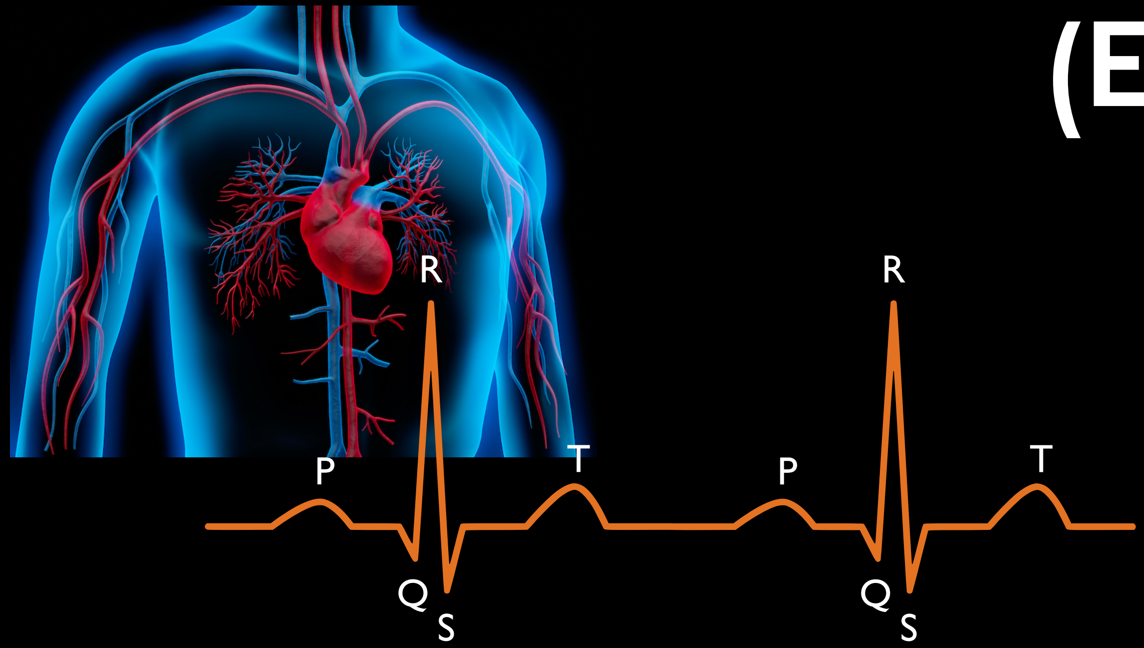
Potsdam Institute for
Climate Impact Research
PIK

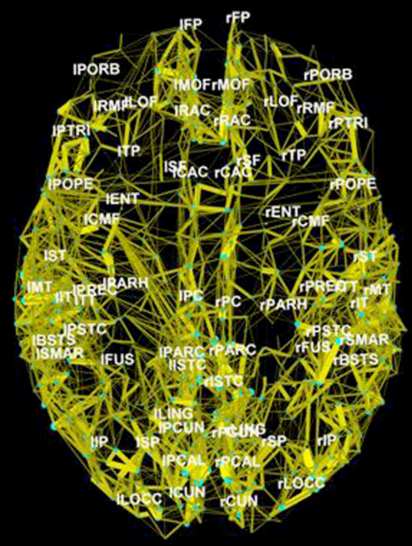
Norbert Marwan
Tobias Braun

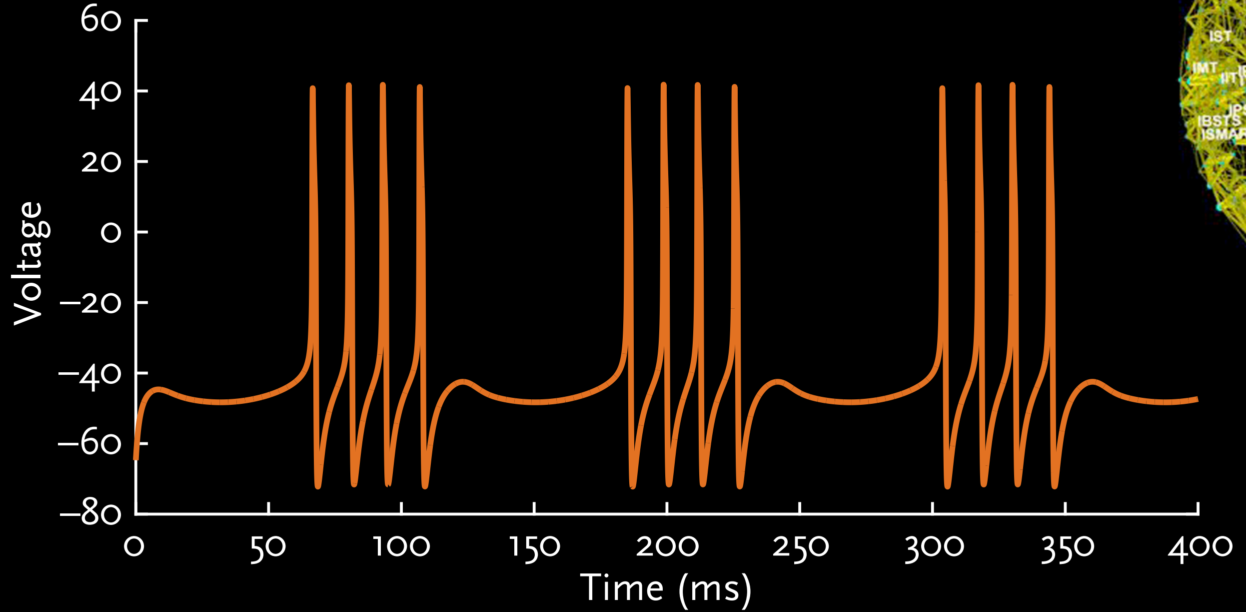
Power Spectrum Estimation for (Extreme) Events Data

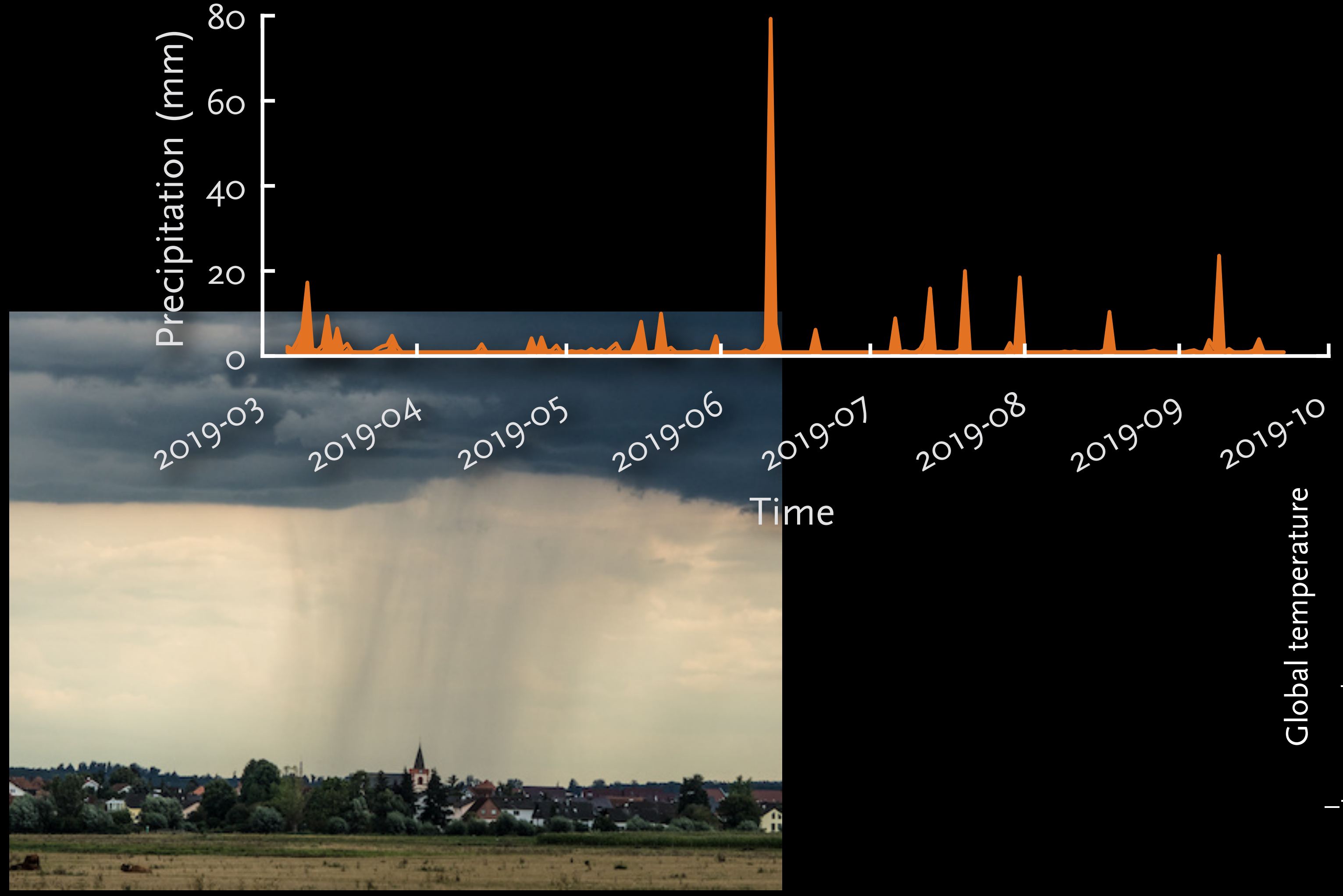
Leibniz-Gemeinschaft

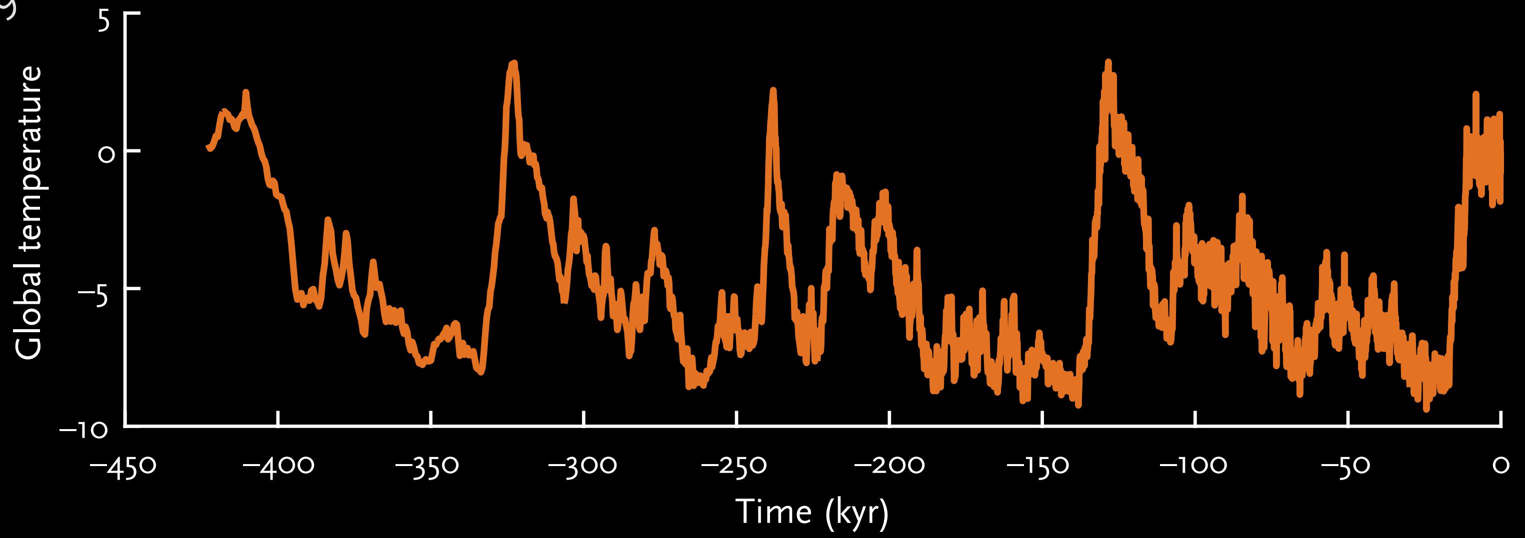
(Extreme) Events Data



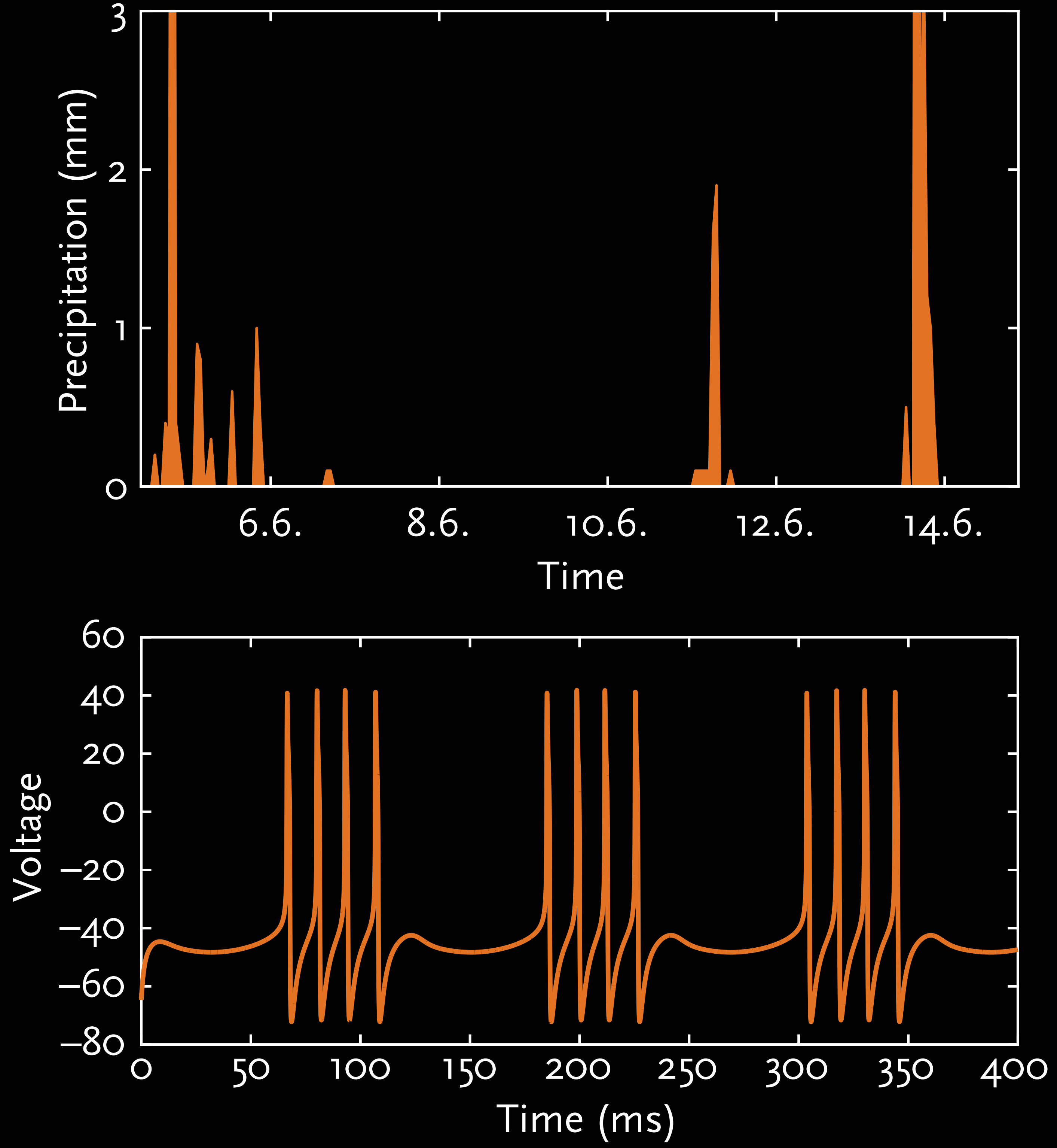








Time Scales of Event Occurrences?



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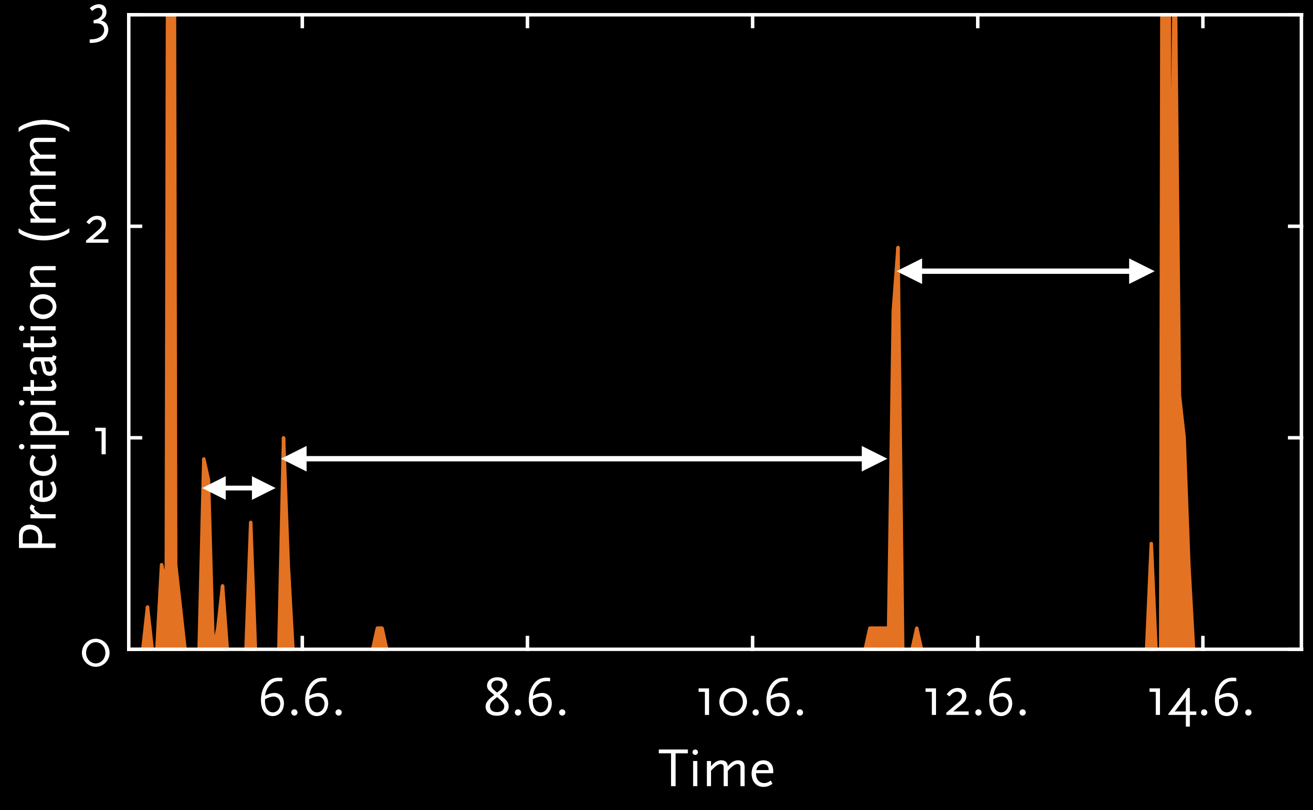
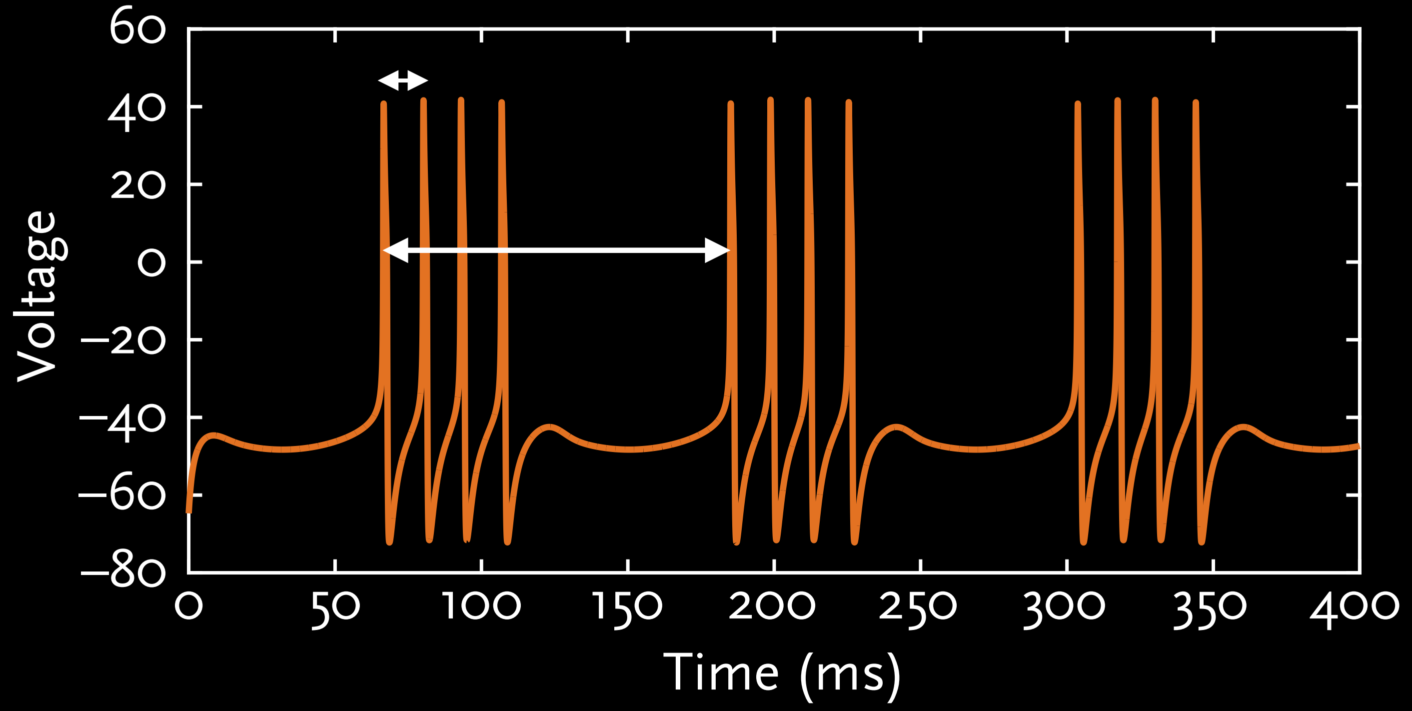
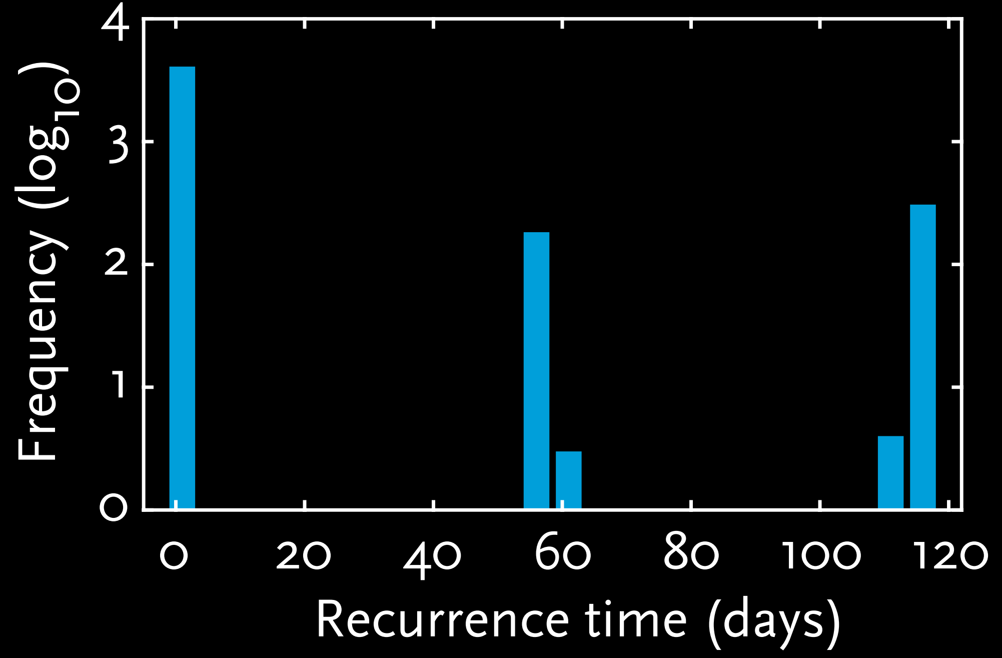


- Interspike intervals (recurrence times)

➡ No unique selection
Discretisation bias





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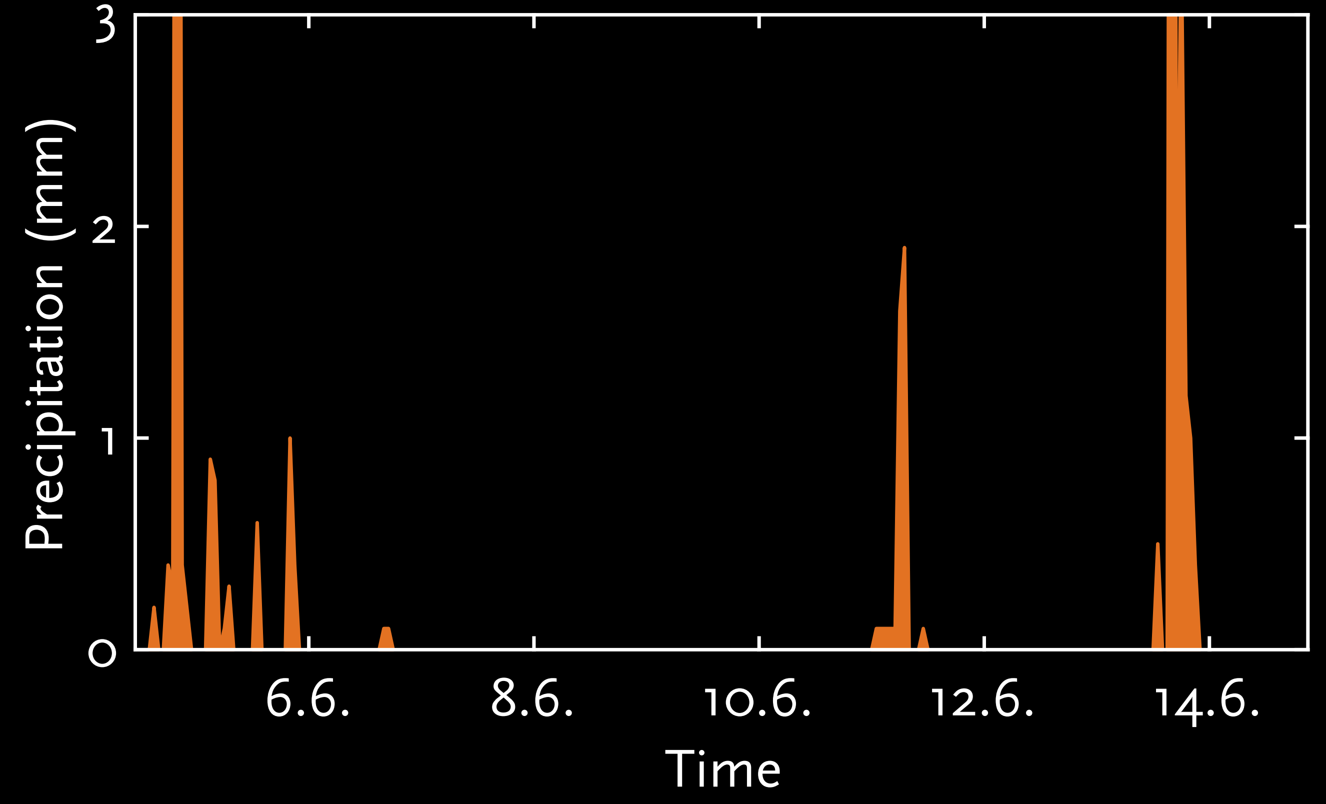
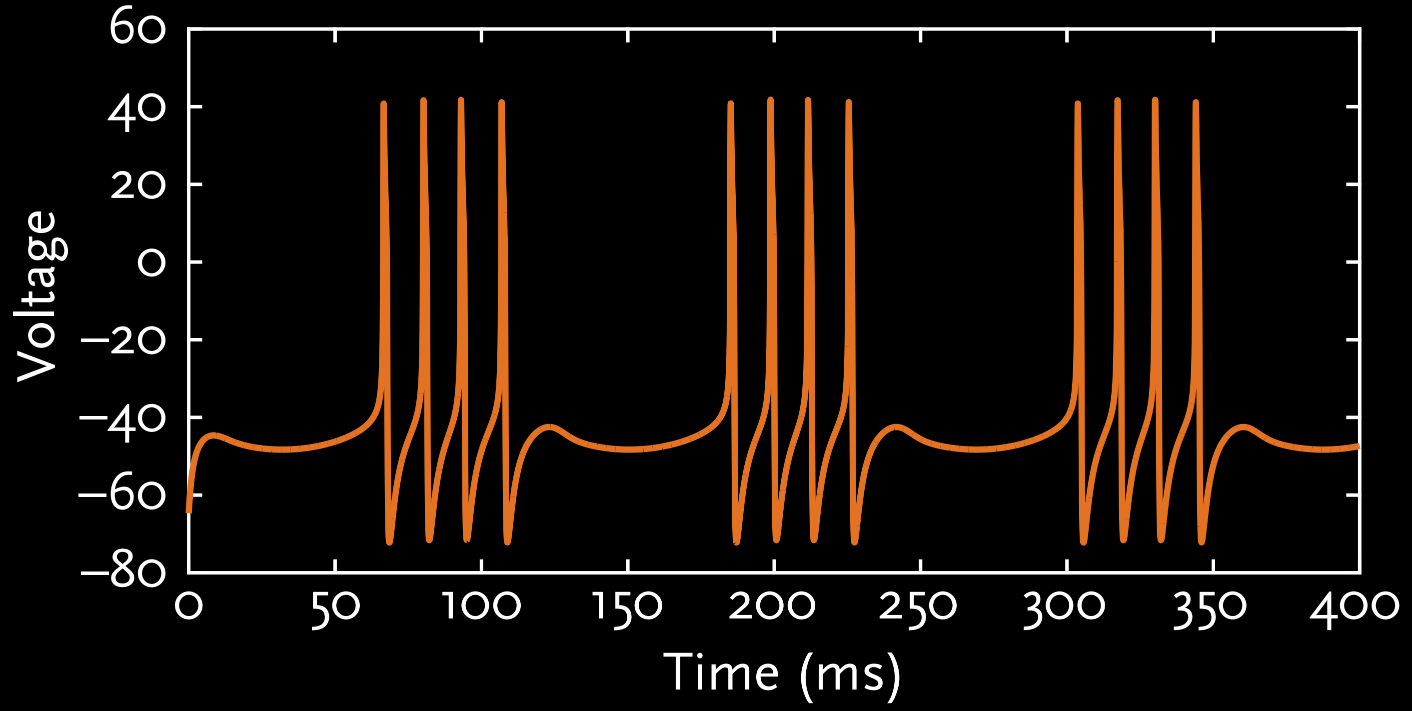
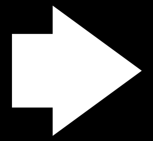
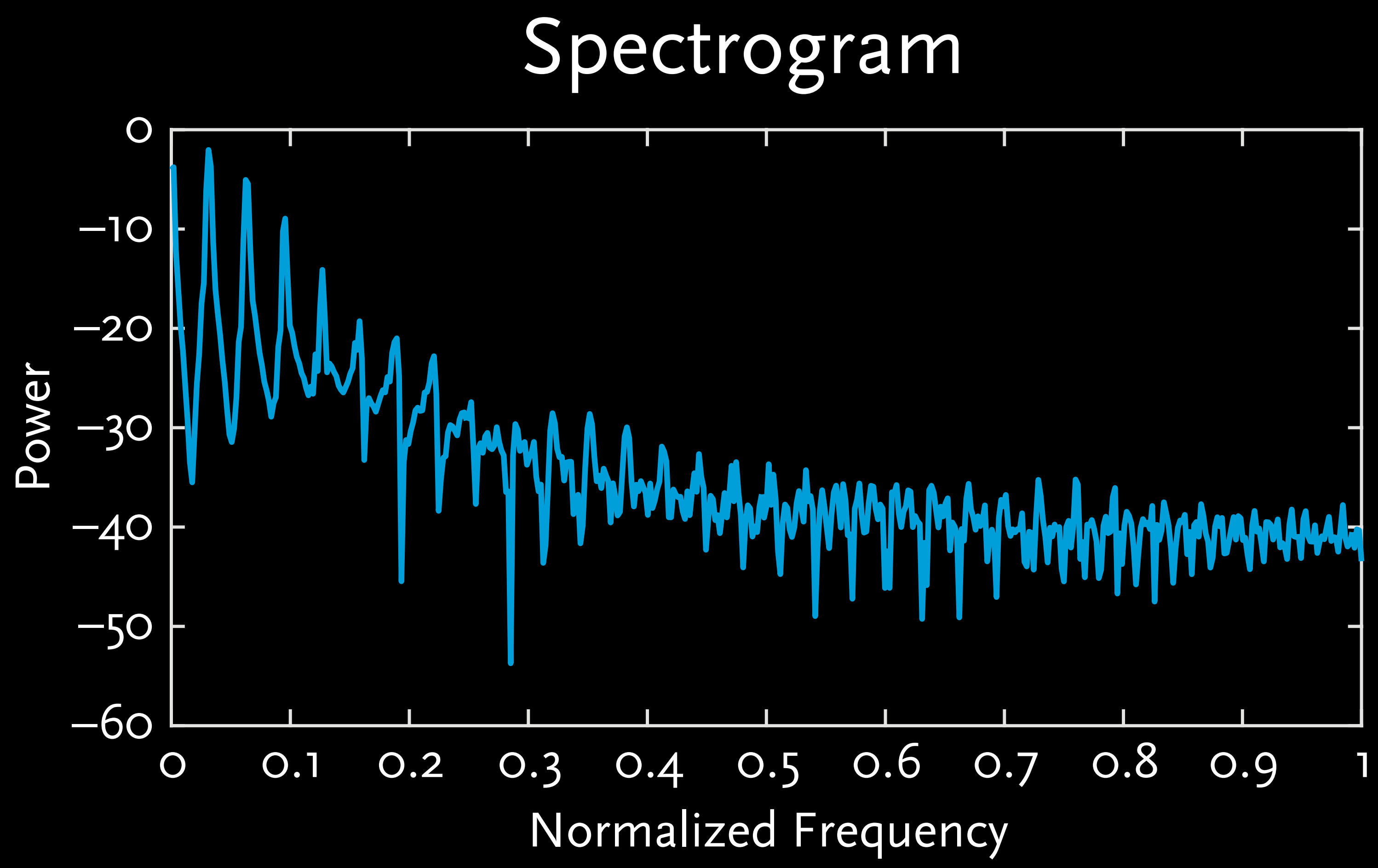


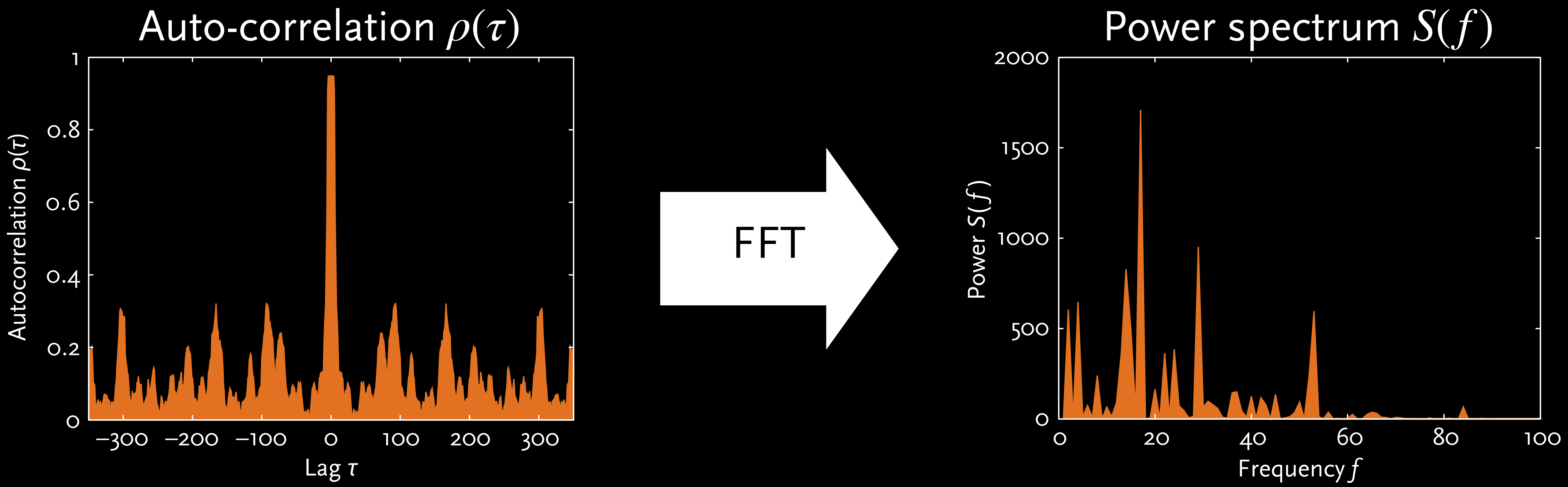






- Simple, no/ few parameters, objective

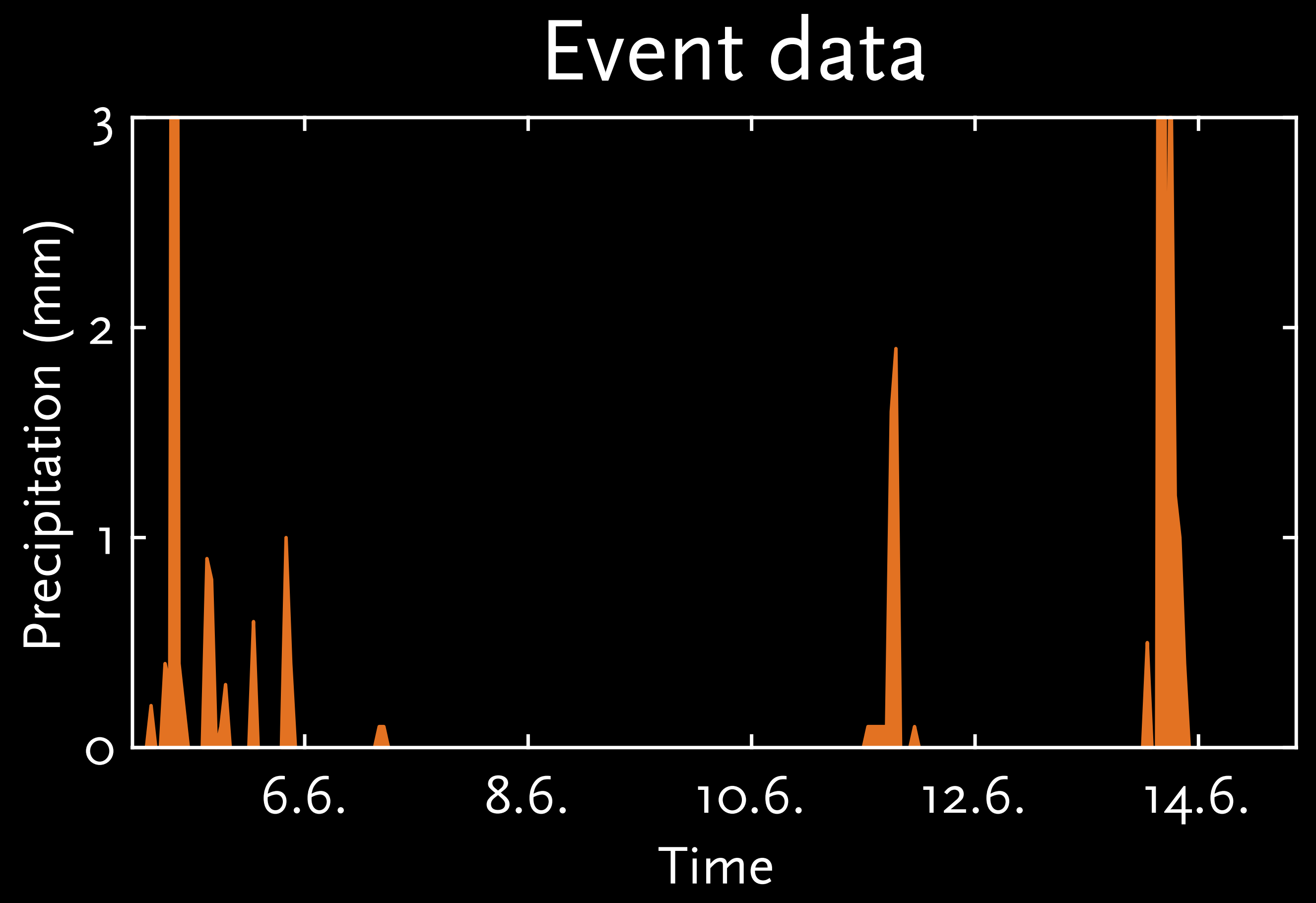
Power Spectrum from Auto-Correlation

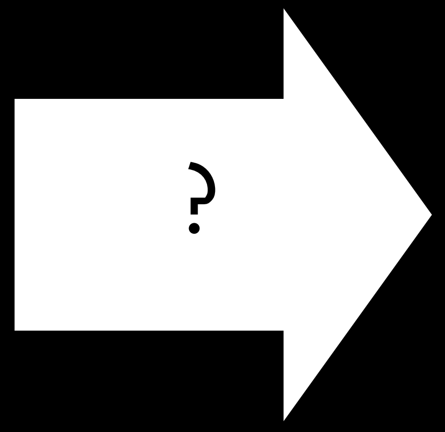


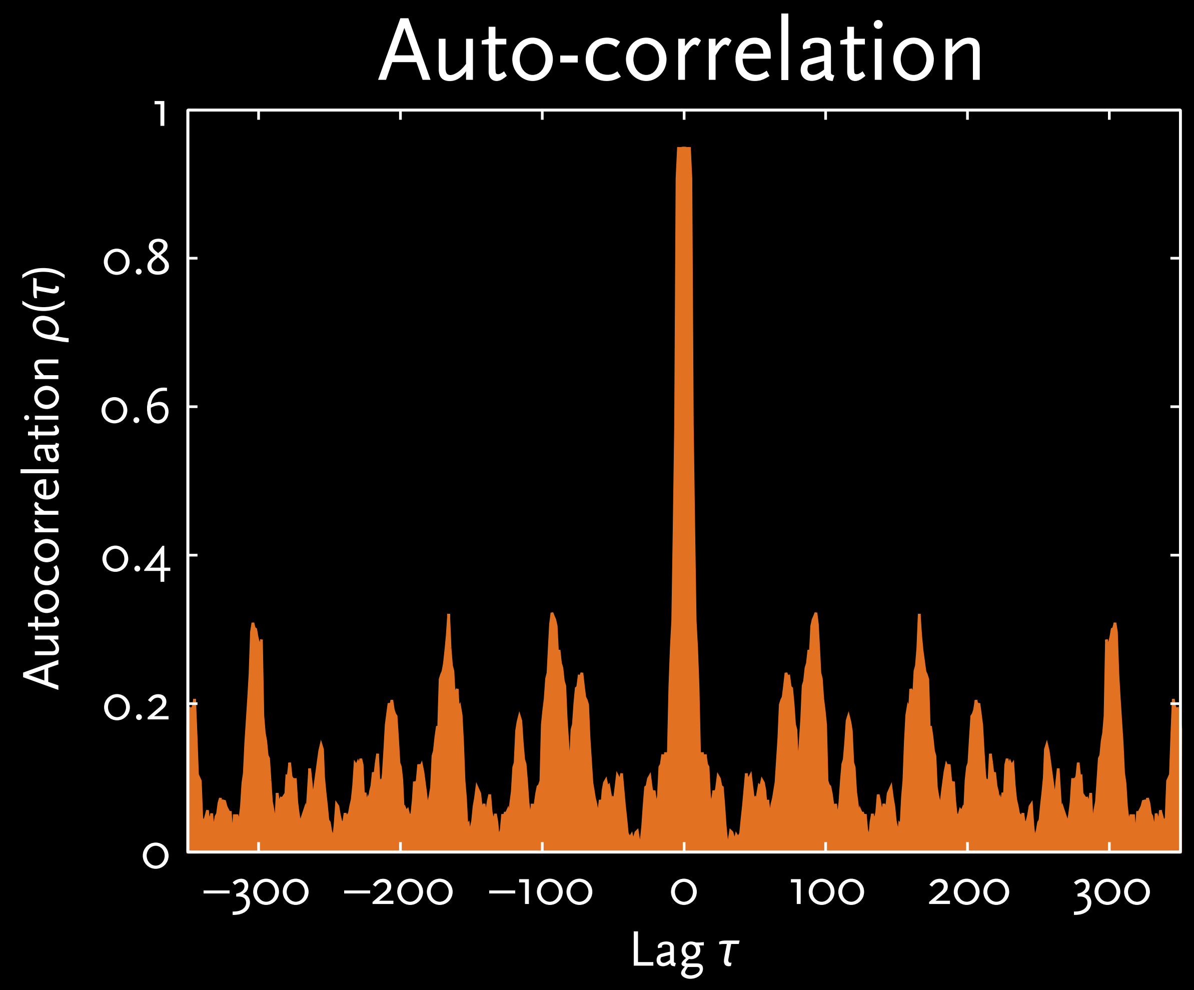
Wiener Khinchin theorem: $$S(f) = \sum_{\tau=-\infty}^{\infty} \rho(\tau) e^{-i2\pi\tau f}$$

Einstein, Archives sciences phys. 37, 1914; Wiener, Acta Math. 55, 1930; Khintchine, Math. Annalen 109, 1934

Auto-Correlation of Event Series



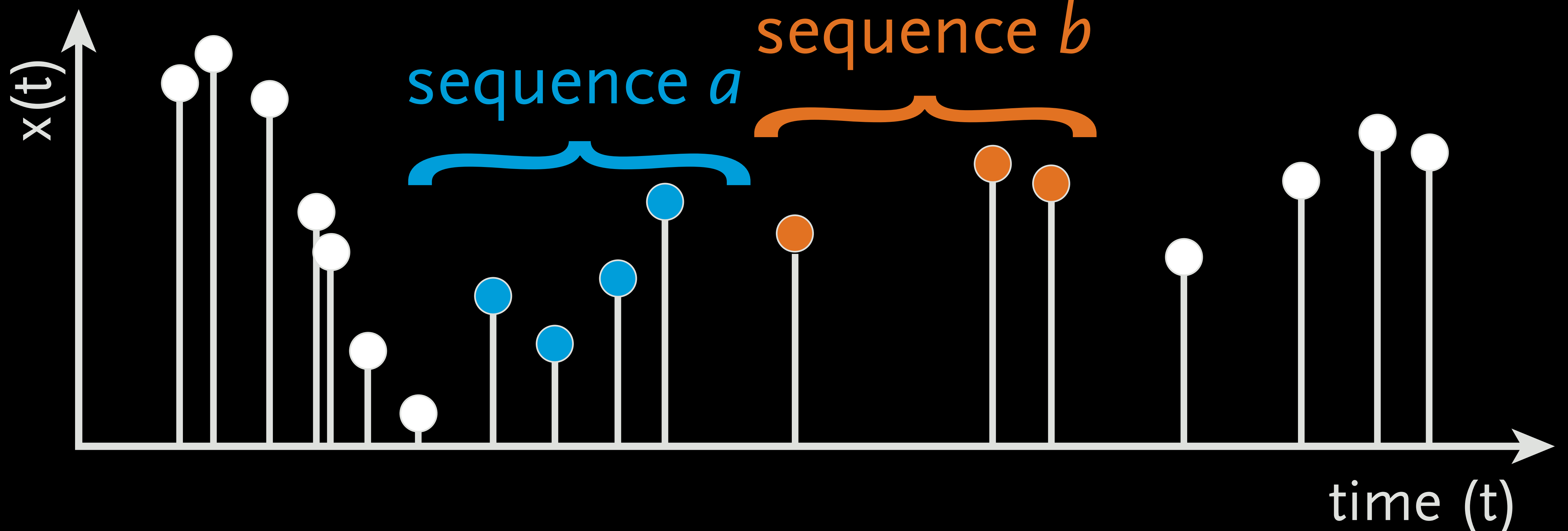




Similarity measure for event data
(e.g., Levenshtein metric, event synchronisation, ...)

Edit Distance*

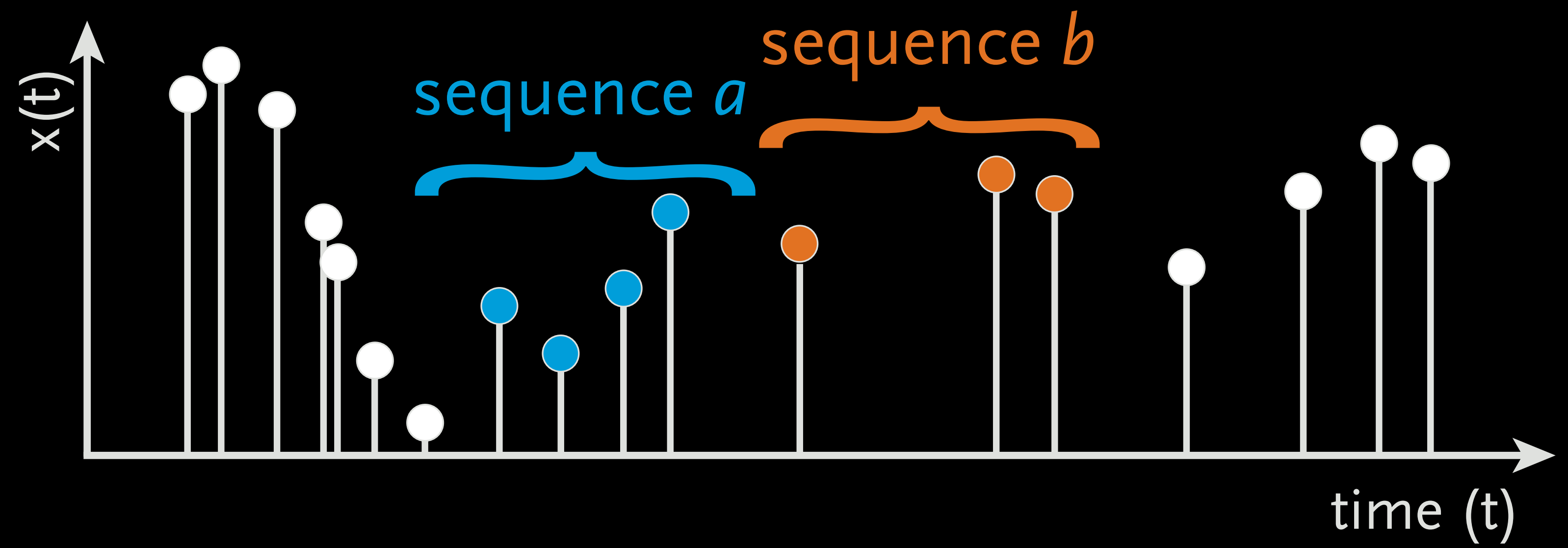
- Distance d = minimize the cost to transform sequence a to sequence b



Victor & Purpura, Network 8, 1997; Suzuki et al., Int. J. Bif. Chaos. 20, 2010

* modified Levenshtein distance

Edit Distance



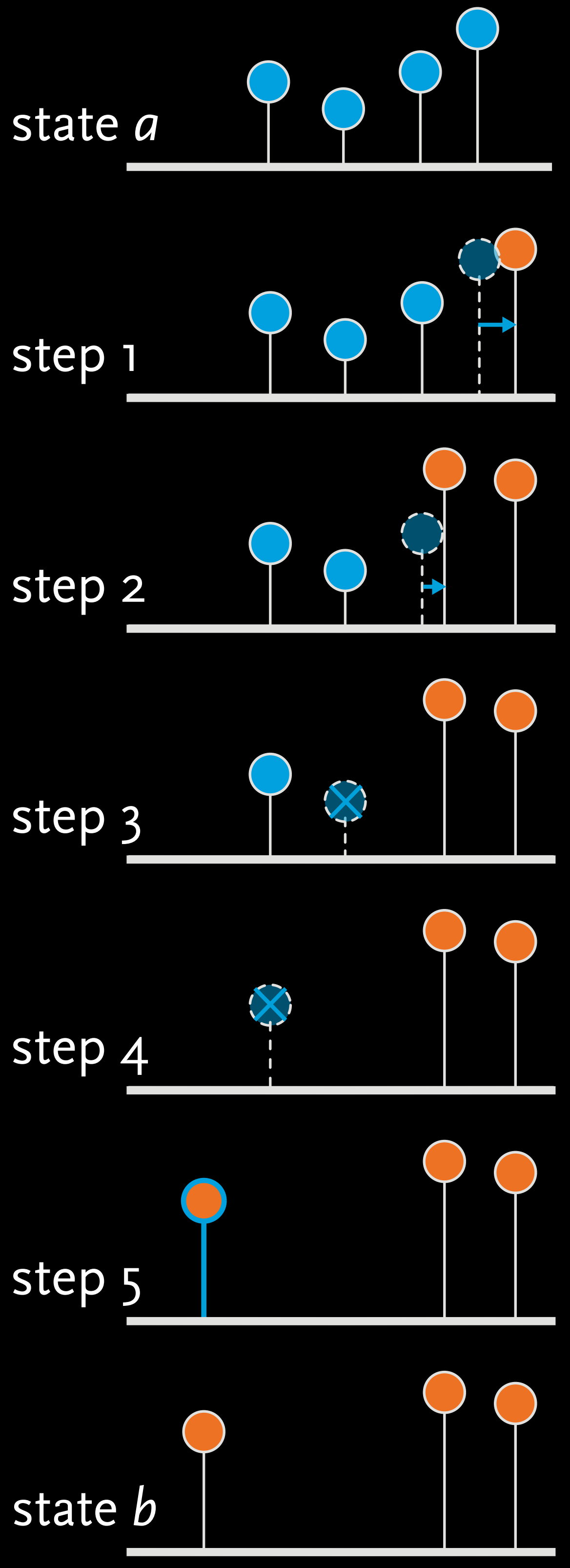
4 Operations:

(1) shifting (cost λ_0)

(2) adding (cost λ_s)

(3) deleting (cost λ_s)

(4) amplitude modification (cost λ_k)



Initial state

$d = 0$

Shifting + amplitude modif.

$+\lambda_0\,|t_a(4) - t_b(3)| + \lambda_k\,|L_a(4) - L_b(3)|$

Shifting + amplitude modif.

$+\lambda_0\,|t_a(3) - t_b(2)| + \lambda_k\,|L_a(3) - L_b(2)|$

Deleting

$+\lambda_s$

Deleting

$+\lambda_s$

Adding

$+\lambda_s$

Final state

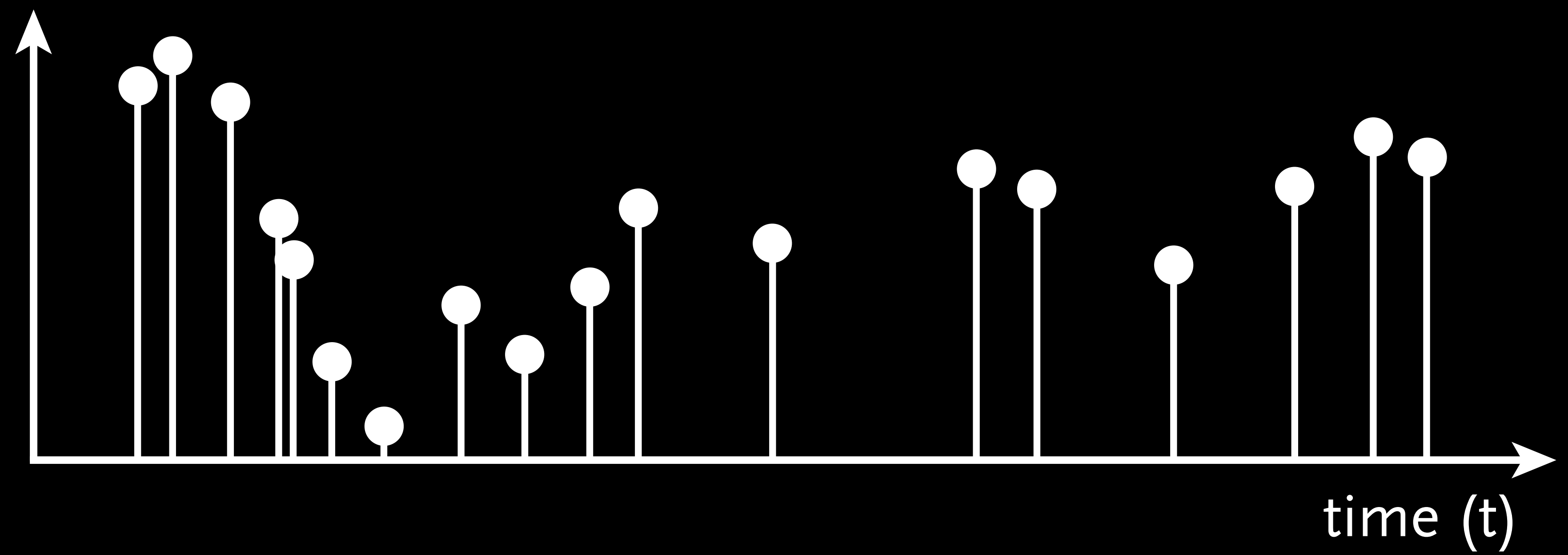
$= d(a,b) \to d(\vec{x}_i, \vec{x}_j)$

Suzuki et al., Int. J. Bif. Chaos. 20, 2010

Edit Distance as Auto-covariance

Edit distance (for binary events):

$$d(\mathcal{S}_a, \mathcal{S}_b) = \min \left\{ \underbrace{\Lambda_S \left(N_a + N_b - 2\,|\mathcal{C}|\right)}_{\text{adding and deleting}} + \sum_{\alpha,\beta \in \mathcal{C}} \underbrace{\Lambda_0 \left\| t_\alpha^{(a)} - t_\beta^{(b)} \right\|}_{\text{shifting}} \right\} \rightarrow d\left(\mathcal{S}, \mathcal{S}(\tau)\right)$$



Marwan & Braun, Chaos, 2023

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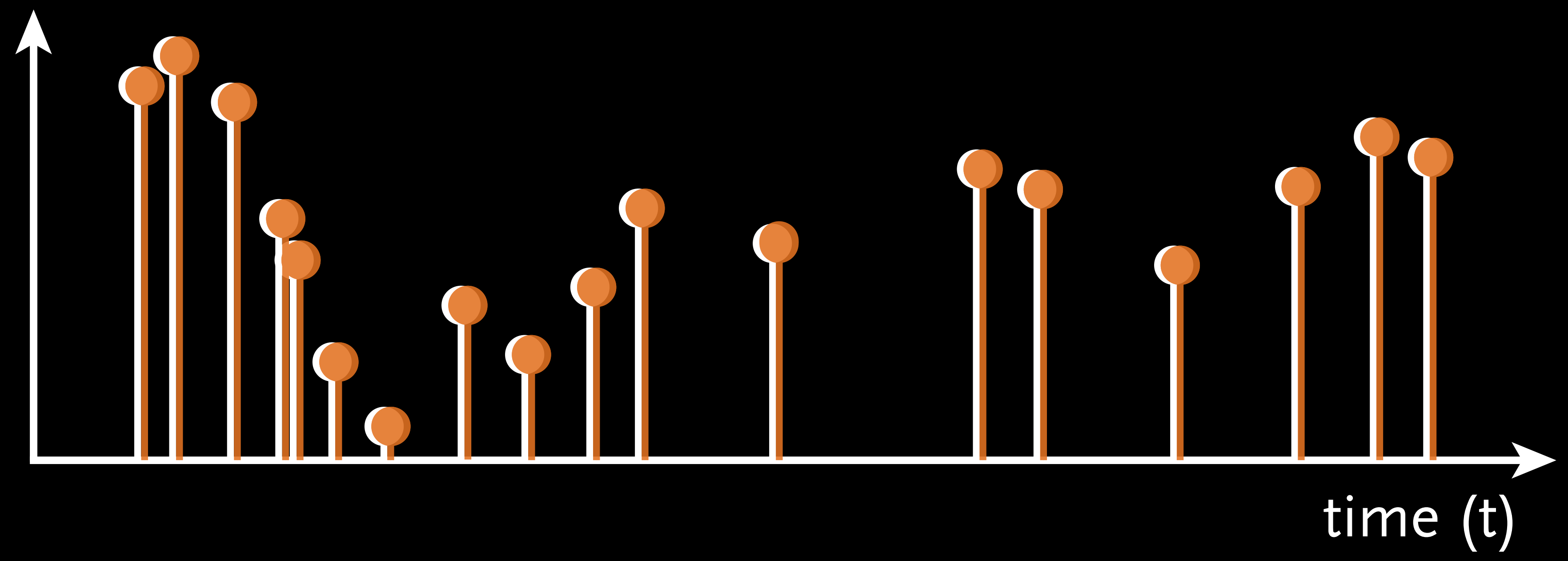

Marwan & Braun, Chaos, 2023

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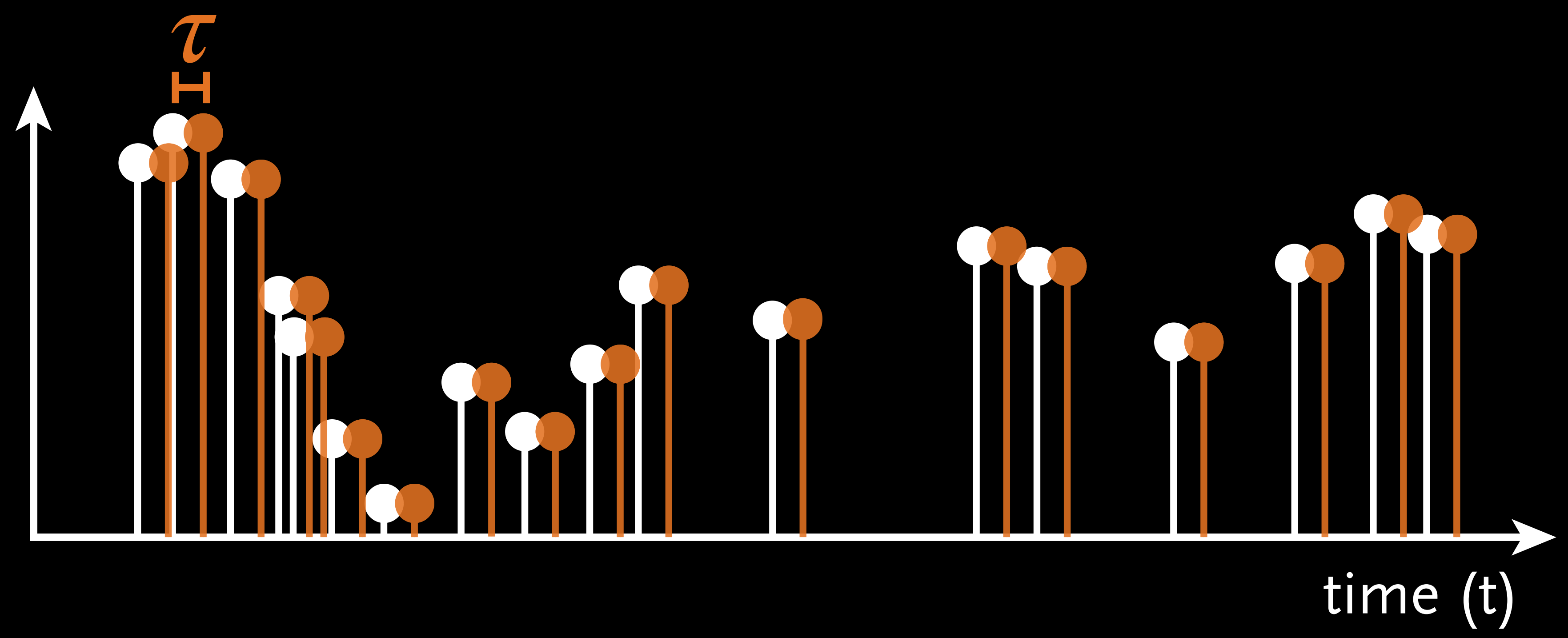

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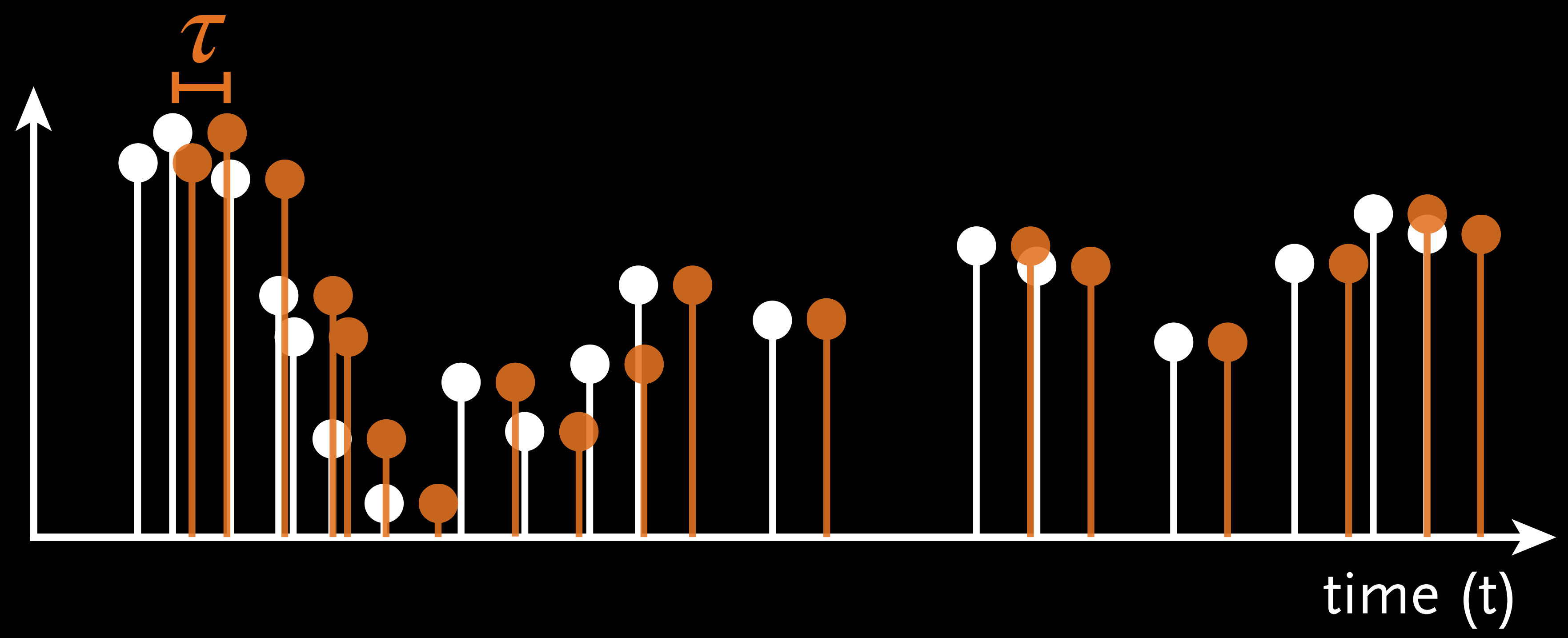

Marwan & Braun, Chaos, 2023

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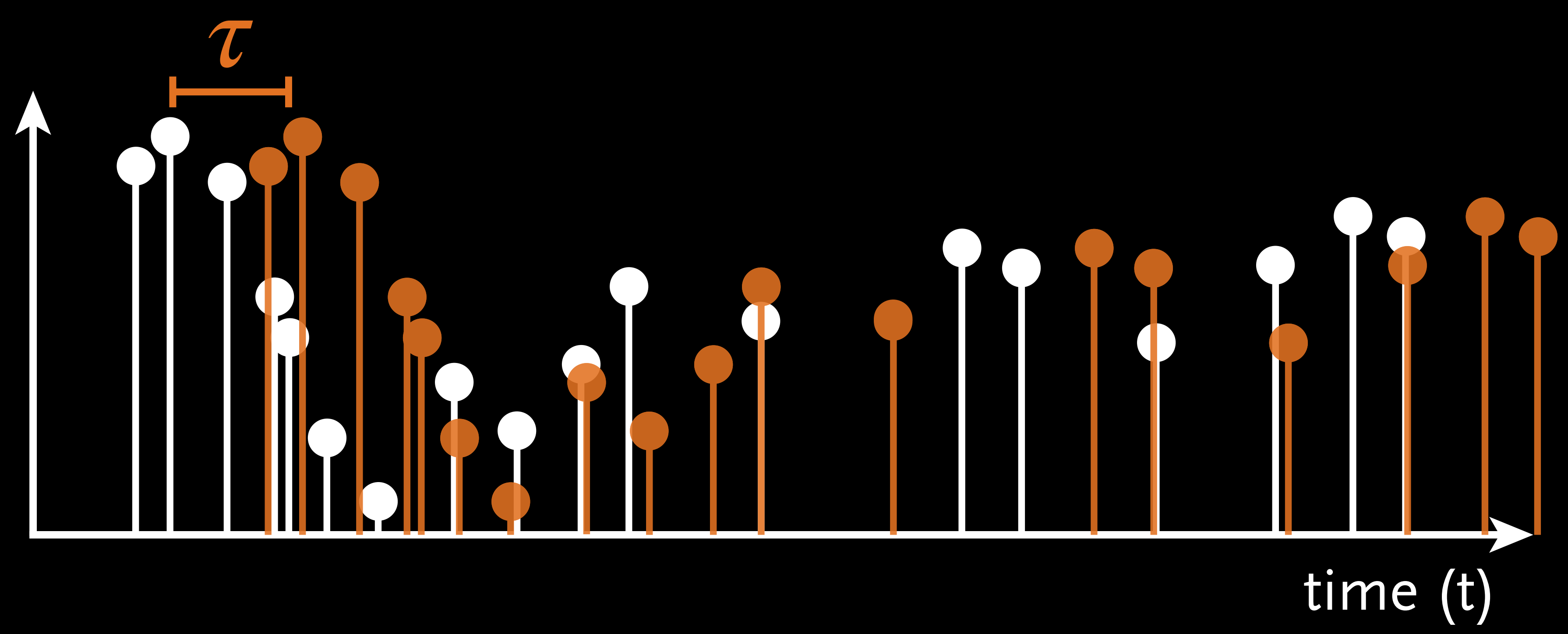

Marwan & Braun, Chaos, 2023

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Combine with Wiener Khinchin theorem:

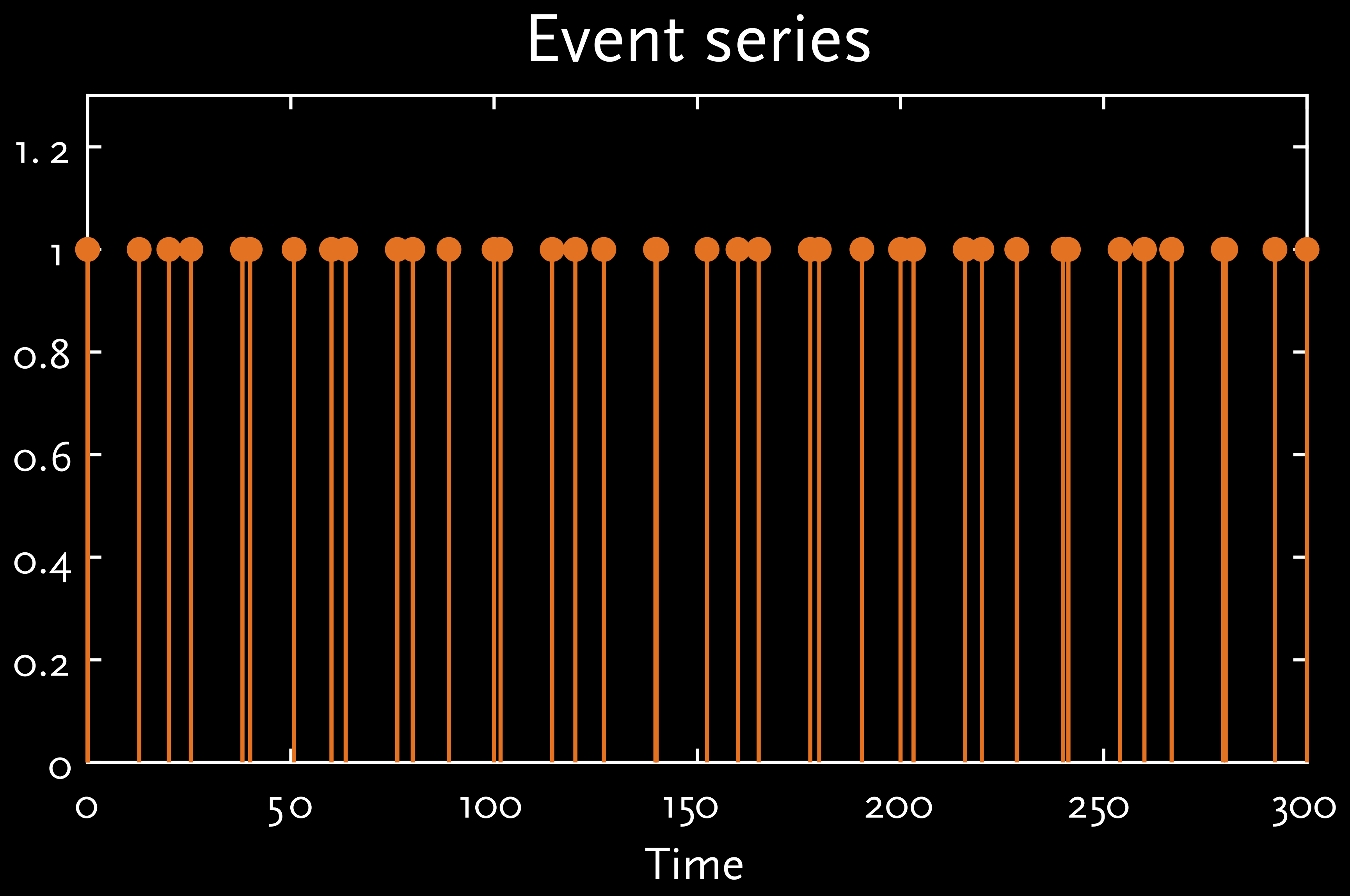
$$S_{\mathcal{S}}^{\text{edit}}(f) = \sum_{\tau=-\infty}^{\infty} \frac{\left(1 - \tilde{d}\left(\mathcal{S}, \mathcal{S}(\tau)\right)\right) - \left\langle 1 - \tilde{d}\left(\mathcal{S}, \mathcal{S}(\tau)\right) \right\rangle}{\text{std}\left(1 - \tilde{d}\left(\mathcal{S}, \mathcal{S}(\tau)\right)\right)} \mathrm{e}^{-j2\pi f\tau}$$

normalised by $\max(d)$

Marwan & Braun, Chaos, 2023

Edit Distance-based Spectrum

Event series with two frequencies ($\omega_1 = 1/20$, $\omega_2 = 1/12.7$):



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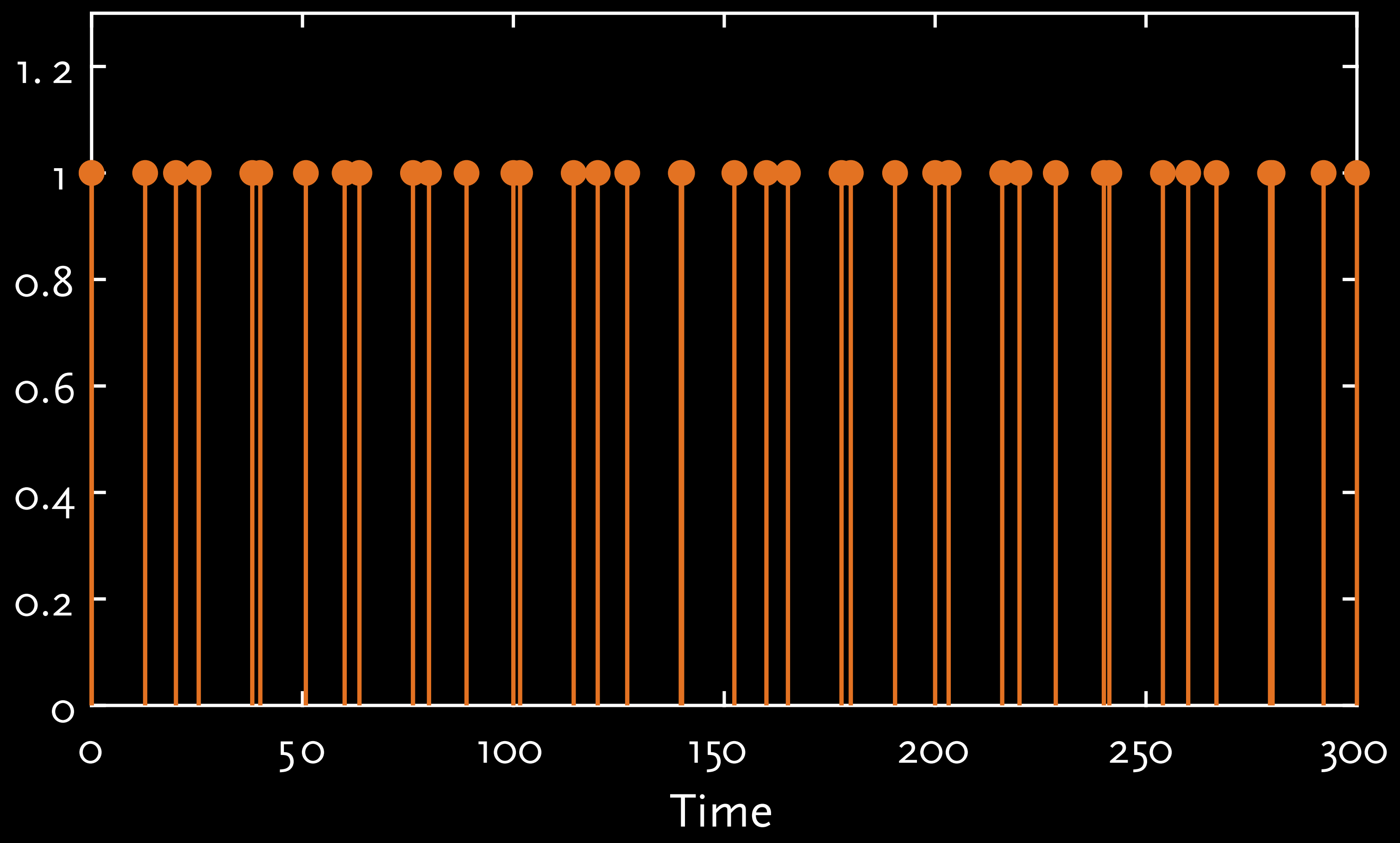
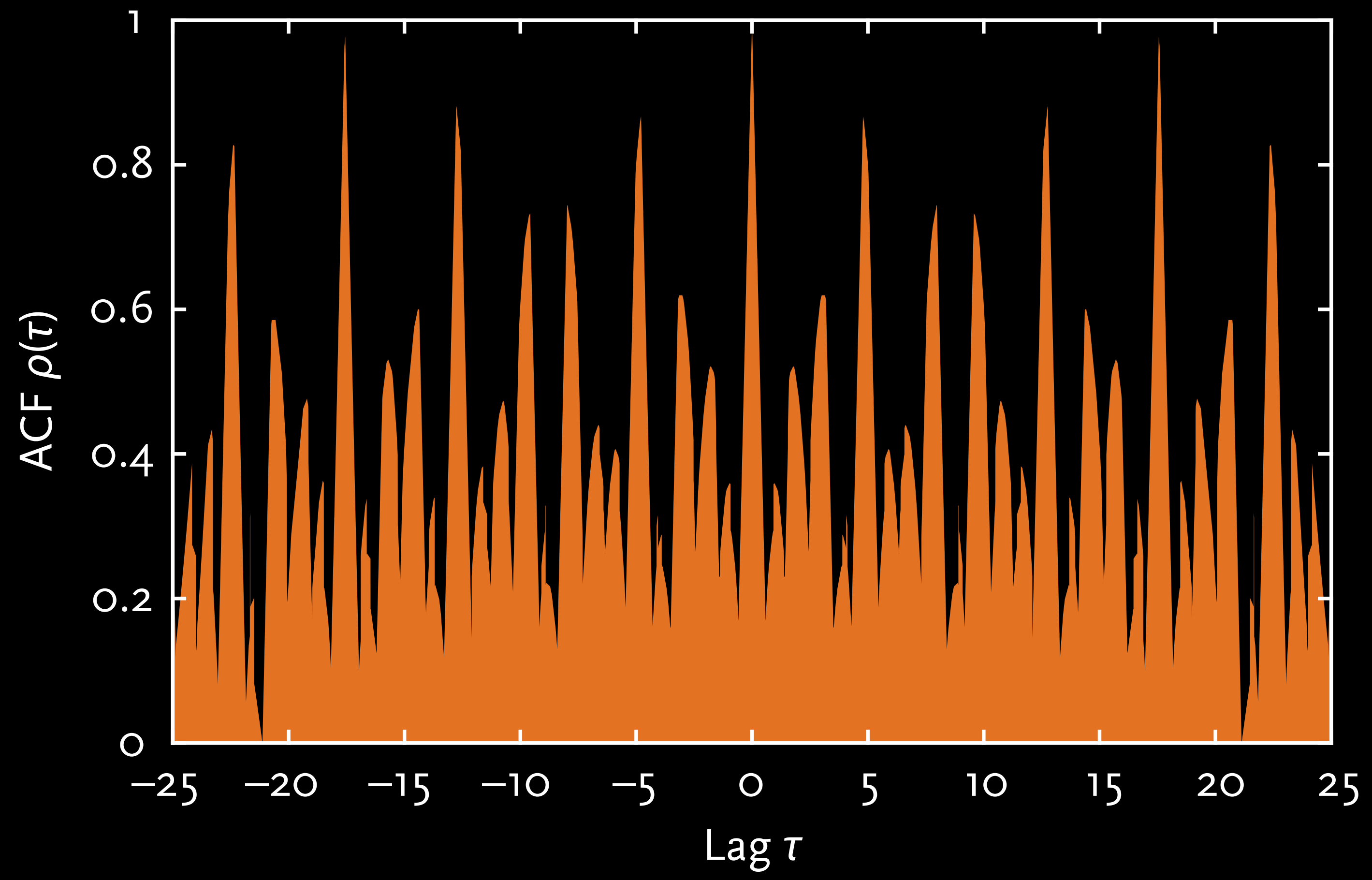
Event series



ED-“ACF”



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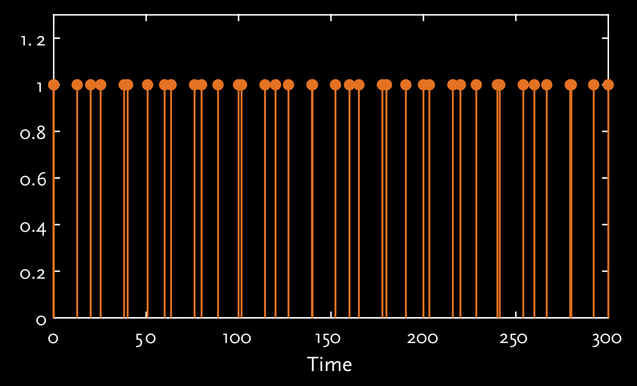

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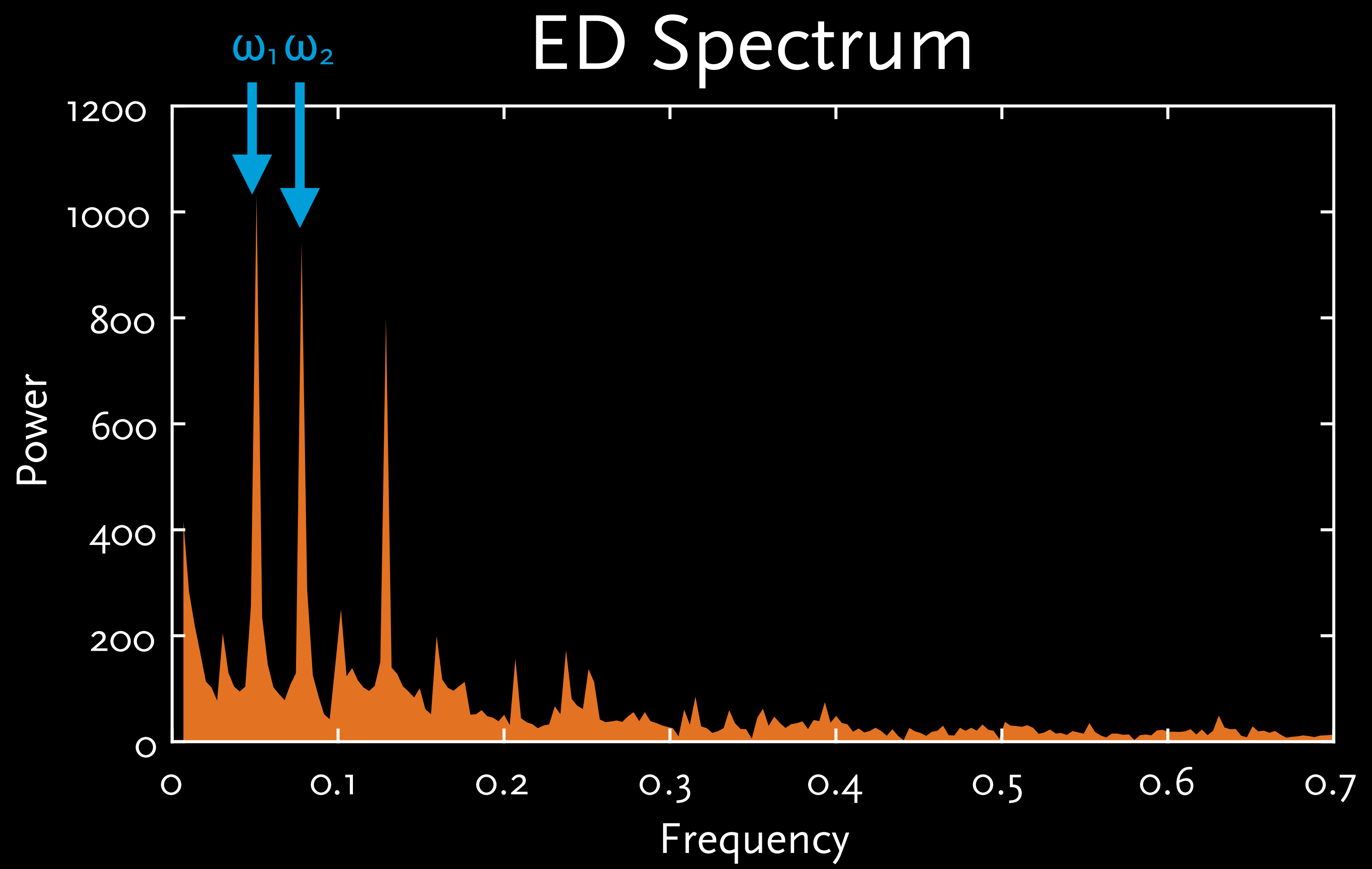

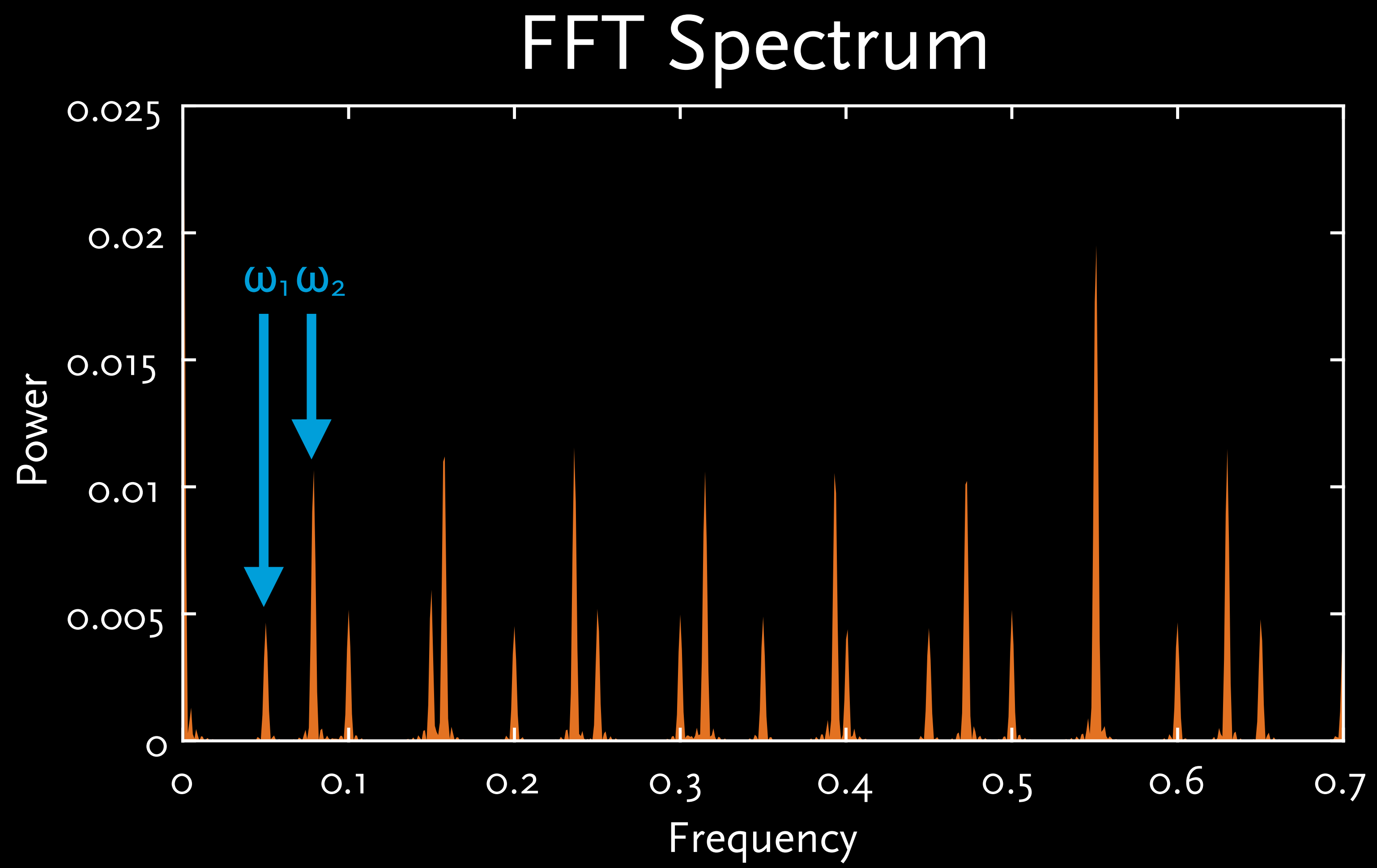

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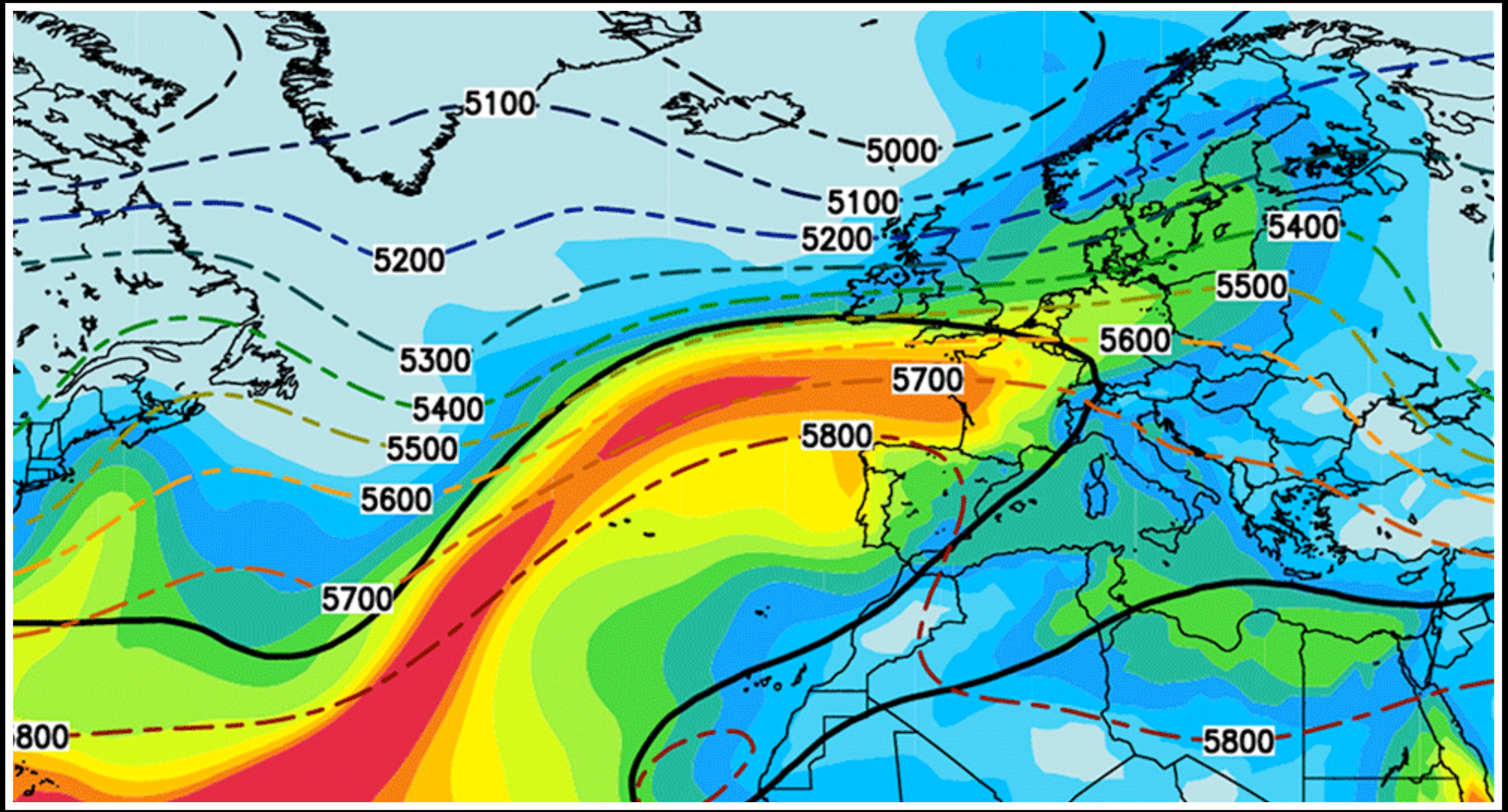
Atmospheric Rivers

Integrated water vapor transport (IVT)

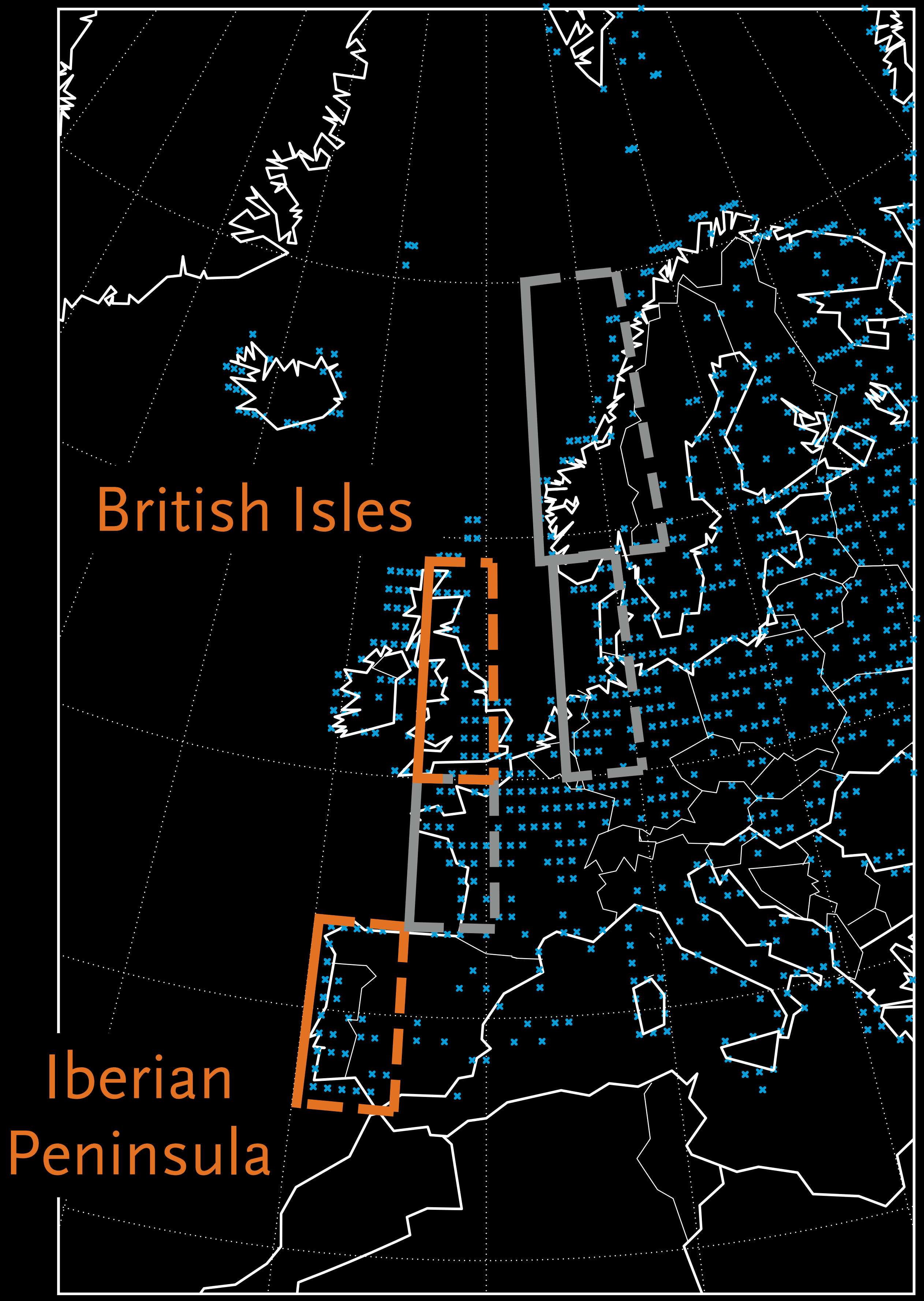


Dec 19, 1993

Ionita et al., HESS 24, 2020

Atmospheric Rivers

- Landfalling atmospheric rivers
- Regional differences?



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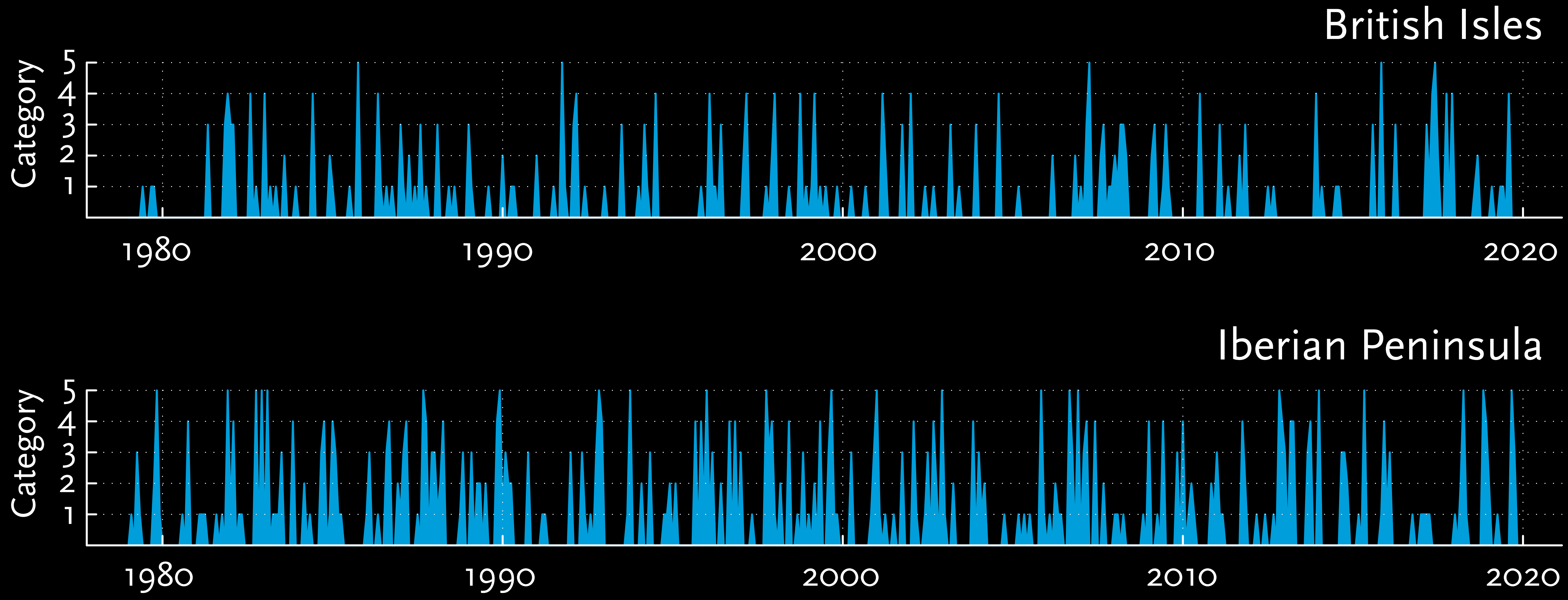
Atmospheric Rivers



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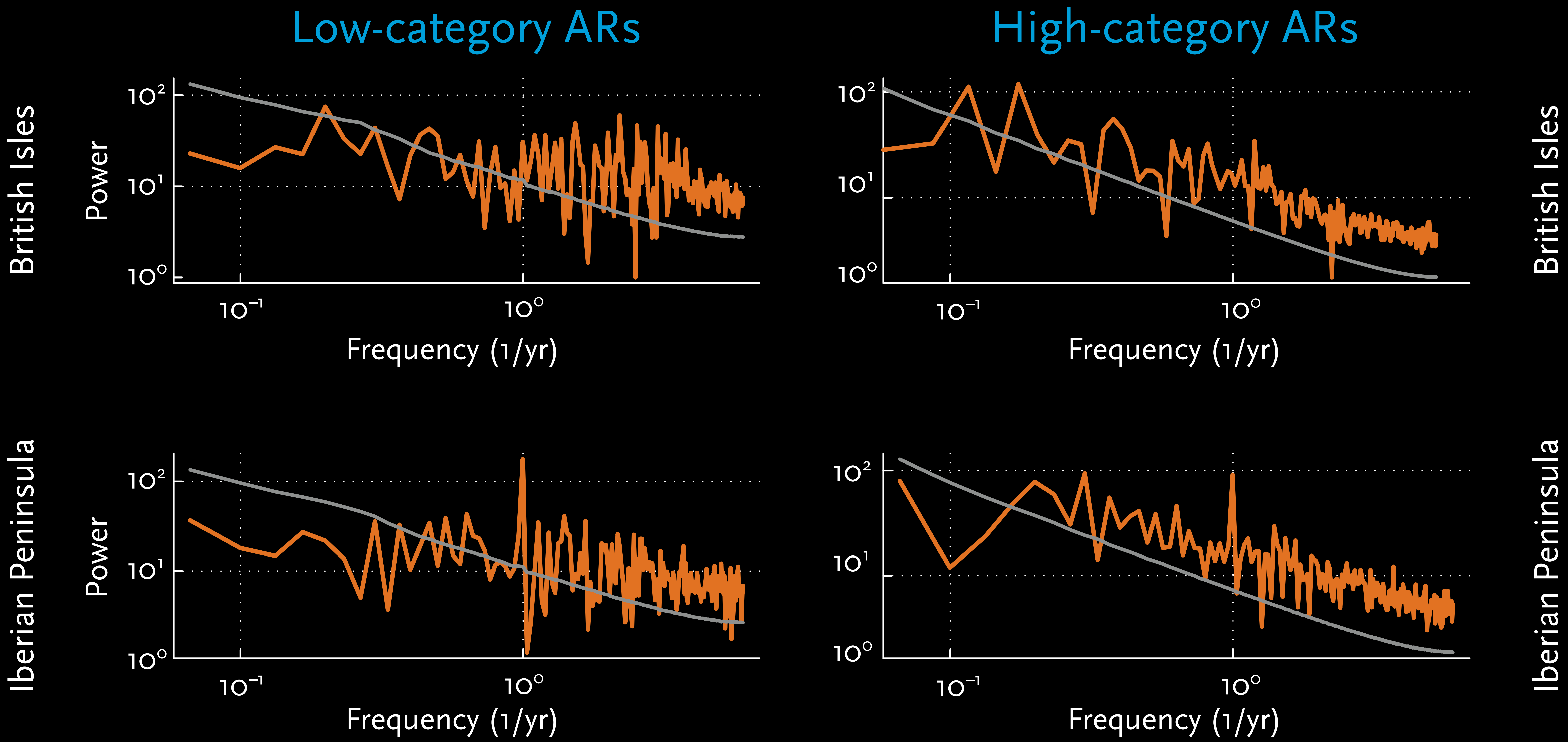
Powerspectra Atmospheric Rivers



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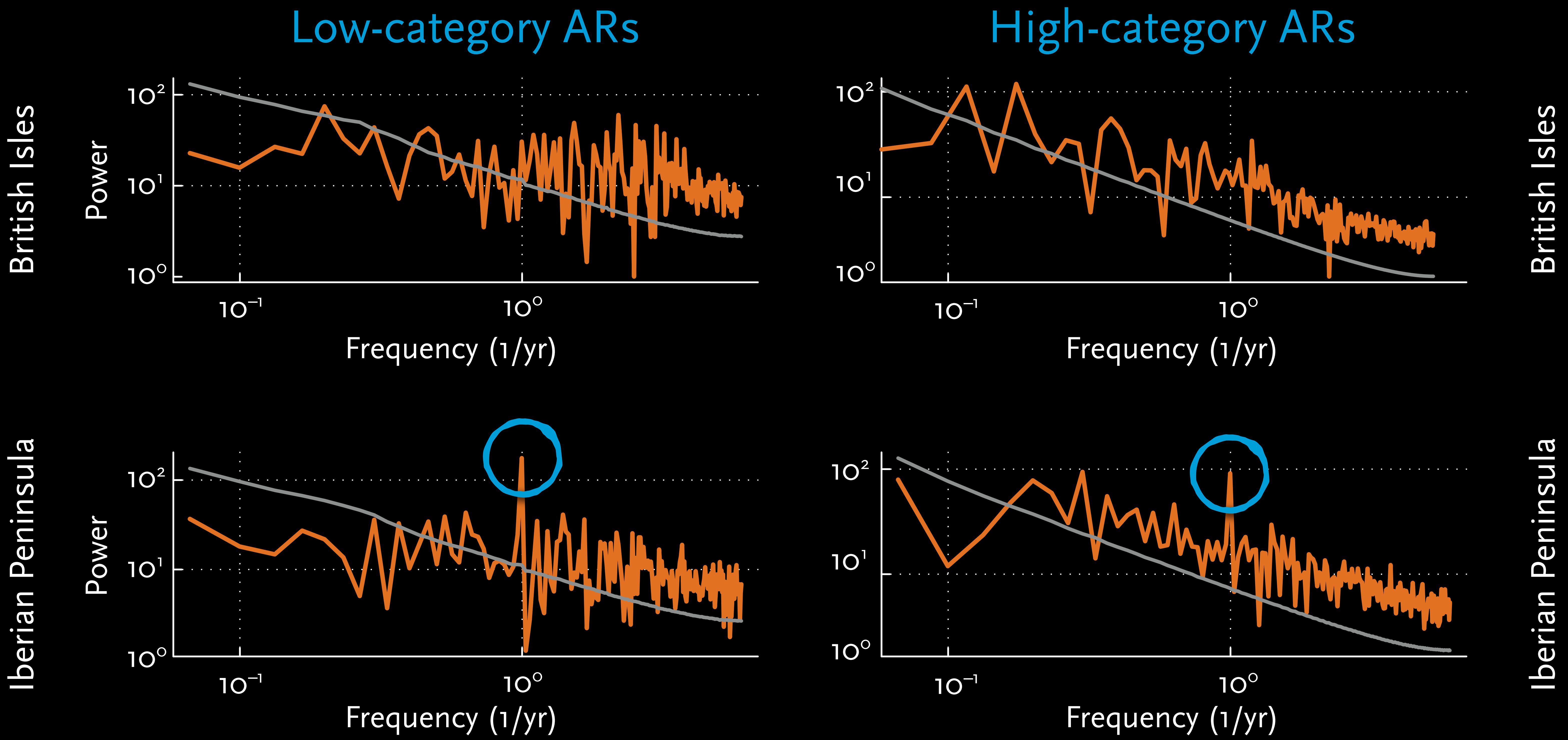
Powerspectra Atmospheric Rivers



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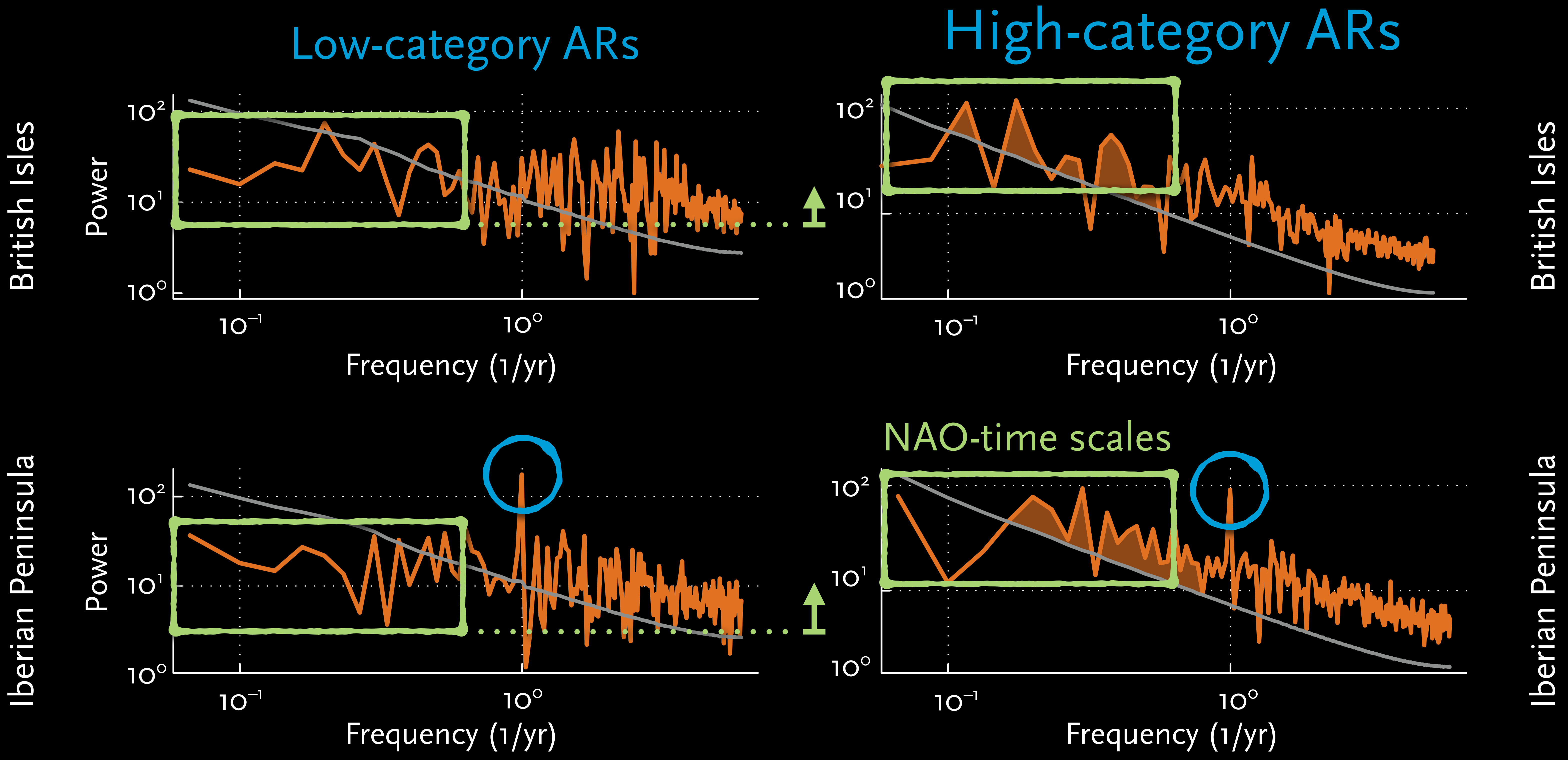
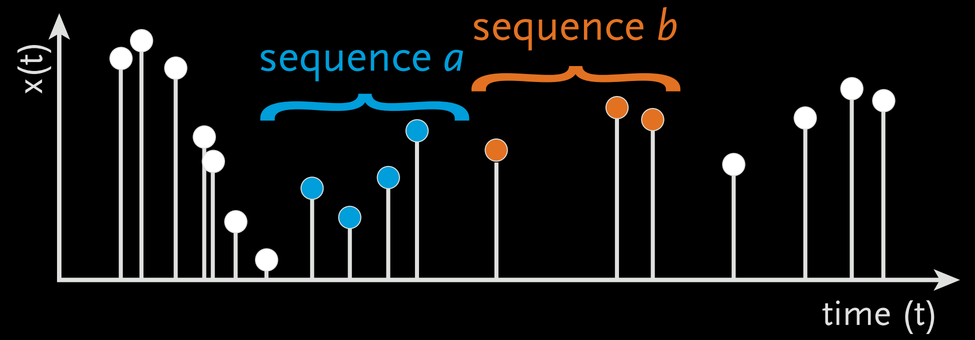
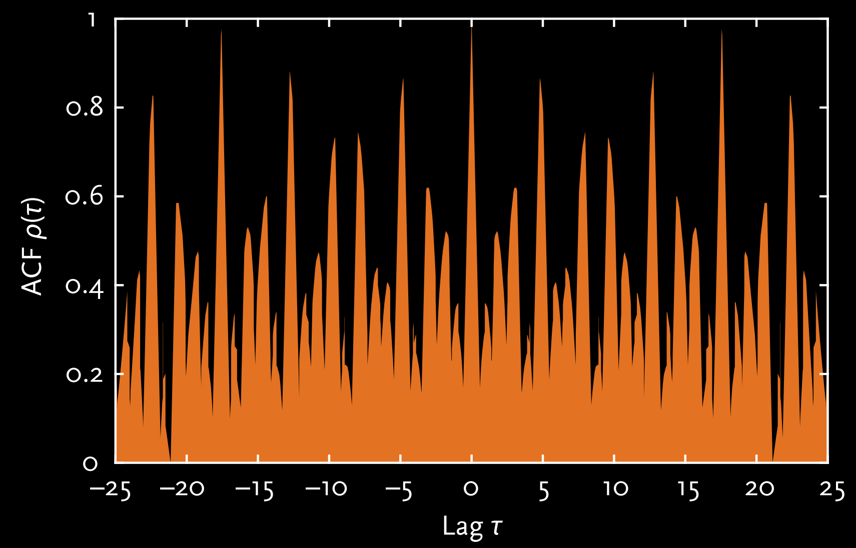


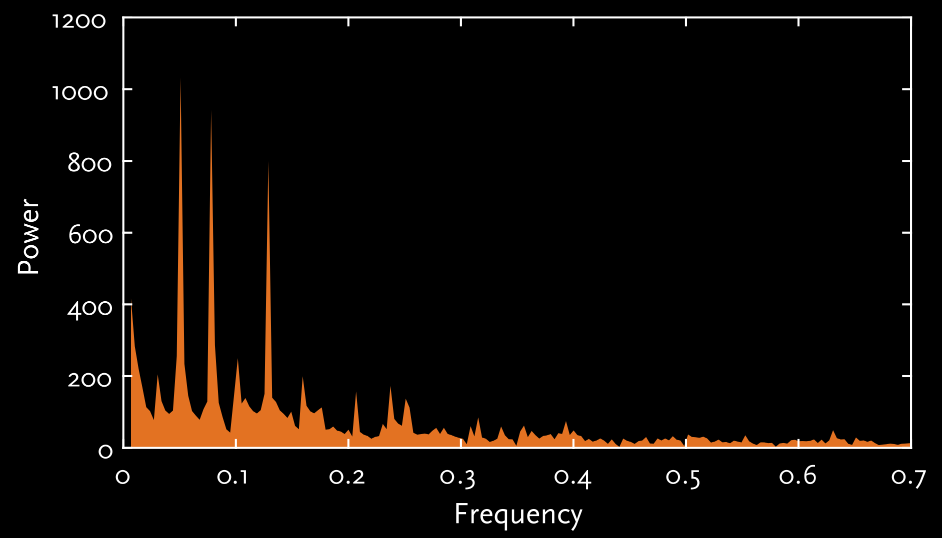
Marwan & Braun, Chaos, 2023

Open Questions

- Non-stationary event distribution, clustered events
- Anti-correlation in event series
- Effect of normalisation in ED-ACF
- Including amplitude variability
- Alternative metrics (ARI-SPIKE, Needleman-Wunsch distance, LCSS)
- Harmonics



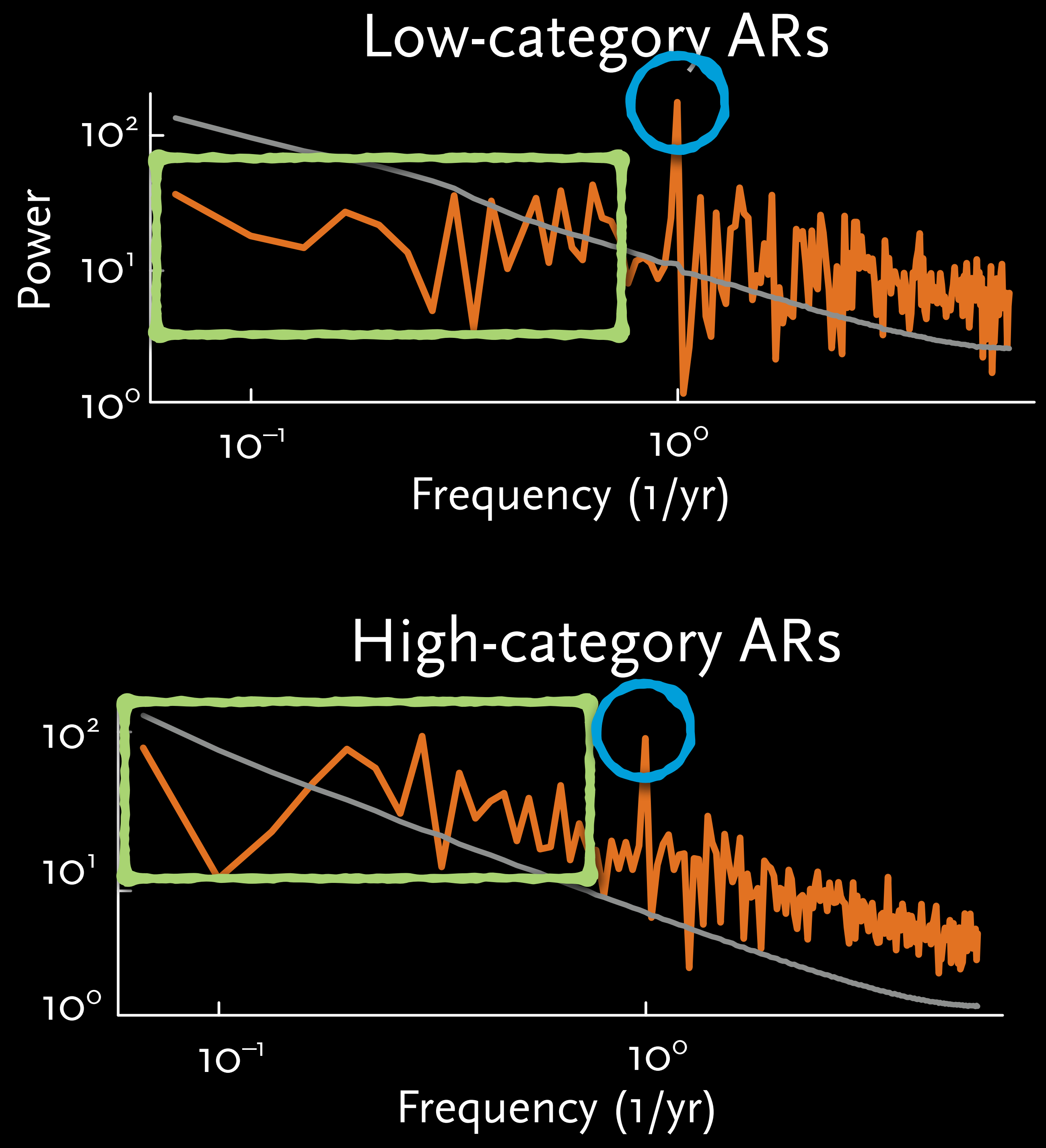




Marwan, Front. Appl. Math. Stat. 9, 2023

Take Home Message

- Simple power spectrum estimation for event data
- Atmospheric rivers in Europe:
  - Clear seasonal cycle, except British isles
  - High spectral power multiannual/ decadal time-scales for high-cat. ARs



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