Recurrence Based Entropies

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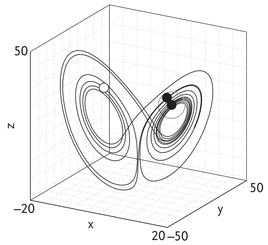
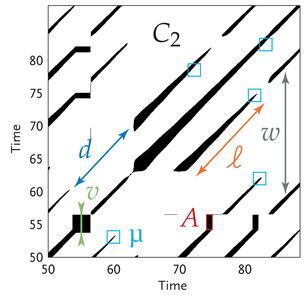
Recurrence plot





Phase space trajectory (left) and recurrence plot with selected features used for entropy calculations (right) – not all entropy variants are presented here.

$$R_k(i,j) = \Theta\left(\varepsilon - \|\vec{x}_k(i) - \vec{x}_k(j)\|\right)$$

examples of microstates μ:

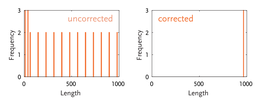
"Standard" RQA entropy

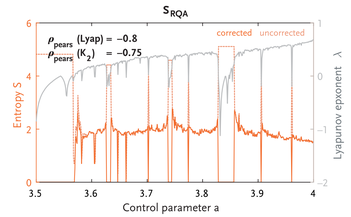
$$S = -\sum_{\ell=1}^{N} p(\ell) \ln p(\ell).$$

- variability of diagonal lines
- mostly used RP based entropy

⚠ border effects (cut diagonal lines), noise, tangential motion

Line histogram periodic signal.



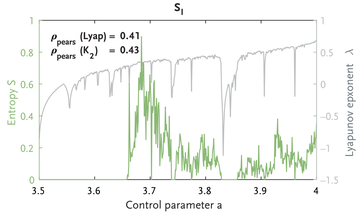


Intermittency entropy

$$S = -\sum_{v=1}^{N} p(v) \ln p(v).$$

- variability of vertical lines, i.e., laminar phases

⚠ only valid for dynamics with intermittent/ laminar regimes; meaning still unclear

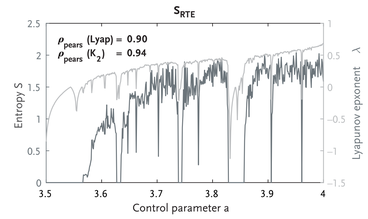


Recurrence time entropy

$$S = -\sum_{w=1}^{N} p(w) \ln p(w).$$

- variability of recurrence times (periods)
- related to KS entropy

⚠ border effects (cut vertical white lines)

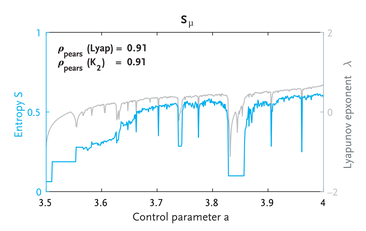


Microstates entropy

$$S_\mu = -\sum_{\mu}^{N^*} p(\mu) \log p(\mu)$$

- variability of short time recurrence patterns
- fast (when using random subset)

⚠ mixing of structures (no clear physical meaning); sensitive to embedding

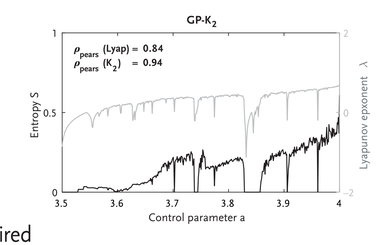


K2 entropy (GP algorithm)

$$S_{\hat{K}_2} = \frac{1}{k\Delta t} \ln \frac{\frac{1}{N_l(N_l-1)} \sum_{i,j} R_{i,j}^{(l)}}{\frac{1}{N_{l+k}(N_{l+k}-1)} \sum_{i,j} R_{i,j}^{(l+k)}}$$

- correlation entropy, mathematically straightforward, using correlation sum C_2

⚠ high embedding dim. and scaling region required

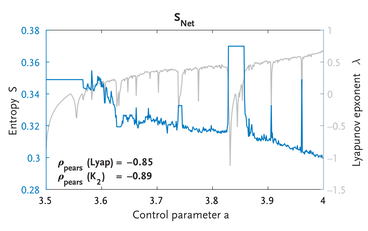


Network entropy

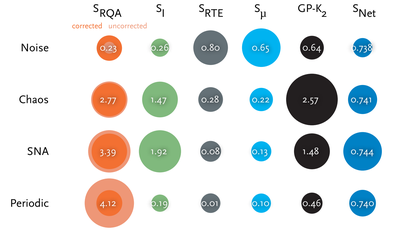
$$S_{\mathrm{Net}} = \frac{1}{N \log(N-1)} \sum_i \log \sum_j (R_{i,j} - \delta_{i,j})$$

- heterogeneity of phase space density

⚠ geometrical, not a dynamical measure



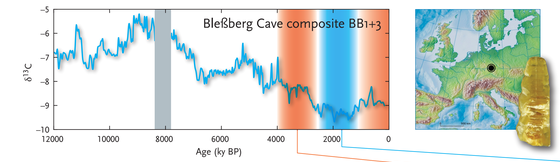
Abilities regarding different dynamics



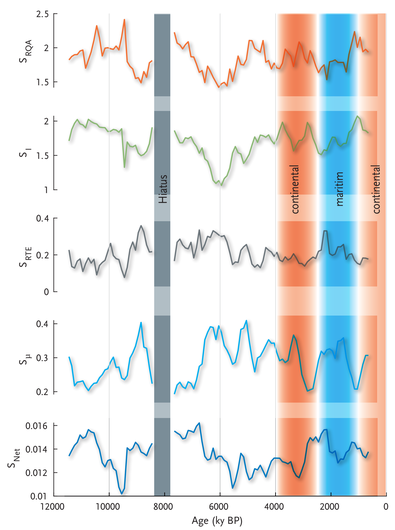
- complexity of recurring structures in the data
- further entropy definitions use block area, white diagonal lines, cumulative diagonal line lengths

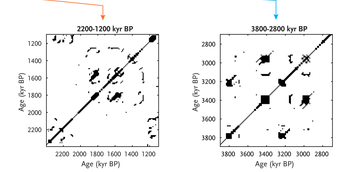
Entropies behave differently for different dynamics. Uncorrected RQA entropy shows too high values for periodic dynamics due to the border effects.

Regime transition in palaeoclimate



Location of Bleßberg Cave and stalagmite BB1, used for palaeoclimate and -environmental interpretations (changes in vegetation densities).





alternating regular and complex dynamics in local climate (indicating the change of maritime and continental influence)

Preliminary results of changing entropies for the palaeoclimate proxy.

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References

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