Quantum Information Processing with Clifford Quantum Cellular Automata

Johannes Gütschow

Abstract

In this thesis, we study quantum information processing under constraints on the available operations and resources. The experimental difficulty in implementing quantum computers with many parallel qubits renders resource-efficient quantum computational schemes with a reduced set of operations a necessity for successful quantum information processing.

We focus on Clifford operations, especially Clifford quantum cellular automata (CQCAs), which are a building block of several quantum computational schemes. By loosening the translation invariance, we incorporate Clifford causal operations and convolutional stabilizer codes into the framework of CQCAs. Memory channels are utilized as a resource-efficient way to implement causal operations and convolutional encoders. Based on this this framework, we study the performance of quantum convolutional codes under resource constraints.

In our analysis of CQCAs we present a complete classification of one-dimensional CQCAs and their time evolution. We determine invariant states and prove that CQCAs generate entanglement at the maximal rate possible for translation-invariant operations. We furthermore show that the spacetime image of a broad class of linear Cellular automata, including CQCAs, exhibits a self similar structure.

It is proven that quasi-local causal operations can be implemented by forgetful memory channels, whereas finite depth causal operations correspond to strictly forgetful channels. We find the required memory dimension equals the index of the causal operation, enabling a resource efficient implementation. Furthermore, we introduce the use of Bratteli diagrams to analyze the memory dynamics. We prove the existence of causal inverses and bounds on their resource requirements. This gives us the means to construct finite depth inverses of memory channels, i.e. operations that recover the sent information with a delay independent of the transmission length.

A theory of Clifford memory channels is established, including a criterion for forgetfulness. We employ this theory to introduce a new channel-based approach to quantum convolutional codes which facilitates a better understanding of catastrophic errors and the construction of non-catastrophic and finite-depth encoders and decoders. We prove a Hamming bound for convolutional codes, finding that convolutional codes have the potential to outperform block codes under resource constraints.