Abstract

Dynamical systems exhibiting *multistability*, characterized by the coexistence of several stable states, are abundant across natural sciences and engineering. Maintaining operation of such systems in a particular stable state in the face of random perturbations, is often critical to their functionality. There has been a persistent drive towards quantifying the *stability* of the multiple stable states of such systems. A major development in this direction was that of *basin stability* (BS), which relates the volume of the basin of attraction of any stable state to the probability of returning to the same in the event of random perturbations. Many complex systems exhibiting multistability involve *complex networks* of interacting oscillators, whereby their *synchronized* dynamics often concurs with the desired operational state of the network. The application of BS to assessing the stability of synchronization and its extension to *single-node basin stability* (SNBS) constitute notable developments. Despite such recent advancements in stability theory, a comprehensive framework for quantifying multistability is still lacking. This fuels the present endeavour, comprising the development of a framework for the assessment of the stability of (multistable) complex (networked) dynamical systems, particularly in the face of random perturbations.

As a first contribution, we propose the framework of *multiple-node basin stability* (MNBS) for gauging the stability of networked dynamical systems in response to non-infinitesimal perturbations simultaneously affecting multiple nodes of the system. We then turn to the theoretical framework of *resilience* in identifying the different aspects characterizing multistability. Inspired by the concept of *ecological resilience*, we assert that the stability of the different attractors of a multistable system is determined by the overall structure of their respective basins of attraction. In particular, we identify the local dynamics of the system in the state space and the relative position of the attractor within the basin, in addition to the volume of the basin of attraction as crucial aspects determining overall stability of an attractor. We combine the aforementioned aspects in proposing the measure of *integral stability* (IS) for holistically quantifying multistability. We also draw inspiration from the concept of *engineering resilience*, which relates to the speed of return of the system to its equilibrium, following a perturbation. In the specific context of networked dynamical systems, we propose the framework of *single-node recovery time* (SNRT) for obtaining an estimate of the relative time scales underlying the transient dynamics of the nodes of a network returning to its desired operational state, following a non-infinitesimal perturbation to any specific node. The conjugation of the concepts of MNBS, IS and SNRT with those of linear stability, BS and SNBS provides a comprehensive framework for quantifying multistability.

Finally, we delve into the explicit investigation of the stability of synchronization on complex dynamical networks exhibiting *small-world* properties and of those, simultaneously displaying *scale-free* behaviour and *hierarchical* organization. The results emanating from these investigations bear important implications in the design of topologies for better synchronizability and in ensuring persistent synchronized operation of dynamical units coupled on them. The aforementioned results open up several new avenues of research directed towards probing the robustness of the synchronized state in complex dynamical networks.