Potsdam Institute for
Climate Impact Research

PIK

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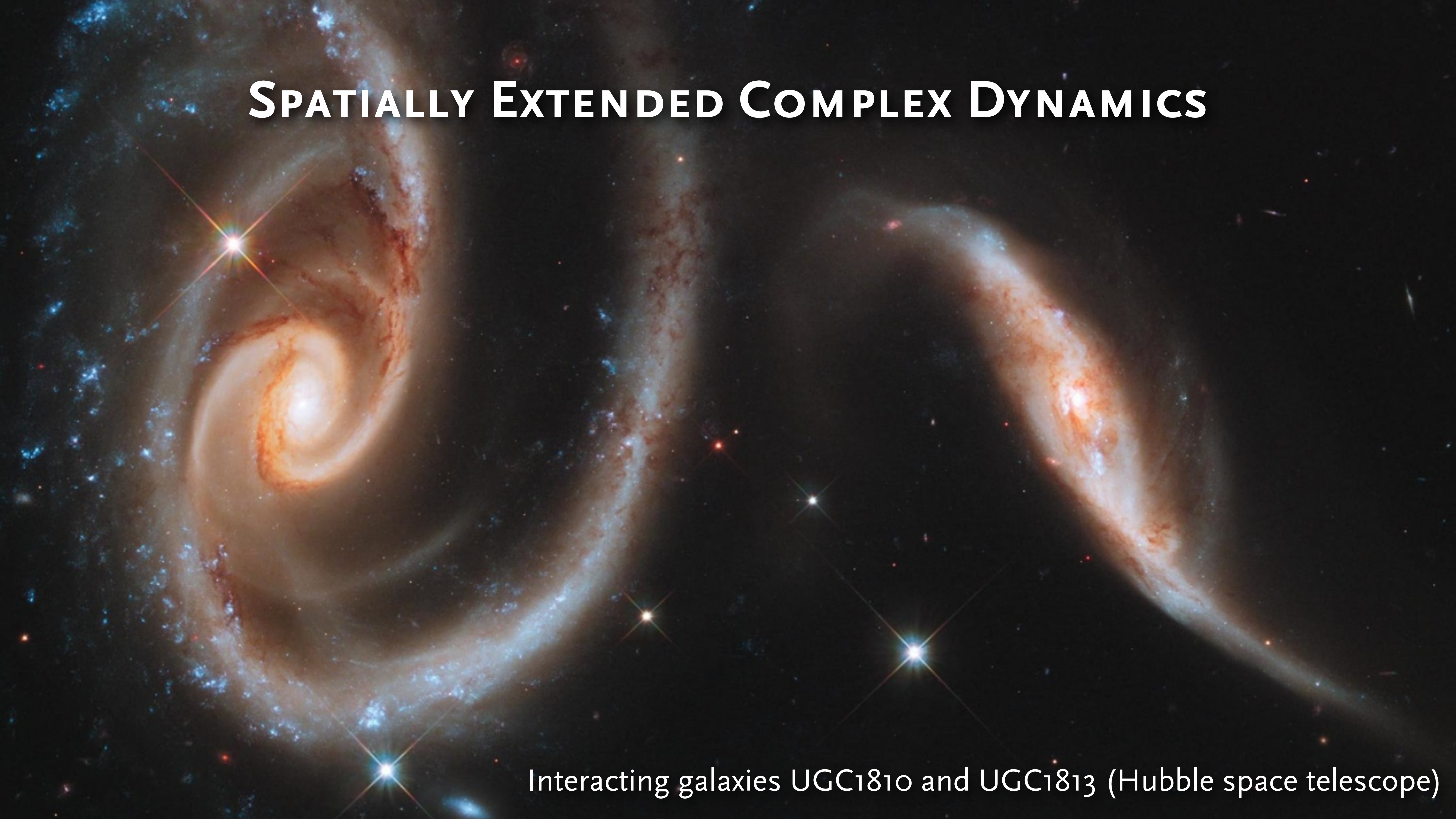
Saskia Foerster, Jürgen Kurths

Recurrence plot analysis of spatially extended high-dimensional dynamics

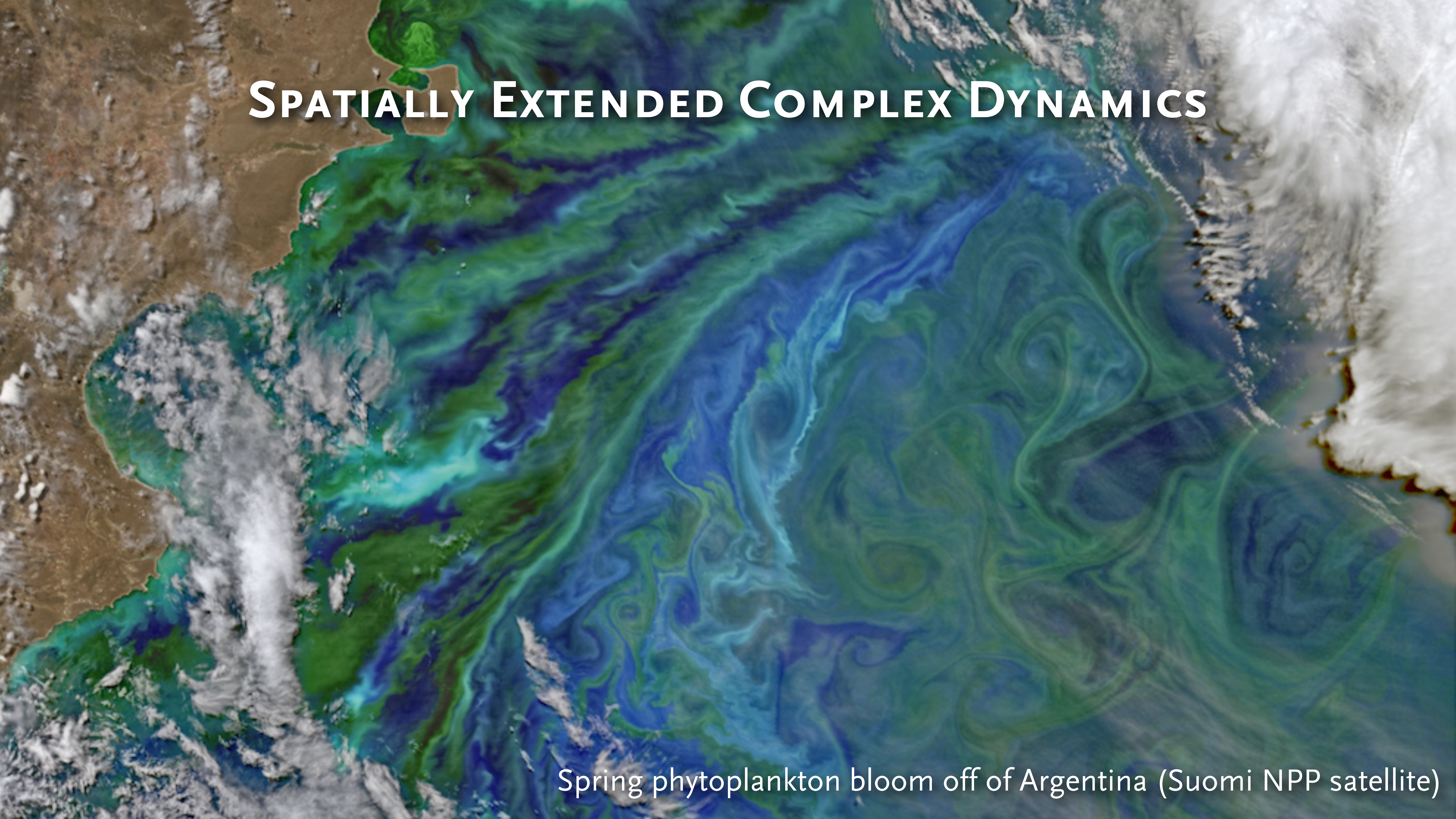
Leibniz-Gemeinschaft

Spatially Extended Complex Dynamics



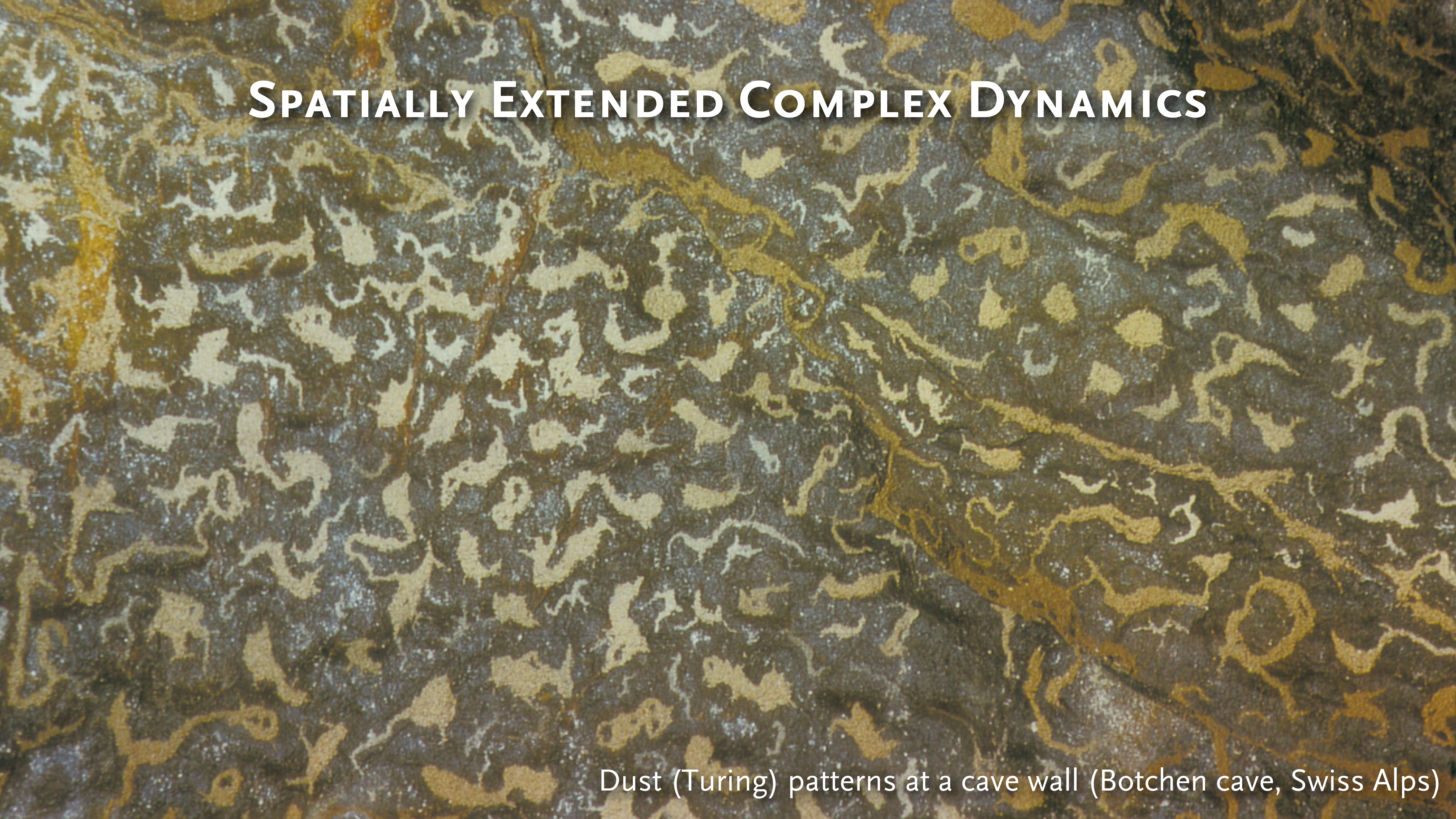
Interacting galaxies UGC1810 and UGC1813 (Hubble space telescope)

Spatially Extended Complex Dynamics



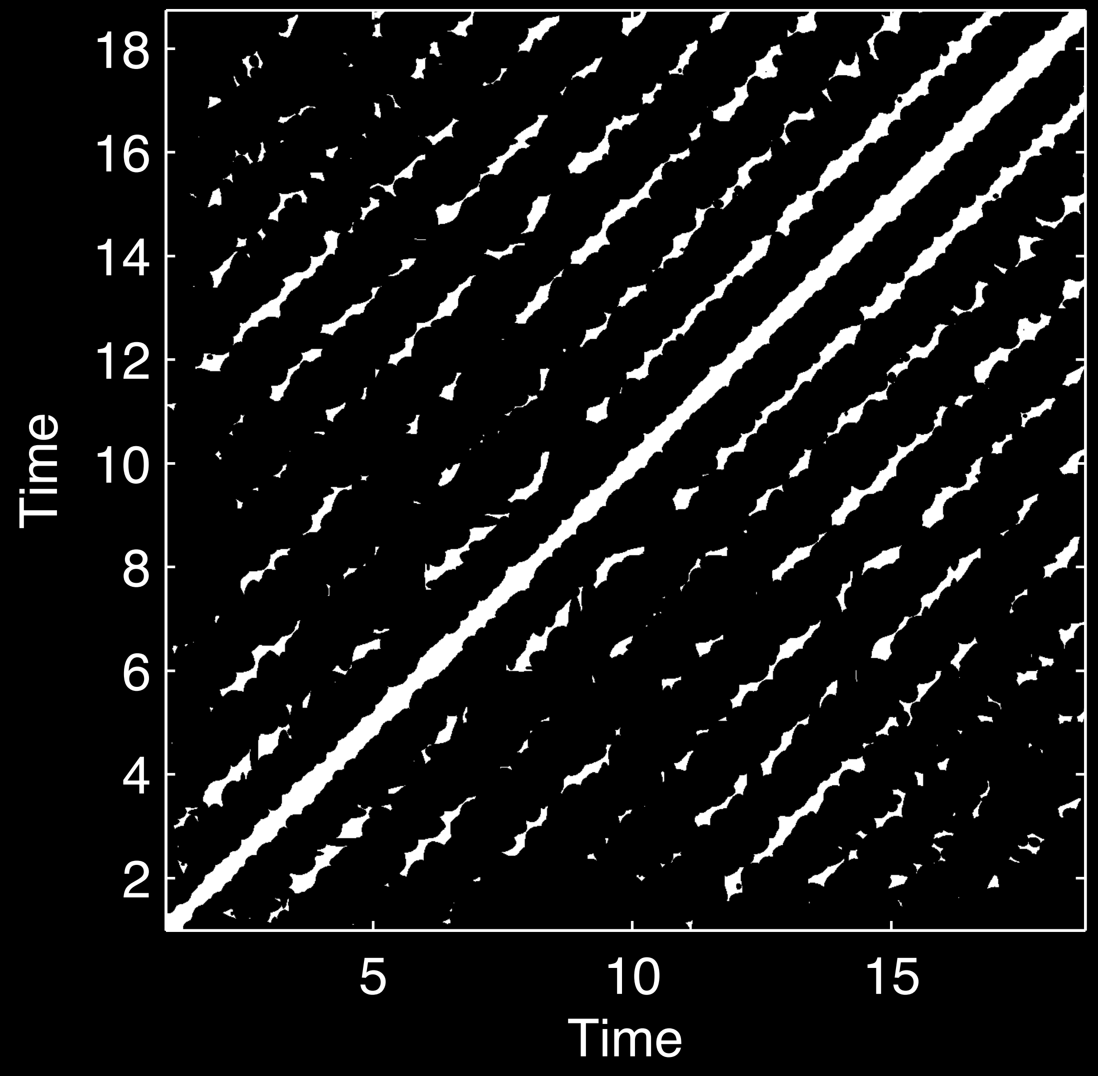
Spring phytoplankton bloom off of Argentina (Suomi NPP satellite)

Spatially Extended Complex Dynamics



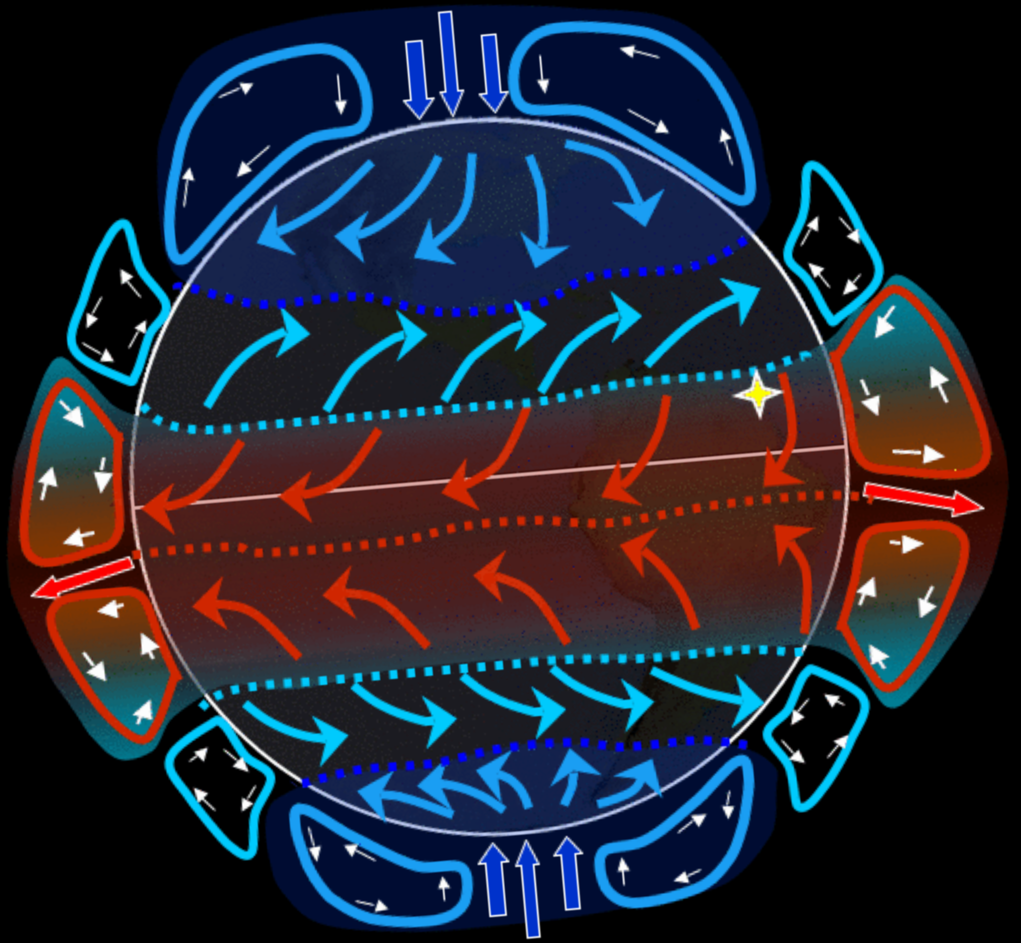
Dust (Turing) patterns at a cave wall (Botchen cave, Swiss Alps)

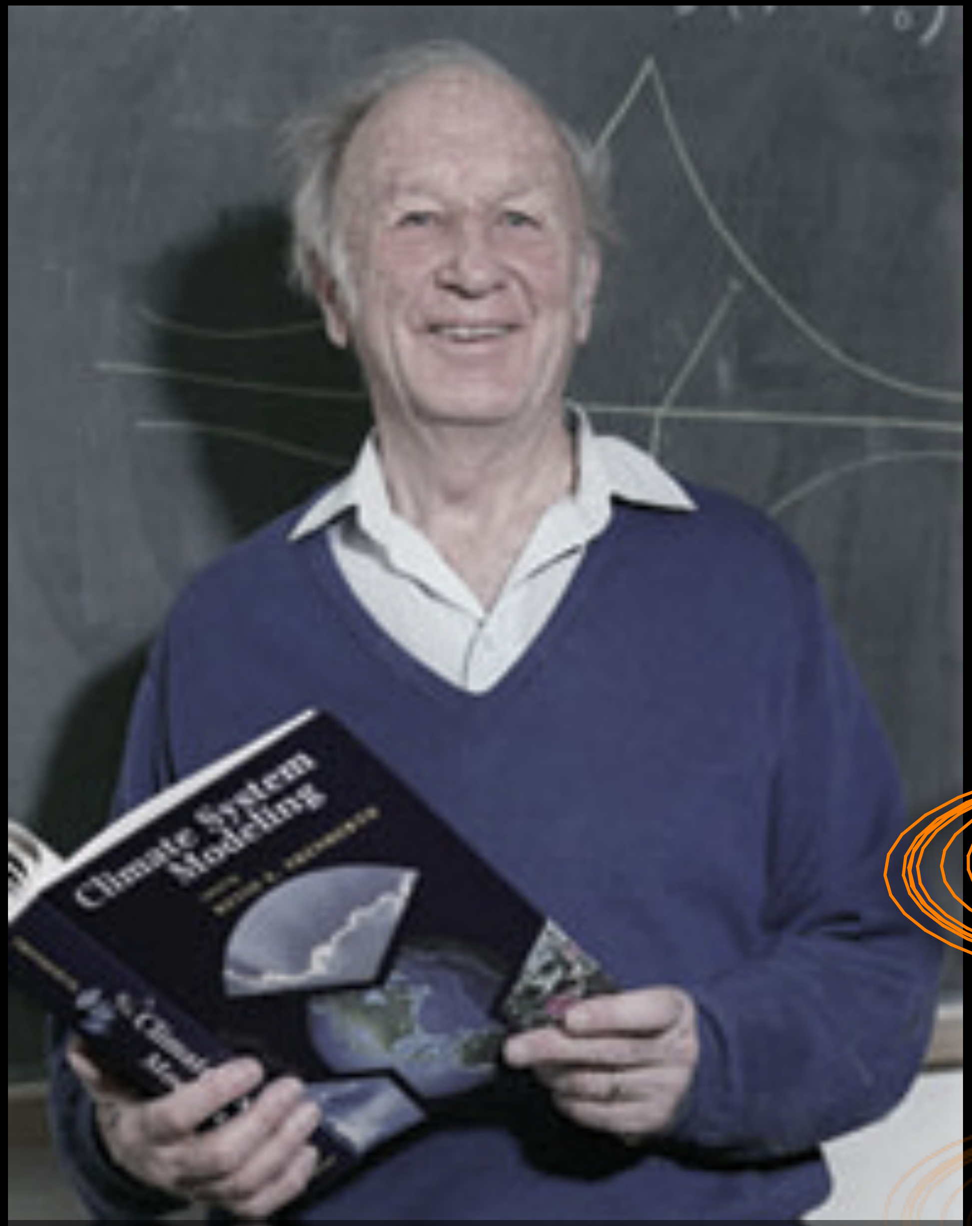
Recurrence Plot Analysis

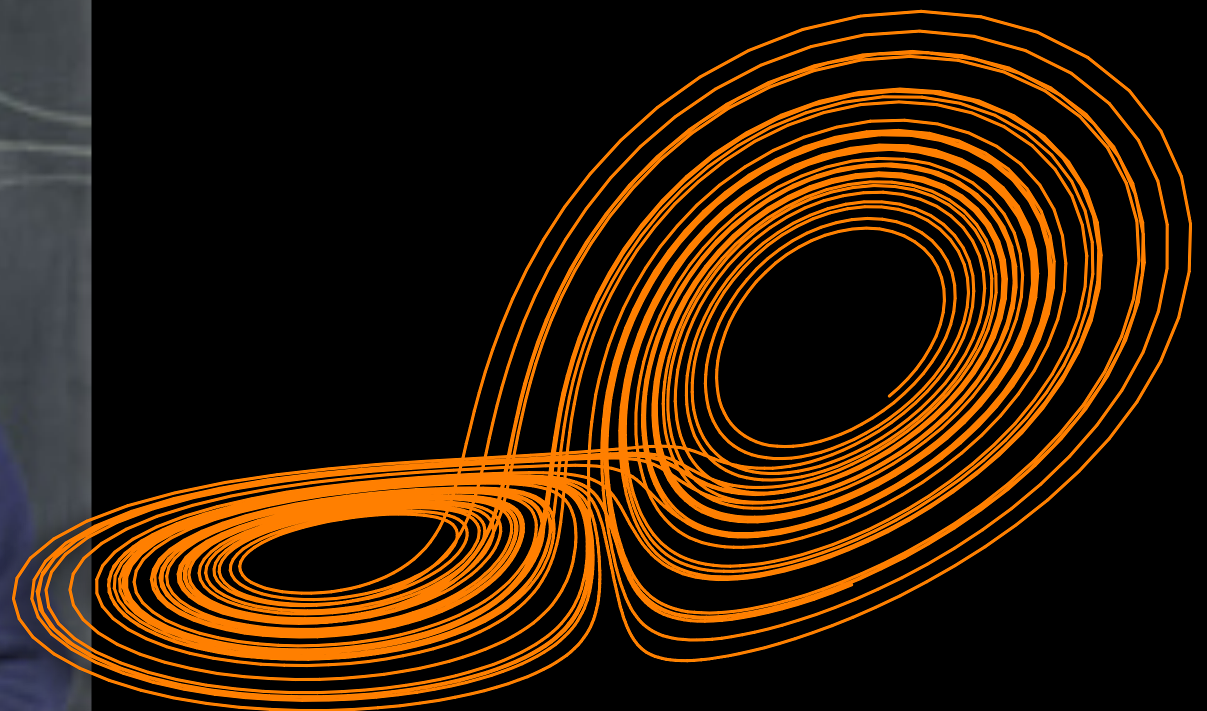


- Suitable for
  - ★ high-dimensional systems?
  - ★ spatio-temporal dynamics?

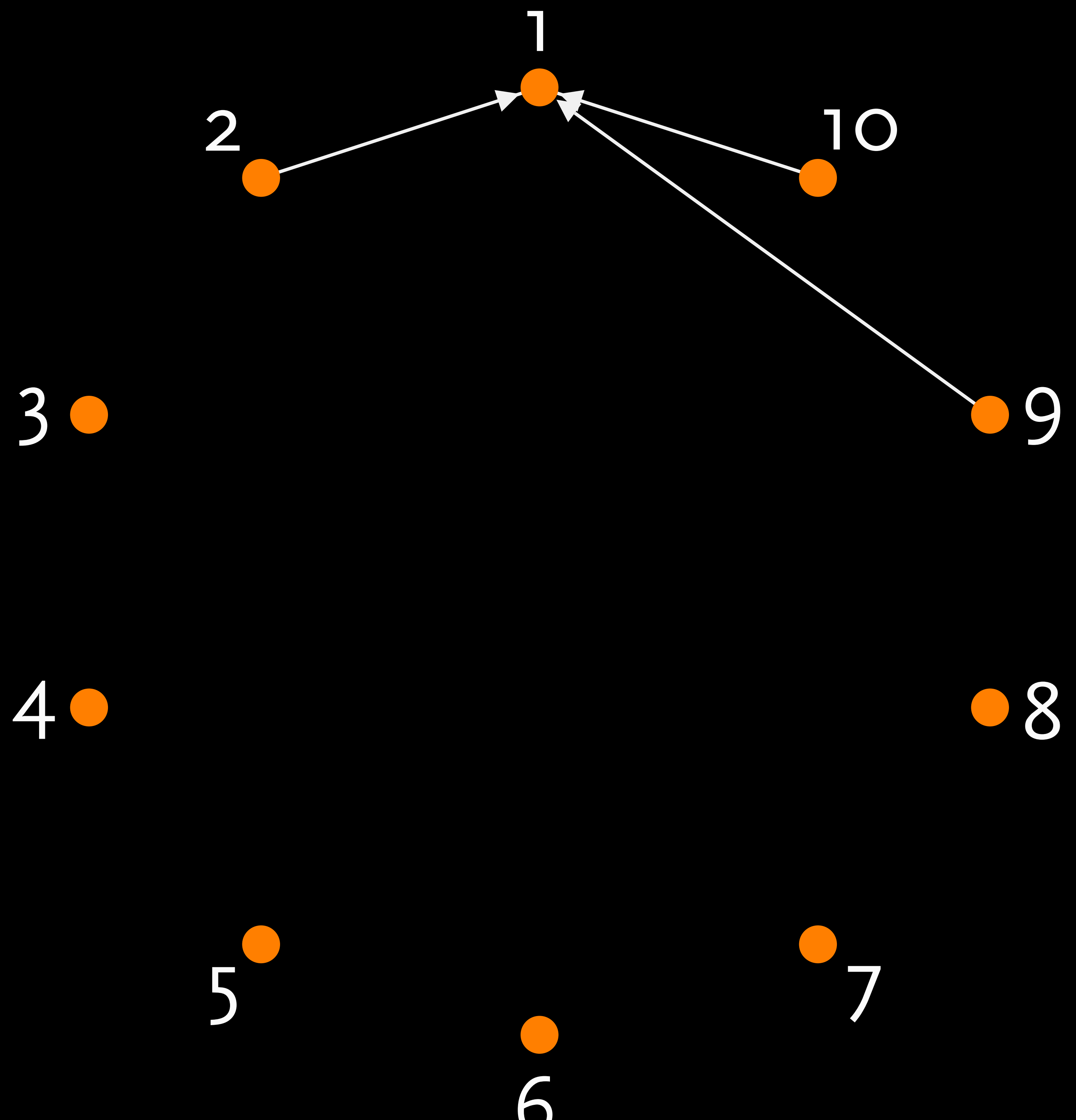
Lorenz96







Lorenz96 – Model



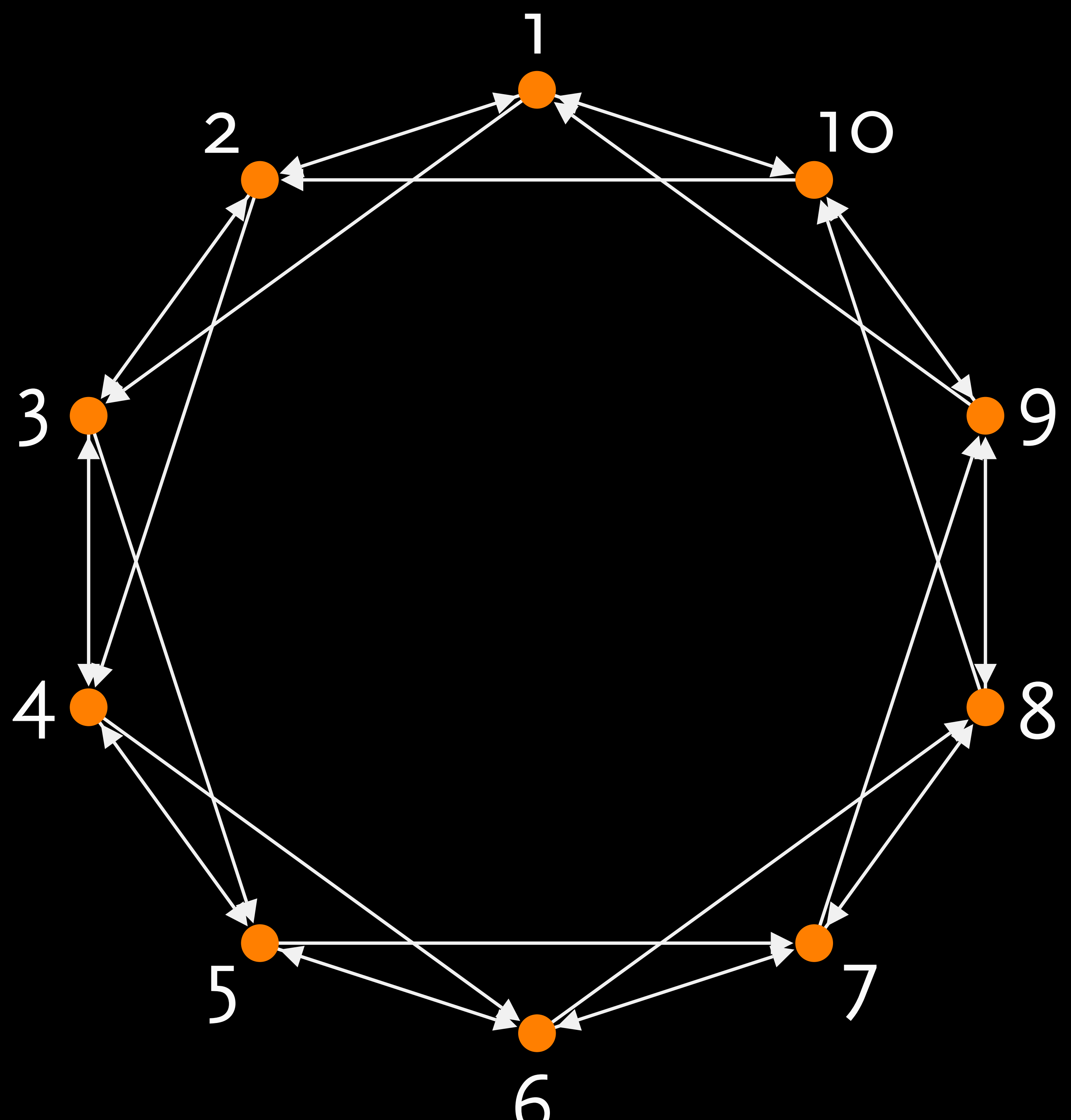
$$\frac{dx_k}{dt} = (x_{k+1} - x_{k-2})x_{k-1} - x_k + f$$

$$x_{N+1} = x_1$$

- Time-continuous linear lattice model
- External forcing f
- System size N

Lorenz, Predictability: A problem partly solved, Vol. 1, ECMWF, Reading, UK, 1996

Lorenz96 – Model



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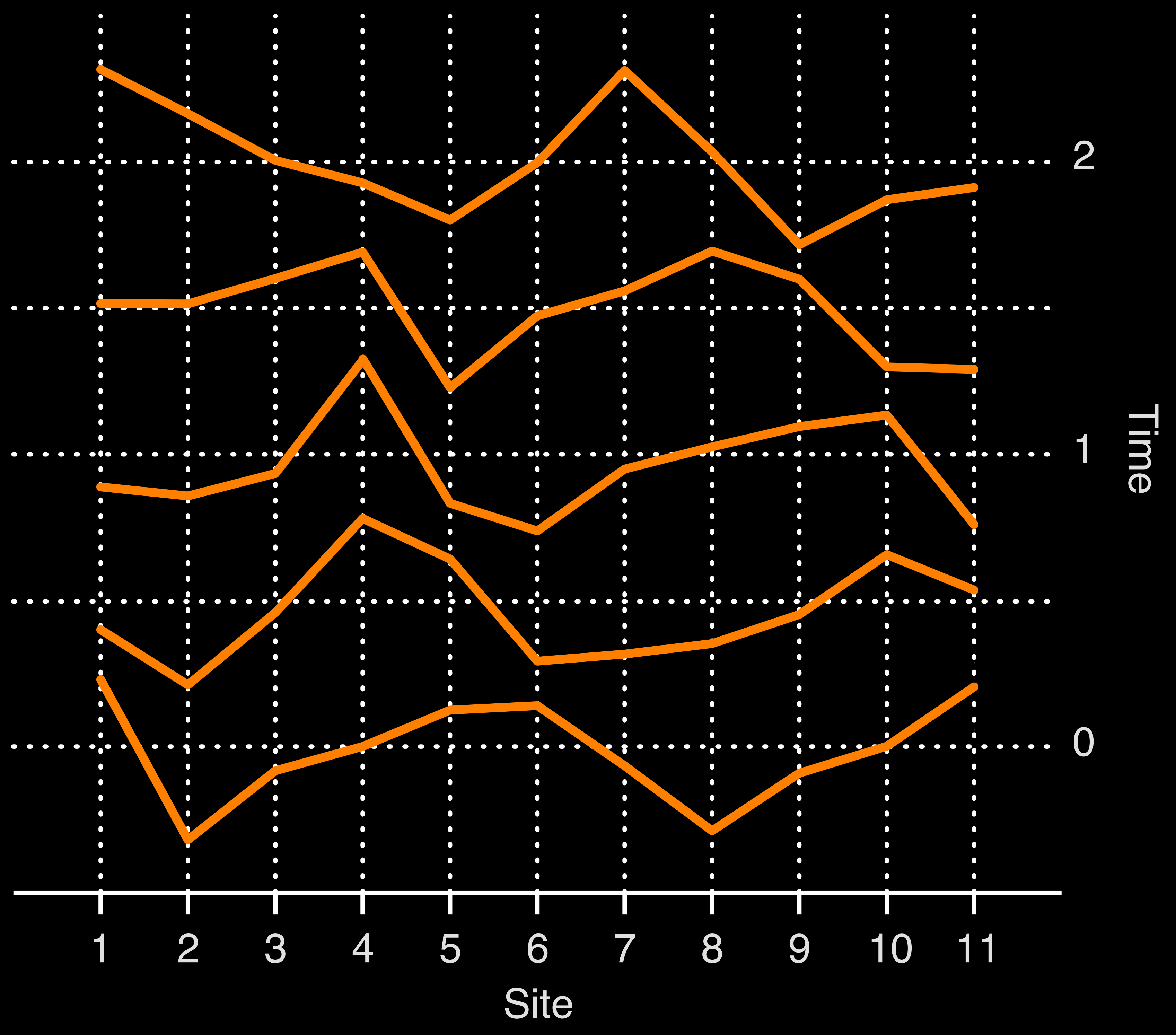

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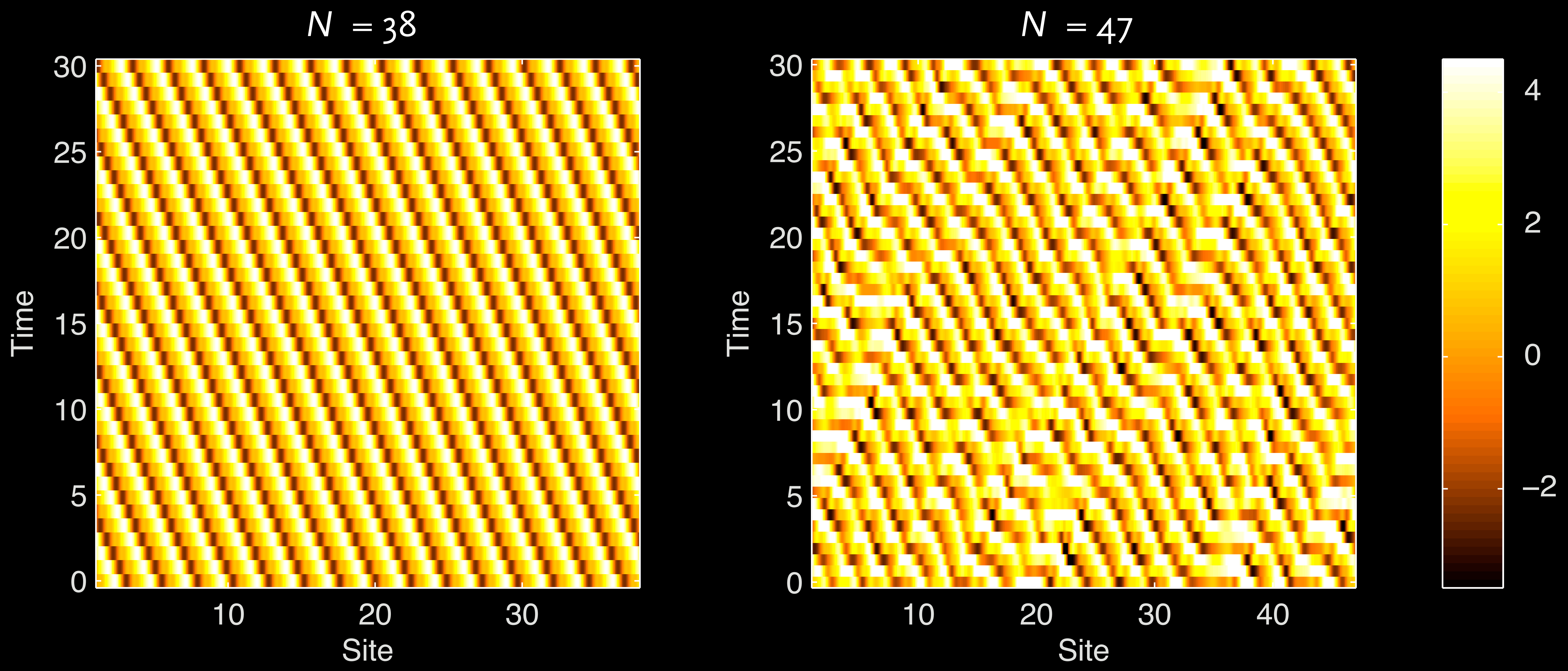
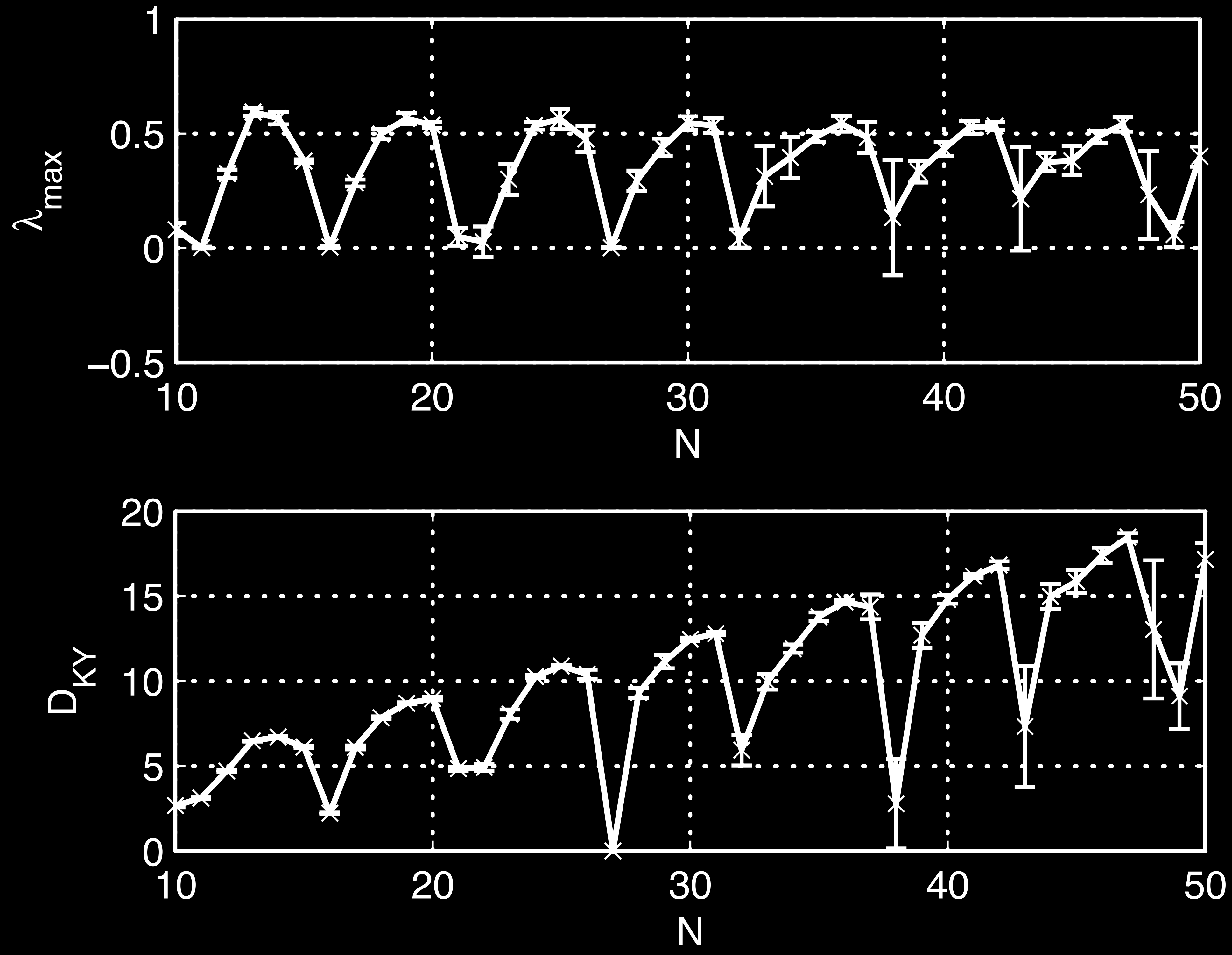
Lorenz96 – Dynamics



- Different dynamics for different system size N

Lorenz96 – Dynamical Properties



- Runge Kutta 4th order
- 200.000 iterations
- 20 different initial conditions

Karimi & Paul, Chaos 20, 2010

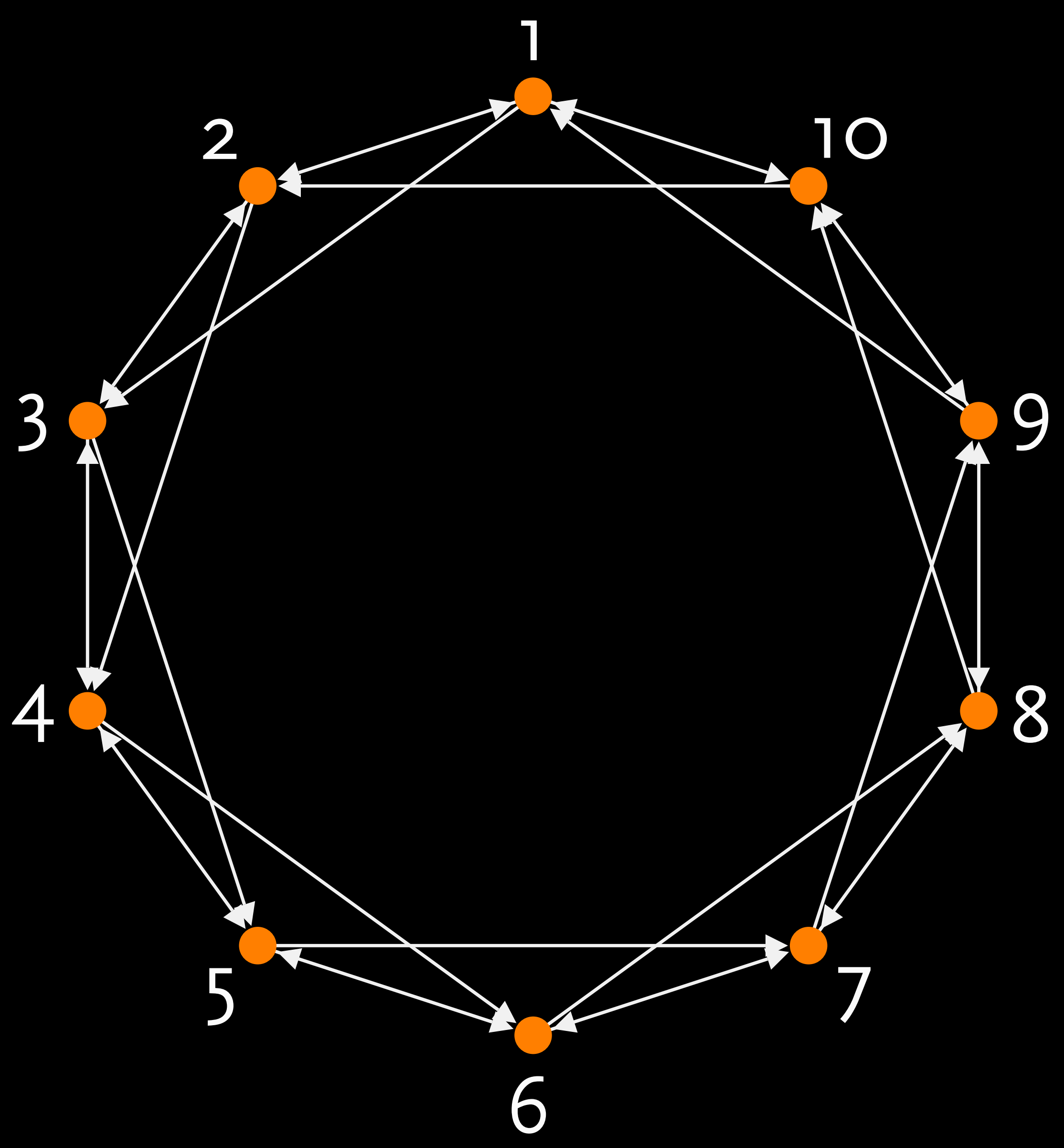
Lorenz96 – Recurrence Analysis

- Phase space vector

$$\vec{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{pmatrix}$$

- Recurrence plot

$$R_{i,j} = \Theta(\varepsilon - \|\vec{x}(t_i) - \vec{x}(t_j)\|)$$



Marwan et al, Phys Lett A 379, 2015

Lorenz96 – Recurrence Analysis

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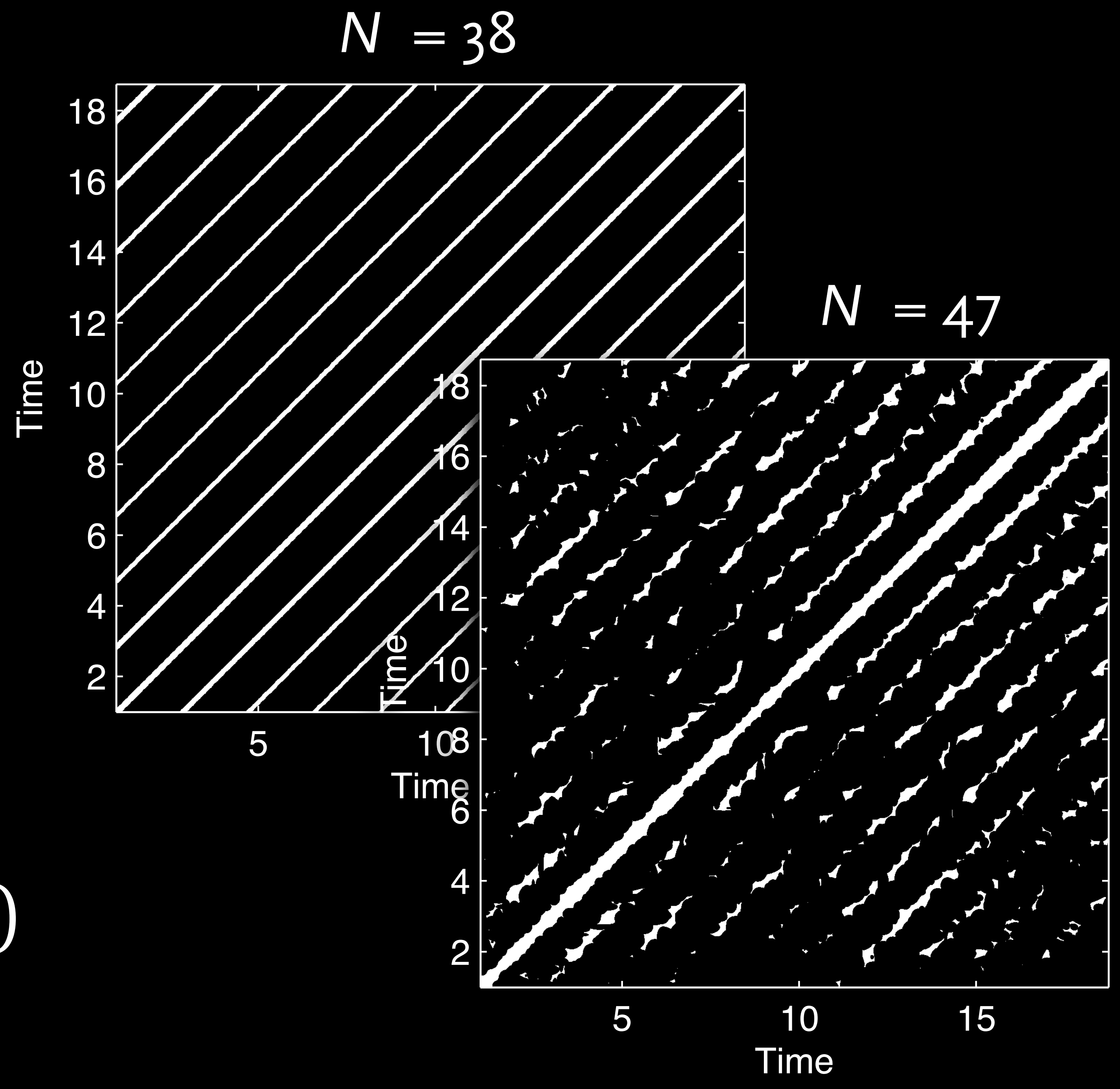
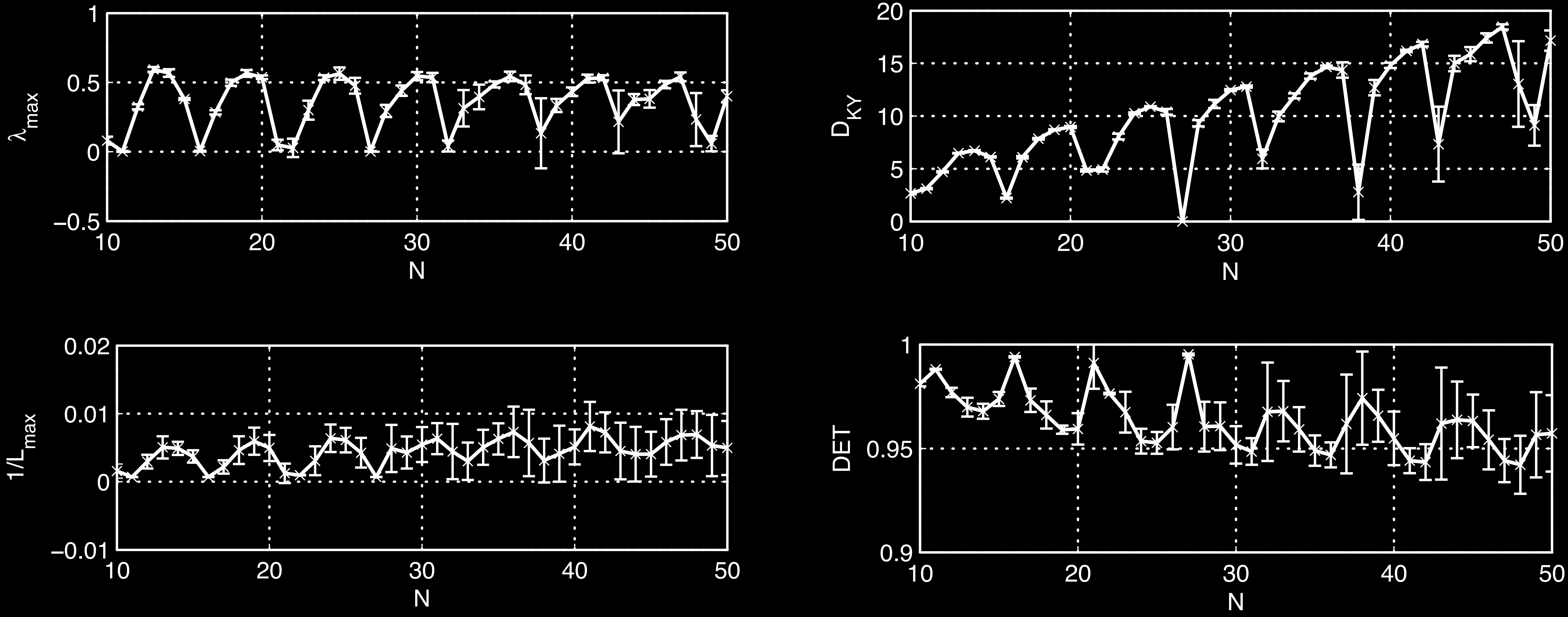


Marwan et al, Phys Lett A 379, 2015

Lorenz96 – Recurrence Analysis

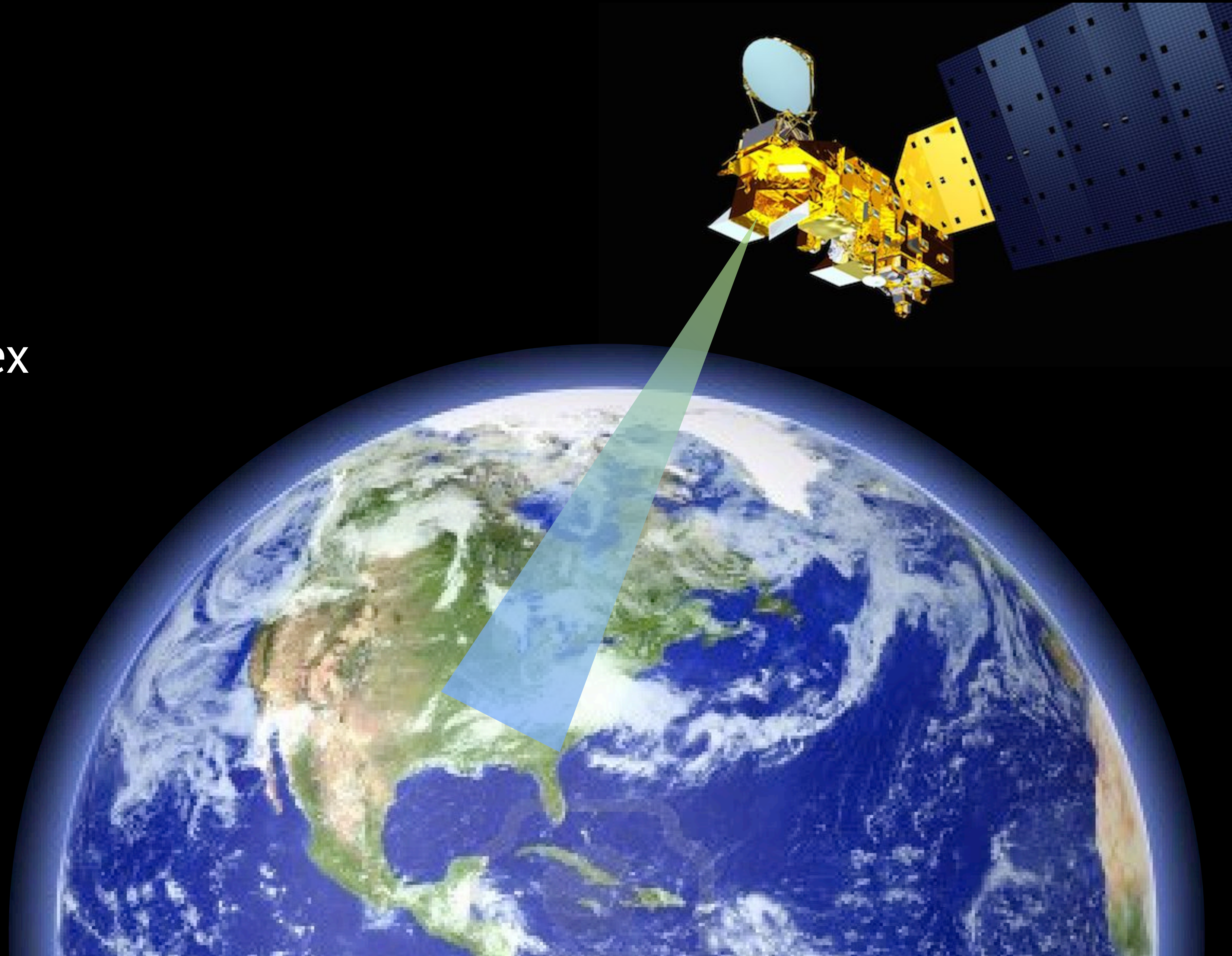


time series length: only 1.500 data points

Marwan et al, Phys Lett A 379, 2015

Modis Satellite Time Series Data

- MODIS-Terra MOD13Q1
- enhanced vegetation index (EVI)
- Feb 2000 – Nov 2013
- 16-day composite image (316 images)
- 250 m spatial resolution



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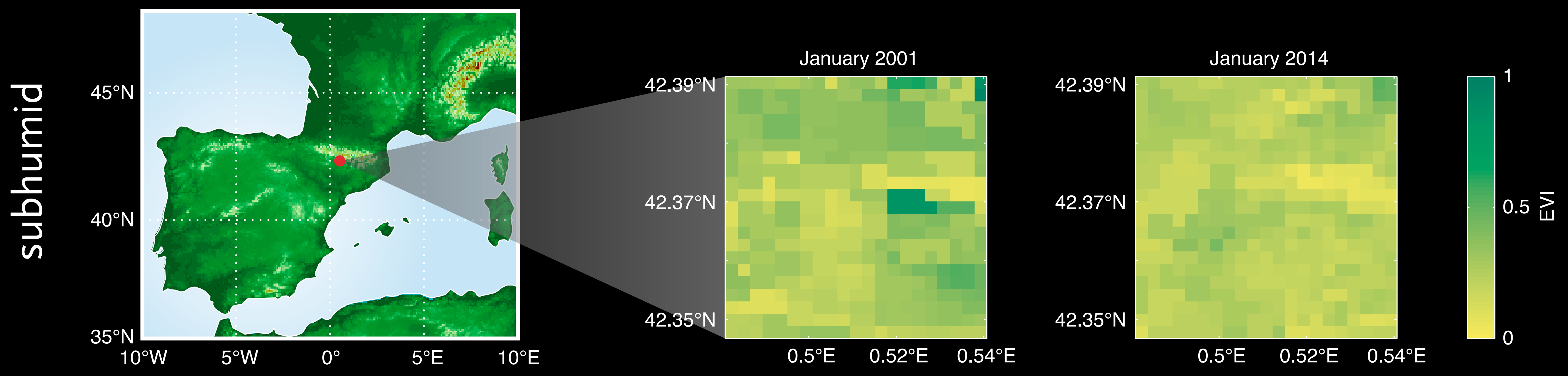
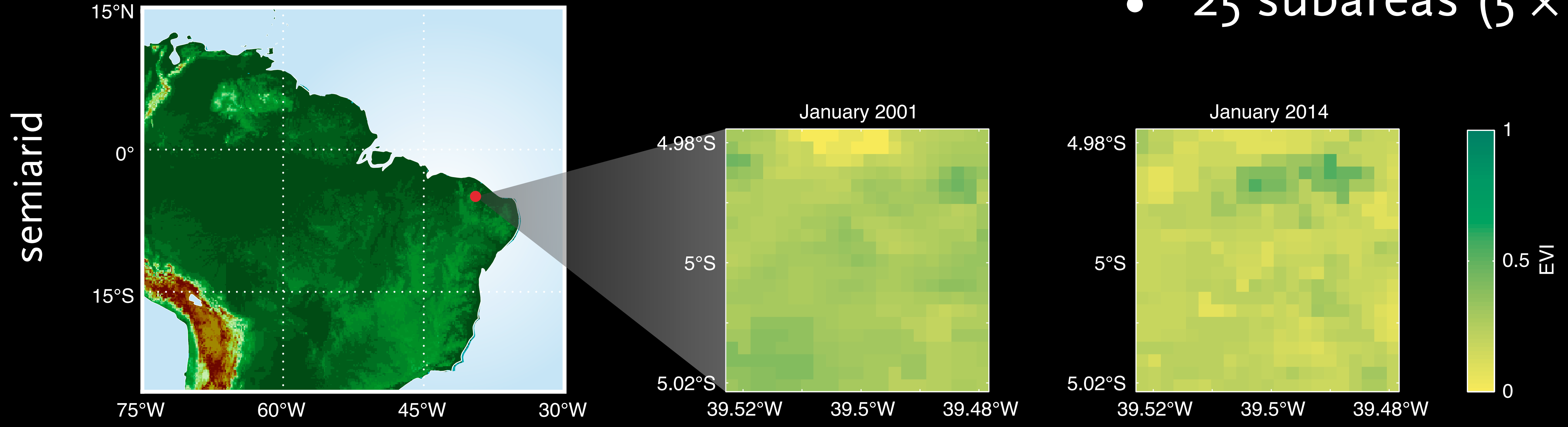


- 25 subareas (5 × 5 km²)



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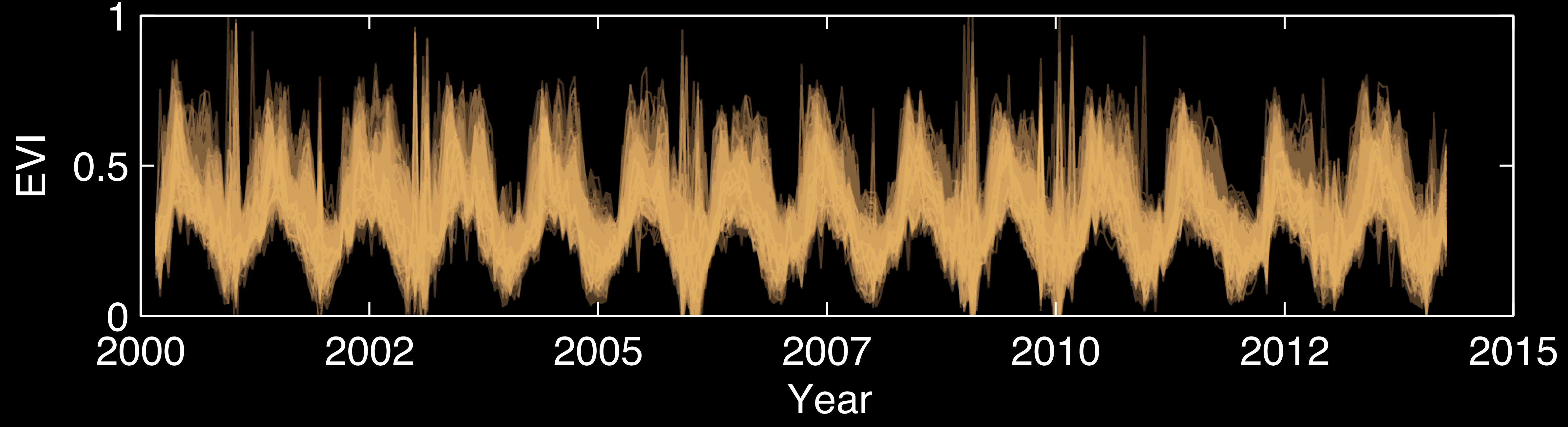

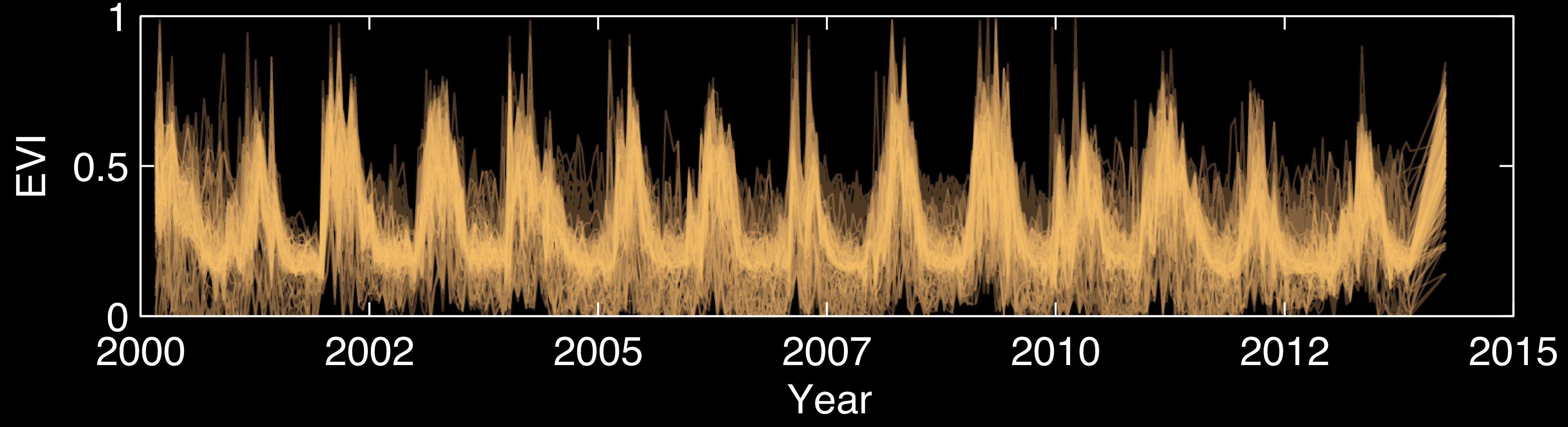

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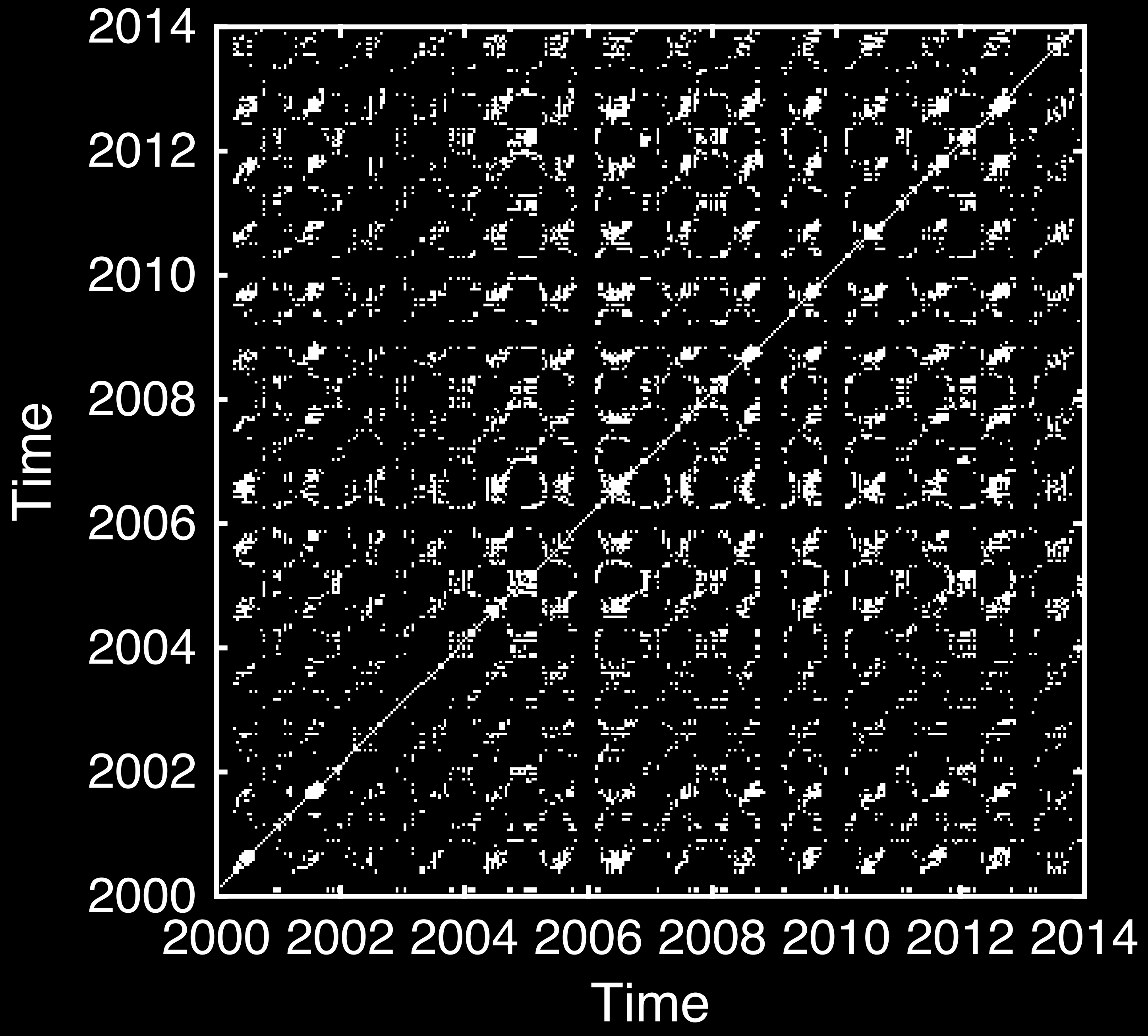

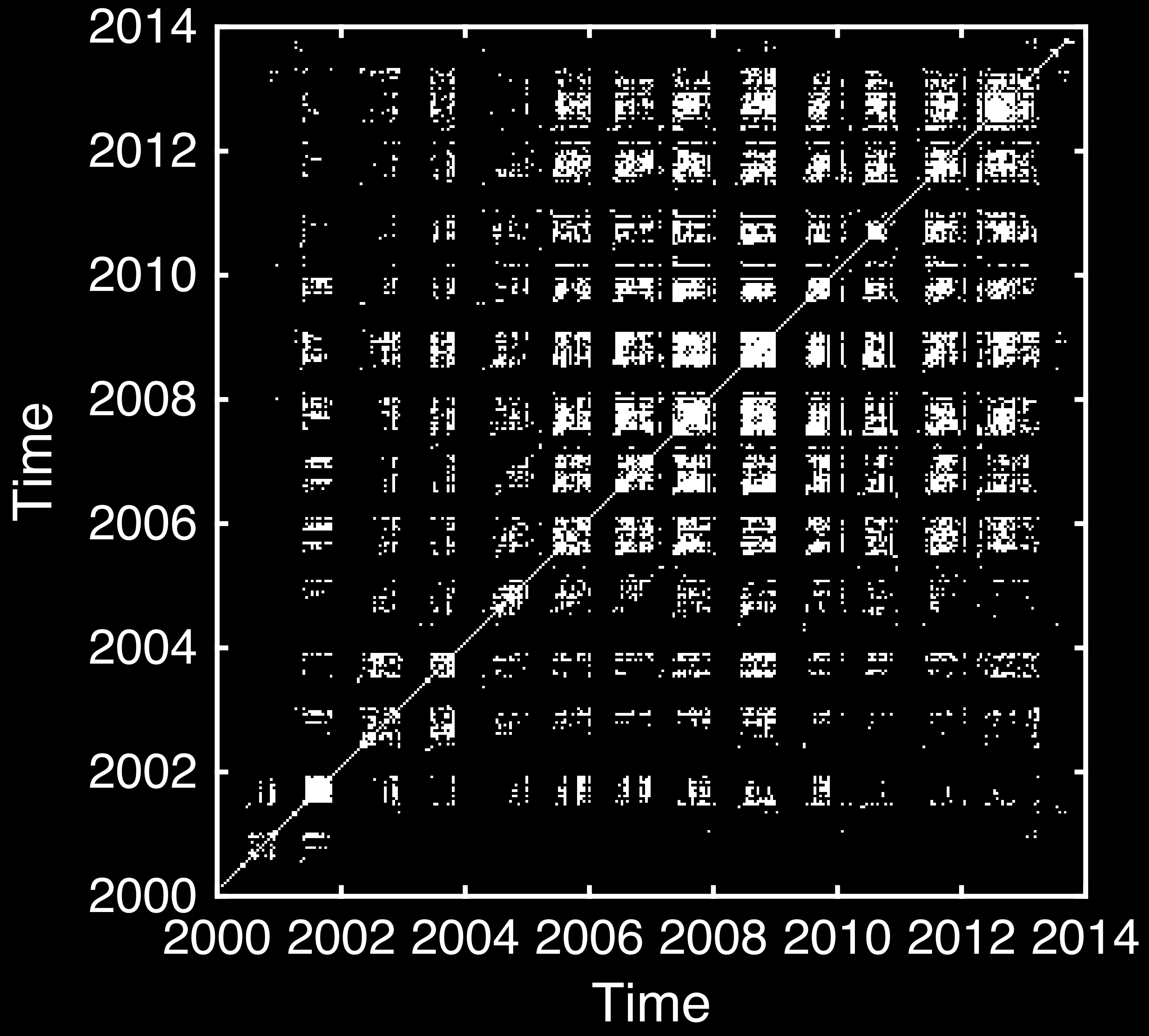

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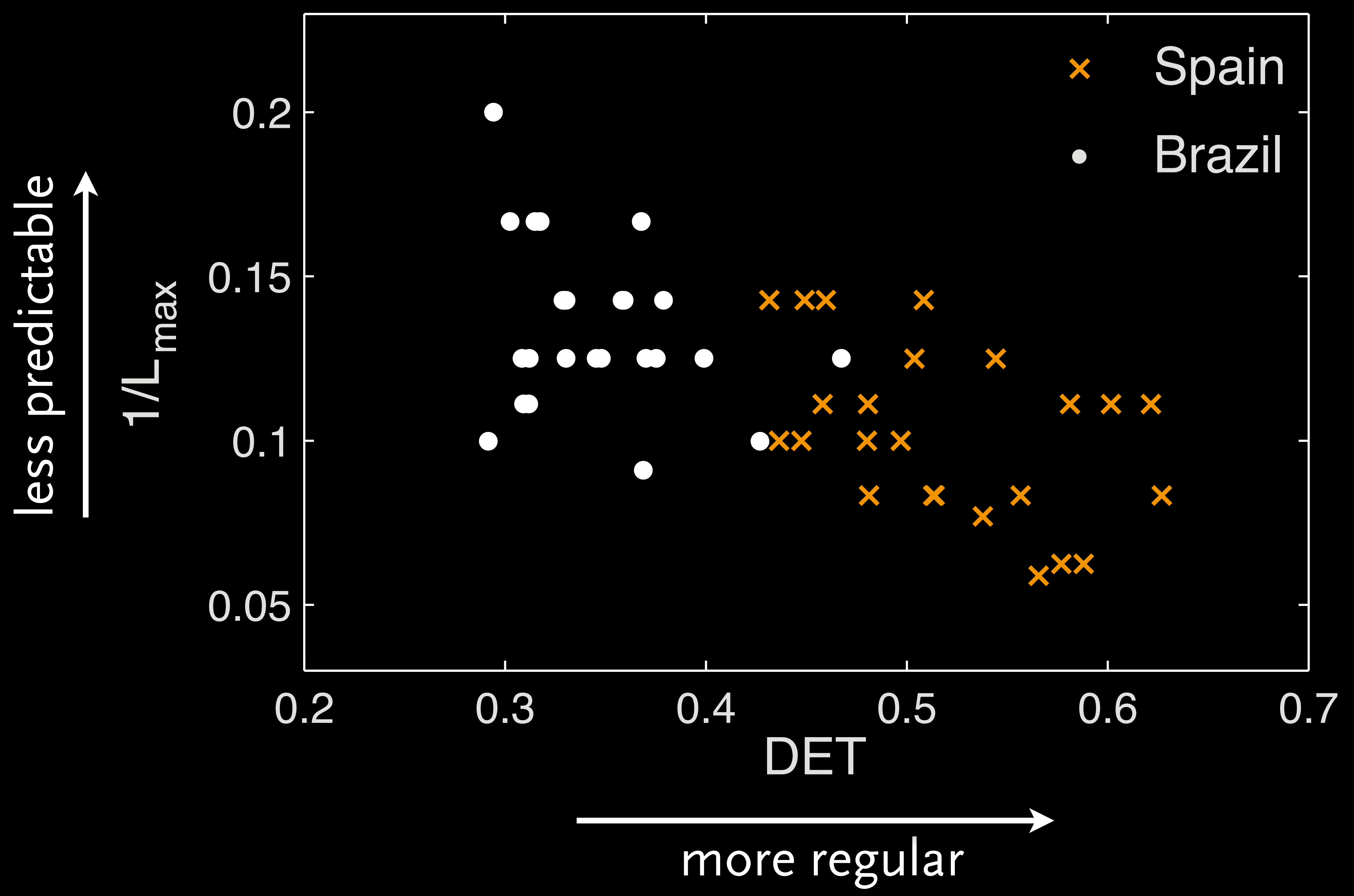

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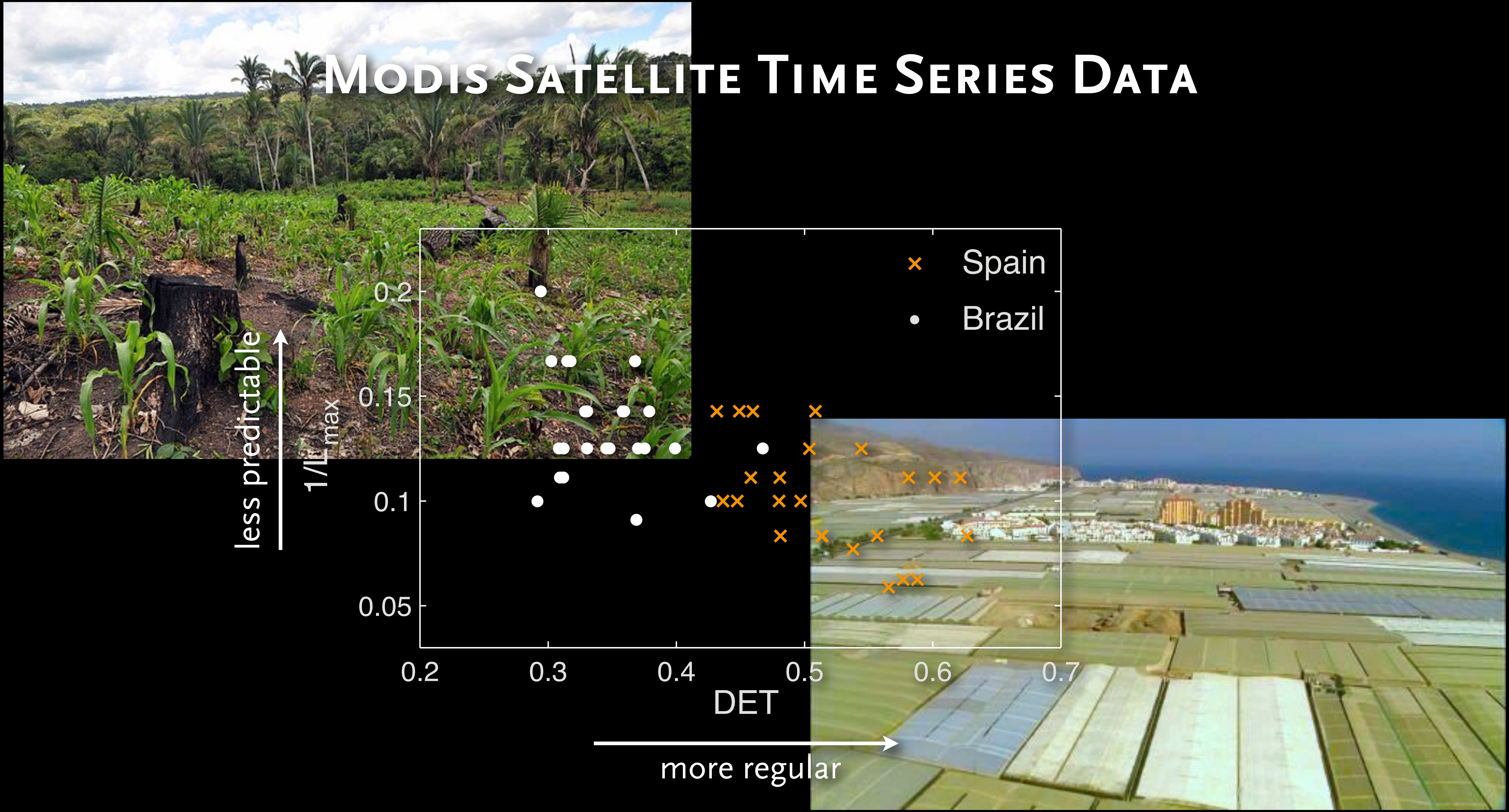

Conclusions

- Recurrence quantification analysis works also for high-dimensional systems
- Identification of different spatio-temporal dynamics
- Potential applications: investigation of multivariate data or spatial dynamics (e.g., landcover change, algae blooms, brain activity, ...)

Issues

- Too sensitive with respect to spatial variations
- Superpositioned dynamics
- Different scales (tunable)