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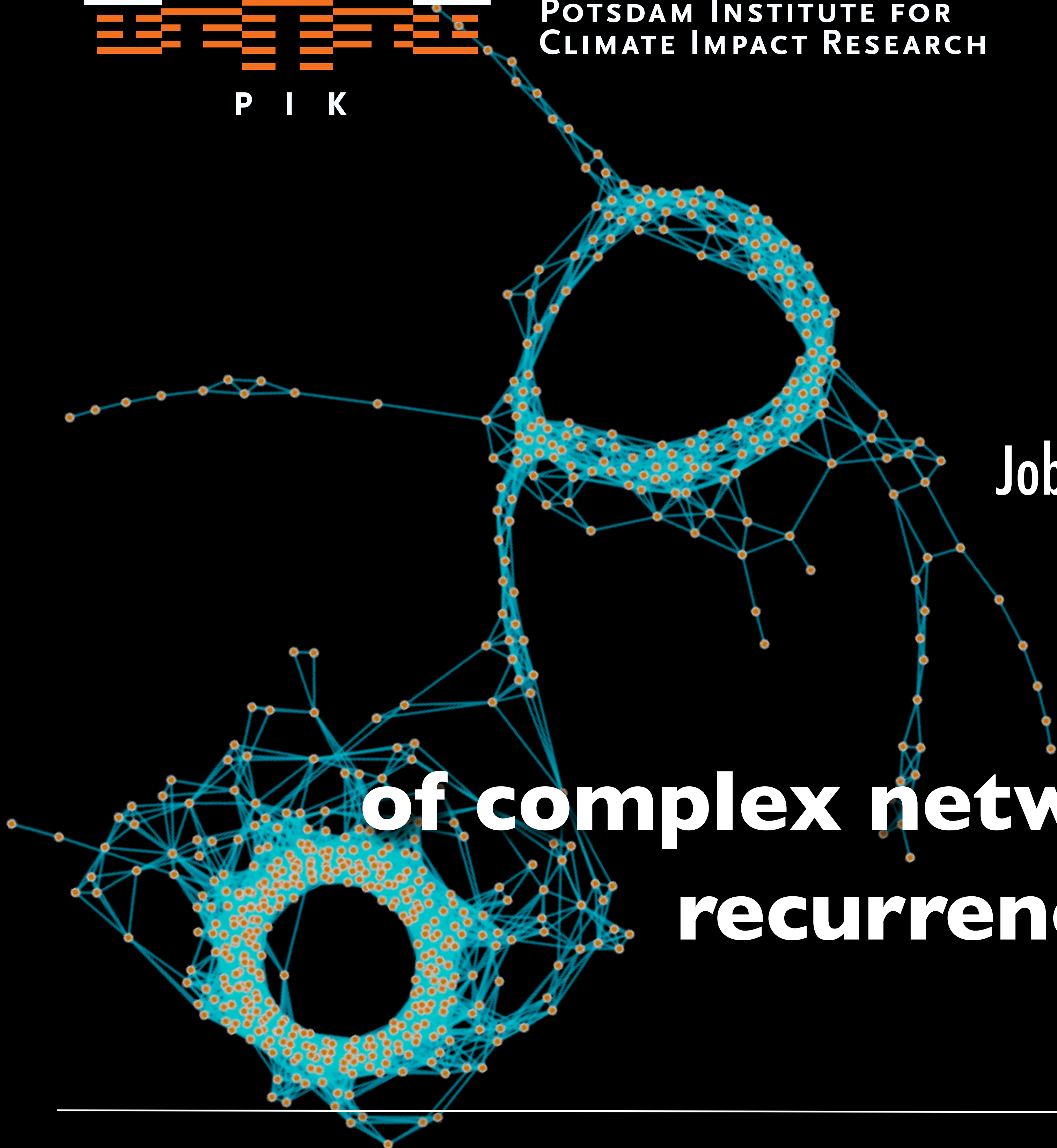


Norbert Marwan

Jonathan Donges, Reik Donner,
Jobst Heitzig, Yong Zou, Jürgen Kurths

Duality of complex network analysis and recurrence based analysis of time series

Leibniz Gemeinschaft

Outline

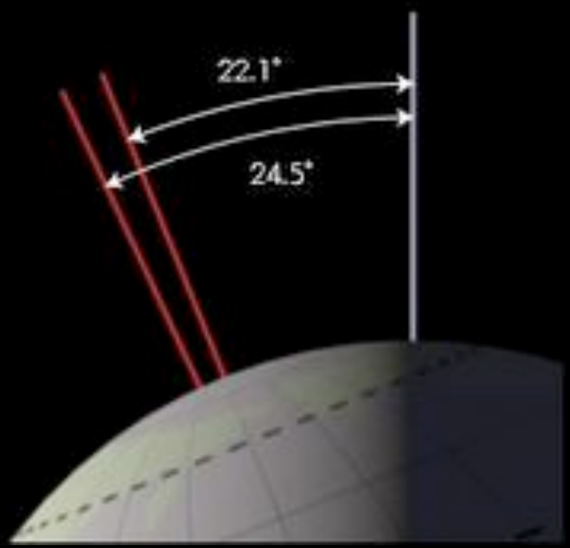
1. Recurrence
2. Recurrence plots
    - definition, structures, quantification, examples
3. Recurrence networks
    - definition, network measures, clustering, examples

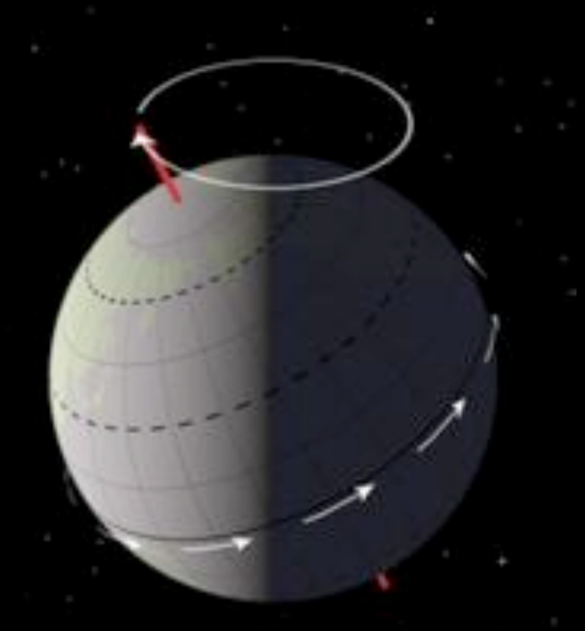
Recurrences

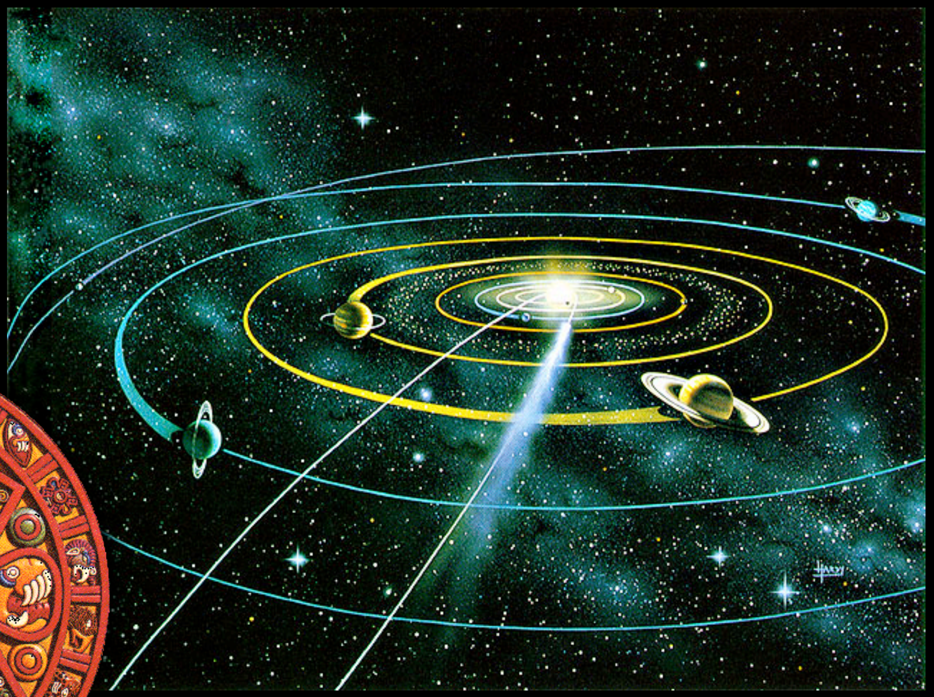
Recurrence

- fundamental characteristic of many dynamical systems
- recurrences in real life:

  Milankovich cycles, weather after storm, El Niño phenomenon, heart beat after exertion, Maya calendar etc.







Recurrence

- Anaxagoras, approx. 450 BC:
  perichoresis: chaotic circular movement

Recurrence

- Poincaré, 1890:

  “a system recurs infinitely many times as close as one wishes to its initial state”

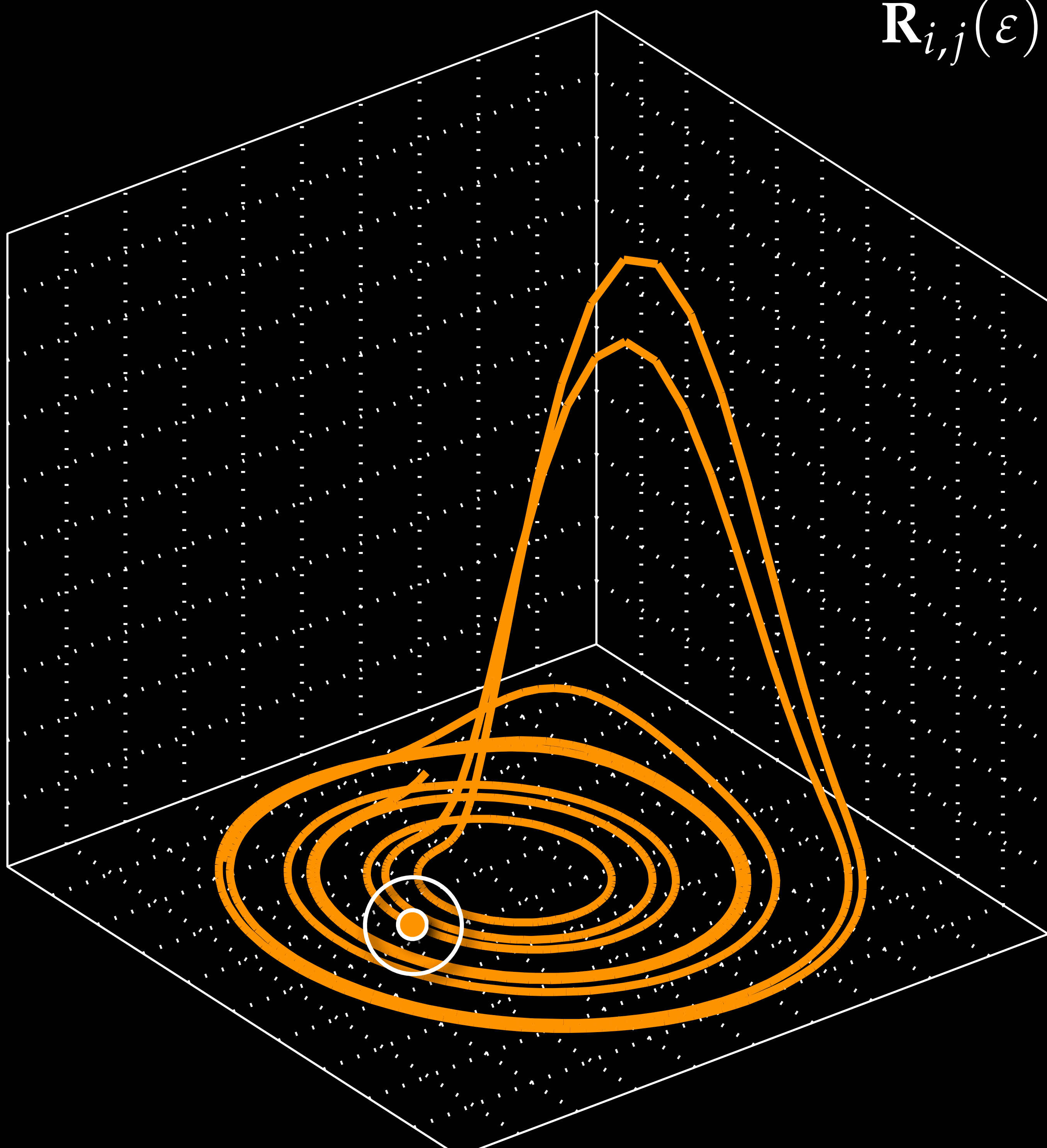
Investigating Recurrence

- Poincaré map
- Recurrence time statistics
- First return map
- Recurrence plot
- Recurrence network

Recurrence Plots

Recurrence Plot

$$\mathbf{R}_{i,j}(\varepsilon) = \Theta\left(\varepsilon - \left\|\vec{x}_i - \vec{x}_j\right\|\right), \quad i, j = 1, \ldots, N$$



J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, Europhysics Letters, 5, 1987

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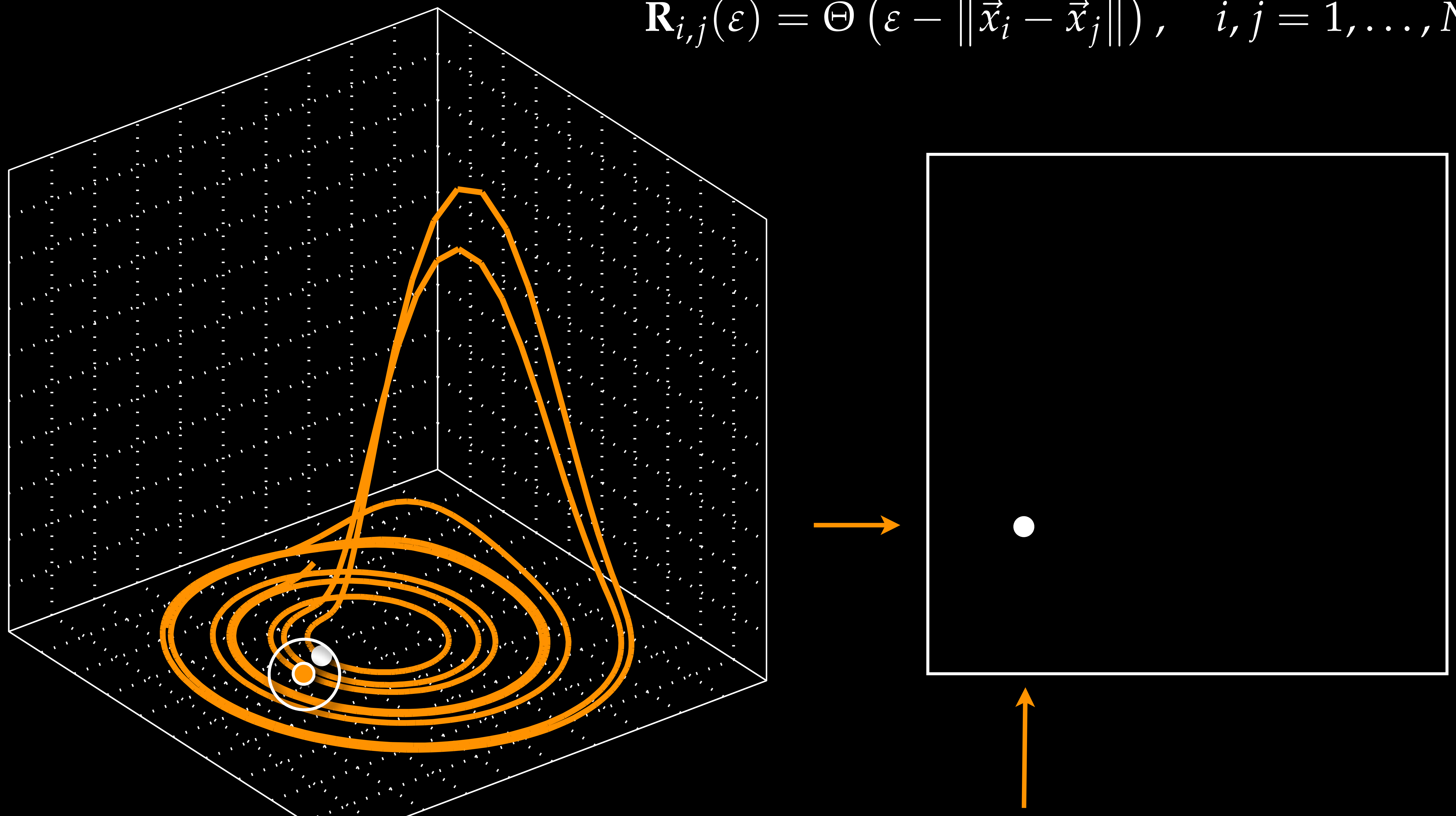

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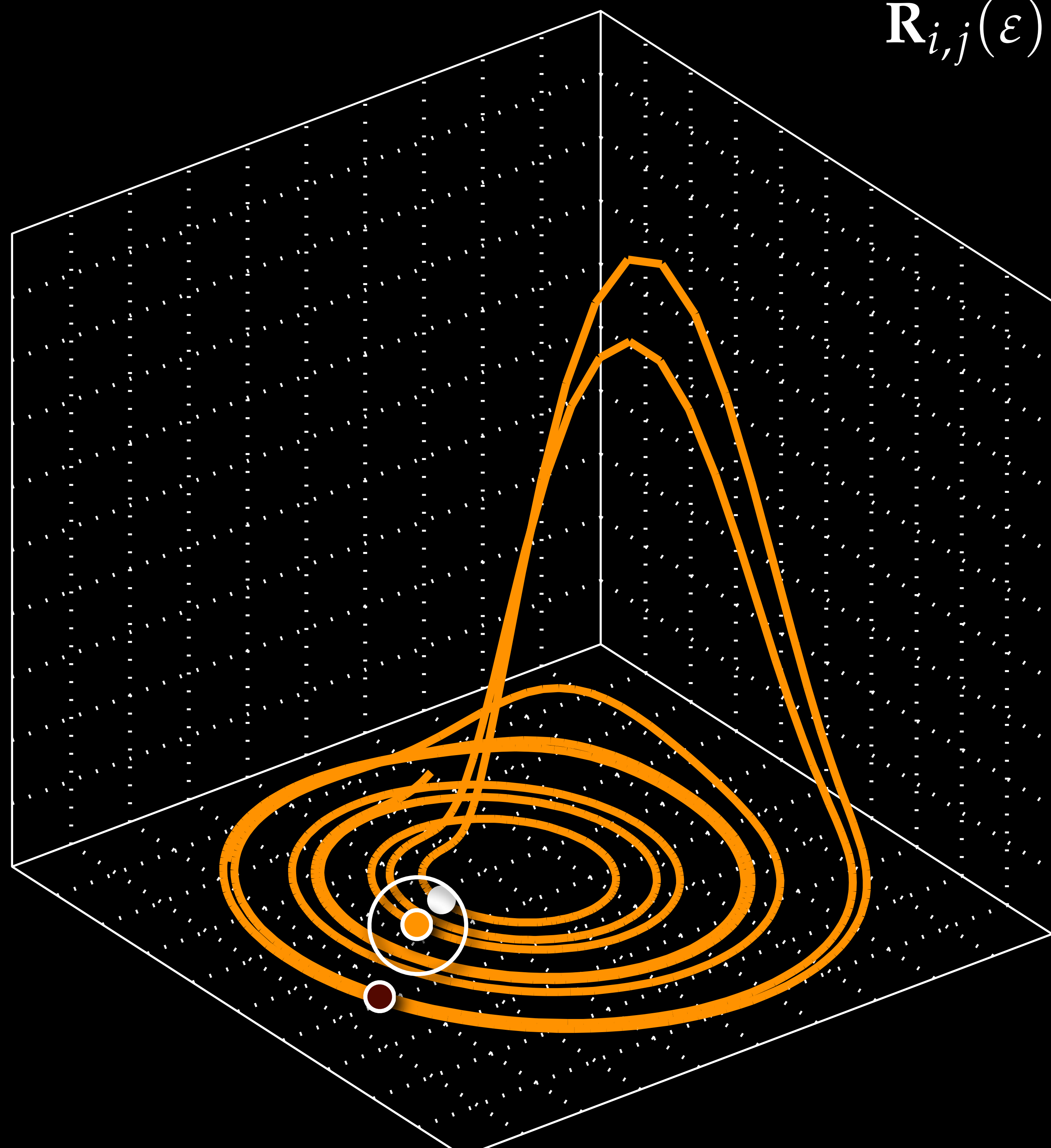

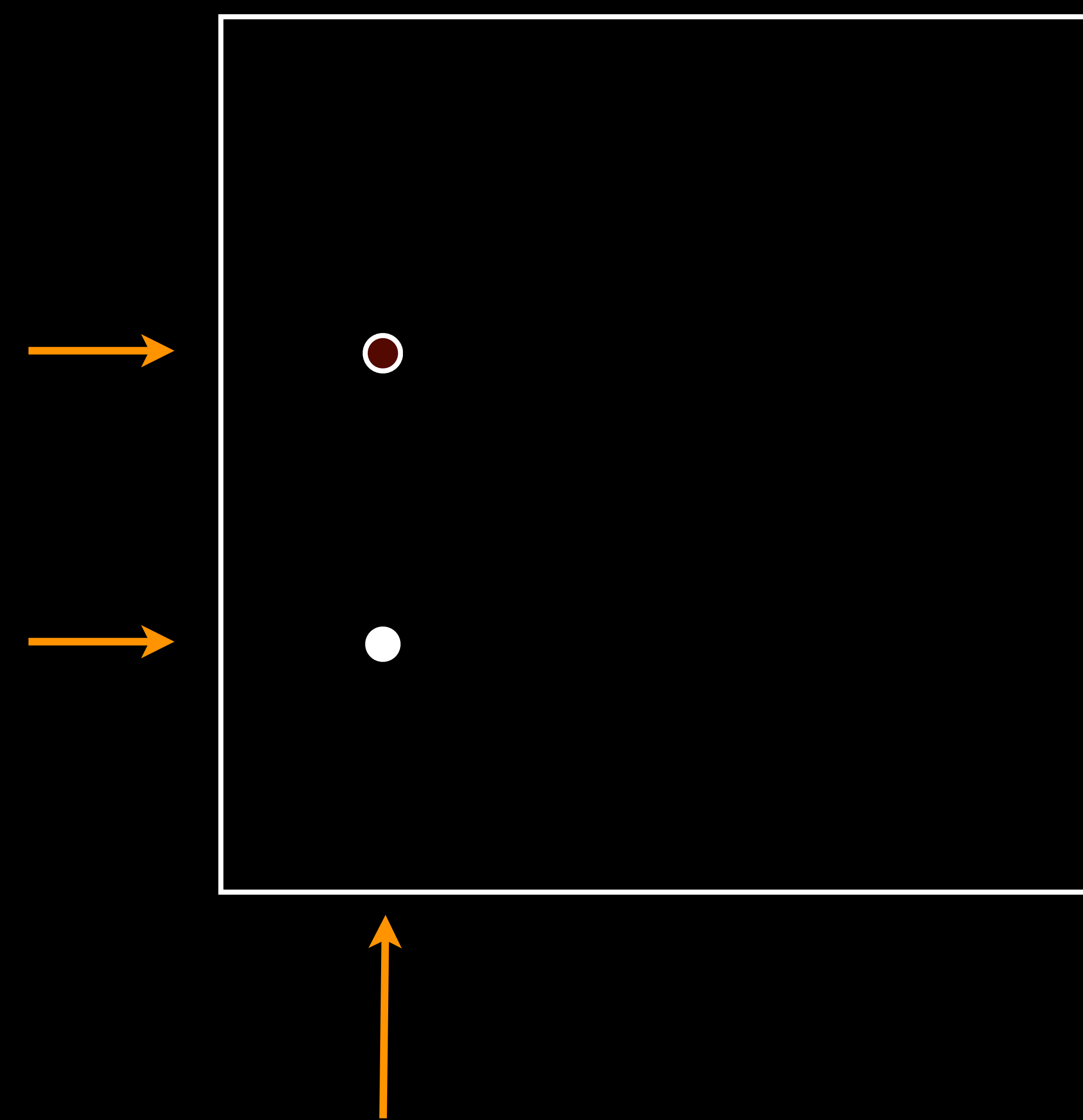

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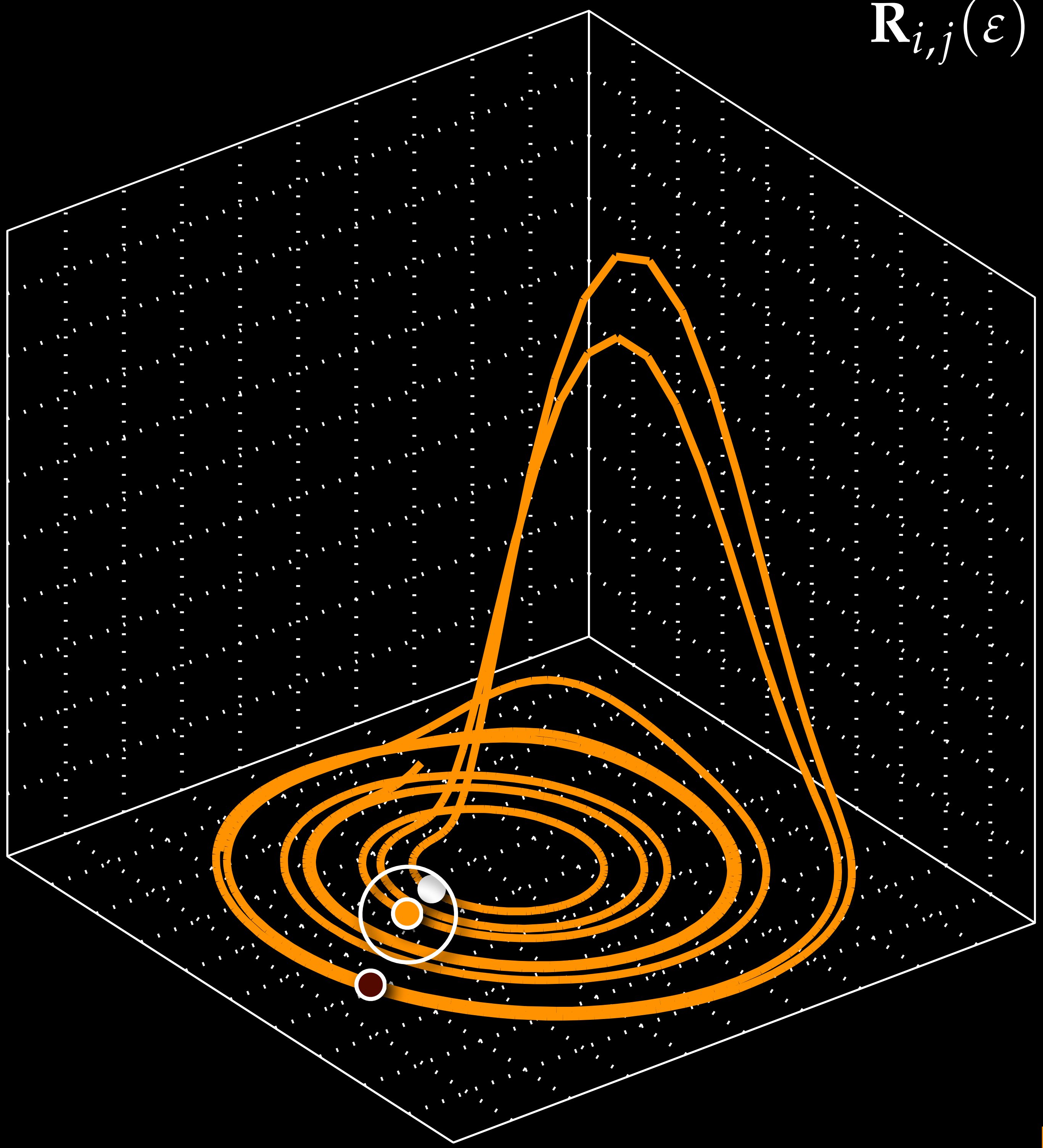
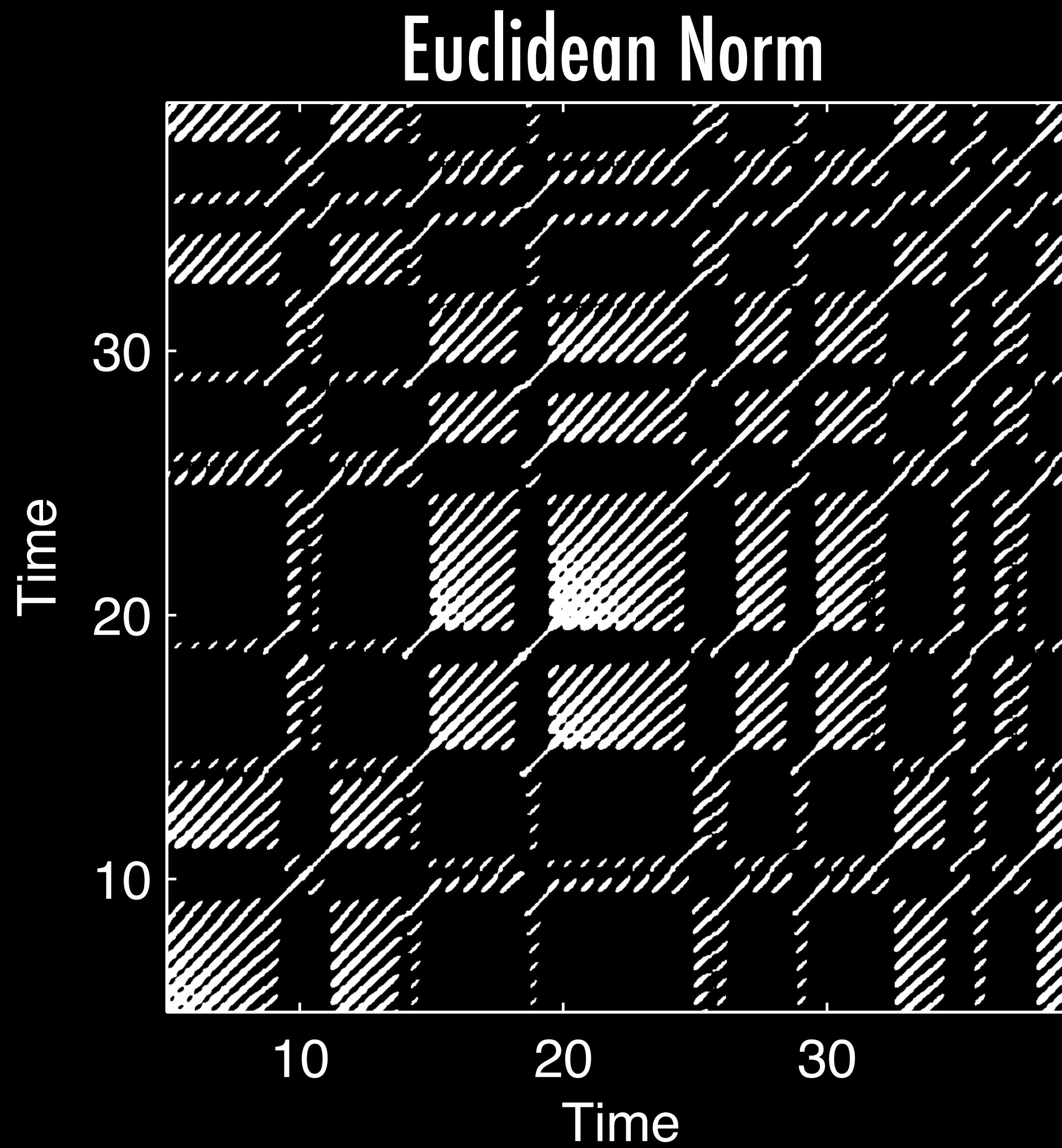




J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, Europhysics Letters, 5, 1987

Recurrence Plot

- to visualise the phase space trajectory by its recurrences

- recurrence matrix:
  - binary
  - symmetric

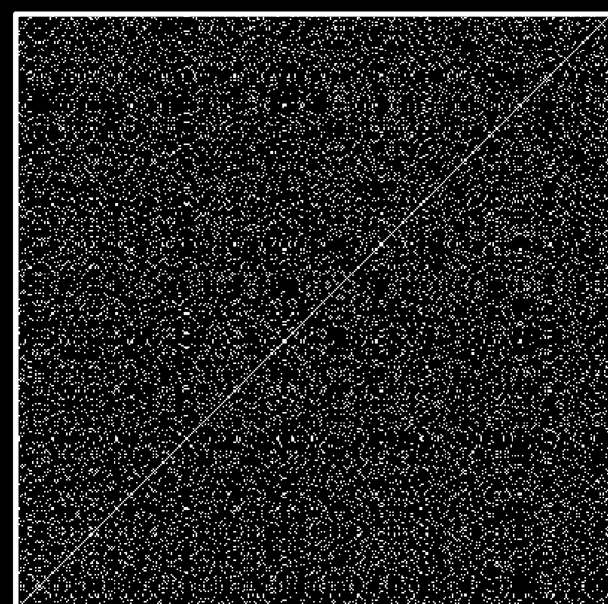
$R_{i,j} =$

1	1	0	0	1
1	1	1	0	1
0	1	1	0	0
0	0	0	1	1
1	1	0	1	1

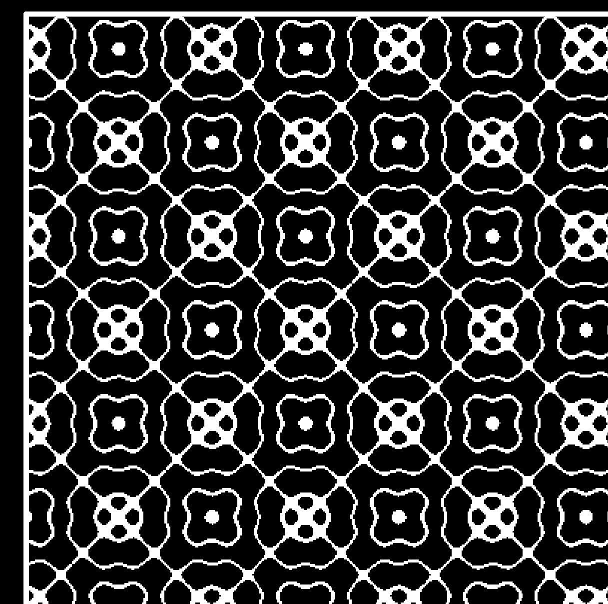
J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, Europhysics Letters, 5, 1987
N. Marwan et al., Physics Reports, 438, 2007

Recurrence Plot Typology

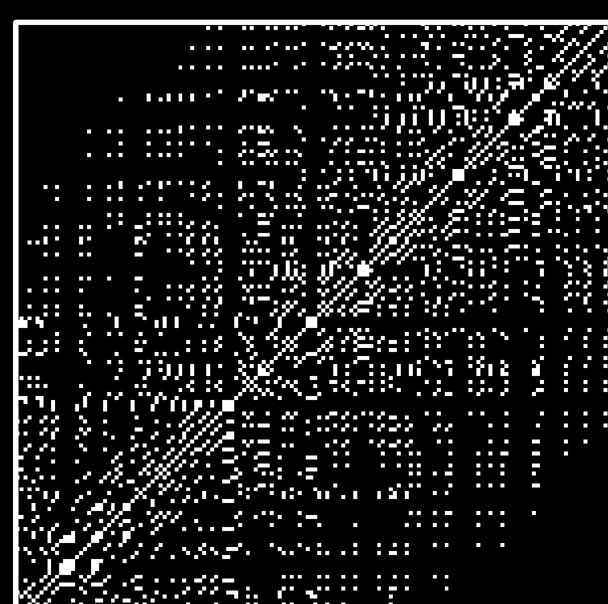
homogeneous



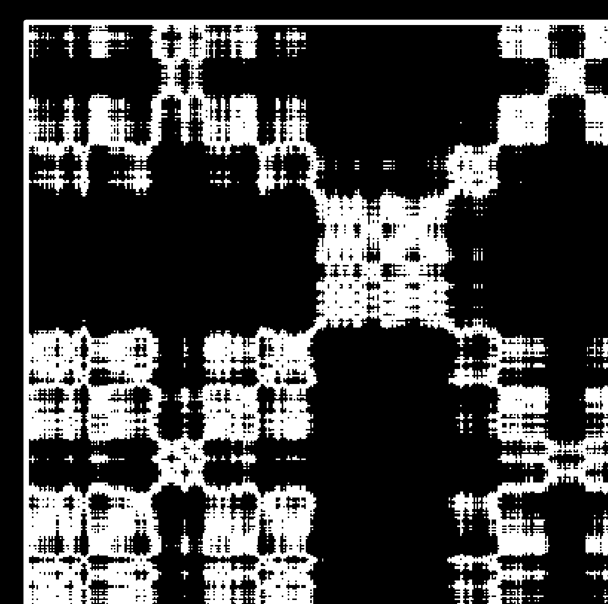
periodic



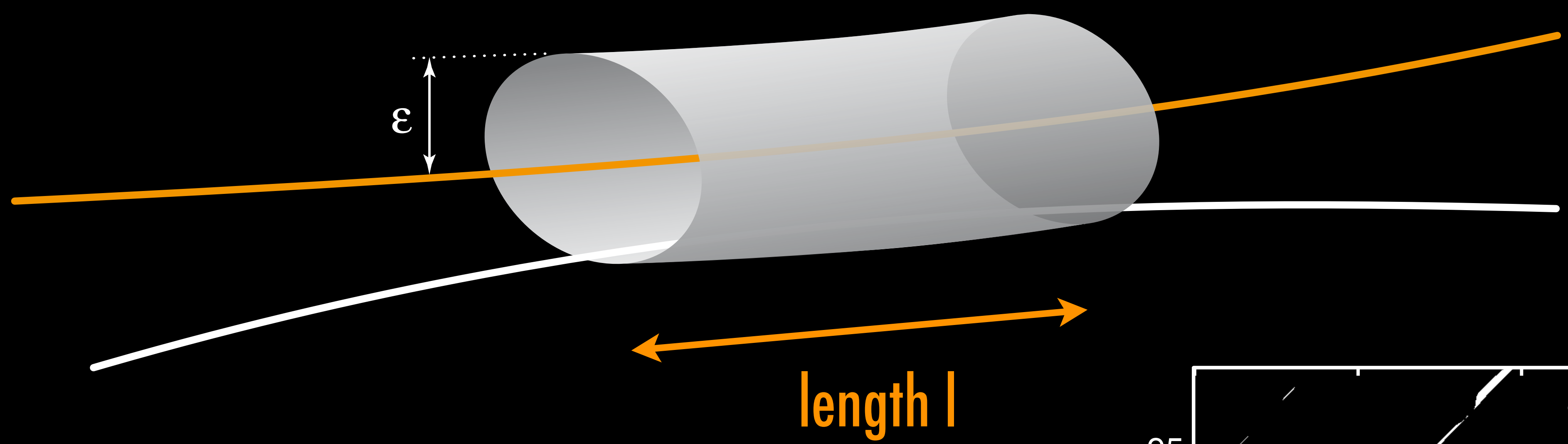
drifty



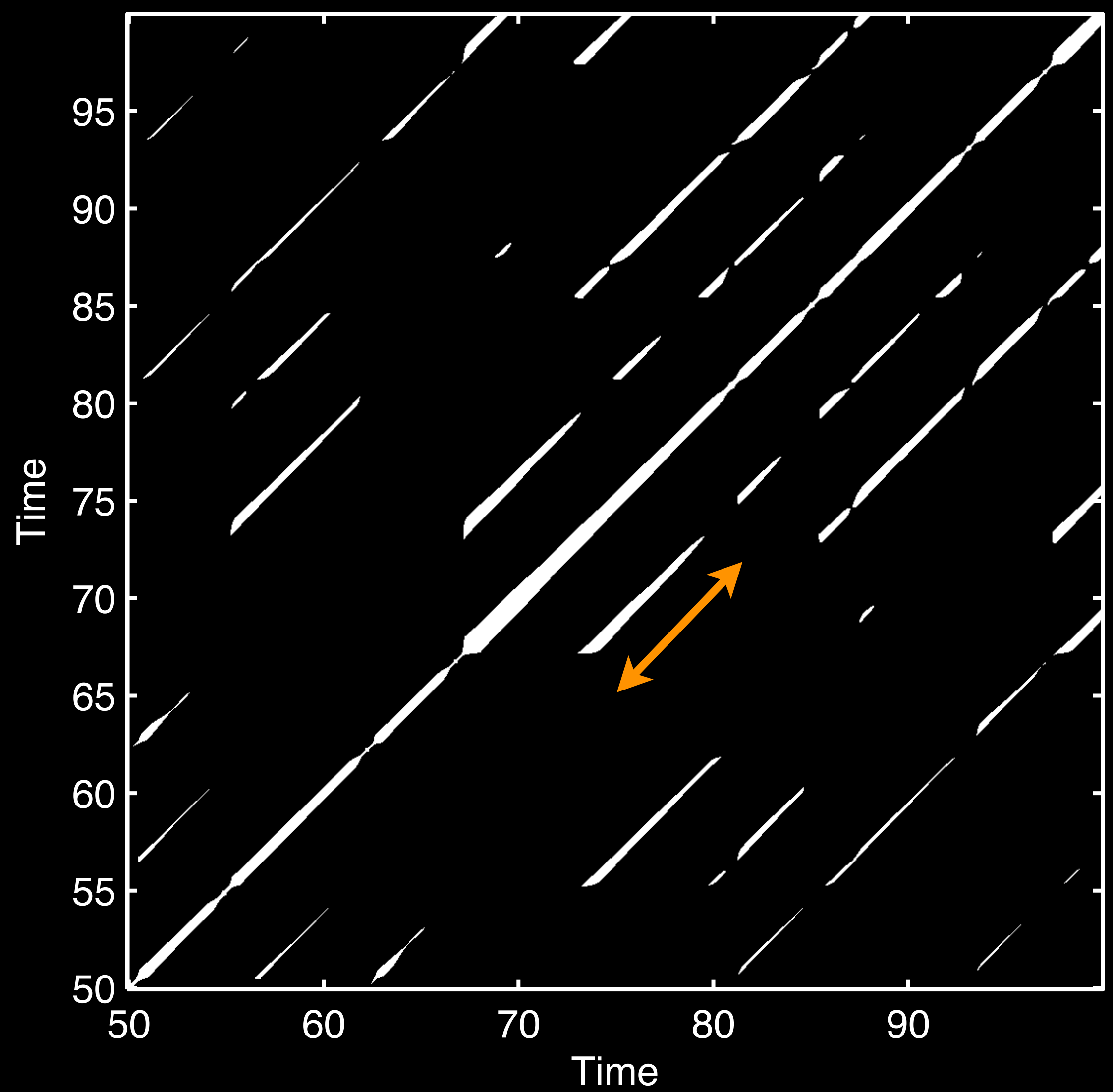
disrupted



Recurrence Quantification



- number of lines of exactly length l
  - histogram P(l)



J. P. Zbilut & C. L. Webber Jr., Phys. Lett. A 171, 1992
N. Marwan et al., Phys. Rev. E 66, 2002

Recurrence Quantification

- Determinism

$$DET = \frac{\sum_{l=l_{\min}}^{N} l\,P(l)}{\sum_{l=1}^{N} l\,P(l)}$$

Probability that recurrences further recur

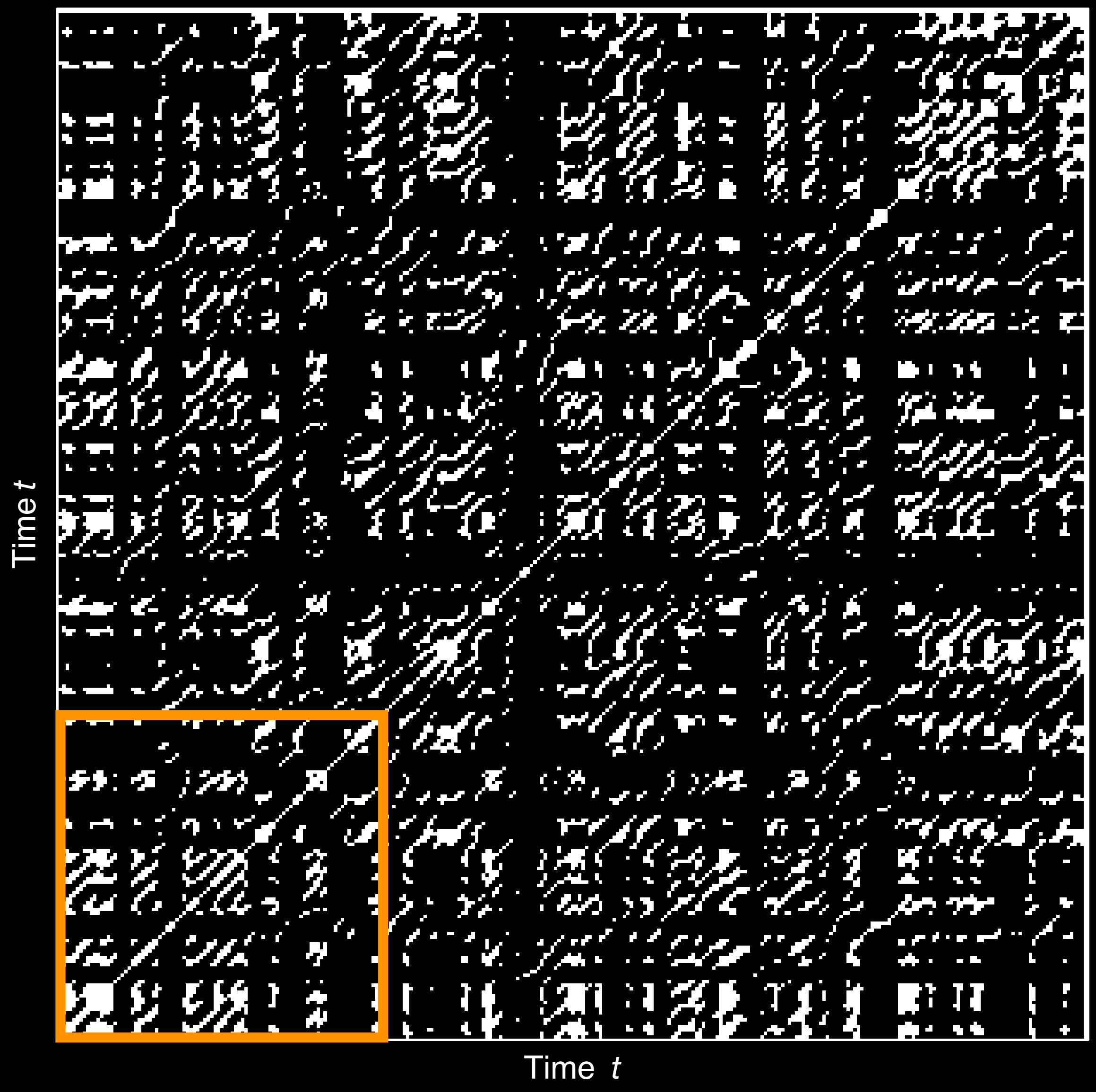
- Laminarity

$$LAM = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)}$$

Probability that a certain recurrent state further recurs

Recurrence Quantification

- Time dependent analysis:
  - sliding windows over RP
- Detection of transitions



Recurrence Quantification

- Time dependent analysis:
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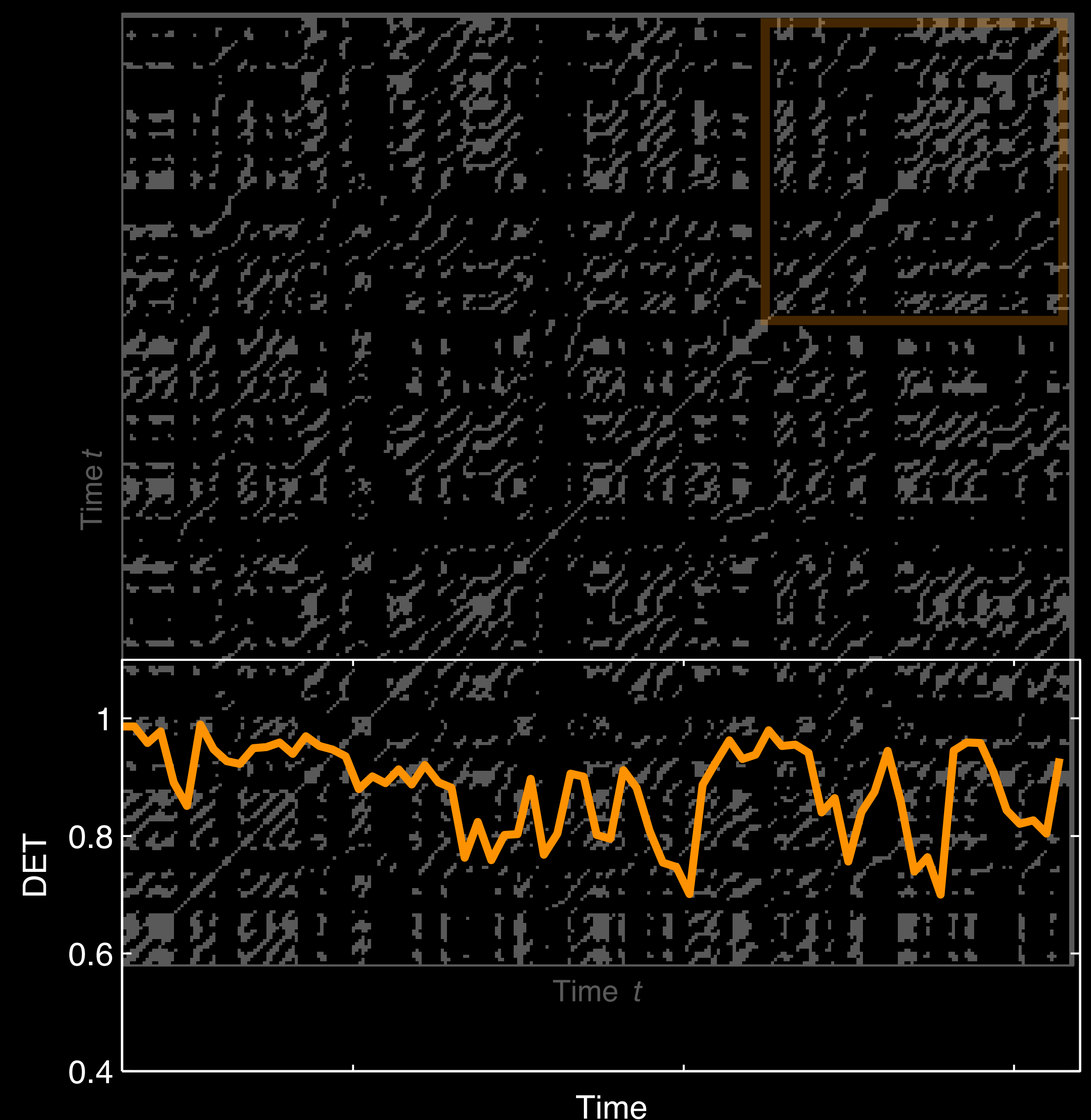
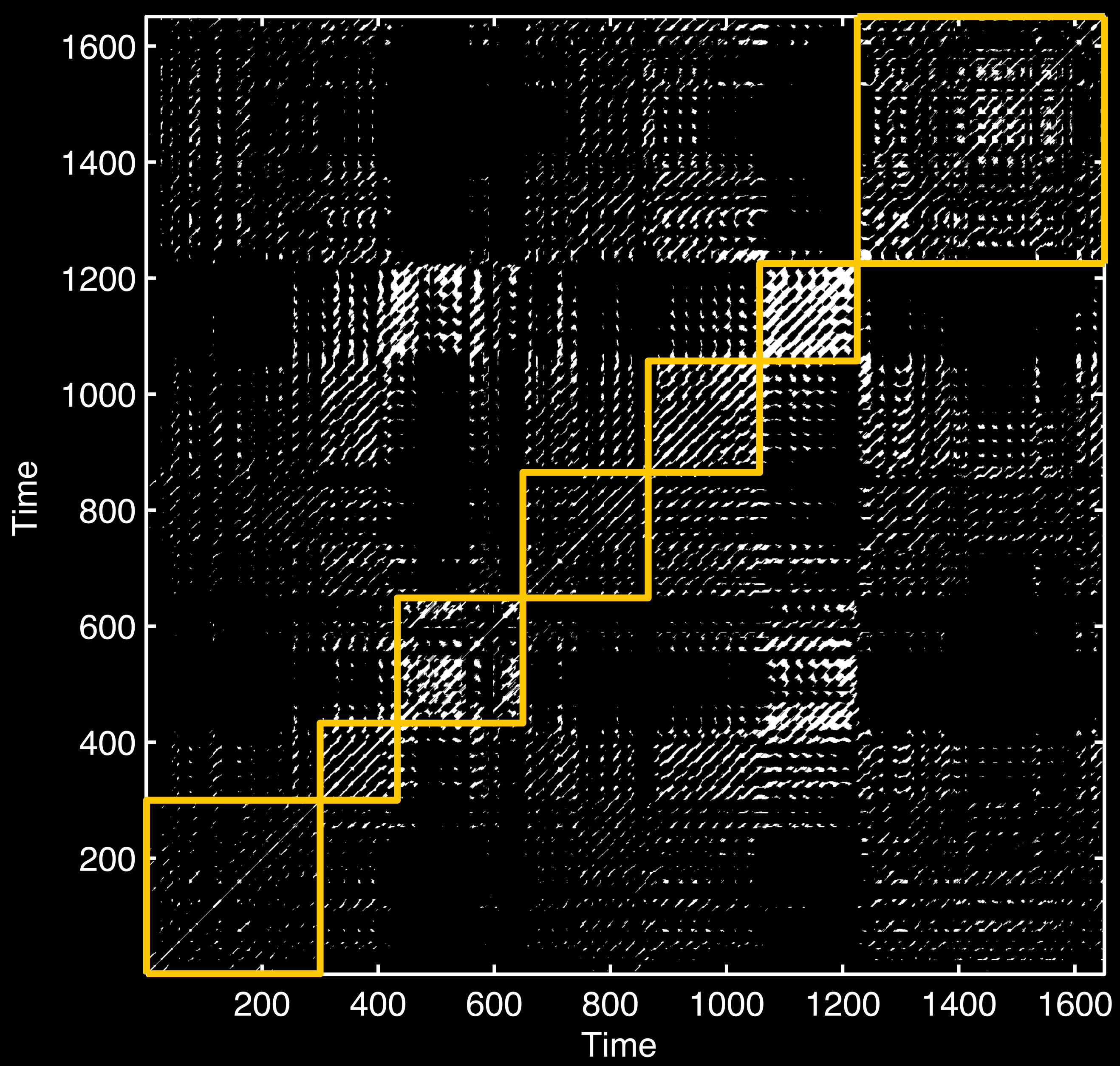
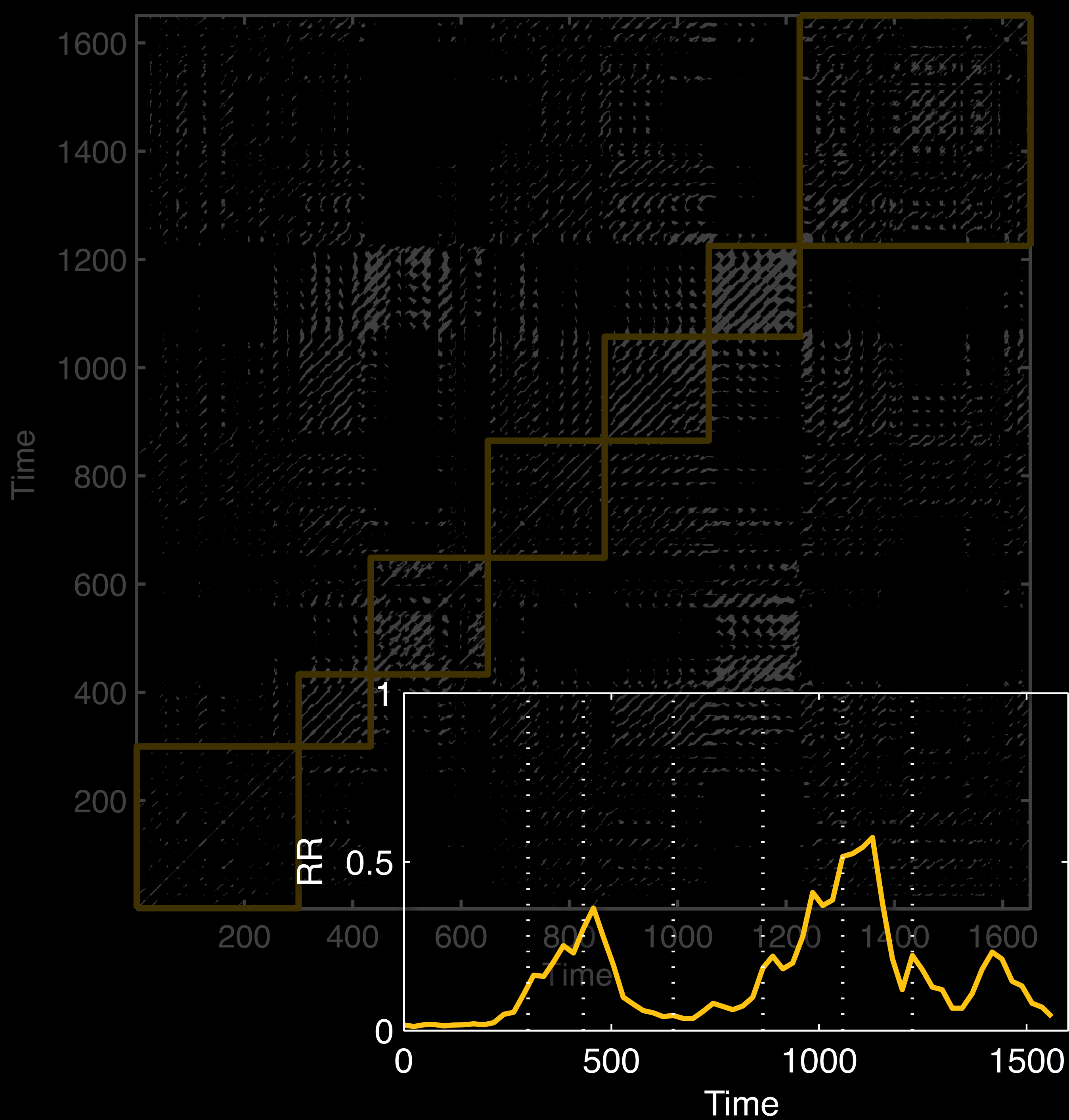


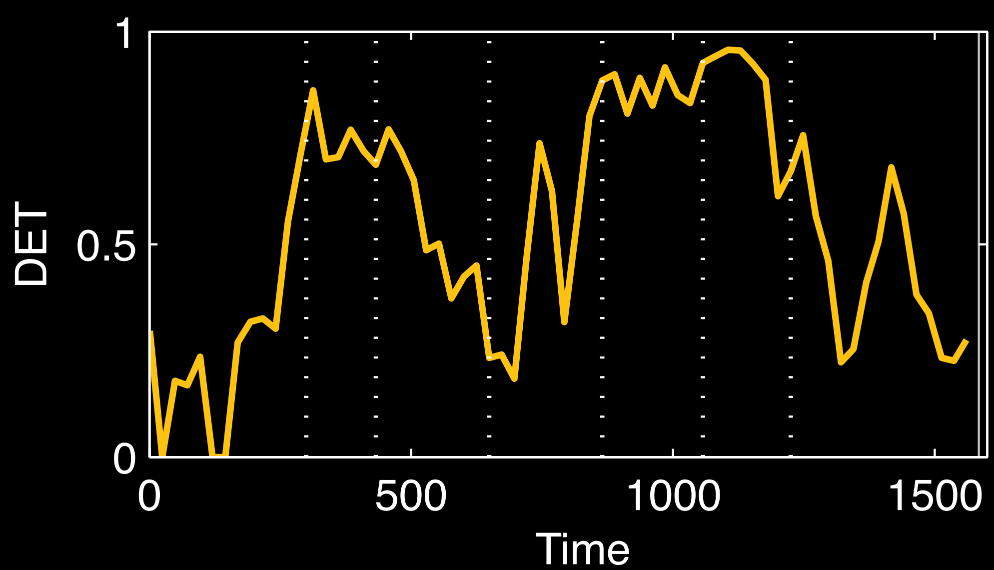
Dynamics of Oxygen Crises in a Lake



Facchini et al., Ecological Modelling, 203, 2007

Dynamics of Oxygen Crises in a Lake

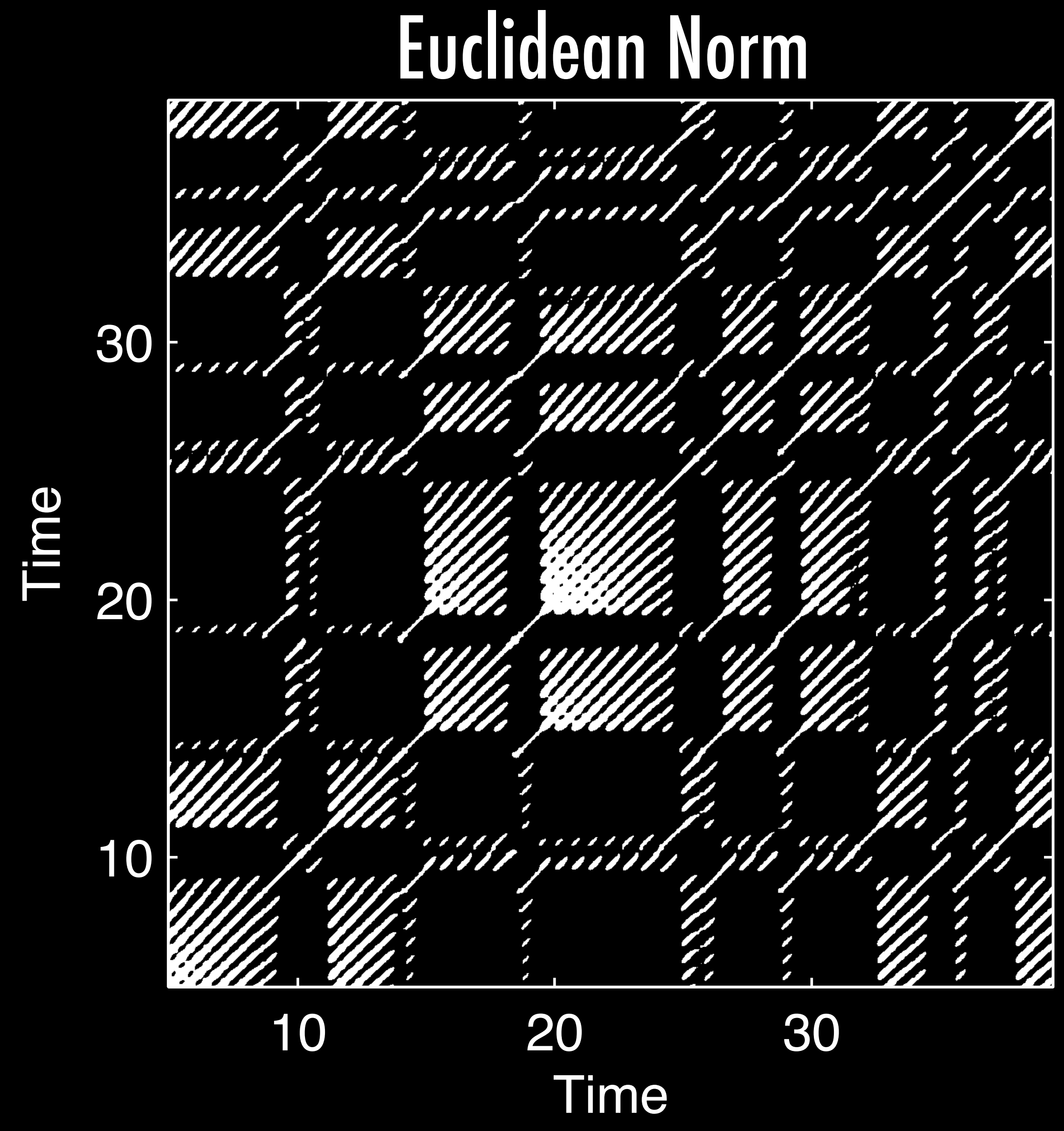




Facchini et al., Ecological Modelling, 203, 2007

Recurrence Plot

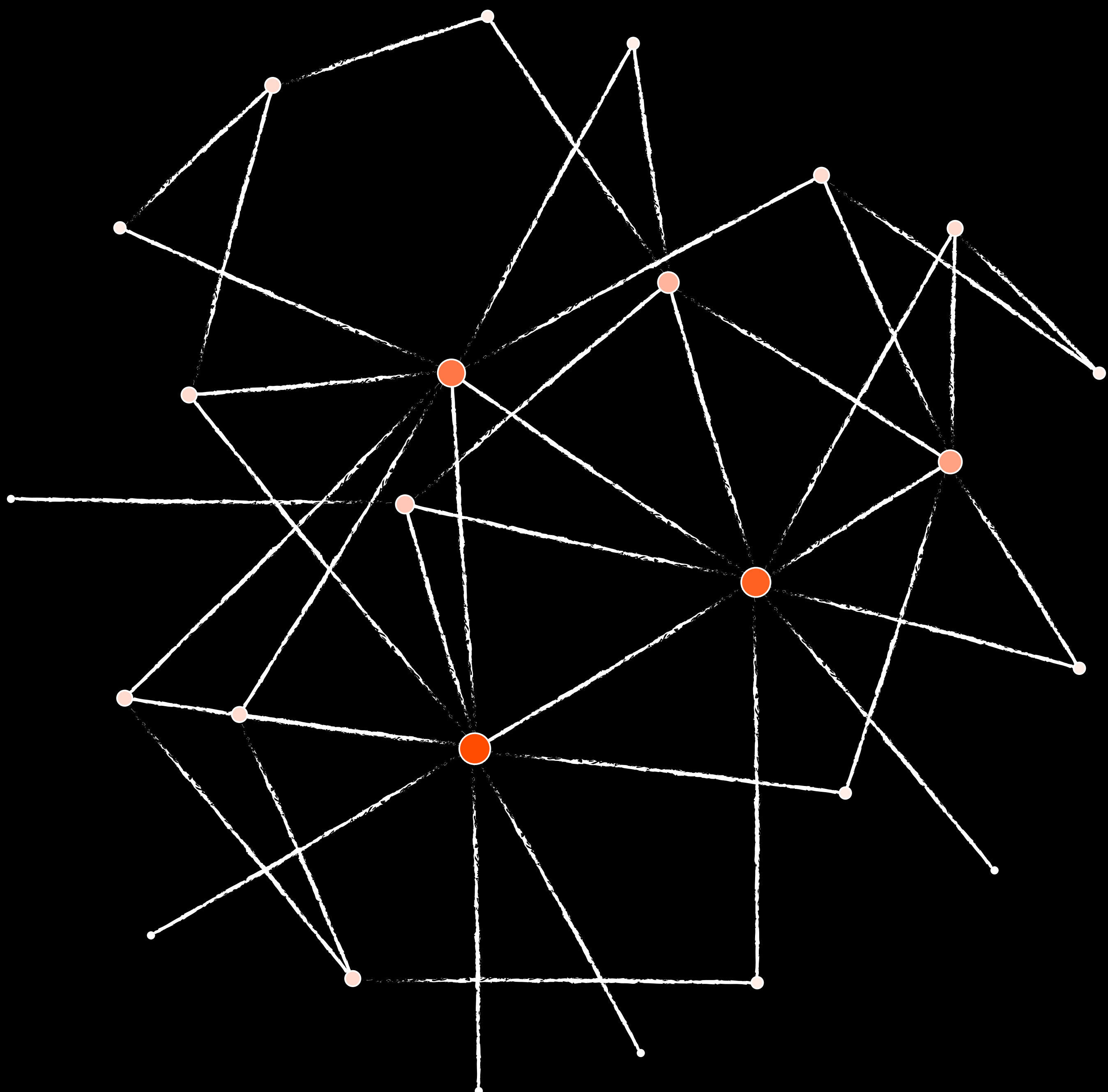
- Transition detection
- Differentiate dynamics
- Finding time scales
- Interrelation detection
- Synchronisation analysis
- Surrogates
- Recurrence time statistics
- etc.



N. Marwan et al., Physics Reports, 438, 2007

Complex Networks

Complex Networks



- link matrix (undirected, unweighted network):
  - binary
  - symmetric

$A_{i,j} =$

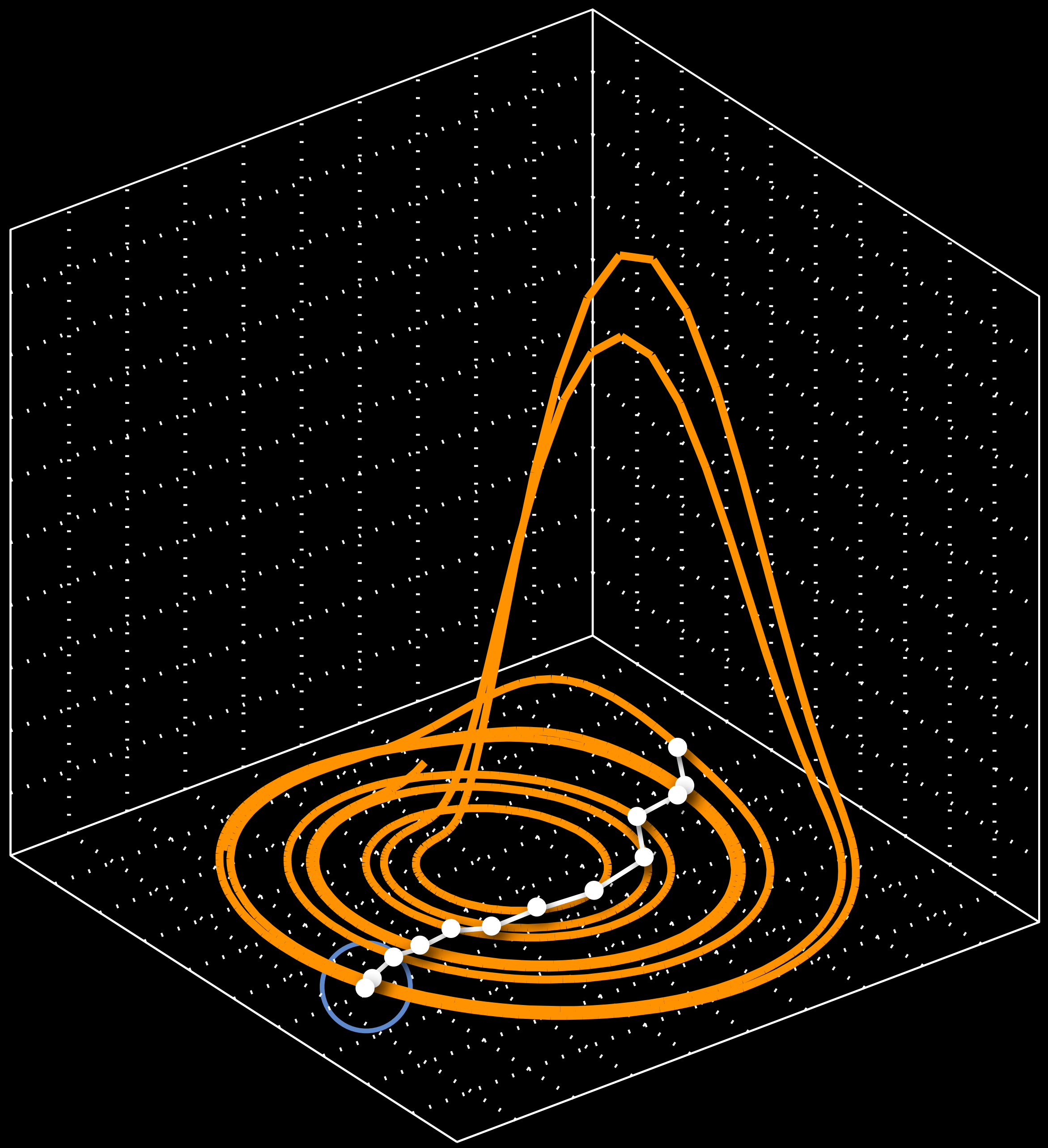
0	1	0	0	1
1	0	1	0	1
0	1	0	0	0
0	0	0	0	1
1	1	0	1	0

- link matrix: similar to recurrence plot

Time Series Analysis using Complex Networks

- Link matrix = recurrence matrix of time series
  - Nodes: states in phase space
  - Links: local neighbours of states (i.e. recurrence)

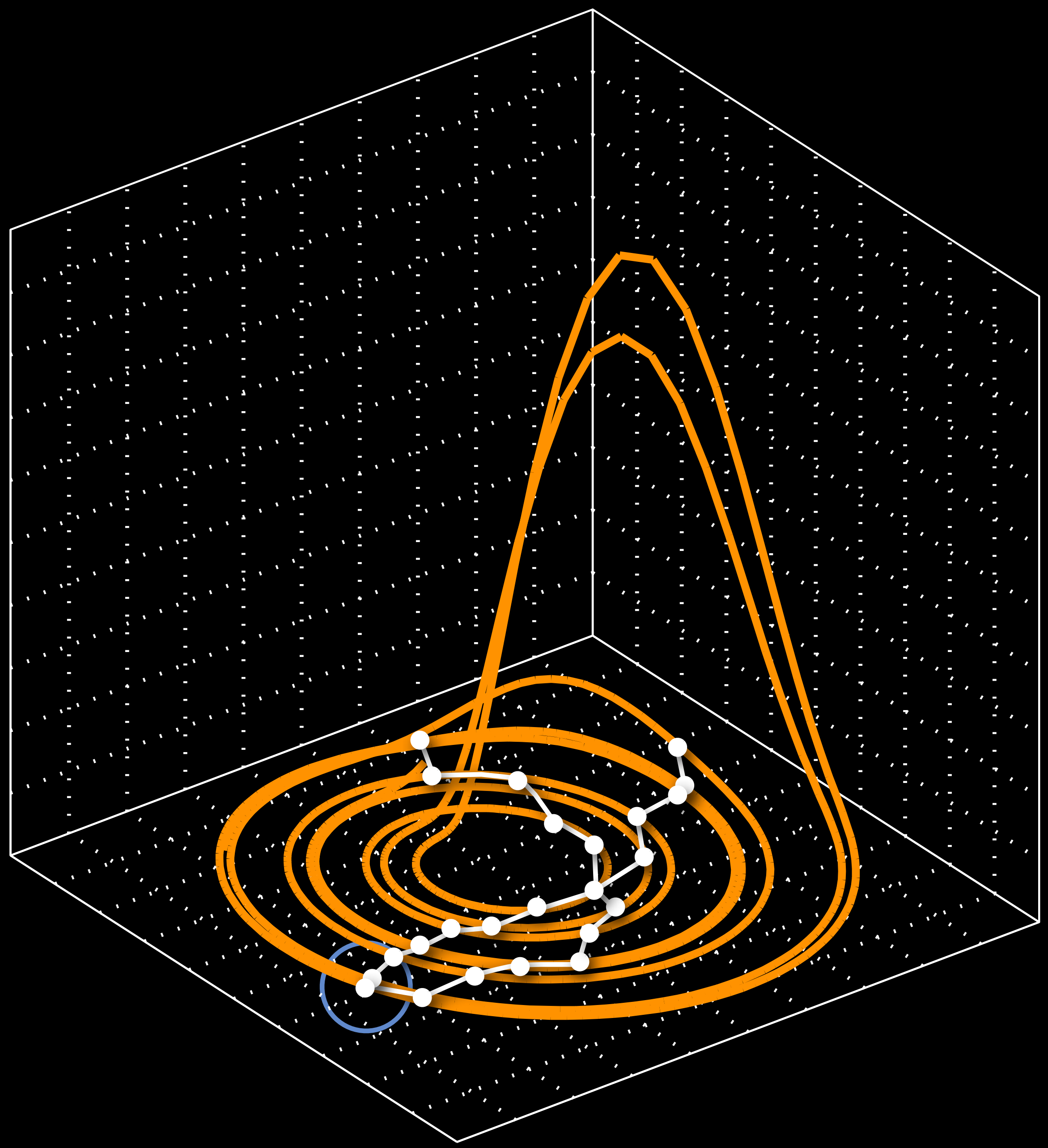
- Path: connected neighbourhoods



Marwan et al., Phys. Lett. A 373, 2009
Donner et al., New J. Phys. 12, 2010

Time Series Analysis using Complex Networks

- Path: connected neighbourhoods
  - no causal path!
  - shortcuts using neighbourhoods
  - no small-world shortcuts



Marwan et al., Phys. Lett. A 373, 2009
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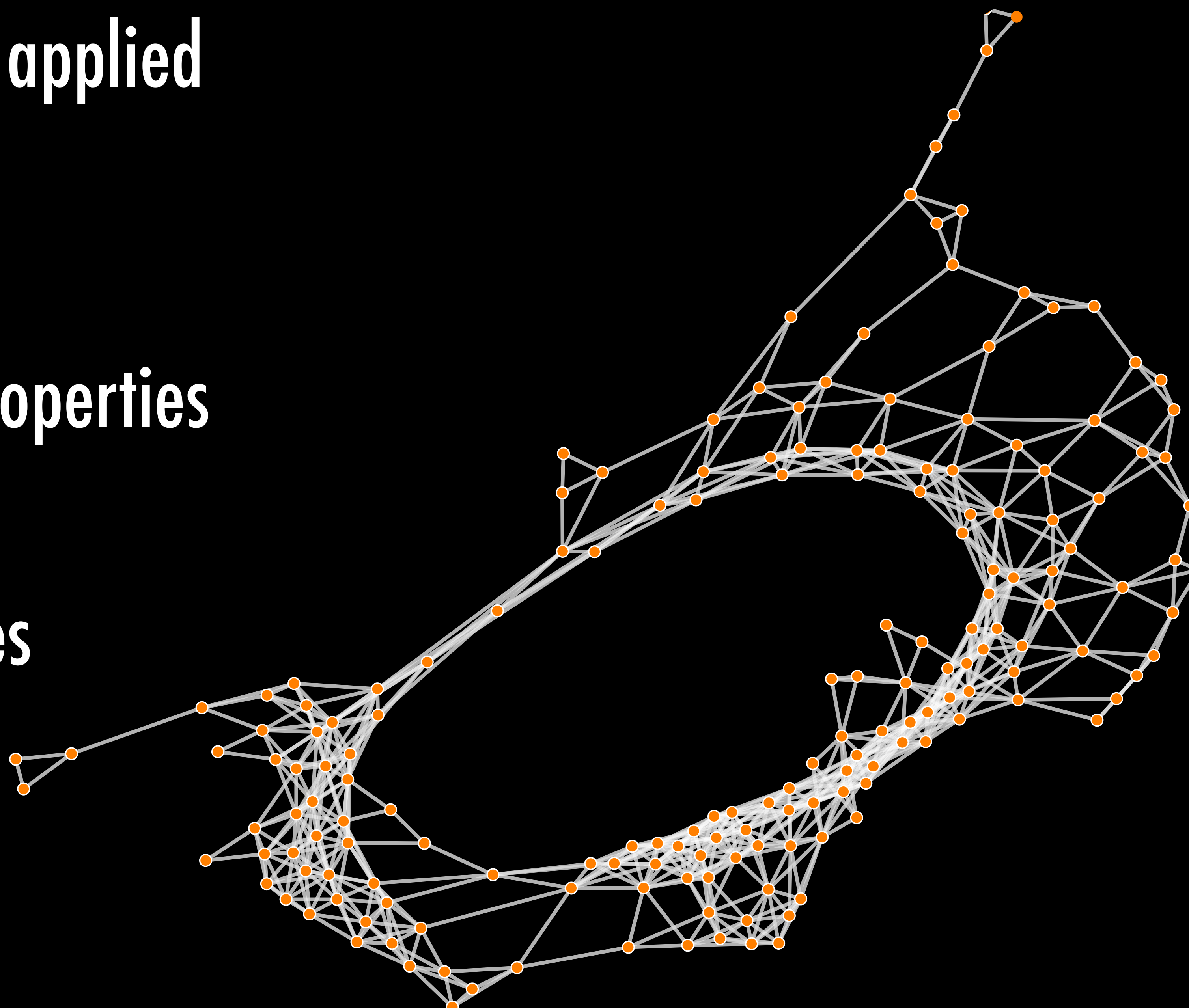
Time Series Analysis using Complex Networks

- Complex network measures applied to recurrence plot
  - measures of complexity explaining topological properties of complex systems
  - local and global measures
- „recurrence network“



Marwan et al., Phys. Lett. A 373, 2009
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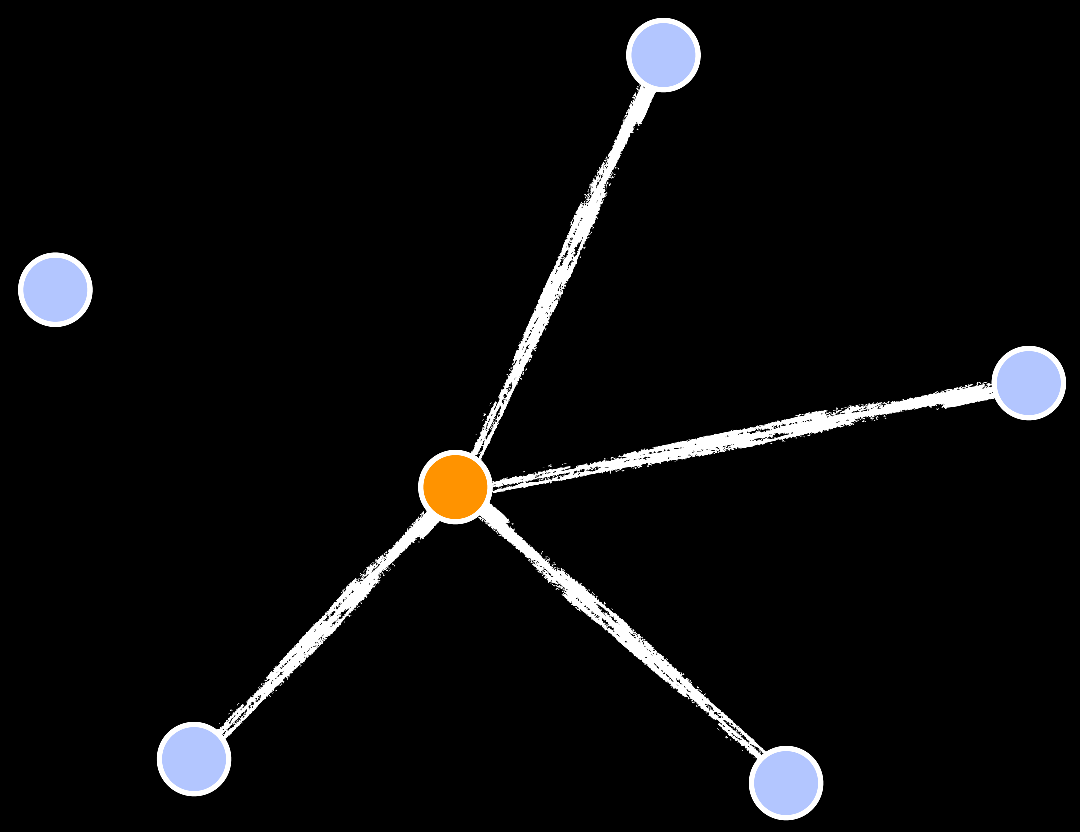
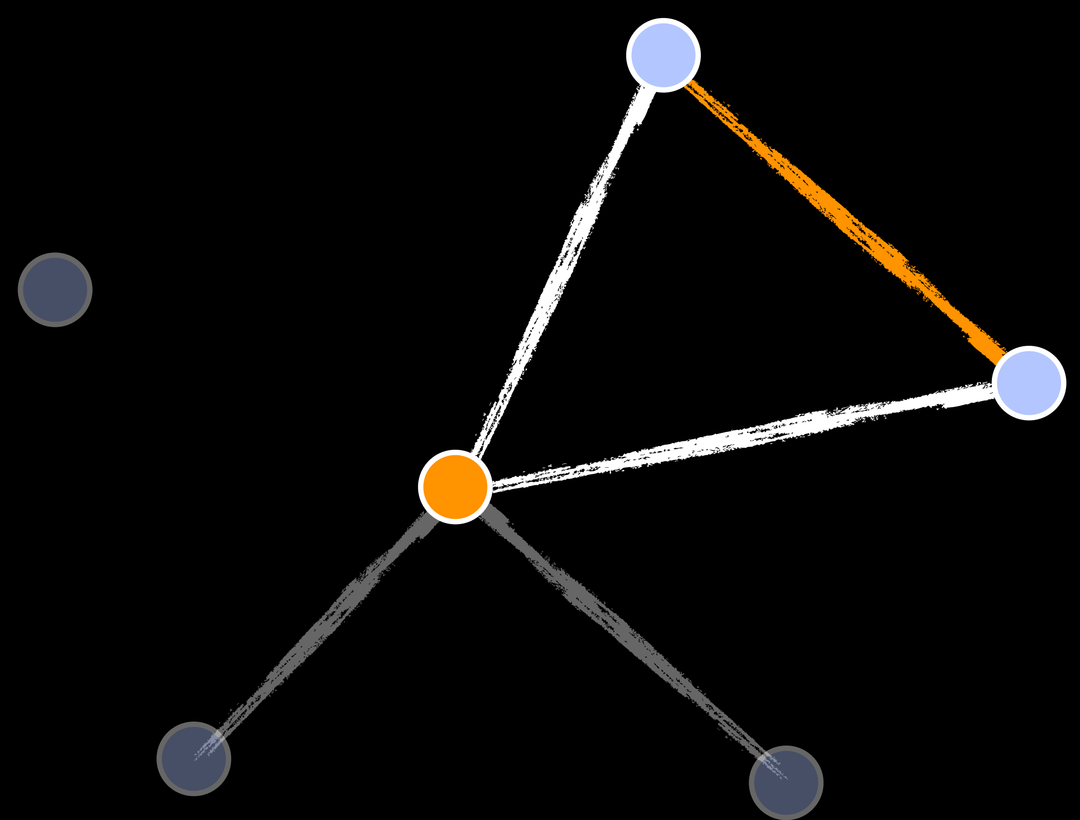
Scale	Network measure	Phase space
Local	link density	global recurrence rate
	degree centrality	local recurrence rate
Intermediate	clustering coefficient	invariant objects, local dimension
	local degree anomaly	local heterogeneity of phase space density
	assortativity	continuity of phase space density
	matching index	twinness
Global	average path length	mean phase space separation
	network diameter	phase space diameter
	closeness centrality	local centeredness in phase space
	betweenness centrality	local attractor fractionation
	global transitivity/ clustering	regular dynamics
	motif distribution	dynamical classification

Clustering Coefficient

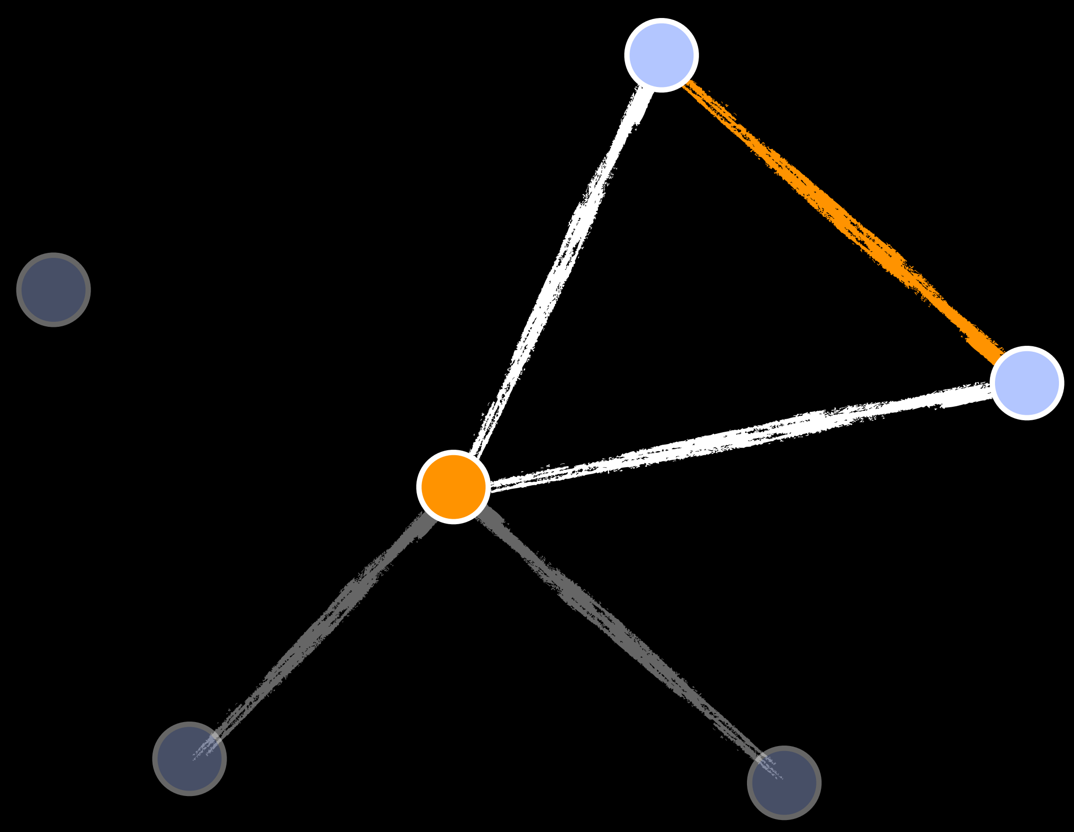


Clustering Coefficient



- probability that neighbours of a node are also connected

Clustering Coefficient

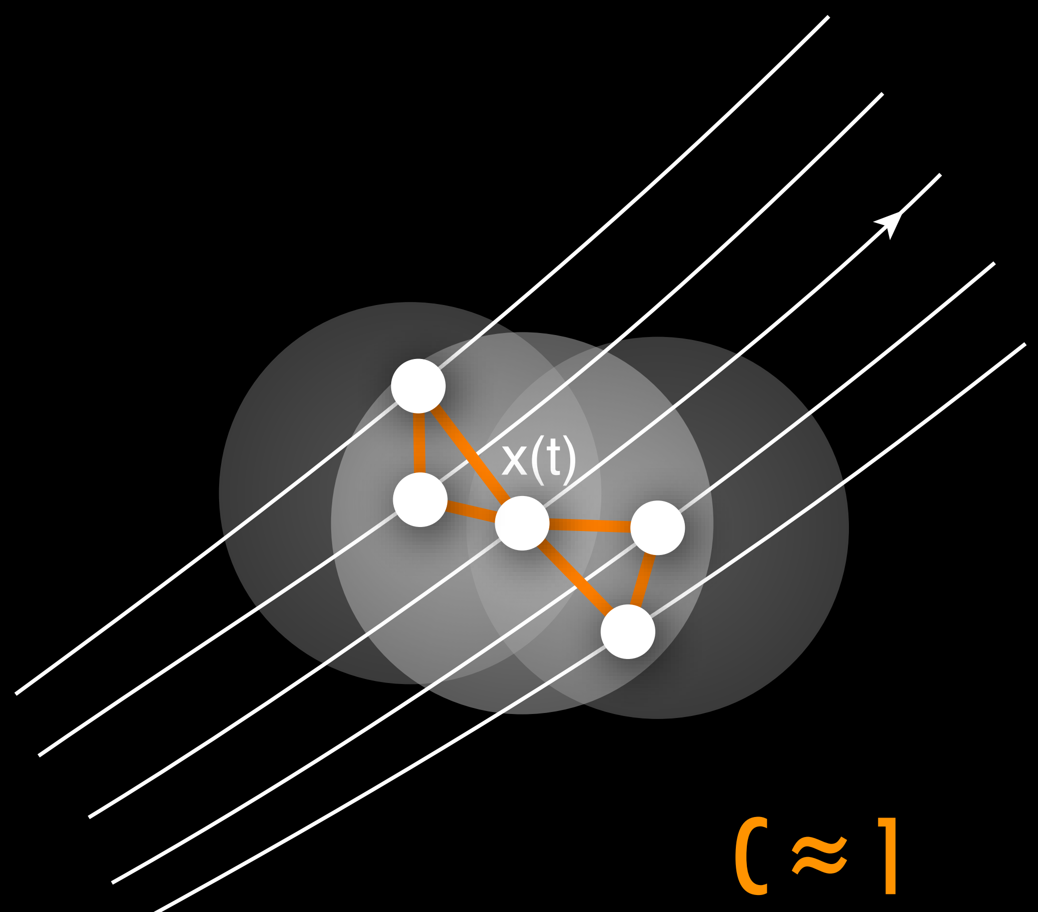


$$C_v = \frac{\sum_{i,j} A_{v,i} A_{i,j} A_{j,v}}{k_v(k_v - 1)}$$

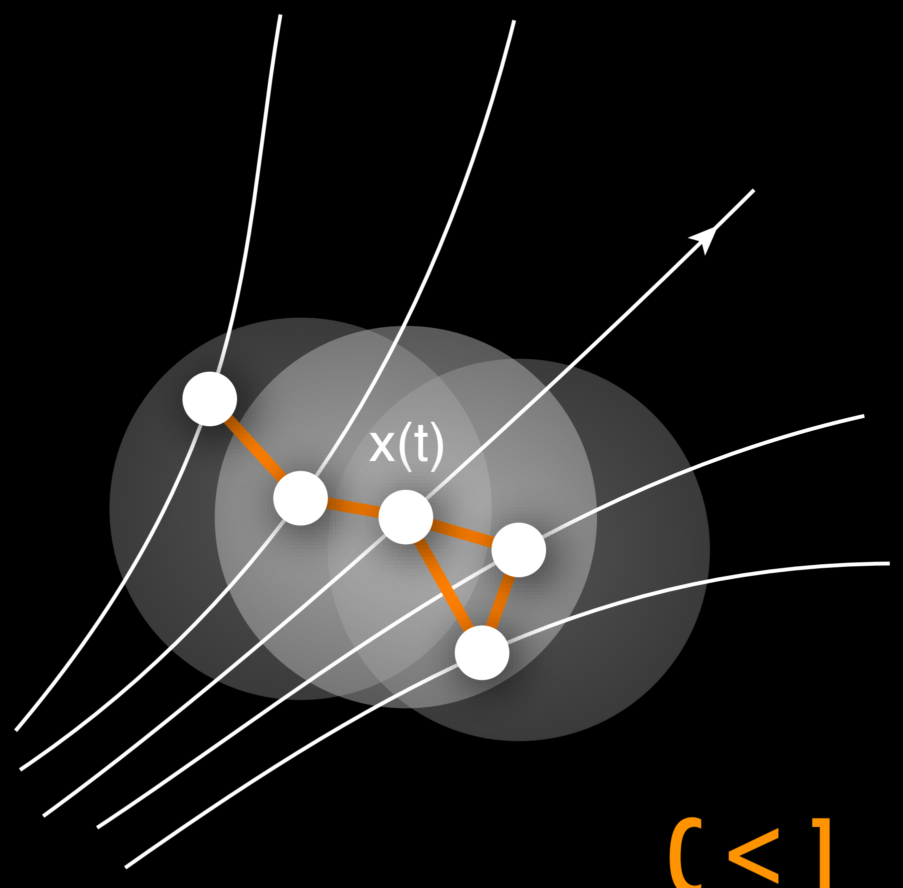
- probability that neighbours of a node are also connected

Clustering Coefficient in Phase Space

Regular/ periodic



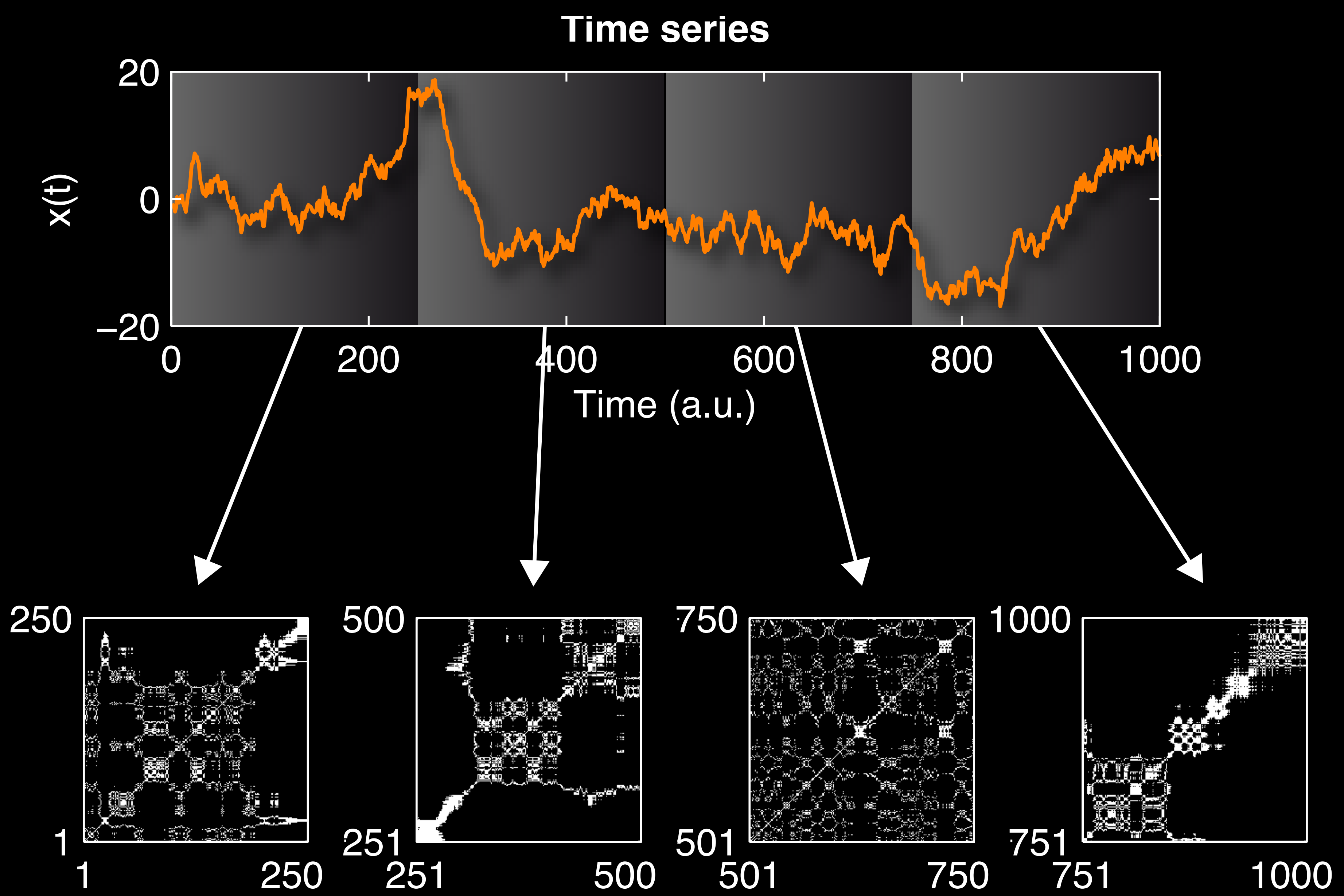
Diverging/ chaotic



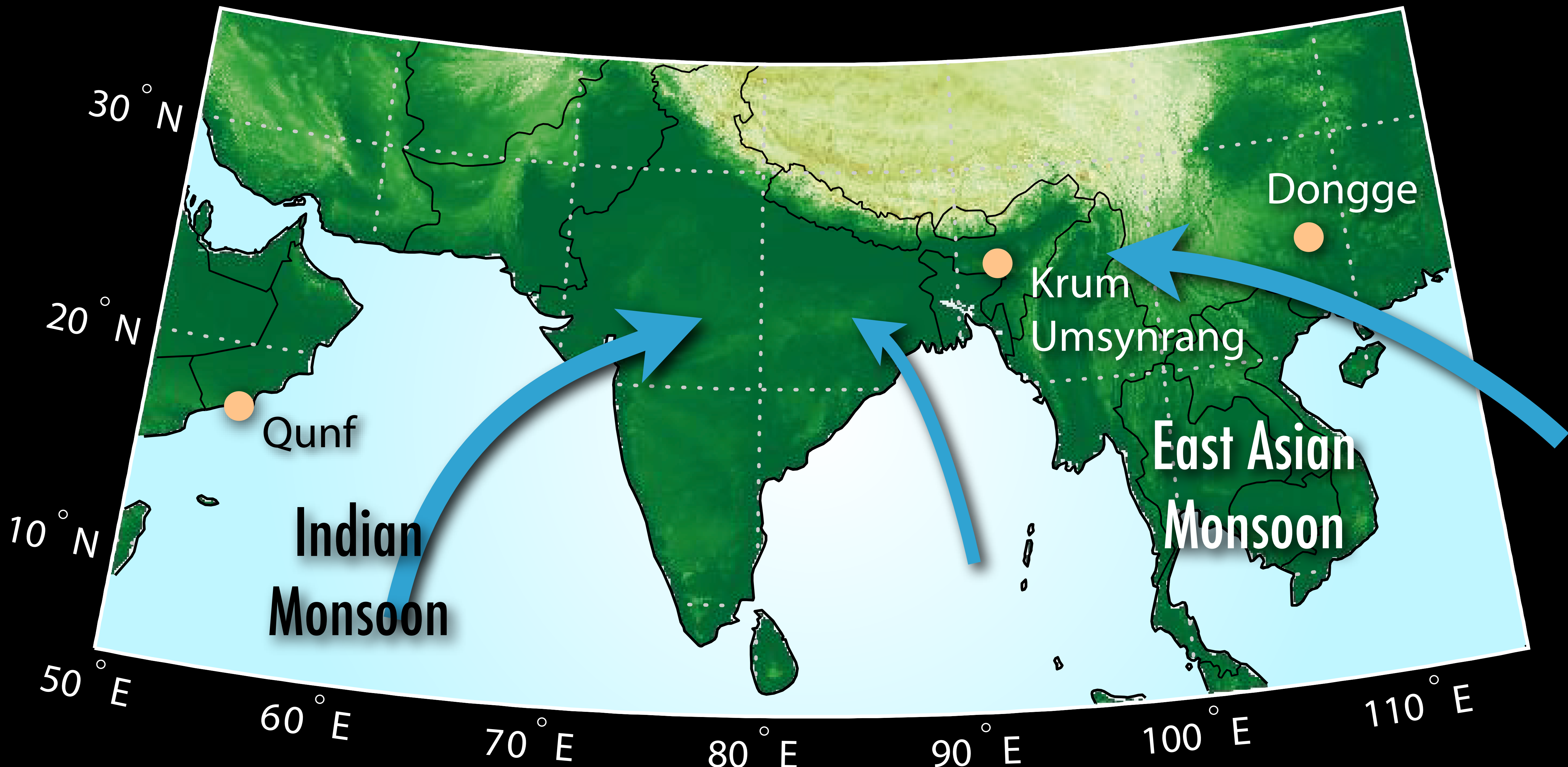
- clustering coefficient: regularity of dynamics, system dimension

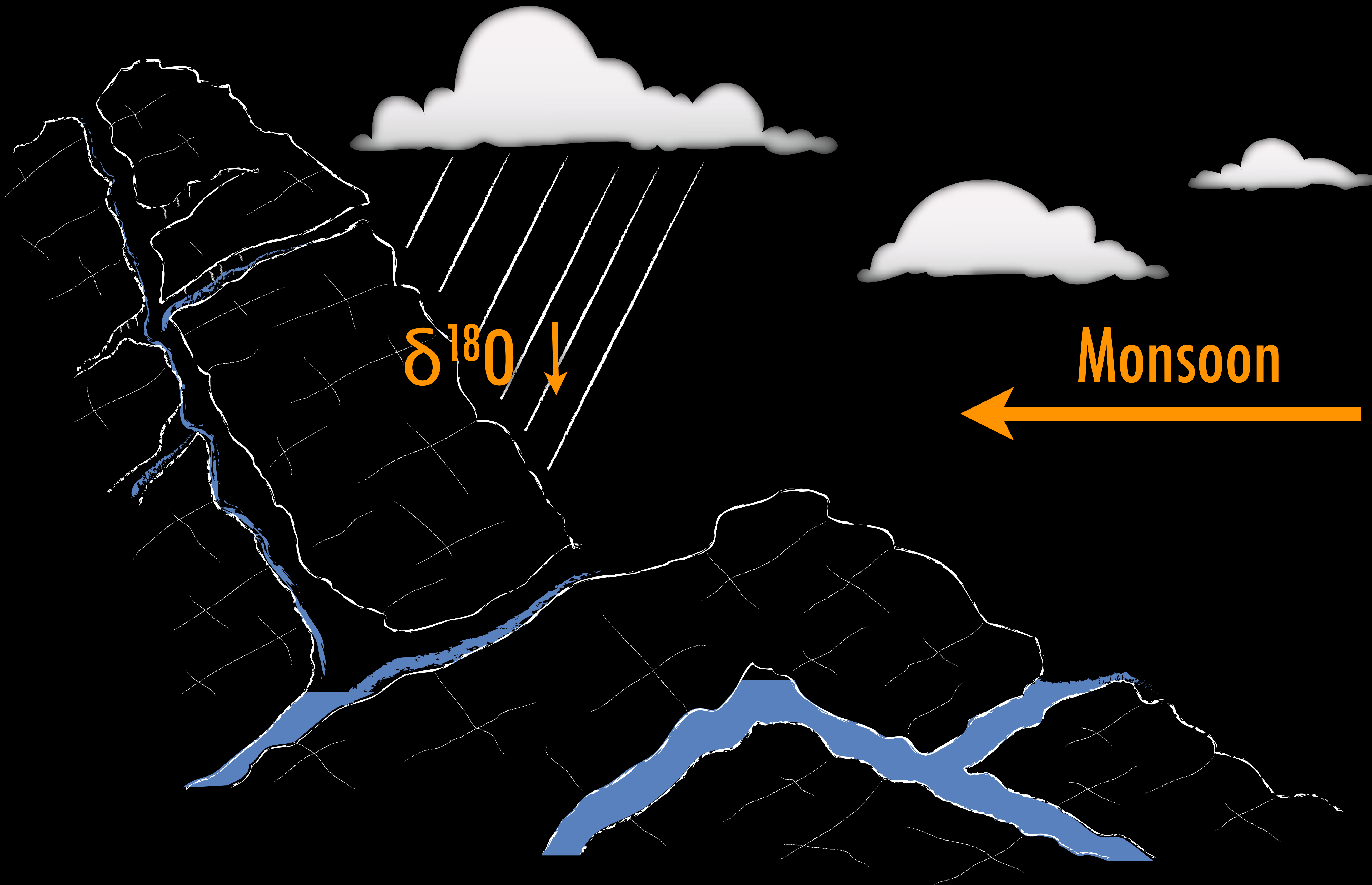
Evolving Complex Networks

- sliding window: detection of dynamical transitions



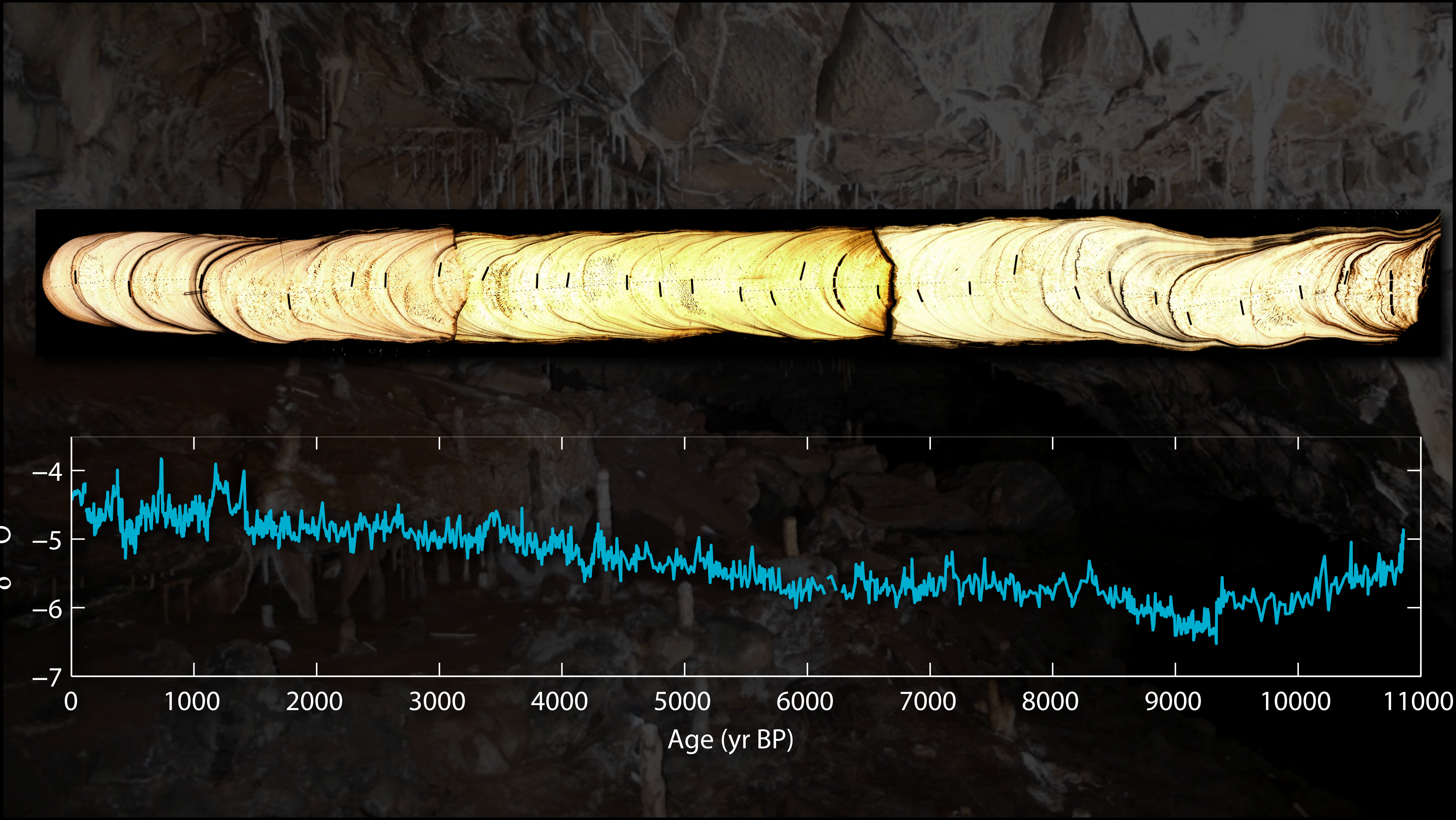
Asian Monsoon




δ18O ↓
Monsoon

Asian Monsoon

Asian Monsoon



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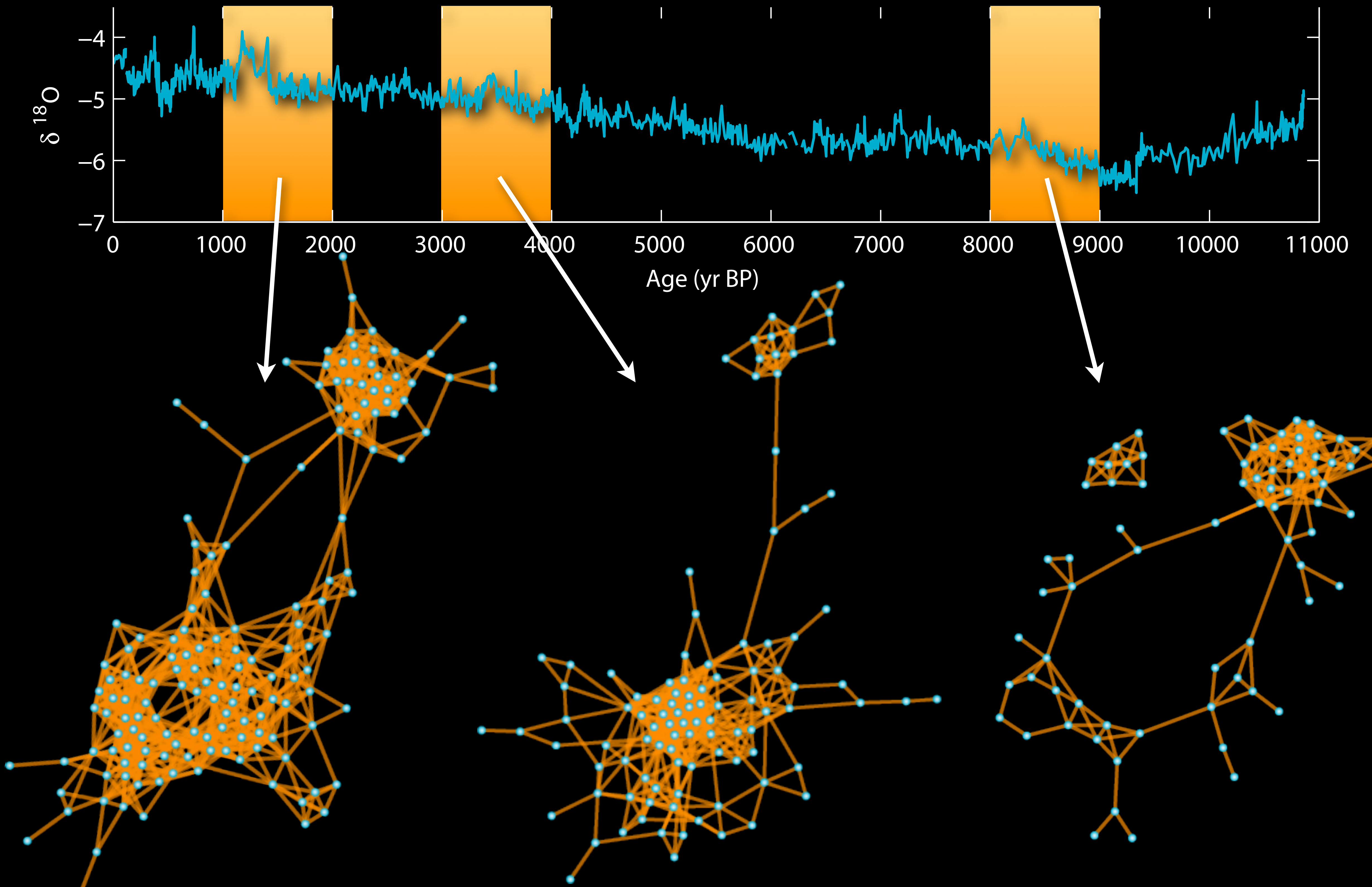
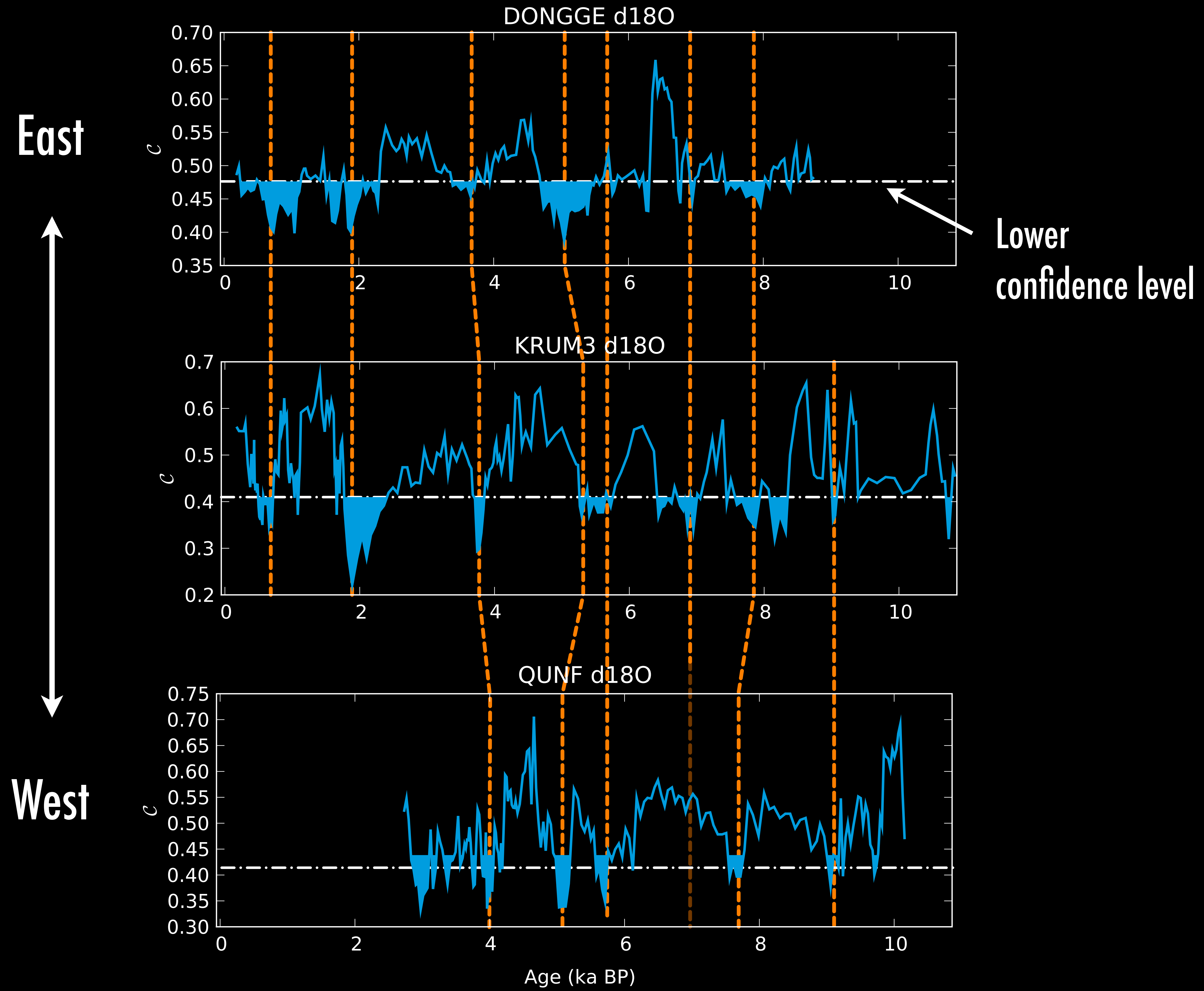
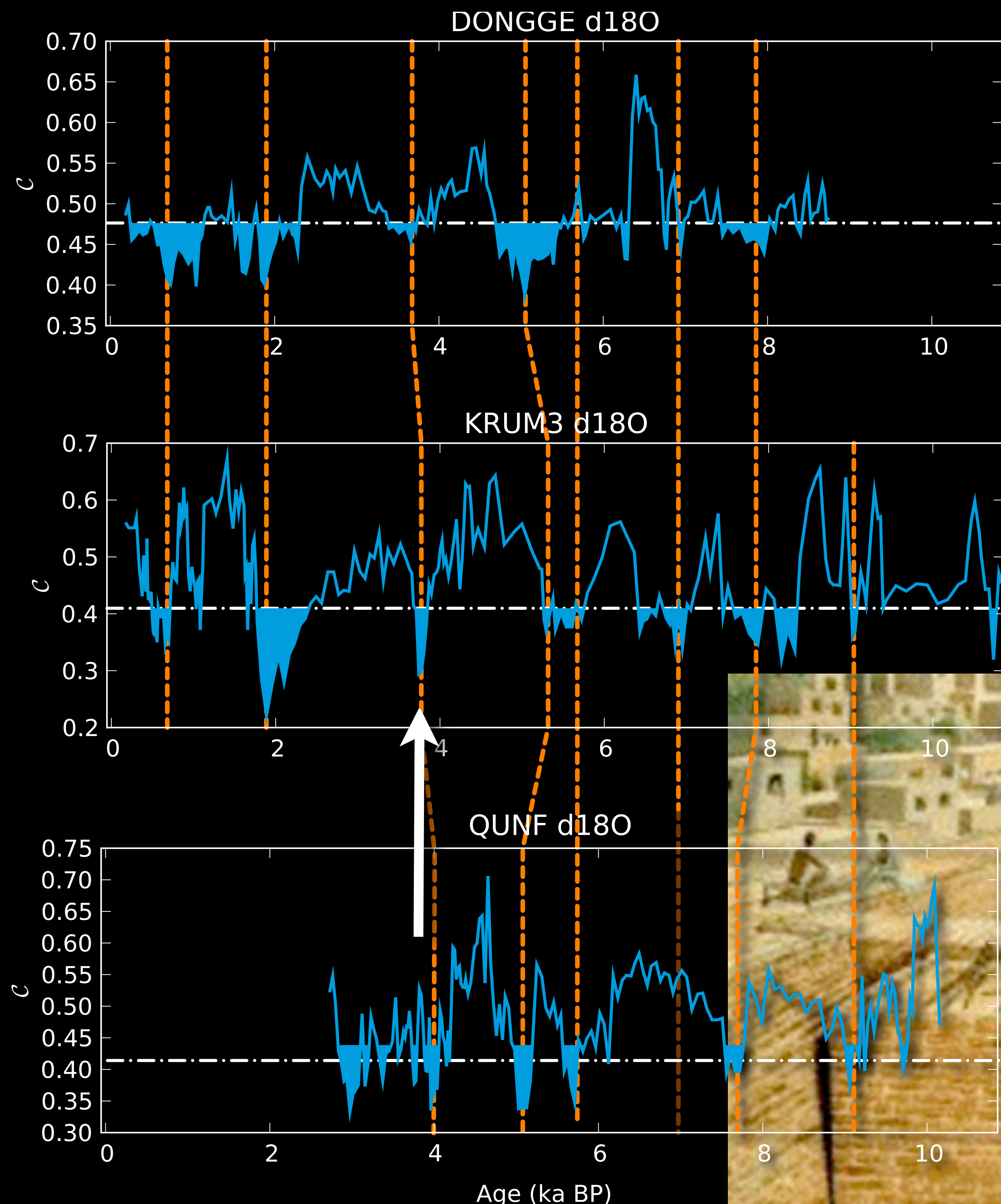



East
West
DONGGE d18O
KRUM3 d18O
QUNF d18O
c
Lower confidence level
Age (ka BP)
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.7
0.6
0.5
0.4
0.3
0.2
0.75
0.30
0
2
4
6
8
10



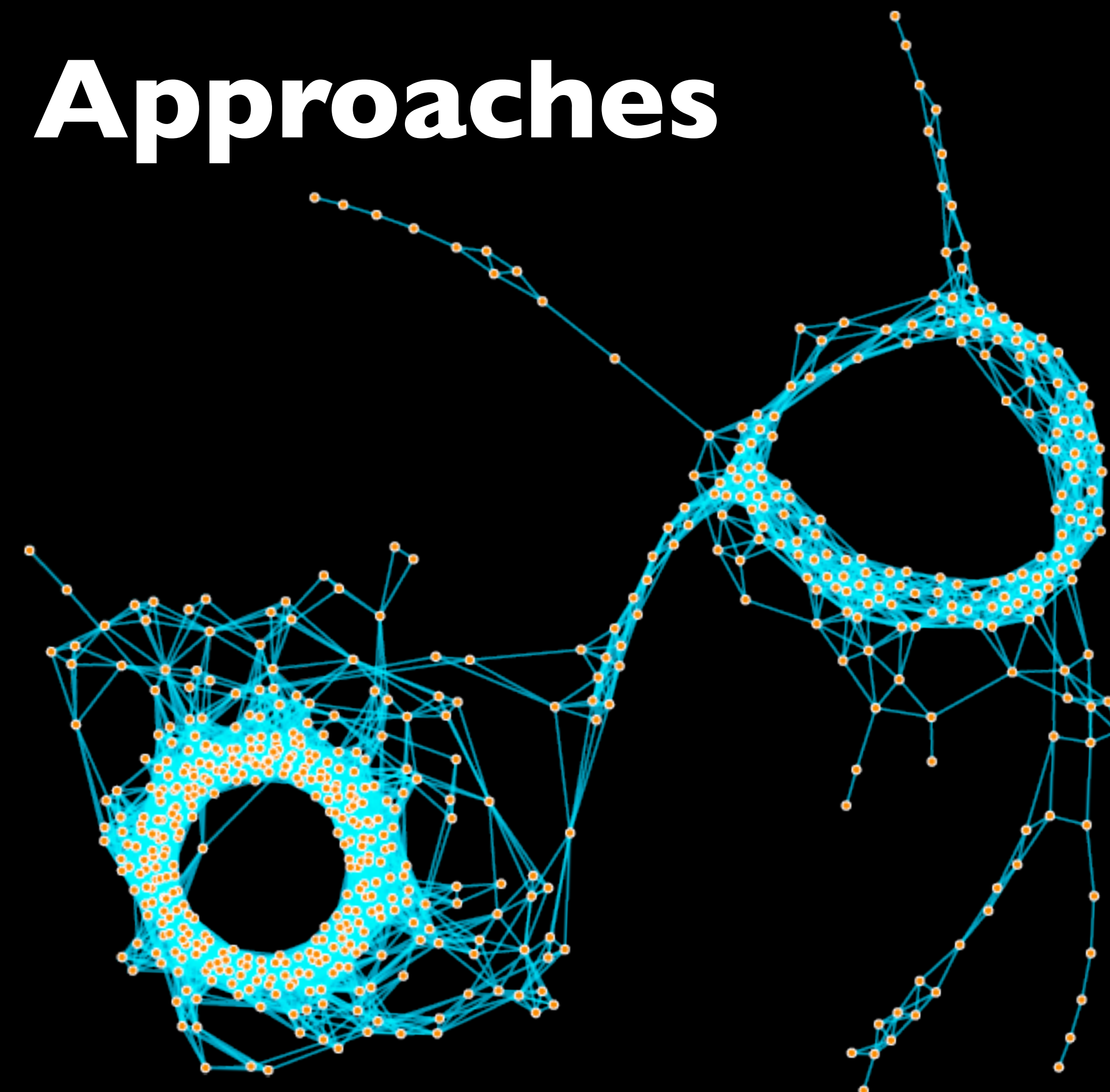
3900-3700 yr BP
Harappan culture vanished

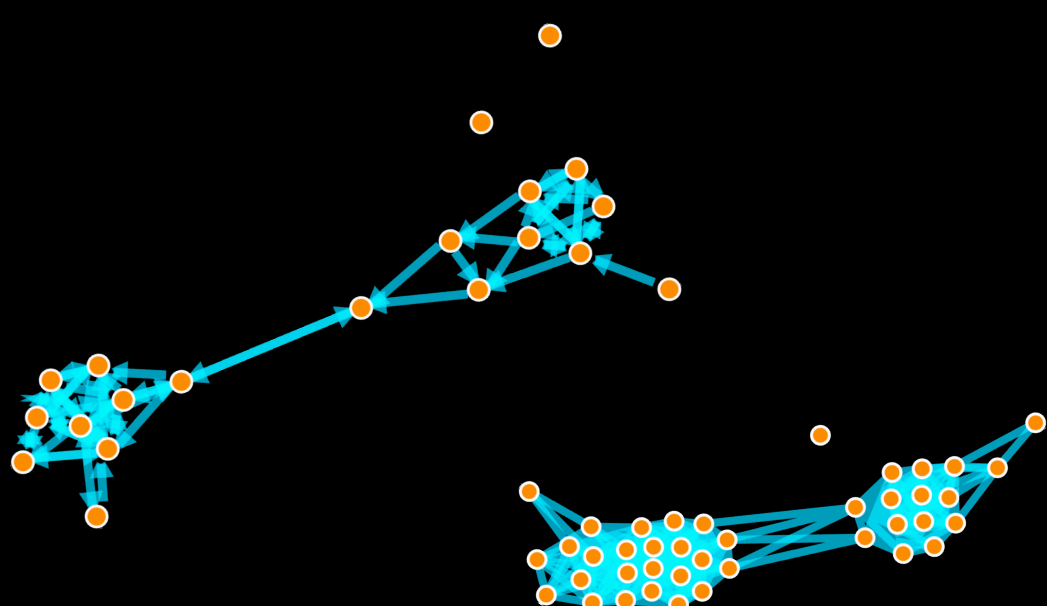
Summary

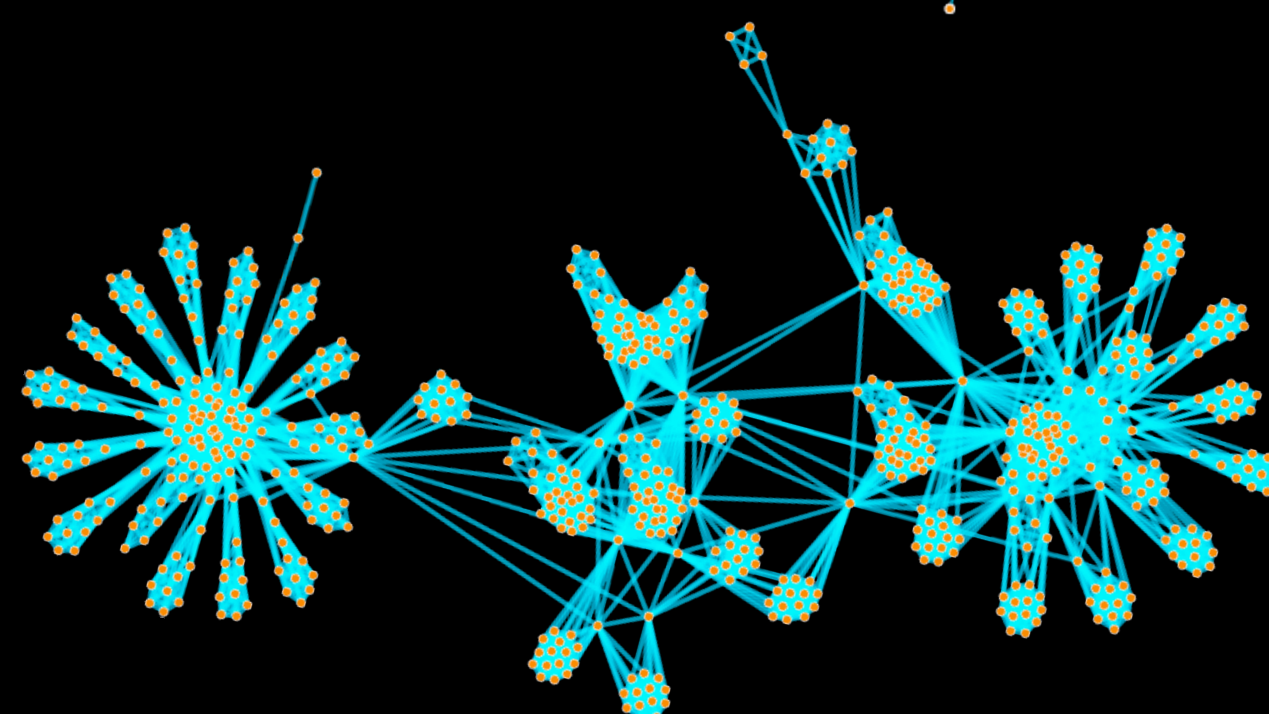
- Complex networks from time series
- Identification and classification of dynamics (regular – chaotic)
- Detection of transitions in dynamics (bifurcations, structural discontinuities)
- Complementary analysis to traditional recurrence analysis

Alternative Approaches

- Visibility graph
- Cycle network
- Correlation network
- Transition network







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Recurrence plots for the analysis of complex systems

Norbert Marwan*, M. Carmen Romano, Marco Thiel, Jürgen Kurths

Nonlinear Dynamics Group, Institute of Physics, University of Potsdam, Potsdam 14415, Germany

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Abstract

Recurrence is a fundamental property of dynamical systems, which can be exploited to characterise the system's behaviour in phase space. A powerful tool for their visualisation and analysis called *recurrence plot* was introduced in the late 1980's. This report is a comprehensive overview covering recurrence based methods and their applications with an emphasis on recent developments. After a brief outline of the theory of recurrences, the basic idea of the recurrence plot with its variations is presented. This includes the quantification of recurrence plots, like the recurrence quantification analysis, which is highly effective to detect, e. g., transitions in the dynamics of systems from time series. A main point is how to link recurrences to dynamical invariants and unstable periodic orbits. This and further evidence suggest that recurrences contain all relevant information about a system's behaviour. As the respective phase spaces of two systems change due to coupling, recurrence plots allow studying and quantifying their interaction. This fact also provides us with a sensitive tool for the study of synchronisation of complex systems. In the last part of the report several applications of recurrence plots in economy, physiology, neuroscience, earth sciences, astrophysics and engineering are shown. The aim of this work is to provide the readers with the know how for the application of recurrence plot based methods in their own field of research. We therefore detail the analysis of data and indicate possible difficulties and pitfalls.

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Recurrence networks—a novel paradigm for nonlinear time series analysis

Reik V Donner[1,2,3,4,6], **Yong Zou**[3], **Jonathan F Donges**[3,5], **Norbert Marwan**[3] **and Jürgen Kurths**[3,5]

[1] Max Planck Institute for Physics of Complex Systems, Dresden, Germany
[2] Institute for Transport and Economics, Dresden University of Technology, Germany
[3] Potsdam Institute for Climate Impact Research, Potsdam, Germany
[4] Graduate School of Science, Osaka Prefecture University, Sakai, Japan
[5] Institute for Physics, Humboldt University of Berlin, Berlin, Germany
E-mail: donner@vwi.tu-dresden.de

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Abstract. This paper presents a new approach for analysing the structural properties of time series from complex systems. Starting from the concept of recurrences in phase space, the recurrence matrix of a time series is interpreted as the adjacency matrix of an associated complex network, which links different