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Climate Impact Research

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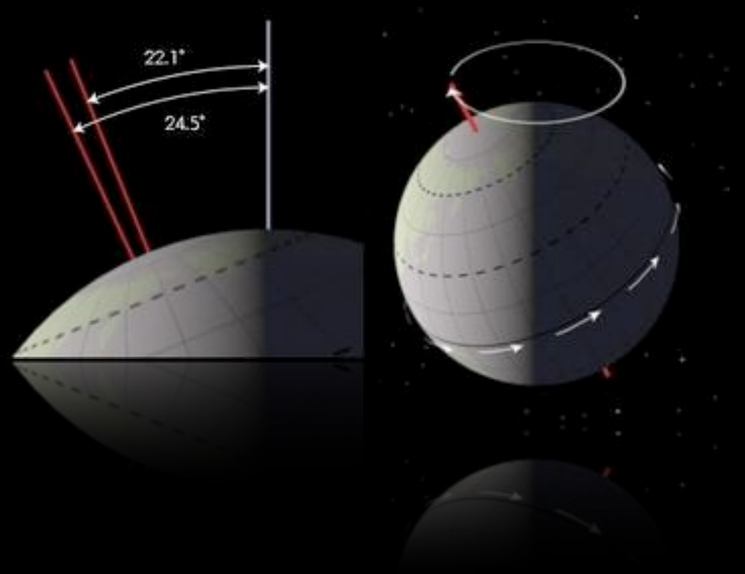
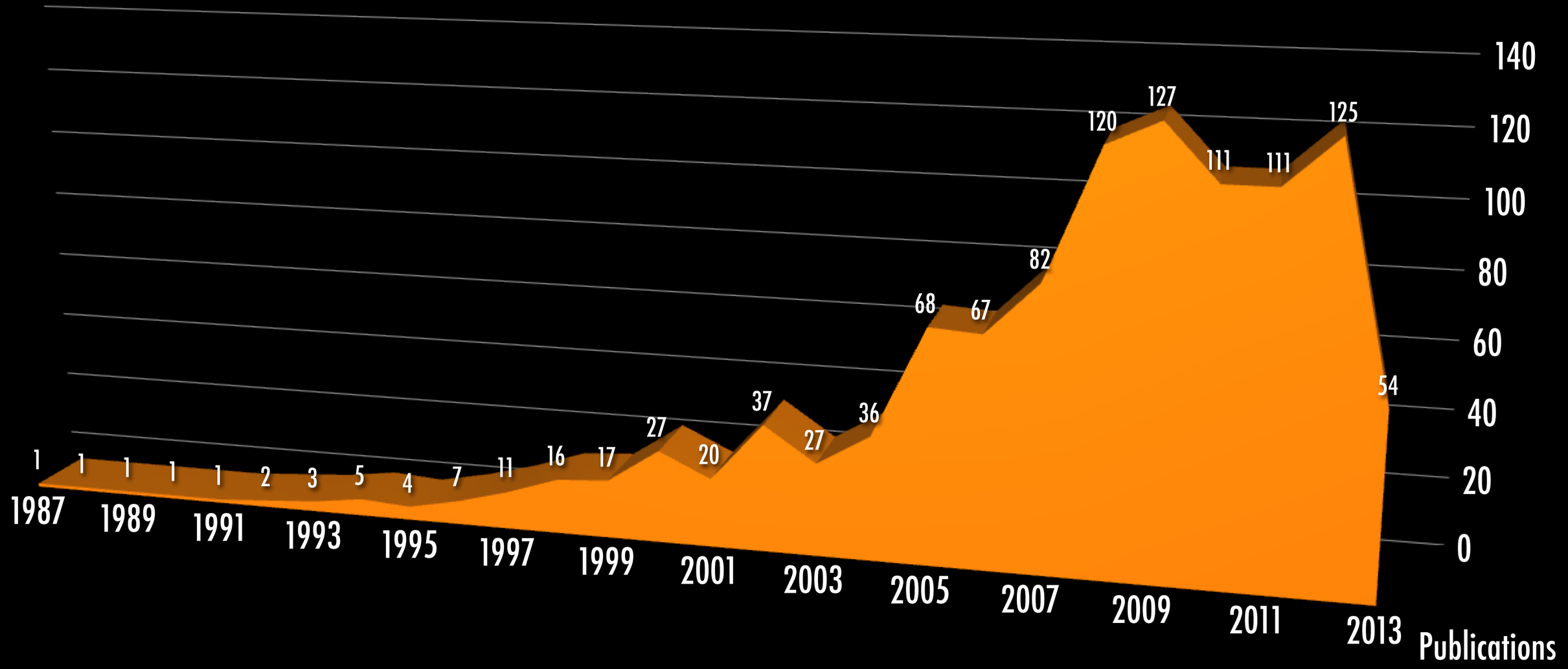
Potential Pitfalls in Recurrence Plot Analysis

Recurrence

- fundamental characteristic of many dynamical systems
- recurrences in real life:
  Milankovich cycles, El Niño phenomenon, extreme floods, heart beat after exertion, predator prey cycles, metal cutting processes, etc.



Recurrence Plot Publications

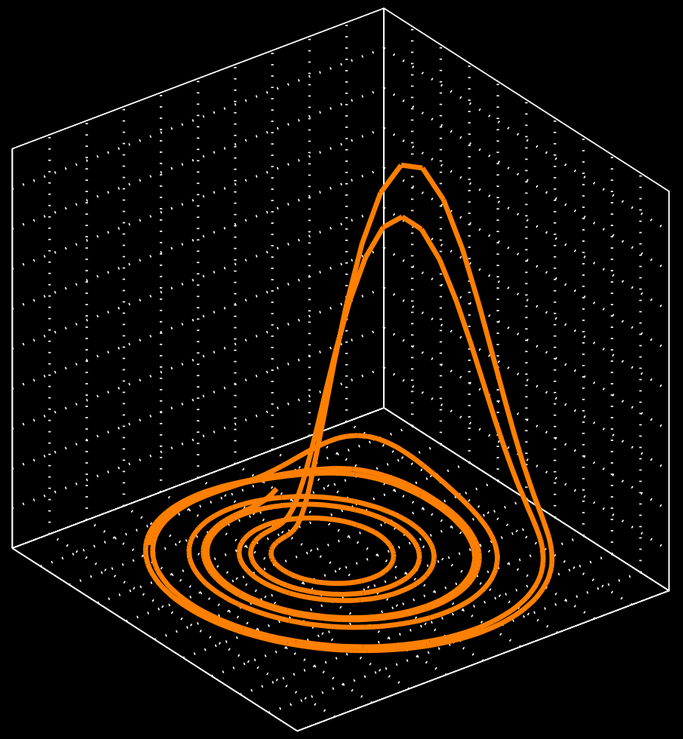


Pitfalls

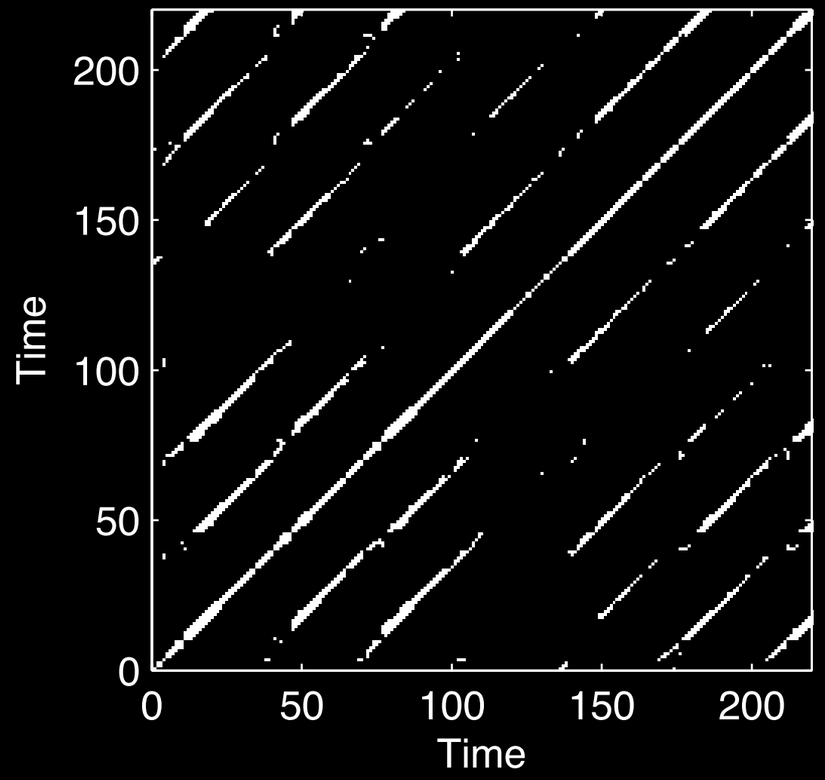
- Choice of parameters (embedding, recurrence plot calculation, RQA parameters)
- Interpretation:
  - visual patterns
  - indicators of determinism, chaos, periodicities, nonstationarity
  - significance
- Further pitfalls (e.g., dynamical invariants, coupling detection, twin surrogates)

Embedding Parameters

Roessler system



x-variable, m = 3, τ = 6

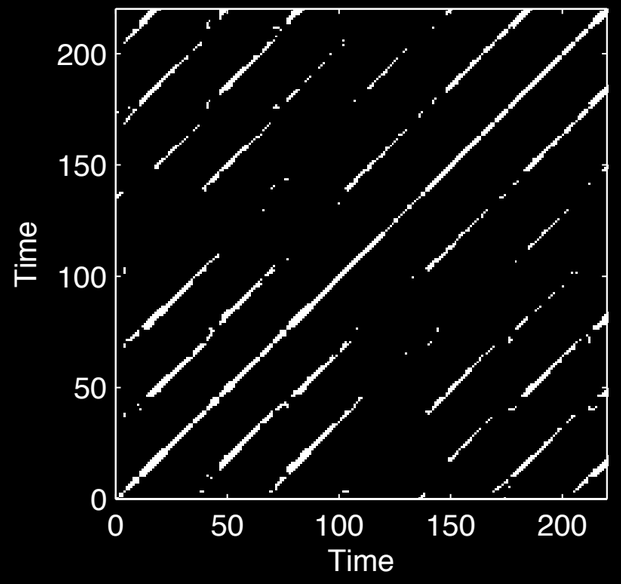


- diagonal lines parallel to LOI
- no perpendicular lines
- continuous lines

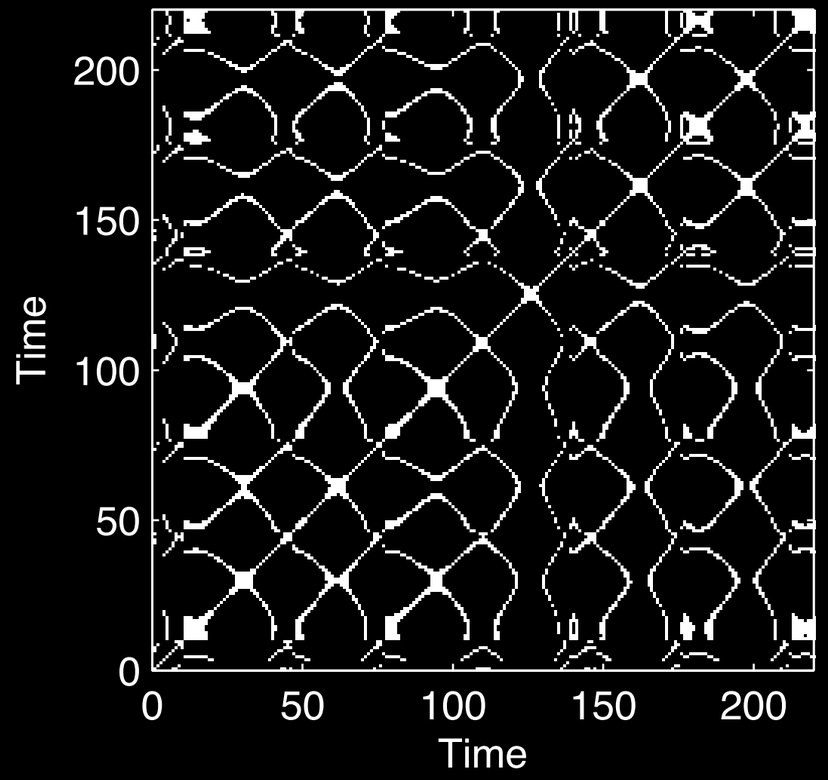
Marwan, Int J Bif Chaos 21, 2011

Embedding Parameters

Optimal RP



x-variable, m = 1, τ = 1



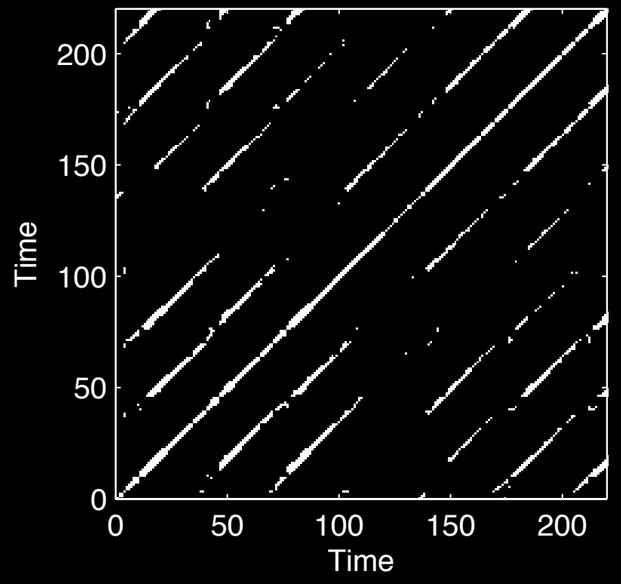
Insufficient embedding dimension:

- perpendicular lines

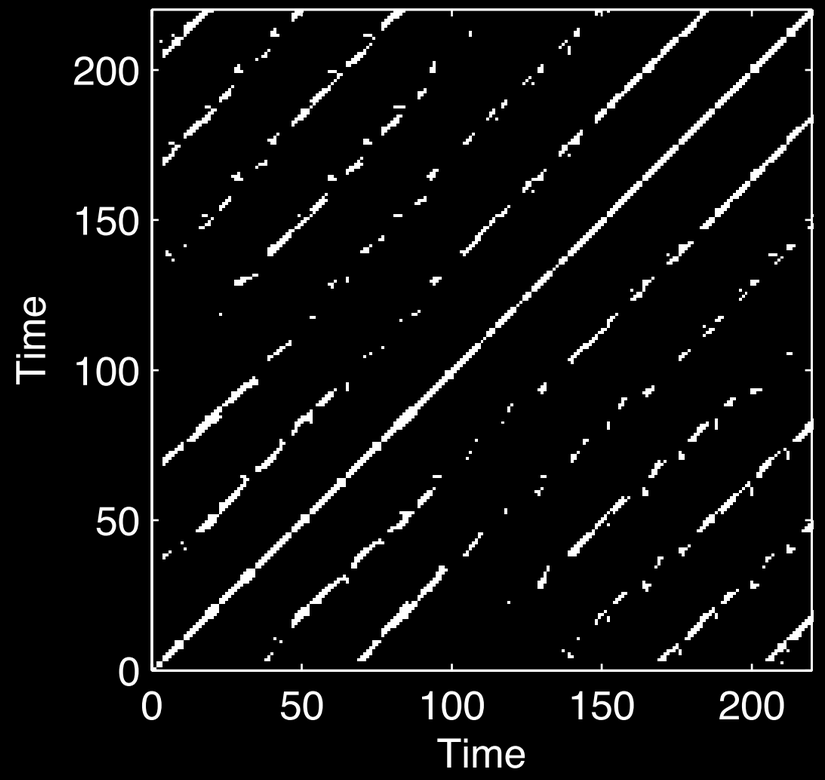
Marwan, Int J Bif Chaos 21, 2011

Embedding Parameters

Optimal RP



x-variable, m = 3, τ = 12



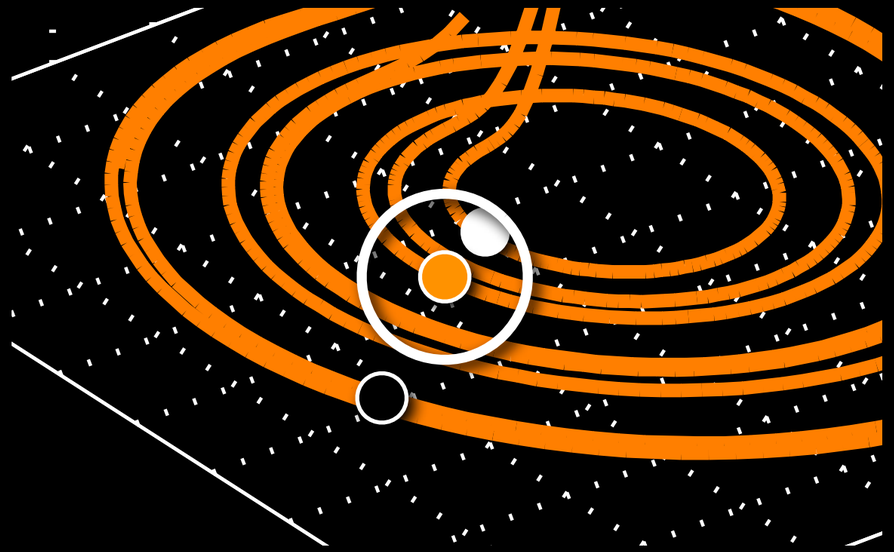
Insufficient time delay:

- interrupted lines
- wobbly lines

➡ check appearance of RP

Marwan, Int J Bif Chaos 21, 2011

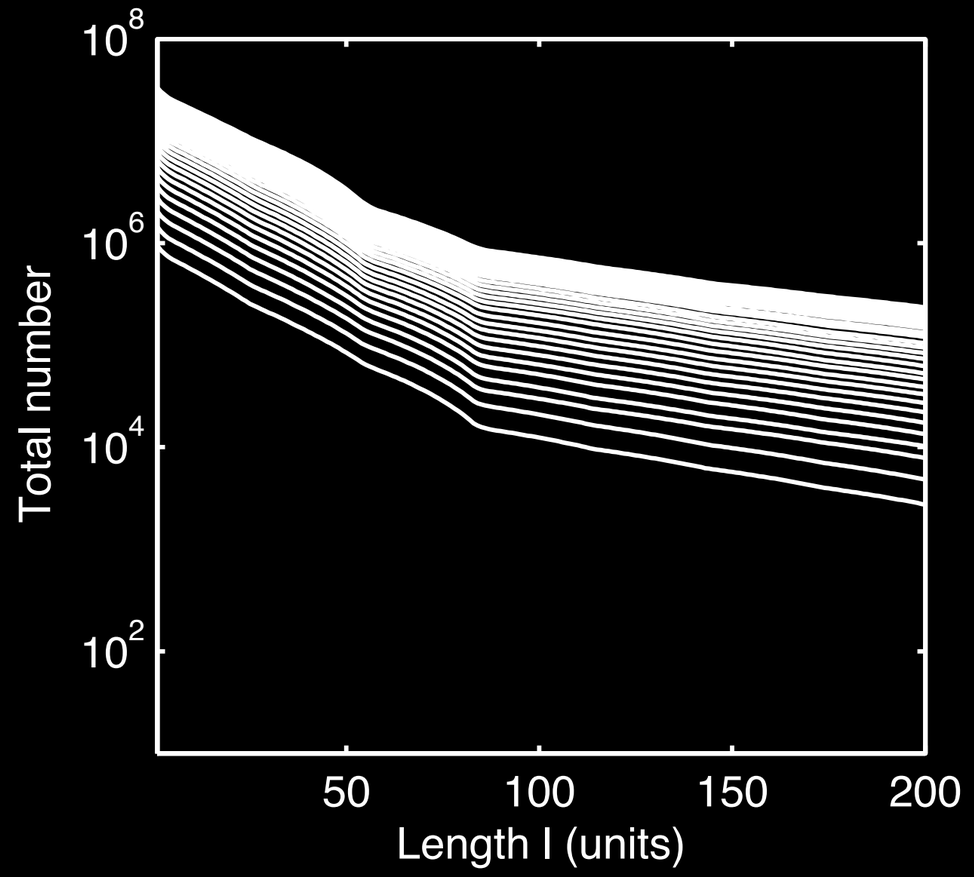
Recurrence Threshold



- no general rule

  ➡ choice depends on application

Recurrence Threshold

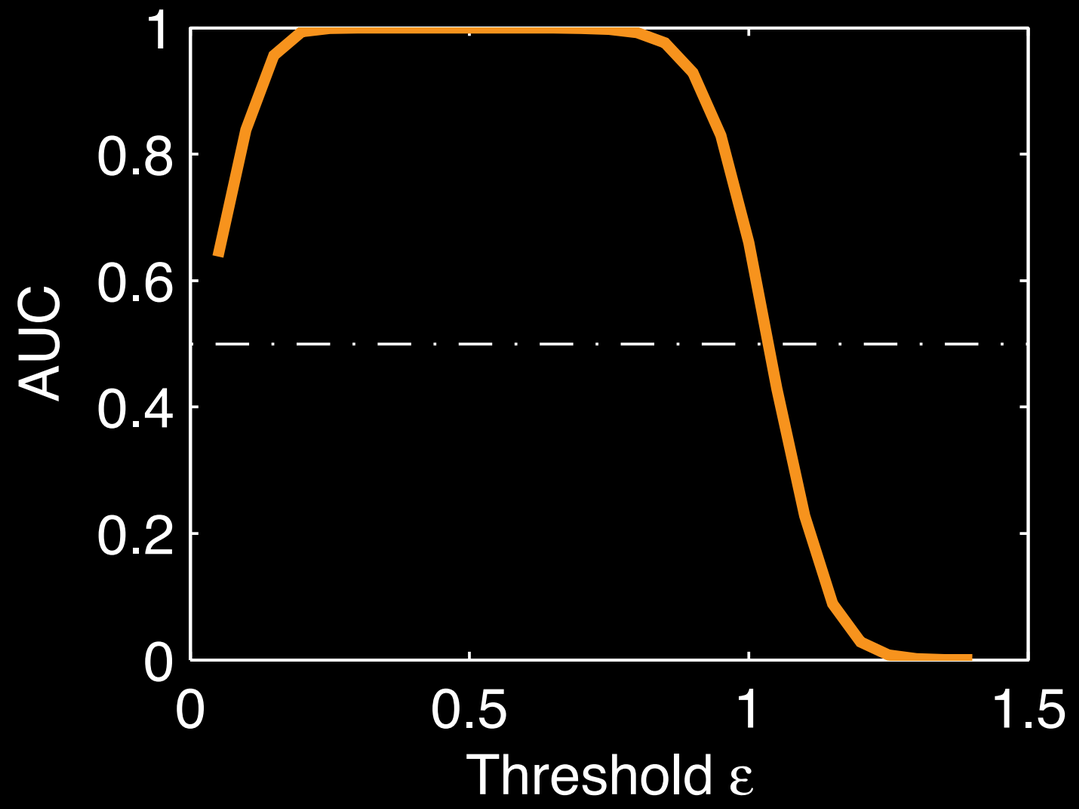


- dynamical invariants

  ➡ threshold as small as possible

Thiel et al, Chaos 14, 2004

Recurrence Threshold

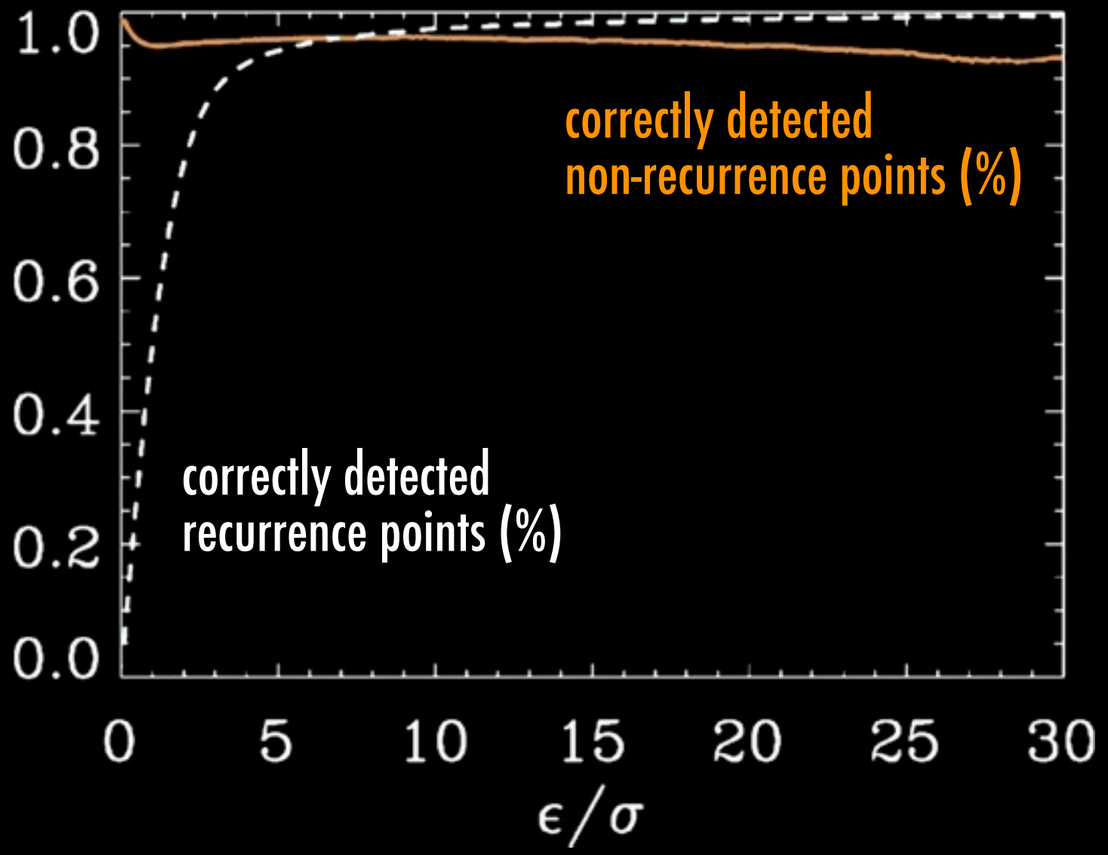


- signal detection from noise
- receiver operator characteristics (ROC) and area under curve (AUC)

➡ range of optimal thresholds $\varepsilon = [0.2 \ldots 0.8]\sigma$

Schinkel et al., European Physical Journal ST 164, 2008

Recurrence Threshold



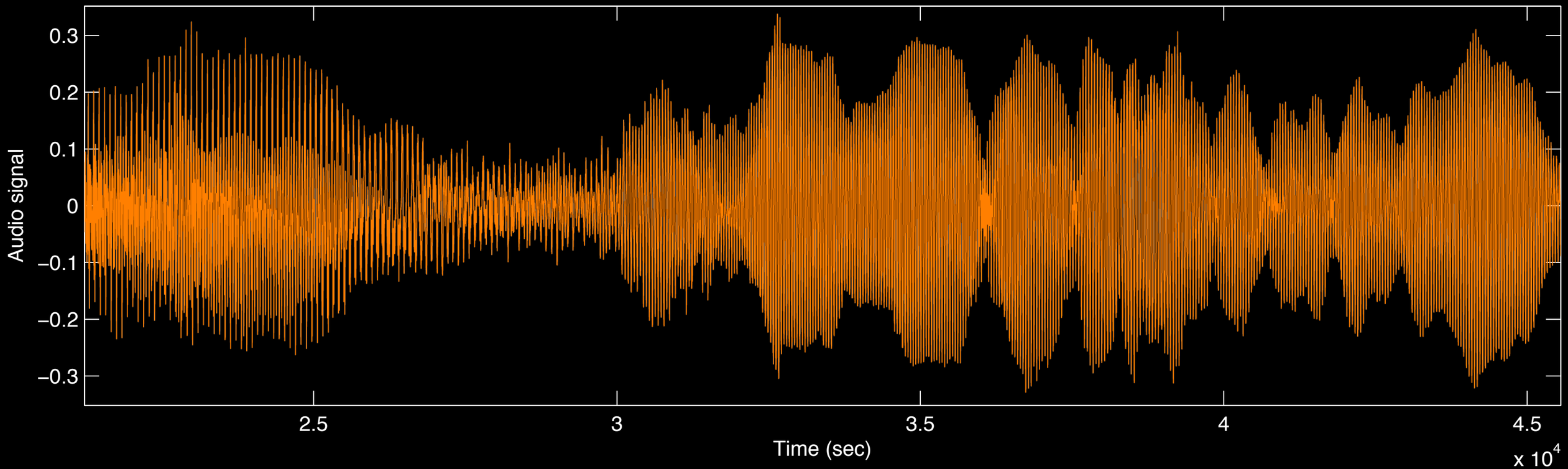
- observational noise

➡ threshold $\varepsilon > 5\sigma$

Thiel et al., Physica D 171, 2002

Macro Structures and Sampling

- interference effect of sampling frequency and signal frequency



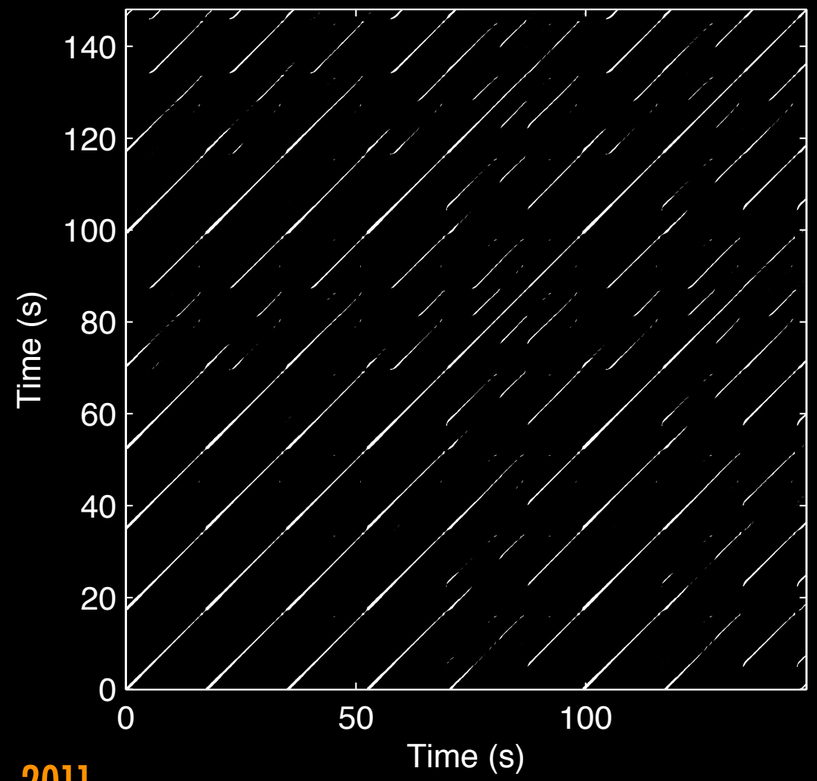
➡ Nyquist-Shannon sampling theorem not enough!

Macro Structures and Sampling

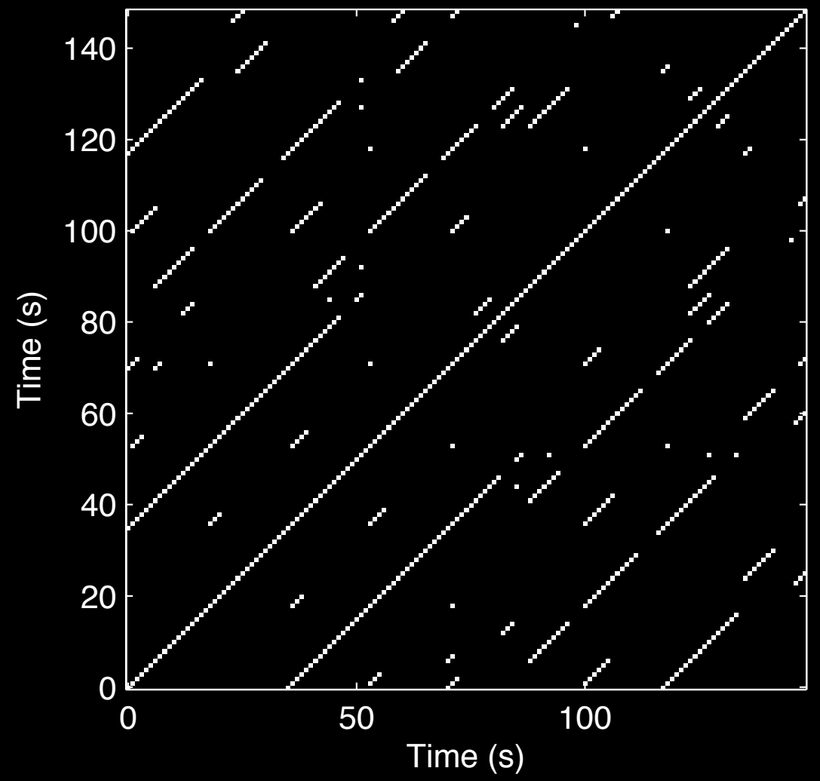
- RPs of Roessler with different sampling

➡ many diagonal lines vanish

$\Delta t = 0.05s$



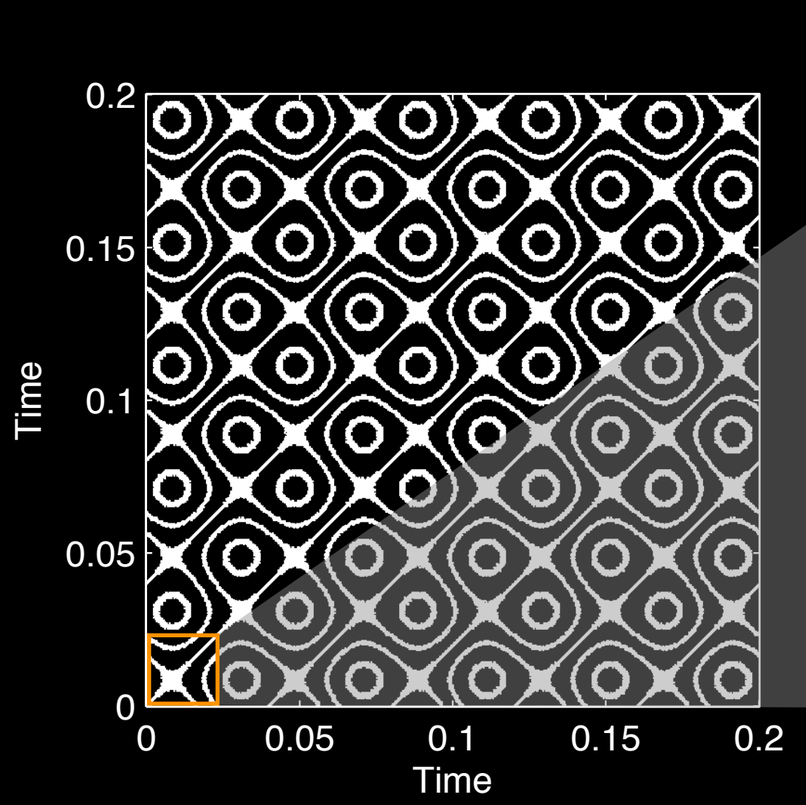
$\Delta t = 1.0s$



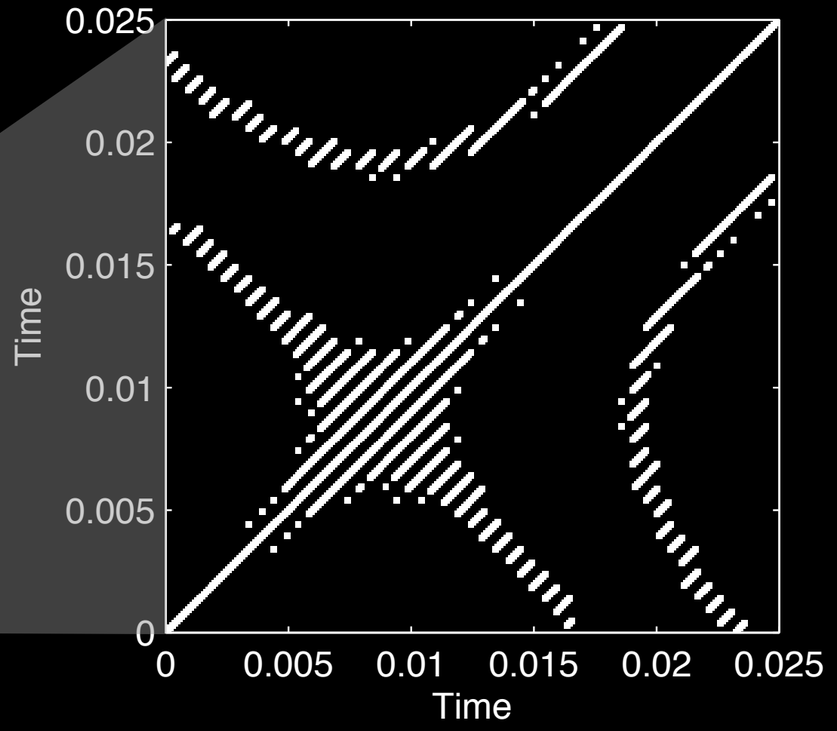
Marwan, Int J Bif Chaos 21, 2011

Macro Structures and Sampling

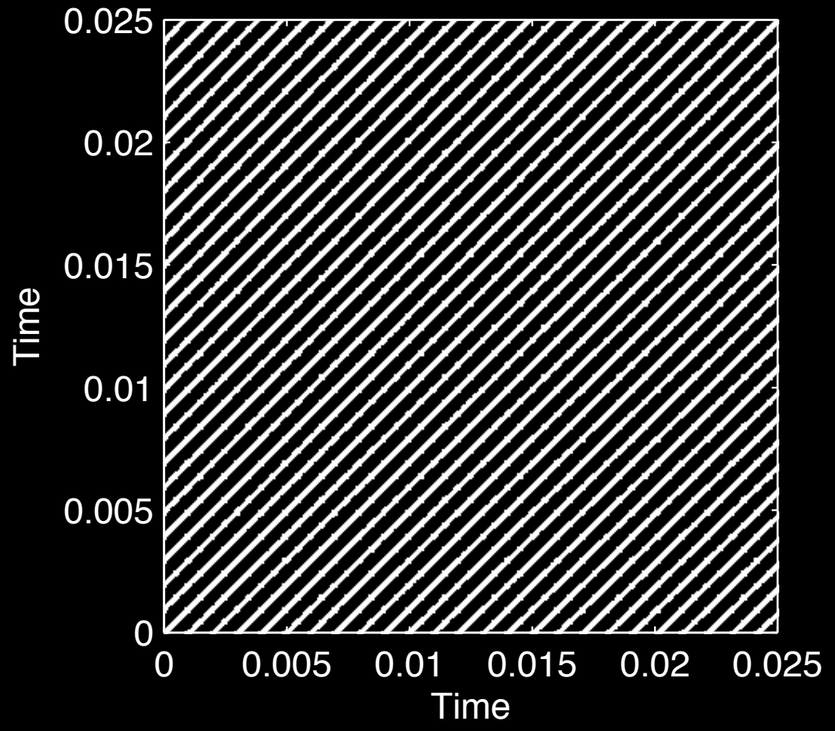
- interference effect of sampling frequency and signal frequency



insufficient sampling



sufficient sampling



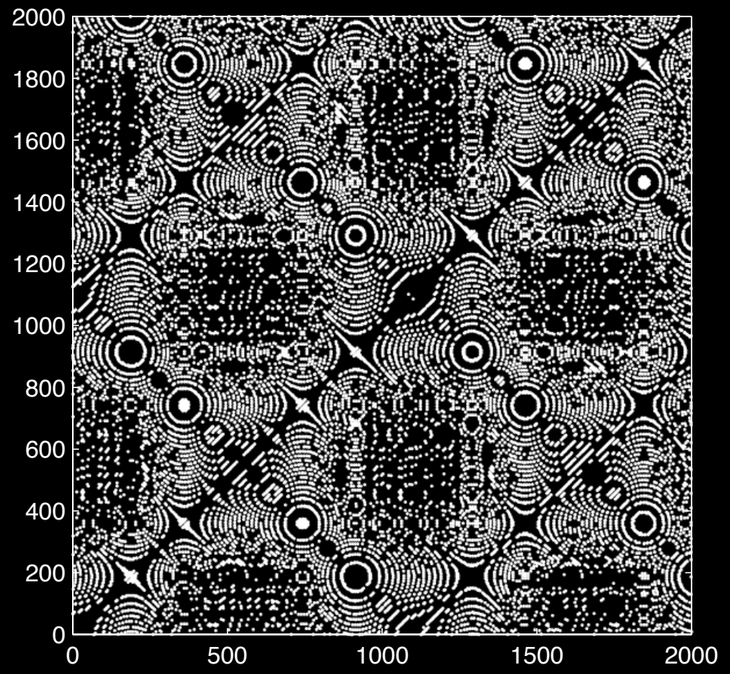
Facchini & Kantz, Phys Rev E 75, 2007
Marwan, Int J Bif Chaos 21, 2011

Macro Structures and Sampling

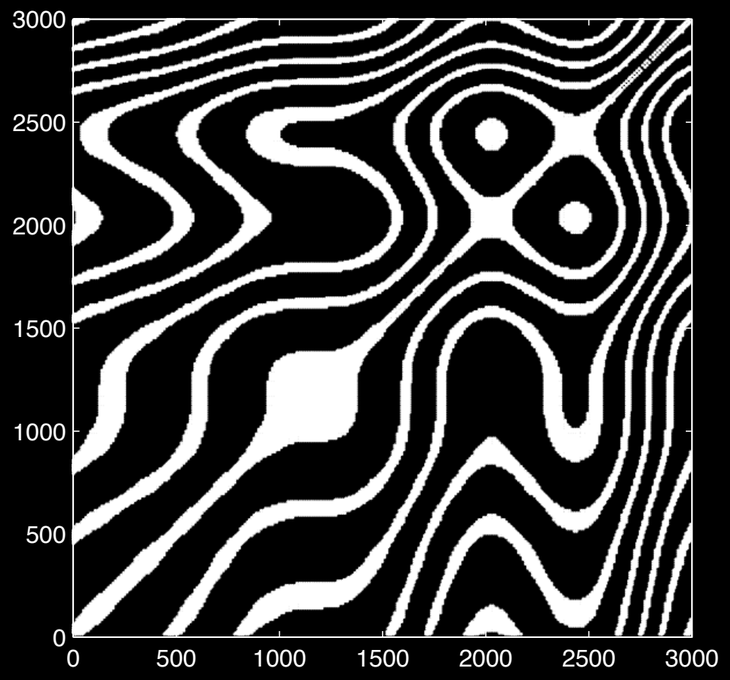
- very sensitive to slight frequency variations

➡ magnification lens to detect tiny frequency modulations

$$\sin(2\pi f_c t + 2\pi \sin(2\pi f_m t))$$



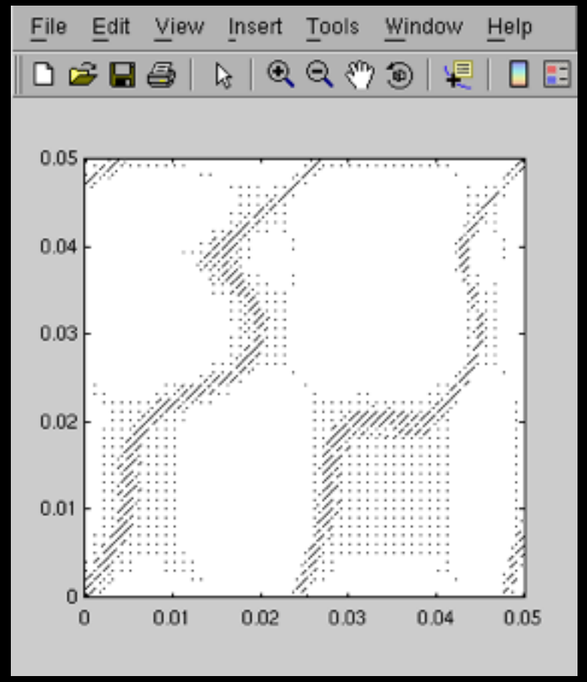
$$\sin(2\pi f_c t + 2\pi \sin(2\pi f_m t) t^{\frac{3}{2}})$$

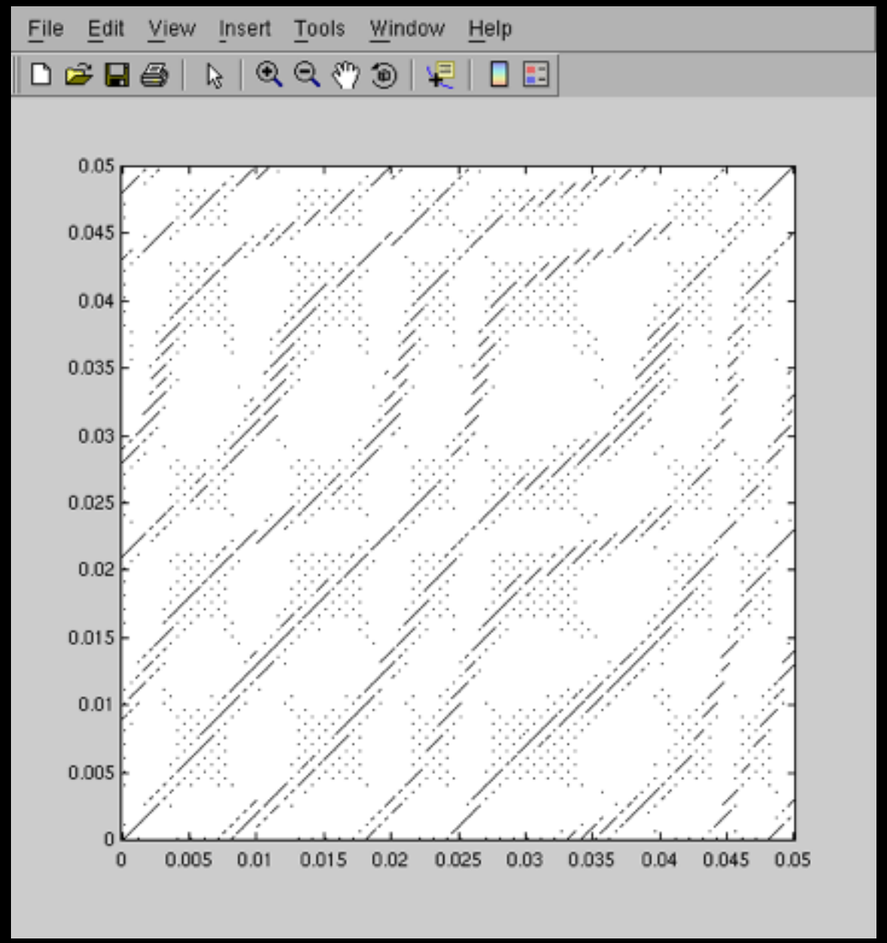


Facchini & Kantz, Phys Rev E 75, 2007

Macro Structures and Sampling

- large RPs (larger than screen resolution)

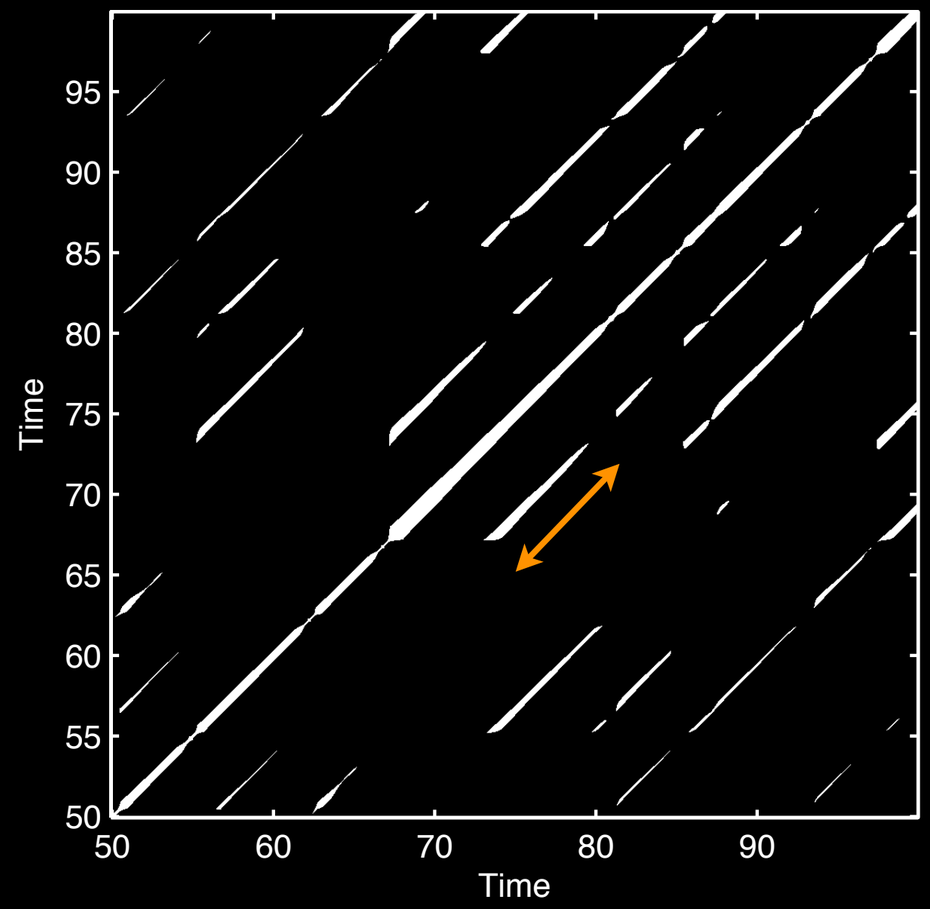


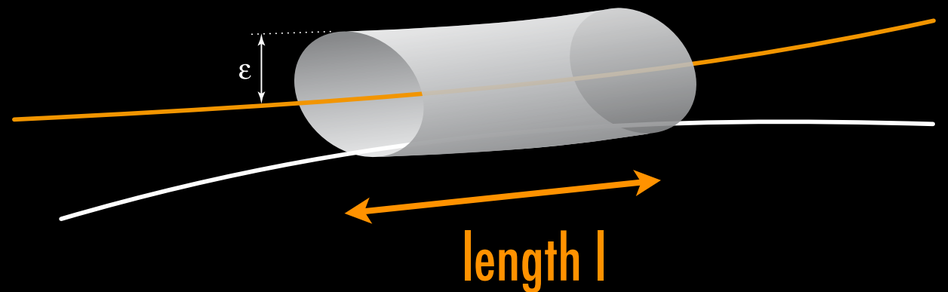


➡ be aware of optimal sampling and size of RP

Marwan, Int J Bif Chaos 21, 2011

Indicators of Determinism and Chaos



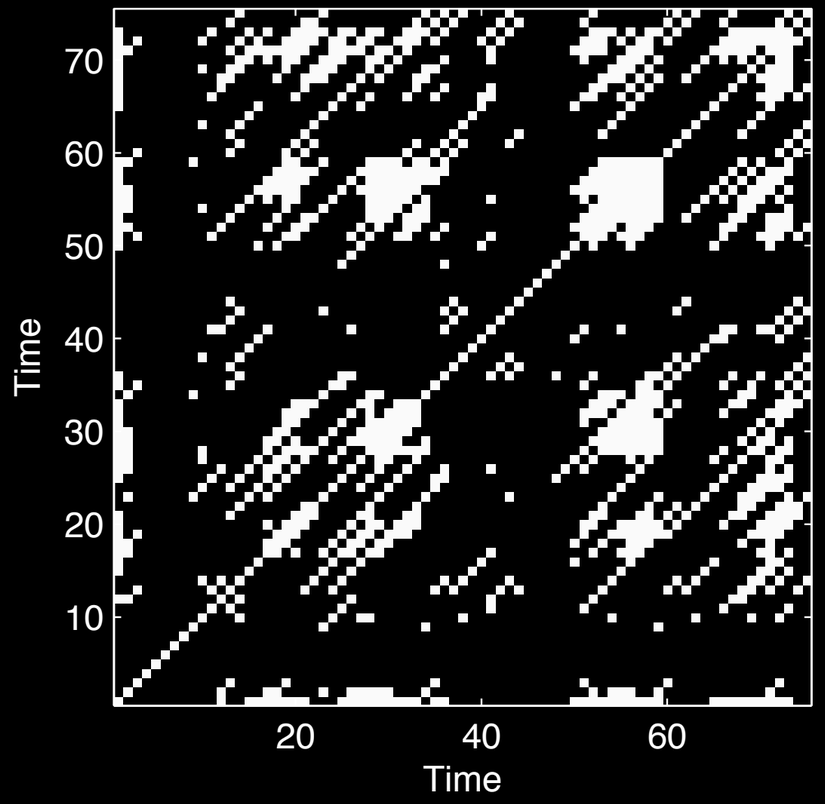


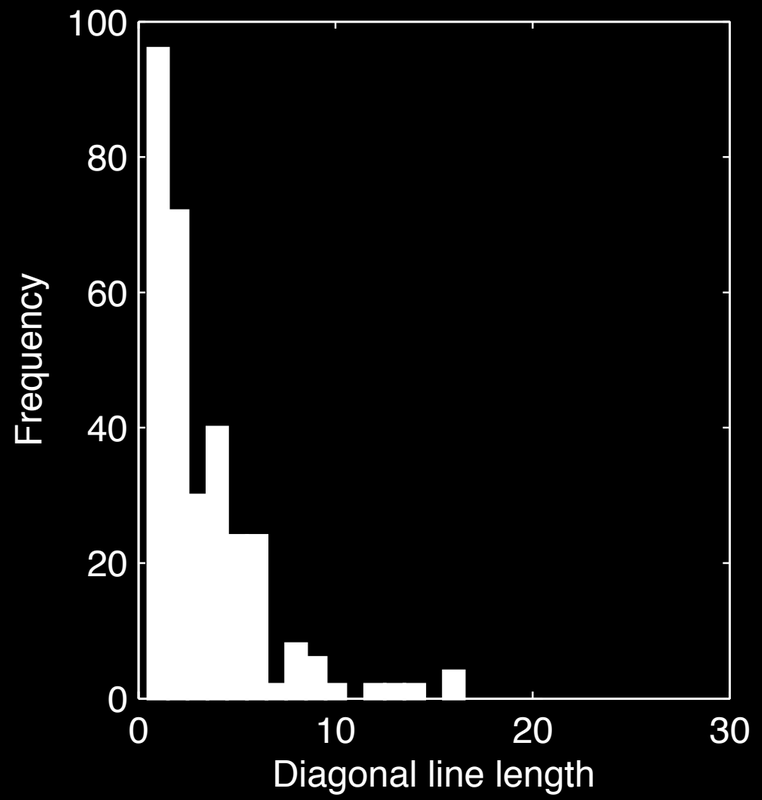
- „close-by" states, divergence behaviour
- heuristic measure for determinism:

$$DET = \frac{\sum_{l=l_{\min}}^{N} l\,P(l)}{\sum_{l=1}^{N} l\,P(l)}$$

➡ not determinism in mathematical sense!

Indicators of Determinism and Chaos

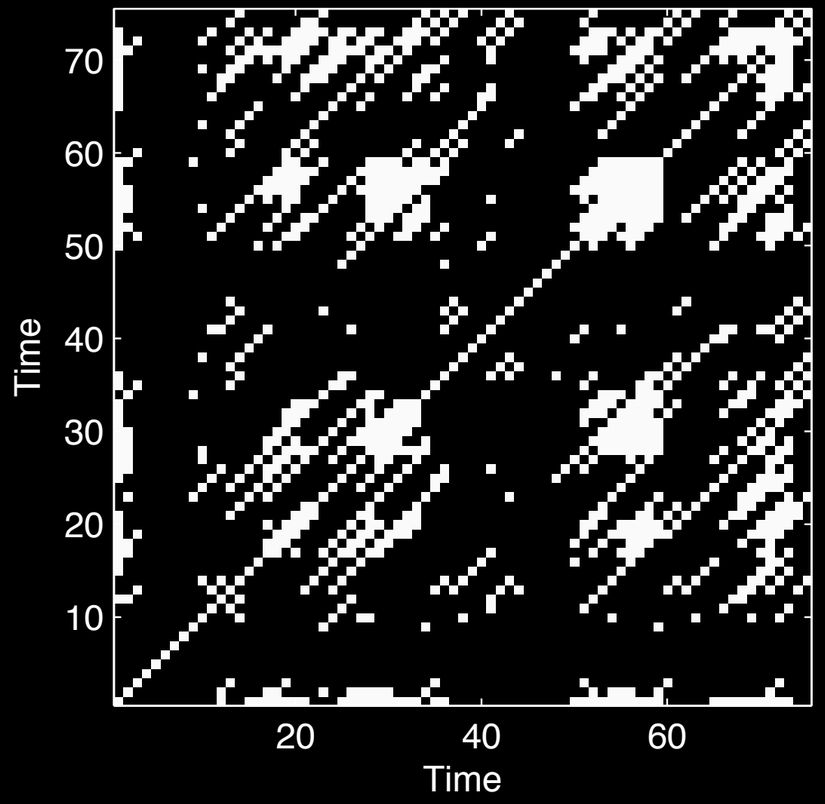


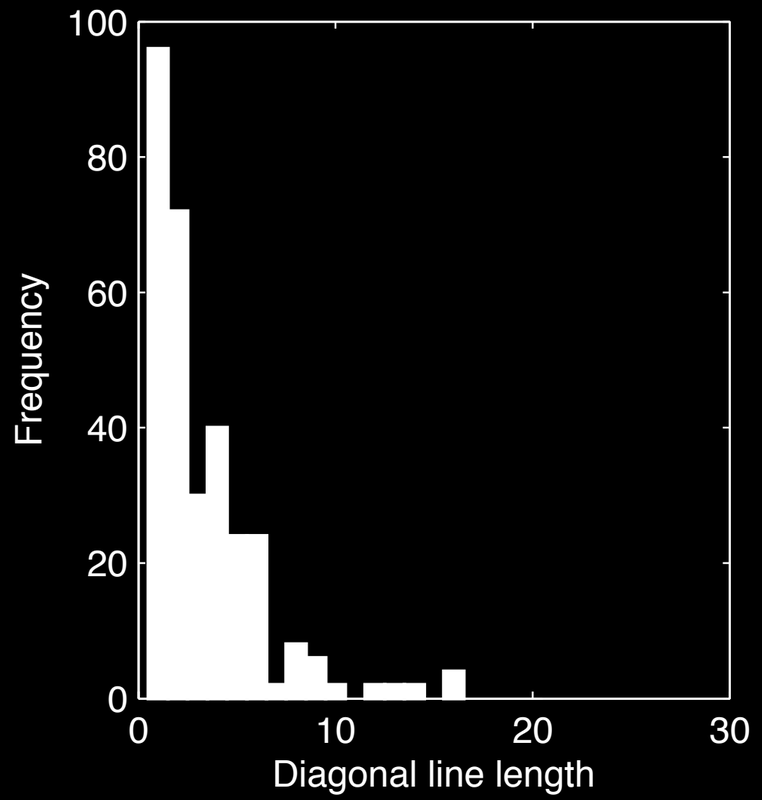


Marwan, Int J Bif Chaos 21, 2011

Indicators of Determinism and Chaos

Gaussian white noise, m = 6, τ = 1





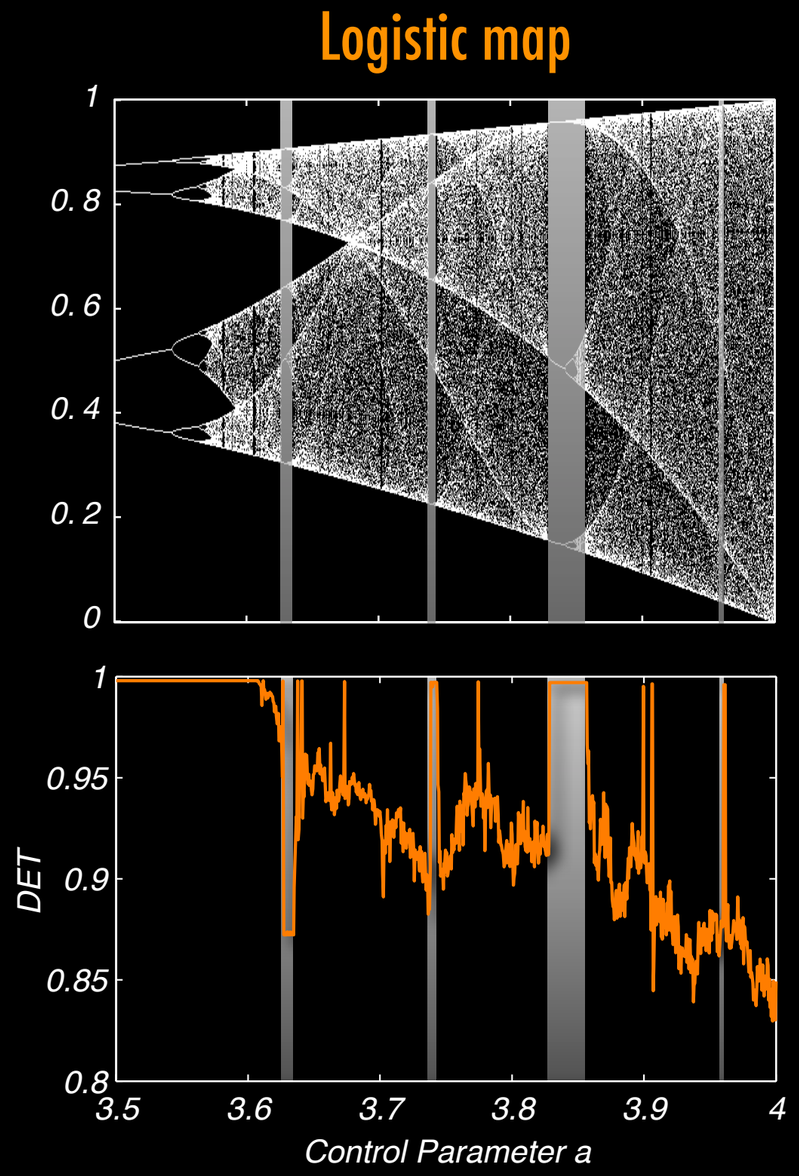
- embedding: spurious correlations
- many long diagonal lines
- L_{max} = 16

➡ alone: not a test for chaos! (apply surrogate test)

➡ use low embedding dimension!

Marwan, Int J Bif Chaos 21, 2011

Indicators of Periodicities

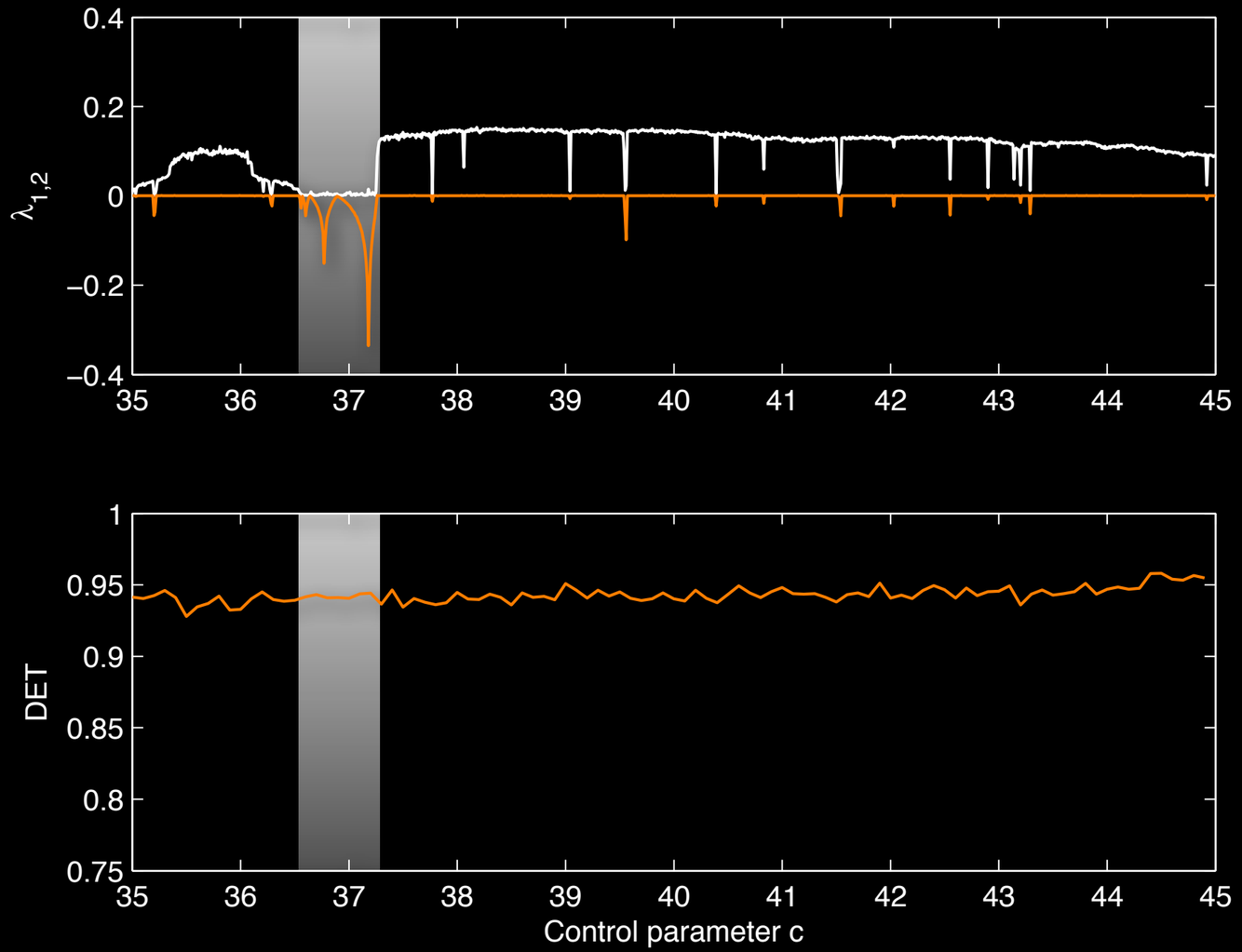


- high determinism: periodic windows

Trulla et al, Phys Lett A 223, 1996
Marwan et al, Phys Rev E 66, 2002

Indicators of Periodicities

Roessler system



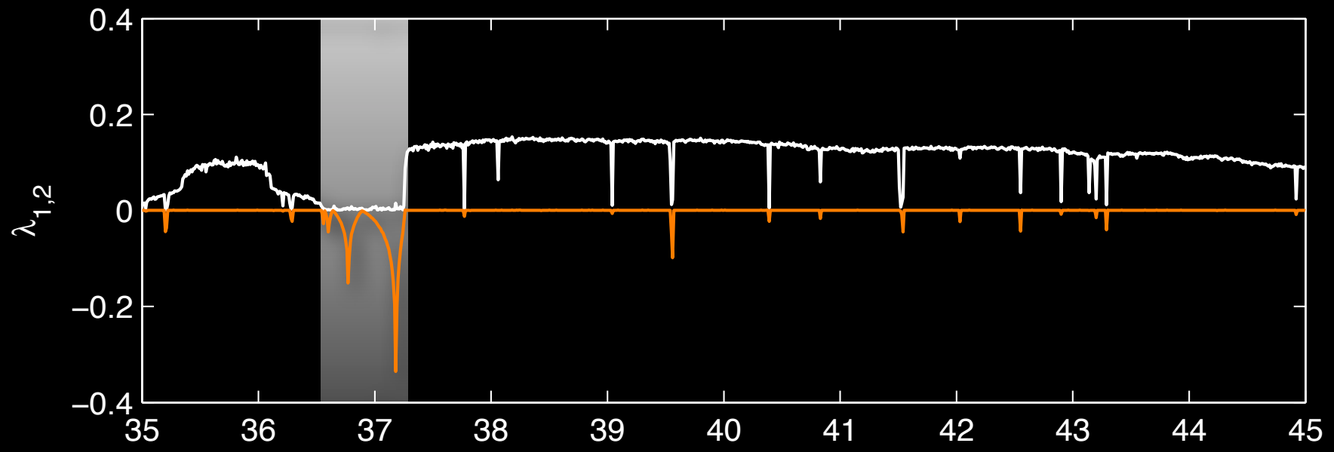
- periodic window not revealed
- continuous system

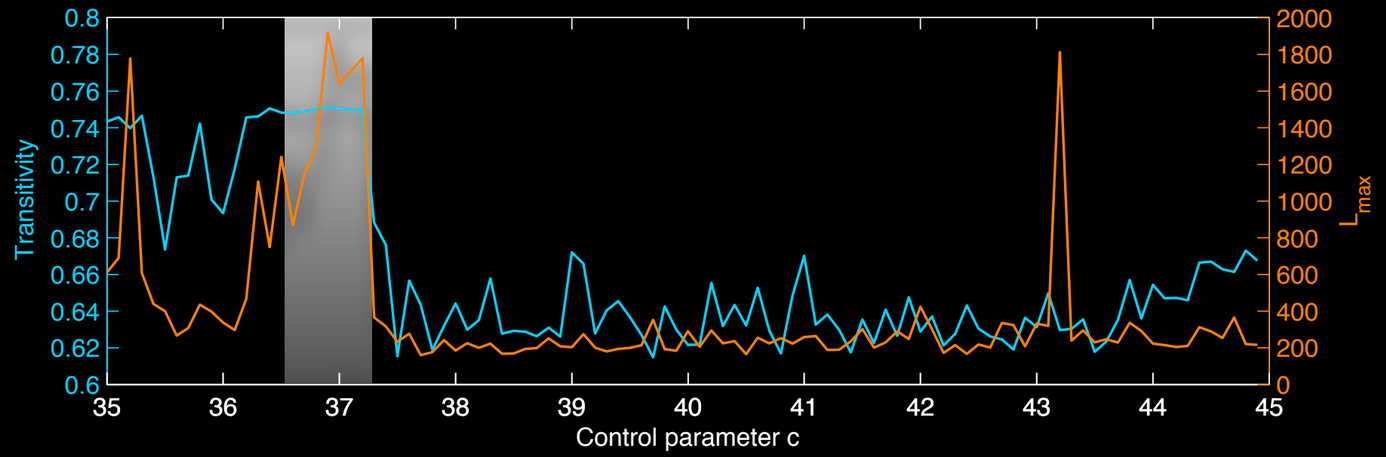
➡ better measures: K_2 entropy or transitivity coefficient

Marwan, Int J Bif Chaos 21, 2011
Zou et al, Chaos 20, 2010

Indicators of Periodicities

Roessler system



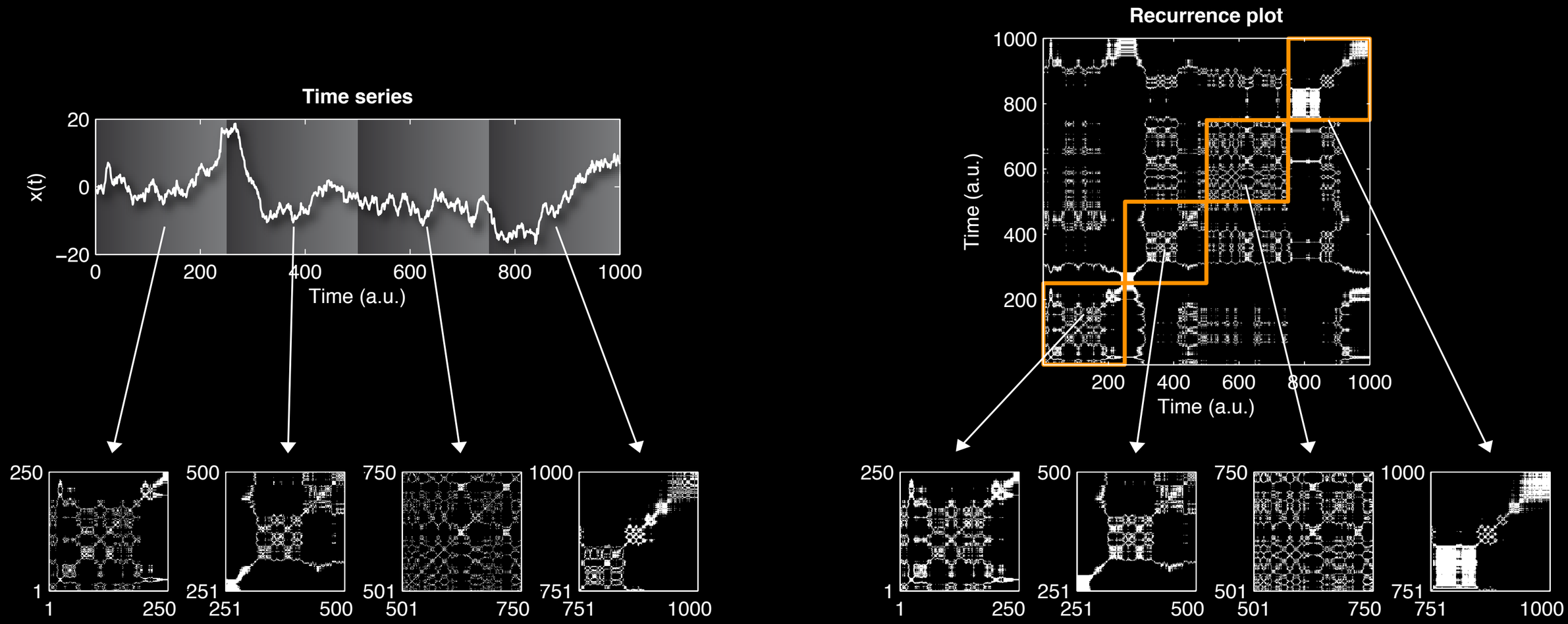


- periodic window not revealed
- continuous system

➡ better measures: K_2 entropy or transitivity coefficient

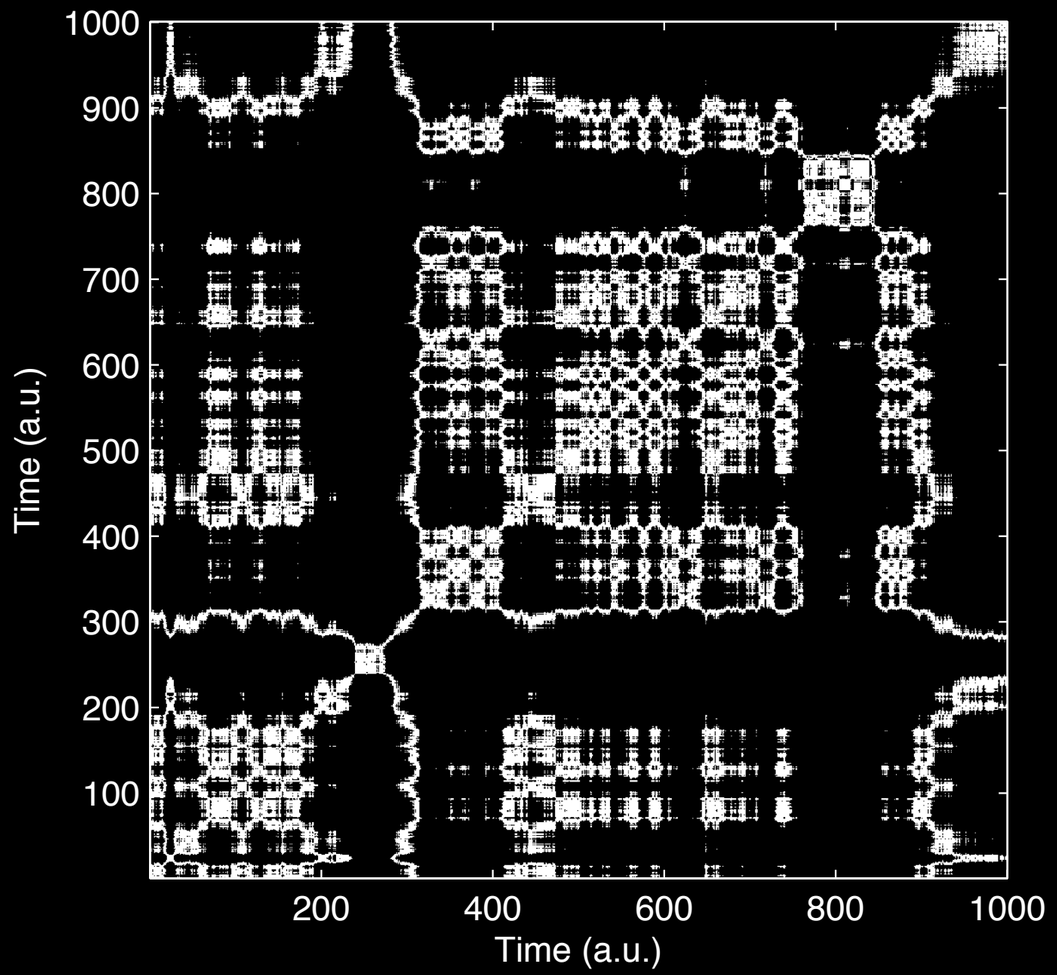
Marwan, Int J Bif Chaos 21, 2011
Zou et al, Chaos 20, 2010

Indicators of Transitions and Nonstationarities



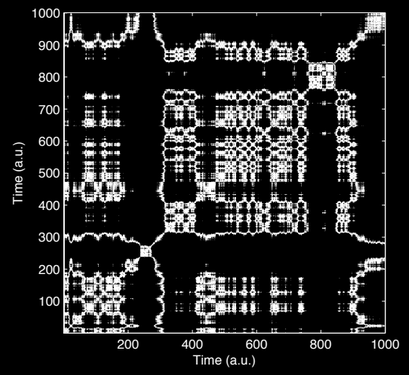
➡ depends on the application and question

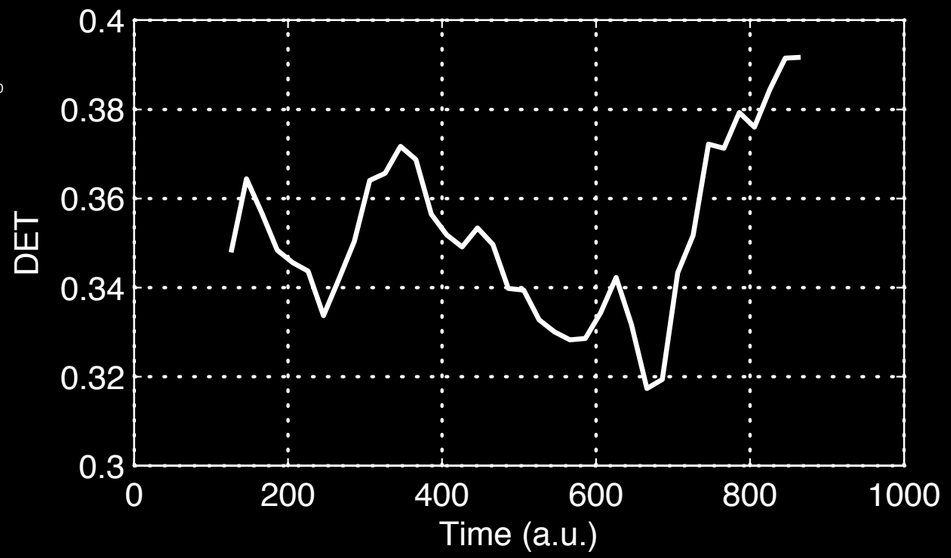
Indicators of Transitions and Nonstationarities

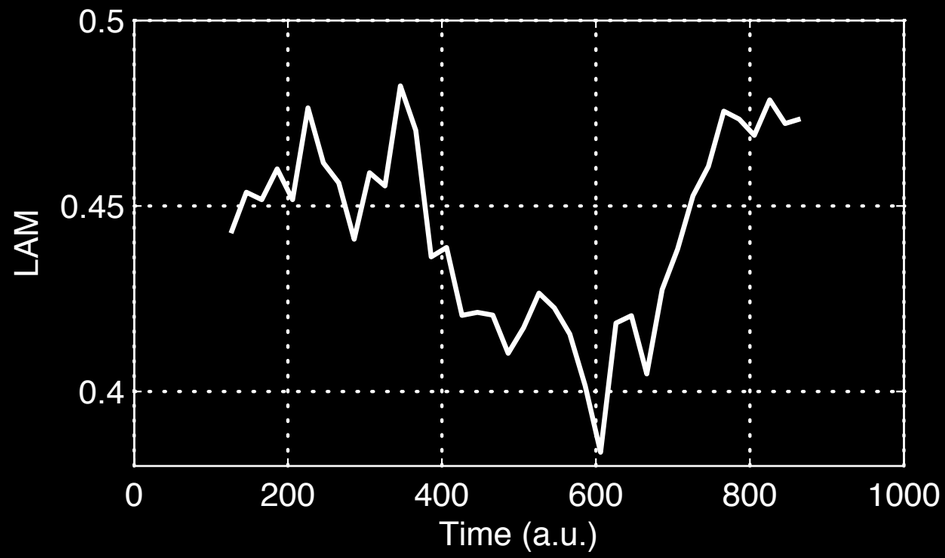


- AR(1): stationary process

Indicators of Transitions and Nonstationarities



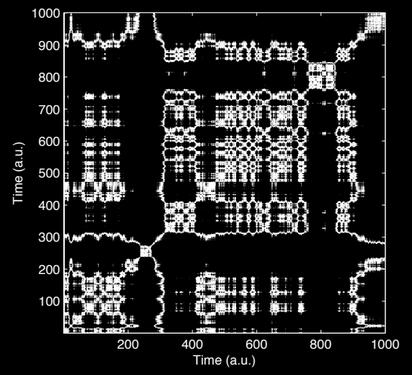


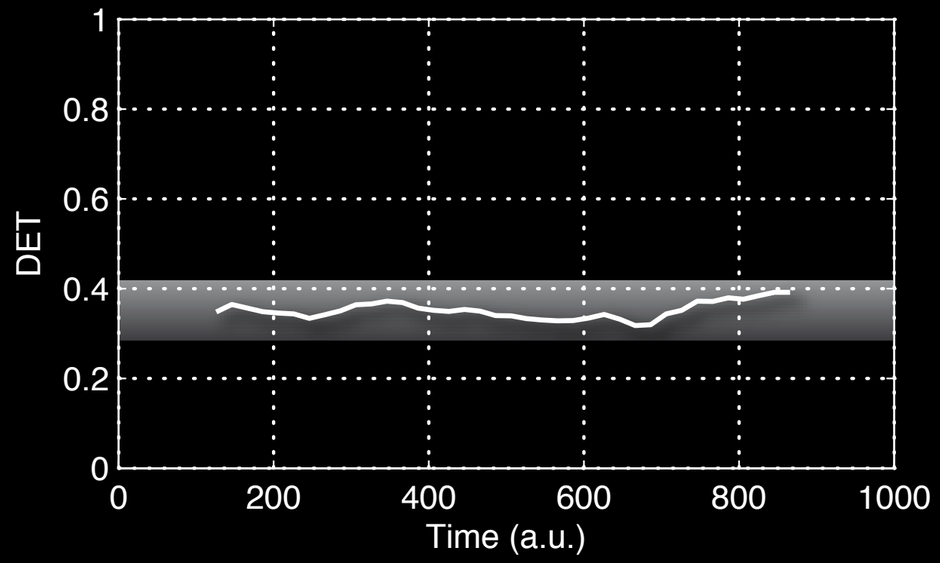


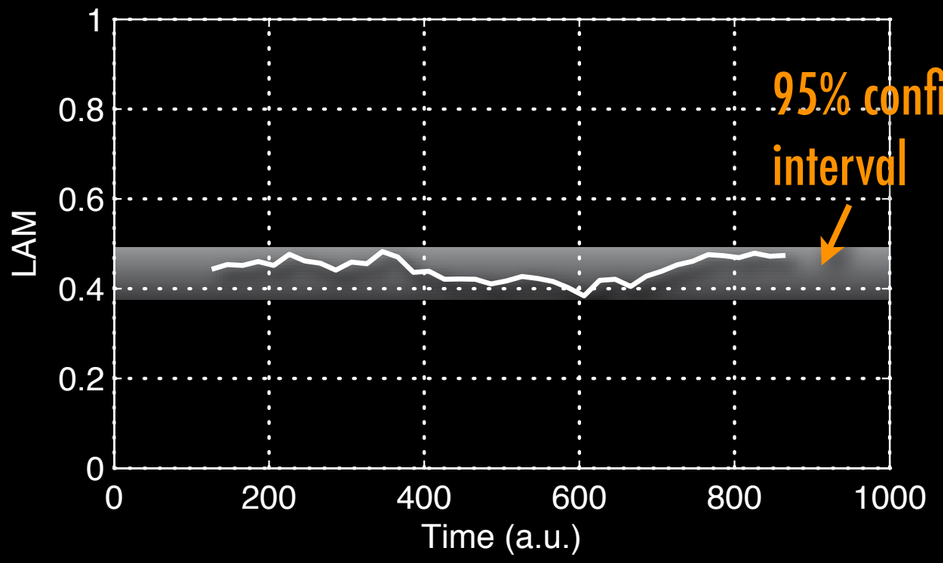
- RQA measures: nonstationarity?

Marwan, Int J Bif Chaos 21, 2011

Indicators of Transitions and Nonstationarities



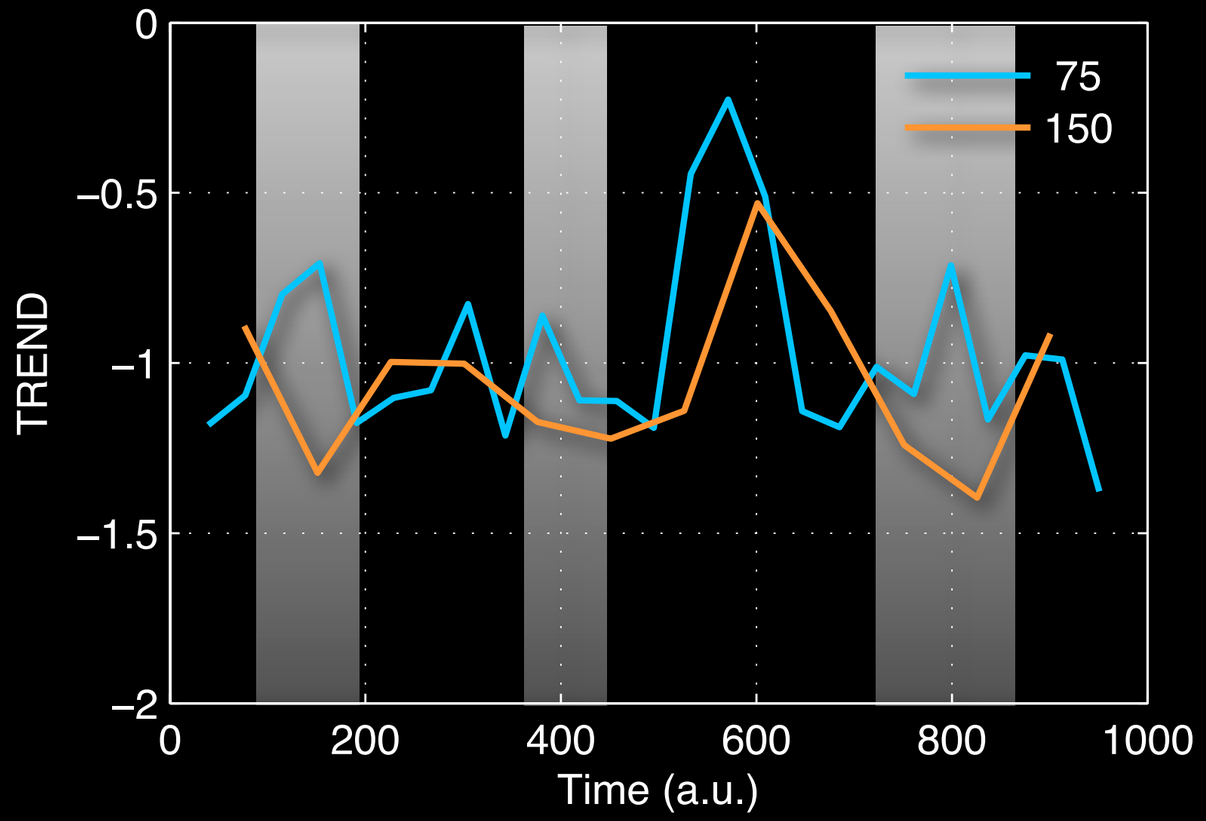




➡ significance test!

Marwan et al, EPL 101, 2013
Marwan, Int J Bif Chaos 21, 2011

Indicators of Transitions and Nonstationarities



- sensitivity on window size
- some RQA measures very sensitive (e.g. TREND)

Marwan, Int J Bif Chaos 21, 2011

Summary

- Careful selection of parameters (embedding, recurrence plot calculation, RQA parameters)
- Careful interpretation:
  - visual patterns
  - indicators of determinism, chaos, periodicities, nonstationarity
  - apply significance test
- Further pitfalls possible!