PIK Report

No. 102

A LONG-TERM MODEL OF THE GERMAN ECONOMY: lagom^{d_sim}

Carlo C. Jaeger

POTSDAM INSTITUTE
FOR
CLIMATE IMPACT RESEARCH (PIK)

Author:
Prof. Dr. Carlo C. Jaeger
Potsdam Institute for Climate Impact Research
P.O. Box 60 12 03, D-14412 Potsdam, Germany
Phone: +49-331-288-2601
Fax: +49-331-288-2600
E-mail: Carlo.Jaeger@pik-potsdam.de

Herausgeber:
Dr. F.-W. Gerstengarbe

Technische Ausführung:
U. Werner

POTSDAM-INSTITUT
FÜR KLIMAFOLGENFORSCHUNG
Telegrafenberg
Postfach 60 12 03, 14412 Potsdam
GERMANY
Tel.: +49 (331) 288-2500
Fax: +49 (331) 288-2600
E-mail-Adresse:pik@pik-potsdam.de

ISSN 1436-0179

POTSDAM, OKTOBER 2005

Abstract

Most models used in climate economics so far are full-employment models. In Germany, as in other countries suffering from persistent unemployment, climate policy can only be successful if it is embedded in an economic policy that generates additional jobs at a large scale. To investigate the possibility for such embedding, a long-term model of the German economy is needed. lagom$^{\text{d_sim}}$ is such a model. It is a dynamic stochastic general equilibrium model; its dynamics is based on a sequence of temporary equilibria with incomplete markets. The model architecture works without needing the problematic fictions of a macro-economic representative agent or a Walrasian auctioneer. The mechanisms of demand and supply on different markets display the frictions of real markets, where price adjustments take time. First simulation results suggest that it is possible to reduce German unemployment by half in a decade by means of suitable incentives for financial markets. This possibility resembles past experiences of Sweden and Austria. If poorly managed, it would result in a massive increase of greenhouse gas emissions. At the same time, it offers opportunities for a Pareto improving climate policy that combines higher growth with lower emissions. To investigate these opportunities in more detail, further model developments are recommended.[1]

[1]The lagom family of models is emerging out of the work of the research group on globalization and financial markets at PIK - currently involving A. Biewald, H. Förster, A. Haas, C.Jaeger, J. Krause, F. Meissner, and M. Welp. The help of Rupert Klein in producing lagom$^{\text{d_sim}}$ has been invaluable. Moreover, two sets of ideas on economic dynamics have been particularly inspiring - developed by K. Hasselmann at MPI in Hamburg, and by S. Hallegatte and J.-C. Hourcade at CIRED in Paris. Finally, I am grateful to the Potsdam University students attending the course on the German economy in summer 2005 for their curiosity, questions, and comments. Responsibility for errors stays firmly with the author.
This document describes lagom$^{\text{d_sim}}$ version 0.1. The model is implemented using the Vensim software for modelling dynamical systems. The code is available from the author upon request. Comments are welcome to carlo.jaeger@pik-potsdam.de.
lagom is a swedish word denoting a sense of balance and harmony, with a flavor of equilibrium but richer, perhaps akin to the chinese dao.

1 Introduction: Progress in Climate Economics

At the beginning of the 21th century, climate policy worldwide has been strongly influenced by the red-green government of Germany. The ratification of the Kyoto protocol and the establishment of the European emissions trading scheme would have happened in very different ways - perhaps not at all - without the high priority given to climate policy by this government. Soon after these milestones of international climate policy, however, the red-green government lost its majority, mainly due to the dismal record of its unemployment policy. This is all the more remarkable as the so-called red-green project was originally based on the vision of a double-dividend policy: environmental policy was supposed to simultaneously protect the environment and generate new jobs. The latter effect was expected from increasing the relative price of energy in comparison with labor. This, it was thought, would lead to a substitution of additional labor for increasingly costly energy in the production process. Moreover, higher energy prices were expected to trigger innovations that would generate new markets at home and for exports. Faced with the demise of the red-green project, however, a fresh look at the relation between climate policy and unemployment seems warranted.

How important it is to improve our understanding of the nexus between climate policy and unemployment is also highlighted by the fate of the national allocation plan for emission rights in Germany. When the European emissions trading scheme had to be implemented at the national level, the very red-green government that had fought hard for that scheme in the first place got entangled in a bitter controversy about the effects of emissions trading on German unemployment. By then, the vision of the double dividend had already faded away, and the fear of generating additional unemployment led to a grandfathering scheme that handed out more emission permits than were required under a business as usual scenario.

Moreover, it is obvious that the relation between climate policy and unemployment is of similar importance in other countries plagued by persistent unemployment, like several European countries, as well as most developing countries. A key fact that deserves careful study is the relevance of unemployment for cost-benefit analyses of climate policy. In a full-employment situation, it is plausible that devoting additional resources to purposes of climate policy will reduce the resources available for other purposes. Estimating the magnitudes of such trade-offs is the business of cost-benefit analysis. In a situation with massive unemployment, however, calculating trade-offs of this kind becomes arbitrary and not very helpful. When 10% of the labor force are unemployed - as is the case in Germany - there are bound to be opportunities for Pareto-improving climate policies, i.e. policies that have a net benefit in comparison with the status quo even in strictly economic terms.

Before this background, the present document outlines a model for studying options to shape the long-term dynamics of the German economy. In climate economics, three main families of models have been used so far. First, there are macro-economic models of the Solow-Ramsey type, pioneered in the climate realm by Bill Nordhaus's DICE model.[2] Second, there are the computable general equilibrium (CGE) models, using algorithms

[2] At PIK, two models of this kind have been developed so far: the economic component of the ICLIPS integrated assessment model, and the MIND model of energy production with endogenous technical change. The work on the lagom family of models has been greatly facilitated by these pioneering efforts; see Edenhofer, O., Bauer, N., Kriegler, E., The impact of technological change on climate protection and welfare: Insights from the model MIND (Ecological economics, 54, 277-292, 2005).

of Scarf, Negishi, and others to simplify the apparatus of general equilibrium theory into structures that can be implemented on today's computers.[3] Third, there are ad-hoc models that model economic phenomena with a combination of intuition and empirical evidence, sometimes in the spirit of econometric research, sometimes along the lines of integrated assessment modelling.[4]

So far, these models have rarely been used to analyse the nexus between climate policy and unemployment. And before rushing to do so it is better to consider some known limitations of these models. Both the Solow-Ramsey models and the CGE models can be formulated as optimization of an aggregated utility function under dynamic constraints. However, the literature in theoretical economics has shown conclusively that this is a special case whose general relevance for the study of market economies is rather doubtful.[5] At a more abstract level, econophysicists have emphasised that in general aggregate utility functions can only be construed as path dependent functionals - and that therefore the selection of an economic trajectory cannot be understood as the result of an aggregated optimization process.[6] The difficulties of aggregation are compounded in the domain of production, where a large - and largely ignored - literature has highlighted the pitfalls of aggregate production functions.[7] For climate economics, a reasonable conclusion is to use these two kinds of models when the focus of inquiry is the neighborhood of a given full-employment situation, and to rely on other tools for other purposes. As for the ad-hoc models mentioned above, it remains to be seen whether they will become more widely used in a debate strongly imbued with theoretical arguments like the one on unemployment.

Clearly, the mentioned limitation concerns the linkage between micro- and macro-economics. Nowadays, a macro-economic model is hardly acceptable without explicit specification of a micro-economic structure it is compatible with. The competing schools of macro-economists - real business cycle theorists, reconstructed keynesians, proponents of monopolistic competition, etc. - all share the effort to spell out suitable micro-macro links. Unfortunately, we are far from a satisfactory understanding of these links, and this makes macro-economic policy advice less sound than one would like.[8]

These theoretical problems are quite relevant for the practical problem of German unemployment. Wage costs per unit of output are not higher in Germany than in directly competing countries, the German export industry is a global leader in the relevant markets, profits are higher than at any time since decades, the fraction of profits going into net investment is at its lowest since decades, government debt - amplified by German re-unification - inflates domestic demand and feeds opportunities to earn interest paid

[3]For the development of the lagom models, the interaction with Claudia Kemfert and her collaborators working with CGE models at DIW have been particularly fruitful; see Kemfert,C., Induced technological change in a multi-regional, multi-sectoral, integrated assessment model (WIAGEM) : impact assessment of climate policy strategies (Ecological economics, 54, 293-305, 2005).

[4]In this respect, the extremely productive exchange with Klaus Hasselmann and his collaborators working with the MADIAM model has provided important insights for our own effort. See Weber, M., Barth, V., Hasselmann, K., A multi-actor dynamic integrated assessment model (MADIAM) of induced technological change and sustainable economic growth (Ecological economics 54, 306-327, 2005). Of course, for specific purposes an ad-hoc model may be superior to models with a more impressive theoretical pedigree.

[5]See Kirman, A., Whom or What Does the Representative Individual Represent? (Journal of Economic Perspectives, 1992, 6, 117-36), for a good introduction to this problematique.

[6]See MacCauley, J., Dynamics of Markets. Econophysics and Finance, Cambridge, 2004, especially section 2.6. This argument is directly relevant to applications of standard cost-benefit analysis to climate policy.

[7]E.g. Herbert Simon in his Nobel lecture, Rational decision making in business organizations, American Economic Review, 69, 493-513, 1979.

[8]This problem is documented with a remarkable piece of self-criticism in A Critical Essay on Modern Macroeconomic Theory by Frank Hahn and Robert Solow (MIT Press, 1997).

from taxes. These facts are hard to reconcile with the two main received views in macroeconomics. On one hand, there is the claim that the whole problem is one of rigid labor markets leading to exaggerated wage costs, on the other hand, the claim that boosting effective demand by increasing government debt could solve the problem. Rather than building climate policy on the hope that somehow these difficulties will not matter, it seems appropriate to investigate the relation between climate policy and unemployment more carefully.

Recently, a research tradition has emerged that holds some promise of progress in this respect, namely dynamic stochastic general equilibrium theory.[9] Here, micro-economic agents are represented as taking their decisions under conditions of inevitable uncertainty. Although it has not yet been done, in such a setting it becomes possible to analyse robust patterns of economic macro-dynamics not as the outcome of a fictitious aggregated optimization process, but as path-dependent, contingent solutions for the co-ordination problems economic micro-agents are faced with. This is the guiding idea of the lagom model family.

The theoretical innovations required are considerable, and so patience is required. In a first step, it is important to set up a model structure suitable for analysing long-term options for German economic dynamics. The task of introducing the emissions side of the economy can only be accomplished once a viable approach to deal with the key modelling challenges outlined above has been found. Therefore, we first develop the basic structure of a model that can be used to analyse possible long-term economic dynamics under conditions of persistent unemployment. In view of the emissions question, the model structure is designed so as to allow for the introduction of additional products and markets. On this basis, production and use of commercial energy as well as the related greenhouse gas emissions shall then be introduced later on. We do not yet take this step in the present paper, but discuss some relevant issues in the concluding section.

In the basic structure presented here, several markets interact with each other and with the production process, each of them is influenced by random events, and the overall trajectory of the system is governed by the dynamic co-ordination of demand and supply for the various goods considered. The model is a macro-model in the sense that optimizing agents are treated implicitly via demand and supply functions. The basic idea is that these functions result from micro-decisions that in turn depend on expected demand and supply conditions.[10]

In the model, markets are incomplete - as they surely are in today's German economy. Therefore, the trajectory of the system describes an evolution of temporary equilibria in the course of which economic agents can update their expectations on the basis of new experience. Markets are imperfect - as they surely are in today's German economy as well. Therefore, producers are faced with falling demand curves for their products, and non-price signals such as unemployment and credit limits influence the working of the economy along with price signals. A defining feature of the model is the attention paid to the relation between market prices and reference prices, well-known from marketing,

[9]So far, there is no introductory textbook to the field. The reader on Dynamic Macroeconomic Analysis. Theory and Policy in General Equilibrium (Altug, S., Chadha J.S., Nolan, C., eds, Cambridge U.P., 2003) provides a good overview. See also Ljungqvist, L, and Sargent, T.J., Recursive Macroeconomic Theory, MIT Press, 2004.

[10]To provide so-called micro-foundations for the model requires an analyis of the individual agents involved - physical persons, households, businesses, and other institutional agents - and of the patterns resulting from their interactions. Such an effort is currently undertaken jointly by researchers working at PIK, DIW, FU Berlin, and further institutions.

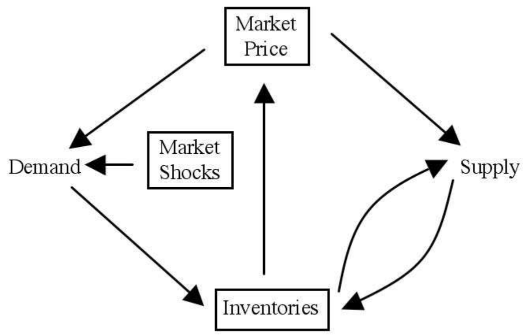


Figure 1: Basic structure of product markets

but all too often neglected in economics.[11]

In the next section, we introduce product markets, represented as simple stochastic dynamical systems (2). Next, we discuss labor markets, using a similar approach, although now the dynamics becomes more complex (3). Then comes the production process, where we take care to avoid the pitfalls of aggregated production functions (4). The dynamics of these three modules will depend on investment decisions that - now avoiding the pitfalls of aggregated utility functions - result form the dynamics of financial markets (5). The conclusion presents some suggestions for how to build on the present model to represent and design possible climate policies (6).

2 Product Markets

On product markets, producers set a price for their products and offer whatever quantity they consider appropriate. Customers are interested to purchase some quantity of the product, depending on the price they are faced with. The quantity they demand may be equal to the quantity offered, but usually the two will not match exactly. If supply exceeds demand, inventories increase, otherwise they decrease. If inventories are small, producers increase the quantity they produce and to some extent the market price of their product, too. Changes in quantities are usually much larger than changes in prices. If inventories are large, the process works the other way round.

The basic structure of product markets is indicated in figure 1. Let P_m be the market price, V inventories, q_d the quantity demanded, and q_s the quantity supplied. Inventories have the dimension "quantity of goods", prices the dimension "amount of money per quantity of goods", demand and supply the dimension "quantity of goods per unit of time", and time the dimension "amount of time". With the usual dot notation, $\dot{P}_m$ and $\dot{V}$ denote the change of P_m and V in time.

Next, let d_q be a *demand function* indicating how much customers will demand at different market prices, s_q a *supply function* indicating the combinations of market price and

[11] The distinction between market prices and reference prices provides a suitable starting point for meeting key challenges of macroeconomic modelling. This starting point is related to the way Adam Smith based his analysis of market dynamics on the distinction between market prices and natural prices in "The Wealth of Nations" (book I, chapter 7).

quantity supplied producers are willing to consider, f_x an *excess demand function*, simply indicating the difference between demand and supply. f_P is a *price adjustment function* indicating how producers change market prices depending on the level of inventories and of market prices. f_V is an *inventories adjustment function* indicating how inventories change depending on excess demand and the level of inventories. Finally, let v_q be market shocks, i.e. unpredictable events that influence the dynamics of demand. Let them follow a stochastic *market shock process* with the derivative $\dot{v}_q$ following some probability distribution g_q based on a probability measure, π. Then one can model a product market with the following functions:

$$\begin{aligned} \dot{P}_m(t) &= f_P(V(t), P_m(t)) \\ \dot{V}(t) &= f_V(f_x(q_d(t), q_s(t)), V(t)) \\ q_d(t) &= d_q(P_m(t), v_q(t)) \\ q_s(t) &= s_q(P_m(t), V(t)) \\ \pi(\dot{v}_q(t) \leq x) &= g_q(x) \\ P_m(0) &= P_{m,0} \ > 0 \\ V(0) &= V_0 \ > 0 \\ P_m(t),\ V(t), q_s(t), q_d(t) &\in I\!R_+,\ v_q(t), x \in I\!R, \end{aligned} \tag{1}$$

Market prices, inventories and market shocks are prognostic variables, i.e. there are functions indicating how these variables will evolve from given values (in figure 1 as in subsequent figures, prognostic variables are marked by boxes). Demand and supply are diagnostic variables, i.e. they are essential for empirical applications of the model but the dynamics of the model can be formulated without making them explicit. In this sense, the model is a three-dimensional stochastic dynamical system that (composing functions and omitting the time variable to simplify notation) can also be written as:

$$\begin{aligned} \dot{P}_m &= f_P(V, P_m) \\ \dot{V} &= f_V(f_x(d_q(P_m, v_q), s_q(P_m, V)), V) \\ \pi(\dot{v}_q(t) \leq x) &= g_q(x) \end{aligned} \tag{2}$$

The auxiliary variables are essential because they gear the mathematical structure to the basic economic phenomena of demand and supply. Thereby, it becomes possible to use the mathematical structure to analyse economic issues and to draw on a wealth of empirical and theoretical research about these issues.

To talk as if on product markets perfectly homogeneous goods were traded at uniform prices is often useful, sometimes dangerous. It can lead one to neglect the important fact that usually prices and quantities cannot be known with arbitrary precision. Therefore, it might sometimes be more appropriate to represent them by means of probability distributions rather than by means of real numbers. In the present setting, this would lead too far away from current practice in theory-driven economic modelling, however. Fortunately, with the emergence of dynamic stochastic general equilibrium models it has become possible to consider at least some important stochastic properties of interdependent markets in a theoretically promising way. Therefore, we include random shocks v_q in the basic model of product markets. For the moment, their mathematical representation is quite arbitrary, it's purpose is to prepare for inquiries about stochastic properties of the system later on.

What can be said about the form of the functions involved in the model? For many purposes, it is reasonable to take the demand function as smooth, and as monotonically decreasing in P_m and in $-v_q$.[12] The supply function can be taken as smooth, monotonically increasing in P_m and V, and asymptotically approaching full capacity utilization with increasing P_m.[13] As mentioned above, the excess demand function simply forms the difference between two real numbers - the economically relevant structure lies in the supply and demand functions yielding those numbers.

In general, the price adjustment function is smooth and monotonically decreasing in V: with large inventories, prices are lowered, with small ones, increased. From an economic point of view, the price adjustment function describes the aggregate outcome of producers taking decisions about an uncertain future. Inventories are costly, so it is reasonable to keep them as small as possible. But the future is uncertain, and it is important to have sufficient inventories to deal with demand shocks. At the micro-level, then, behind the price adjustment function lies a rather difficult decision problem. It can be represented by dynamic stochastic optimization procedures, and one may investigate how different agents cope with it. Here, we are satisfied with the aggregate description. The same holds for the role of reference prices. At the micro-level, producers are cautions when considering deviations from the reference price because such deviations involve the risk of losing the trust of customers. Therefore, at the aggregate level we take the price adjustment function as smooth and decreasing in P_m, with a zero when inventories have the desired level and the market price is equal to the reference price.

As long as inventories are not zero, the inventories adjustment function is just the negative excess demand function. When inventories are zero, this is still the case when supply exceeds demand, otherwise, inventories stay empty and some customers don't get what they want at current prices (at the micro level, this then implies some kind of rationing process).

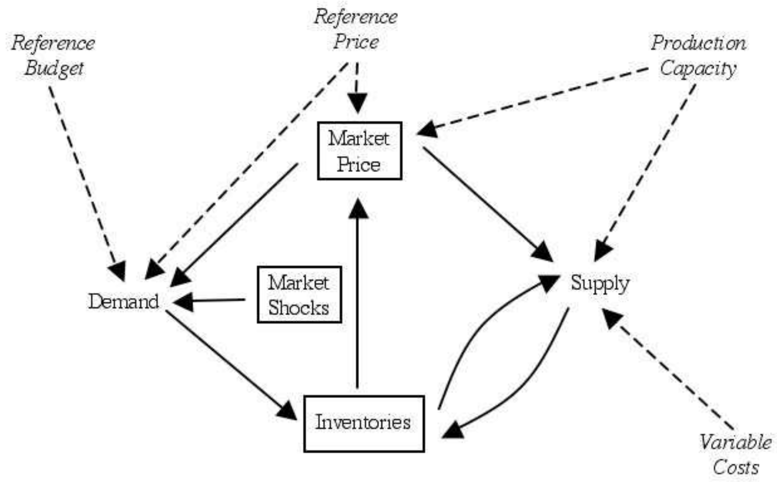
For some domain of positive prices, demand and supply functions share a co-domain of positive quantities - otherwise there would be no market for the product in question. The co-domain of f_P includes negative and positive values - prices can increase and decrease. Finally, the distribution of v_q may be taken to be unimodal with a variance that is small in comparison with the interval of possible prices.

The model we are faced with is a three-dimensional non-linear stochastic dynamical system. With the functional forms just specified and disregarding for a moment market shocks, there is a positive price and a positive level of inventories that form the unique fixed point of the system: if it is there, it will stay there until random shocks or exogenous developments drive it away. With random shocks, the market will fluctuate around its fixed point.

It looks natural to call this fixed point an equilibrium. When thinking about it, it is important to keep in mind that this equilibrium usually comes with positive inventories and that it depends on parameters that are likely to change in the course of time. Next, we identify these parameters. They are important because they are exogenous only as

[12]We call a function smooth if it has continuos first derivatives over the whole domain of its independent variables.

[13]With increasing returns, this means that higher prices go along with lower unit costs and correspondingly higher profits. What stops producers from increasing production indefinitely is the fact that at each moment in time each one of them is faced with a falling demand curve.
On a different note, it is useful to keep in mind that production capacity is not as clear cut a number as one might think - nevertheless it is a barrier of key importance for the working of markets.



Boxes denote prognostic variables, plain text labels diagnostic variables, labels in italics parameters.

Figure 2: Product market with parameters

long as a product market is considered in isolation. Once different components of the economy are considered together, they become endogenous variables.

A case in point is the role of income for the determination of demand. Demand certainly somehow depends on the amount of money available to customers. However, for many reasons this is not a clear-cut figure. A good way to represent it is by defining reference budgets for different goods, Y_q. Income elasticities can then be taken into account by letting reference budgets change with growing incomes.

The other parameter of considerable relevance for the determination of demand is given by reference prices, P_r. In marketing, the notion of reference prices is used to describe the fact that on a given market there usually is a shared sense of what is a reasonable price for the relevant products. In the course of time, reference prices are adjusted on the basis of new experience without completely forgetting the past.

Reference prices are equally important for the determination of supply - common knowledge about what constitutes a reasonable price for a given product is essential if producers are to take advantage of opportunities arising on the market and to avoid the danger of being driven out of business.

Another important parameter for the determination of supply is given by productive capacity, $\hat{K}$. In the short run, there is an upper limit to the output that can be produced, and this of course influences the form of the supply function. Finally, the combinations of product prices and quantities supplied a producers are willing to consider also depend on their variable costs, c_v (in particular the wage level W_m that we will consider in the next section).

Together with the random shocks, these parameters shape the dynamics of the system. The resulting structure of product markets is indicated in figure 2.

Finally, we list functional forms whose general shape is familiar from the literature and which provide a possible starting point for looking at product markets in Germany:[14]

$$
\begin{aligned}
\dot{P}_m &= 0.03 * P_m \left(1 - \frac{P_m}{P_r} * \frac{V}{0.05 * \hat{K}}\right) \\
\dot{V} &= q_s - q_d, \quad \text{if } q_s - q_d > -V \\
& \quad 0 \ \text{ otherwise} \\
q_s &= \hat{K} * \left(0.5 + 0.5 * \tanh\left(5 * \frac{P_m}{c_v} - 10 * \frac{V}{\hat{K}} - 4.1\right)\right) \qquad (3) \\
q_d &= -\frac{Y_q}{P_r} * 0.53 * \ln(0.15 * \frac{P_m}{P_r}) * (1 + 0.5 * v_q) \\
\pi(\dot{v}_q \leq x) &= \text{uniform distribution over the interval } (-1, 1) \\
\hat{K} &= 8000, \ \ Y_q = 5000, \ \ P_r \ = \ 0.666.., \ \ c_v \ = \ 0.56 \\
P_m(0) &= 0.666.., \ V(0) \ = \ 300, \ v_q(0) = 0
\end{aligned}
$$

It is perhaps useful to collect a few rough figures on the state of the German economy as of 2005 (in billion Euros):

Gross production value	5000
Gross social product	2000
Total depreciation	300
Net social product	1700
Total wage costs	1200
Total profits	500

This yields the following unit cost breakdown in percent. For purely illustrative purposes, a unit price of 0.66.. has been assumed. This leads to a unit cost breakdown in monetary units and figures for gross production and gross social product in units of generic good[15]. Finally, orders of magnitude are suggested for productive capacity and inventories, again in units of generic good:

Unit intermediate input costs	60% (= 0.4)
Unit depreciation	6% (= 0.04)
Unit wage costs	24% (= 0.16)
Unit profits	10% (= 0.066..)

Productive capacity	8000

[14] We use dimensionless notation.
A useful toolbox when specifying functional forms for this kind of systems is the family of functions that can be built from the exponential function by algebraic operations. One of these is the hyperbolic tangens, a function that gets arbitrarily close to 1 for increasing positive numbers and arbitrarily close to -1 for decreasing negative numbers: $\tanh(x) = (e^{2x} - 1)/(e^{2x} + 1)$. In (3), the numerical values of the parameters have been chosen so as to yield an illustrative pattern. Estimating parameter values from empirical data is one of the interesting challenges arising in developing the model further.
To start with, market shocks are supposed to happen at high frequency with regular intervals of time. More sophisticated approaches are possible, but not needed for the present outline.

[15] For the purposes of formal analysis, the generic good can be defined as an eigenvector of a comprehensive input-output table (i.e. including capital stocks) representing the German economy, with the rate of economic growth being equal to the corresponding eigenvalue minus 1. This approach is developed in the literature on the von Neumann model of general equilibrium - see e.g. the seminal treatise by Sraffa, P., Production of Commodities by Means of Commodities, Cambridge U.P., 1960.

Inventories	400
Physical gross production	7500 (=demand=supply)
Physical gross social product	3000

An array of product markets can now be modelled by introducing an index for each market, using it to distinguish the corresponding variables, functions, and parameters, and assigning suitable forms and values to functions and parameters. This, however, will usually lead to situations where the short-term modifications of reference budgets do not cancel out. As a result, the total budget is not consistent with total income - a problem that is likely to arise anyway due to short-term fluctuations in total income. To address this problem, which lies at the heart of general equilibrium theory, it is necessary to look at income formation and savings decisions. Therefore, and in order to start thinking about unemployment, we next consider labor markets.

3 Labor Markets

When analyizing labor markets, one can take the same starting point as with product markets: a demand and a supply curve depending on a price variable - in this case the market wage. From there one can again move towards a dynamic analysis by looking at how differences between demand and supply - via the rate of unemployment - influence the change of that variable. However, wage dynamics depend not simply on the unemployment rate, but rather on the relation between this rate and a reference rate of unemployment that is often referred to as the Nairu, the non-accelerating-inflation-rate-of-unemployment (more about this below). Figure 3 illustrates the resulting structure.

On labor markets, unemployment plays a similar role to inventories on product markets. There are important differences, though. They are related to the fact that labor is like electricity in so far as it cannot be stored. This has led to the habit of talking about labor as if it had the dimension "time", i.e. talking about hours of labor. The example of electricity shows that this way of talking makes sense only if these hours are characterized - at least implicitly - by some kind of intensity measure. When talking of electricity, we talk about "kilowatt-hours", not just hours. Clearly, the electrical current flowing for one hour through a 50 watt bulb has a different price from the current flowing for one hour through a 100 watt bulb. Moreover, electricity as an economic good is also characterized by further characteristics like voltage, reliability of delivery, and more.

With human labor, the basic characteristic is given in terms of persons: what matters are hours of activity of human beings, person-hours. And these persons must come with certain capabilities, including perhaps the ability to write programs, or to sell shoes, and to do so according to cultural standards of reliability. The dimension of labor then is not time, but activity of people, and the basic unit is the activity of a person with certain characteristics.

What counts as labor cannot be specified as a physical process, labor is as culturally defined as telling jokes or dancing. Where we will need a general term for the relevant activity, we will talk about economic activity of some number of people. Therefore, the quantities of labor supplied, labor demanded, and labor employed, l_s, l_d, l_e, have the

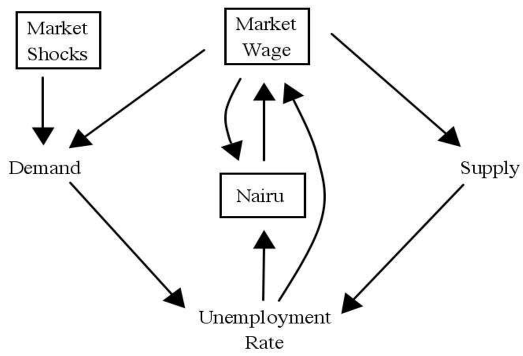


Figure 3: Basic structure of labor markets

dimension economic activity of a number of people.

Because labor cannot be stored, unemployment - unlike inventories - is not a stock of a good traded on some market. Unemployment indicates potential, but unrealized economic activity. The dimension of the market wage, W_m, is money per (economic activity of one person times one unit of time). This is the gross wage, indicating the costs incurred by the employer; its relation with net wages, the cash actually received by the employee, will be discussed in section 5, where we will investigate monetary matters. When discussing economic activity, an hour is usually a better time unit than a month or a year, because different person-months and -years may involve quite different numbers of hours. The dimension of wage costs, $l_e * W_m$ is money per hour: economic activity of a number of people times (money per (economic activity of one person times one hour)). Finally, it is useful at this stage to introduce output per hour, z, also called labor productivity. This is a key characteristic of production processes with the dimension output per (economic activity of one person times one hour).

While these elaborations may appear tedious, they can help to avoid serious conceptual confusions. For modelling purposes, a simple consequence is that unemployment differs from inventories by not being a state variable in an elementary model of labor markets. Unemployment would be a state variable in another kind of model, namely a demographic one where one would be interested to track the fate of people getting, having, losing, and missing jobs. And something akin to unemployment is an inventory in a slave economy: the number of slaves not put to work by their owners.

Let s_l and d_l be the *supply and demand functions* for labor and f_W the *bargaining function* for the market wage (as in section 2, f_x is the excess demand function). The bargaining function represents a bargaining process taking place partly as institutionalized negotiation between trade unions and employers associations, partly as an on-going and often informal negotiation process between individual employees or potential employees and managers. In this bargaining process, the level of unemployment is a critical variable, as it influences the bargaining strength of both sides - the higher the level of unemployment, the weaker the bargaining position of employees.

Labor supply depends on the wage level via the decision of households on what amount of labor they want (and need) to supply at the wage level they are faced with. Labor demand depends on the wage level in a more indirect way: once producers have decided

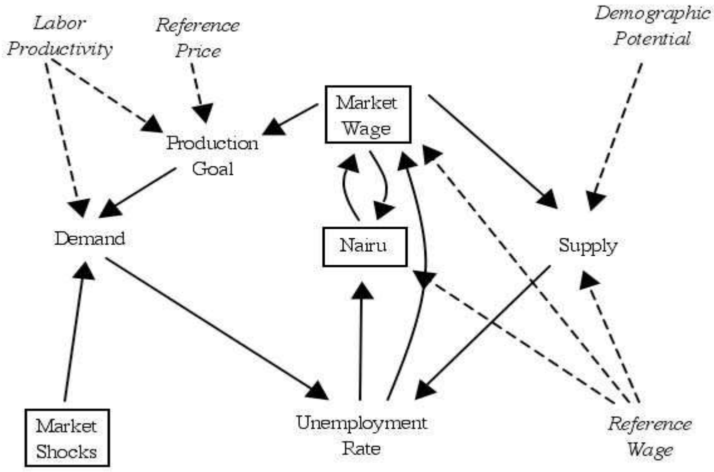


Figure 4: Labor market with parameters

how much they want to produce, their demand results from the current level of output per hour (output, that is, of the good to be produced). Wage costs in turn influence the decision about the production goal, q_q. When considering the labor market in isolation, we represent this influence by a product supply function s_q yielding the production goal as a response to wage costs. It is obtained from the supply function on the product market by freezing all variables determined on the product market at some given level while allowing variable costs to vary with wage costs. Later in this section, we will couple labor and product markets and include the interaction with the variables determined on the product market in an explicit way.

Next, let N_u be the Nairu, and f_N an adjustment function showing how it changes with the long-term development of unemployment. Finally, let v_l represent labor market shocks that influence the outcome of this bargaining process, again characerized by a probability distribution g_l for the derivative of these shocks.

With these tools one can model a labor market with the following functions:

$$\begin{array}{rcl} \dot{W}_m & = & f_W(N_u, u_l, v_l) \\ \dot{N}_u & = & f_N(N_u, u_l) \\ u_l & = & f_\mu(f_x(l_d, l_s)) = \max(l_s - l_d, 0) \\ l_d & = & d_l(q_q) \\ q_q & = & s_q(W_m) \\ l_s & = & s_l(W_m) \\ \pi(\dot{v}_l \leq x) & = & g_l(x) \\ W_m(0) & = & W_{m,0} \ > 0 \\ \multicolumn{3}{c}{W_m, N_u, u_l, l_s, l_d, q_q \in I\!R_+, \ v_q, x \in I\!R} \end{array} \tag{4}$$

This is a three-dimensional dynamical system that can also be written as:

$$
\begin{aligned}
\dot{W}_m &= f_W(N_u, f_\mu(f_x(s_d(s_q(W_m)), s_l(W_m))), v_l) \\
\dot{N}_u &= f_N(N_u, f_\mu(f_x(s_d(s_q(W_m)), s_l(W_m)))) \\
\pi(\dot{v}_l \leq x) &= g_l(x)
\end{aligned} \tag{5}
$$

Figure 4 shows the parameters involved in modelling labor markets. Labor demand by producers depends on the level of labor productivity, L_z. The production goal of producers depends on the reference price and on labor productivity (the impact of the market wage on production decisions varies with these parameters). Labor supply depends on demographic potential, l_p, even if this potential is less of a clear-cut figure than one might think, in particular because of migration possibilities, changes in retirement schemes, etc. Moreover, labor supply as well as the dynamics of market wages and of the Nairu depend on reference wages, W_r - on few markets do ideas of fairness play such an important role as on labor markets, and reference wages embody such ideas along with experiences from the past and expectations for the future.

In line with the model of the product market, for a start it is reasonable to take s_l, d_l, s_q, f_W, and f_N as smooth functions, with d_l, s_l and f_N monotonically increasing, s_q monotonically decreasing.

The wage barganining function, f_W, deserves a more differentiated treatment. Historical experience suggests that two different regimes should be distinguished, one with a low and one with a high Nairu. Currently, the former lies around 5%, the latter around 10%. Theoretical research has led to an understanding of the importance of multiple equilibria in economics that so far has been underutilized in applied research[16]. The economic worries of Germany as well as the challenge of climate policy are instances where the study of multiple equilibria can provide important insights. Over the past decades, Germany has moved from a low-Nairu regime to a high Nairu regime. Most OECD countries have experienced a similar transition starting around 1970. Some countries have found a way back, most notably the U.S. and the UK, but also Sweden, Australia, and Japan. Studying the possibility of such transitions is a key task when thinking about the economic future of Germany. When it comes to climate policy, it is quite obvious that without a transition to a low-Nairu regime it will be all but impossible to implement effective measures to lower greenhouse gas emissions. For these reasons, we use a wage bargaining function with two equilibria for the Nairu, one at 5% and one at 10%.

When defining possible functional forms for the labor market, two cases must be distinguished. Modelled as a stand-alone module, the labor market takes exogenous parameters that become endogenous variables when the labor market is coupled with other modules. We immediately consider the latter case:[17]

$$
\begin{aligned}
\dot{W}_m &= W_m * \left(1 - \frac{W_m}{W_r} * \frac{u_l}{N_u}\right) * (1 + v_l) \\
\dot{N}_u &= 1000 * \left(-N_u^3 + 0.225N_u^2 - 0.01625N_u + 0.000375\right) + \\
&\quad 0.05 * \left(\frac{u_l}{N_u} - 1\right)
\end{aligned}
$$

[16] The key result is the Sonnenschein-Mantel-Debreu theorem, see Sonnenschein, H. (1972), Market Excess Demand Functions, Econometrica, 40, 549-563. See also the literature on sunspot equilibria, e.g. Evans, G.W., Sakari, S.M. (2003) Existence of adaptively stable sunspot equilibria near an indeterminate steady state. Journal of Economic Theory, 111, 125-134.

[17] We will use the same procedure with the subsequent modules.

$$
\begin{aligned}
l_s &= l_p * \tanh\left(0.1 * \frac{W_m}{W_r} + 1\right) \\
l_d &= \frac{q_q}{z} \qquad (6)\\
q_q &= \hat{K} * \\
&\quad \left(0.5 + 0.5 * \tanh\left(5 * \frac{P_m}{c_u + (W_m/z)} - 10 * \frac{V}{\hat{K}} - 4.1\right)\right) \\
\pi(\dot{v}_l \leq x) &= g(x) \\
&\quad \text{with } g \text{ the uniform distribution over the} \\
&\quad \text{interval } (-1,1) \text{ and } v_l \in (-1,1) \\
\text{with} \quad l_p = 50, &\quad W_r = 33.33.., \; z = 208.33.., \; \hat{K} = 8000, \; P_m = 0.66.., \\
&\quad c_u = 0.4, \; W_m(0) = 0.33.., \; N_u(0) = 0.09, \; v_l(0) = 0
\end{aligned}
$$

Again, it may be useful to mention a few rough figures characterizing the German economy (in million people):

Employed	36
Unemployed	4
Unemployment ratio	10%

To make the figures used in the representation of the product market fit with those used here, a labor productivity of 208.33.. (unit of generic good per person employed) has been assumed along with a labor potential of 50 (million people).

Much as with the model of the product market, this system converges to a neighborhood of an equilibrium value for any admissible initial value. But there are two important differences. First, the Nairu adjustment function now leads to a non-linear system with two stable equilibria. Random shocks or exogenous influences can move it from one to the other. Second, the model represents the fact that labor markets hardly ever clear the way product markets often (not always) do. With the parameters chosen, the equilibrium configuration is one of persistent unemployment with an unemployment rate of about 10% for the high-Nairu regime and about 5% for the low-Nairu regime.

The key difference between product and labor markets in this regard is that on product markets the price adjustment function is equal to zero when supply equals demand, while on labor markets the wage adjustment function is equal to zero in a situation where supply is larger than demand. In the model of product markets, inventories are built into the supply function, while in the model of labor markets their analogue, unemployment, results from the bargaining function. This is related to the fact that while inventories exist mainly as a tool to manage the uncertainties of future market dynamics, unemployment is often much larger than would be needed to deal with market frictions. There is all the difference in the world between persistent unemployment of 2 or 3%, as experienced in the "golden age" of industrialized countries after World War II, and unemployment rates of 10% and more, as experienced in Germany and some other countries today. The present model shall help to understand this difference and what one might do about it.

Before investigating these matters, however, we need to cover more ground. First of all, we couple the product and the labor market. In the labor market, the only change consists in the fact that now the market price is not a parameter any more but rather a variable taken from the product market. In the product market, there is an analogous change because variable costs cease to be a parameter and come to depend on wage costs as taken from the labor market. This implies that the production goal becomes a variable jointly determined by the two markets.

Moreover, the production goal may turn out to be unrealistic because of insufficient labor supply. Therefore, we distinguish between a production goal function f_q and a supply function s_q. The supply function now simply takes the minimum of the production goal q_q and the product of labor supply and output per hour. The production goal function inherits the structure of the product supply function considered so far, with both market price and the market wage as independent variables.

Those parameters that have not turned into variables stay the same. With these specifications, the two coupled markets again reach one of the two equilibria with a Nairu of 10% or 5%.

The coupled model of product and labor markets yields a non-linear stochastic dynamical system with four deterministic state variables - market price, inventories, market wage, Nairu - and two stochastic components representing random shocks on each market.

With suitable modifications of the parameters, such a system is sufficient to create all sorts of oscillations and chaotic dynamics, and these may show interesting patterns in their own right. However, in economic terms this two-market system is useful mainly as a prelude to a model including investment and finance. Without these elements, there is little to be said about the German economy and its dynamics. Therefore, we now turn to the production process and then to financial markets.

4 Production Processes

When modelling production processes, it is important to distinguish between two time-scales. Inventory management and capacity utilisation evolve in the short term, capacity expansion and increase of labor productivity evolve in the long term.

The short term is related to market dynamics: if for some reasons firms are faced with falling demand, they react by filling up inventories and reducing production without immediately scrapping part of their productive capacity. With increasing demand, on the other hand, inventories are emptied and production increases until capacity limits are reached. Both ways, capacity utilisation shifts over time scales of weeks, not years. At this time-scale, which we considered in the section on product and labor markets, the production process can be represented by a *short-run production function* describing the quantity produced as the product of labor employed and output per head, with the constraint that the quantity produced cannot exceed the given productive capacity:

$$q_s = l_e * z,\ q_s \leq \hat{K}$$

Fluctuations in output then correspond to fluctuations in employment.[18]

The long term is related to production dynamics - to the expansion of productive capacity and the enhancement of labor productivity.[19] The relevant structure of production processes is represented in figure 1.

[18] The labor demand function considered in section 3 is based on the inverse of the short-run production function. In section 2, the constraint $q \leq \hat{K}$ set by production capacity is embodied in the form of the supply function.

[19] The first thorough analysis of the long-term expansion of productive capacity was provided by Marx in his "Capital",

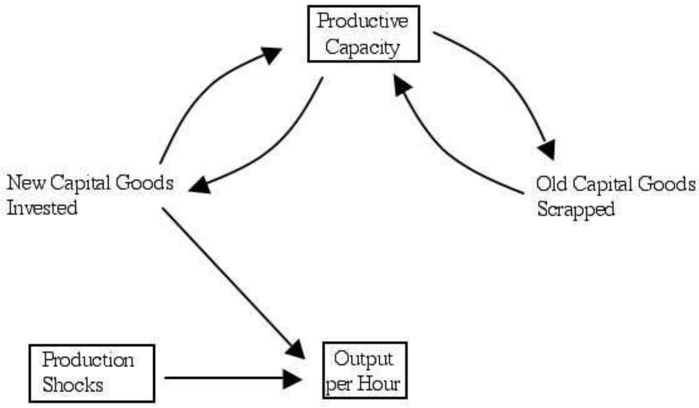


Figure 5: Basic structure of production processes

Consider productive capacity first. It is maintained and expanded by gross investment, I_q. This is the flow of new capital goods accumulated in the stock of such goods. Part of it is needed to offset the scrapping of old facilities, Γ_q, the rest increases productive capacity. The change of productive capacity results from the difference between gross investment and scrapped capital goods, each multiplied by its respective capital output ratio. The two ratios may or may not coincide. In the long run, their fluctuations seem to cancel out, so that for a start it seems reasonable to treat them as identical and constant.

Gross investment results from an *investment function* f_I. This function represents one of the most important processes in economic dynamics: the decision about how much to invest. When considering a production process in isolation, the subtleties of this decision get lost. In a preliminary way, then, we will represent investment as a linear function of total output, in the style of Solow growth models. Total output in turn results from productive capacity and a parameter for the degree of capacity utilization, β_q. When coupling the production process with the product market, this will become a variable. But eventually, the investment function will have to be defined in the context of financial markets, to be discussed in section 5.

The dimension of gross investment is units of new capital goods per unit of time. The dimension of productive capacity is units of output goods per unit of time. Gross investment is related to changes in productive capacity via the capital-output ratio for new capital goods, κ_I. As we keep intermediate products explicit, the capital output ratio refers to gross production; the ratio for gross social product is larger, depending on the proportion of intermediate products in gross production.

vol.2. He saw the importance of a concept of capital that would relate the dynamics of production to the dynamics of financial markets. But this requires mathematical tools to deal with vector spaces of different dimensions - one, many, infinite - that have been developed only in the 20^{th} century. One might say that he provided the Euclidean geometry of capitalist development - a geometry shared by the Ramsey-Solow-Romer family of growth models, whose dynamics is governed by an even more simplified concept of capital than the one used by Marx. Given the disasters his ideas contributed to, however, it will take some time until it will be possible to treat Marx in economics with the detachment that mathematicians display when talking about Euclid.

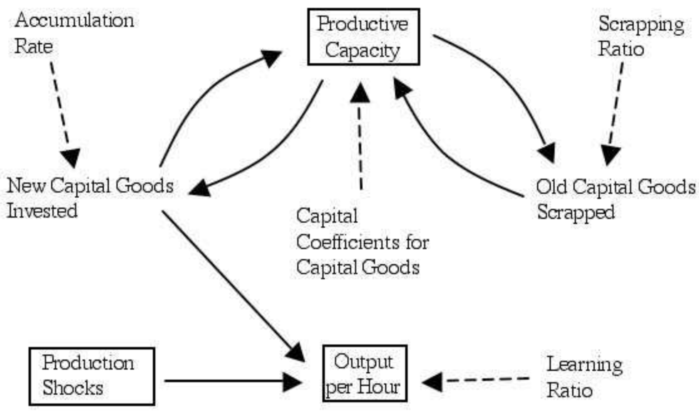


Figure 6: Production process with parameters

As long as we consider generic goods only, the capital-output ratio is a dimensionless number. Once we get interested in differences between goods, the capital stock becomes a vector, as do gross investment and scrapping. The capital output ratio for new goods then becomes a matrix ($\kappa_I[i,j]$) whose elements have the dimension product of type j per capital good of type i. All the formulas introduced can be generalized from scalar to vector format.

Scrapped old capital goods, Γ_q result from a *scrapping function* f_Γ. Again, when considering a production process in isolation, we are left with a very coarse representation of scrapping decisions. Still, a simple linear relation between productive capacity and scrapping provides a useful start for analysis. The dimension of scrapped goods is old capital goods per unit of time, scrapping is related to changes in productive capacity via the capital-output ratio for scrapped capital goods, κ_Γ.

Gross investment affects not only productive capacity, but also labor productivity, or, with a more sober term, output per hour of labor, z. A solid body of empirical research has shown that gross investment systematically fosters learning by doing, thereby enhancing output per hour. In particular, the effect of learning by doing on productivity growth is larger than the effect of expenditures for research and development.[20] Although the relation with gross investment is quite robust, the dynamics of output per hour still is quite erratic. We represent this dynamics by a learning function, f_z depending on gross investment and a random term v_p for production shocks, with an analogous notation to the one used for market shocks.

Figure 6 shows the model structure including the parameters used. With these tools,

[20]This line of research originated with ideas expressed in Arrow, K.J. (1962) Economic Implications of Learning by Doing, Review of Economic Studies 29,155-173; key empirical findings are due to Lieberman, M. (1987) Patents, Learning by Doing, and Market Structure in the Chemical Processing Industries, International Journal of Industrial Organization, 5, 257-276; a comprehensive overview has been provided by Scott, M. Fg. (1989) A new view of economic growth, Clarendon Press, Oxford; detailed U.S. data can be found in Bessen, J. (1997) Productivity Adjustments and Learning-by-Doing as Human Capital, Discussion Paper 97-17, Center for Economics Studies, U. S. Census Bureau, Washington D.C.

we can analyze the dynamics behind the so-called substitution of labor by capital. In the short run, varying degrees of capacity utilization lead to varying amounts of labor being employed with a given stock of capital. In the long run, it is instructive to look at full-capacity employment. Two polar cases can be distinguished. If gross investment is just sufficient to replace scrapping, the capital stock does not grow while output per hour increases due to learning by doing, leading to a decreasing amount of labor being necessary to operate the existing capital stock at full capacity. If productive capacity is increased, employment at full capacity may increase or decrease depending on the shape of the learning function.

Next, we retain the structure of the resulting model of production processes:

$$\begin{aligned}
\dot{\hat{K}} &= I_q * \kappa_I - \Gamma_q * \kappa_\Gamma \\
\dot{z} &= f_z(I_q, v_p) \\
I_q &= f_I(\hat{K}) \\
\Gamma_q &= f_\Gamma(\hat{K}) \\
\pi(\dot{v}_p \leq x) &= g_p(x) \\
&\hat{K}, I_q, z, \Gamma_q, \in I\!R_+,\ v_p, x \in I\!R
\end{aligned} \tag{7}$$

Written as a three-dimensional dynamical system the model looks as follows:

$$\begin{aligned}
\dot{\hat{K}} &= f_I(\hat{K}) * \kappa_I - f_\Gamma(\hat{K}) * \kappa_\Gamma \\
\dot{z} &= f_z(f_I(\hat{K}), v_p) \\
\pi(\dot{v}_p \leq x) &= g_p(x)
\end{aligned} \tag{8}$$

To start with, it is reasonable to assume all functions involved as smooth, increasing in each argument, and yielding zero when all arguments are zero. This leads to a three-dimensional linear stochastic system driven by a one-dimensional subsystem following an exponential trend. The rate of growth is determined by the investment share. For the moment, this share is an exogenous parameter; in the next section, we will turn it into a variable whose value depends on the interplay of the system considered so far with financial markets.

A simple set of functional forms and parameter values that offers a starting point for analysis is the following ($\tilde{I}_v$, discussed in section 5, is the actual investment share):

$$\begin{aligned}
\dot{\hat{K}} &= I_q * \kappa_I - \Gamma_q * \kappa_\Gamma \\
\dot{z} &= \alpha_z * I_q * (1 + v_p) \\
I_q &= \tilde{I}_v * \hat{K} * \beta_q \\
\Gamma_q &= \alpha_\Gamma * \frac{\hat{K}}{\kappa} \\
\pi(\dot{v}_p \leq x) &= g_p(x) \\
& \text{with } g \text{ the uniform distribution over the interval } (-1,1) \text{ and } v_p \in (-1,1) \\
\hat{K}(0) = 8000, \quad \kappa_I = \kappa_\Gamma = 1, \quad \alpha_\Gamma &= 0.056, \alpha_z = 0.007, \\
\beta_q = 0.94, \quad z(0) = 208.33.., \quad & v_p(0) = 0
\end{aligned} \tag{9}$$

Orders of magnitude that may be used to characterize the aggregate production process of the German economy are (again using a fictitious price of 0.66 for conversion into units of generic good):

growth rate	1%
investment share	7.5%
value of gross investment	380 billion
depreciation	300 billion
value of net investment	80 billion
value of productive capacity	8000 billion
capital stock in units of generic good	12000
new capital goods invested	570
thereof replacing scrapped capital	450
net investment in units of generic good	120

For the dynamics of technical progress, a learning rate of 0.007 is roughly consistent with the data. As mentioned in the previous sections, a productive capacity of 8000 units of generic goods, gross production of 7500 units, a gross social product (i.e. without intermediary inputs) of 3000 units and a labor productivity of 208.33.. units of generic good per person employed have been assumed.

To couple the production process with the product and labor market, five steps are needed. First, capacity utilisation is turned from a parameter into a variable; it now enters the production process as the result of the interplay between demand and supply on the product market. Second, in an analogous fashion labor productivity enters the labor market as the result of learning by doing in the production process. Third, reference wages grow with labor productivity and with reference prices.

While these steps are sound representations of economic realities, the next two are more provisional. To model the long-term dynamics of the product market, we need a dynamics of reference budgets. Budgets depend on incomes, of course, but between incomes and expenditures lie the dynamics of money and credit. For the moment we bypass these by Say's law - an analytical device that should rather be known as Say's shortcut. Here we use it by setting reference budgets equal to product supply times reference prices. A link taking account of money and credit will be established in the next section.

Reference prices depend on inflation expectations, and these are still not well understood[21]. To start, we let reference prices grow according to an exogenous inflation expectation (2% is a good guess for Germany). Variable costs except wage costs are then geared to the increase in reference prices.

The coupling of the three components developed so far yields a non-linear seven-dimensional stochastic dynamical system with two locally stable trajectories of exponential growth.

[21] Keynes, an economist with a thorough mathematical training in probability theory, changed the world with a set of ideas that included bold and somewhat obscure conjectures about economic expectations and animal spirits. Friedman challenged these ideas with claims about adaptive expectations of future inflation, claims that are easier to understand at first sight, have a wealth of data as background, and lead into a theoretical quagmire. Lucas and Sargent revolutionized the field with the elegant, if somewhat oversold idea of rational expectations. The next wave in this area of research is no doubt in the making. Examples of current contributions are Andolfatto, D.,Hendry, S., Moran, K. (2002) Inflation Expectations and Learning about Monetary Policy, Bank of Canada, Working Paper 2002-30; and Mankiw, N.G., Reis, R., Wolfers, J. (2003) Disagreement about Inflation Expectations, Harvard Institute for Economic Research, Working Paper 2011. For the time being, a mix of adaptive and rational expectations seems a good way to go - and on this basis for the long-term analyis of a country like Germany the assumption of a constant inflation expectation is a useful first approximation

The low-growth trajectory comes with growth rates of less than 2% and a persistent unemployment rate that starts around 10% and grows with irregular steps of about 3%. The high-growth trajectory comes with growth rates of 2.5% or more and unemployment rates below 4%. Between the two trajectories lies a region where the economy slowly and erratically meanders between these two possibility. Which behavior is found depends on the level of net investment, which in turn depends to a large extent on the dynamics of financial markets.

5 Financial Markets

What is the role of financial markets in the determination of investment? Financial markets generate demand for investment goods on the basis of expectations about future returns. A new airplane is not built to satisfy present demand for plane tickets in the future, but to satisfy present demand for airplanes based on expectations of future ticket sales. And the relevant expectations are those of airlines who purchase planes, banks who give or deny credit to airlines, and financial investors - including banks - who buy or sell stock of those airlines.

In the economy we live in, investment is not determined by a set of complete future markets - markets on which contracts for all conceivable future contingencies would be traded today -, but rather by financial markets. They generate investment decisions based on social expectations where price signals don't exist. In the short run, these expectations refer to a very large extent to the internal dynamics of financial markets themselves - each financial investor tries to guess what will happen as the result of the guesses of all other investors. Therefore, the short-run dynamics of financial markets are extremely complex and hard to grasp.[22] In the long run, the internal dynamics is much simpler, leading to an amazing stability of the interest rate over centuries and even millennia.

We can discuss this dynamics by looking at three auxiliary variables: the share of investment, new credit, and the rate of profits (see figure 7). Their dynamics depends on six state variables: desired investment share, depreciation, volume of credits, credit limits, value of productive capacity, and financial shocks. The interface to other components of the economy is defined by parameters like revenues and costs. Finally, there are parameters that can be influenced by policy decisions. These include taxation schemes for property income, new government debt, and the rate of interest set by central banks (see figure 8 for the role of these parameters).

With regard to the generation of investment demand, the role of stock and credit markets are quite similar: the expectations of financial investors, including banks, determine to what extent the expectations of other businesses are translated into investment decisions[23]. Basically, financial investors select those - necessarily risky - entrepreneurial investment projects they consider sufficiently promising in comparison to the low-risk return to be expected from government bonds.

Credit is quite different from typical economic goods because credit transactions are based on assessments of default risk that require information about the individual parties in-

[22] For an instructive discussion of these complexities and their relevance for modelling short-term financial market dynamics see Derman, E. (2004) My Life as a Quant : Reflections on Physics and Finance, Wiley.

[23] Therefore, we do not represent stock markets explicitly at the present stage of model development, but limit ourselves to the representation of credit - the basic mechanism remains the same with stock markets.

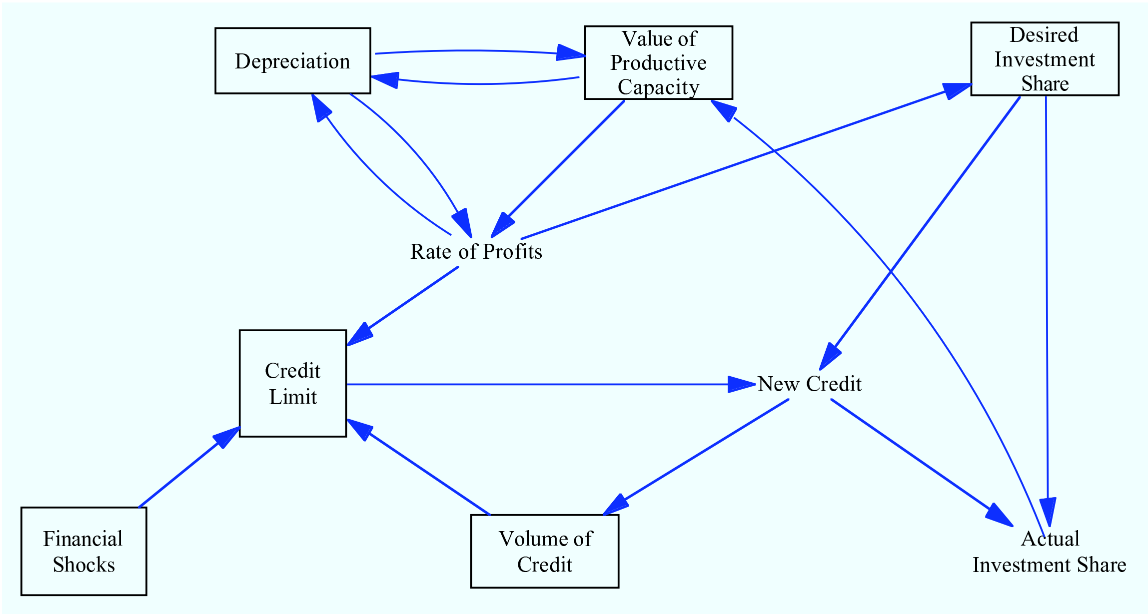


Figure 7: Basic structure of financial markets

volved[24]. If somebody wants to buy a typical economic good it is sufficient to pay the price to do so. If somebody wants a credit at the ruling rate of interest the whole point of the transaction is that he will have to pay the credit back at a later stage - and therefore the creditor needs to assess whether the debitor will actually be able to fulfill the promise he is willing to make in order to get the credit.

We represent the process of investment determination by distinguishing between gross investment as defined in section and the quantity of gross investment business would envisage in the absence of financial constraints. We express both quantities as shares of total output evaluated at market prices, in order to get constant numbers if changes in investment reflect economic growth rather than changing investment decisions. We write $\tilde{I}_v$ for the actual investment share and $\tilde{I}_d$ for the desired one. The actual share then results from an *investment selection function*, f_s. Writing C_b for the total amount of credit, the amount of new credit made available for investment purposes can be represented by the net amount of new credit made available by the banking system, $\dot{C}_b$, minus the net amount of new debt desired by government, D_n. Clearly, government debt is a key parameter from a policy point of view.

What determines desired investment? Expected profits as assessed by the business in question. We represent this assessment by an *investment adjustment function* f_I. This function yields changes in desired investment; actual investment then results from applying the selection function to desired investment. Of course, when assessing expected profits, business is aware of actual profits as well as of the rate of interest, defining the cost of credit as well as the return on low-risk government bonds. In the German economy of the present, the rate of interest, ι is a parameter, mainly depending on decisions of the European central bank. As profits are riskier than interests, investors will want the rate

[24]For an inspiring analysis of the specificities of financial markets see Stiglitz, J.E., Greenwald, B. (2003) Towards a New Paradigm in Monetary Economics, Cambridge U.P.

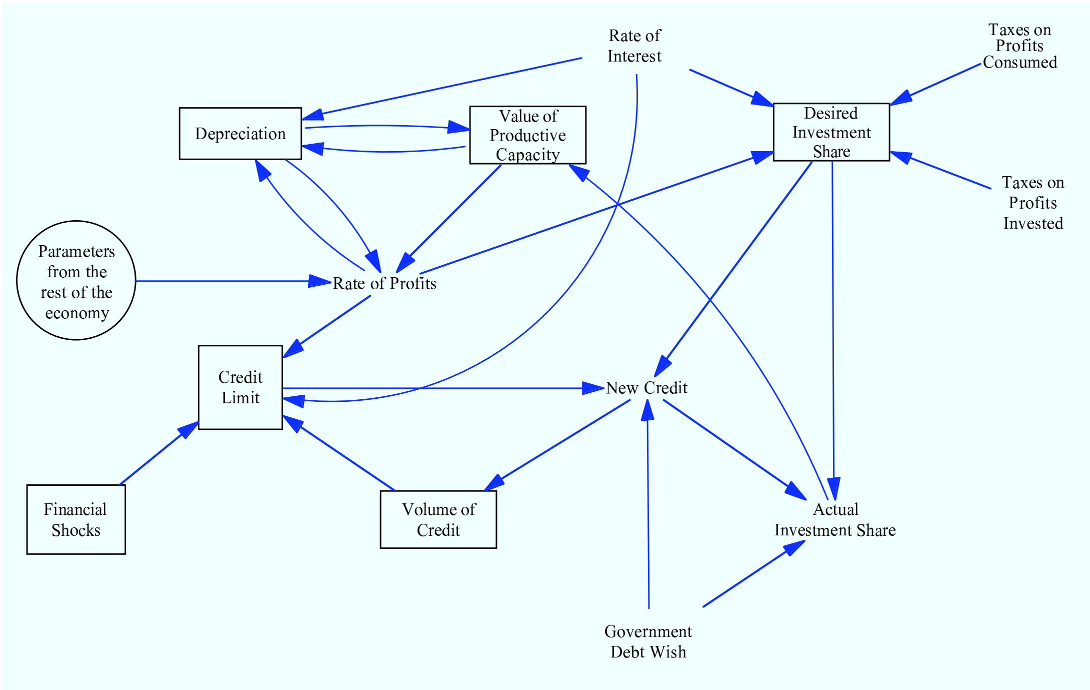


Figure 8: Financial market with parameters

of profit to be considerably higher than the rate of interest.

Desired investment is also influenced by the taxation of property income, the source out of which net investment is financed nearly exclusively in Germany. Property income invested in capacity expansion is taxed at a different rate than property income invested in government bonds or spent for consumption. We represent this influence by two parameters, again of key interest from a policy point of view: the taxation of property income spent on investment in productive capacity, τ_I, and the taxation of property income spent for other purposes (mainly consumption or government bonds), τ_x.

One might think that the rate of profits is a simple signal passed from the production process to financial markets, but this would be a rather misleading picture. Reasonable assessments of the rate of profits of any business are rarely to be found in its books. There are many problems here, but the most important one is how to evaluate productive capacity in the course of time. Just adding up the historical costs incurred in creating that capacity will not do: what matters for the valuation of productive capacity are not past costs, but future profits, and even if they could be known with certainty, these two magnitudes usually lead to very different valuations. Assessing what a business is worth is one of the key functions performed by financial markets, and therefore the rate of profit is a signal produced jointly by the production process and these markets. In figure 8 this is represented by letting the interest incentive - and thereby the relation between the rate of profits and the rate of interest - influence depreciation D_K (and thereby the valuation of productive capacity). We will represent this influence by a *depreciation function* f_D.

As for new credit, the problem of default risk means that the rate of interest does not suffice as co-ordination signal on credit markets: whatever the rate of interest, there will always be a default risk that can be further reduced by refusing and/or limiting credits to risky clients. This means that on credit markets, prices will be systematically complemented by non-price signals, with the result that these markets are routinely governed by a combination of price incentives and rationing processes. We model this situation by a credit limit $\bar{C}_b$. Its change resuls from a *risk management function* f_r depending on the given credit limit, the volume of credit, the interest incentive, and financial shocks. New credit then is the minimum between the credit limit and demand for new credit, i.e. the sum of desired government debt and desired investment. We write f_C for the *credit function* that yields new credit along these lines.

On the basis of these arguments, we can represent financial markets with the following functions:

$$\begin{aligned}
\dot{\tilde{I}}_d &= f_I(\tilde{I}_d, r) \\
\dot{K} &= \phi_K(\tilde{I}_v, D_K) \\
\dot{D}_K &= f_D(D_K, K, \tilde{I}_v, r) \\
\dot{C}_b &= f_C(\bar{C}_b, \tilde{I}_d) \\
\dot{\bar{C}}_b &= f_r(\bar{C}_b, C_b, r, v_f) \\
\pi(\dot{v}_f \leq x) &= g_f(x) \\
r &= \frac{P}{K} \\
P &= \phi_P(\dot{K}) \\
\tilde{I}_v &= f_s(\tilde{I}_d, C_b)
\end{aligned} \tag{10}$$

These functions yield a six-dimensional stochastic dynamic system. Depending on the functional forms assumed, such a model can lead to highly involved dynamic patterns. And when looking at data of short term dynamics, this is what one would want to get. Given the remarkably stable long term dynamics of the German economy, however, for our present purposes it is more reasonable to assume functional forms and initial values that lead to a single stable trajectory for a rather wide range of values of the exogenous parameters.

The following specifications lead to trajectories akin to those observed in the past. They are geared to the other components of the economy via the parameters $q_s, q_d, P_m, c_u, l_s, l_d$, and W_m. When considering the financial market in isolation, one can set initial values and growth rates for these parameters at typical levels taken from the coupled simulation of the other model components. Notice how financial markets are coupled to the rest of the economy: they synthesize information about the rest of the economy via the rate of profits and influence its course via the investment share.

$$\begin{aligned}
\dot{\tilde{I}}_d &= \tilde{I}_d * \left(1 - \frac{\tilde{I}_d}{i_\tau * (0.01 * i_\iota + 0.99)}\right) \\
\dot{K} &= I_v - D_K \\
\dot{D}_K &= 0.001 * \frac{K}{i_\iota} \\
\dot{C}_b &= \min(I_b + D_n, \bar{C}_b) \\
\dot{\bar{C}}_b &= \min(0.035 * \bar{C}_b * (0.5 * i_\iota + 0.5) *
\end{aligned}$$

$$
\begin{aligned}
& & & (1+v_f), 0.07 * C_b) \\
r &= & & \frac{P}{K} \qquad (11) \\
P &= & & \min(q_s, q_d) * (P_m - c_u) - \\
& & & \min(l_s, l_d) * W_m - D_K \\
\tilde{I}_v &= & & \min\left(\tilde{I}_d, \frac{\max(\dot{C}_b - D_n, 0)}{0.25 * q_s * P_m}\right) \\
i_\iota &= & & \frac{r}{3 * \iota} \\
i_\tau &= & & 0.1 * ((0.5 * \tau_x^2 + 0.5) * (1-\tau_I)^2) \\
& & & + 0.05 \\
I_b &= & & 0.25 * \tilde{I}_d * q_s * P_m \\
I_v &= & & \tilde{I}_v * q_s * P_m \\
\pi(\dot{v}_f \leq x) &= & & g_f(x): \\
& & & \text{uniform distribution over}(-1,1) \\
\tilde{I}_d(0) = 0.082, & \quad K(0) = 5333.33.., & & D_K(0) = 266.66.., \ C_b(0) = 4000, \\
\bar{C}_b(0) = 200, & \quad v_f(0) = 0, & & \tau_I = 0.25, \ \tau_x = 0.35, \\
D_n(0) = 100, & \quad \dot{D}_n = 0.035 * D_n, & & \iota = 0.04
\end{aligned}
$$

Together with the other three model components, and with the parameter values indicated so far, this yields long-term growth of about 1% with an unemployment rate erratically fluctuating around 10% (see fig. 9) - thereby reproducing key stylized facts of the current German economy in an intelligible way.

6 The challenge of emissions reduction

From a climate policy point of view, a second finding is crucial. The system can also follow an alternative path with unemployment fluctuating around somewhat less than 5% and economic growth increasing to more than 2%. This is clearly desirable on grounds of economic policy - but it can easily lead to significant increases in greenhouse gas emissions.

To deal with this potential conflict between economic and climate policy, it is essential to analyse how a transition from the low-growth, high-unemployment path to the high-growth, low-unemployment path might be achieved. Neither a neo-classically inspired policy of decreasing wage costs nor a policy of deficit spending in the keynesian tradition can achieve this. Rather, a combination of three measures is required. First, tax incentives must be shifted so as to encourage investment and discourage consumption from profit incomes.[25] Second, the steady increase of government debt must be stopped.[26] Third, the rate of interest must be reduced by about a percentage point - a relatively trivial step if the second measure is realized. Together, these measures amount to a financial transition whereby the power of financial markets is used to mobilize idle economic resources.

[25] Such a policy has been pursued in Sweden and Austria, and in both these two rather different economies it has yielded very positive results.

[26] Germany had achieved this goal before the poorly managed process of economic re-unification, the U.S. before the Iraq war. Decreasing government debt would be helpful, but not required.

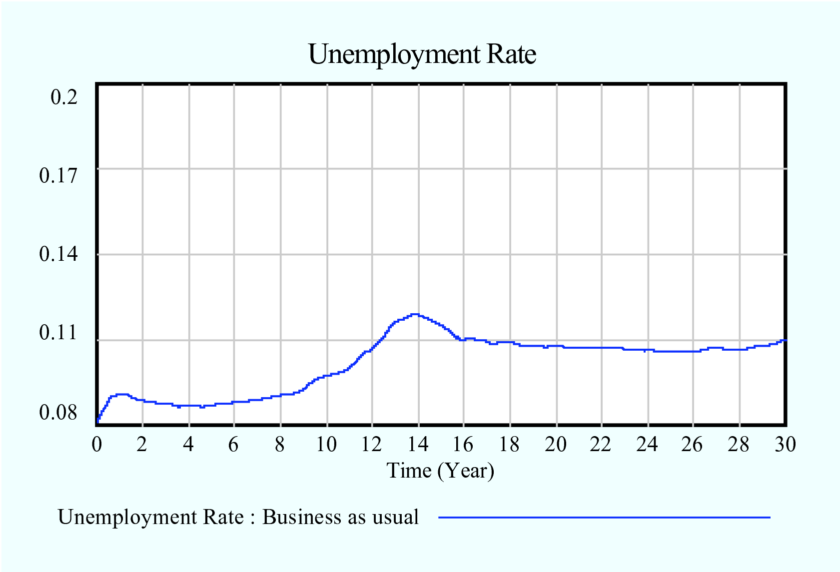


Figure 9: The low-growth, high-unemployment trajectory

With such a transition, it seems possible to achieve a shift from the first to the second path in about ten years, reducing the unemplyoment rate by more than 0.5 percentage points per year during that decade (see fig. 10). This means a Pareto improvement over that period. It is triggered by an increase in investment that induces an increase in effective demand while the growth of government debt decreases.

From a climate policy point of view, this offers a striking opportunity: the additional resources mobilized in the process can be used to reduce the energy intensity of the German economy. Over the relevant time-horizon of a decade, it would be impossible to transform the whole system of German power plants. But it seems possible to systematically reduce the energy needed for heating and for transport to an extent that would result in a significant overall reduction of greenhouse gases.[27] In the longer term, then, additional opportunities involving major changes in the production of commercial energy can be seized.

The key point is the stark contrast between an economic policy that exacerbates the emission problem in order to shift the system to a different growth-trajectory and the same policy that eases the emission problem by setting appropriate incentives for the use of the additional economic resources mobilized in the transition.

The financial transition then offers opportunities for Pareto improving structural changes that are desirable from a climate policy point of view. To analyse these opportunities, the model will have to be expanded so as to include markets and production processes relevant for reducing greenhouse gas emissions.[28]

When doing so, it will be useful to keep in mind the distinction between models as used

[27] One needs not to agree on every item in the long list of emissions reduction opportunities provided by S.Pacala and R.Socolow, Stabilization Wedges: Solving the Climate Problem for the Next 50 Years with Current Technologies (Science, 305, 968-972, 2004) to see that significant opportunities do in fact exist both in the shorter and the longer run.

[28] This will require additional enhancements, in particular concerning the dynamics of reference prices and reference budgets when new goods enter the market.

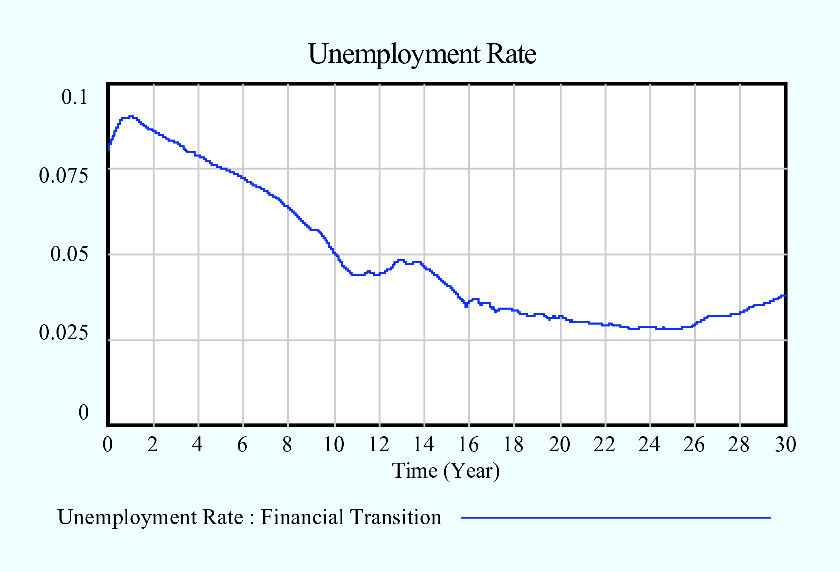


Figure 10: The high-growth, low-unemployment trajectory

in, say, meteorology and in architecture. In the former case, it is possible and desirable to provide a model describing with considerable accuracy processes taking place quite independently of human action. In the latter case, the goal is not to provide a model of all actual and even less all possible buildings, but to use a model to organize specific courses of action.

Both the reduction in unemployment and the reduction in emissions envisaged here are of a kind that will hardly ever be amenable to precise quantitative forecast. What an economic model like lagom$^{\text{d_sim}}$ can realistically provide is a toolbox to sketch possible courses of action, a toolbox that can be used first to design a reasonable policy and then to improve it on the basis of the experiences made with its implementation. In this perspective, the current version prepares the ground for some structured thinking about an economic landscape relevant for climate policy. If the ground proves to be fertile, subsequent versions should be useful to develop current climate policies in countries like Germany so as to enhance their effectiveness and credibility in the years to come.

7 Appendix: Symbol List

α_Γ	scrapping rate
α_I	investment rate
α_z	learning rate
β_q	capacity utilization
C_b	total amount of credit
$\bar{C}_b$	credit limit
c_v	variable costs
c_u	variable costs without wages
D_K	depreciation
D_n	desired net change of government debt
d_l	demand function for labor
d_q	demand function for product
f_C	function for new credit
f_D	depreciation function
f_Γ	scrapping function
f_I	investment adjustment function
f_μ	function taking the maximum of a variable and zero
f_N	adjustment function for Nairu
f_P	adjustment function for market prices
f_q	production goal function
f_r	risk management function for credit
f_s	selection function for investment
f_V	adjustment function for inventories
f_W	bargaining function for market wage
f_x	excess demand function
f_z	learning function
g_f	probability distribution for financial market shocks
g_l	probability distribution for labor market shocks
g_p	probability distribution for production shocks
g_q	probability distribution for product market shocks
Γ_q	scrapping of old facilities
I_b	desired investment credit
$\tilde{I}_d$	desired investment share
I_q	gross investment
I_v	value of gross investment
$\tilde{I}_v$	actual investment share
i_ι	interest incentive
i_τ	tax incentive
ι	rate of interest
K	value of productive capacity
$\hat{K}$	productive capacity
κ	Total production per average capital goods
κ_I	Total production per new capital goods
κ_Γ	Total production per scrapped capital goods
l_d	labor demanded

l_e	labor employed
l_p	demographic labor potential
l_s	labor supplied
L_z	labor productivity
N_u	Nairu
P	Profits
P_m	market price
P_r	reference price
ϕ_K	capital valuation function
ϕ_P	profit function
π	probability measure on the real line
q_d	quantity demanded
q_q	production goal
q_s	quantity supplied
s_l	supply function for labor
r	rate of profits
s_q	supply function for product
τ_I	tax on profits spent for entrepreneurial investment
τ_x	tax on profits not spent for entrepreneurial investment
u_l	rate of unemployment
V	inventories
v_f	financial market shocks
v_l	labor market shocks
v_p	production shocks
v_q	market shocks on product markets
W_m	market wage
W_r	reference wage
x	real number
Y_q	reference budget for product q
z	output per hour of labor

References

[1] S. Altug, J.S. Chadha, and C. Nolan (eds). *Dynamic Macroeconomic Analysis; Theory and Policy in General Equilibrium.* Cambridge, U.P., 2003.

[2] D. Andolfatto, S. Hendry, and K. Moran. Inflation expectations and learning about monetary policy. *Bank of Canada*, Working Paper 2002-03, 2002.

[3] K. J. Arrow. Economic implications of learning by doing. *Review of Economic Studies*, 29:155–173, 1962.

[4] J. Bessen. Productivity adjustments and learning-by-doing as human capital. *U.S. Census Bureau, Washington D.C.*, Economic Studies 97-17, 1997.

[5] E. Derman. *My Life as a Quant: Reflections on Physics and Finance*. Wiley, 2004.

[6] O. Edenhofer, N. Bauer, and E.Kriegler. The impact of technological change on climate protection and welfare: Insights from the model mind. *Ecological economics*, 54:277–292, 2005.

[7] G.W. Evans and S.M. Sakari. Existence of adaptively stable sunspot equilibria near an indeterminate steady state. *Journal of Economic Theory*, 111:125–134, 2003.

[8] F. Hahn and R. Solow. *A Critical Essay on Modern Macroeconomic Theory.* MIT Press, 1997.

[9] C. Kemfert. Induced technological change in a multi-regional, multi-sectoral, integrated assessment model (wiagem) : impact assessment of climate policy strategies. *Ecological economics*, 54:293–305, 2005.

[10] A. Kirman. Whom or what does the representative individual represent? *Journal of Economic Perspectives*, 6:117–36, 1992.

[11] M. Lieberman. Patents, learning by doing, and market structure in the chemical processing industries. *International Journal of Industrial Organization*, 5:257–276, 1987.

[12] L. Ljungqvist and T.J. Sargent. *Recursive Macroeconomic Theory.* MIT Press, 2004.

[13] J. MacCauley. *Econophysics and Finance*. Cambridge U.P., 2004.

[14] N. G. Mankiw, R. Reis, and N. Wolfers. Disagreement about inflation expectations. *Harvard Institute for Economic Research*, Working Paper 2011, 2003.

[15] K. Marx. *Das Kapital, vol. 2.* Hamburg, 1885.

[16] S. Pacala and R. Socolow. Stabilization wedges: Solving the climate problem for the next 50 years with current technologies. *Science*, 305:968–972, 2004.

[17] M. F. Scott. *A New View of Economic Growth.* Clarendon Press, Oxford, 1989.

[18] H. Simon. Rational decision making in business organizations. *American Economic Review*, 69:493–513, 1979.

[19] A. Smith. *The Wealth of Nations I.* Edinburgh, 1776.

[20] H. Sonnenschein. Market excess demand functions. *Econometrica*, 40:549–563, 1972.

[21] P. Sraffa. *Production of Commodities by Means of Commodities*. Cambridge, U.P., 1960.

[22] J.E. Stiglitz and B. Greenwald. *Towards a New Paradigm in Monetary Economics*. Cambridge U.P., 2003.

[23] M. Weber, V. Barth, and K. Hasselmann. A multi-actor dynamic integrated assessment model (madiam) of induced technological change and sustainable economic growth. *Ecological economics*, 54:306–327, 2005.

PIK Report-Reference:

No. 1 3. Deutsche Klimatagung, Potsdam 11.-14. April 1994
Tagungsband der Vorträge und Poster (April 1994)

No. 2 Extremer Nordsommer '92
Meteorologische Ausprägung, Wirkungen auf naturnahe und vom Menschen beeinflußte Ökosysteme, gesellschaftliche Perzeption und situationsbezogene politisch-administrative bzw. individuelle Maßnahmen (Vol. 1 - Vol. 4)
H.-J. Schellnhuber, W. Enke, M. Flechsig (Mai 1994)

No. 3 Using Plant Functional Types in a Global Vegetation Model
W. Cramer (September 1994)

No. 4 Interannual variability of Central European climate parameters and their relation to the large-scale circulation
P. C. Werner (Oktober 1994)

No. 5 Coupling Global Models of Vegetation Structure and Ecosystem Processes - An Example from Arctic and Boreal Ecosystems
M. Plöchl, W. Cramer (Oktober 1994)

No. 6 The use of a European forest model in North America: A study of ecosystem response to climate gradients
H. Bugmann, A. Solomon (Mai 1995)

No. 7 A comparison of forest gap models: Model structure and behaviour
H. Bugmann, Y. Xiaodong, M. T. Sykes, Ph. Martin, M. Lindner, P. V. Desanker, S. G. Cumming (Mai 1995)

No. 8 Simulating forest dynamics in complex topography using gridded climatic data
H. Bugmann, A. Fischlin (Mai 1995)

No. 9 Application of two forest succession models at sites in Northeast Germany
P. Lasch, M. Lindner (Juni 1995)

No. 10 Application of a forest succession model to a continentality gradient through Central Europe
M. Lindner, P. Lasch, W. Cramer (Juni 1995)

No. 11 Possible Impacts of global warming on tundra and boreal forest ecosystems - Comparison of some biogeochemical models
M. Plöchl, W. Cramer (Juni 1995)

No. 12 Wirkung von Klimaveränderungen auf Waldökosysteme
P. Lasch, M. Lindner (August 1995)

No. 13 MOSES - Modellierung und Simulation ökologischer Systeme - Eine Sprachbeschreibung mit Anwendungsbeispielen
V. Wenzel, M. Kücken, M. Flechsig (Dezember 1995)

No. 14 TOYS - Materials to the Brandenburg biosphere model / GAIA
Part 1 - Simple models of the "Climate + Biosphere" system
Yu. Svirezhev (ed.), A. Block, W. v. Bloh, V. Brovkin, A. Ganopolski, V. Petoukhov, V. Razzhevaikin (Januar 1996)

No. 15 Änderung von Hochwassercharakteristiken im Zusammenhang mit Klimaänderungen - Stand der Forschung
A. Bronstert (April 1996)

No. 16 Entwicklung eines Instruments zur Unterstützung der klimapolitischen Entscheidungsfindung
M. Leimbach (Mai 1996)

No. 17 Hochwasser in Deutschland unter Aspekten globaler Veränderungen - Bericht über das DFG-Rundgespräch am 9. Oktober 1995 in Potsdam
A. Bronstert (ed.) (Juni 1996)

No. 18 Integrated modelling of hydrology and water quality in mesoscale watersheds
V. Krysanova, D.-I. Müller-Wohlfeil, A. Becker (Juli 1996)

No. 19 Identification of vulnerable subregions in the Elbe drainage basin under global change impact
V. Krysanova, D.-I. Müller-Wohlfeil, W. Cramer, A. Becker (Juli 1996)

No. 20 Simulation of soil moisture patterns using a topography-based model at different scales
D.-I. Müller-Wohlfeil, W. Lahmer, W. Cramer, V. Krysanova (Juli 1996)

No. 21 International relations and global climate change
D. Sprinz, U. Luterbacher (1st ed. July, 2n ed. December 1996)

No. 22 Modelling the possible impact of climate change on broad-scale vegetation structure - examples from Northern Europe
W. Cramer (August 1996)

No. 23 A methode to estimate the statistical security for cluster separation
F.-W. Gerstengarbe, P.C. Werner (Oktober 1996)
No. 24 Improving the behaviour of forest gap models along drought gradients
H. Bugmann, W. Cramer (Januar 1997)
No. 25 The development of climate scenarios
P.C. Werner, F.-W. Gerstengarbe (Januar 1997)
No. 26 On the Influence of Southern Hemisphere Winds on North Atlantic Deep Water Flow
S. Rahmstorf, M. H. England (Januar 1977)
No. 27 Integrated systems analysis at PIK: A brief epistemology
A. Bronstert, V. Brovkin, M. Krol, M. Lüdeke, G. Petschel-Held, Yu. Svirezhev, V. Wenzel (März 1997)
No. 28 Implementing carbon mitigation measures in the forestry sector - A review
M. Lindner (Mai 1997)
No. 29 Implementation of a Parallel Version of a Regional Climate Model
M. Kücken, U. Schättler (Oktober 1997)
No. 30 Comparing global models of terrestrial net primary productivity (NPP): Overview and key results
W. Cramer, D. W. Kicklighter, A. Bondeau, B. Moore III, G. Churkina, A. Ruimy, A. Schloss, participants of "Potsdam '95" (Oktober 1997)
No. 31 Comparing global models of terrestrial net primary productivity (NPP): Analysis of the seasonal behaviour of NPP, LAI, FPAR along climatic gradients across ecotones
A. Bondeau, J. Kaduk, D. W. Kicklighter, participants of "Potsdam '95" (Oktober 1997)
No. 32 Evaluation of the physiologically-based forest growth model FORSANA
R. Grote, M. Erhard, F. Suckow (November 1997)
No. 33 Modelling the Global Carbon Cycle for the Past and Future Evolution of the Earth System
S. Franck, K. Kossacki, Ch. Bounama (Dezember 1997)
No. 34 Simulation of the global bio-geophysical interactions during the Last Glacial Maximum
C. Kubatzki, M. Claussen (Januar 1998)
No. 35 CLIMBER-2: A climate system model of intermediate complexity. Part I: Model description and performance for present climate
V. Petoukhov, A. Ganopolski, V. Brovkin, M. Claussen, A. Eliseev, C. Kubatzki, S. Rahmstorf (Februar 1998)
No. 36 Geocybernetics: Controlling a rather complex dynamical system under uncertainty
H.-J. Schellnhuber, J. Kropp (Februar 1998)
No. 37 Untersuchung der Auswirkungen erhöhter atmosphärischer CO_2-Konzentrationen auf Weizenbestände des Free-Air Carbondioxid Enrichment (FACE) - Experimentes Maricopa (USA)
T. Kartschall, S. Grossman, P. Michaelis, F. Wechsung, J. Gräfe, K. Waloszczyk, G. Wechsung, E. Blum, M. Blum (Februar 1998)
No. 38 Die Berücksichtigung natürlicher Störungen in der Vegetationsdynamik verschiedener Klimagebiete
K. Thonicke (Februar 1998)
No. 39 Decadal Variability of the Thermohaline Ocean Circulation
S. Rahmstorf (März 1998)
No. 40 SANA-Project results and PIK contributions
K. Bellmann, M. Erhard, M. Flechsig, R. Grote, F. Suckow (März 1998)
No. 41 Umwelt und Sicherheit: Die Rolle von Umweltschwellenwerten in der empirisch-quantitativen Modellierung
D. F. Sprinz (März 1998)
No. 42 Reversing Course: Germany's Response to the Challenge of Transboundary Air Pollution
D. F. Sprinz, A. Wahl (März 1998)
No. 43 Modellierung des Wasser- und Stofftransportes in großen Einzugsgebieten. Zusammenstellung der Beiträge des Workshops am 15. Dezember 1997 in Potsdam
A. Bronstert, V. Krysanova, A. Schröder, A. Becker, H.-R. Bork (eds.) (April 1998)
No. 44 Capabilities and Limitations of Physically Based Hydrological Modelling on the Hillslope Scale
A. Bronstert (April 1998)
No. 45 Sensitivity Analysis of a Forest Gap Model Concerning Current and Future Climate Variability
P. Lasch, F. Suckow, G. Bürger, M. Lindner (Juli 1998)
No. 46 Wirkung von Klimaveränderungen in mitteleuropäischen Wirtschaftswäldern
M. Lindner (Juli 1998)
No. 47 SPRINT-S: A Parallelization Tool for Experiments with Simulation Models
M. Flechsig (Juli 1998)

No. 48 The Odra/Oder Flood in Summer 1997: Proceedings of the European Expert Meeting in Potsdam, 18 May 1998
A. Bronstert, A. Ghazi, J. Hladny, Z. Kundzewicz, L. Menzel (eds.) (September 1998)

No. 49 Struktur, Aufbau und statistische Programmbibliothek der meteorologischen Datenbank am Potsdam-Institut für Klimafolgenforschung
H. Österle, J. Glauer, M. Denhard (Januar 1999)

No. 50 The complete non-hierarchical cluster analysis
F.-W. Gerstengarbe, P. C. Werner (Januar 1999)

No. 51 Struktur der Amplitudengleichung des Klimas
A. Hauschild (April 1999)

No. 52 Measuring the Effectiveness of International Environmental Regimes
C. Helm, D. F. Sprinz (Mai 1999)

No. 53 Untersuchung der Auswirkungen erhöhter atmosphärischer CO_2-Konzentrationen innerhalb des Free-Air Carbon Dioxide Enrichment-Experimentes: Ableitung allgemeiner Modellösungen
T. Kartschall, J. Gräfe, P. Michaelis, K. Waloszczyk, S. Grossman-Clarke (Juni 1999)

No. 54 Flächenhafte Modellierung der Evapotranspiration mit TRAIN
L. Menzel (August 1999)

No. 55 Dry atmosphere asymptotics
N. Botta, R. Klein, A. Almgren (September 1999)

No. 56 Wachstum von Kiefern-Ökosystemen in Abhängigkeit von Klima und Stoffeintrag - Eine regionale Fallstudie auf Landschaftsebene
M. Erhard (Dezember 1999)

No. 57 Response of a River Catchment to Climatic Change: Application of Expanded Downscaling to Northern Germany
D.-I. Müller-Wohlfeil, G. Bürger, W. Lahmer (Januar 2000)

No. 58 Der "Index of Sustainable Economic Welfare" und die Neuen Bundesländer in der Übergangsphase
V. Wenzel, N. Herrmann (Februar 2000)

No. 59 Weather Impacts on Natural, Social and Economic Systems (WISE, ENV4-CT97-0448) German report
M. Flechsig, K. Gerlinger, N. Herrmann, R. J. T. Klein, M. Schneider, H. Sterr, H.-J. Schellnhuber (Mai 2000)

No. 60 The Need for De-Aliasing in a Chebyshev Pseudo-Spectral Method
M. Uhlmann (Juni 2000)

No. 61 National and Regional Climate Change Impact Assessments in the Forestry Sector - Workshop Summary and Abstracts of Oral and Poster Presentations
M. Lindner (ed.) (Juli 2000)

No. 62 Bewertung ausgewählter Waldfunktionen unter Klimaänderung in Brandenburg
A. Wenzel (August 2000)

No. 63 Eine Methode zur Validierung von Klimamodellen für die Klimawirkungsforschung hinsichtlich der Wiedergabe extremer Ereignisse
U. Böhm (September 2000)

No. 64 Die Wirkung von erhöhten atmosphärischen CO_2-Konzentrationen auf die Transpiration eines Weizenbestandes unter Berücksichtigung von Wasser- und Stickstofflimitierung
S. Grossman-Clarke (September 2000)

No. 65 European Conference on Advances in Flood Research, Proceedings, (Vol. 1 - Vol. 2)
A. Bronstert, Ch. Bismuth, L. Menzel (eds.) (November 2000)

No. 66 The Rising Tide of Green Unilateralism in World Trade Law - Options for Reconciling the Emerging North-South Conflict
F. Biermann (Dezember 2000)

No. 67 Coupling Distributed Fortran Applications Using C++ Wrappers and the CORBA Sequence Type
T. Slawig (Dezember 2000)

No. 68 A Parallel Algorithm for the Discrete Orthogonal Wavelet Transform
M. Uhlmann (Dezember 2000)

No. 69 SWIM (Soil and Water Integrated Model), User Manual
V. Krysanova, F. Wechsung, J. Arnold, R. Srinivasan, J. Williams (Dezember 2000)

No. 70 Stakeholder Successes in Global Environmental Management, Report of Workshop, Potsdam, 8 December 2000
M. Welp (ed.) (April 2001)

No. 71 GIS-gestützte Analyse globaler Muster anthropogener Waldschädigung - Eine sektorale Anwendung des Syndromkonzepts
M. Cassel-Gintz (Juni 2001)

No. 72 Wavelets Based on Legendre Polynomials
J. Fröhlich, M. Uhlmann (Juli 2001)

No. 73 Der Einfluß der Landnutzung auf Verdunstung und Grundwasserneubildung - Modellierungen und Folgerungen für das Einzugsgebiet des Glan
D. Reichert (Juli 2001)

No. 74 Weltumweltpolitik - Global Change als Herausforderung für die deutsche Politikwissenschaft
F. Biermann, K. Dingwerth (Dezember 2001)

No. 75 Angewandte Statistik - PIK-Weiterbildungsseminar 2000/2001
F.-W. Gerstengarbe (Hrsg.) (März 2002)

No. 76 Zur Klimatologie der Station Jena
B. Orlowsky (September 2002)

No. 77 Large-Scale Hydrological Modelling in the Semi-Arid North-East of Brazil
A. Güntner (September 2002)

No. 78 Phenology in Germany in the 20th Century: Methods, Analyses and Models
J. Schaber (November 2002)

No. 79 Modelling of Global Vegetation Diversity Pattern
I. Venevskaia, S. Venevsky (Dezember 2002)

No. 80 Proceedings of the 2001 Berlin Conference on the Human Dimensions of Global Environmental Change "Global Environmental Change and the Nation State"
F. Biermann, R. Brohm, K. Dingwerth (eds.) (Dezember 2002)

No. 81 POTSDAM - A Set of Atmosphere Statistical-Dynamical Models: Theoretical Background
V. Petoukhov, A. Ganopolski, M. Claussen (März 2003)

No. 82 Simulation der Siedlungsflächenentwicklung als Teil des Globalen Wandels und ihr Einfluß auf den Wasserhaushalt im Großraum Berlin
B. Ströbl, V. Wenzel, B. Pfützner (April 2003)

No. 83 Studie zur klimatischen Entwicklung im Land Brandenburg bis 2055 und deren Auswirkungen auf den Wasserhaushalt, die Forst- und Landwirtschaft sowie die Ableitung erster Perspektiven
F.-W. Gerstengarbe, F. Badeck, F. Hattermann, V. Krysanova, W. Lahmer, P. Lasch, M. Stock, F. Suckow, F. Wechsung, P. C. Werner (Juni 2003)

No. 84 Well Balanced Finite Volume Methods for Nearly Hydrostatic Flows
N. Botta, R. Klein, S. Langenberg, S. Lützenkirchen (August 2003)

No. 85 Orts- und zeitdiskrete Ermittlung der Sickerwassermenge im Land Brandenburg auf der Basis flächendeckender Wasserhaushaltsberechnungen
W. Lahmer, B. Pfützner (September 2003)

No. 86 A Note on Domains of Discourse - Logical Know-How for Integrated Environmental Modelling, Version of October 15, 2003
C. C. Jaeger (Oktober 2003)

No. 87 Hochwasserrisiko im mittleren Neckarraum - Charakterisierung unter Berücksichtigung regionaler Klimaszenarien sowie dessen Wahrnehmung durch befragte Anwohner
M. Wolff (Dezember 2003)

No. 88 Abflußentwicklung in Teileinzugsgebieten des Rheins - Simulationen für den Ist-Zustand und für Klimaszenarien
D. Schwandt (April 2004)

No. 89 Regionale Integrierte Modellierung der Auswirkungen von Klimaänderungen am Beispiel des semi-ariden Nordostens von Brasilien
A. Jaeger (April 2004)

No. 90 Lebensstile und globaler Energieverbrauch - Analyse und Strategieansätze zu einer nachhaltigen Energiestruktur
F. Reusswig, K. Gerlinger, O. Edenhofer (Juli 2004)

No. 91 Conceptual Frameworks of Adaptation to Climate Change and their Applicability to Human Health
H.-M. Füssel, R. J. T. Klein (August 2004)

No. 92 Double Impact - The Climate Blockbuster 'The Day After Tomorrow' and its Impact on the German Cinema Public
F. Reusswig, J. Schwarzkopf, P. Polenz (Oktober 2004)

No. 93 How Much Warming are we Committed to and How Much Can be Avoided?
B. Hare, M. Meinshausen (Oktober 2004)

No. 94 Urbanised Territories as a Specific Component of the Global Carbon Cycle
A. Svirejeva-Hopkins, H.-J. Schellnhuber (Januar 2005)

No. 95 GLOWA-Elbe I - Integrierte Analyse der Auswirkungen des globalen Wandels auf Wasser, Umwelt und Gesellschaft im Elbegebiet
F. Wechsung, A. Becker, P. Gräfe (Hrsg.) (April 2005)

No. 96 The Time Scales of the Climate-Economy Feedback and the Climatic Cost of Growth
S. Hallegatte (April 2005)

No. 97 A New Projection Method for the Zero Froude Number Shallow Water Equations
S. Vater (Juni 2005)

No. 98 Table of EMICs - Earth System Models of Intermediate Complexity
M. Claussen (ed.) (Juli 2005)

No. 99 KLARA - Klimawandel - Auswirkungen, Risiken, Anpassung
M. Stock (Hrsg.) (Juli 2005)

No. 100 Katalog der Großwetterlagen Europas (1881-2004) nach Paul Hess und Helmut Brezowsky - 6., verbesserte und ergänzte Auflage
F.-W. Gerstengarbe, P. C. Werner (September 2005)

No. 101 An Asymptotic, Nonlinear Model for Anisotropic, Large-Scale Flows in the Tropics
S. Dolaptchiev (September 2005)

No. 102 A Long-Term Model of the German Economy: lagom^{d_sim}
C. C. Jaeger (Oktober 2005)