

## Summary of the Results

An important feature of dynamical systems is the transient phase of a trajectory, i.e. the part of the trajectory distant from the attractor. In this PhD thesis, I tackle two questions concerning this transient analysis: (i) “How can we properly quantify the time to reach a system’s attractor?” and (ii) “Can we avoid transgressing certain boundaries and stay safe (& just)?” In particular, I consider these questions in the context of sustainability science and transformations to a sustainable future, focusing to natural and socio-economic models.

The original version of the question (i) was: “How long does a trajectory spend in the transient phase?” Observing that for most systems the mathematically precise answer is infinity leads to the reformulation stated as (i). In this thesis, I analyze several problems that come up when quantifying such transient times and define four conditions that a metric answering question (i) should fulfill. Further, I introduce two metrics, *Area under Distance Curve* ( $D$ ) and *Regularized Reaching Time* ( $T_{RR}$ ), capturing two complementary aspects of the transient dynamics.  $D$  is the distance of the trajectory to the attractor integrated over time and measures how “reluctant” or “eager” it is. In contrast,  $T_{RR}$  quantifies the additional time (positive or negative) a trajectory starting at a chosen initial condition needs in order to approach the attractor with comparison to some reference trajectory. It measures how much “earlier” or “later” a trajectory approaches the attractor. Further, I extend the definition of  $T_{RR}$  to networks by creating the metric *single-node recovery time* ( $T_{RR}^N$ ), analyzing how quickly system can recover after a perturbation at a node. I apply them to multiple paradigmatic example and discuss in detail how they fulfill the aforementioned conditions.

Question (ii) concerns with a different point of view on the transient dynamics. Often, there are distinctions of the state space in desirable and undesirable. In particular in the context of sustainability there are such regions, e.g. defined by “planetary boundaries”, “tolerable environment and development window”, “guardrails”, or the “safe (and just) operating space for humanity”. To understand systems with such boundaries, there is not only the need to consider the intrinsic dynamics but also the possible ways of influence, i.e. managing it. I present a variant definition of the mathematical framework *Topology of Sustainable Management* (TSM) that was developed as a tool to analyze models with respect to their intrinsic dynamics, possible management and desirable states. I illustrate the concepts with multiple two-dimensional, manually-analyzed example systems. The variant definition is built on concepts from viability theory (VT). This opens the possibility to use the tools from VT, in particular the Saint-Pierre algorithm (SPA). Hence, I go the step from two-dimensional examples to higher-dimensional models by extending the application of the SPA to TSM. Furthermore, I present an extension of SPA to compute implicitly defined capture basins, a notion from VT that is more elaborated in the thesis, as these come up in TSM. I use a three-dimensional model focusing on climate change, economic output and energy transformation to demonstrate the applicability of this approach. Two common problems of estimations in VT (using SPA) are substantial for this example: (i) an unbounded state space and (ii) highly varying time scales. I solve both by introducing appropriate, general coordinate transformations and apply them to the three-dimensional example system.

Finally, I present work that was originally devised to apply the aforementioned ideas to the El Niño-Southern Oscillation (ENSO) using *functional climate networks*. However, during the process the direction of this work deviated. I use *functional climate networks* to analyze how major perturbations – the El Niño and La Niña phases of ENSO and three largest volcanic eruptions since the middle of the 20<sup>th</sup> century, Mount Pinatubo (1991), Mount Agung (1963) and El Chichon (1982) – influence the teleconnectivity structure of the surface area temperature field (SAT). The results confirm the existence of global effects of ENSO by breaking down the modular structure of the global SAT field, signifying the emergence of strong teleconnections. I find similar effects after the aforementioned volcanic eruptions on regional scales.