Analysing Spatially Extended High-dimensional Dynamics by Recurrence Plots

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Recurrence plots and their quantification are modern and powerful tools of nonlinear time series analysis – however, can this approach be applied to high-dimensional systems?

$$\mathbf{R}_{i,j} = \Theta(\varepsilon - \|\vec{x}_i - \vec{x}_j\|)$$

Phase space representation and corresponding recurrence plot.
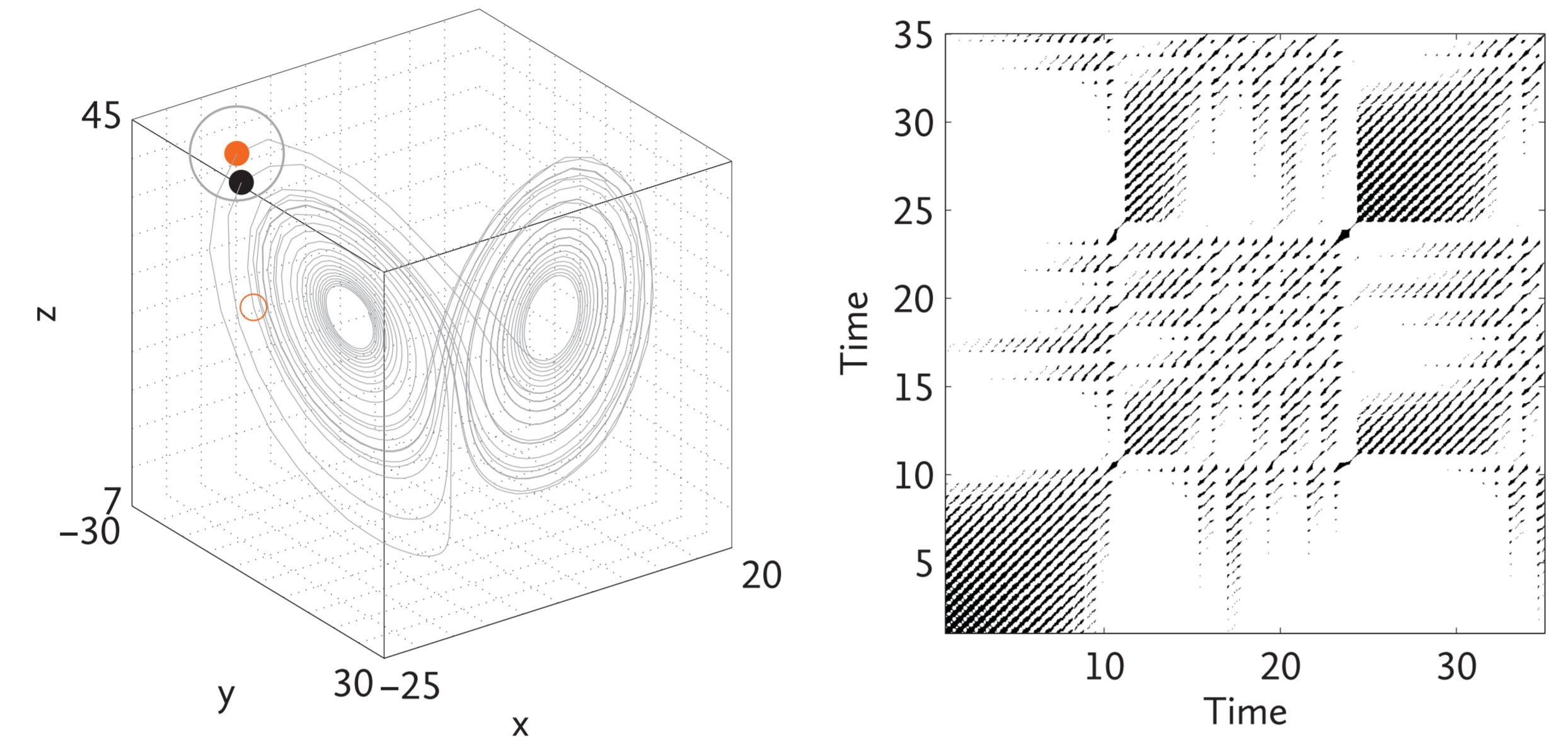
PROTOTYPICAL EXAMPLE: LORENZ96

$$\frac{dx_k}{dt} = (x_{k+1} - x_{k-2})x_{k-1} - x_{k-1} + f$$

shows different types of spatio-temporal dynamics (chaotic, periodic), high-dimensional (x_k – state at node k, f – external force)
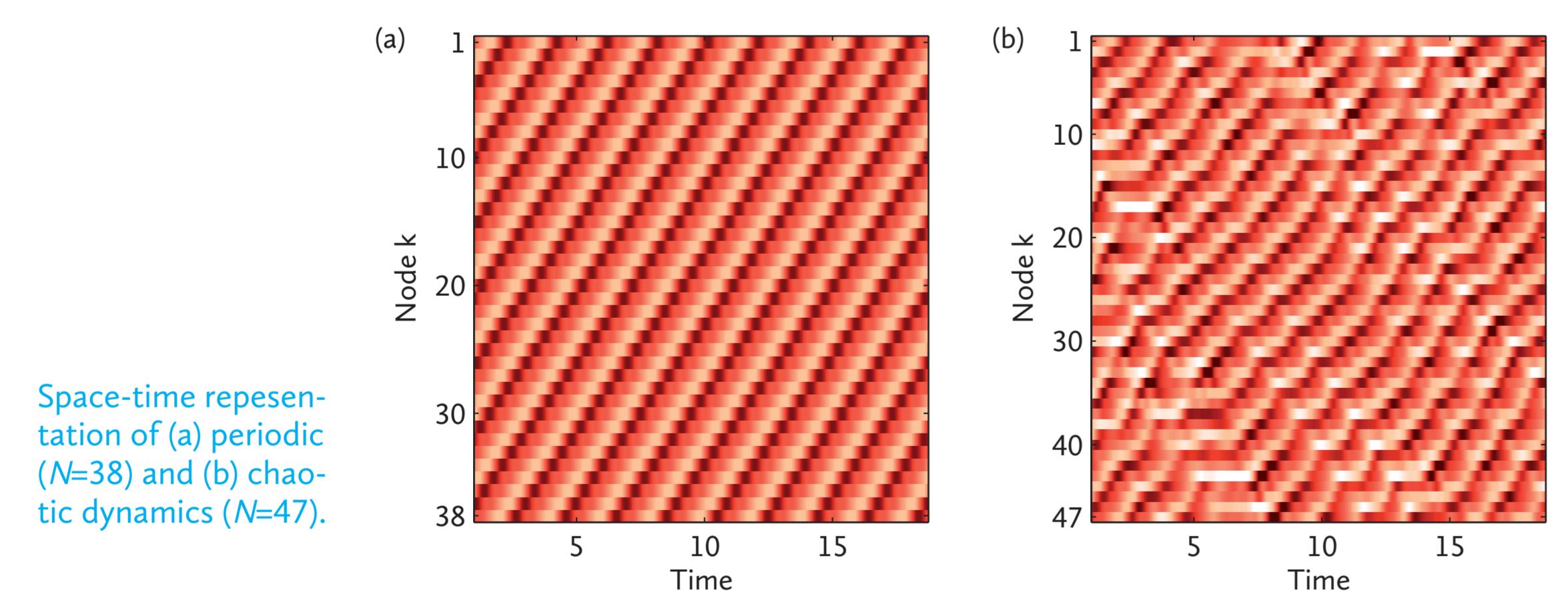

Space-time repesentation of (a) periodic (N=38) and (b) chaotic dynamics (N=47).


(a) Maximal Lyapunov exponent and (b) Kaplan-Yorke dimension calculated from 200,000 iterations; recurrence measures (c) determinism and (d) transitivity dimension are calculated from only 1500 data points, but nevertheless reflect the dynamical regime changes.

Recurrence plots of (a) periodic (N=38) and (b) chaotic dynamics (N=47).

REAL WORLD APPLICATION: SATELLITE TIME SERIES (EVI)

investigate the difference in the spatio-temporal vegetation dynamics in a subhumid (Spain) and in a semiarid (Brazil) climate
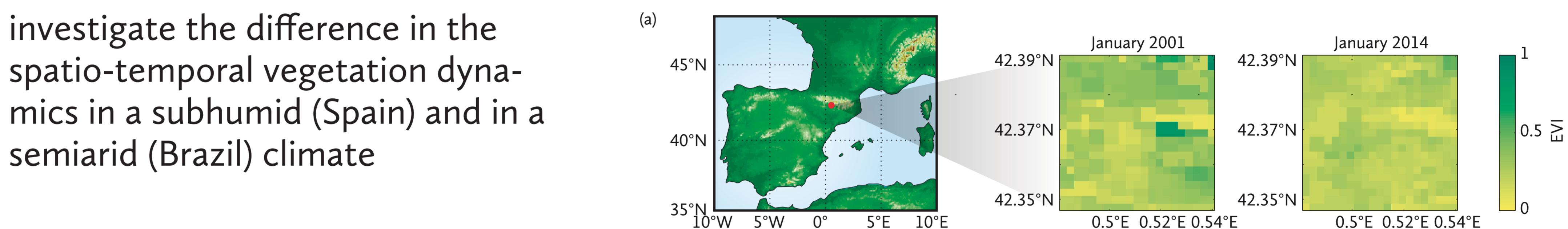

Geographical location and MODIS extended vegetation index (EVI) within the 5 × 5 km² subarea used for the analysis for the regions (a) NE Spain and (b) NE Brazil.
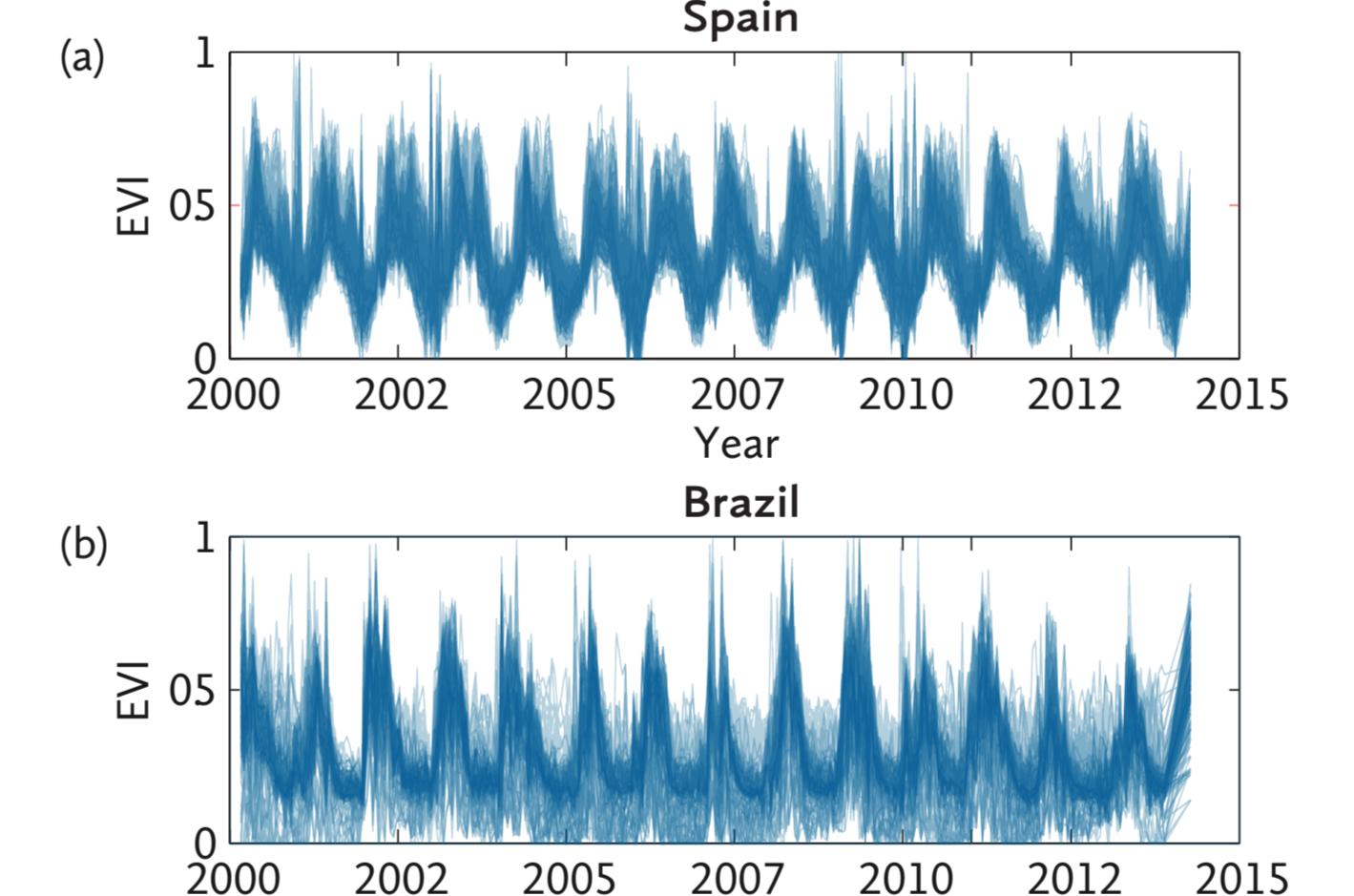
EVI time series of all pixels in the subareas of (a) NE Spain and (b) NE Brazil.
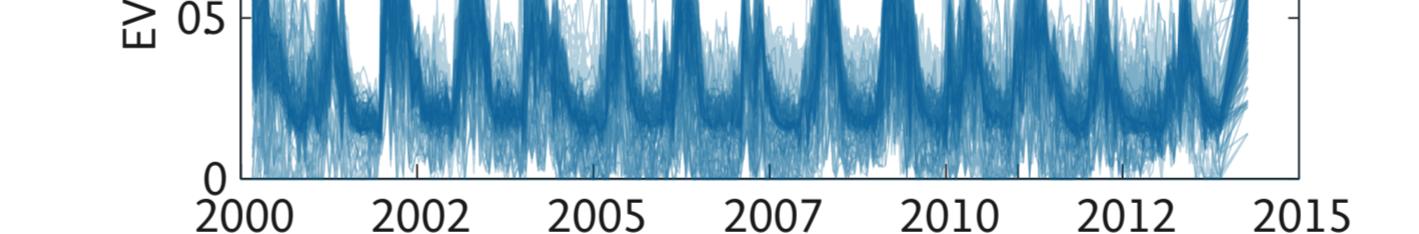
Recurrence plots of (a) NE Spain and (b) NE Brazil.
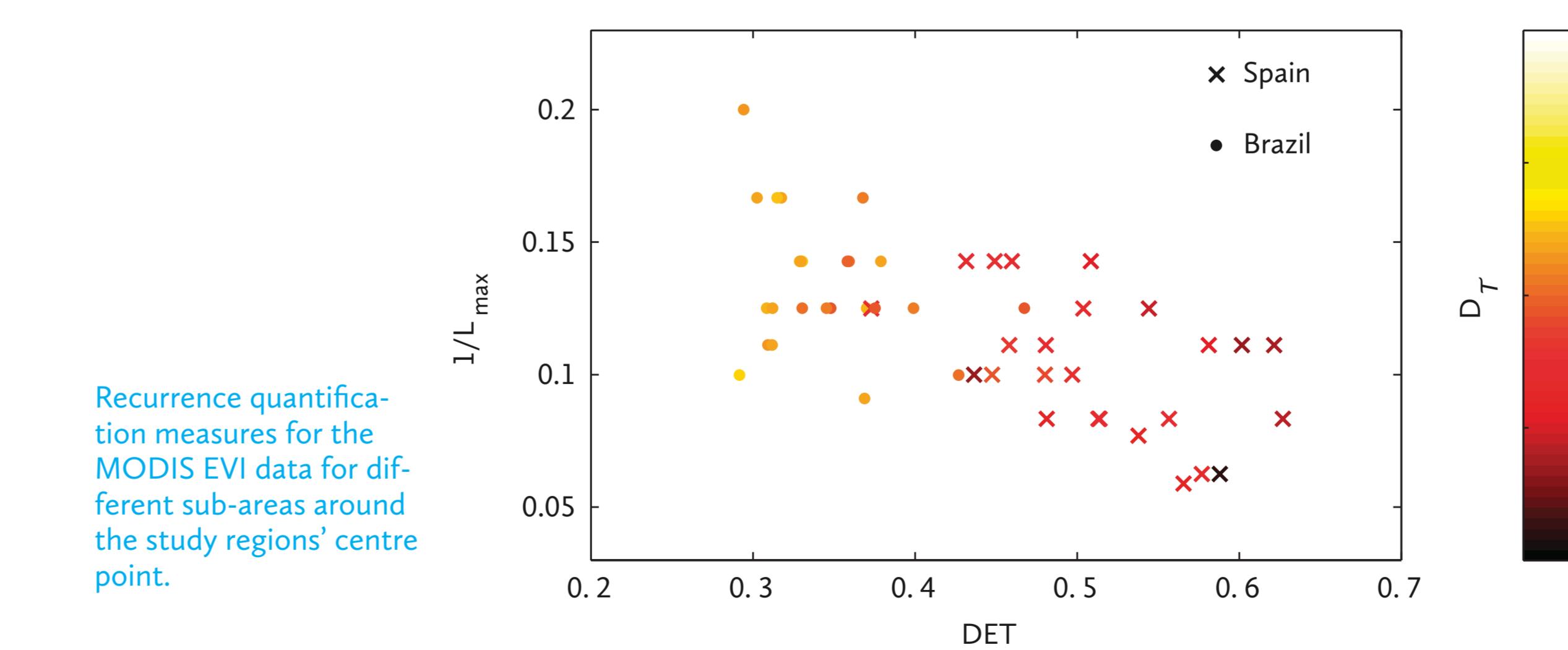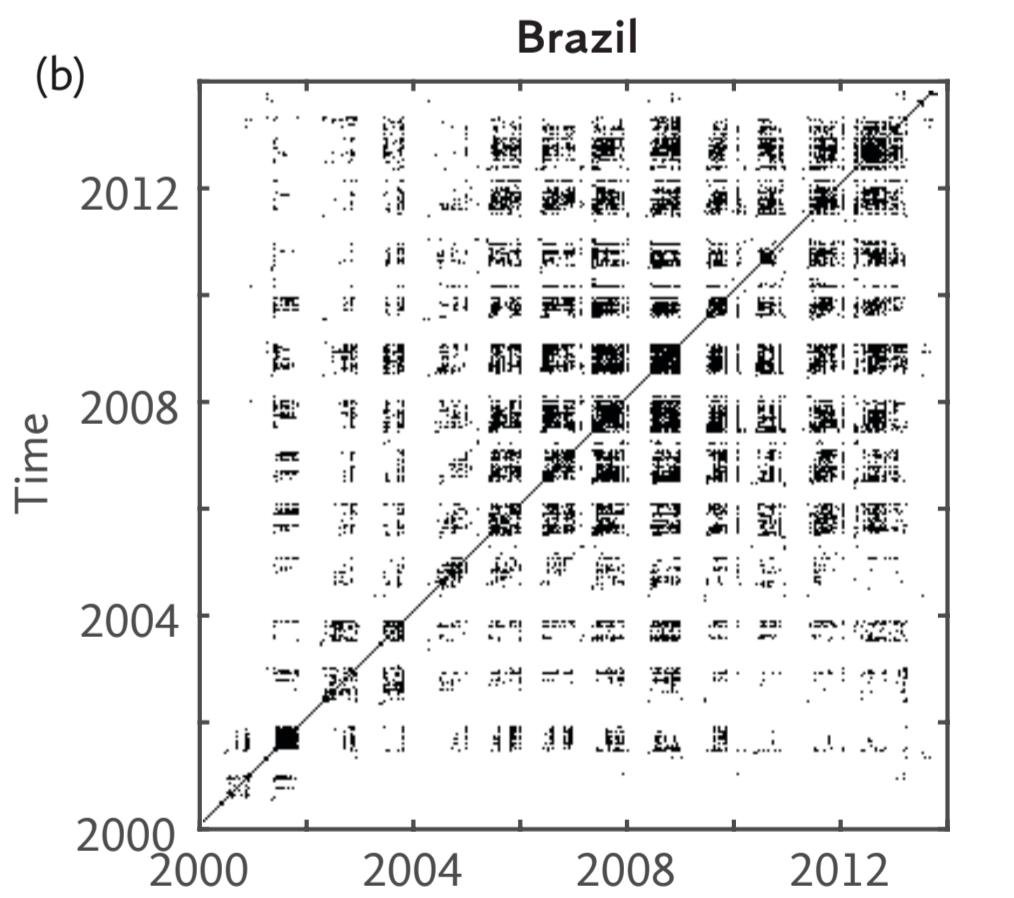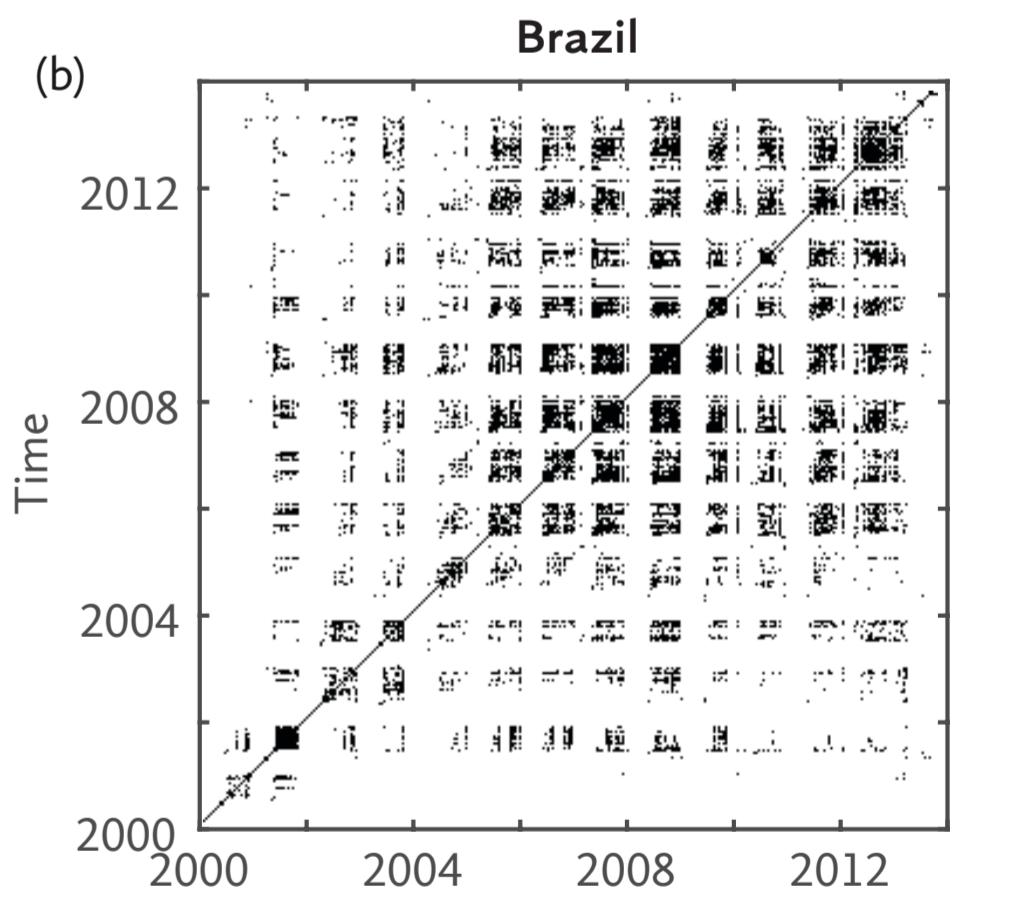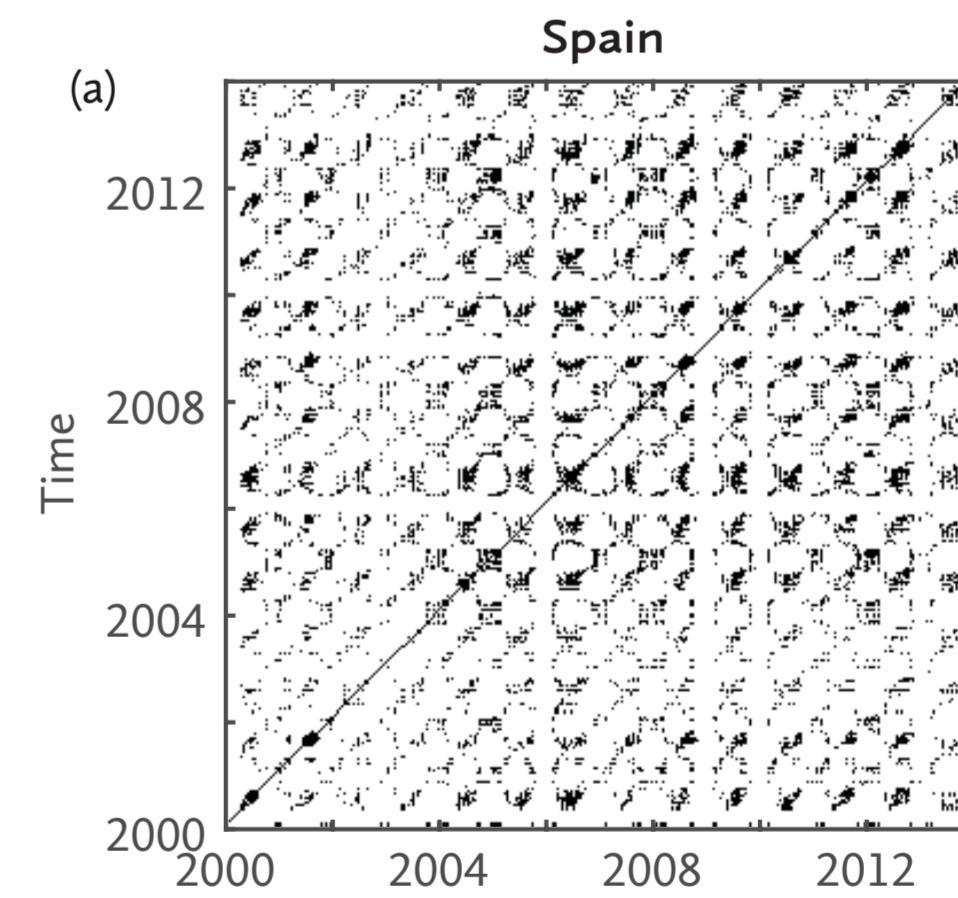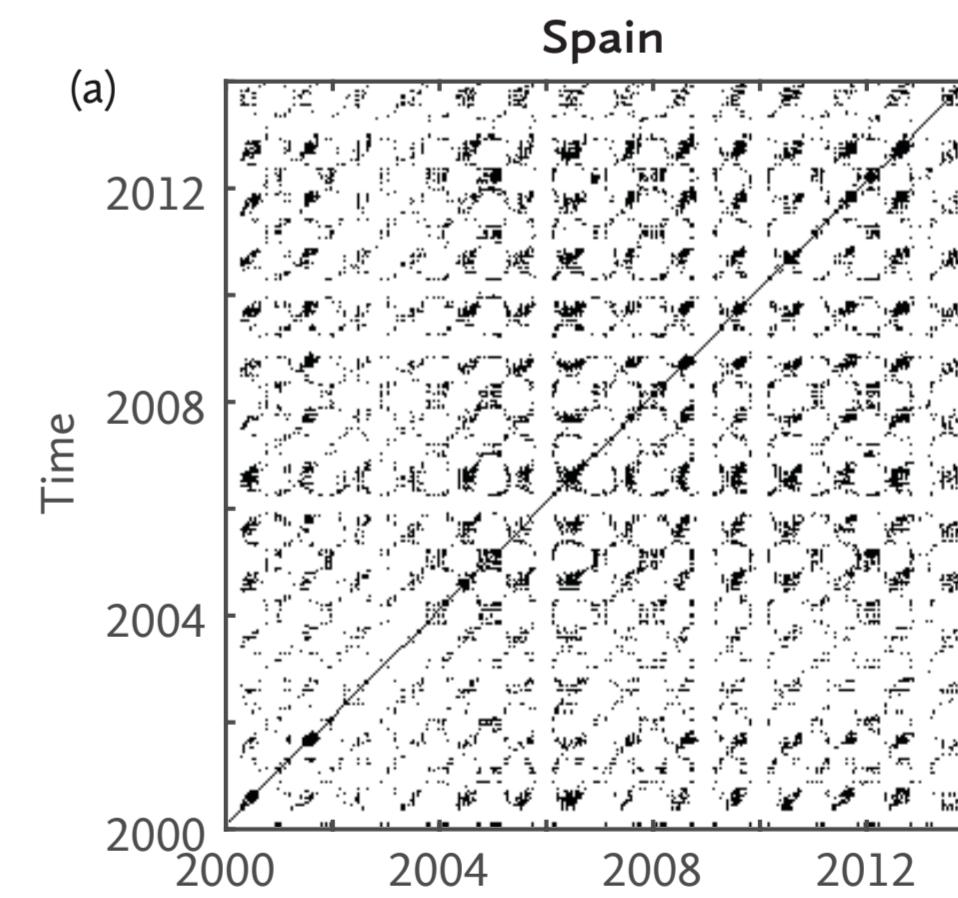
Recurrence quantification measures for the MODIS EVI data for different sub-areas around the study regions' centre point.

clear difference in the spatio-temporal vegetation dynamics where Spain shows a more regular pattern, whereas Brazil is characterized by a more irregular and less predictable behavior

References: Karimi, A., & Paul, M. R. (2010). *Extensive chaos in the Lorenz-96 model.* Chaos, 20(4), 043105.

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