \widetilde{(w_a \rho' u')}}{\rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-4}$

[A34] $\frac{\nabla_z \widetilde{(w_a \rho' v')}}{\rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-4}$

[A35] $\frac{\rho_0 \widetilde{u_g v_g} \tan\phi}{a \rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-1} \tan\phi$

[A36] $\frac{\rho_0 \widetilde{u_g^2} \tan\phi}{a \rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-1} \tan\phi$

[A37] $\frac{\rho_0 \widetilde{u_g v_a} \tan\phi}{a \rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-2} \tan\phi$

[A38] $\frac{\rho_0 \widetilde{u_a v_g} \tan\phi}{a \rho_0 \widetilde{f U_a}} \approx 5. \cdot 10^{-2} \tan\phi$

[A39] $\frac{\rho_0 \widetilde{u_a v_a} \tan\phi}{a \rho_0 \widetilde{f U_a}} \approx 1.5 \cdot 10^{-2} \tan\phi$

[A40] $\frac{\widetilde{\rho_0 u_g u_a \tan\phi}}{a\rho_0 \widetilde{f U_a}} \approx 5.\cdot 10^{-2}\tan\phi$

[A41] $\frac{\widetilde{\rho_0 u_a^2 \tan\phi}}{a\rho_0 \widetilde{f U_a}} \approx 1.5\cdot 10^{-2}\tan\phi$

[A42] $\frac{\widetilde{\rho_0 u'^2 \tan\phi}}{a\rho_0 \widetilde{f U_a}} \approx 1.5\cdot 10^{-1}\tan\phi$

[A43] $\frac{\widetilde{\rho_0 u'v' \tan\phi}}{a\rho_0 \widetilde{f U_a}} \approx 5.\cdot 10^{-2}\tan\phi$

[A44] $\frac{\widetilde{F_{\lambda,z}}}{\rho_0 \widetilde{f U_a}} \approx 2.5\cdot 10^{-1}$

[A45] $\frac{\widetilde{F_{\lambda,H}}}{\rho_0 \widetilde{f U_a}} \approx 1.\cdot 10^{-1}$

[A46] $\frac{\widetilde{F_{\phi,z}}}{\rho_0 \widetilde{f U_a}} \approx 2.5\cdot 10^{-1}$

[A47] $\frac{\widetilde{F_{\phi,H}}}{\rho_0 \widetilde{f U_a}} \approx 1.\cdot 10^{-1}$

The $\rho_0 f u_a$ and $\rho_0 f v_a$ terms are of the zero order (i.e., the ratio of their characteristic magnitude to $\rho_0 \widetilde{f U_a}$ is of the order of 10^0).

Applying estimations [A1]–[A47] to the eqs. (3.5) for u and v with the extracted geostrophic terms $\nabla_\lambda p$, $\rho_0 f v_g$, $\nabla_\phi p$, $\rho_0 f u_g$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we arrive at the model basic equations (4.35), (4.36) for v_a and u_a.

Dividing the eq.(3.5) for $y_1 = T$ by $\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}$, applying the scale/magnitude estimates (4.1) – (4.34), (4.49)–(4.55) and accounting for eqs. (4.44), (4.45), (4.47) provide the following evaluation of the order of terms in the eq. (3.5) for T

[A48] $\frac{\widetilde{\partial_t \rho_0 T}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 4.5\cdot 10^{-1}$

[A49] $\frac{\nabla_H\cdot\widetilde{(\rho_0 T^* \vec{v}_n)}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 4.5\cdot 10^{-1}$

[A50] $\frac{\nabla_H\cdot\widetilde{(\rho_0 T' \vec{v}'_H)}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 4.5\cdot 10^{-1}$

[A51] $\frac{\widetilde{\nabla_z \rho T w_n}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 4.5\cdot 10^{-1}$

[A52] $\frac{\widetilde{\nabla_z \rho T' w'}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 4.5\cdot 10^{-1}$

[A53] $\frac{\widetilde{\nabla_z T \rho' w'}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 7.5\cdot 10^{-2}$

[A54] $\frac{\widetilde{\nabla_z w \rho' T'}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 5.\cdot 10^{-2}$

[A55] $\frac{\widetilde{\nabla_\lambda \rho \overline{T} u_{T,eff}}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 5.\cdot 10^{-1}$

[A56] $\frac{\widetilde{\nabla_\lambda \rho T^* u_{T,eff}}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 3.\cdot 10^{-1}$

[A57] $\frac{\widetilde{\frac{1}{c_v} p\, div\vec{v}}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx \frac{\rho w_T \widetilde{(\Gamma_a - \Gamma)}}{\rho_0 \widetilde{\nabla_\phi \overline{T} U_a}} \approx 3.\cdot 10^0$

Consider the characteristic value of the diabatic term $\widetilde{q^T}$ to be of the same order as $\widetilde{\frac{1}{c_v} p\, div\vec{v}}$. Finally, the term $\nabla_H \cdot (\rho_0 \overline{T} \vec{v}_n)$ is of the zero order (i.e., the ratio of its characteristic value to $\rho_0 \widetilde{\nabla_\phi \overline{T}} U_a$ is of the order of 10^0).

Applying estimations [A48]–[A57] to the eq. (3.5) for $y_1 = T$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we obtain the model basic equation (4.56) for T.

Dividing the eq. (3.5) for $y_2 = q_v$ by $\nabla_z \widetilde{\rho q_v} w_T$ and then applying the scale/magnitude estimates (4.1) – (4.34), (4.58)–(4.68), (4.49)– (4.55), (3.25) provide the following evaluation of the order of terms in that equation

[A58] $\quad \frac{\partial_t \widetilde{\rho_0 q_v}}{\nabla_z \widetilde{\rho q_v} w_T} \approx 1.5 \cdot 10^{-1}$

[A59] $\quad \frac{\nabla_H \cdot \widetilde{(\rho_0 q_v \vec{v}_g)}}{\nabla_z \widetilde{\rho q_v} w_T} \approx 3. \cdot 10^{-1}$

[A60] $\quad \frac{\nabla_H \cdot \widetilde{(\rho_0 q_v \vec{v}_{H,a})}}{\nabla_z \widetilde{\rho q_v} w_T} \approx 1. \cdot 10^{-1}$

[A61] $\quad \frac{\nabla_z \widetilde{\rho q_v} w_n}{\nabla_z \widetilde{\rho q_v} w_T} \approx 5. \cdot 10^{-1}$

[A62] $\quad \frac{\nabla_H \cdot \widetilde{(\rho_0 q_v' \vec{v}'_H)}}{\nabla_z \widetilde{\rho q_v} w_T} \approx 1.5 \cdot 10^{-1}$

[A63] $\quad \frac{\nabla_z \widetilde{q_v \rho' w'}}{\nabla_z \widetilde{\rho q_v} w_T} \approx 5. \cdot 10^{-2}$

[A64] $\quad \frac{\nabla_z \widetilde{\rho q_v' w'}}{\nabla_z \widetilde{\rho q_v} w_T} \approx 2. \cdot 10^{-1}$

[A65] $\quad \frac{\nabla_z \widetilde{w_T \rho' q_v'}}{\nabla_z \widetilde{\rho q_v} w_T} \approx 5. \cdot 10^{-2}$

[A66] $\quad \frac{\nabla_z \widetilde{w_n \rho' q_v'}}{\nabla_z \widetilde{\rho q_v} w_T} \approx 1.5 \cdot 10^{-2}$

The characteristic value of the source/sink term $\widetilde{q^v}$ is assumed to be of the same order as $\nabla_z \widetilde{\rho q_v} w_T$.

Applying estimations [A58]–[A66] to the eq. (3.5) for $y_2 = q_v$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we come to the model basic equation (4.69) for q_v.

Appendix B

Dividing the eq. (3.17) for u'^2 by $\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}$ and applying the scale/magnitude estimates (4.1) – (4.34) provide the following evaluation of the order of terms in that equation

$$[B1] \qquad \frac{\frac{1}{2}\widetilde{\rho_0\partial_t u'^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 1.\cdot 10^{-1}$$

$$[B2] \qquad \frac{\rho_0\widetilde{u'^2\nabla_\lambda u}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 7.5\cdot 10^{-2}$$

$$[B3] \qquad \frac{\frac{1}{2}\rho_0\widetilde{u'v'\nabla_\phi u}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 7.5\cdot 10^{-2}$$

$$[B4] \qquad \frac{\frac{1}{2}\rho_0\widetilde{u^*\nabla_\lambda u'^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 1.\cdot 10^{-1}$$

$$[B5] \qquad \frac{\frac{1}{2}\rho_0\widetilde{v\nabla_\phi u'^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 1.\cdot 10^{-1}$$

$$[B6] \qquad \frac{\frac{1}{2}\rho\widetilde{w\nabla_z u'^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 7.5\cdot 10^{-2}$$

$$[B7] \qquad \frac{\rho\widetilde{u'w'\nabla_z u}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 3.\cdot 10^{-1}$$

$$[B8] \qquad \frac{\frac{1}{2}\widetilde{u'^2\nabla_z\rho'w'}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 3.\cdot 10^{-1}$$

$$[B9] \qquad \frac{\rho_0\widetilde{\tan\phi v u'^2}}{\frac{1}{2}a\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 1.\cdot 10^{-1}\tan\phi$$

$$[B10] \qquad \frac{\rho_0\widetilde{\tan\phi u u'v'}}{\frac{1}{2}a\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 7.5\cdot 10^{-2}\tan\phi$$

$$[B11] \qquad \frac{\frac{1}{2}\widetilde{\Delta(u'^2)}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'^2}} \approx 1.\cdot 10^{-1}$$

Applying estimations [B1]–[B11] to the eq. (3.17) for u'^2 and retaining the terms of the zero (10^0) and first (10^{-1}) order we arrive at the model basic equation (5.2) for u'^2.

Analogously, dividing the eq. (3.17) for v'^2 by $\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'^2}$ and applying the scale/magnitude estimates (4.1) – (4.34) give the following evaluation of the order of terms in that equation

$$[B12] \qquad \frac{\frac{1}{2}\widetilde{\rho_0\partial_t v'^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'^2}} \approx 1.\cdot 10^{-1}$$

$$[B13] \qquad \frac{\rho_0\widetilde{v'^2\nabla_\phi v}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'^2}} \approx 5.\cdot 10^{-2}$$

$$[B14] \qquad \frac{\rho_0\widetilde{u'v'\nabla_\lambda v}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'^2}} \approx 5.\cdot 10^{-3}$$

$$[B15] \qquad \frac{\frac{1}{2}\rho_0 \widetilde{u^* \nabla_\lambda v'^2}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda v'^2}} \approx 1. \cdot 10^{-1}$$

$$[B16] \qquad \frac{\frac{1}{2}\rho_0 \widetilde{v \nabla_\phi v'^2}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda v'^2}} \approx 1. \cdot 10^{-1}$$

$$[B17] \qquad \frac{\frac{1}{2}\rho \widetilde{w \nabla_z v'^2}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda v'^2}} \approx 7.5 \cdot 10^{-2}$$

$$[B18] \qquad \frac{\rho \widetilde{v' w' \nabla_z v}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda v'^2}} \approx 3. \cdot 10^{-1}$$

$$[B19] \qquad \frac{\frac{1}{2} \widetilde{v'^2 \nabla_z \rho' w'}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda v'^2}} \approx 3. \cdot 10^{-1}$$

$$[B20] \qquad \frac{2\rho_0 \widetilde{\tan\phi u u' v'}}{\frac{1}{2} a \rho_0 \widetilde{\overline{u_g} \nabla_\lambda v'^2}} \approx 1 \cdot 10^{-1} \tan\phi$$

$$[B21] \qquad \frac{\frac{1}{2} \widetilde{\Delta(v'^2)}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda v'^2}} \approx 1. \cdot 10^{-1}$$

Applying estimations [B12]–[B21] to the eq. (3.17) for v'^2 and retaining the terms of the zero (10^0) and first (10^{-1}) order we get the model basic equation (5.3) for v'^2.

Dividing the eq. (3.17) for T'^2 by $\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}$ and applying the scale/magnitude estimates (4.1) – (4.34), (4.49)–(4.55) allow one to perform the following evaluation of the order of terms in that equation

$$[B22] \qquad \frac{\frac{1}{2}\rho_0 \widetilde{\partial_t T'^2}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 1. \cdot 10^{-1}$$

$$[B23] \qquad \frac{\rho_0 \widetilde{u' T' \nabla_\lambda T}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 6. \cdot 10^{-2}$$

$$[B24] \qquad \frac{\rho_0 \widetilde{v' T' \nabla_\phi T}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 6. \cdot 10^{-2}$$

$$[B25] \qquad \frac{\frac{1}{2}\rho_0 \widetilde{u^* \nabla_\lambda T'^2}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 1. \cdot 10^{-1}$$

$$[B26] \qquad \frac{\frac{1}{2}\rho_0 \widetilde{v \nabla_\phi T'^2}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 1. \cdot 10^{-1}$$

$$[B27] \qquad \frac{\frac{1}{2}\rho \widetilde{w \nabla_z T'^2}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 7.5 \cdot 10^{-2}$$

$$[B28] \qquad \frac{\rho \widetilde{T' w' \nabla_z T}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 5. \cdot 10^{-1}$$

$$[B29] \qquad \frac{\frac{1}{2} \widetilde{T'^2 \nabla_z \rho' w'}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 3. \cdot 10^{-1}$$

$$[B30] \qquad \frac{\frac{\rho}{T} \widetilde{w T'^2 \nabla_z T}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 5. \cdot 10^{-3}$$

$$[B31] \qquad \frac{\frac{1}{2} \nabla_z (\widetilde{\frac{\rho}{T} T'^2 T' w'})}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 1.5 \cdot 10^{-3}$$

$$[B32] \qquad \frac{T' q_1' - \widetilde{(1/c_v) T' p' \nabla \cdot \vec{v}}}{\frac{1}{2}\rho_0 \widetilde{\overline{u_g} \nabla_\lambda T'^2}} \approx 1. \cdot 10^{-1}$$

$$[B33] \qquad \frac{\frac{1}{2}\widetilde{\Delta(T'^2)}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda T'^2}} \approx 1.\cdot 10^{-1}$$

Applying estimations [B22]–[B33] and assumption 2. in Chapter 4 to the eq. (3.17) for T'^2 and retaining the terms of the zero (10^0) and first (10^{-1}) order we obtain the model basic equation (5.7) for T'^2.

Dividing the eq. (3.17) for ${q'_v}^2$ by $\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}$ and applying the scale/magnitude estimates (4.1) – (4.34), (4.49)–(4.55), (4.58)– (4.68) provide the following evaluation of the order of terms in that equation

$$[B34] \qquad \frac{\frac{1}{2}\rho_0\widetilde{\partial_t {q'_v}^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 1.\cdot 10^{-1}$$

$$[B35] \qquad \frac{\rho_0\widetilde{u' q'_v\nabla_\lambda q_v}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 6.\cdot 10^{-2}$$

$$[B36] \qquad \frac{\rho_0\widetilde{v' q'_v\nabla_\phi q_v}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 6.\cdot 10^{-2}$$

$$[B37] \qquad \frac{\frac{1}{2}\rho_0\widetilde{u^*\nabla_\lambda {q'_v}^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 1.\cdot 10^{-1}$$

$$[B38] \qquad \frac{\frac{1}{2}\rho_0\widetilde{v\nabla_\phi {q'_v}^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 1.\cdot 10^{-1}$$

$$[B39] \qquad \frac{\frac{1}{2}\rho\widetilde{w\nabla_z {q'_v}^2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 7.5\cdot 10^{-2}$$

$$[B40] \qquad \frac{\rho\widetilde{q'_v w'\nabla_z q_v}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 5.\cdot 10^{-1}$$

$$[B41] \qquad \frac{\frac{1}{2}\widetilde{{q'_v}^2\nabla_z\rho' w'}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 3.\cdot 10^{-1}$$

$$[B42] \qquad \frac{\frac{\rho}{T}\widetilde{w q'_v T'\nabla_z q_v}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 5.\cdot 10^{-3}$$

$$[B43] \qquad \frac{\frac{1}{2}\nabla_z(\frac{\rho}{T}\ \widetilde{q'_v T'}\ \ q'_v w')}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 1.5\cdot 10^{-3}$$

$$[B44] \qquad \frac{\widetilde{q'_v q'_2}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 5.\cdot 10^{-1}$$

$$[B45] \qquad \frac{\frac{1}{2}\widetilde{\Delta({q'_v}^2)}}{\frac{1}{2}\rho_0\widetilde{\overline{u_g}\nabla_\lambda {q'_v}^2}} \approx 1.\cdot 10^{-1}$$

Applying estimations [B34]–[B45] and assumption 2. in Chapter 4 to the eq. (3.17) for ${q'_v}^2$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we arrive at the model basic equation (5.11) for ${q'_v}^2$.

Dividing the eq. (3.16) for $u'v'$ by $\rho_0\widetilde{v'^2\nabla_\phi\overline{u_g}}$ and applying the scale/magnitude estimates (4.1) – (4.34) imply the following estimation of the order of terms in that equation

$$[B46] \qquad \frac{\rho_0\widetilde{\partial_t u'v'}}{\rho_0\widetilde{v'^2\nabla_\phi\overline{u_g}}} \approx 1.\cdot 10^{-1}$$

$$[B47] \qquad \frac{\rho_0\widetilde{u'v'\nabla_\lambda u}}{\rho_0\widetilde{v'^2\nabla_\phi\overline{u_g}}} \approx 2.\cdot 10^{-2}$$

$$[B48] \qquad \frac{\rho_0\widetilde{u'v'\nabla_\phi v}}{\rho_0\widetilde{v'^2\nabla_\phi\overline{u_g}}} \approx 2.\cdot 10^{-2}$$

[$B49$] $\frac{\rho_0 v \widetilde{\nabla_\phi u'v'}}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 1. \cdot 10^{-1}$

[$B50$] $\frac{\rho_0 \widetilde{v'^2 \nabla_\phi} u^*}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 3. \cdot 10^{-1}$

[$B51$] $\frac{\rho_0 \widetilde{u'^2 \nabla_\lambda} v}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 3. \cdot 10^{-1}$

[$B52$] $\frac{\rho_0 u \widetilde{\nabla_\lambda u'v'}}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 1. \cdot 10^{-1}$

[$B53$] $\frac{\frac{\rho}{T} w \widetilde{u'T'} \nabla_z v}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 3. \cdot 10^{-2}$

[$B54$] $\frac{\frac{\rho}{T} w \widetilde{v'T'} \nabla_z u}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 7.5 \cdot 10^{-2}$

[$B55$] $\frac{\rho w \widetilde{\nabla_z u'v'}}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 7.5 \cdot 10^{-2}$

[$B56$] $\frac{\widetilde{u'w'\rho} \nabla_z v}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 3. \cdot 10^{-1}$

[$B57$] $\frac{\widetilde{v'w'\rho} \nabla_z u}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 3. \cdot 10^{-1}$

[$B58$] $\frac{\nabla_z (\frac{\rho}{T} \quad \widetilde{T'w'} \quad u'v')}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 3. \cdot 10^{-2}$

[$B59$] $\frac{u'v' \widetilde{\nabla_z \rho' w'}}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 3. \cdot 10^{-1}$

[$B60$] $\frac{\rho_0 \widetilde{\tan \phi u v'^2}}{a \rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 1. \cdot 10^{-1} \tan \phi$

[$B61$] $\frac{\rho_0 \widetilde{\tan \phi v u'v'}}{a \rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 3. \cdot 10^{-2} \tan \phi$

[$B62$] $\frac{\rho_0 \widetilde{\tan \phi u u'^2}}{a \rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 1. \cdot 10^{-1} \tan \phi$

[$B63$] $\frac{\Delta \widetilde{(u'v')}}{\rho_0 \widetilde{v'^2} \nabla_\phi \overline{u_g}} \approx 1. \cdot 10^{-1}$

Applying estimations [B46]–[B63] to the eq. (3.16) for $u'v'$ and retaining the terms of the zero (10^0) and first (10^{-1}) order one can obtain the model basic equation (5.4) for $u'v'$.

Dividing the eq. (3.15) for $u'T'$ by $\rho_0 \overline{u_g} \widetilde{\nabla_\lambda} u'T'$ and applying the scale/magnitude estimates (4.1) – (4.34), (4.49)–(4.55) provide the following evaluation of the order of terms in that equation

[$B64$] $\frac{\rho_0 \widetilde{\partial_t u'T'}}{\rho_0 \overline{u_g} \nabla_\lambda u'T'} \approx 1.5 \cdot 10^{-1}$

[$B65$] $\frac{\rho_0 \widetilde{u'^2 \nabla_\lambda} T}{\rho_0 \overline{u_g} \nabla_\lambda u'T'} \approx 0.5 \cdot 10^{0}$

[$B66$] $\frac{\rho_0 \widetilde{u'v' \nabla_\phi} T}{\rho_0 \overline{u_g} \nabla_\lambda u'T'} \approx 7.5 \cdot 10^{-2}$

[$B67$] $\frac{\rho_0 \widetilde{T' \vec{v'}_H} \cdot \nabla_H u}{\rho_0 \overline{u_g} \nabla_\lambda u'T'} \approx 7.5 \cdot 10^{-2}$

[$B68$] $\frac{\rho_0 \vec{v}_H \cdot \widetilde{\nabla_H u'T'}}{\rho_0 \overline{u_g} \nabla_\lambda u'T'} \approx 1. \cdot 10^{-1}$

$[B69]$ $\quad \frac{\rho\widetilde{\{A_W\}_{1,1}+(1/c_v)u'p'\nabla\cdot\vec{v}}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'T'}} \approx 1.\cdot 10^{-3}$

$[B70]$ $\quad \frac{\widetilde{u'q_1'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'T'}} \approx 3.\cdot 10^{-2}$

$[B71]$ $\quad \frac{\rho w\widetilde{\nabla_z u'T'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'T'}} \approx 7.5\cdot 10^{-2}$

$[B72]$ $\quad \frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'w'\ \ u'T'})}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'T'}} \approx 1.\cdot 10^{-3}$

$[B73]$ $\quad \frac{\widetilde{u'T'\nabla_z\rho'w'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'T'}} \approx 3.\cdot 10^{-1}$

$[B74]$ $\quad \frac{w\frac{\rho}{T}(\widetilde{u'T'\nabla_z T+T'^2\nabla_z u})}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'T'}} \approx 1.\cdot 10^{-3}$

$[B75]$ $\quad \frac{\rho_0\tan\phi(\widetilde{uv'T'+vu'T'})}{a\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'T'}} \approx 3.\cdot 10^{-1}\tan\phi$

$[B76]$ $\quad \frac{\Delta(\widetilde{u'T'})}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'T'}} \approx 1.\cdot 10^{-1}$

Applying estimations [B64]–[B76] to the eq. (3.15) for $u'T'$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we arrive at the model basic equation (5.5) for $u'T'$.

Dividing the eq. (3.15) for $v'T'$ by $\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}$ and applying the scale/magnitude estimates (4.1) – (4.34), (4.49)–(4.55) provide the following evaluation of the order of terms in that equation

$[B77]$ $\quad \frac{\rho_0\widetilde{\partial_t v'T'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 1.5\cdot 10^{-1}$

$[B78]$ $\quad \frac{\rho_0\widetilde{u'v'\nabla_\lambda T}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 7.5\cdot 10^{-2}$

$[B79]$ $\quad \frac{\rho_0\widetilde{v'^2\nabla_\phi T}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 0.5\cdot 10^{0}$

$[B80]$ $\quad \frac{\rho_0\widetilde{T'\vec{v}'_H\cdot\nabla_H v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 2.5\cdot 10^{-2}$

$[B81]$ $\quad \frac{\rho_0\widetilde{\vec{v}_H\cdot\nabla_H v'T'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 1.\cdot 10^{-1}$

$[B82]$ $\quad \frac{\rho\widetilde{\{A_W\}_{2,1}+(1/c_v)v'p'\nabla\cdot\vec{v}}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 1.\cdot 10^{-3}$

$[B83]$ $\quad \frac{\widetilde{v'q_1'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 3.\cdot 10^{-2}$

$[B84]$ $\quad \frac{\rho w\widetilde{\nabla_z v'T'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 7.5\cdot 10^{-2}$

$[B85]$ $\quad \frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'w'\ \ v'T'})}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 1.\cdot 10^{-3}$

$[B86]$ $\quad \frac{\widetilde{v'T'\nabla_z\rho'w'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 3.\cdot 10^{-1}$

$[B87]$ $\quad \frac{w\frac{\rho}{T}(\widetilde{v'T'\nabla_z T+T'^2\nabla_z\ v})}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'T'}} \approx 1.\cdot 10^{-3}$

$[B88]$ $\frac{\rho_0 \tan\widetilde{\phi 2uu'T'}}{a\rho_0\overline{u_g}\widetilde{\nabla_\lambda v'T'}} \approx 3.\cdot 10^{-1}\tan\phi$

$[B89]$ $\frac{\Delta(\widetilde{v'T'})}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda v'T'}} \approx 1.\cdot 10^{-1}$

Applying estimations [B77]–[B89] to the eq. (3.15) for $v'T'$ and retaining the terms of the zero (10^0) and first (10^{-1}) order one can obtain the model basic equation (5.6) for $v'T'$. Let us notice here that in the estimations [B69], [B82] we additionally accounted for, firstly, the proximity of the extratropical free atmosphere to the state correspondent to the "baroclinic adjustment" [210] and, secondly, the closeness of the characteristic value of , in that part of the atmosphere to $\frac{\Gamma_a}{2}$.

Dividing the eq. (3.22) for $T'q'_v$ by $\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}$ and applying the scale/magnitude estimates (4.1) – (4.34), (4.49)–(4.55), (4.58)–(4.68) provide the following evaluation of the order of terms in that equation

$[B90]$ $\frac{\rho_0\widetilde{\partial_t T'q'_v}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 1.5\cdot 10^{-1}$

$[B91]$ $\frac{\rho_0\widetilde{T'u'\nabla_\lambda q_v}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 7.5\cdot 10^{-2}$

$[B92]$ $\frac{\rho_0\widetilde{T'v'\nabla_\phi q_v}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 7.5\cdot 10^{-2}$

$[B93]$ $\frac{\rho_0\widetilde{q'_v\vec{v'}_H\cdot\nabla_H T}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 7.5\cdot 10^{-2}$

$[B94]$ $\frac{\rho_0\vec{v^*}_H\cdot\widetilde{\nabla_H T'q'_v}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 1.\cdot 10^{-1}$

$[B95]$ $\frac{\frac{\rho}{T}\widetilde{T'^2w\nabla_z q_v}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 1.\cdot 10^{-3}$

$[B96]$ $\frac{\rho w\widetilde{\nabla_z T'q'_v}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 7.5.\cdot 10^{-2}$

$[B97]$ $\frac{\frac{\rho}{T}\widetilde{wT'q'_v\nabla_z T}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 7.5\cdot 10^{-3}$

$[B98]$ $\frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'w'}\ \ T'q'_v)}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 1.\cdot 10^{-3}$

$[B99]$ $\frac{\widetilde{T'q'_v\nabla_z\rho'w'}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 3.\cdot 10^{-1}$

$[B100]$ $\frac{\widetilde{\rho q'_v w'\nabla_z T+\rho T'w'\nabla_z q_v}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 3.\cdot 10^{-1}$

$[B101]$ $\frac{T'q'_2+q'_vq'_1-\widetilde{\frac{q'_vp'}{c_v}}\nabla\cdot\vec{v}}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 1.\cdot 10^{-1}$

$[B102]$ $\frac{\Delta(\widetilde{T'q'_v})}{\rho_0\overline{u_g}\widetilde{\nabla_\lambda T'q'_v}} \approx 1.\cdot 10^{-1}$

Applying estimations [B90]–[B102] to the eq. (3.22) for $T'q'_v$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we get the model basic equation (5.8) for $T'q'_v$.

Dividing the eq. (3.15) for $u'q'_v$ by $\rho_0\overline{u_g}\widetilde{\nabla_\lambda u'q'_v}$ and applying the scale/magnitude estimates (4.1) – (4.34), (4.49)–(4.55), (4.58)–(4.68) provide the following evaluation of the order of terms in that equation

$[B103]\qquad \frac{\rho_0\widetilde{\partial_t u'q'_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 1.5\cdot 10^{-1}$

$[B104]\qquad \frac{\rho_0\widetilde{u'^2\nabla_\lambda q_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 0.5\cdot 10^{0}$

$[B105]\qquad \frac{\rho_0\widetilde{u'v'\nabla_\phi q_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 7.5\cdot 10^{-2}$

$[B106]\qquad \frac{\rho_0\widetilde{q'_v\vec{v'}_H\cdot\nabla_H u}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 7.5\cdot 10^{-2}$

$[B107]\qquad \frac{\rho_0\vec{v}_H\cdot\widetilde{\nabla_H u'q'_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 1.\cdot 10^{-1}$

$[B108]\qquad \frac{\rho\{\widetilde{A_W}\}_{1,2}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 1.\cdot 10^{-3}$

$[B109]\qquad \frac{\widetilde{u'q'_2}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 3.\cdot 10^{-2}$

$[B110]\qquad \frac{\rho w\widetilde{\nabla_z u'q'_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 7.5\cdot 10^{-2}$

$[B111]\qquad \frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'w'\ \ u'q'_v})}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 1.\cdot 10^{-3}$

$[B112]\qquad \frac{u'q'_v\widetilde{\nabla_z\rho'w'}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 3.\cdot 10^{-1}$

$[B113]\qquad \frac{w\frac{\rho}{T}(u'T'\widetilde{\nabla_z q_v + q'_v T'}\nabla_z u)}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 1.\cdot 10^{-3}$

$[B114]\qquad \frac{\rho_0\tan\phi(\widetilde{uv'q'_v + vu'q'_v})}{a\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 3.\cdot 10^{-1}\tan\phi$

$[B115]\qquad \frac{\Delta(\widetilde{u'q'_v})}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda u'q'_v}} \approx 1.\cdot 10^{-1}$

Applying estimations [B103]–[B115] to the eq. (3.15) for $u'q'_v$ and retaining the terms of the zero (10^0) and first (10^{-1}) order gives the model basic equation (5.9) for $u'q'_v$.

Dividing the eq. (3.15) for $v'q'_v$ by $\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'q'_v}$ and applying the scale/magnitude estimates (4.1) – (4.34), (4.49)–(4.55), (4.58)–(4.68) provide the following evaluation of the order of terms in that equation

$[B116]\qquad \frac{\rho_0\widetilde{\partial_t v'q'_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'q'_v}} \approx 1.5\cdot 10^{-1}$

$[B117]\qquad \frac{\rho_0\widetilde{u'v'\nabla_\lambda q_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'q'_v}} \approx 7.5\cdot 10^{-2}$

$[B118]\qquad \frac{\rho_0\widetilde{v'^2\nabla_\phi q_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'q'_v}} \approx 0.5\cdot 10^{0}$

$[B119]\qquad \frac{\rho_0\widetilde{q'_v\vec{v'}_H\cdot\nabla_H v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'q'_v}} \approx 2.5\cdot 10^{-2}$

$[B120]\qquad \frac{\rho_0\vec{v}_H\cdot\widetilde{\nabla_H v'q'_v}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'q'_v}} \approx 1.\cdot 10^{-1}$

$[B121]\qquad \frac{\rho\{\widetilde{A_W}\}_{2,2}}{\rho_0\widetilde{\overline{u_g}\nabla_\lambda v'q'_v}} \approx 1.\cdot 10^{-3}$

[B122] $\frac{\widetilde{v'q_2'}}{\rho_0\overline{u_g}\nabla_\lambda v'q_v'} \approx 3.\cdot 10^{-2}$

[B123] $\frac{\rho w\widetilde{\nabla_z v'q_v'}}{\rho_0\overline{u_g}\nabla_\lambda v'q_v'} \approx 7.5\cdot 10^{-2}$

[B124] $\frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'w'\ \ v'q_v'})}{\rho_0\overline{u_g}\nabla_\lambda v'q_v'} \approx 1.\cdot 10^{-3}$

[B125] $\frac{\widetilde{v'q_v'\nabla_z\rho'w'}}{\rho_0\overline{u_g}\nabla_\lambda v'q_v'} \approx 3.\cdot 10^{-1}$

[B126] $\frac{w\frac{\rho}{T}(v'T'\nabla_z\widetilde{q_v+q_v'}T'\nabla_z\ \ v)}{\rho_0\overline{u_g}\nabla_\lambda v'q_v'} \approx 1.\cdot 10^{-3}$

[B127] $\frac{\rho_0\tan\widetilde{\phi 2uu'q_v'}}{a\rho_0\overline{u_g}\nabla_\lambda v'q_v'} \approx 3.\cdot 10^{-1}\tan\phi$

[B128] $\frac{\Delta(\widetilde{v'q_v'})}{\rho_0\overline{u_g}\nabla_\lambda v'q_v'} \approx 1.\cdot 10^{-1}$

Applying estimations [B116]–[B128] to the eq. (3.15) for $v'q_v'$ and retaining the terms of the zero (10^0) and first (10^{-1}) order one can arrive at the model basic equation (5.10) for $v'q_v'$.

Dividing the eq. (3.18) for $u'w'$ by $\rho\widetilde{w'^2\nabla}_z\overline{u_g}$ and applying the scale/magnitude estimates (3.14), (4.1)–(4.34), (5.31)– (5.33) provide the following evaluation of the order of terms in that equation

[B129] $\frac{\rho_0 w'\widetilde{\vec{v}'_H\cdot\nabla_H}u}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 5.\cdot 10^{-2}$

[B130] $\frac{\rho_0\vec{v}_H\cdot\widetilde{\nabla_H u'w'}}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 1.\cdot 10^{-1}$

[B131] $\frac{\rho_0\vec{v}_H\cdot\widetilde{(u'\nabla_H w')}}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 2.5\cdot 10^{-2}$

[B132] $\frac{\rho w\widetilde{\nabla_z u'w'}}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 1.\cdot 10^{-2}$

[B133] $\frac{\rho w\widetilde{u'\nabla_z w'}}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 2.5\cdot 10^{-2}$

[B134] $\frac{\frac{\rho}{T}\widetilde{T'w'}w\nabla_z u}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 5.\cdot 10^{-2}$

[B135] $\frac{\frac{1}{2}\frac{\rho}{T}\widetilde{u'T'}\nabla_z w'^2}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 1.\cdot 10^{-2}$

[B136] $\frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'w'\ \ u'w'})}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 1.\cdot 10^{-2}$

[B137] $\frac{\widetilde{u'w'\nabla_z\rho'w'}}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 3.\cdot 10^{-1}$

[B138] $\frac{\rho_0\tan\widetilde{\phi uv'w'}}{a\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 1.5\cdot 10^{-2}$

[B139] $\frac{\rho_0\tan\widetilde{\phi vu'w'}}{a\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 5.\cdot 10^{-3}$

[B140] $\frac{\Delta(\widetilde{u'w'})}{\rho\widetilde{w'^2}\nabla_z\overline{u_g}} \approx 1.\cdot 10^{-1}$

Applying estimations [B129]–[B140] to the eq. (3.18) for $u'w'$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we obtain the model basic equation (5.45) for $u'w'$.

Dividing the eq. (3.19) for $v'w'$ by $\rho\widetilde{w'^2}\nabla_z v^*$ and applying the scale/magnitude estimates (3.14), (4.1)–(4.34), (5.34)– (5.36) provide the following evaluation of the order of terms in that equation

[B141] $\quad \frac{\rho_0 \widetilde{w'\vec{v'}_H \cdot \nabla_H v}}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 5. \cdot 10^{-2}$

[B142] $\quad \frac{\rho_0 \vec{v}_H \cdot \widetilde{\nabla_H v'w'}}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 3. \cdot 10^{-1}$

[B143] $\quad \frac{\rho_0 \vec{v}_H \cdot \widetilde{(v'\nabla_H w')}}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 7.5 \cdot 10^{-2}$

[B144] $\quad \frac{\rho w \widetilde{\nabla_z v'w'}}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 3. \cdot 10^{-2}$

[B145] $\quad \frac{\rho w \widetilde{v'\nabla_z w'}}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 7.5 \cdot 10^{-2}$

[B146] $\quad \frac{\frac{\rho}{T}\widetilde{T'w'}w\nabla_z v}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 5. \cdot 10^{-2}$

[B147] $\quad \frac{\frac{1}{2}\frac{\rho}{T}\widetilde{v'T'}\nabla_z \widetilde{w'^2}}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 3. \cdot 10^{-2}$

[B148] $\quad \frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'w'\ \ v'w'})}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 3. \cdot 10^{-2}$

[B149] $\quad \frac{\widetilde{v'w'\nabla_z \rho' w'}}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 7.5 \cdot 10^{-1}$

[B150] $\quad \frac{2\rho_0 \tan\phi\, u\widetilde{u'w'}}{a\rho\widetilde{w'^2}\nabla_z v^*} \approx 7.5 \cdot 10^{-2}$

[B151] $\quad \frac{\Delta\widetilde{(u'w')}}{\rho\widetilde{w'^2}\nabla_z v^*} \approx 3. \cdot 10^{-1}$

Applying estimations [B141]–[B151] to the eq. (3.19) for $v'w'$ and retaining the terms of the zero (10^0) and first (10^{-1}) order give the model basic equation (5.46) for $v'w'$.

Dividing the eq. (3.20) for $T'w'$ by $\rho\widetilde{w'^2}\nabla_z T$ and applying the scale/magnitude estimates (3.14), (4.1)–(4.34), (4.49)– (4.55) ,(5.37)– (5.39) provide the following evaluation of the order of terms in that equation

[B152] $\quad \frac{\rho_0 \widetilde{w'\vec{v'}_H \cdot \nabla_H T}}{\rho\widetilde{w'^2}\nabla_z T} \approx 1. \cdot 10^{-2}$

[B153] $\quad \frac{\rho_0 \vec{v}_H \cdot \widetilde{(w'\nabla_H T')}}{\rho\widetilde{w'^2}\nabla_z T} \approx 1. \cdot 10^{-3}$

[B154] $\quad \frac{\rho w \widetilde{w'\nabla_z T'}}{\rho\widetilde{w'^2}\nabla_z T} \approx 3. \cdot 10^{-3}$

[B155] $\quad \frac{\frac{\rho}{T}\widetilde{T'w'}w\nabla_z T}{\rho\widetilde{w'^2}\nabla_z T} \approx 3. \cdot 10^{-4}$

[B156] $\quad \frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'^2\ \ w'^2})}{\rho\widetilde{w'^2}\nabla_z T} \approx 7.5 \cdot 10^{-2}$

[B157] $\quad \frac{\widetilde{T'w'\nabla_z \rho' w'}}{\rho\widetilde{w'^2}\nabla_z T} \approx 3. \cdot 10^{-1}$

$[B158]$ $\frac{\frac{1}{2}\frac{\rho}{T}\widetilde{T'^2\nabla_z w'^2}}{\rho\widetilde{w'^2}\nabla_z T} \approx 4. \cdot 10^{-2}$

$[B159]$ $\frac{\widetilde{\frac{w'p'}{c_v}\nabla\cdot\vec{v}}}{\rho\widetilde{w'^2}\nabla_z T} \approx 1. \cdot 10^{-3}$

$[B160]$ $\frac{\widetilde{w'q'_1}}{\rho\widetilde{w'^2}\nabla_z T} \approx 3. \cdot 10^{-1}$

$[B161]$ $\frac{\Delta(\widetilde{T'w'})}{\rho\widetilde{w'^2}\nabla_z T} \approx 1. \cdot 10^{-1}$

Applying estimations [B152]–[B161] to the eq. (3.20) for $T'w'$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we arrive at the model basic equation (5.43) for $T'w'$.

Dividing the eq. (3.21) for $q'_v w'$ by $\rho\widetilde{w'^2\nabla}_z q_v$ and applying the scale/magnitude estimates (3.14), (4.1)–(4.34), (4.49)– (4.55) , (4.58)–(4.68), (5.40)–(5.42) provide the following evaluation of the order of terms in that equation

$[B162]$ $\frac{\rho_0\widetilde{w'\vec{v'}_H\cdot\nabla_H q_v}}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 1. \cdot 10^{-2}$

$[B163]$ $\frac{\rho_0\vec{v}_H\cdot(\widetilde{w'\nabla_H q'_v})}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 1. \cdot 10^{-3}$

$[B164]$ $\frac{\rho w\widetilde{w'\nabla_z q'_v}}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 3. \cdot 10^{-3}$

$[B165]$ $\frac{\frac{\rho}{T}\widetilde{T'w'}w\nabla_z q_v}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 3. \cdot 10^{-4}$

$[B166]$ $\frac{\nabla_z(\frac{\rho}{T}\ \widetilde{T'w'}\ \ q'_v w')}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 7.5 \cdot 10^{-2}$

$[B167]$ $\frac{q'_v\widetilde{w'\nabla_z\rho'}w'}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 3. \cdot 10^{-1}$

$[B168]$ $\frac{\frac{1}{2}\frac{\rho}{T}\widetilde{T'q'_v\nabla_z w'^2}}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 4. \cdot 10^{-2}$

$[B169]$ $\frac{\widetilde{w'q'_2}}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 3. \cdot 10^{-1}$

$[B170]$ $\frac{\Delta(\widetilde{q'_v w'})}{\rho\widetilde{w'^2}\nabla_z q_v} \approx 1. \cdot 10^{-1}$

Applying estimations [B162]–[B170] to the eq. (3.21) for $q'_v w'$ and retaining the terms of the zero (10^0) and first (10^{-1}) order we obtain the model basic equation (5.44) for $q'_v w'$.

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