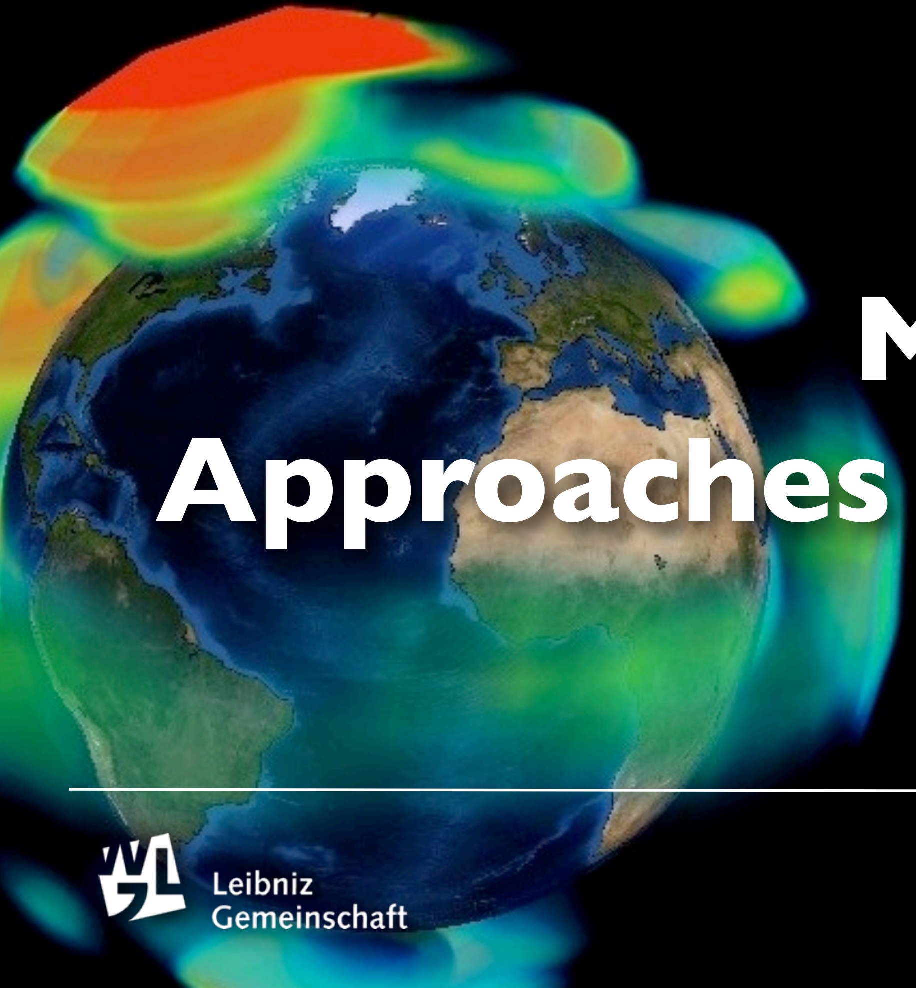
PIK
POTSDAM INSTITUTE FOR
CLIMATE IMPACT RESEARCH

Norbert Marwan

# Modern Nonlinear Approaches for Data Analysis



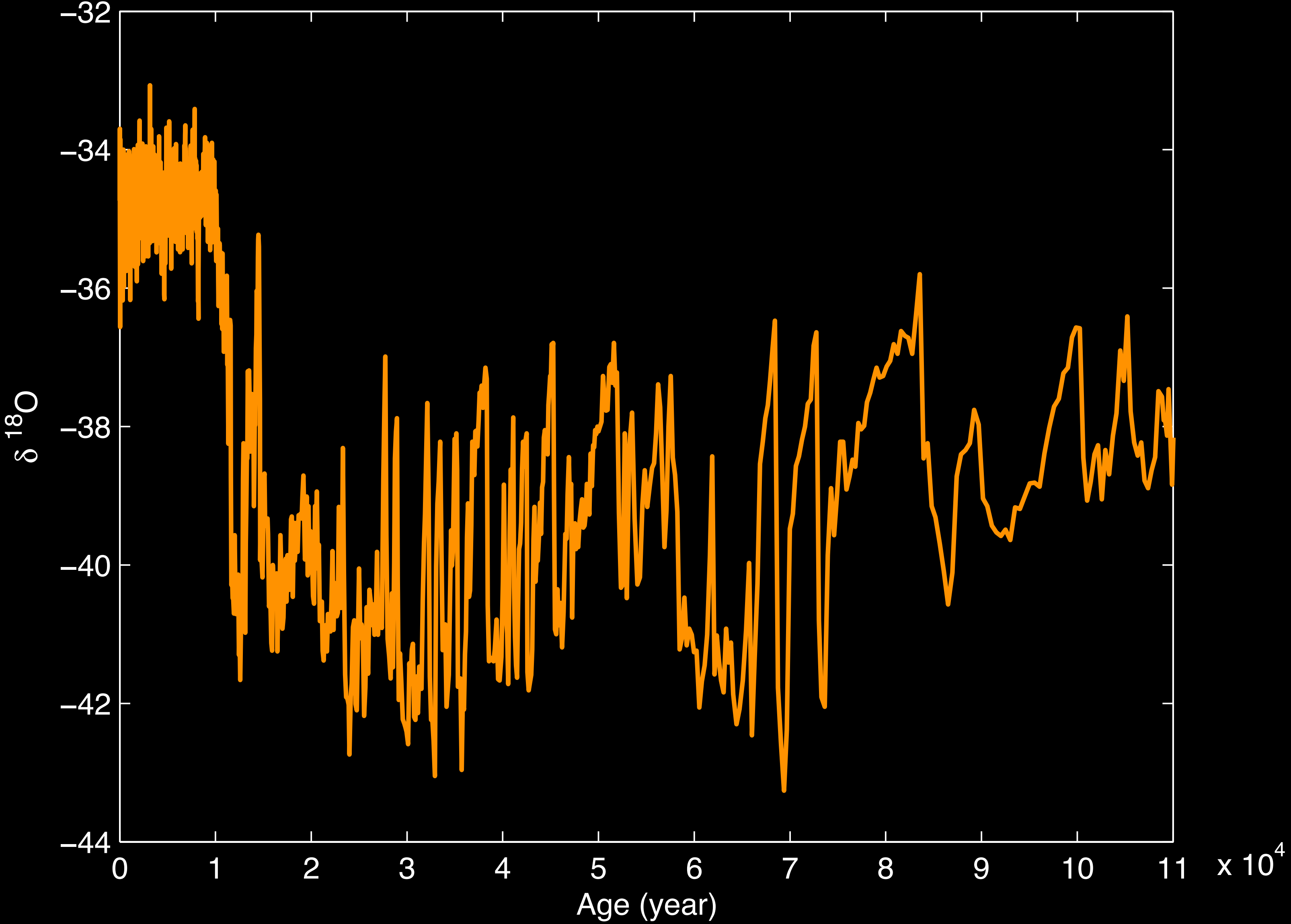
Leibniz
Gemeinschaft

# Literature

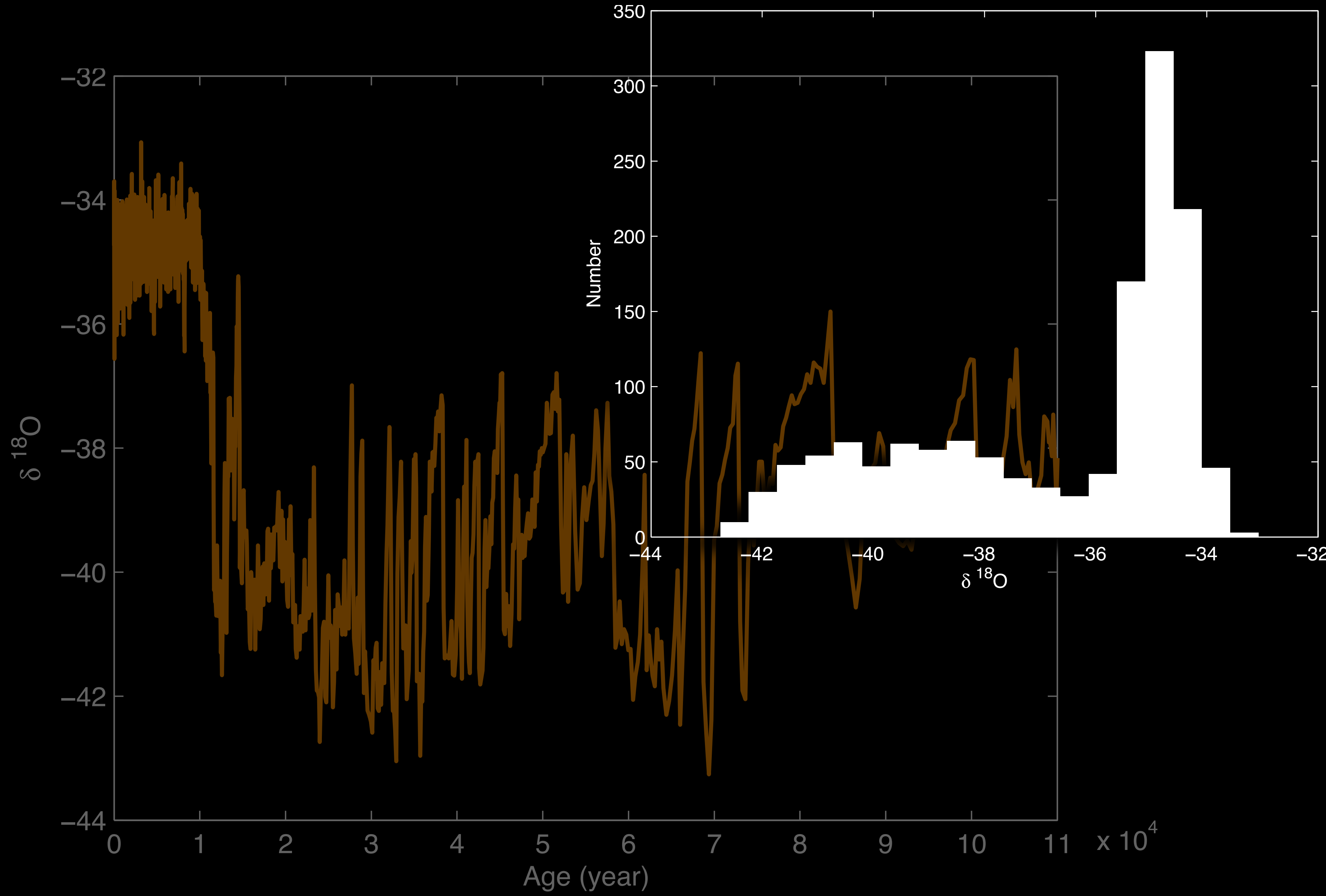
- Kantz, Schreiber: Nonlinear Time Series Analysis, Cambridge University Press
- Sprott: Chaos and time-series analysis, Oxford University Press
- Tong: Non-linear time series, Oxford University Press
- Trauth: Matlab Recipes for Earth Sciences, Springer
- Kruhl: Fractals and Dynamic Systems in Geoscience, Springer
- Middleton, Plotnick, Rubin: Nonlinear Dynamics and Fractals - New Numerical Techniques for Sedimentary Data, SEPM Short Course No. 36

# Circulation, Turbulence

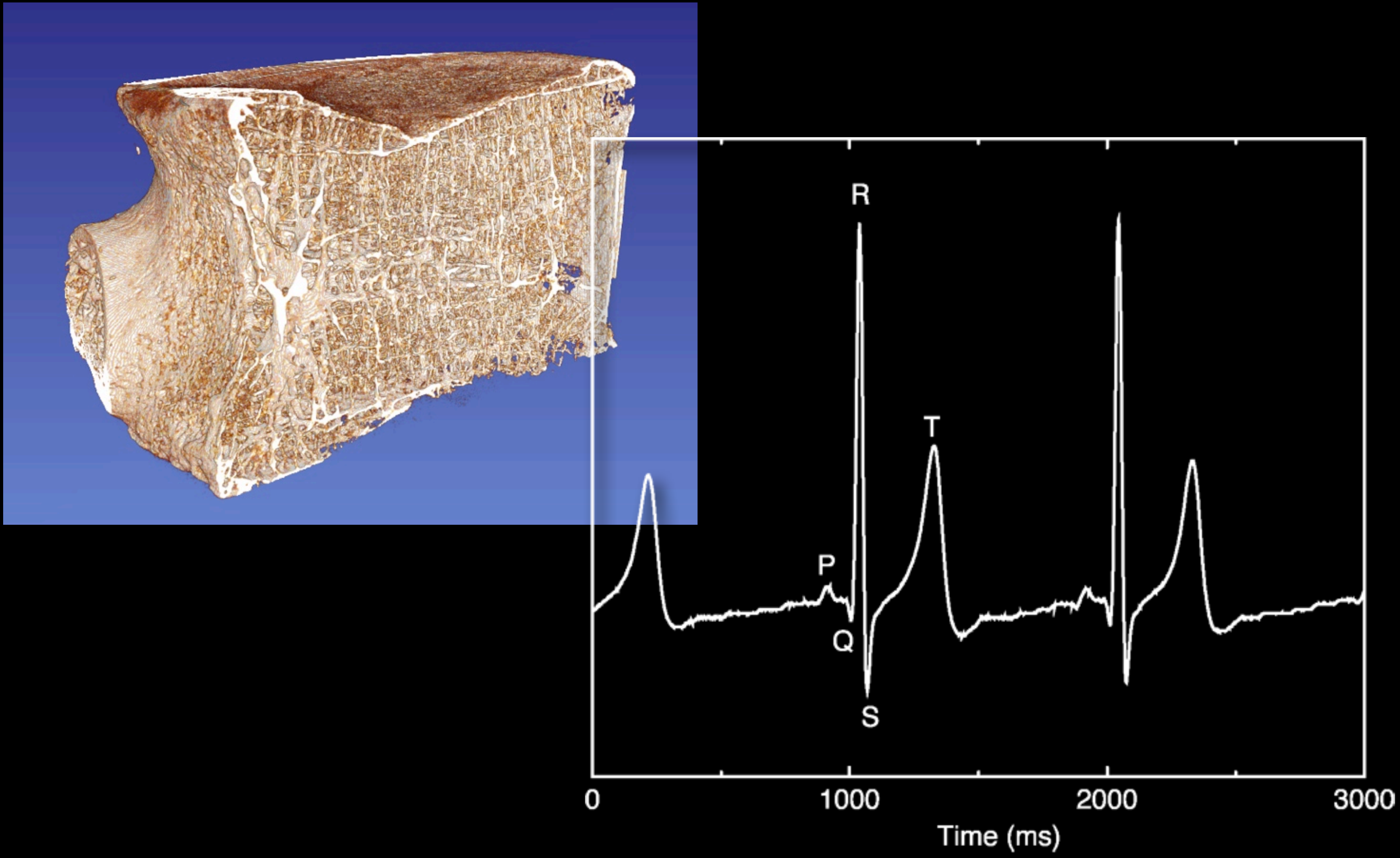
# Palaeo-climate Proxy Records



# Palaeo-climate Proxy Records

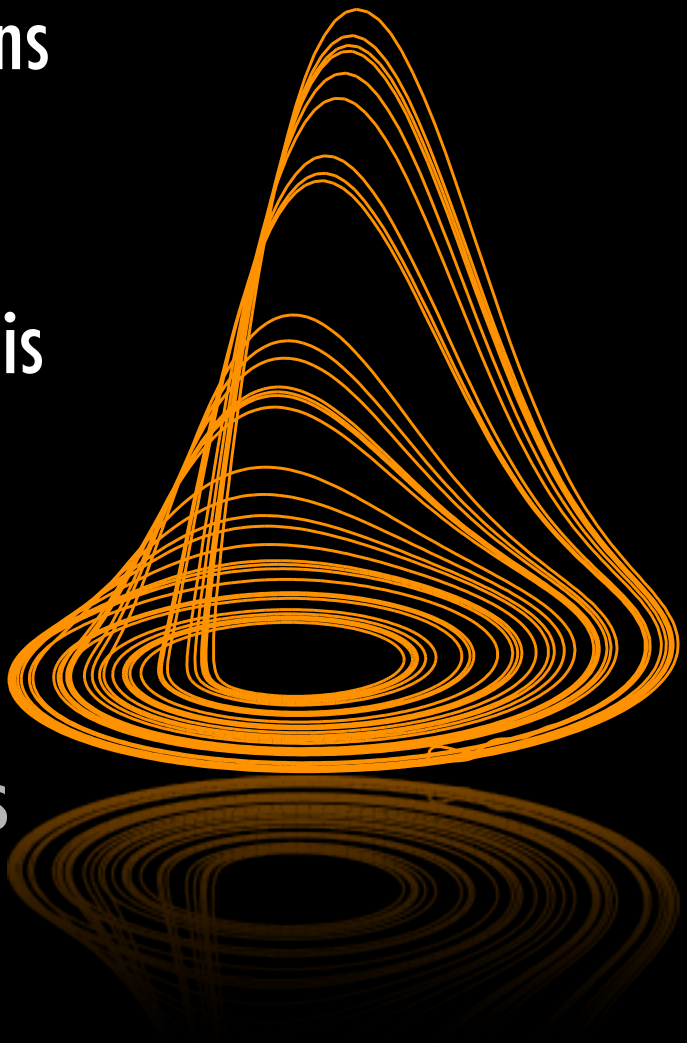


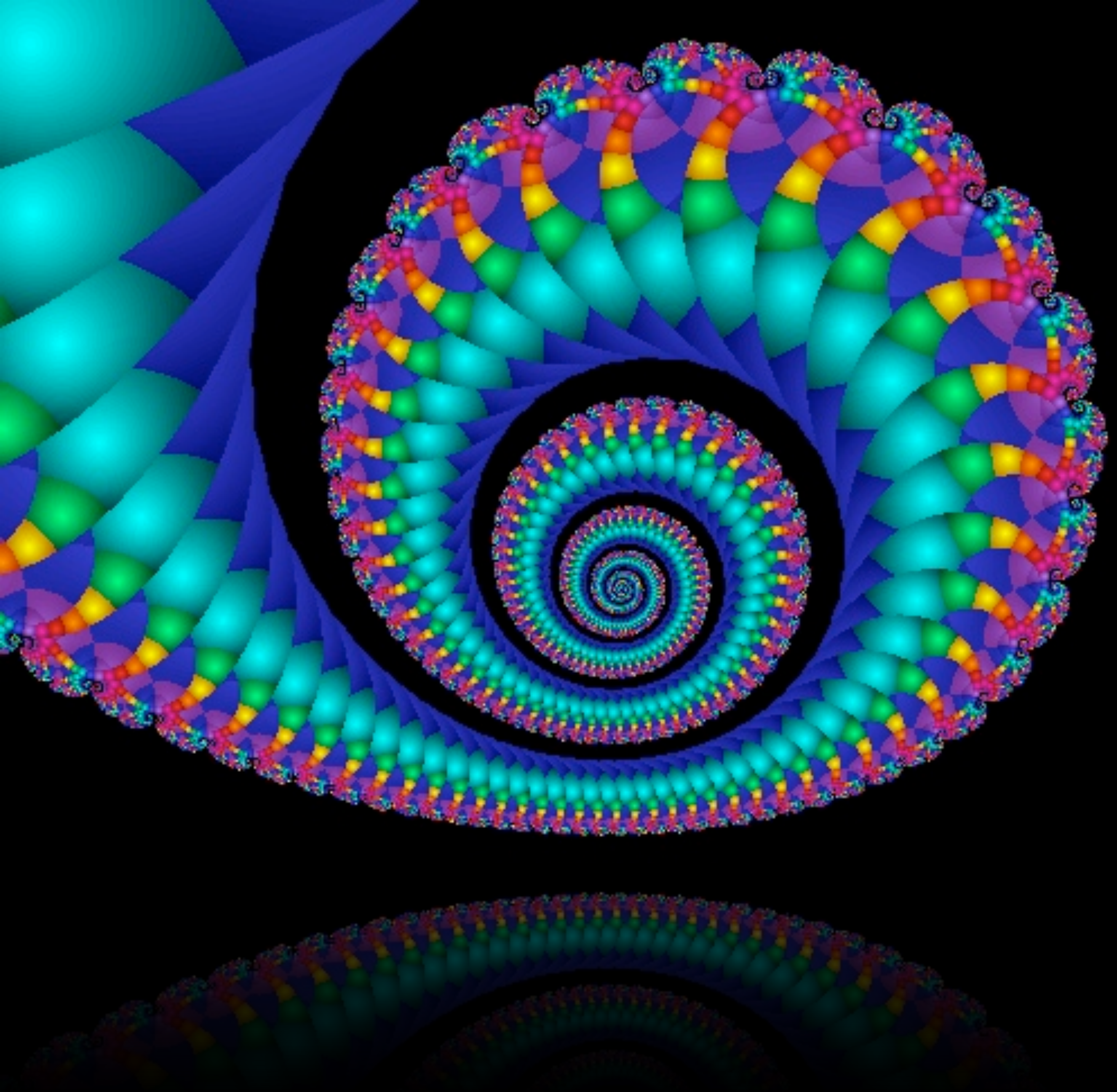
# Complex Biological Structures



# Concepts of Nonlinear Data Analysis

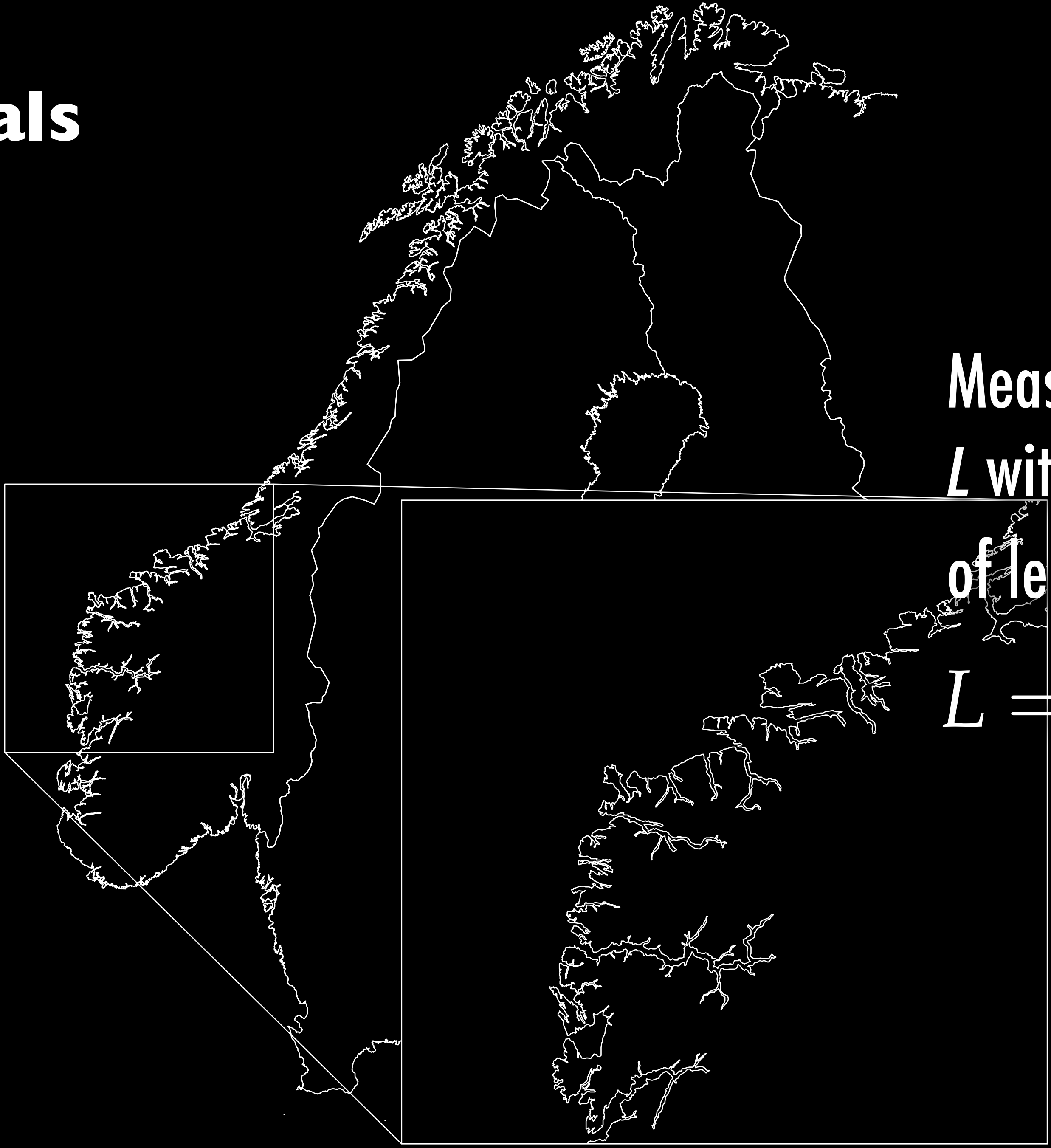
- Fractals, self-similarity & dimensions
- Symbolic dynamics
- Phase space and recurrence analysis
- Complex networks
- Synchronisation & interrelations
- Modes, decompositions, frequences





# Fractals and Dimensions

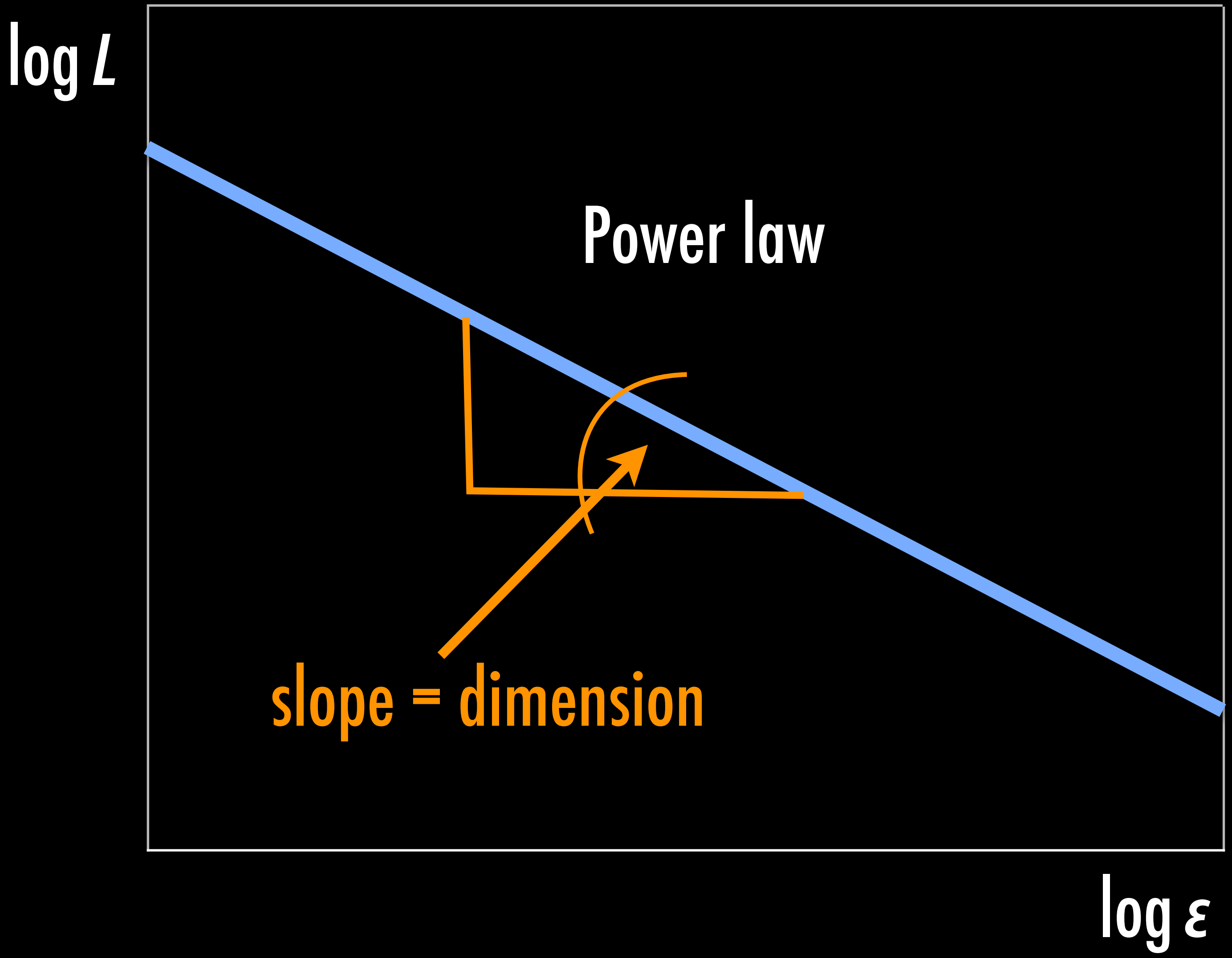
# Fractals



Measure the length *L* with a certain ruler of length *ε*

$$L = N \cdot \varepsilon$$

# Fractals



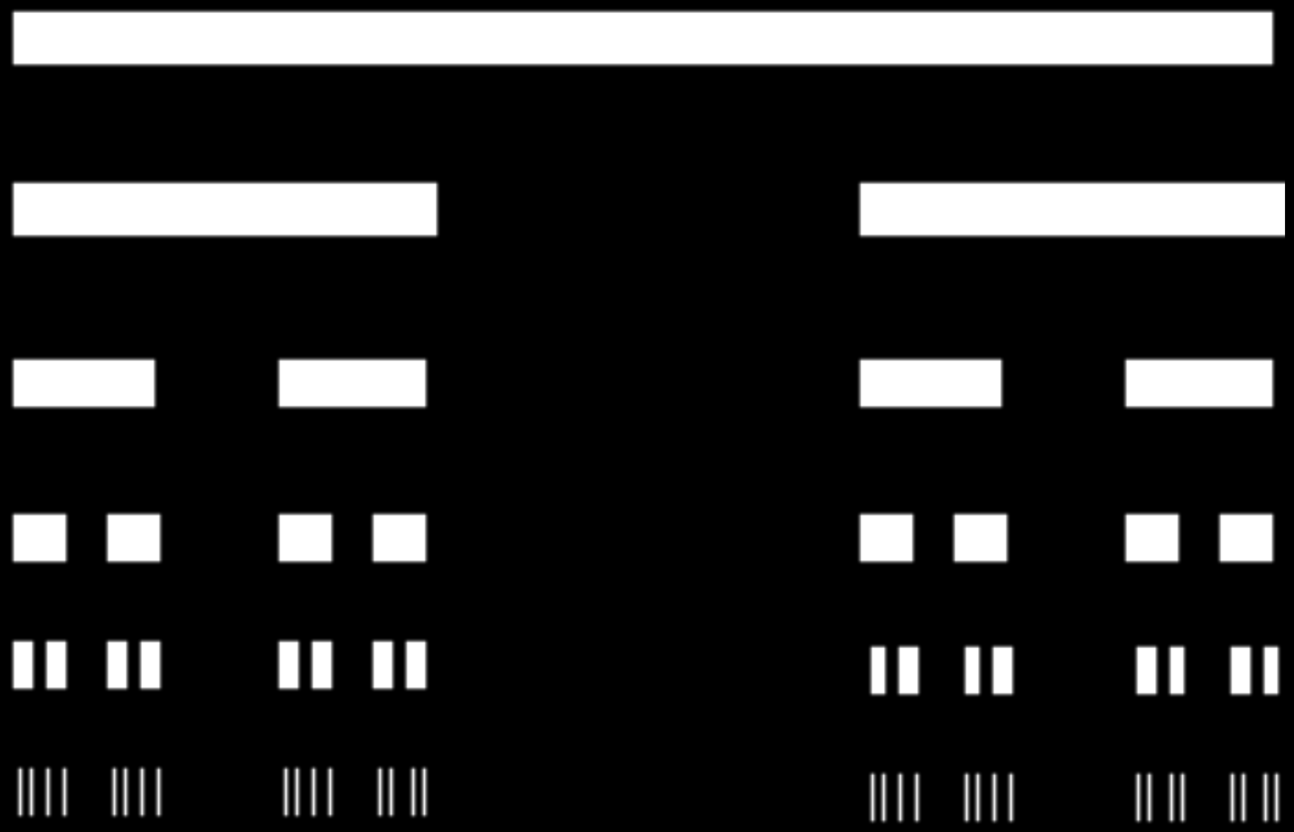
Measure the length *L* with a certain ruler of length ε

$$L = N \cdot \varepsilon$$

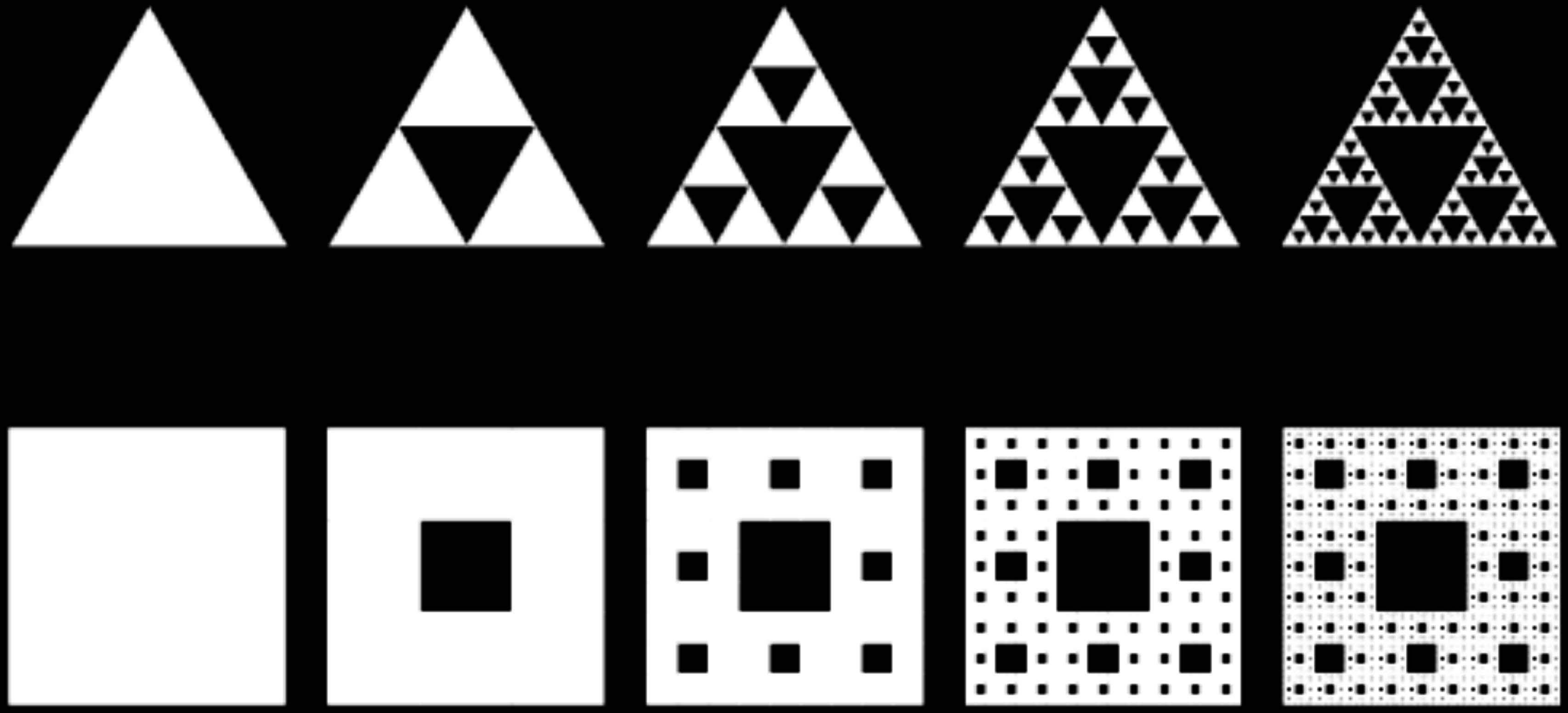
$$N \sim \varepsilon^{-D}$$

Fractal dimension

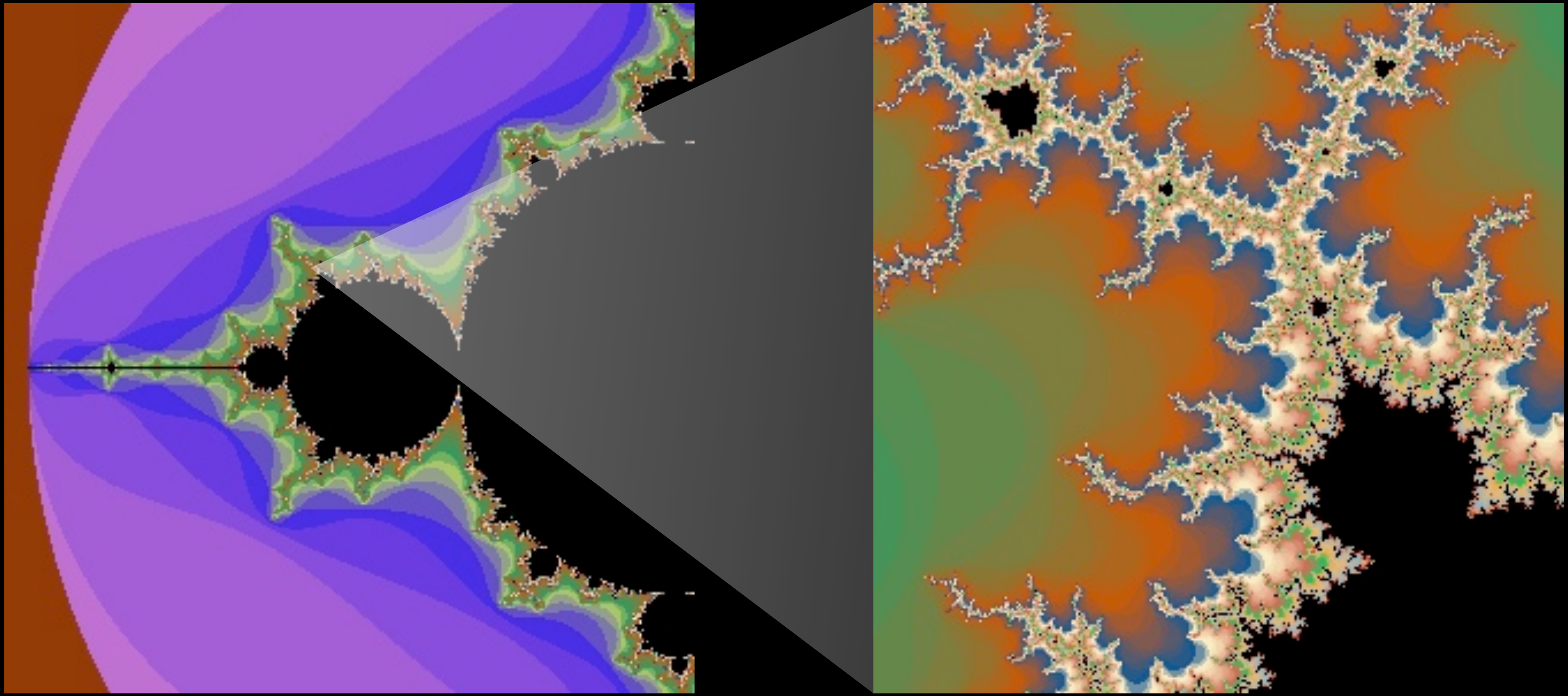
# Cantor Dust

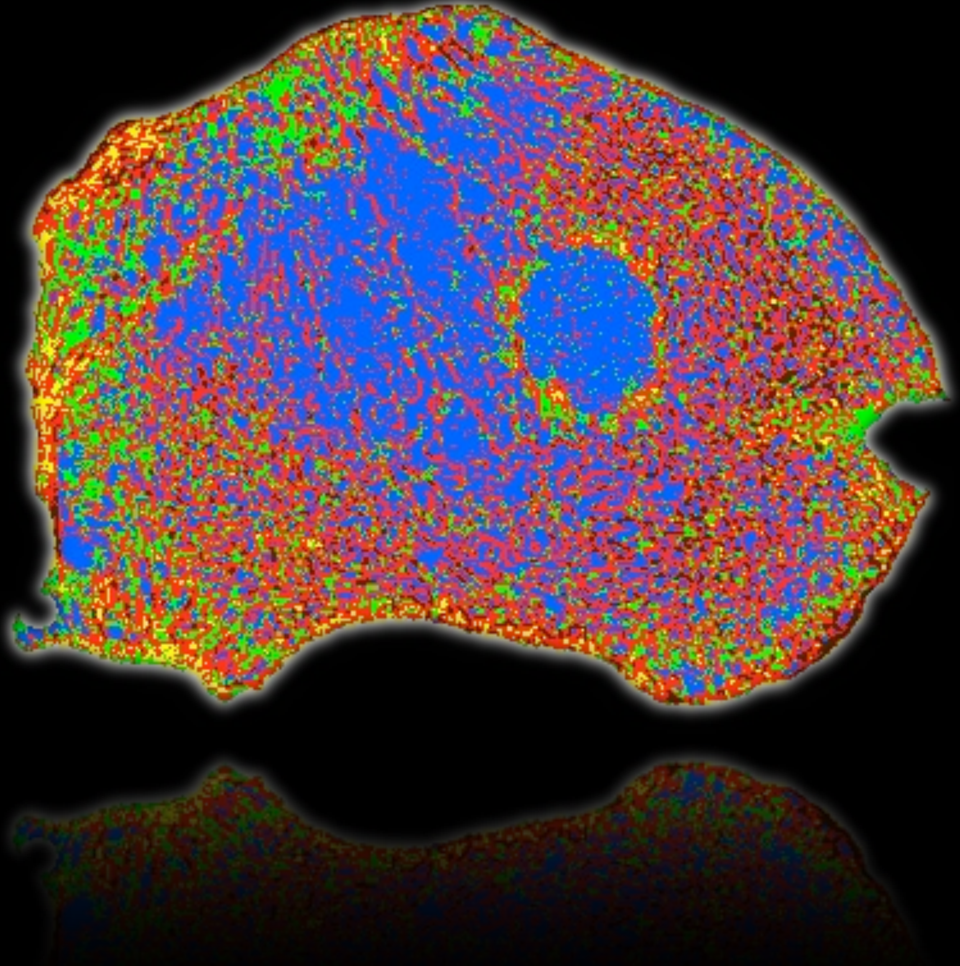


# Sierpinsky Gasket/Carpet

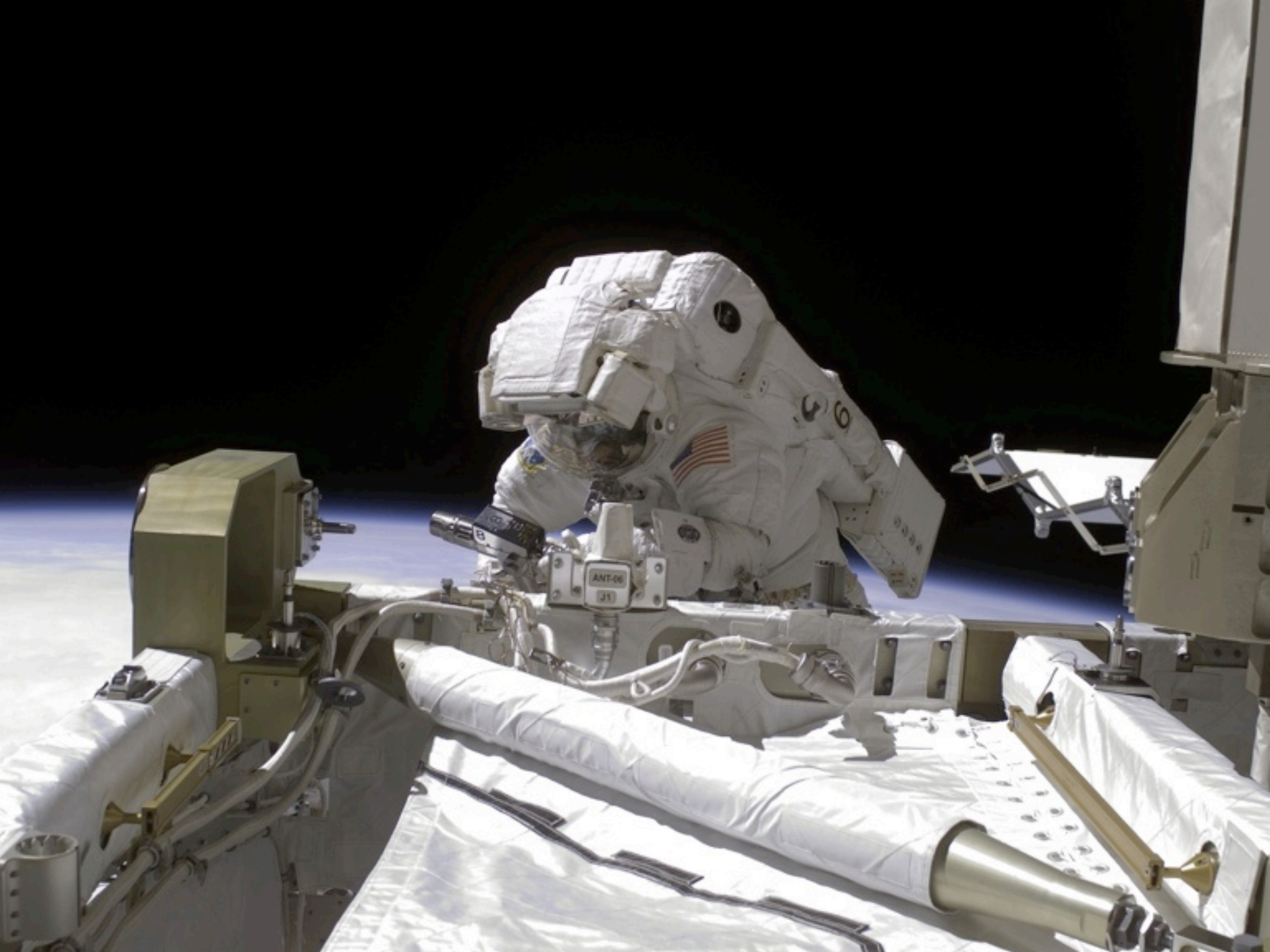


# Mandelbrodt Fractal





# Symbolic Dynamics


ANT-06
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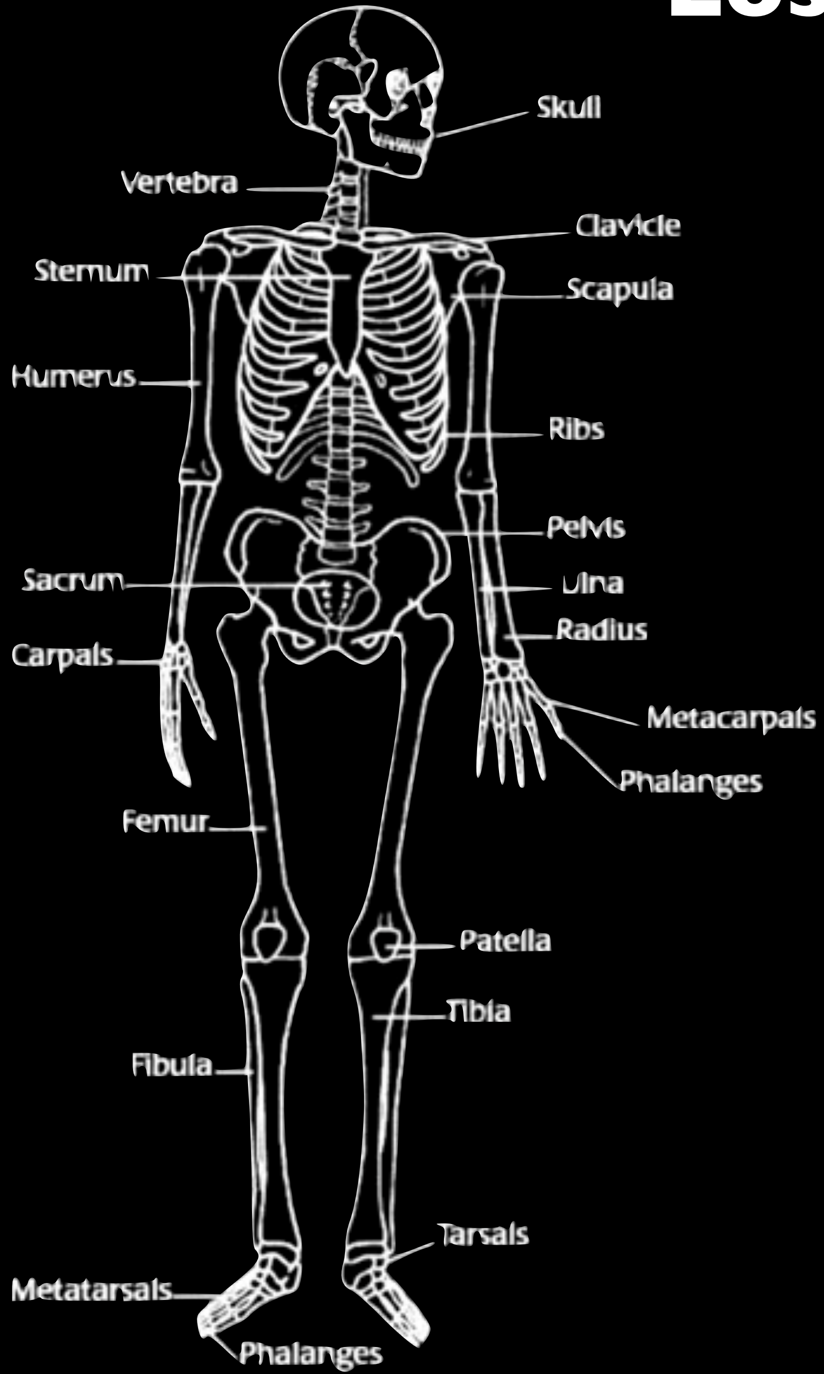
# Bone Loss in Space

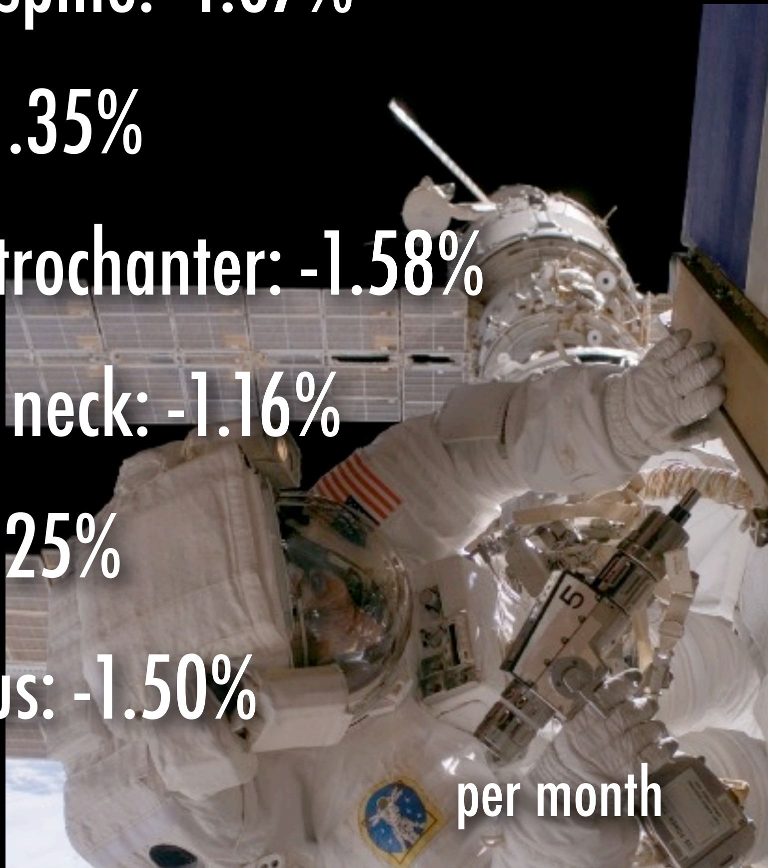
- 2nd important problem after radiation for the manned (long term) spaceflight
- bone loss in space: 1.5% per month

© Courtesy of NASA, 2005

# Lost in Space



- Skull: +0.6%
- Arm: +0.1%
- Lumbar spine: -1.07%
- Pelvis: -1.35%
- Greater trochanter: -1.58%
- Femoral neck: -1.16%
- Tibia: -1.25%
- Calcaneus: -1.50%



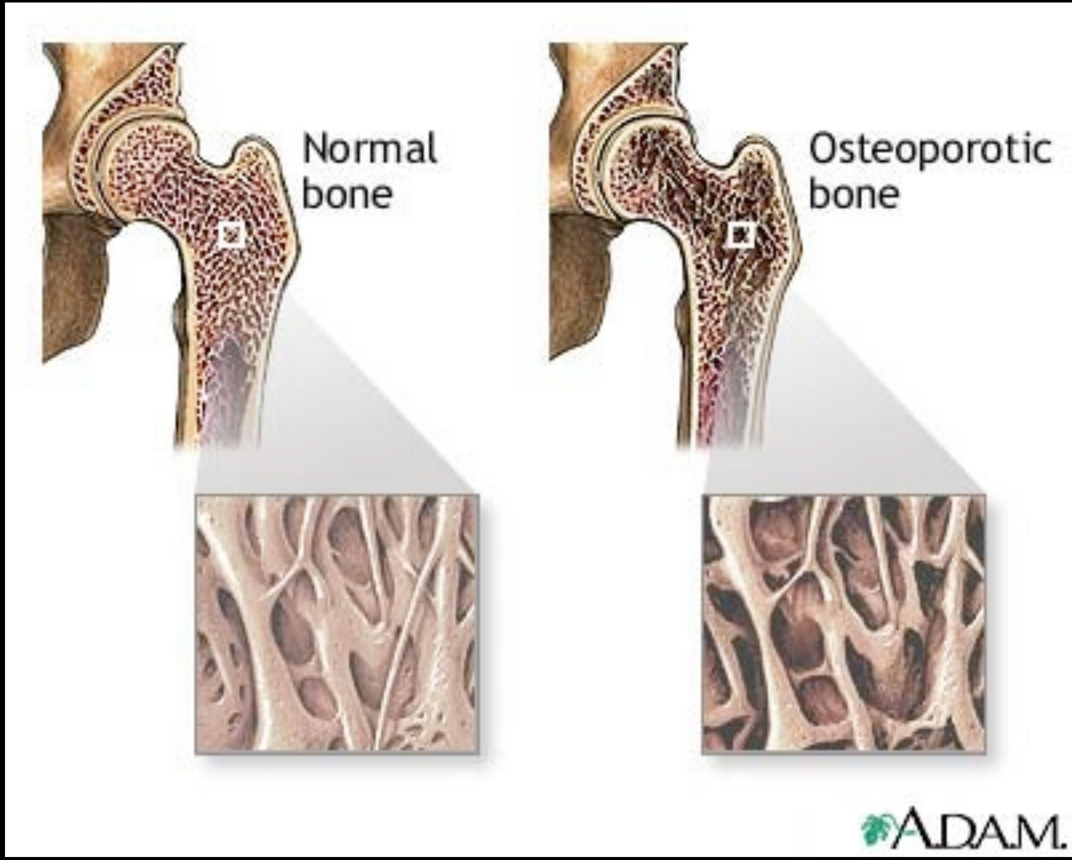
per month

# Reduce Fracture Risk

- monitoring bone alterations during space flights
- exercises
- medical countermeasures

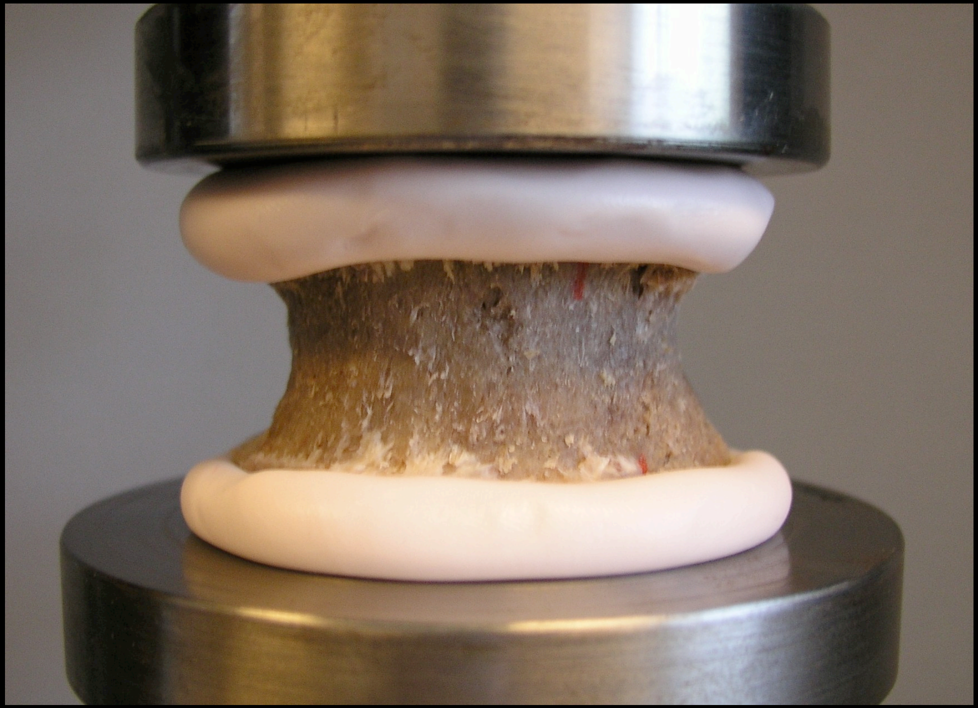
# Osteoporosis

- global problem
- more than 50 % of post-menopausal woman
- fractures occur in up to 16 % of the women



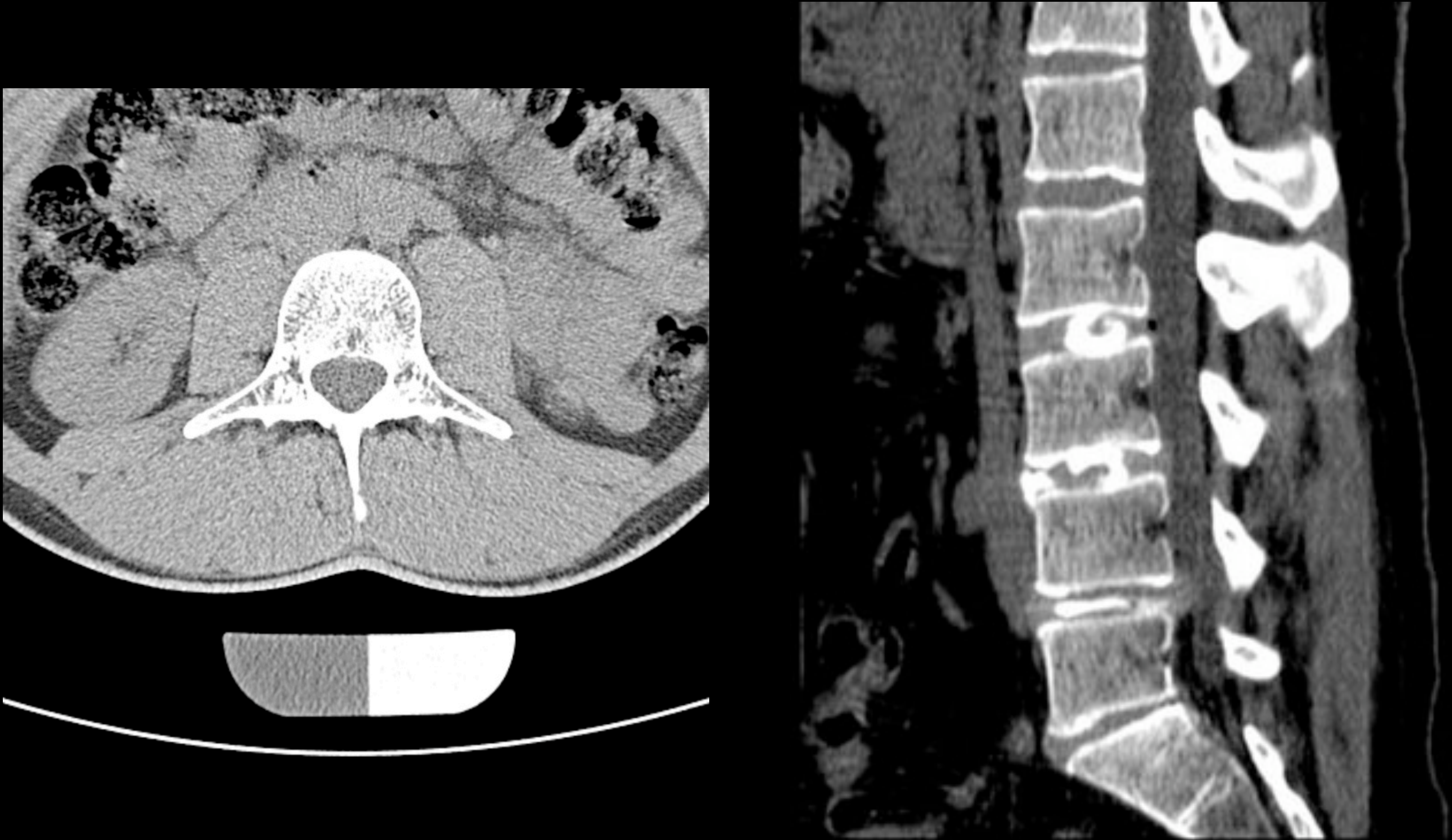
# Bone Strength

- bone composition
- bone density
- internal bone structure



# Diagnosis

Bone Mineral Density (BMD)



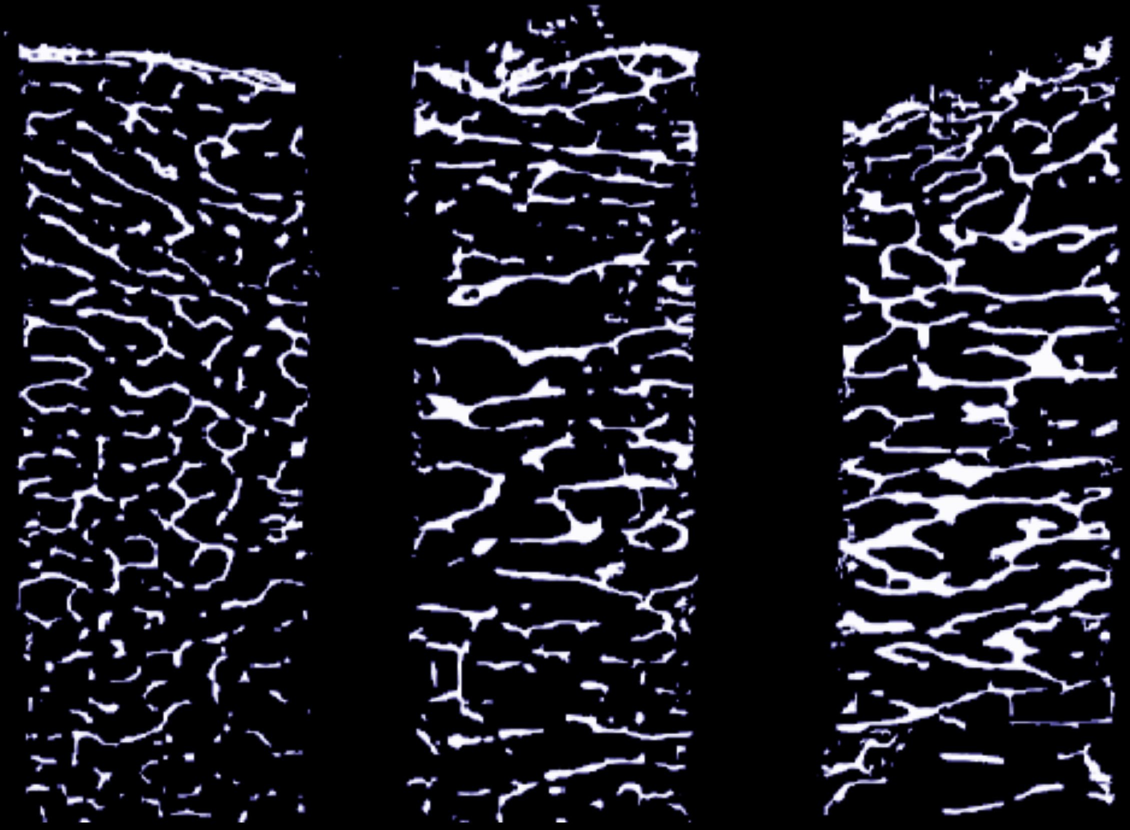
# Trabecular Bone Structure

- plays important role for bone strength
- changes during development of osteoporosis or in microgravity

# Structural Quantification

## Histomorphometry

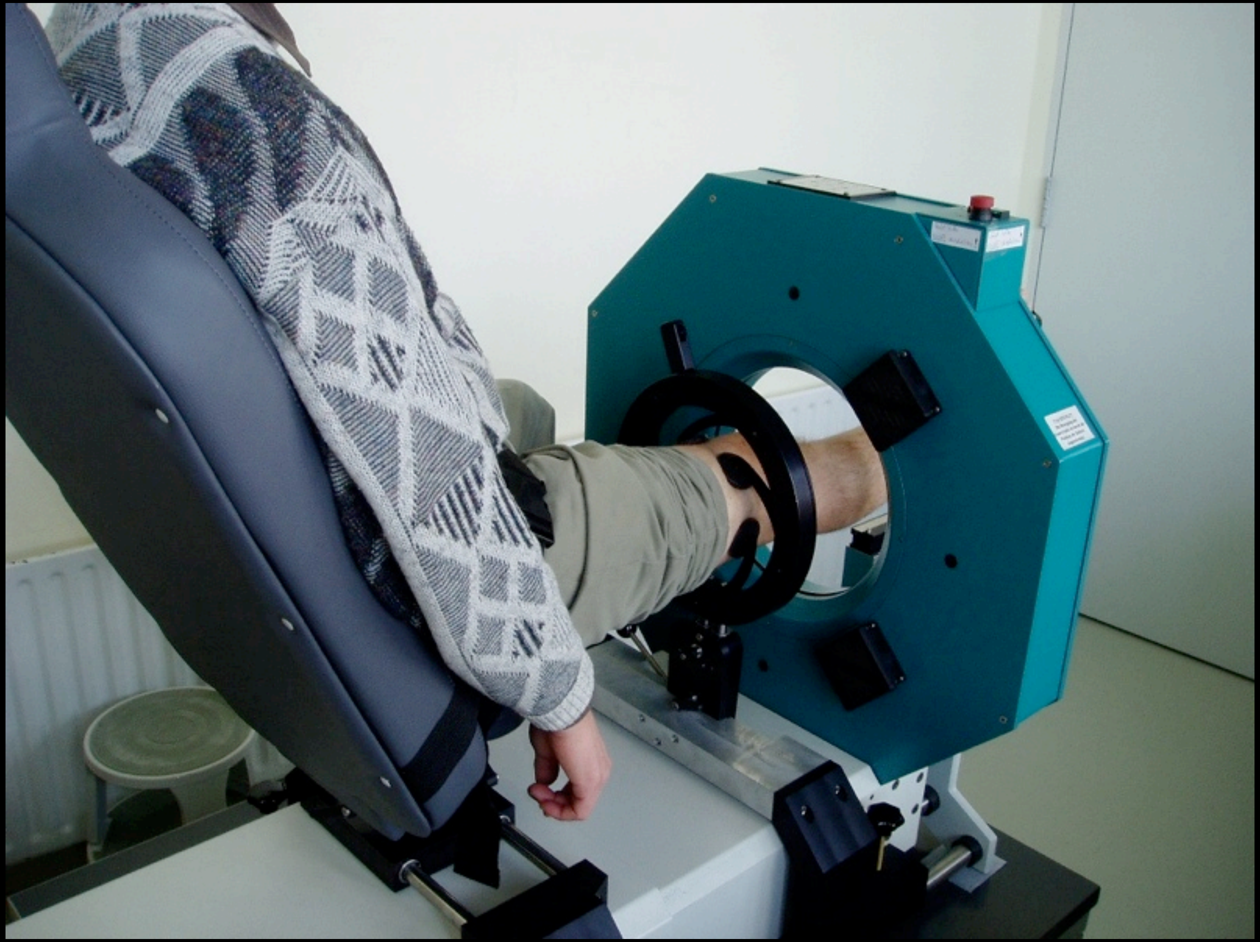
- “gold standard”
- invasive method

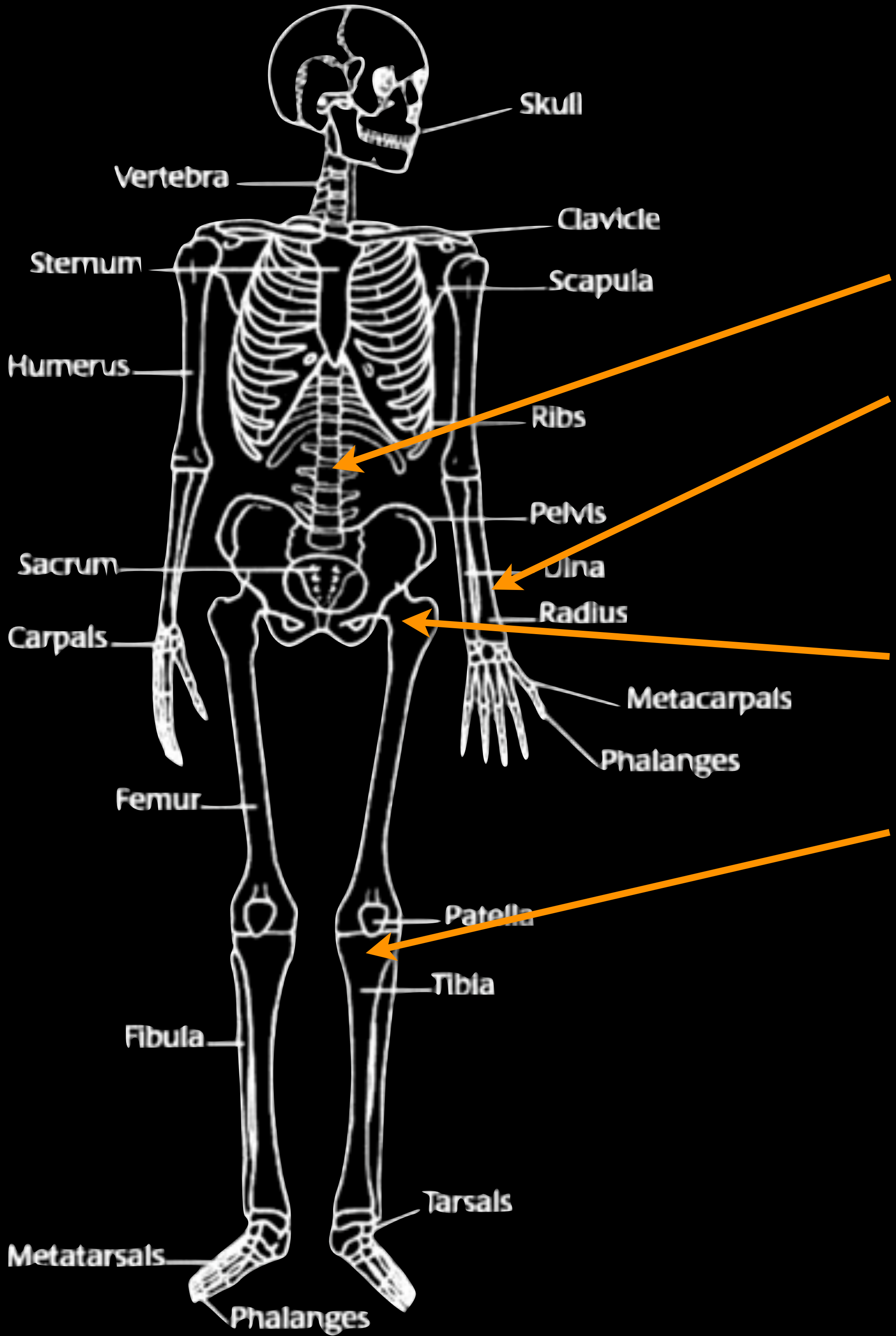


# Structural Quantification

## Symbolic Analysis

- pQCT (peripheral quantitative computer tomography)



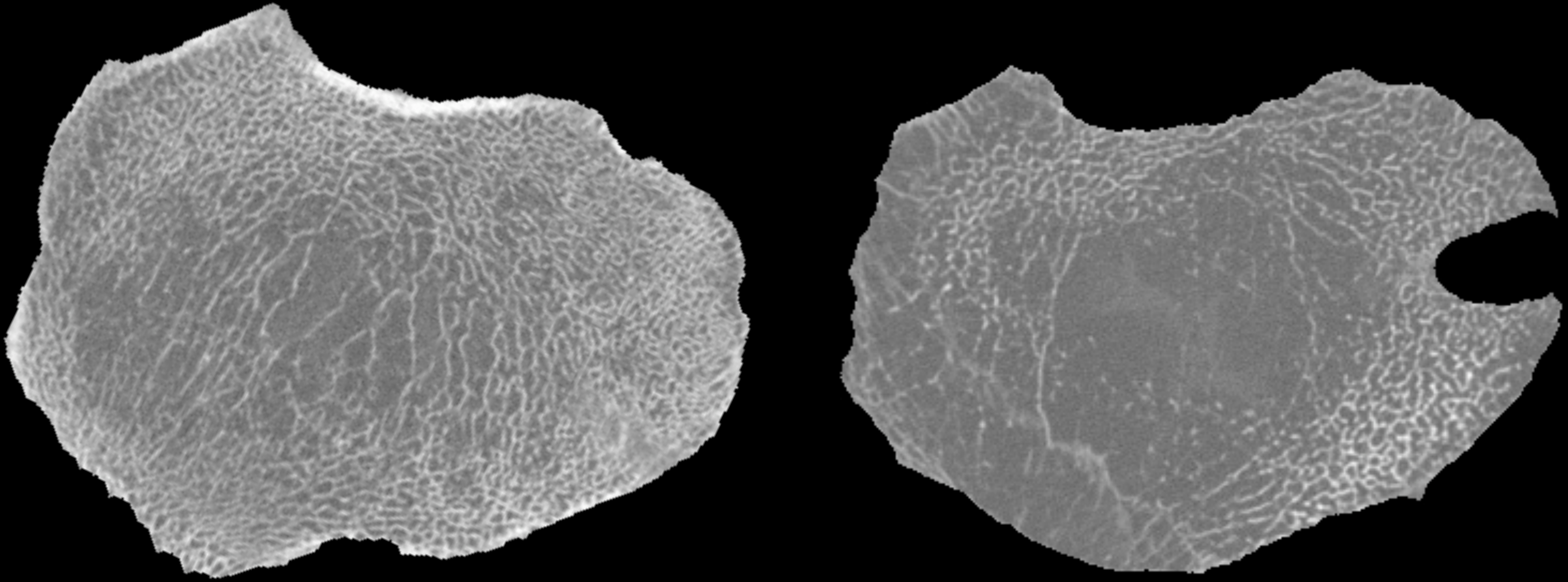


lumbar vertebrae

radius

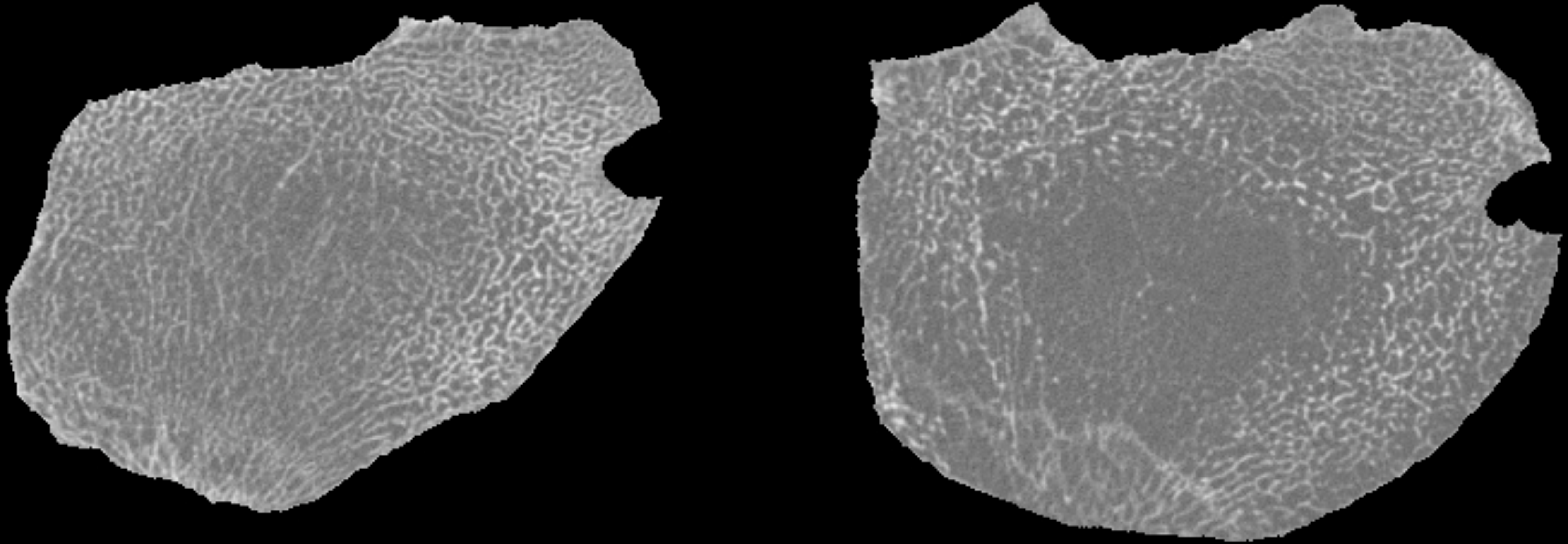
femoral neck

proximal tibia



Normal

Osteoporotic

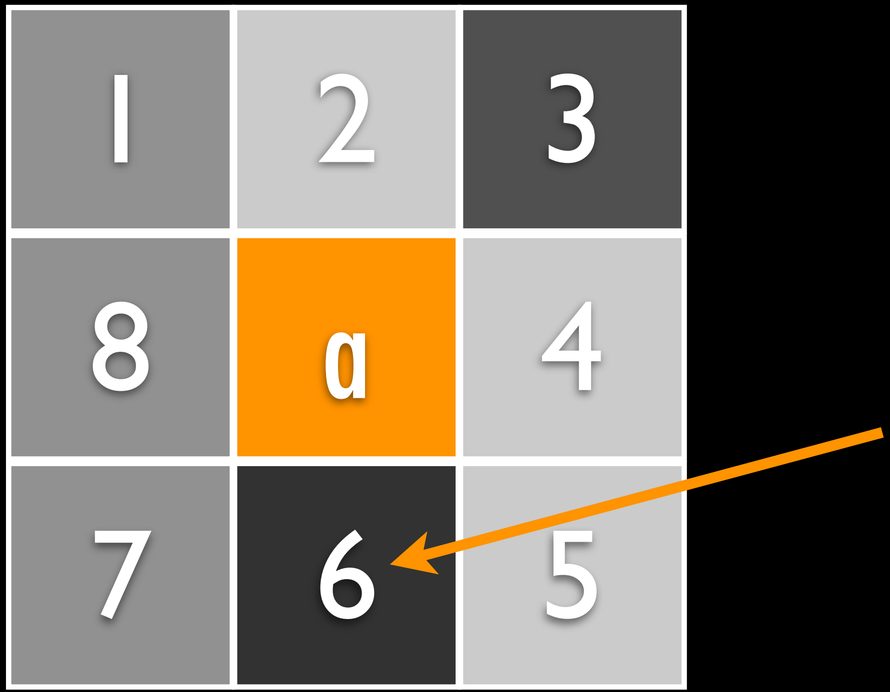


# Symbolisation

- 2 steps:
  1. attenuation threshold
  2. edge threshold

# Symbolisation

- edge image E



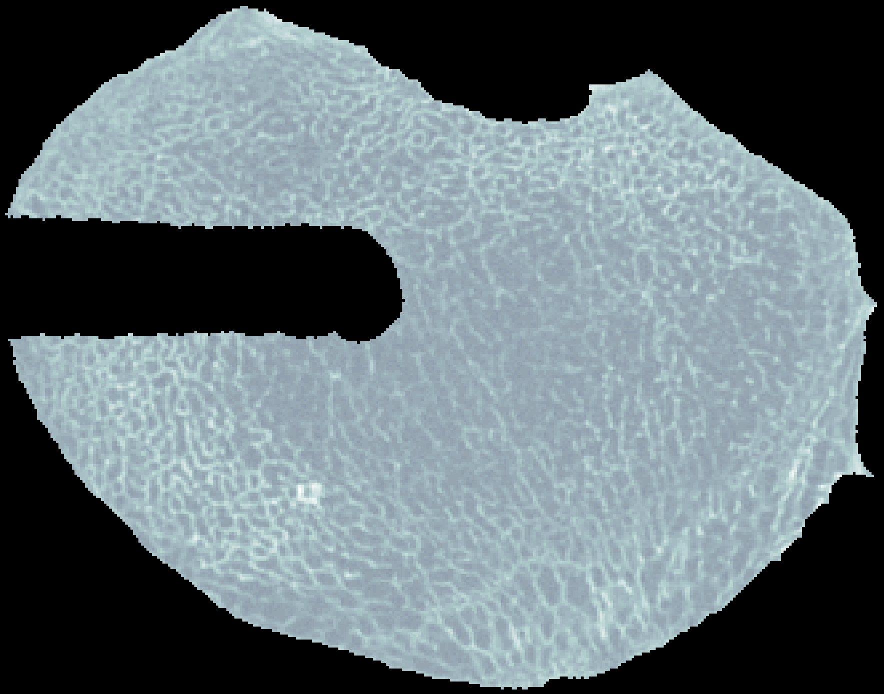
| | | |
|---|---|---|
| 1 | 2 | 3 |
| 8 | a | 4 |
| 7 | 6 | 5 |

e = a – min. neighbour

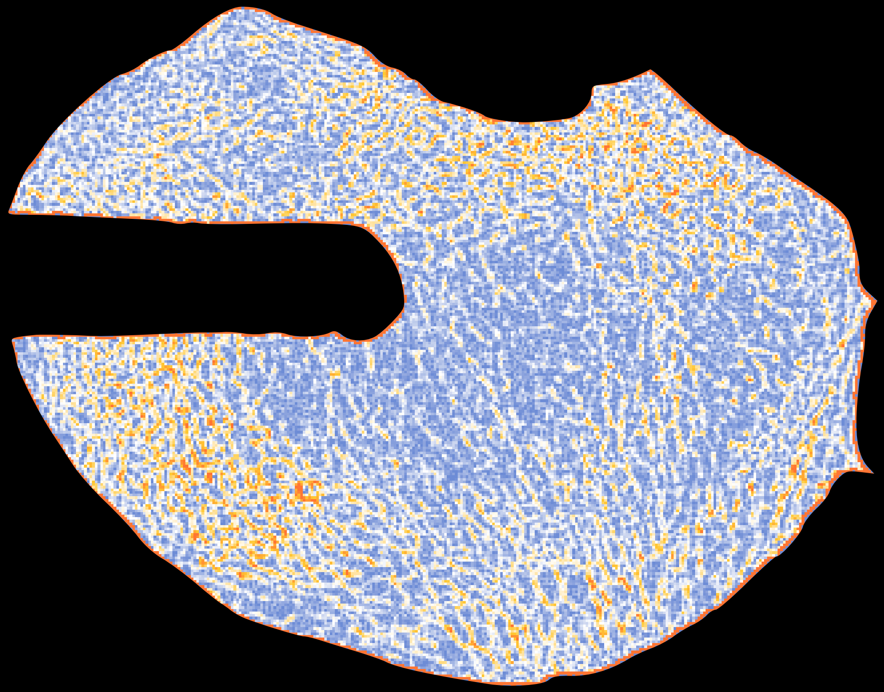
minimal value

# Symbolisation

- edge image E

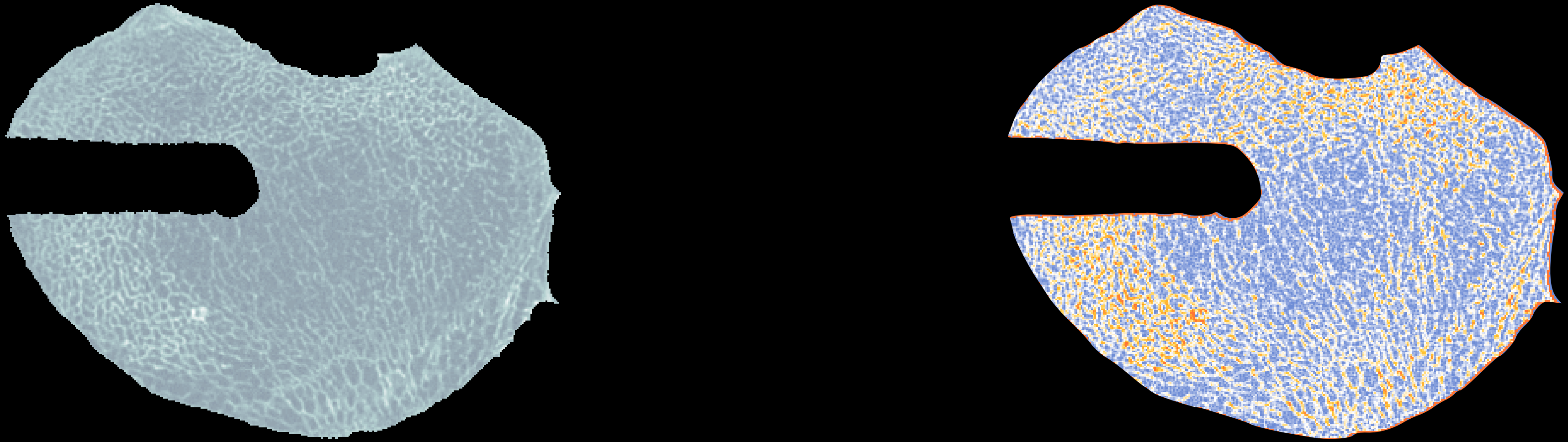


raw data



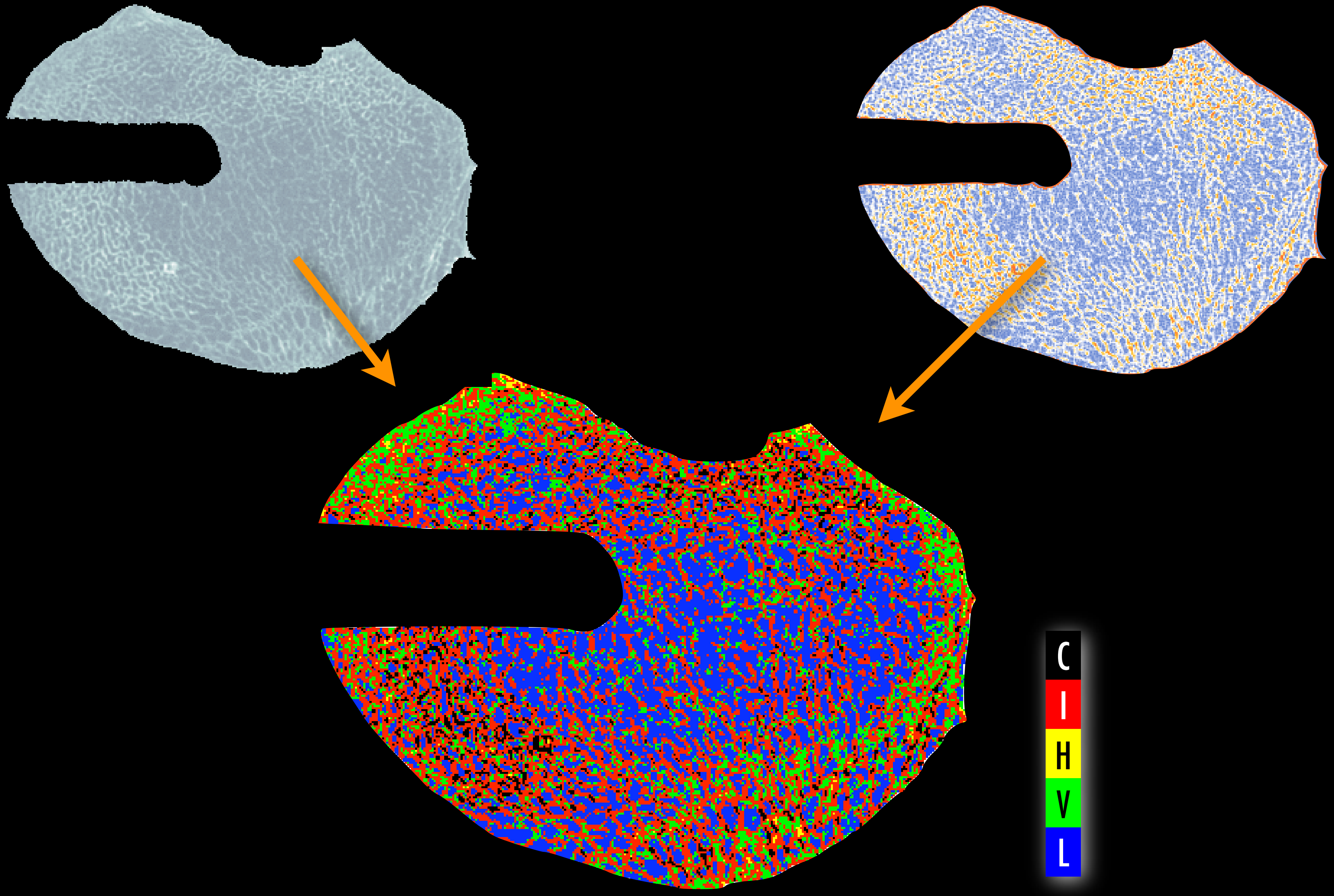
edge image

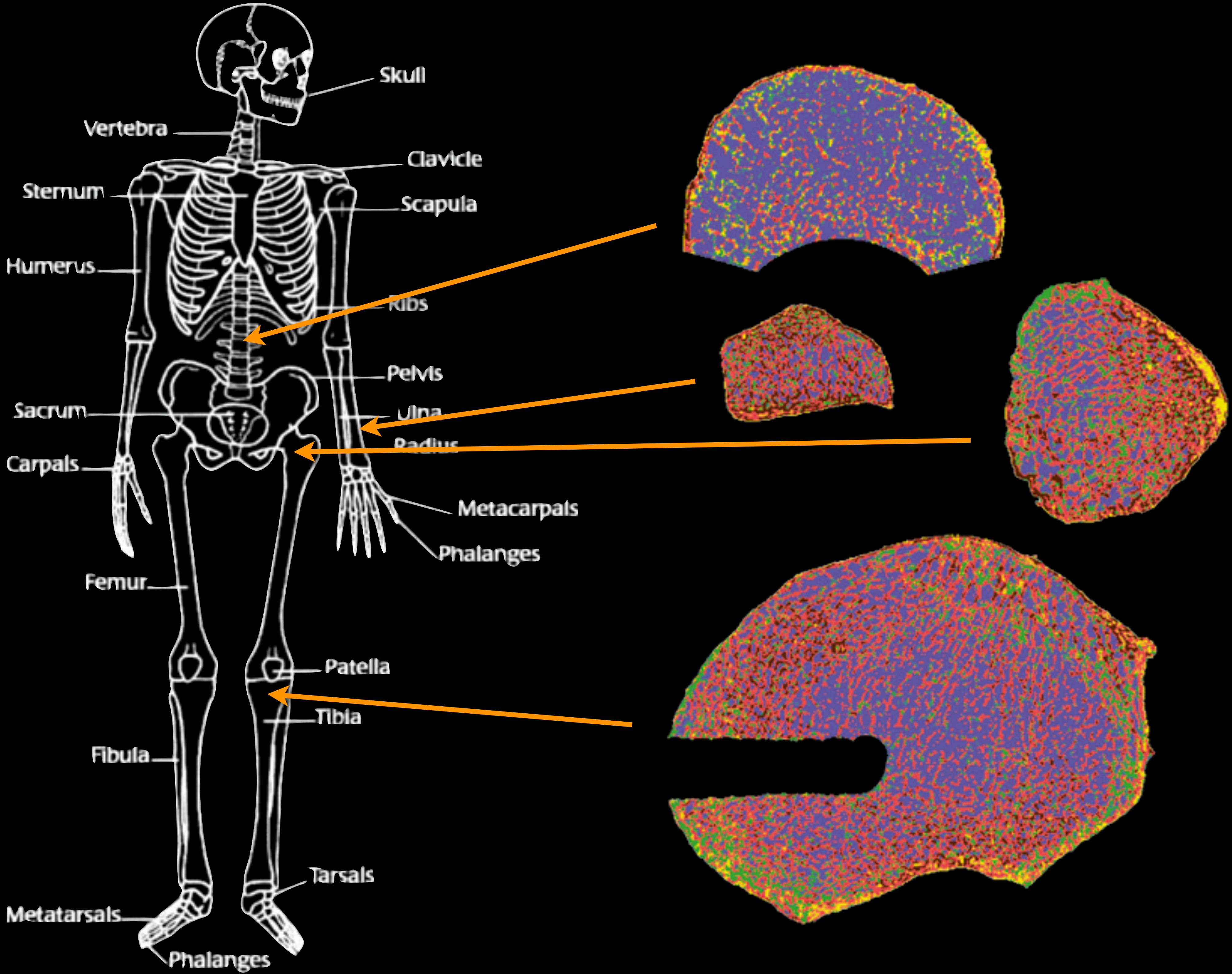
# Symbolisation



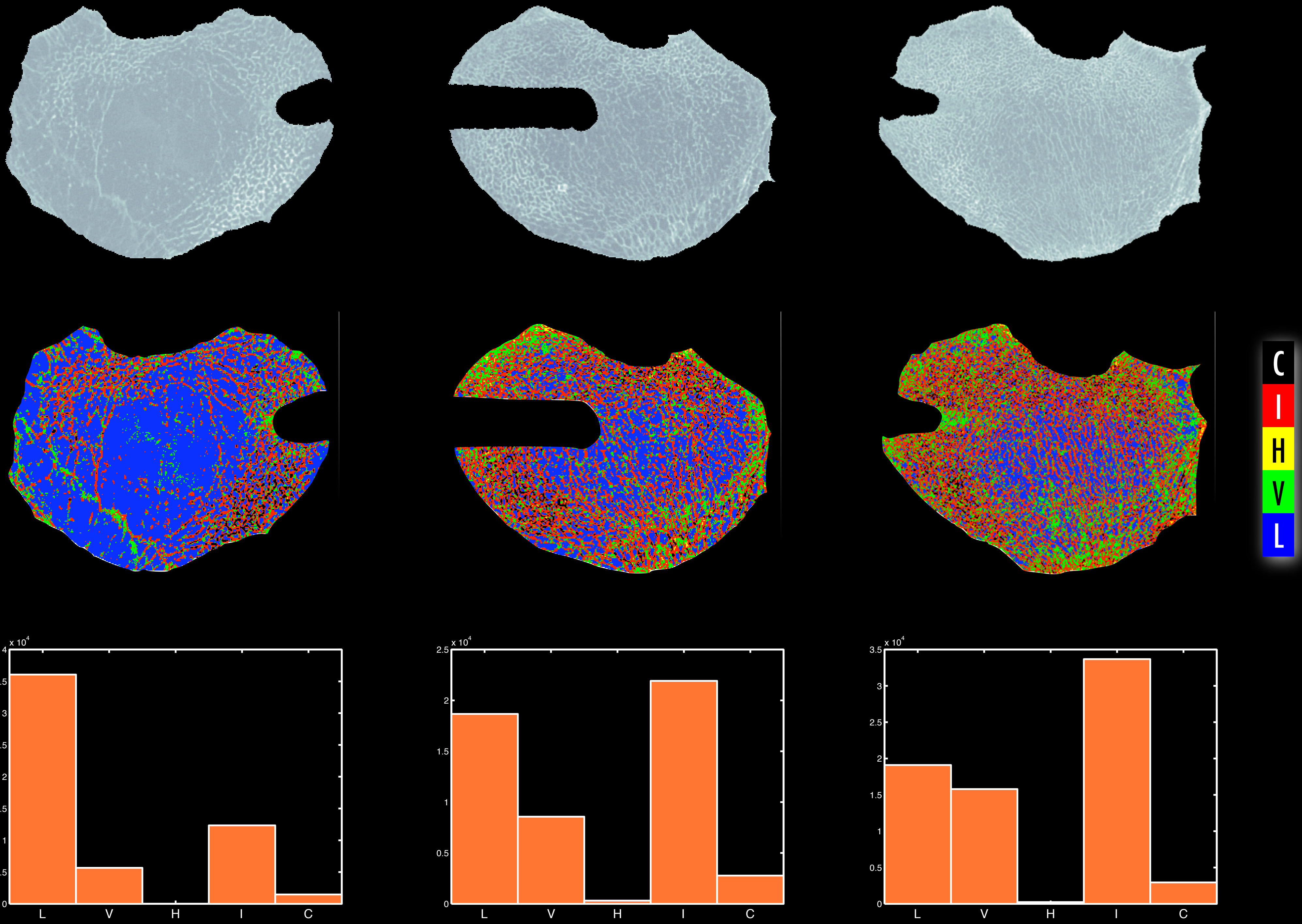
- thresholding attenuation values:
  1. Lake: $a < T$
  2. Valley: $T < a \lesssim \bar{a}$
  3. Highland: $\bar{a} < a$
- thresholding edge values:
  1. Incline: $T \lesssim e \lesssim 3T$
  2. Cliff: $3T < e$

# Symbolisation




Skull
Vertebra
Clavicle
Sternum
Scapula
Humerus
Ribs
Pelvis
Ulna
Sacrum
Radius
Carpals
Metacarpals
Phalanges
Femur
Patella
Tibia
Fibula
Tarsals
Metatarsals
Phalanges

osteoporotic osteopenic normal



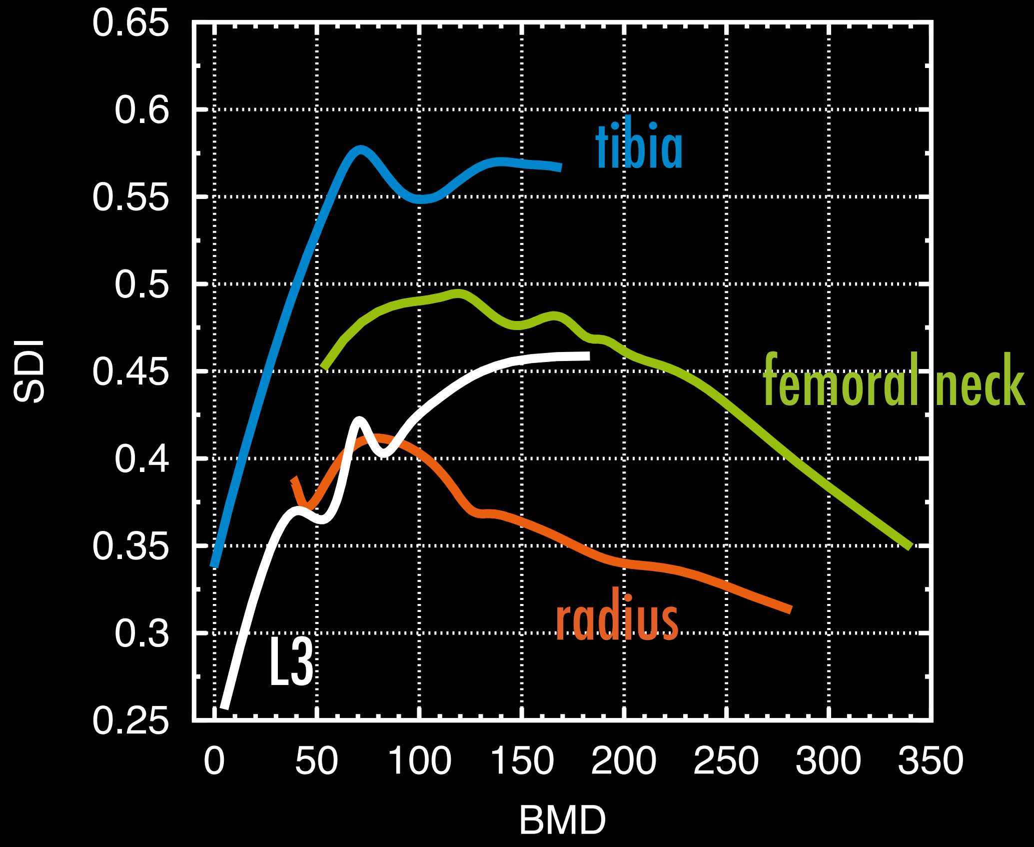
# Structural Measures

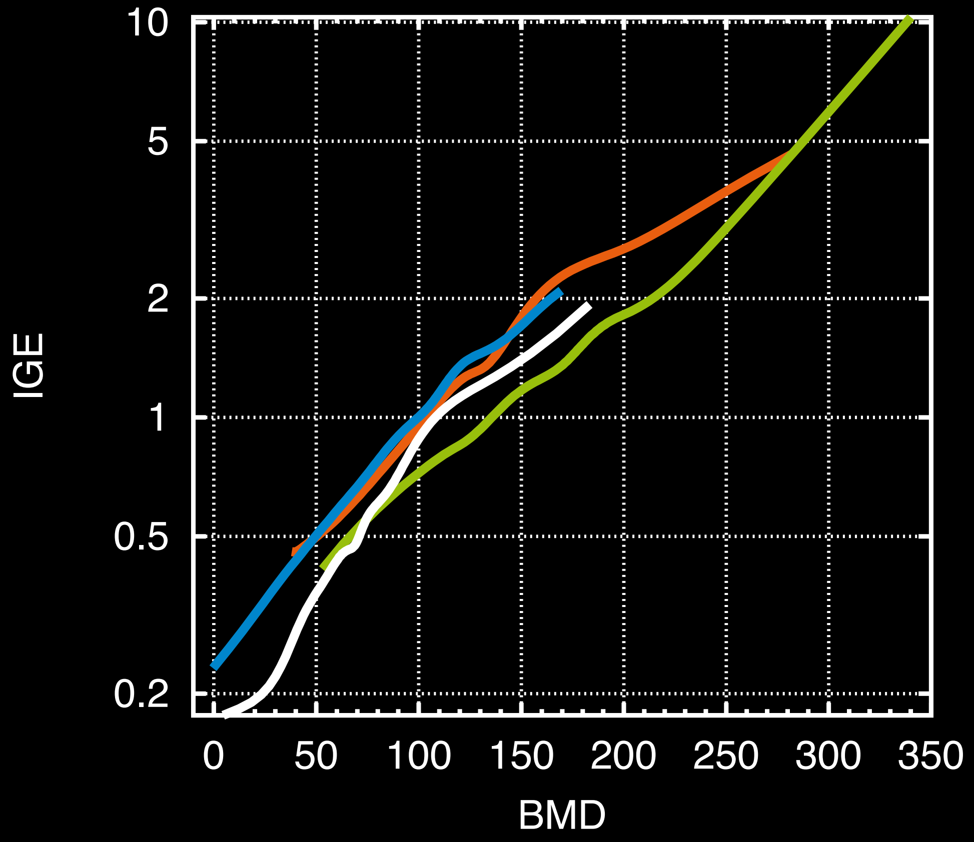
- Complexity measures based on distribution of symbols L, V, H, I, C
- Structure Disorder Index (SDI): 3D Shannon entropy of {p(L), p(I | C), p(V | H)} – disorder of the structure
- Index of Global Ensemble (IGE):

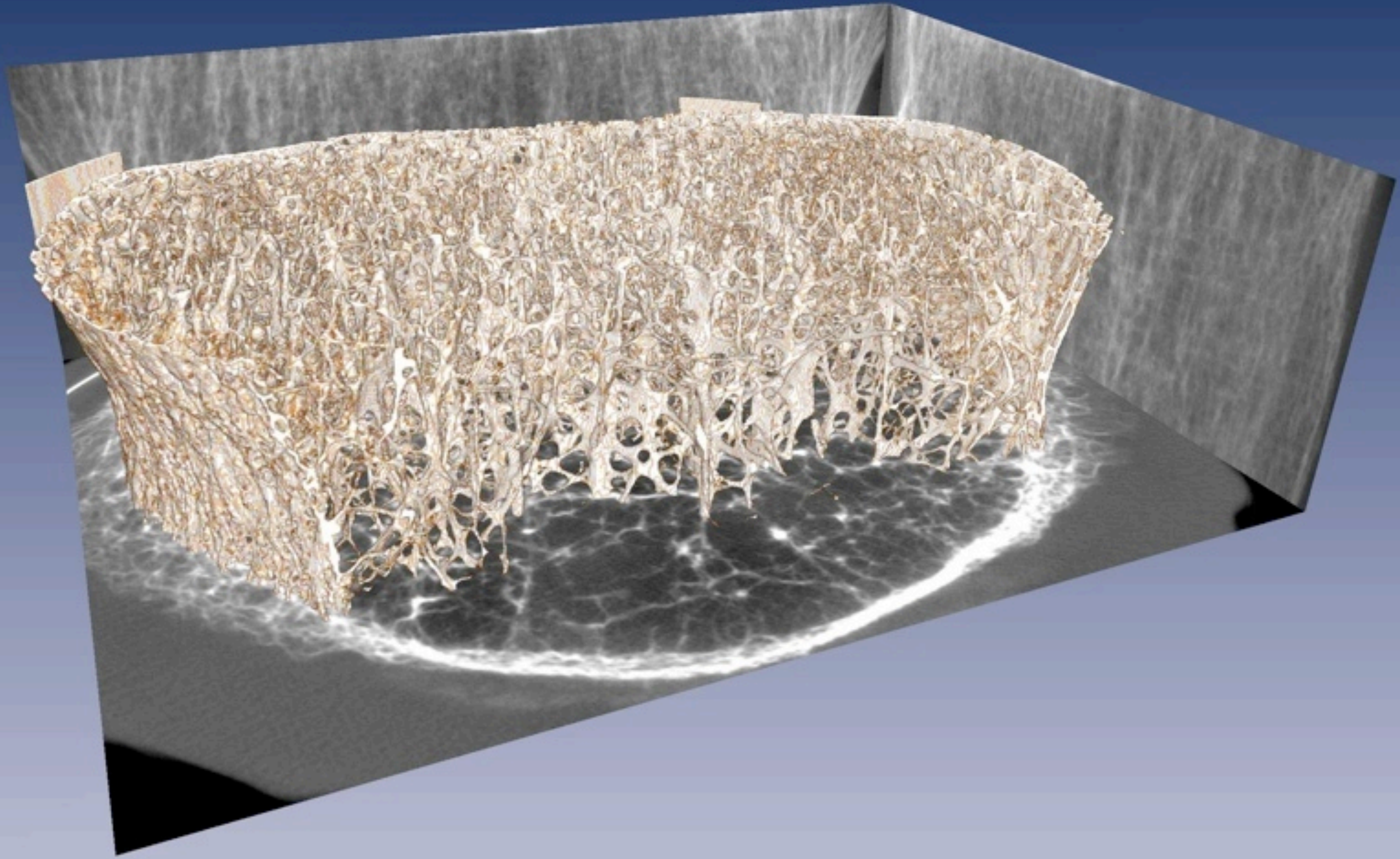
$$IGE = \frac{p(I) + p(C)}{p(L)}$$

measuring the composition of bone

# Structural Measures

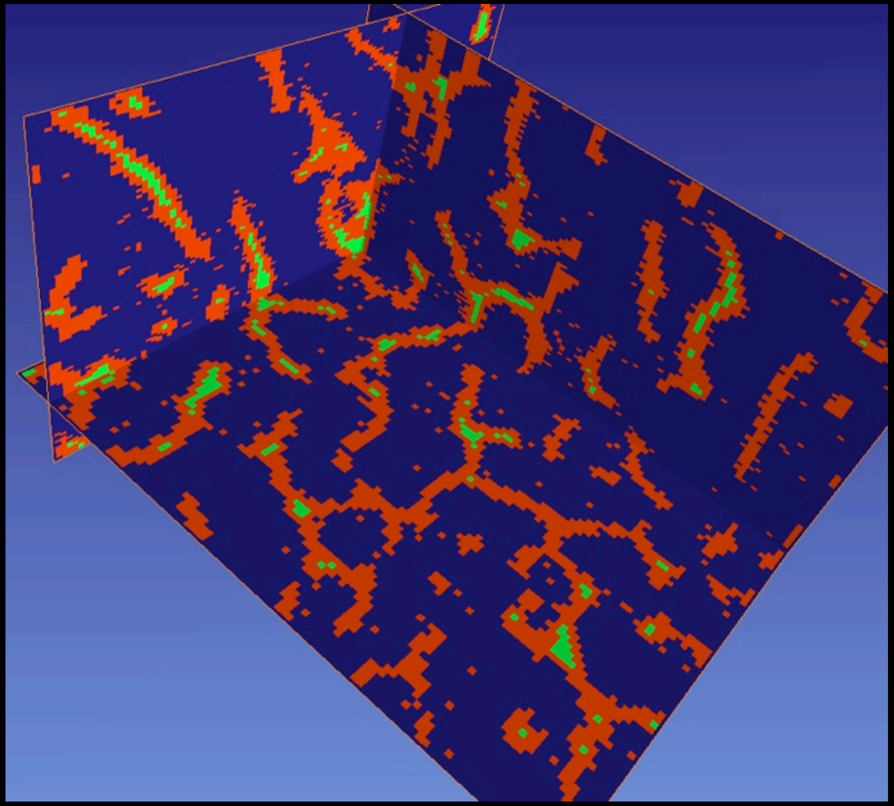




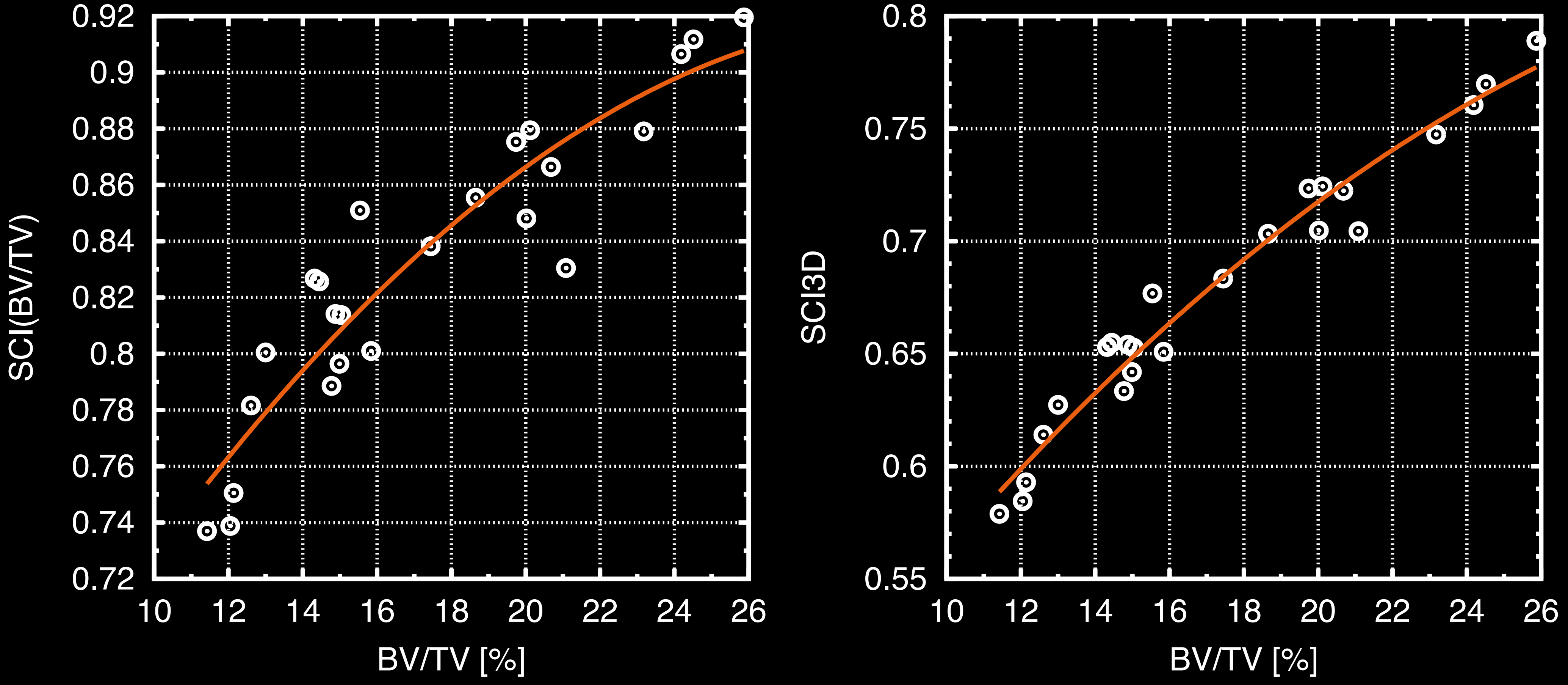


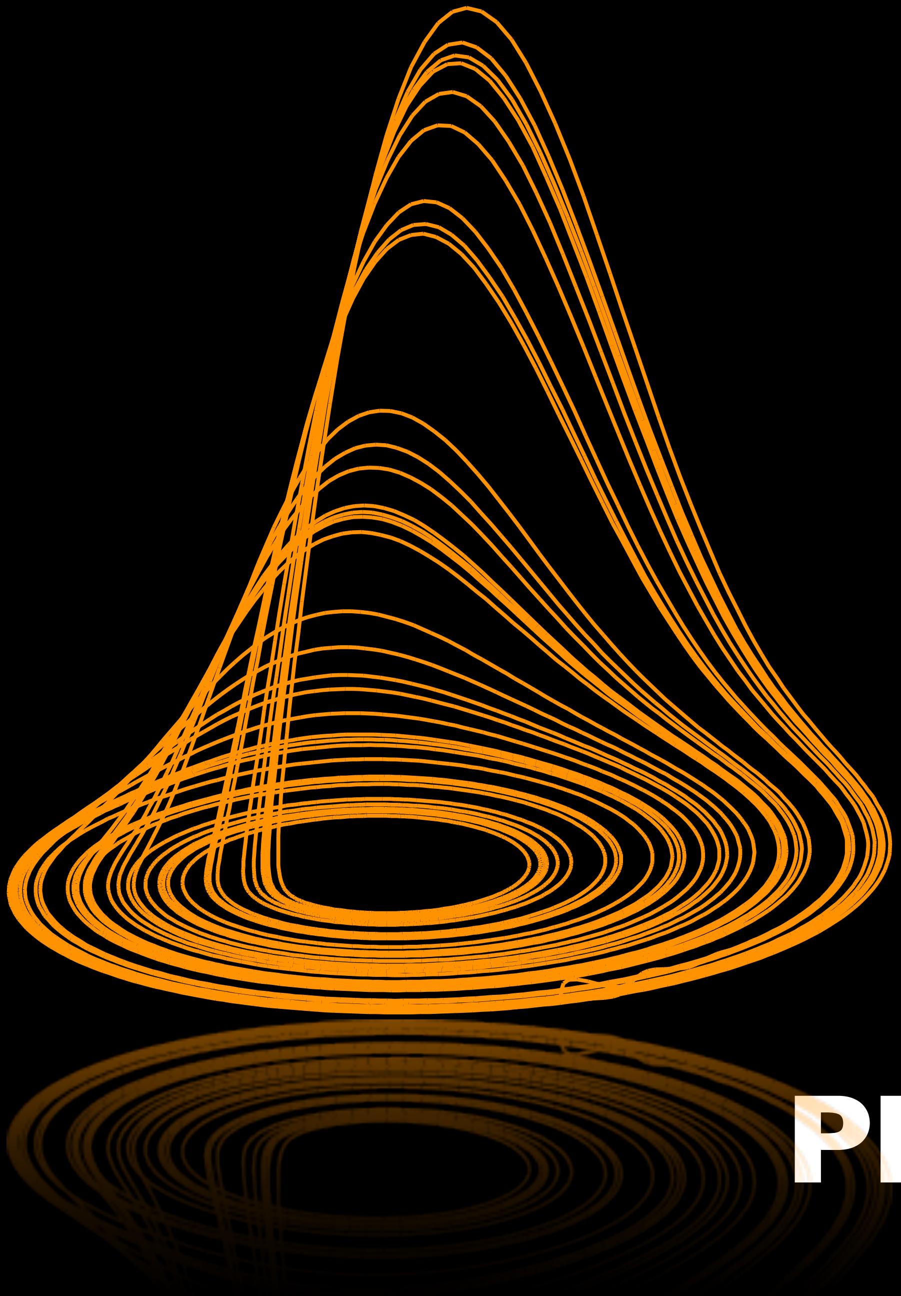
# Extension in 3D

- three symbols
- marrow (blue)
- internal bone (green)
- surface (red)



# Structural Measures in 3D





# Phase Space and Recurrences

# Phase Space

- System: differential equations

$$\dot{\vec{x}}(t) = \vec{F}(\vec{x}(t)), \quad F:\ \mathbb{R}^d \rightarrow \mathbb{R}^d$$

- Trajectory in phase space (dimension d)

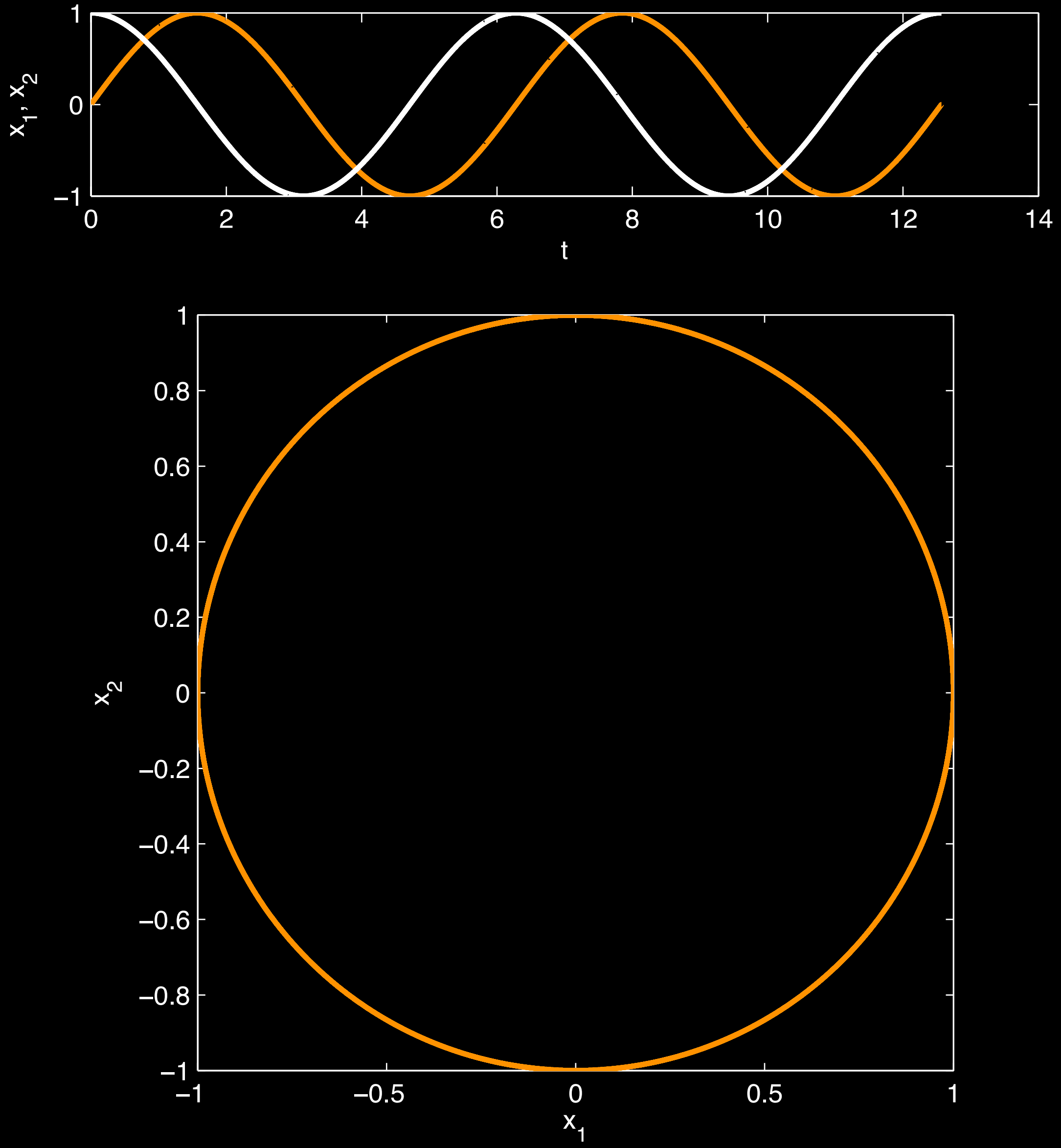
$$\vec{x}(t) \in \mathbb{R}^d$$

- Example:

temperatur and pressure

speed and location (pendulum)

# Phase Space



# Phase Space Reconstruction

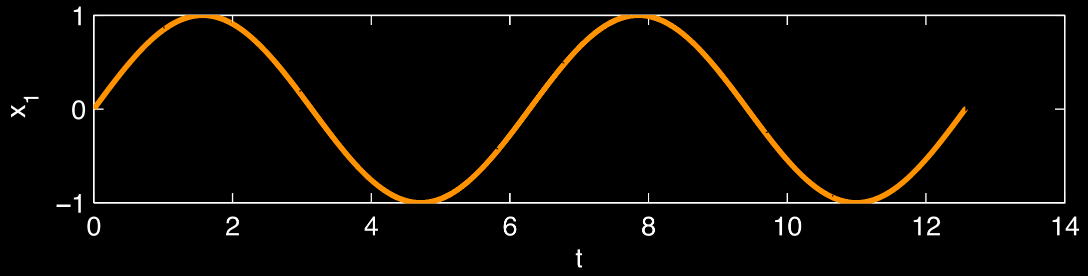
- measurement:
  only one variable available, or other variables not known
- reconstruction by using time delay embedding:

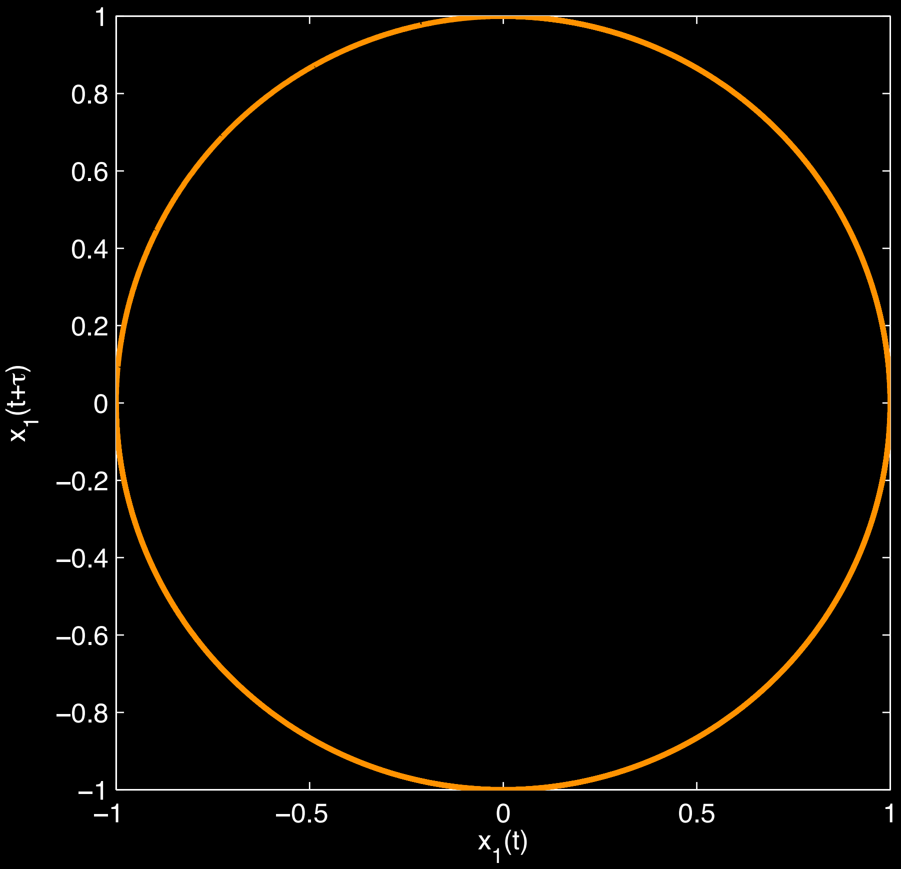
$$\vec{x}(i) = \left(u_i, u_{i+\underset{\uparrow}{\tau}}, u_{i+2\tau}, \ldots, u_{i+(\underset{\uparrow}{m}-1)\tau}\right)$$

time delay

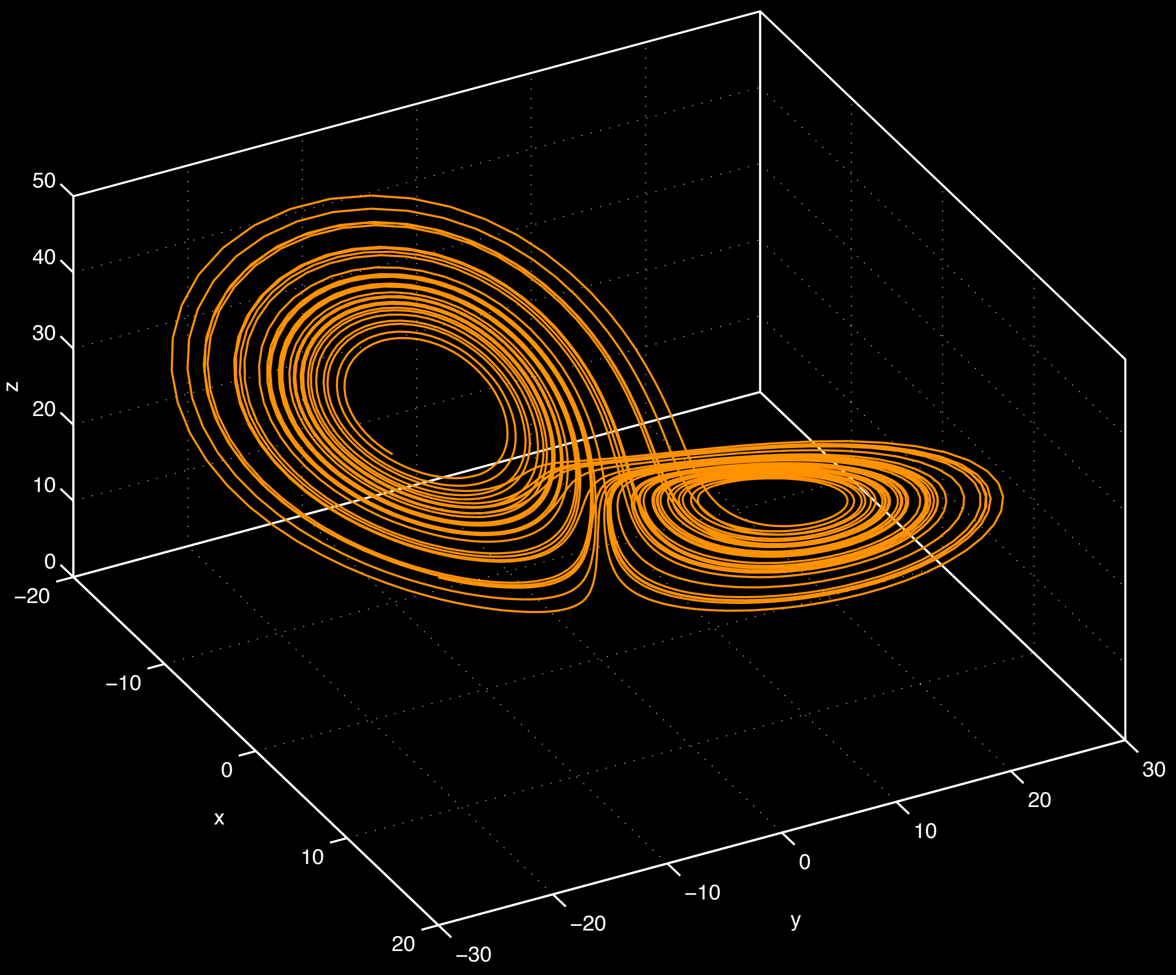
embedding dimension

# Phase Space Reconstruction





# Phase Space



- representation of the system's variables
- trajectory represents the dynamics of the system
- many dynamical properties can be derived from the phase space trajectory

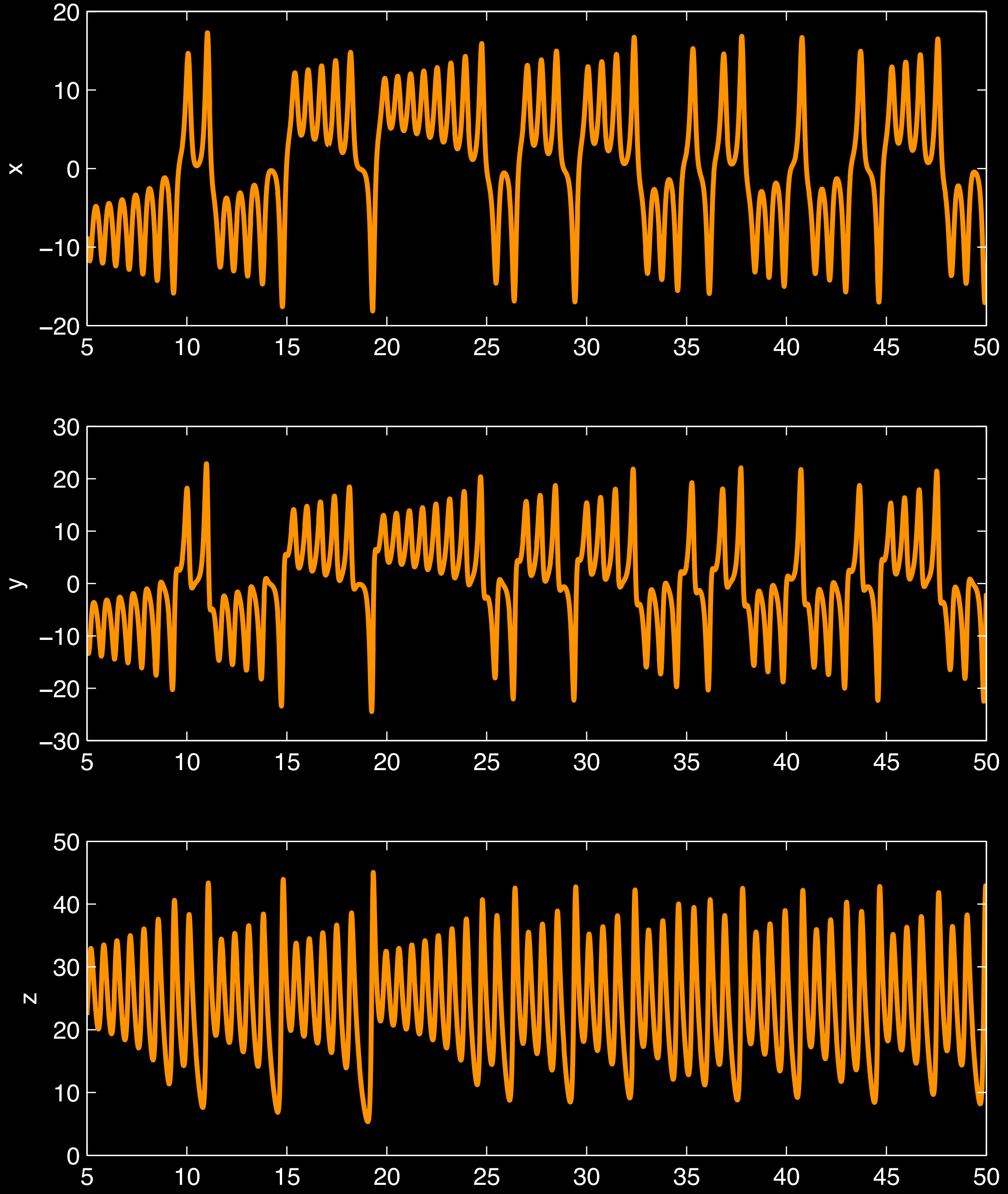
# Lorenz System

$$\dot{x}_1 = \sigma(x_2 - x_1)$$

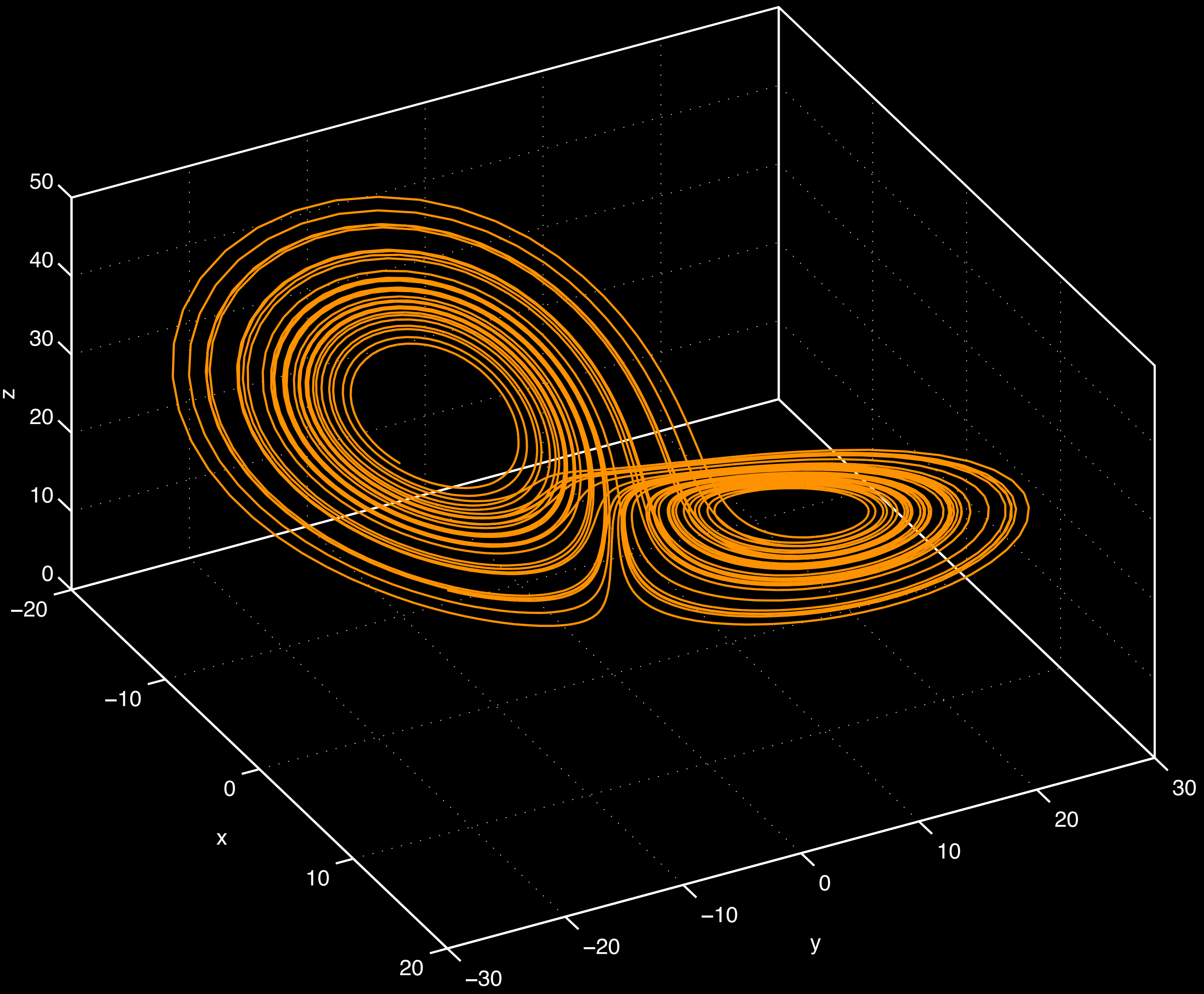
$$\dot{x}_2 = \rho x_1 - x_2 - x_1 x_3$$

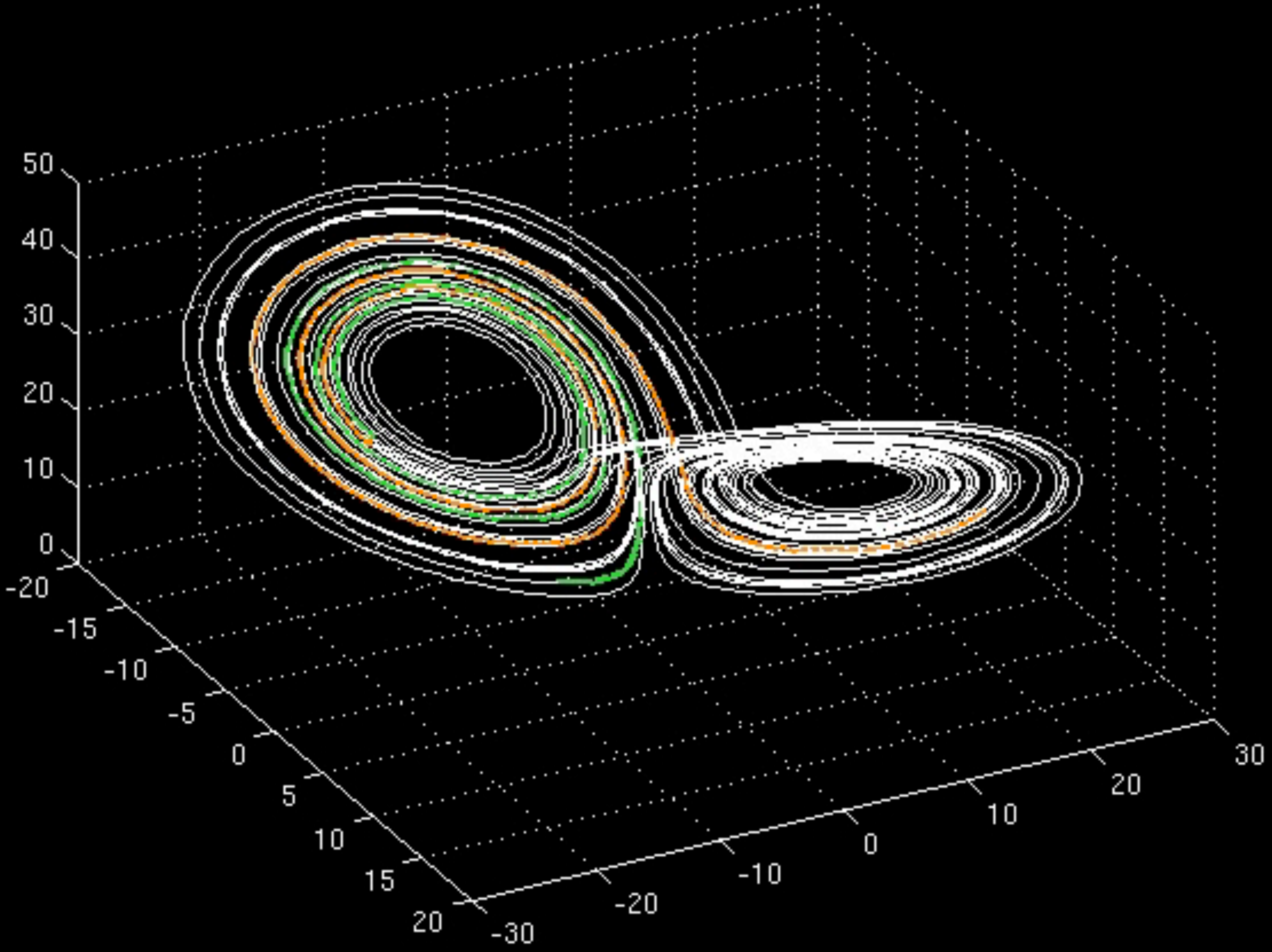
$$\dot{x}_3 = x_1 x_2 - \beta x_3$$

$$(\sigma = 10, \rho = 28, \beta = 8/3)$$

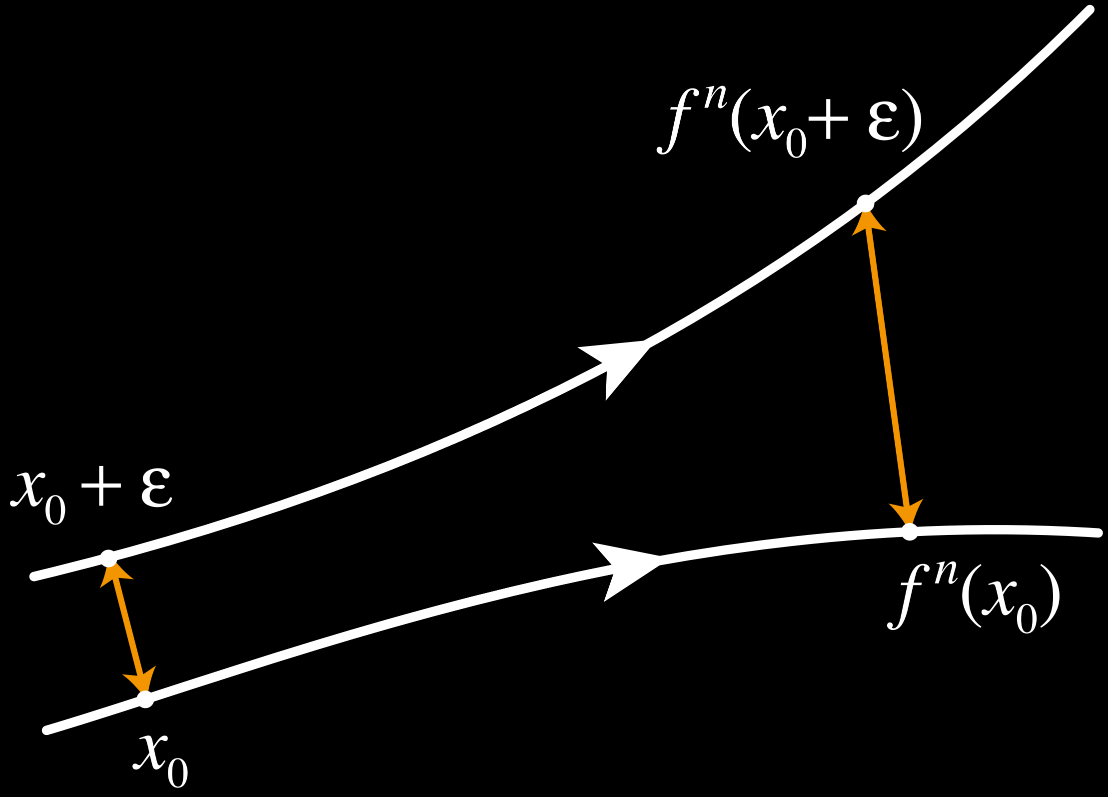


# Lorenz Attractor




50
40
30
20
10
0
-20
-15
-10
-5
0
5
10
15
20
-30
-20
-10
0
10
20
30

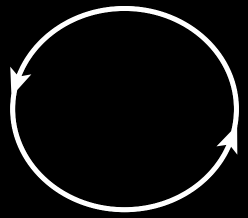
# Lyapunov Exponent



Lyapunov exponent

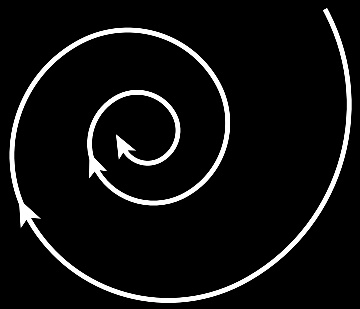
$$|f^n(x_0 + \varepsilon) - f^n(x_0)| = \varepsilon e^{n\lambda}$$

# Lyapunov Exponent



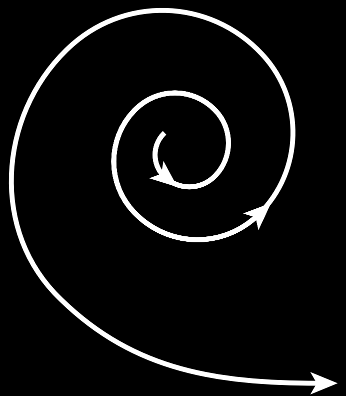
$\lambda = 0$ (distortions remain)

instable fixpoint



$\lambda < 0$ (distortions exponentially shrink)

stable fixpoint

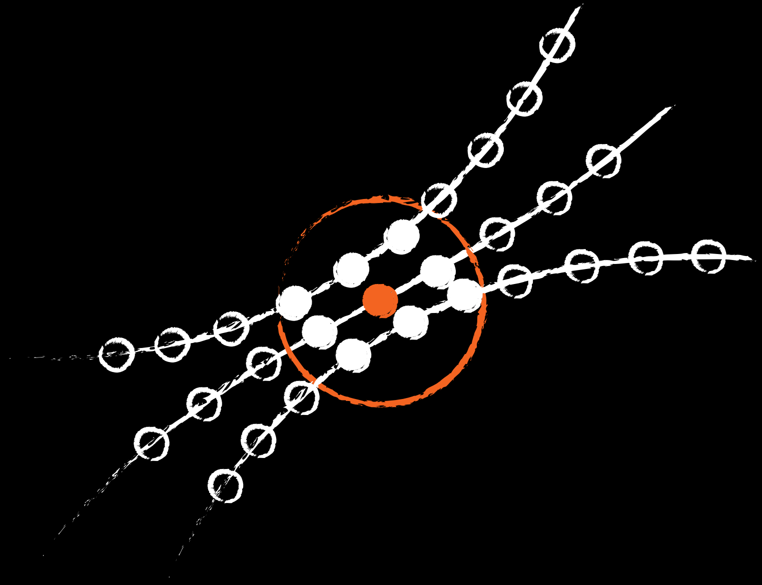


$\lambda > 0$ (distortions exponentially grow)

chaos

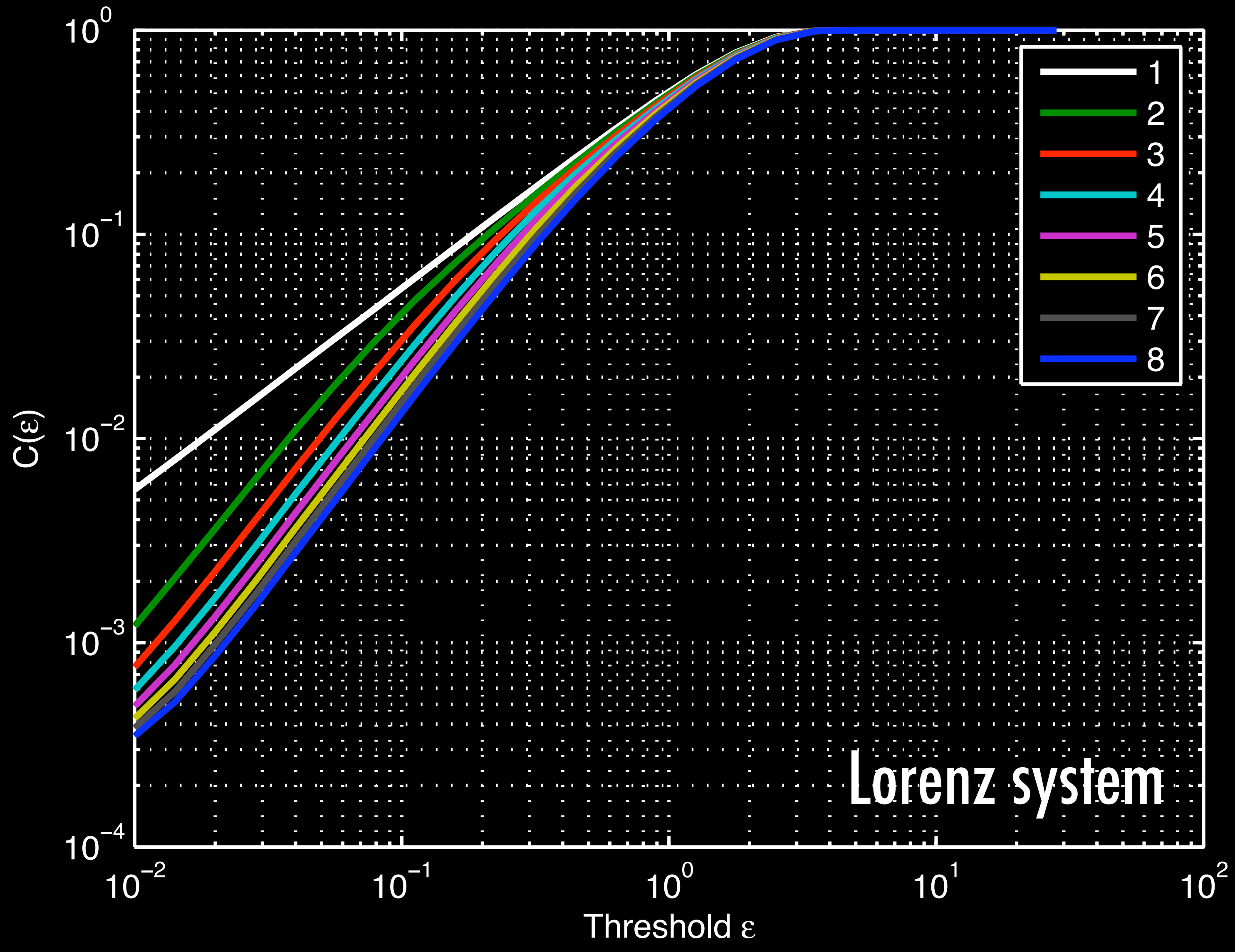
# Correlation Sum

$$C_2 = \frac{1}{N(N-1)} \sum_{i \neq j} \Theta(\varepsilon - ||\vec{x}_i - \vec{x}_j||)$$

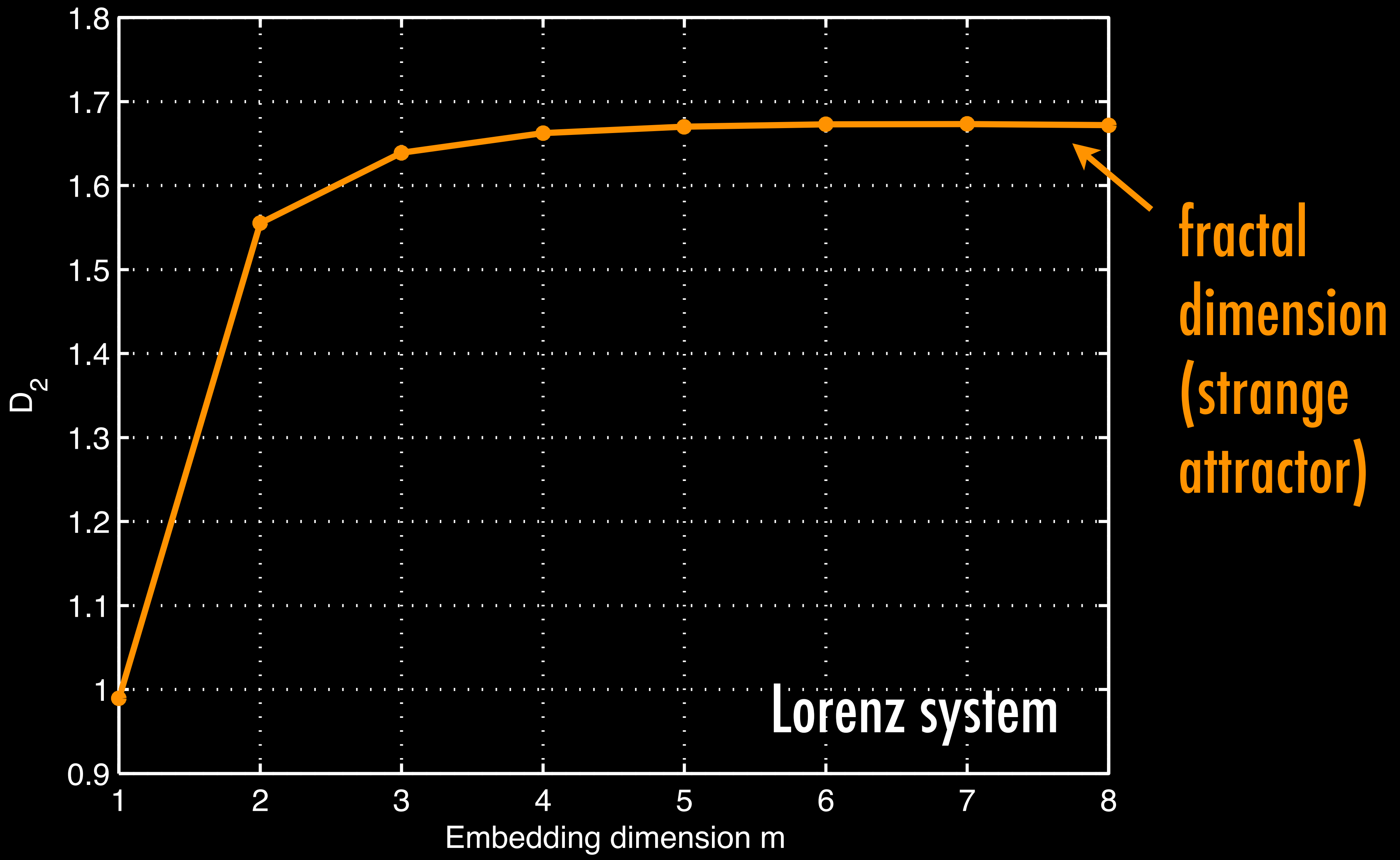


Neighbours in phase space

# Correlation Sum



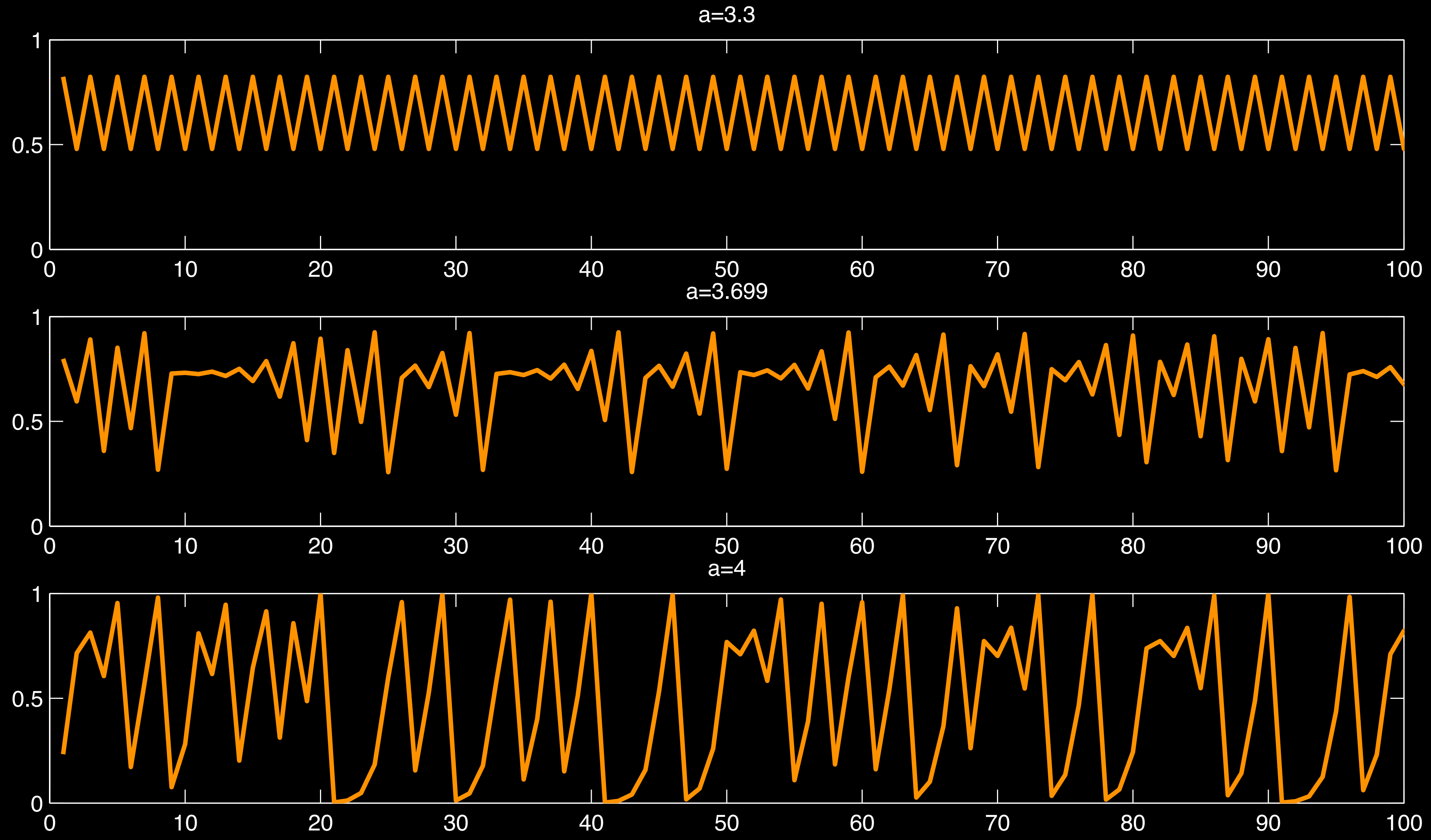
# Correlation Dimension

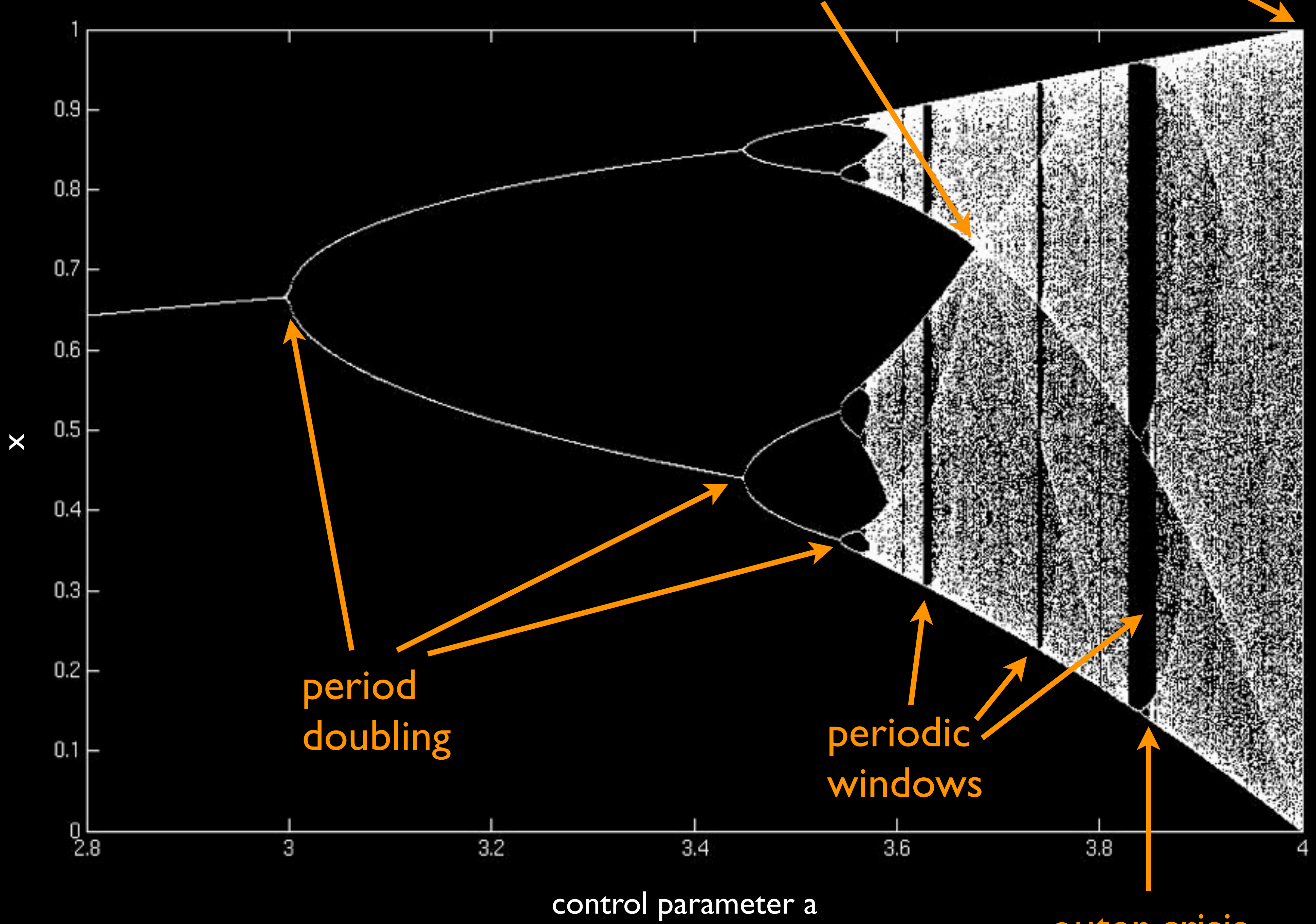


fractal dimension (strange attractor)

# Logistic Map

$$x_{i+1} = a\,x_i(1 - x_i)$$





band merging

outer crisis

period doubling

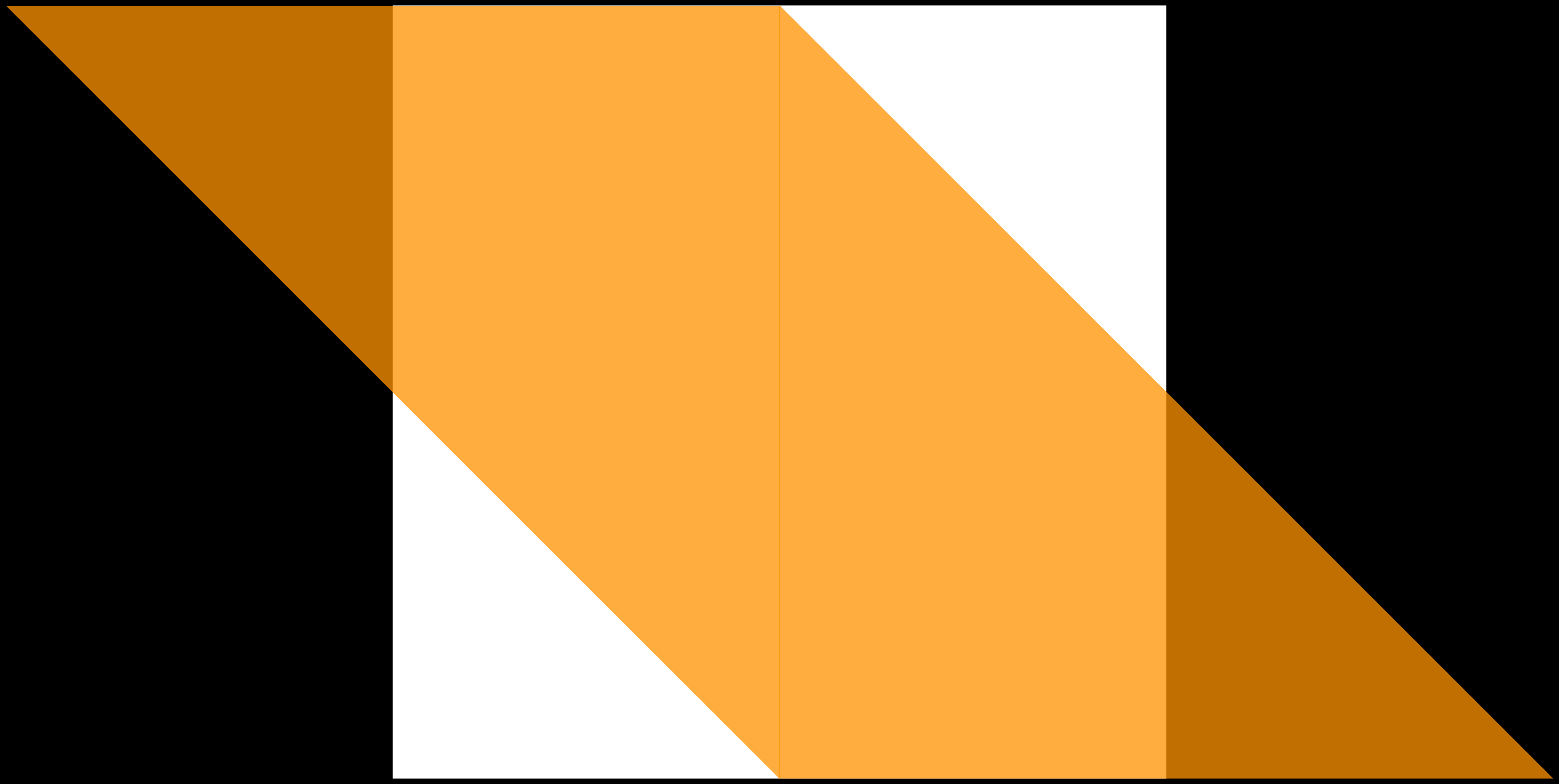
periodic windows

outer crisis

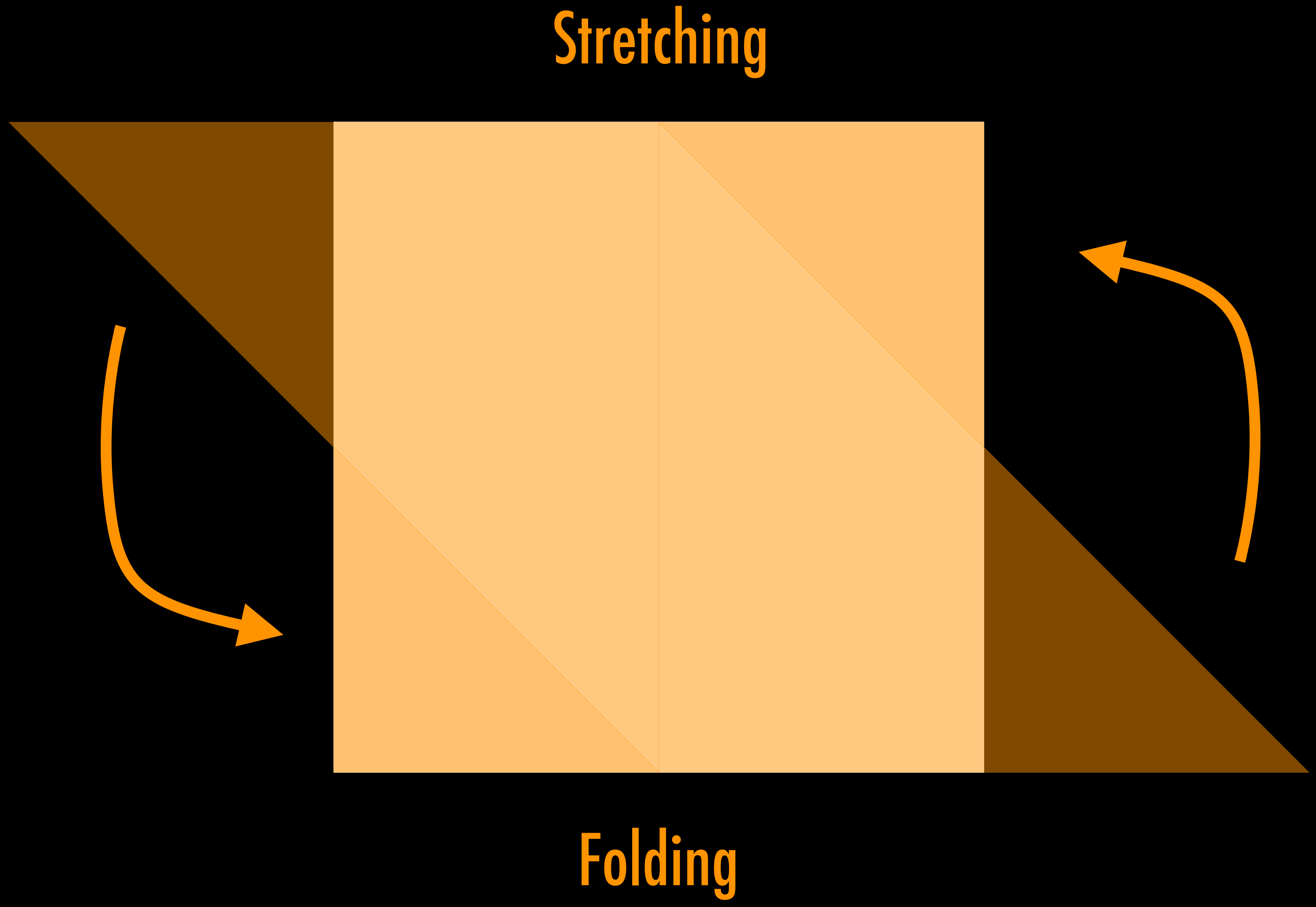
# Nonlinear Processes

# Nonlinear Processes

## Stretching



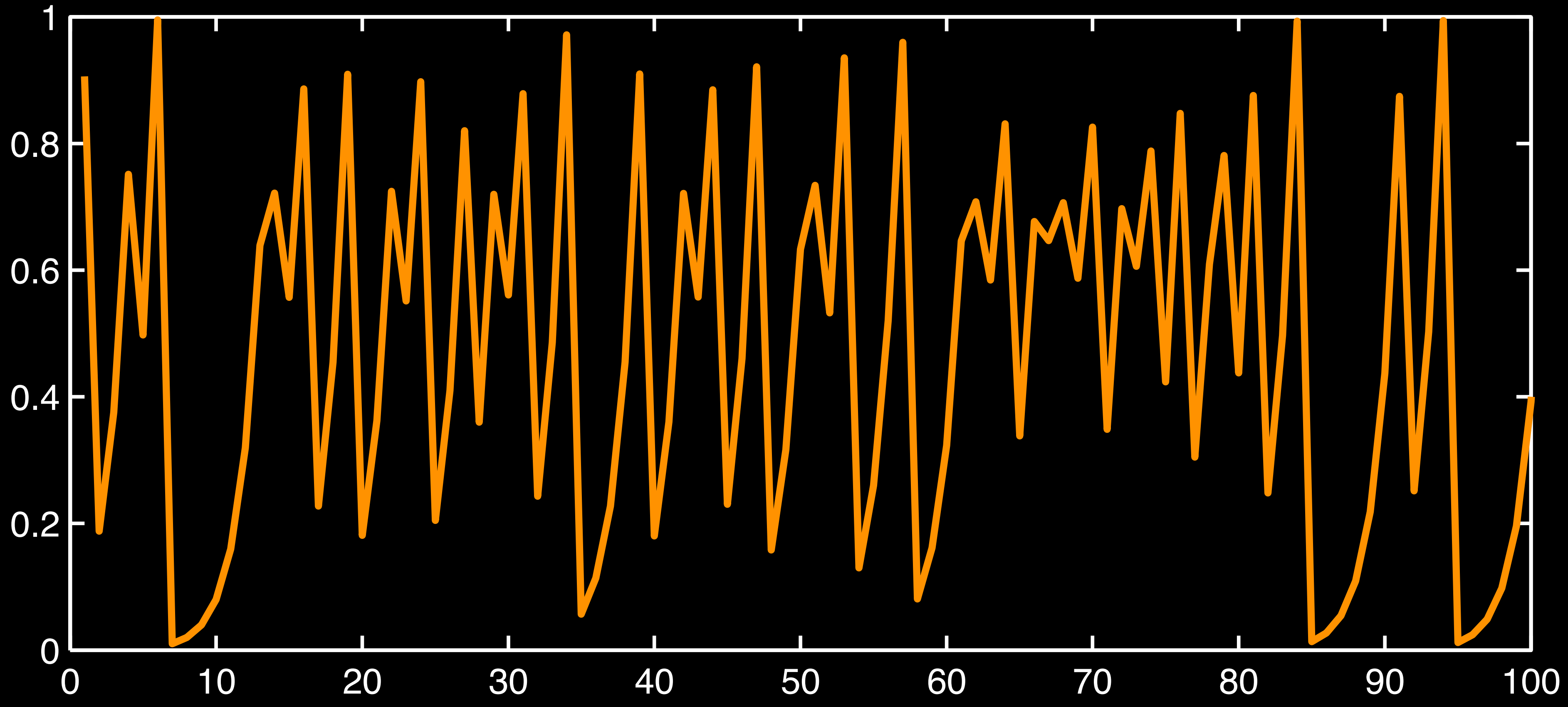
# Nonlinear Processes



# Recurrence by a Nonlinear Process

# Tent Map

$$x_{i+1} = \begin{cases} 2x_i & x_i < 0.5 \\ 2(1-x_i) & x \geq 0.5 \end{cases}$$



# Error Growth in Tent Map

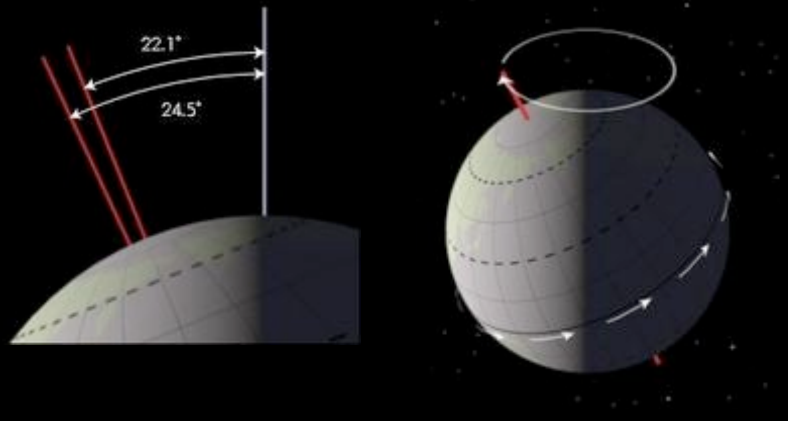
| x | 2x<br>2(1-x) | x | 2x<br>2(1-x) | Fehler |
|---|---|---|---|---|
| 0,8 | 0,4 | 0,8+0,0001 | 0,3998 | 0,0002 |
| 0,4 | 0,8 | 0,3998 | 0,7996 | 0,0004 |
| 0,8 | 0,4 | 0,7996 | 0,4008 | 0,0008 |
| 0,4 | 0,8 | 0,4008 | 0,8016 | 0,0016 |

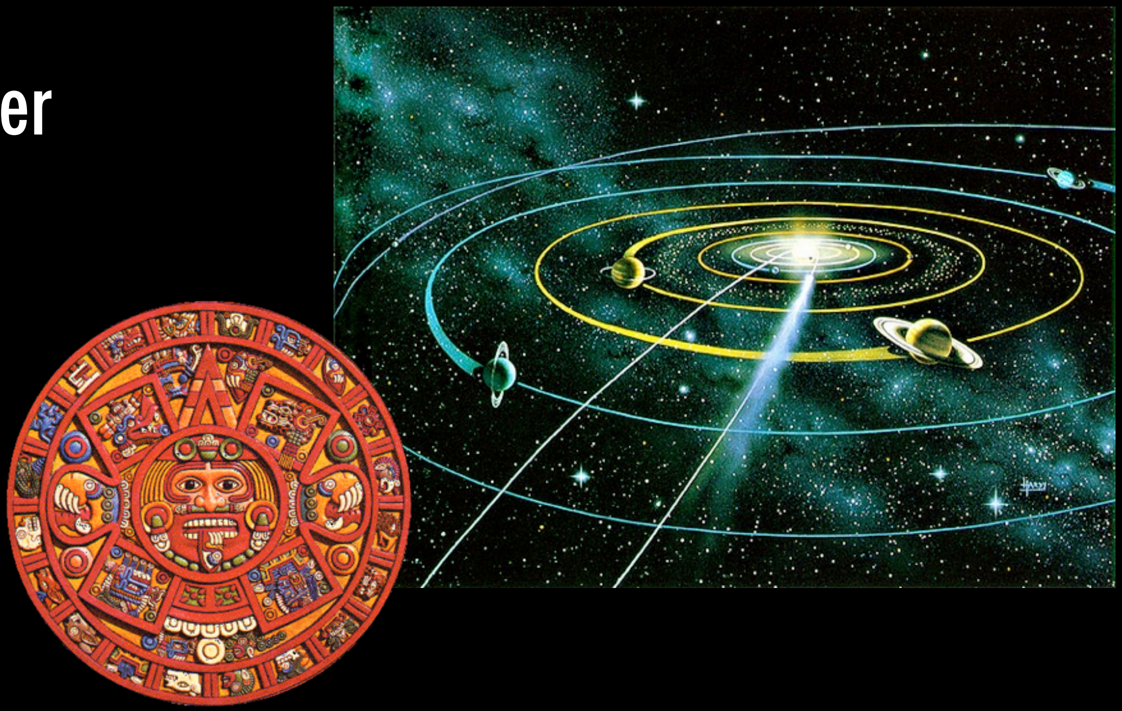
exponential error growth ($2^4$ after 4 steps)

# Recurrence

- fundamental characteristic of many dynamical systems
- recurrences in real life:

  Milankovich cycles, weather after storm, El Niño phenomenon, heart beat after exertion, Maya calendar etc.





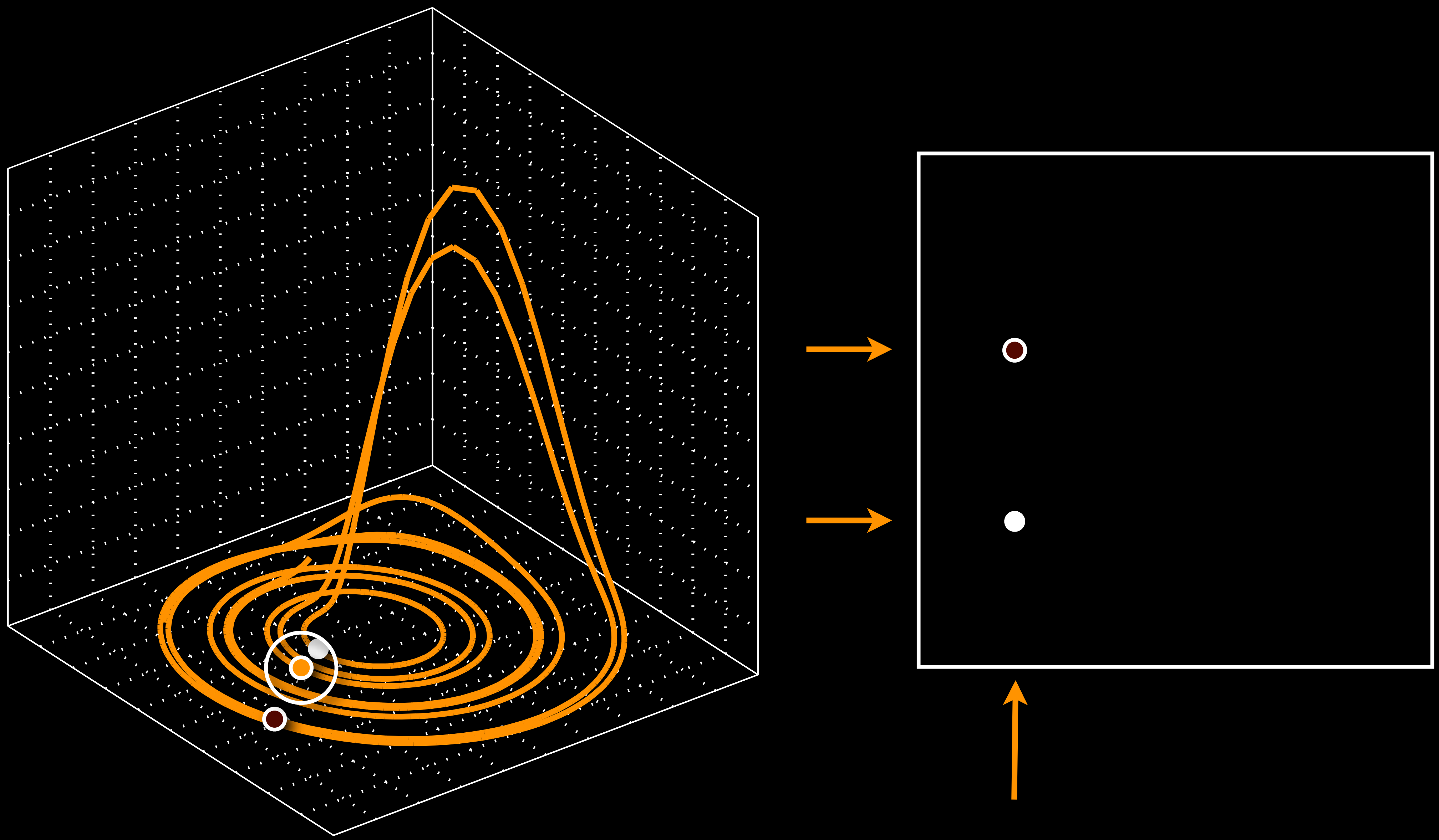
# Recurrence

- Poincaré, 1890
- study of the three-body system (planetary motion)
- “a system recurs infinitely many times as close as one wishes to its initial state”
- won price by King Oscar of Sweden II on the occassion of his 60th birthday

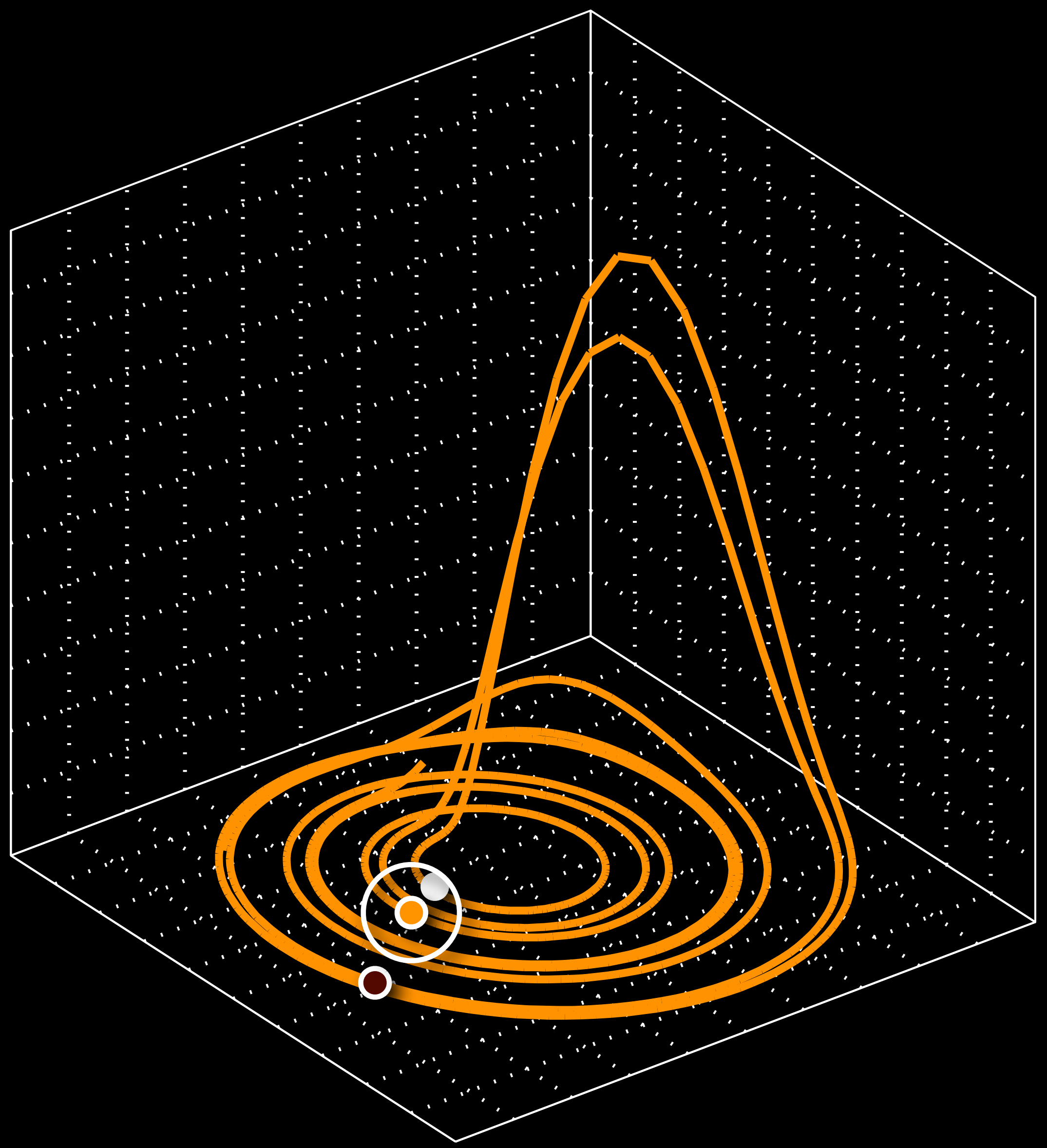
# Investigating Recurrence

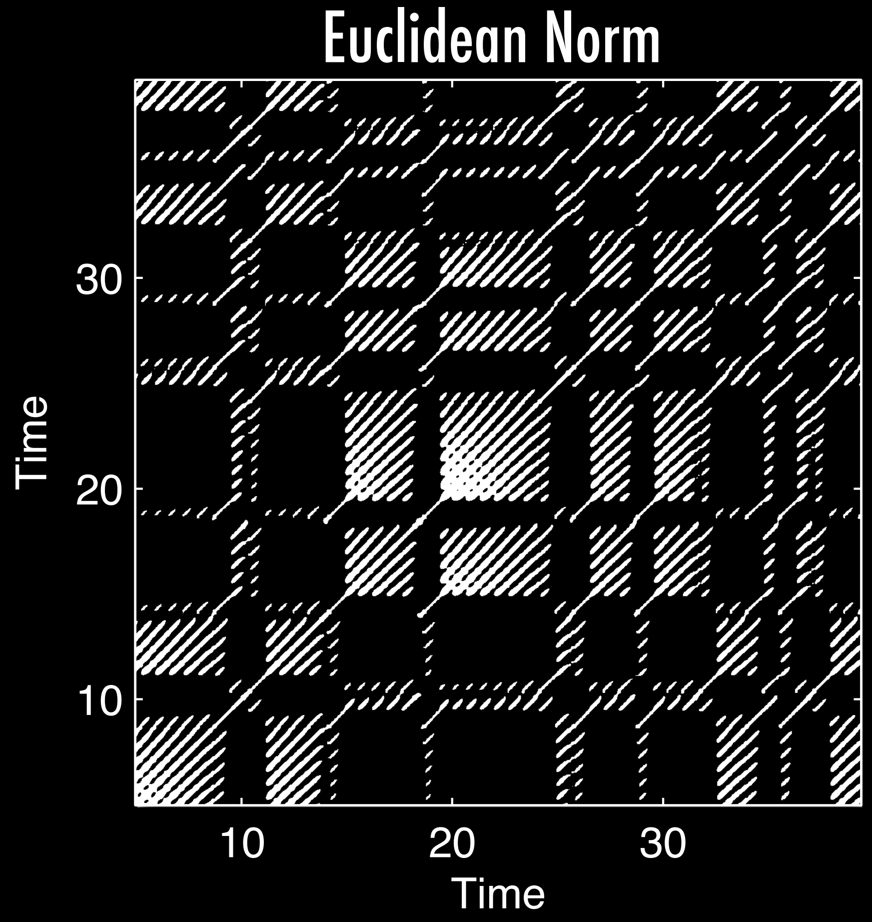
- Poincaré map
- Recurrence time statistics
- First return map
- Recurrence plot

# Recurrence Plot



# Recurrence Plot





# Recurrence Plot

- to visualise the phase space trajectory by its recurrences

$$\mathbf{R}_{i,j} = \begin{cases} 1 : \vec{x}_i \approx \vec{x}_j \\ \\ 0 : \vec{x}_i \not\approx \vec{x}_j \end{cases} \qquad\qquad i,j = 1,\ldots,N$$

- formal

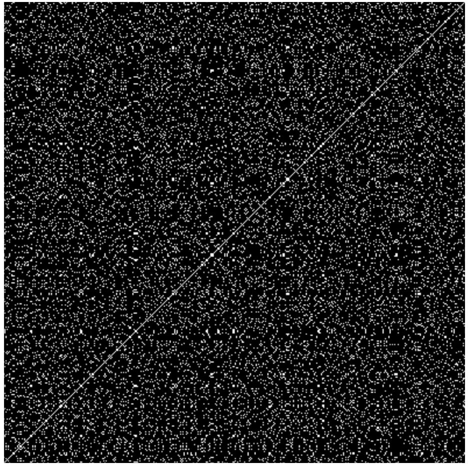
Heaviside function

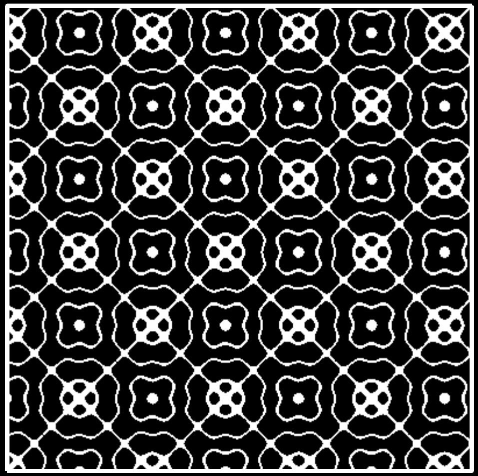
$$\mathbf{R}_{i,j}(\varepsilon) = \Theta\left(\varepsilon - \left\|\vec{x}_i - \vec{x}_j\right\|\right), \quad i,j = 1,\ldots,N$$

J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, Europhysics Letters, 5, 1987

# Recurrence Plot Typology

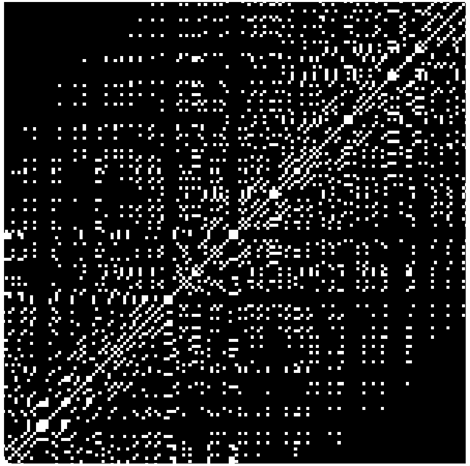
homogeneous

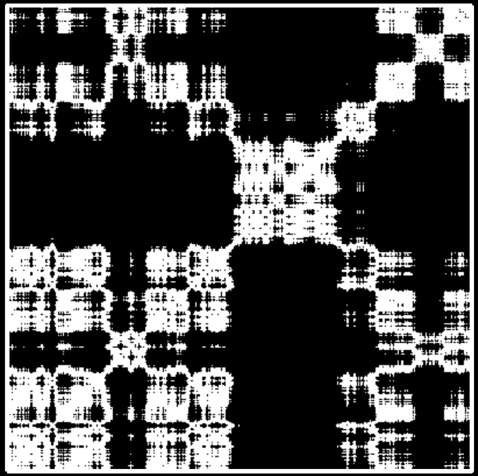




periodic

drifty

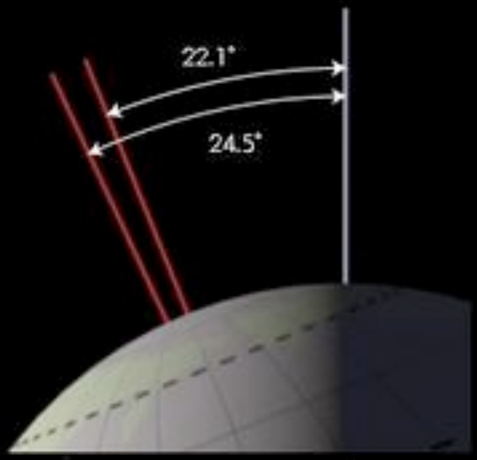




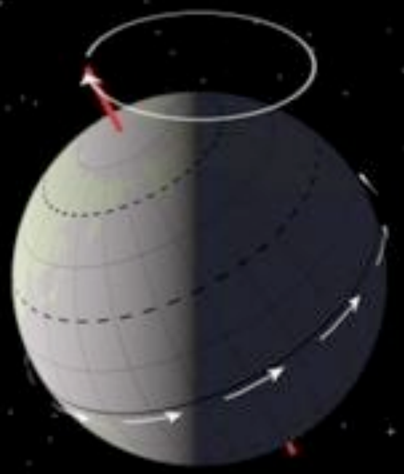
disrupted

# Solar Insolation

Eccentricity (413 kyr)



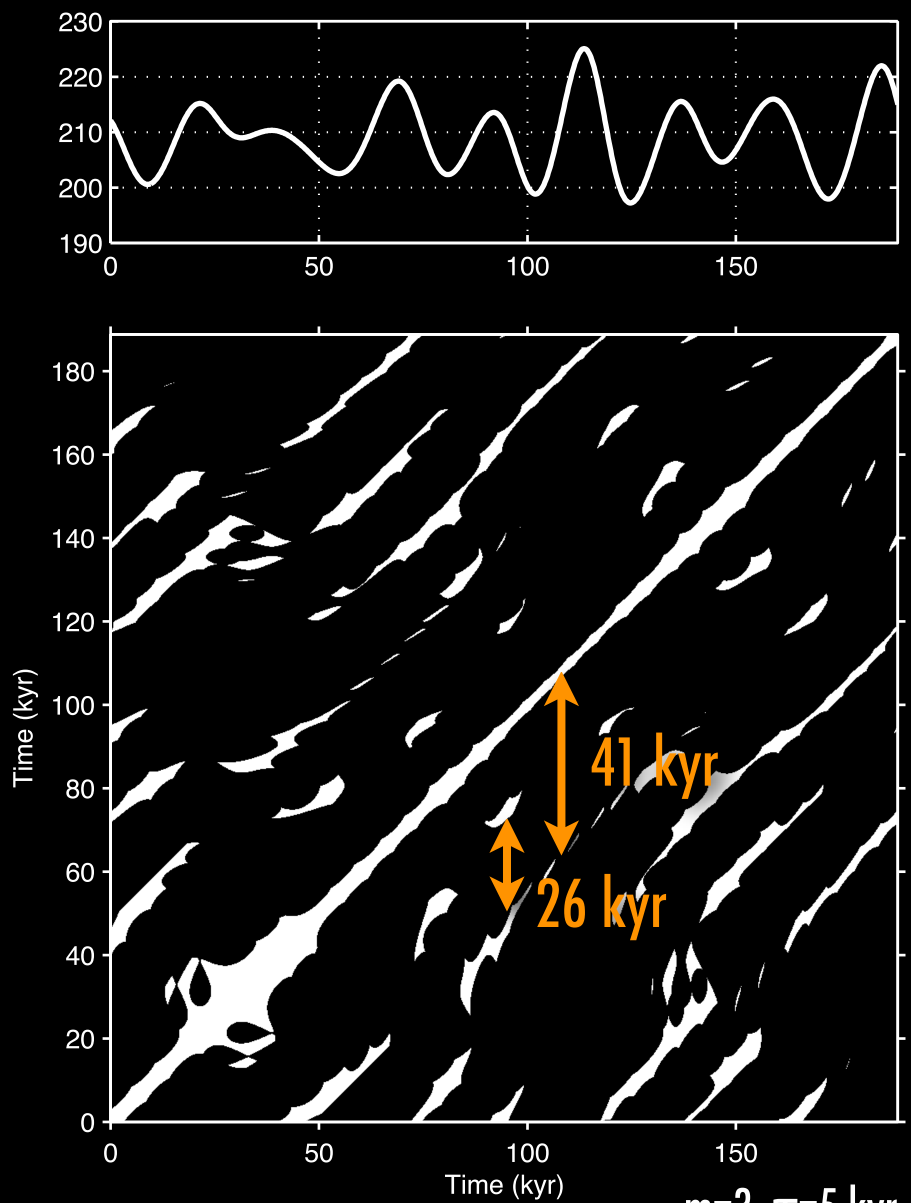
Axial tilt (41 kyr)



Precession (26 kyr)

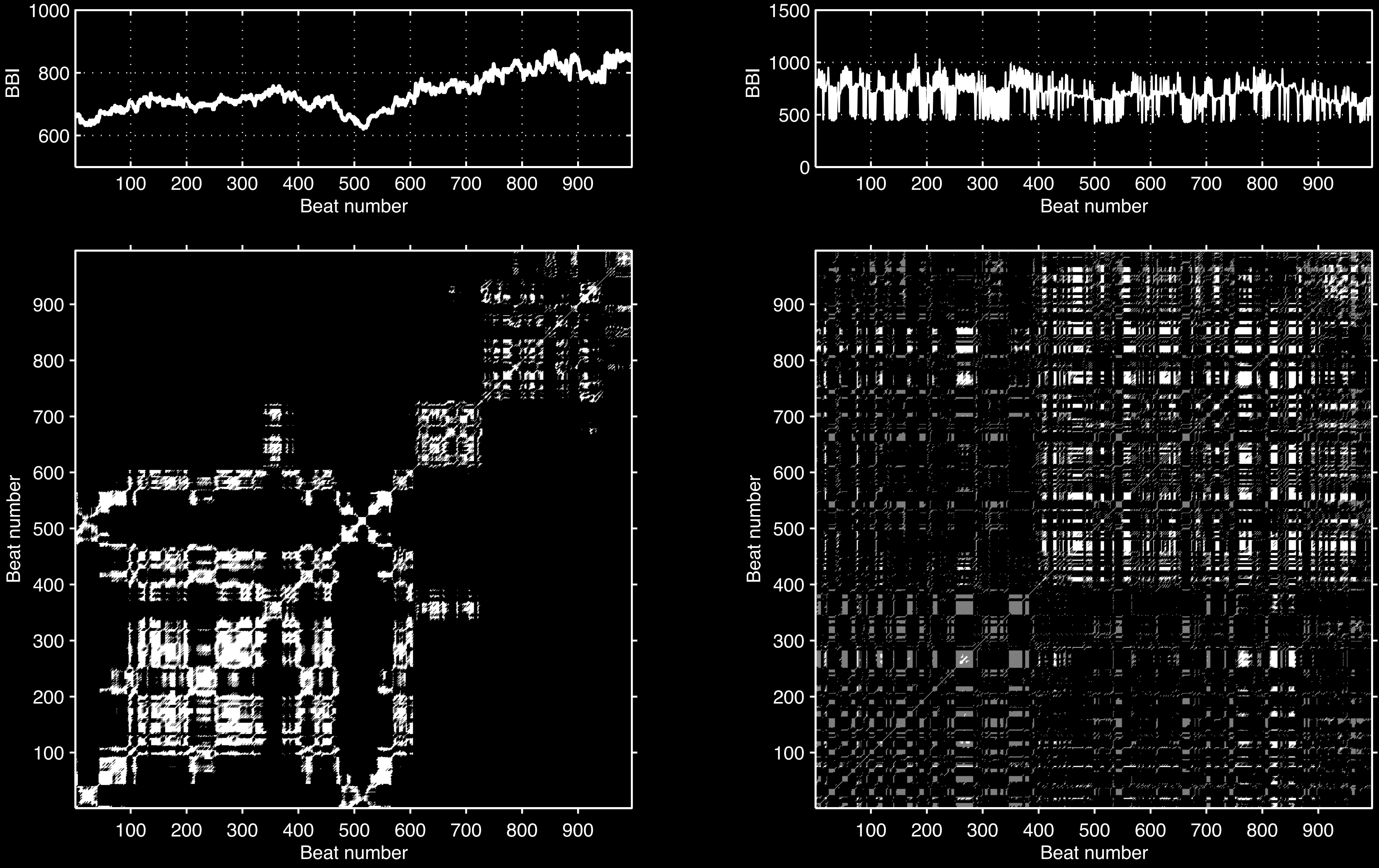
Milankovich cycles

## January insolation (44°N)



m=3, τ=5 kyr

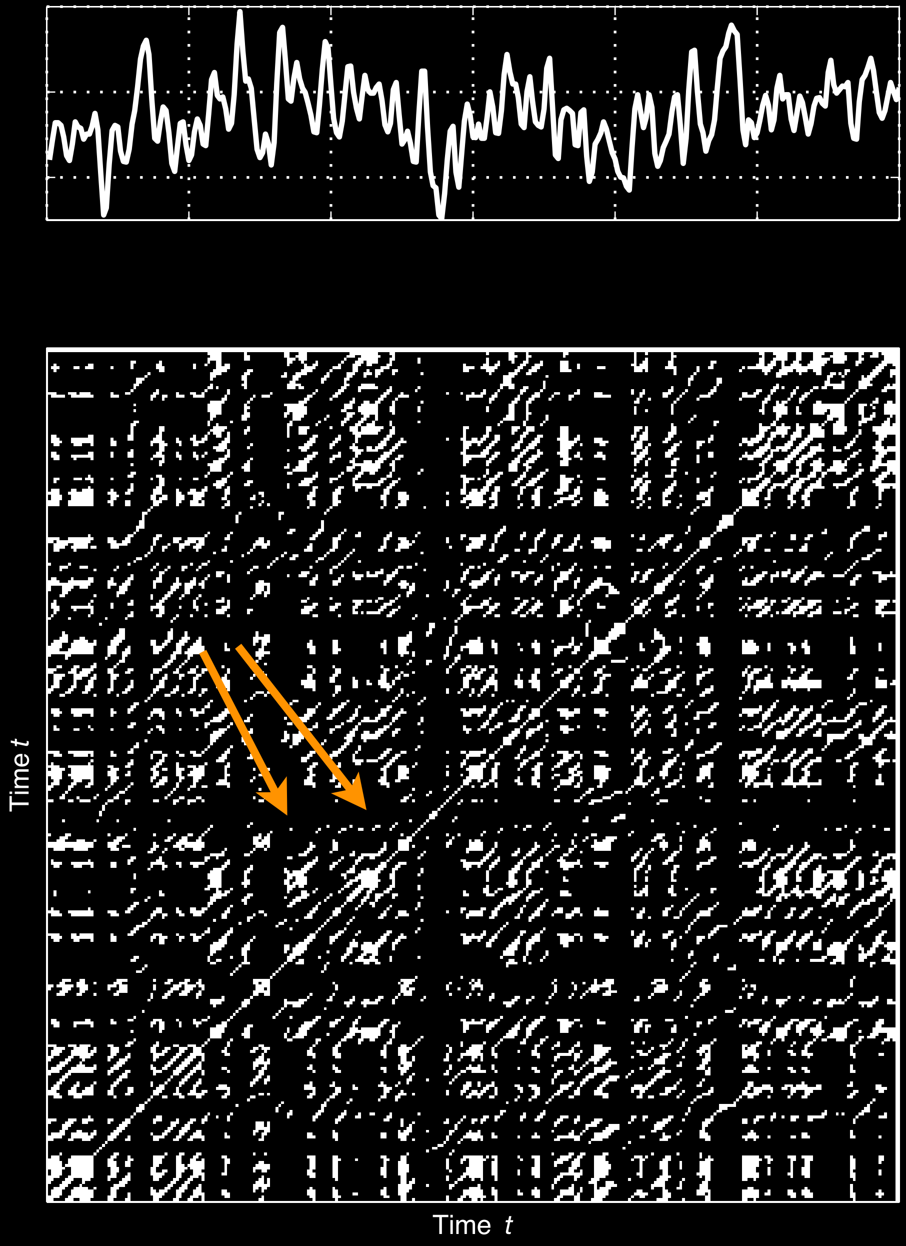
# Heart beat variability



m=5, τ=1

# Line Structures in Recurrence Plots

- Single dots
- Diagonal lines
- Vertical lines
- other line-like structures



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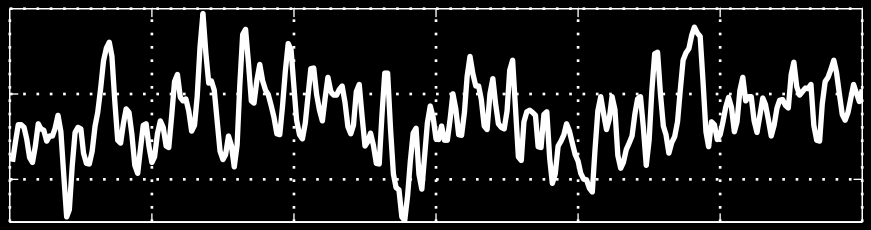

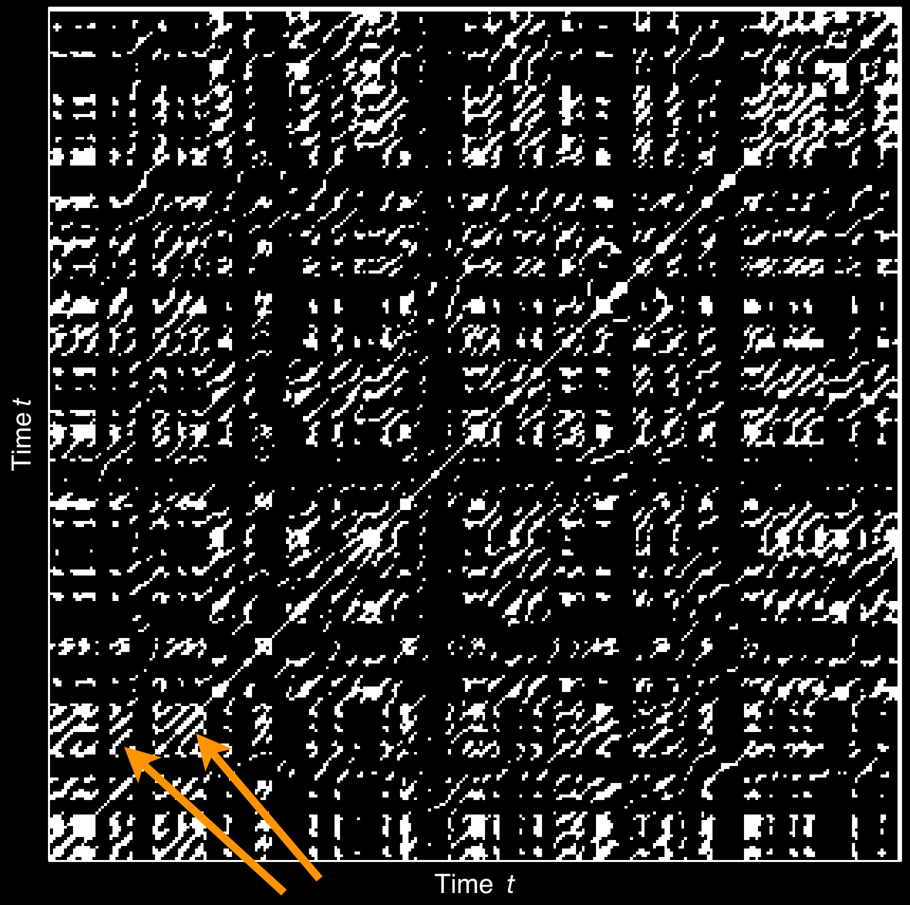

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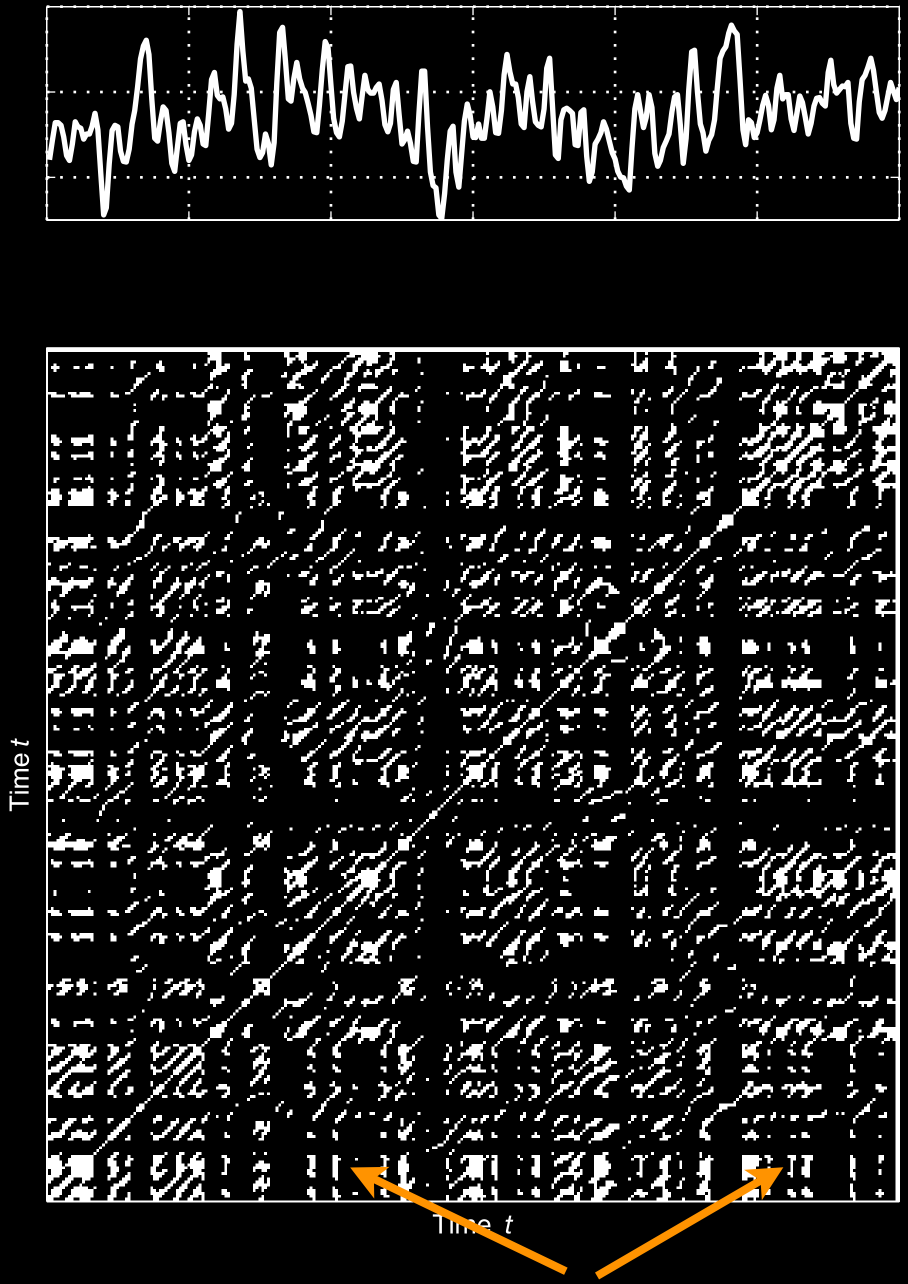
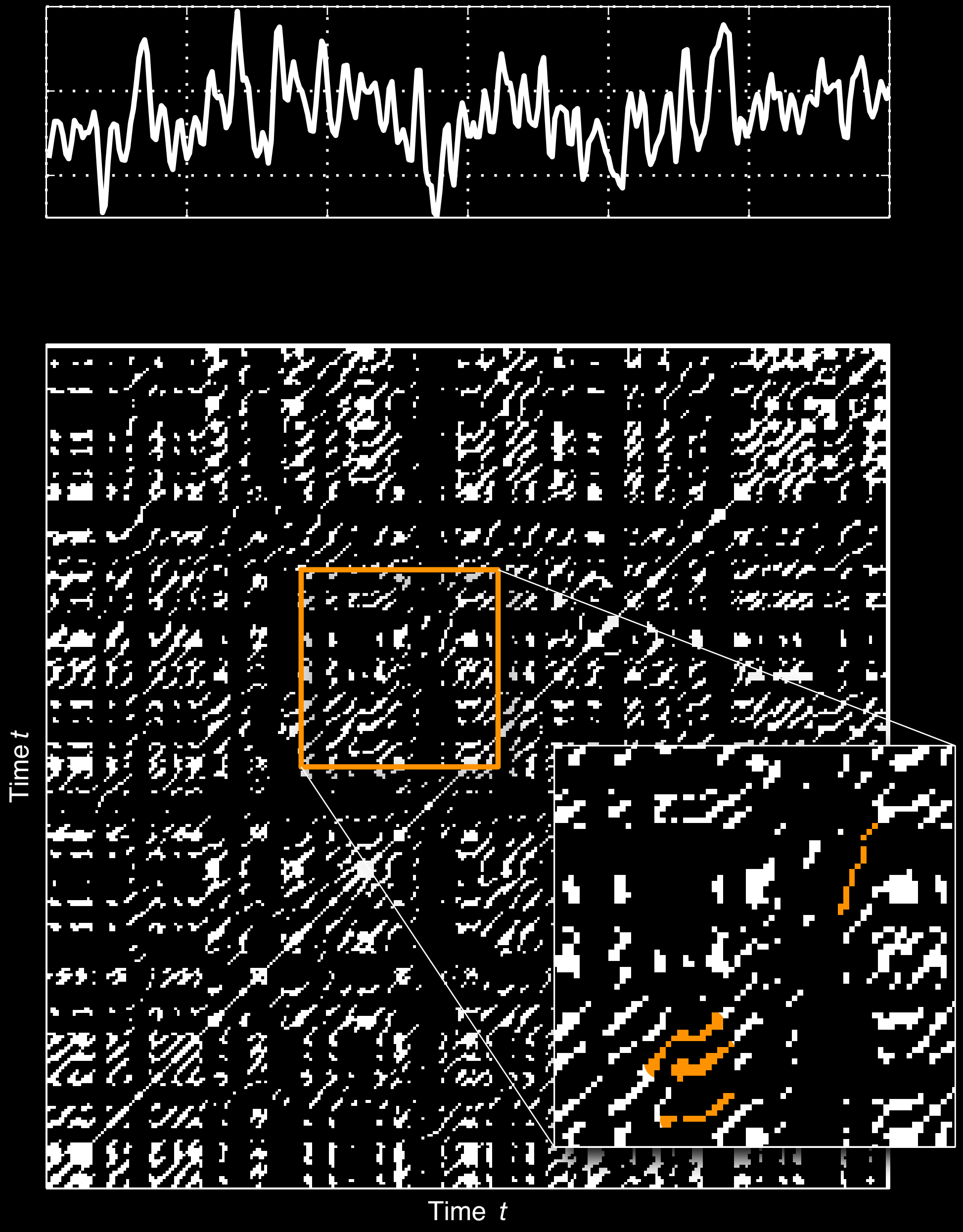


# Line Structures in Recurrence Plots



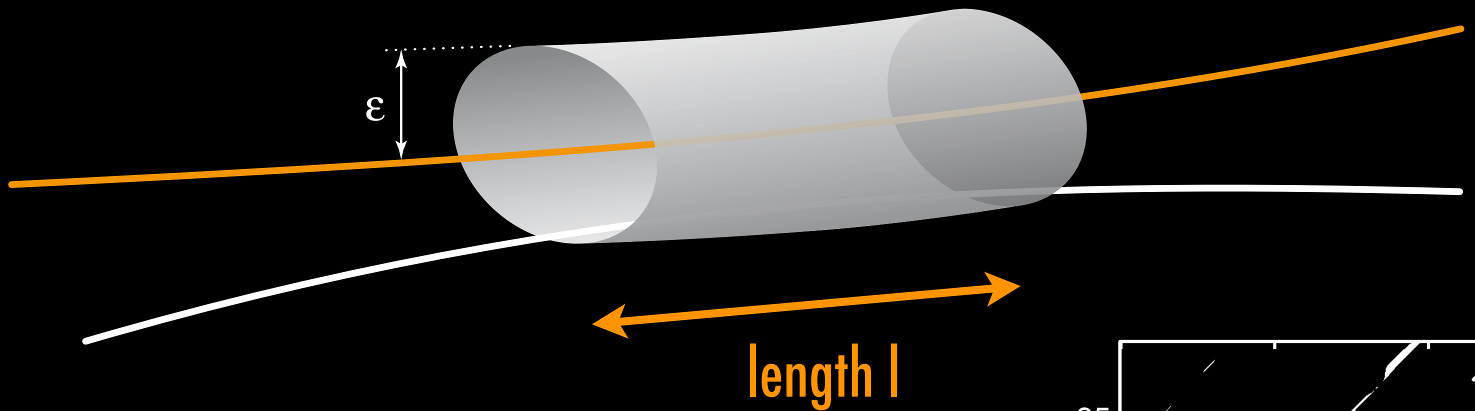
- Single dots
- Diagonal lines
- Vertical lines
- other line-like structures

# Recurrence Quantification

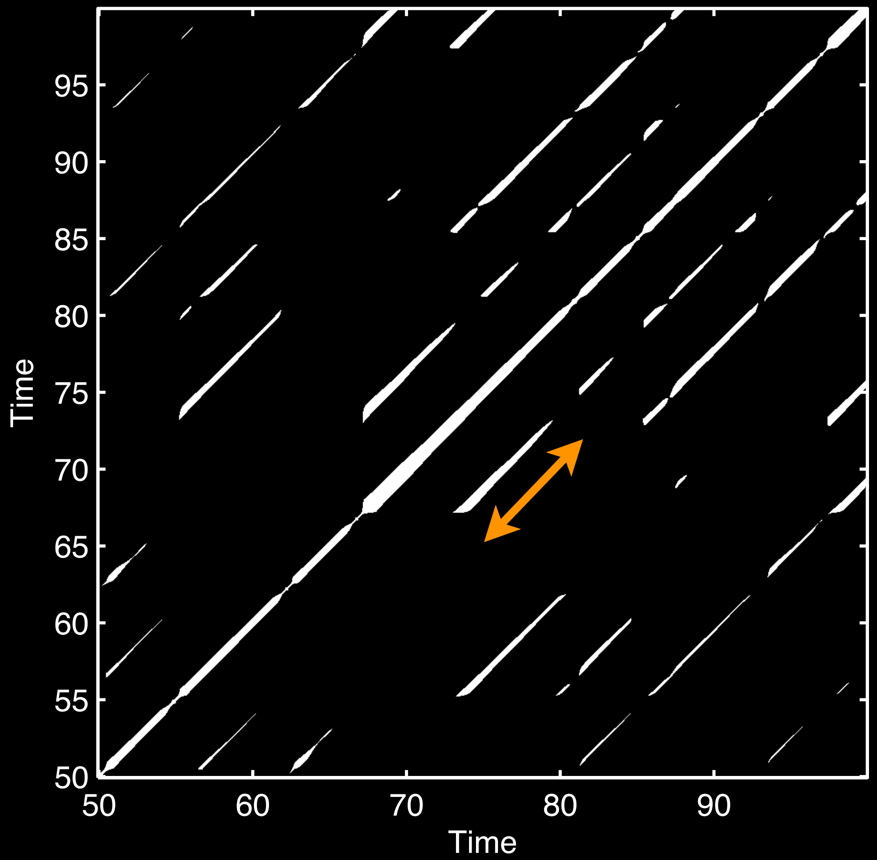
- quantitative description of RPs
- based on
  - recurrence point density
  - diagonal lines
  - vertical lines

C. L. Webber Jr., J. P. Zbilut, Journal of Applied Physiology, 76, 1994
N. Marwan, N. Wessel, U. Meyerfeldt, A. Schirdewan, J. Kurths, Physical Review E, 66, 2002

# Recurrence Quantification



- number of lines of exactly length l
  - histogram P(l)



# Recurrence Quantification

- Determinism DET

$$DET = \frac{\sum_{l=l_{\min}}^{N} l\, P(l)}{\sum_{l=1}^{N} l\, P(l)}$$

Probability that recurrences further recur

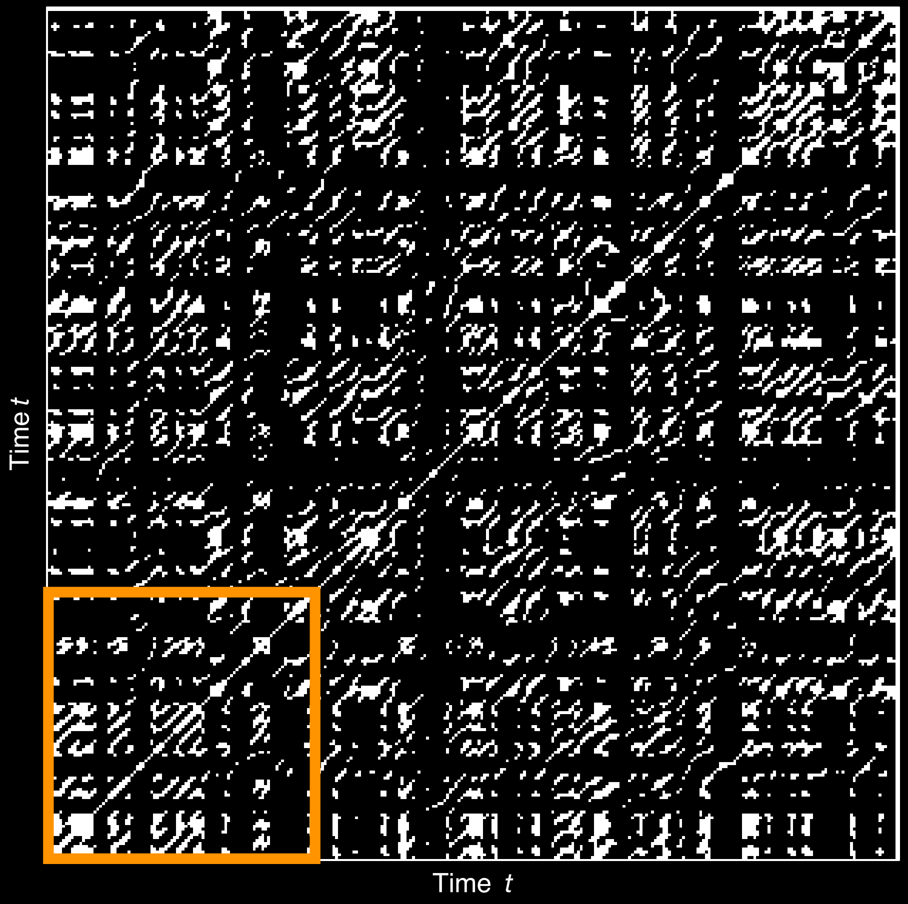
- Laminarity LAM

$$LAM = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)}$$

Probability that a certain recurrent state further recurs

# Recurrence Quantification

- Time dependent analysis:
  - sliding windows over RP
- Detection of transitions



# Recurrence Quantification

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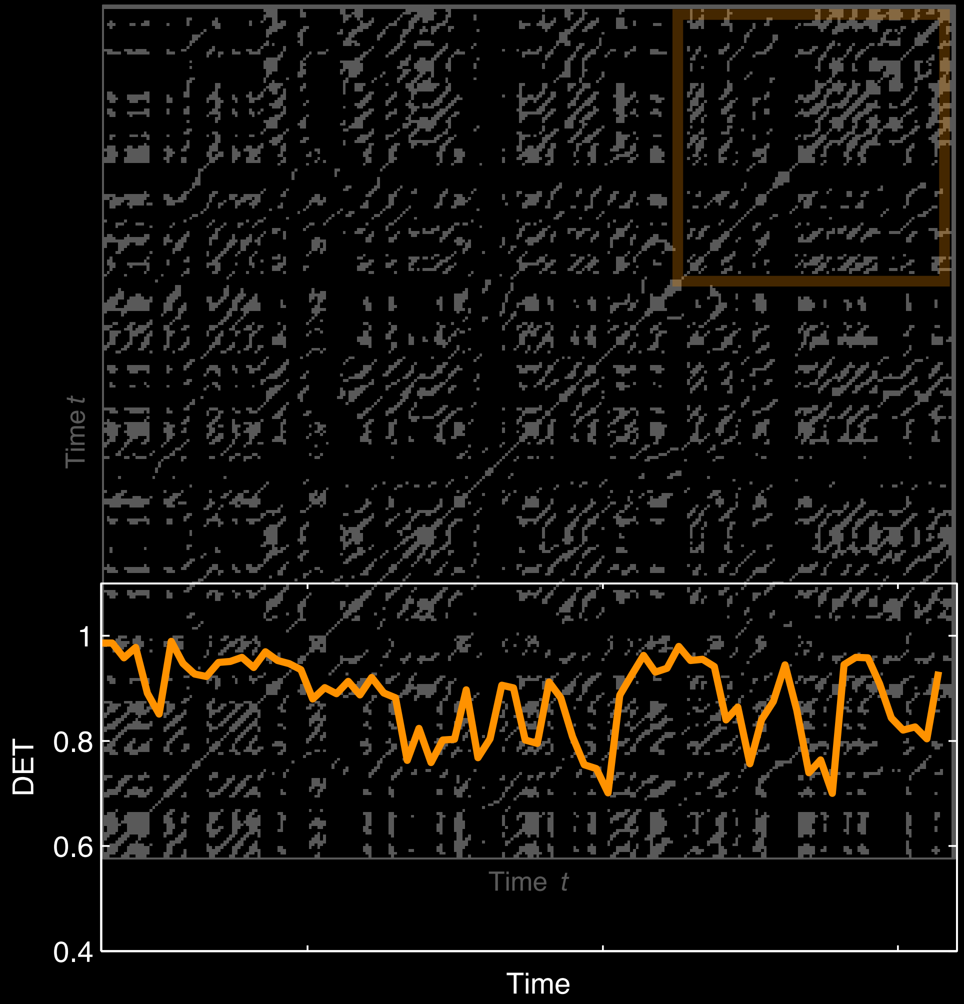
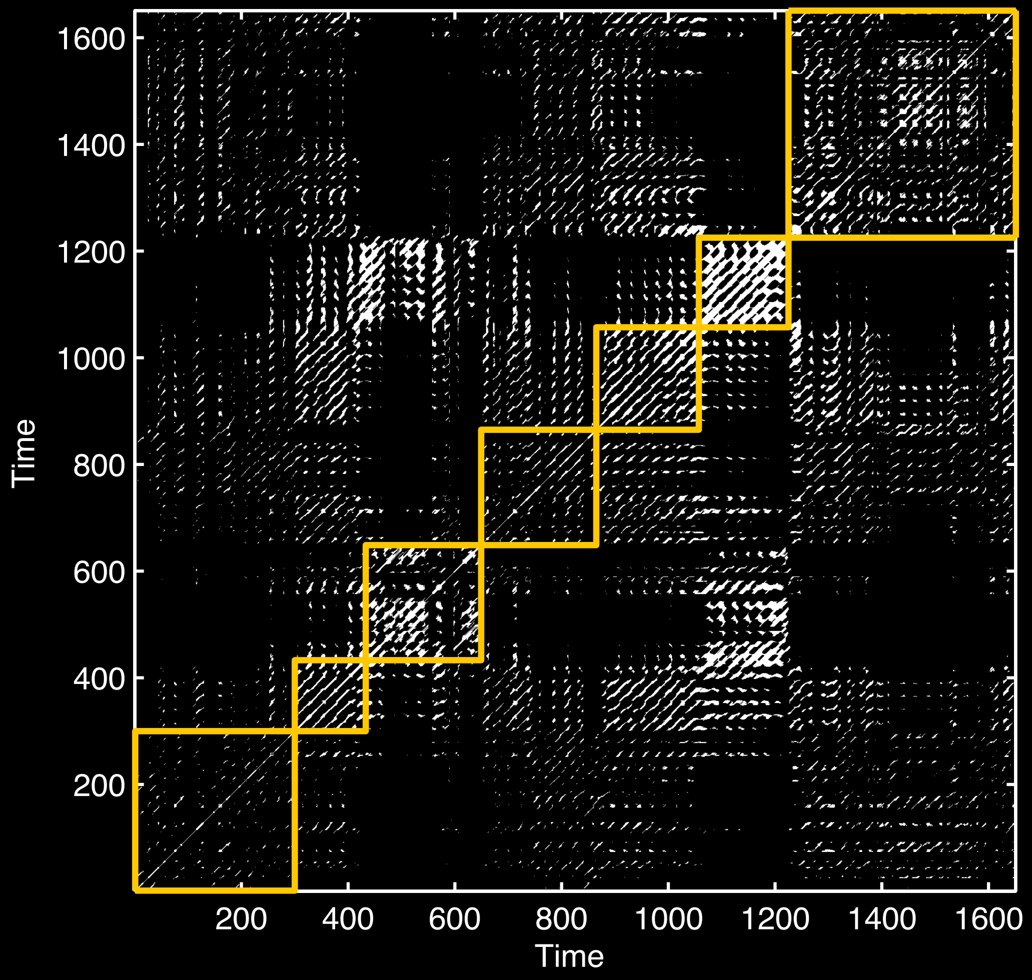


# Dynamics of Oxygen Crises in Lakes



Facchini et al., Ecological Modelling, 203, 2007

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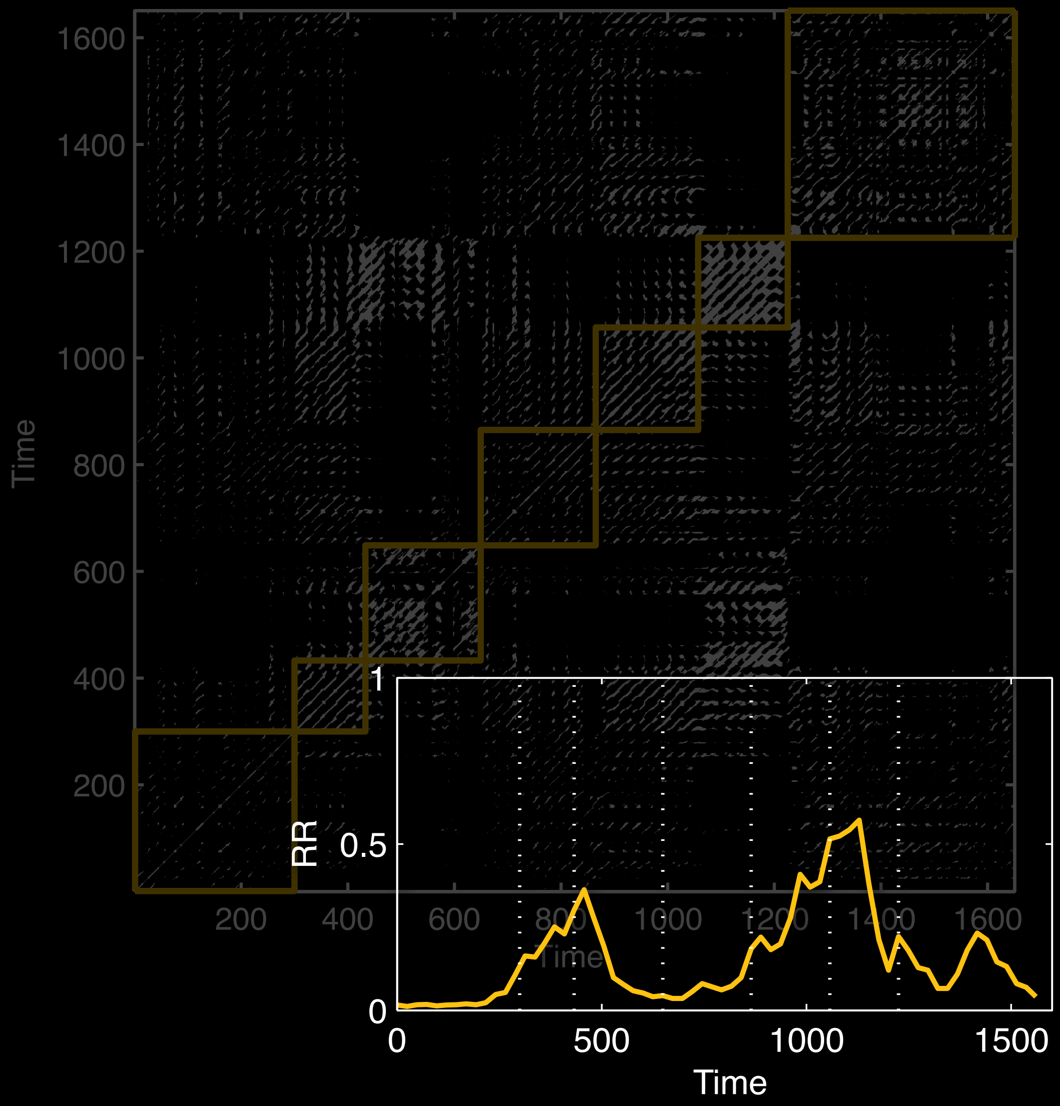
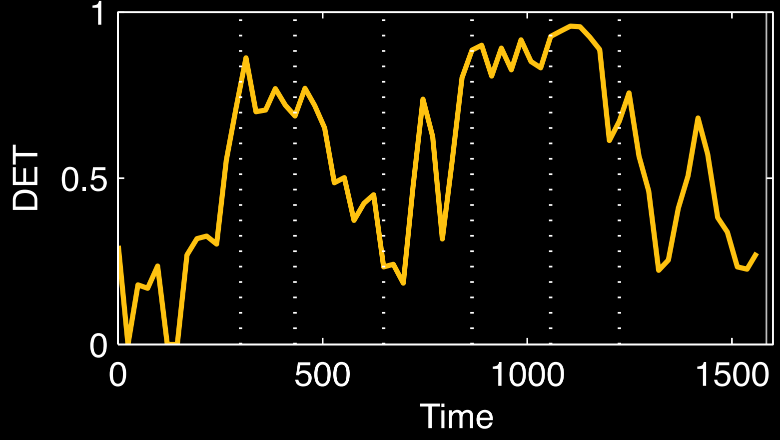
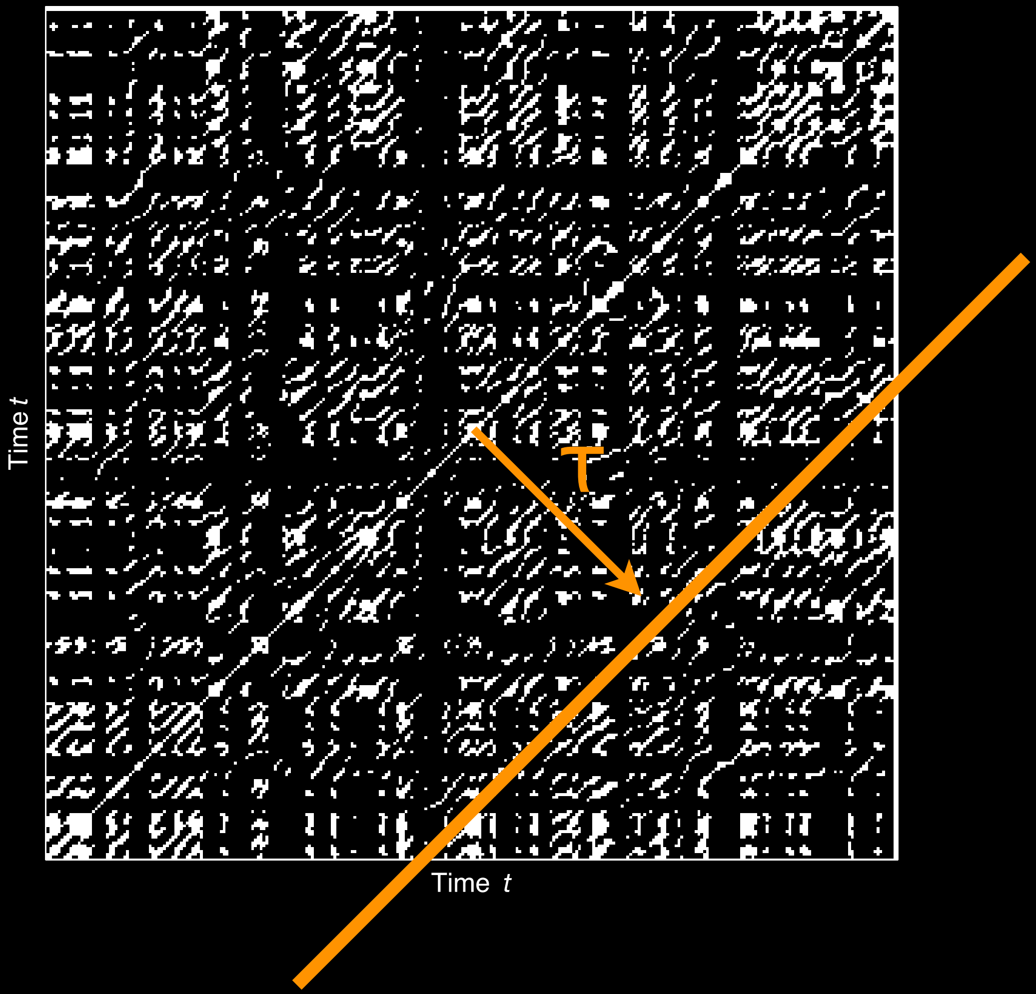




Facchini et al., Ecological Modelling, 203, 2007

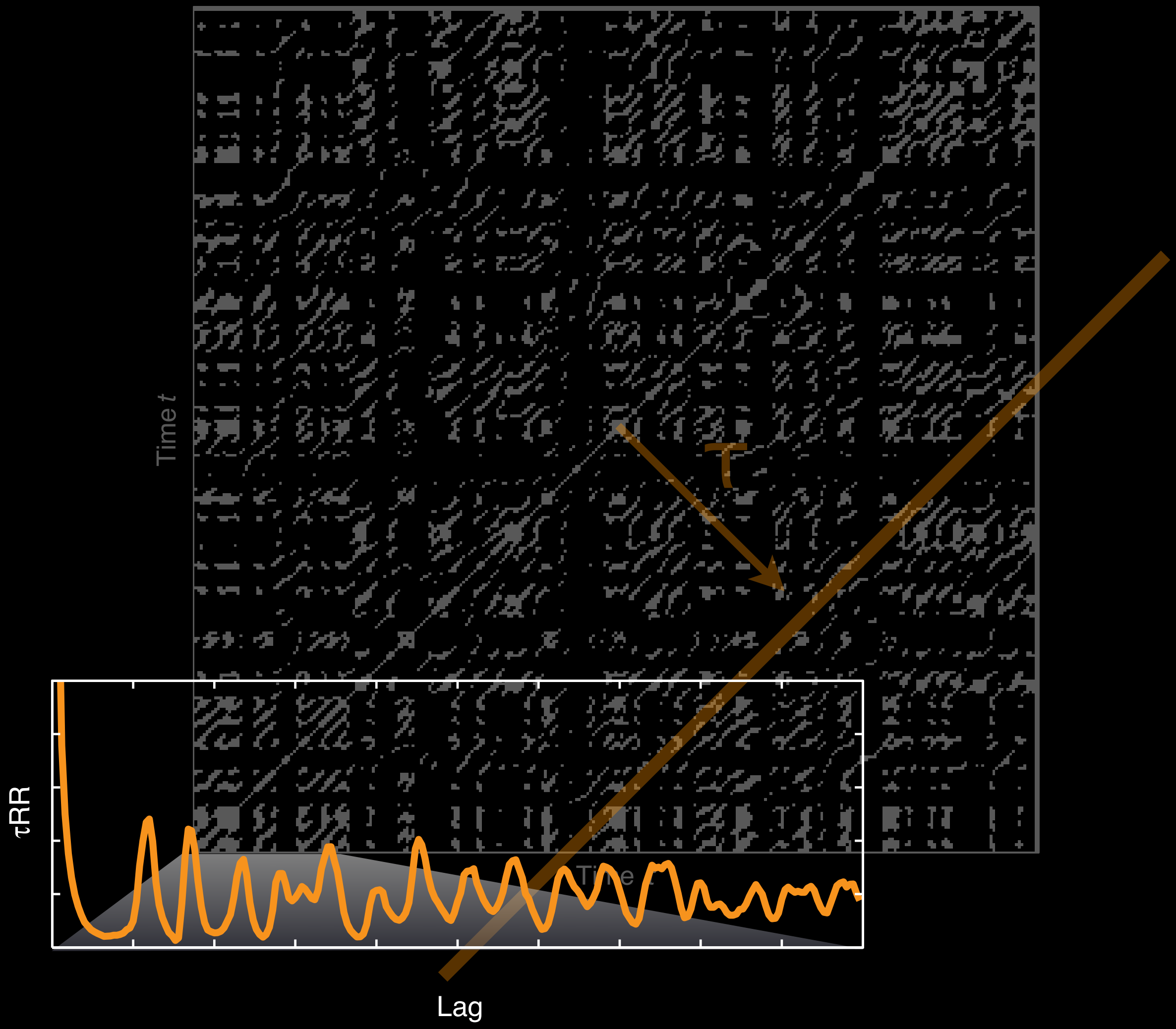
# Phase Synchronisation

- Probability that system recurs after time τ



# Phase Synchronisation

- Probability that system recurs after time τ



# Phase Synchronisation

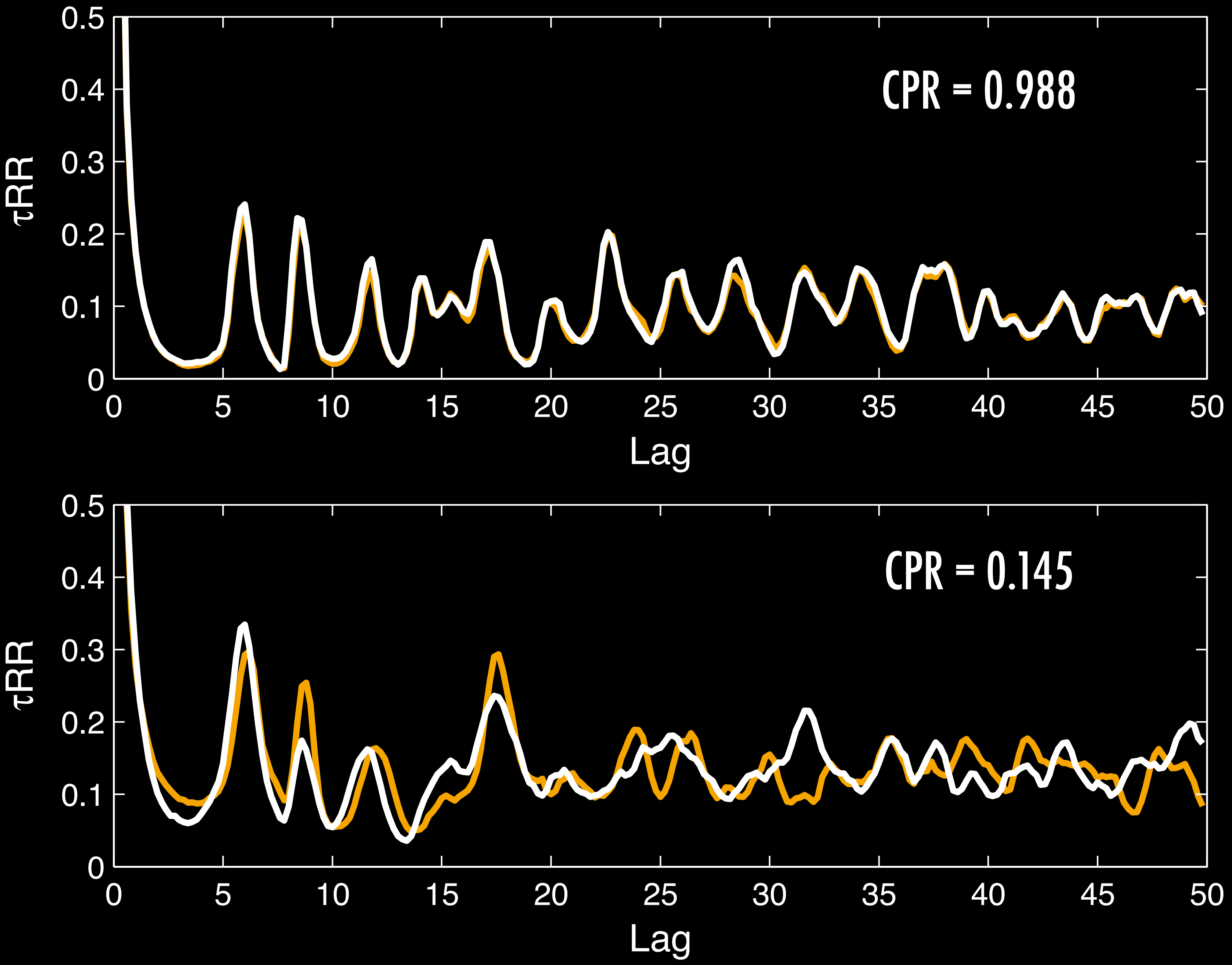
- Correlation coefficient

$$CPR = \langle \bar{p}_{\vec{x}}(\tau)\bar{p}_{\vec{y}}(\tau)\rangle$$

- high-dimensional systems, non phase coherent attractors

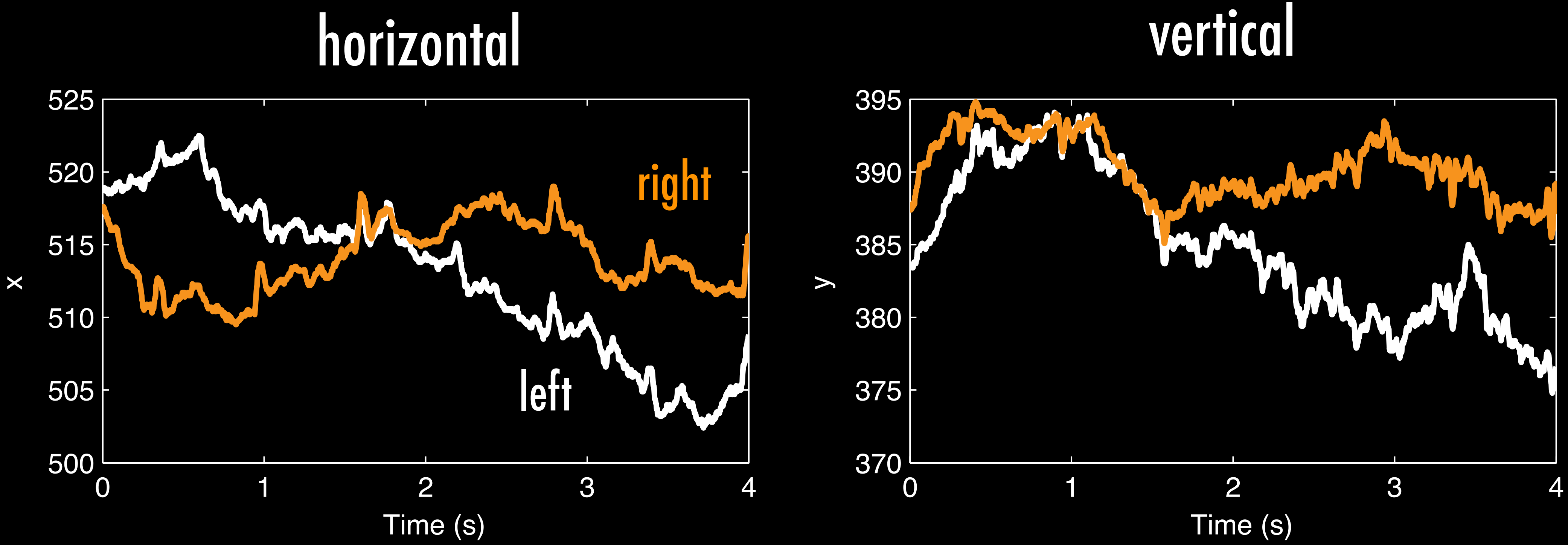
M. C. Romano, M. Thiel, J. Kurths, I. Z. Kiss, J. Hudson, Europhysics Letters, 71, 2005

# Synchronisation Analysis

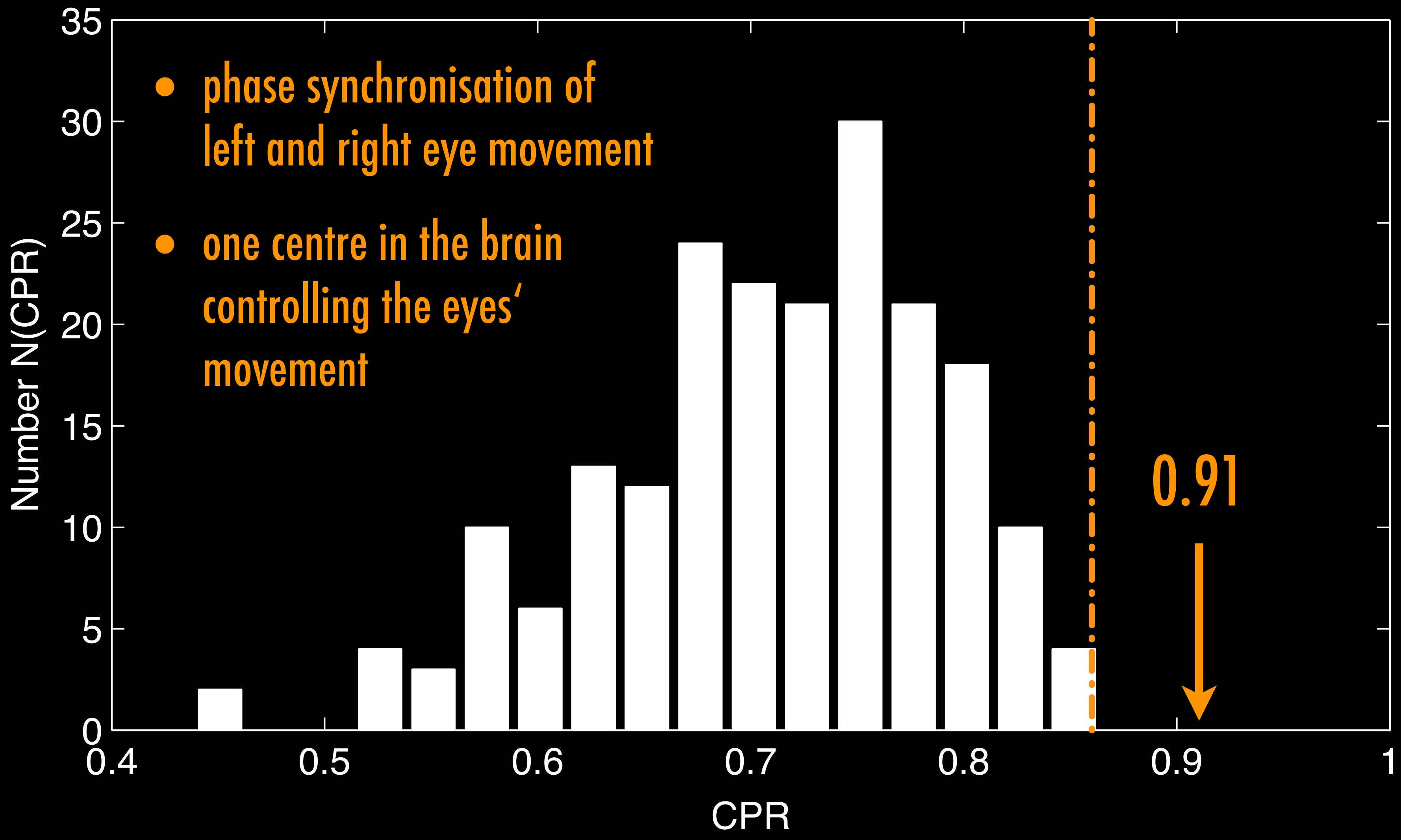


# Fixational Eye Movements

# Fixational Eye Movements



# Fixational Eye Movements



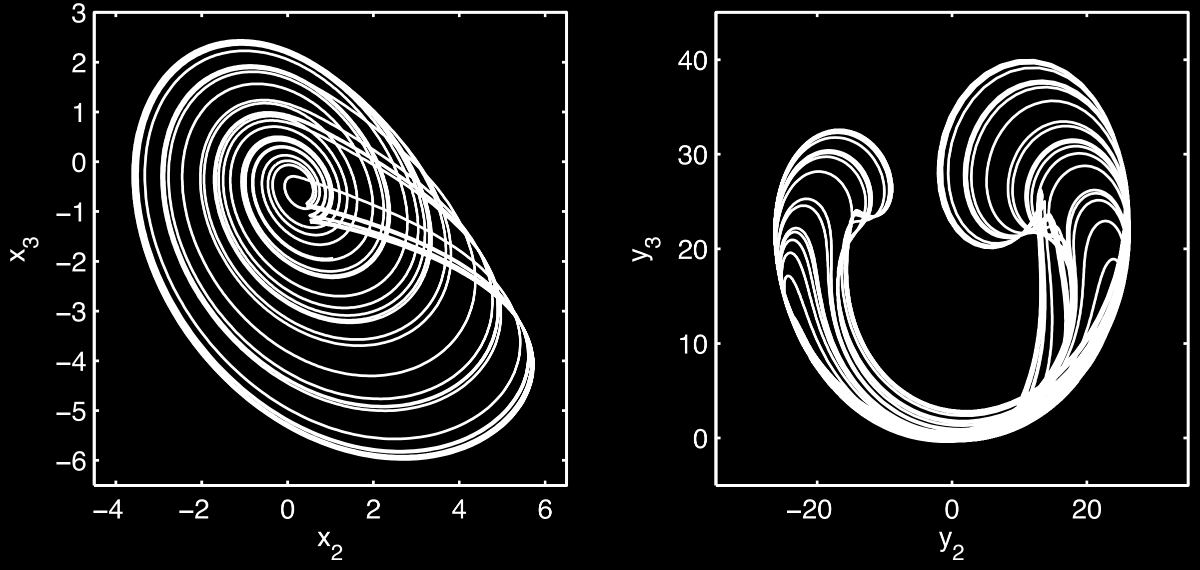
- phase synchronisation of left and right eye movement
- one centre in the brain controlling the eyes‘ movement

# Coupling Direction

Rössler oscillator drives Lorenz oscillator

$$\begin{aligned} \dot{x}_1 &= b + x_1(x_2 - c) \\ \dot{x}_2 &= -x_1 - x_3 \\ \dot{x}_3 &= x_2 + a x_3 \end{aligned} \qquad \begin{aligned} \dot{y}_1 &= -\sigma(y_1 - y_2) \\ \dot{y}_2 &= r\,u - y_2 - u\,y_3 \\ \dot{y}_3 &= u\,y_2 - b\,y_3 \end{aligned}$$

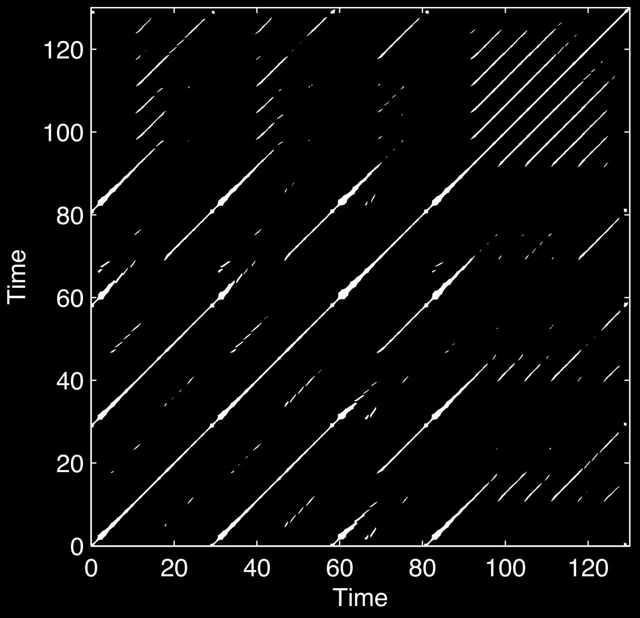
$$\text{where} \quad u = x_1 + x_2 + x_3$$

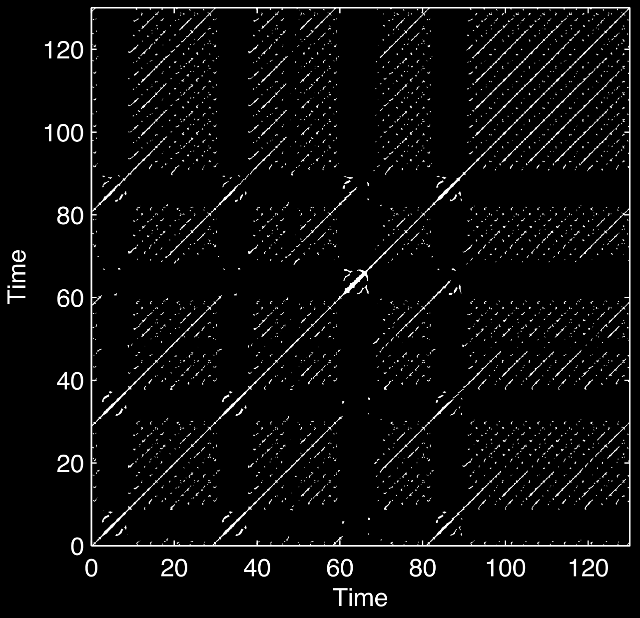


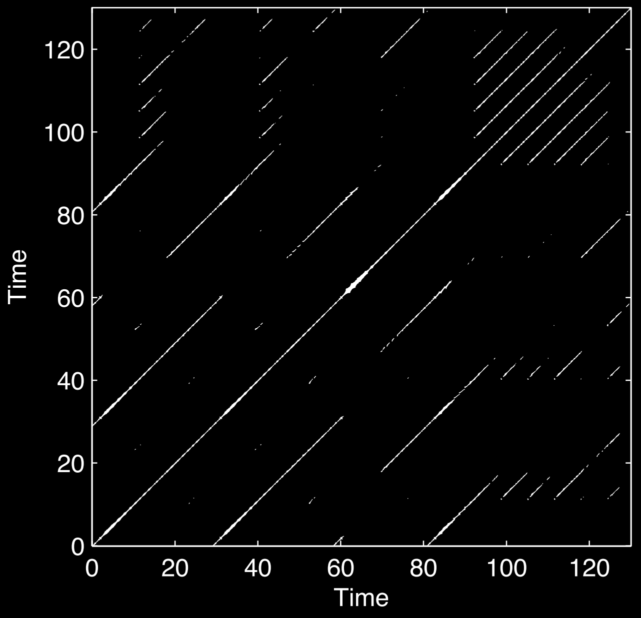
# Coupling Direction

- Joint recurrence plot:

$$\mathbf{JR}_{i,j}(x,y) = \mathbf{R}_{i,j}(x) \cdot \mathbf{R}_{i,j}(y)$$


RP Rössler


RP Lorenz


JRP

# Coupling Direction

prob. of recurrence of state $x_i$

$$p(x_i) = \sum_{j=1}^{N} \mathbf{R}_{i,j}(x)$$

joint prob. of recurrence of state $x_i$ and $y_i$

$$p(x_i, y_i) = \sum_{j=1}^{N} \mathbf{JR}_{i,j}(x,y)$$

mean conditional prob. of recurrence between x and y

$$MCR(y|x) = \frac{1}{N} \sum_{i=1}^{N} p(y_i|x_i) = \frac{1}{N} \sum_{i=1}^{N} \frac{\sum_{j=1}^{N} \mathbf{JR}_{i,j}(x,y)}{\sum_{j=1}^{N} \mathbf{R}_{i,j}(x)}$$

Romano et al., Physical Review E, 76, 2007

# Coupling Direction

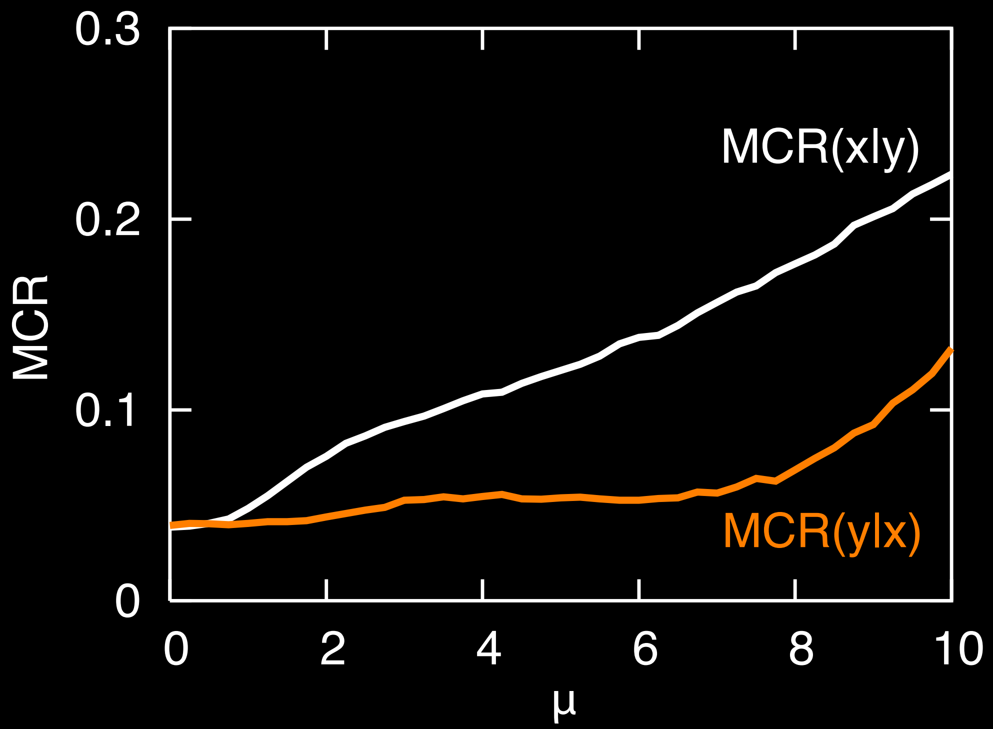
- $MCR(y|x) < MCR(x|y)$

  ➡ x drives y

- $MCR(x|y) < MCR(y|x)$

  ➡ y drives x

weakly coupled, non-identical Lorenz oscillators



Romano et al., Physical Review E, 76, 2007

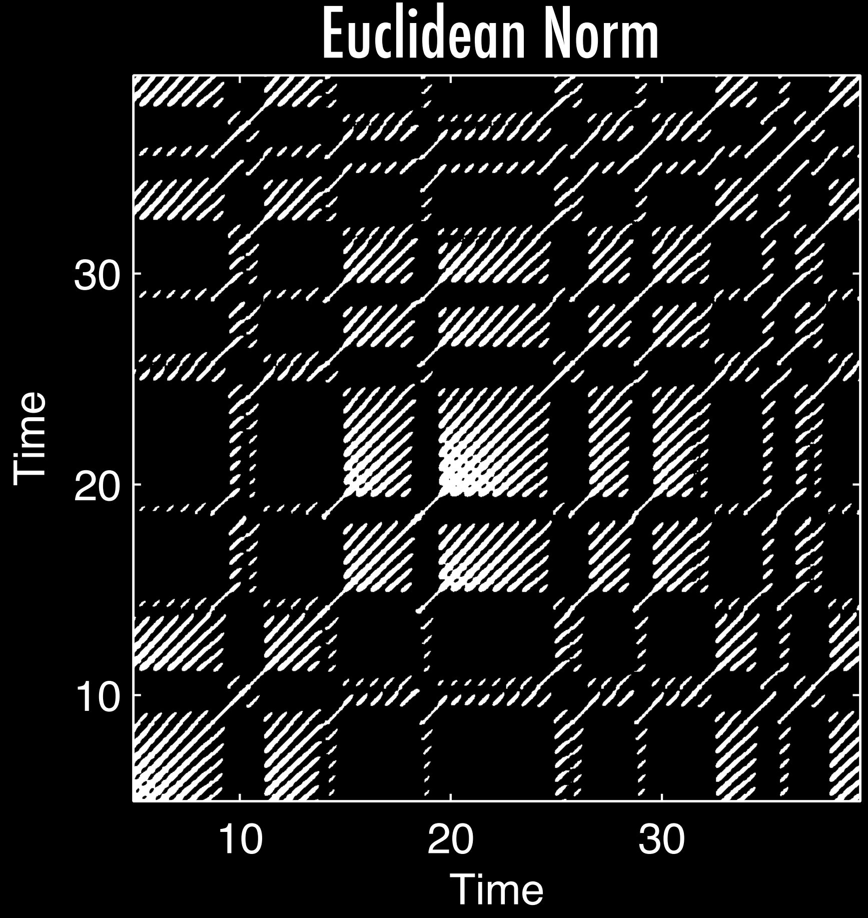
# Coupling Direction

- Joint recurrences:

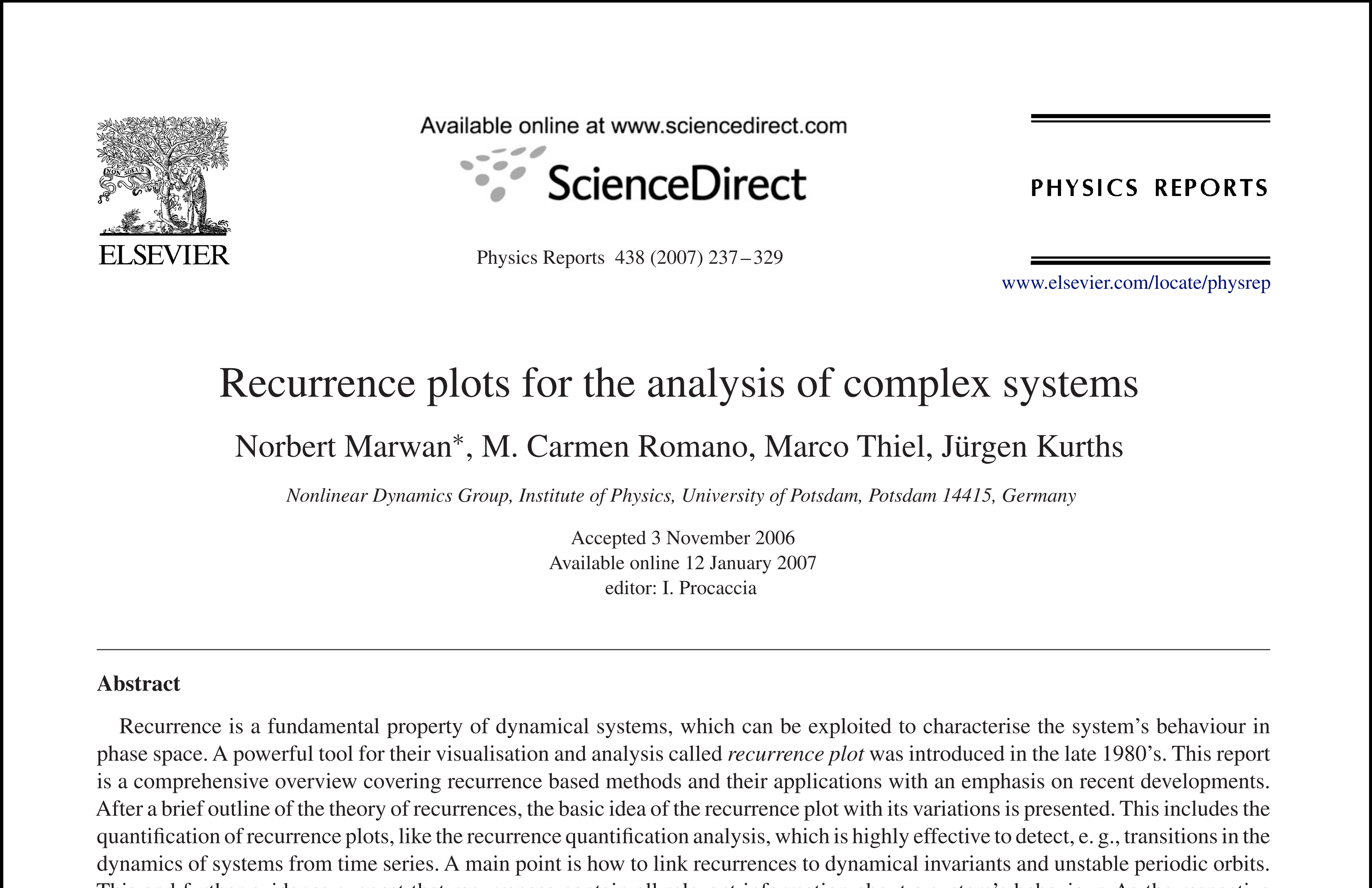
  promising approach for detection of coupling directions

# Recurrence Plot

- Transition detection
- Differentiate dynamics
- Finding time scales
- Interrelation detection
- Synchronisation analysis
- Surrogates
- Recurrence time statistics
- Noise reduction
- Detection of weak frequency changes



# Literature



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## Recurrence plots for the analysis of complex systems

Norbert Marwan*, M. Carmen Romano, Marco Thiel, Jürgen Kurths

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Accepted 3 November 2006
Available online 12 January 2007
editor: I. Procaccia

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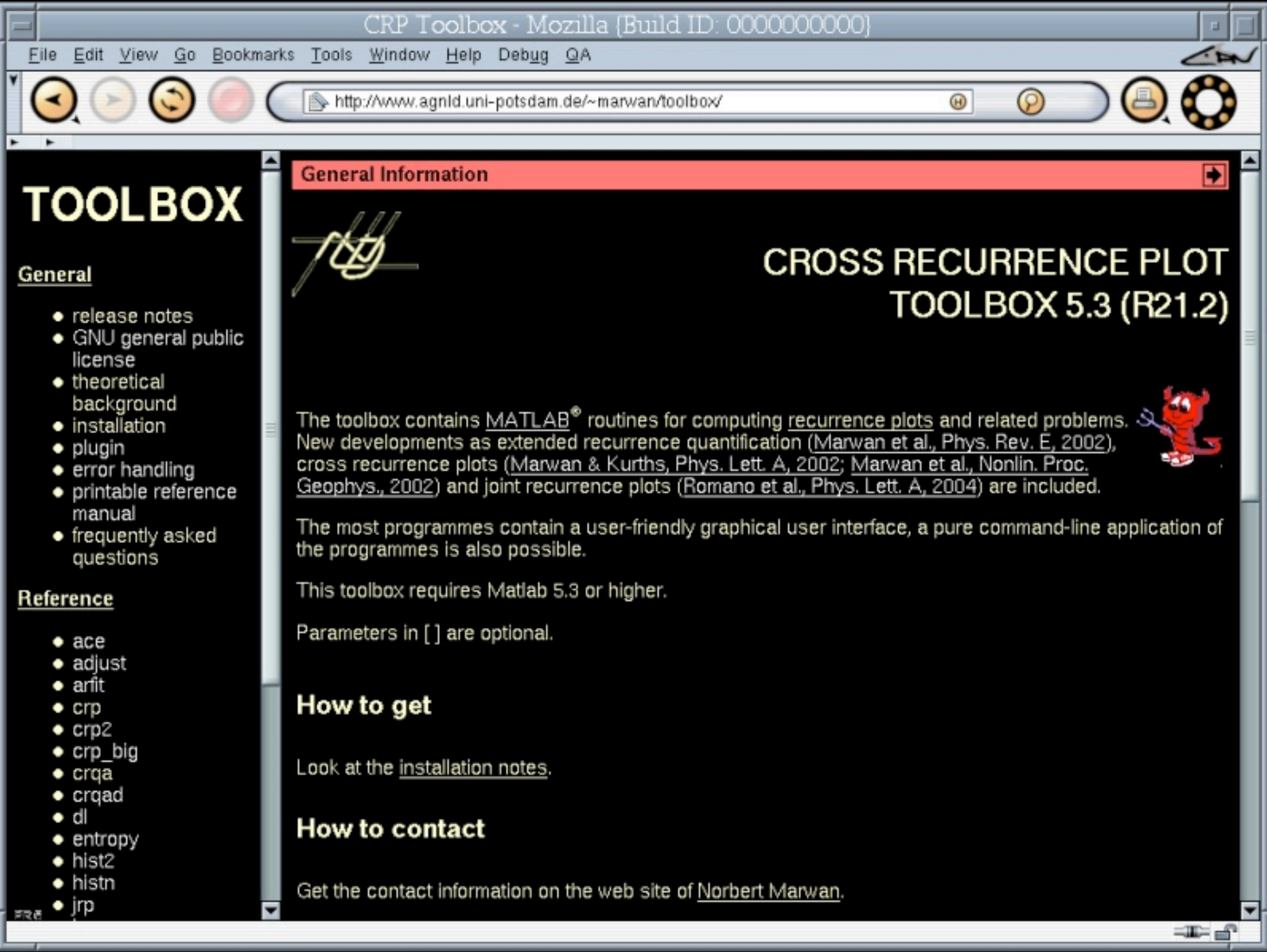
### Abstract

Recurrence is a fundamental property of dynamical systems, which can be exploited to characterise the system's behaviour in phase space. A powerful tool for their visualisation and analysis called *recurrence plot* was introduced in the late 1980's. This report is a comprehensive overview covering recurrence based methods and their applications with an emphasis on recent developments. After a brief outline of the theory of recurrences, the basic idea of the recurrence plot with its variations is presented. This includes the quantification of recurrence plots, like the recurrence quantification analysis, which is highly effective to detect, e. g., transitions in the dynamics of systems from time series. A main point is how to link recurrences to dynamical invariants and unstable periodic orbits.



# TOOLBOX

**General**

- release notes
- GNU general public license
- theoretical background
- installation
- plugin
- error handling
- printable reference manual
- frequently asked questions

**Reference**

- ace
- adjust
- arfit
- crp
- crp2
- crp_big
- crqa
- crqad
- dl
- entropy
- hist2
- histn
- jrp

**General Information**

## CROSS RECURRENCE PLOT TOOLBOX 5.3 (R21.2)

The toolbox contains MATLAB® routines for computing recurrence plots and related problems. New developments as extended recurrence quantification (Marwan et al., Phys. Rev. E, 2002), cross recurrence plots (Marwan & Kurths, Phys. Lett. A, 2002; Marwan et al., Nonlin. Proc. Geophys., 2002) and joint recurrence plots (Romano et al., Phys. Lett. A, 2004) are included.

The most programmes contain a user-friendly graphical user interface, a pure command-line application of the programmes is also possible.

This toolbox requires Matlab 5.3 or higher.

Parameters in [ ] are optional.

### How to get

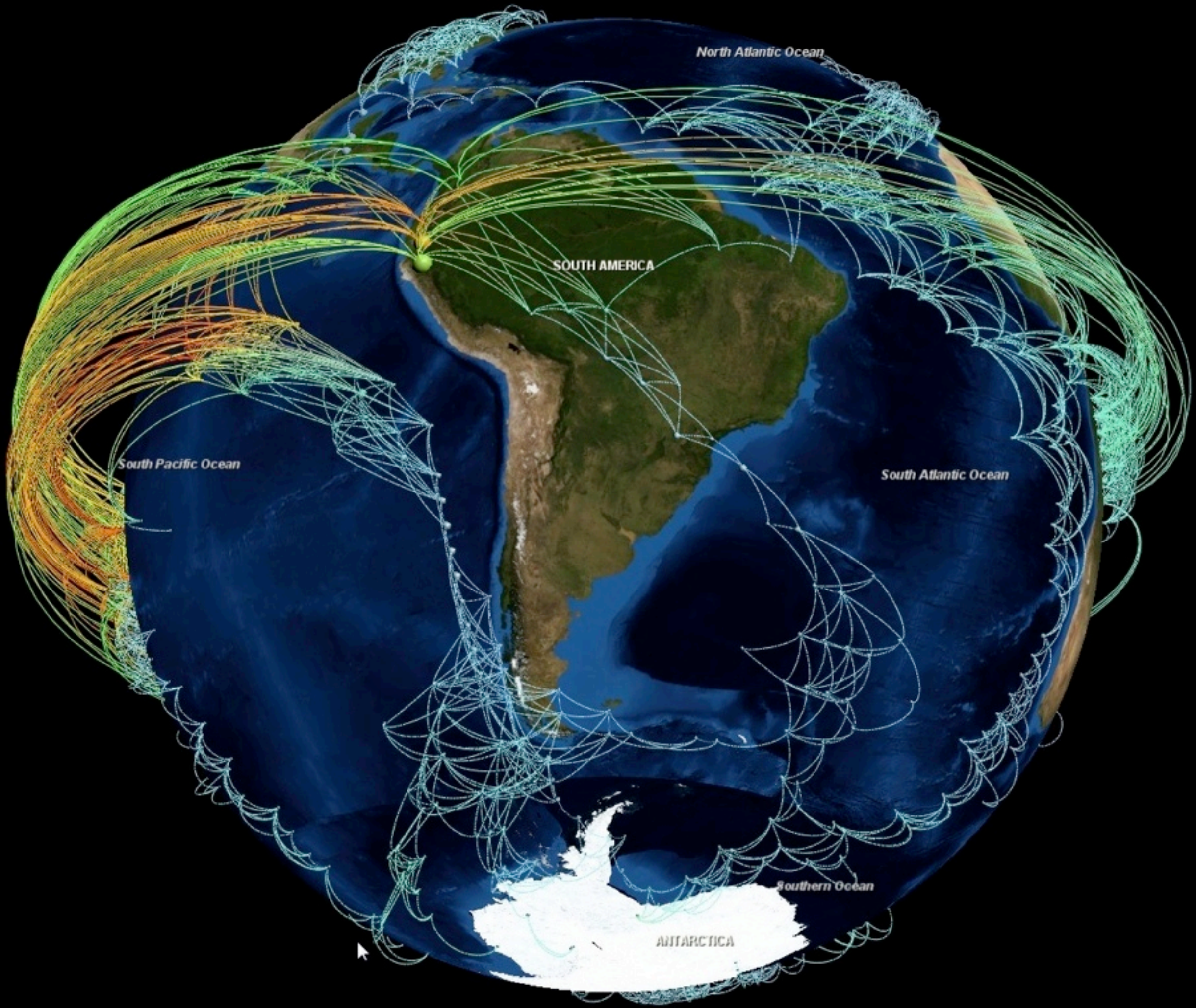
Look at the installation notes.

### How to contact

Get the contact information on the web site of Norbert Marwan.

# Installation

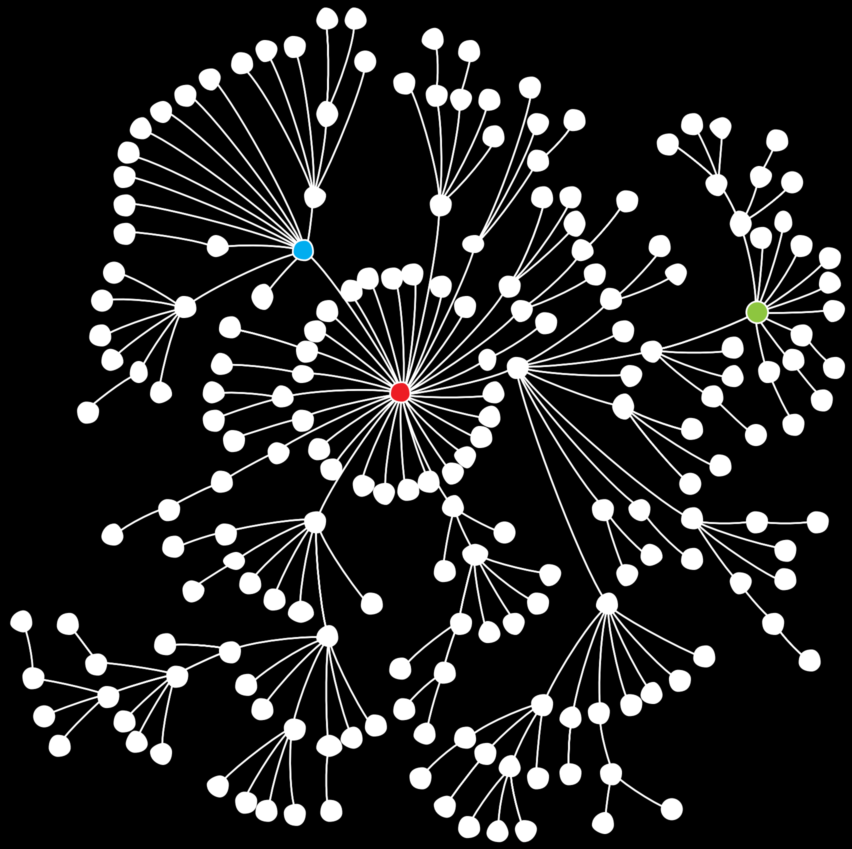
- web site:
  http://tocsy.agnld.uni-potsdam.de (CRP Toolbox)
- request access data
- download installation file install.m
- call install on the Matlab commandline



# Complex Networks

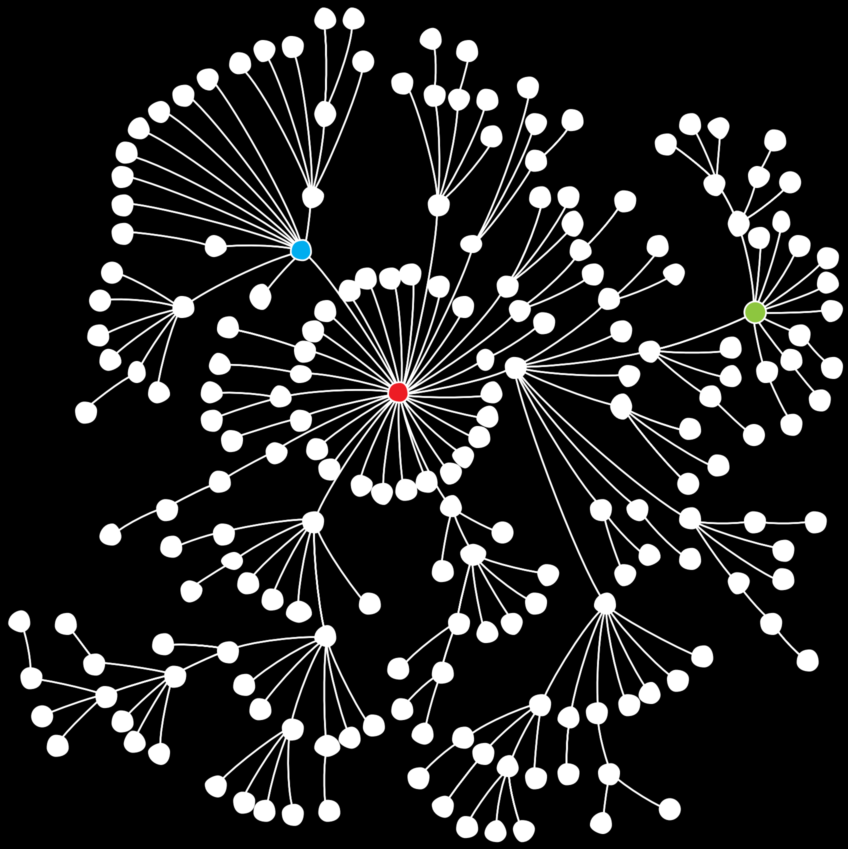
# Complex Networks

- spatio-temporal analysis
- different classes of networks (random, scale-free, small world, regular, modular)
- identification of hubs, stability, clusters, self-organisation

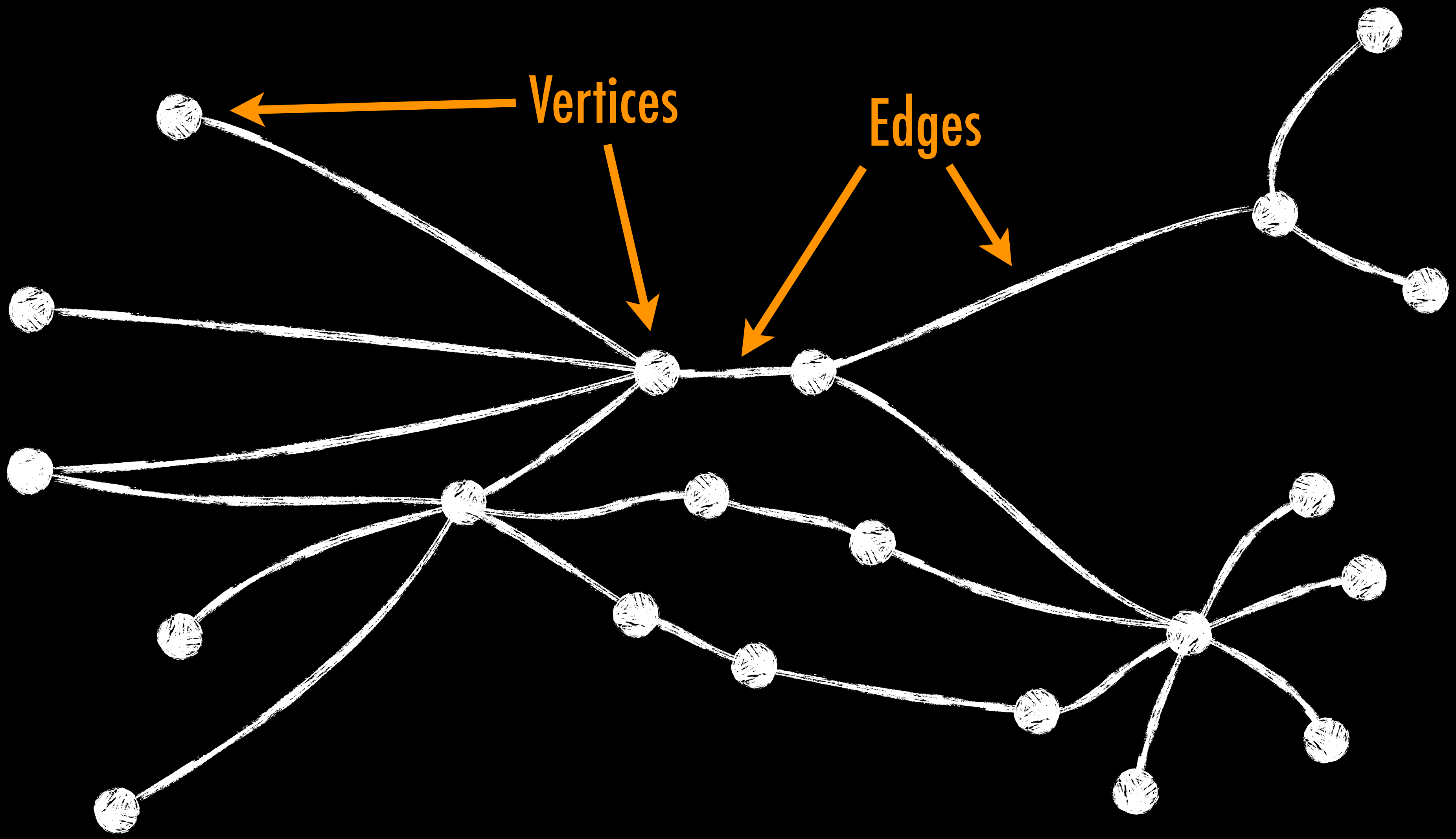


# Complex Networks

- Successful applications in many fields
  - social networks
  - brain dynamics
  - power grids
  - metabolic networks etc.

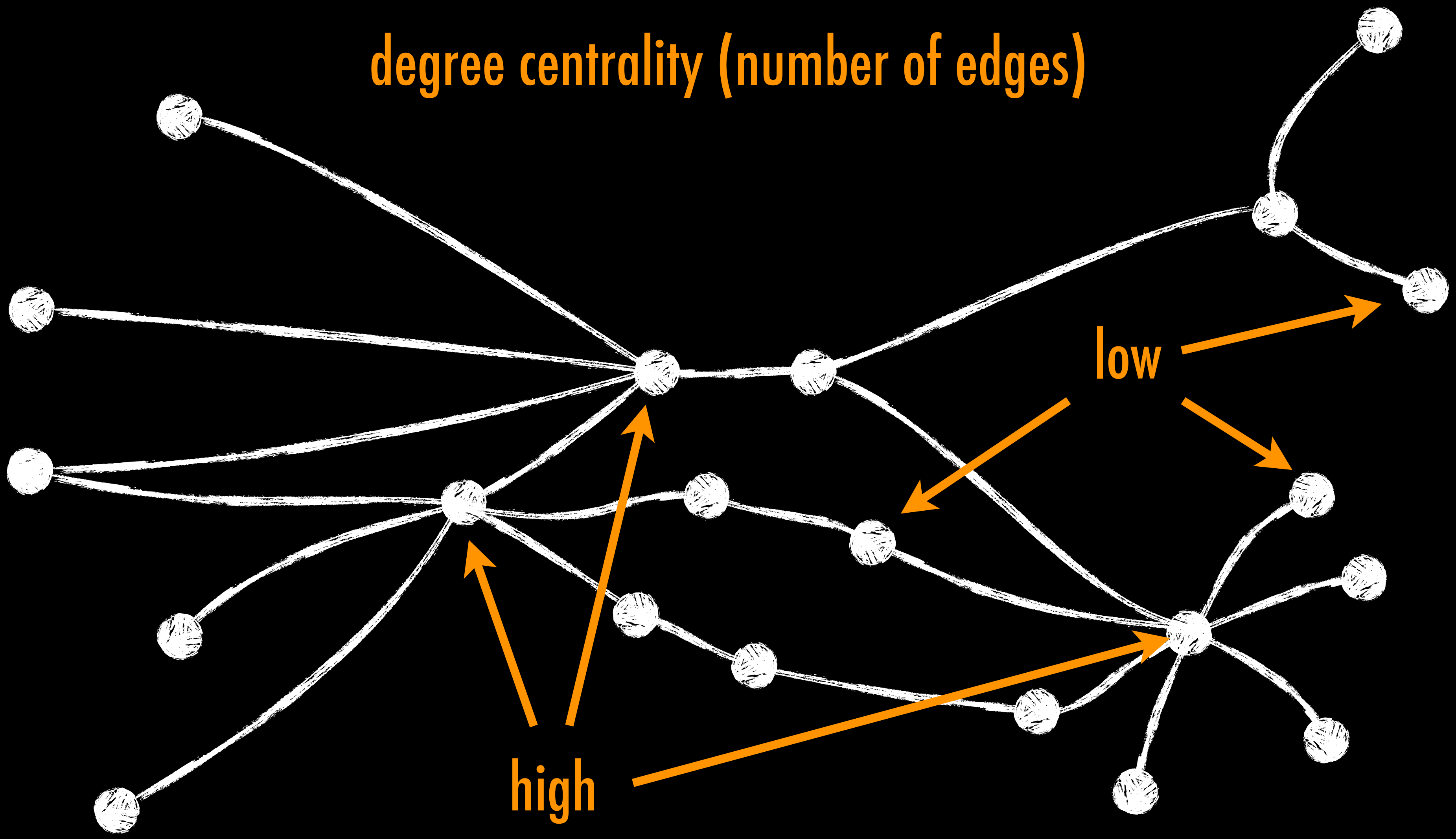


# Network Properties



# Network Properties

degree centrality (number of edges)



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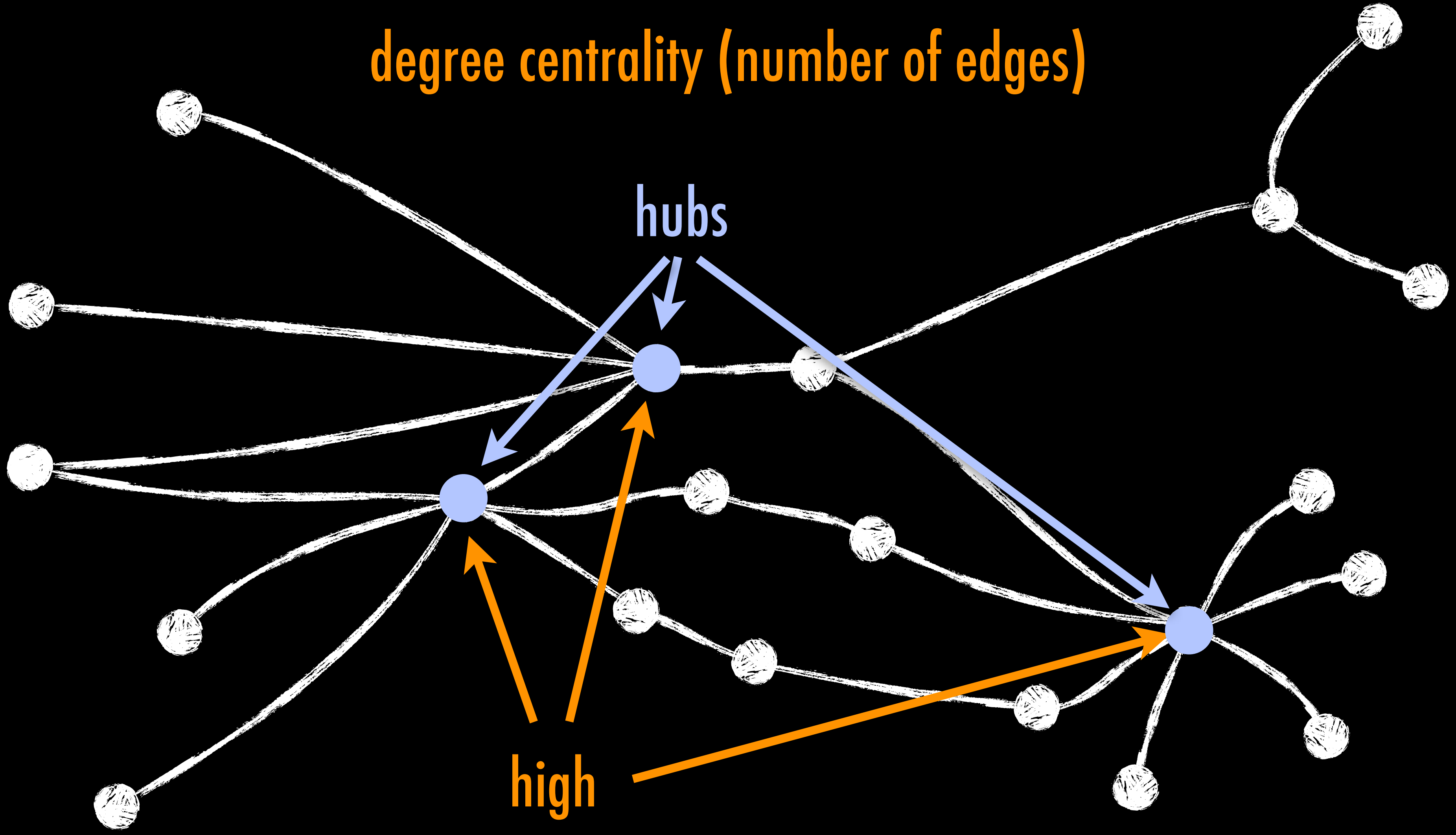
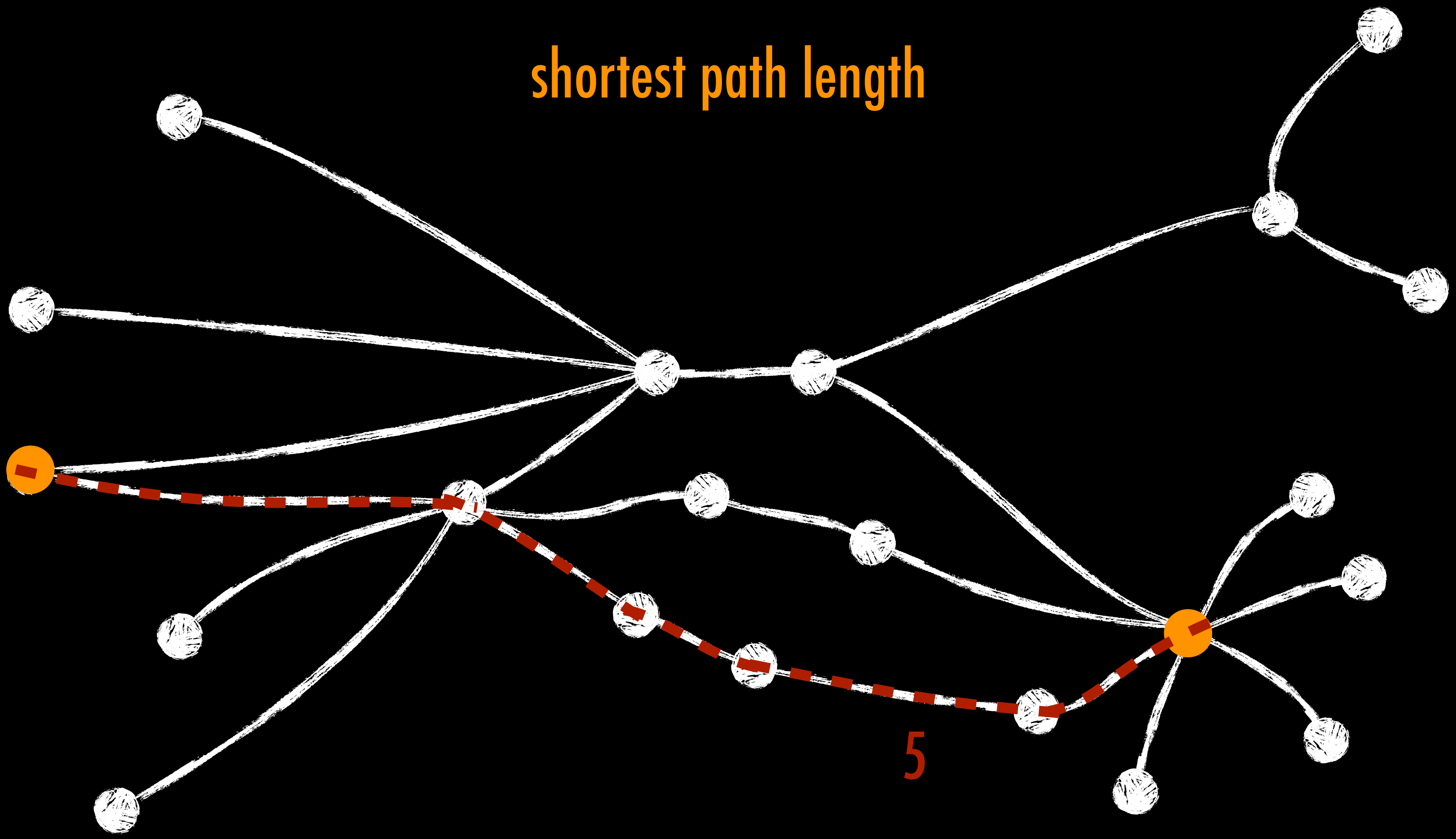


- importance of vertex for network

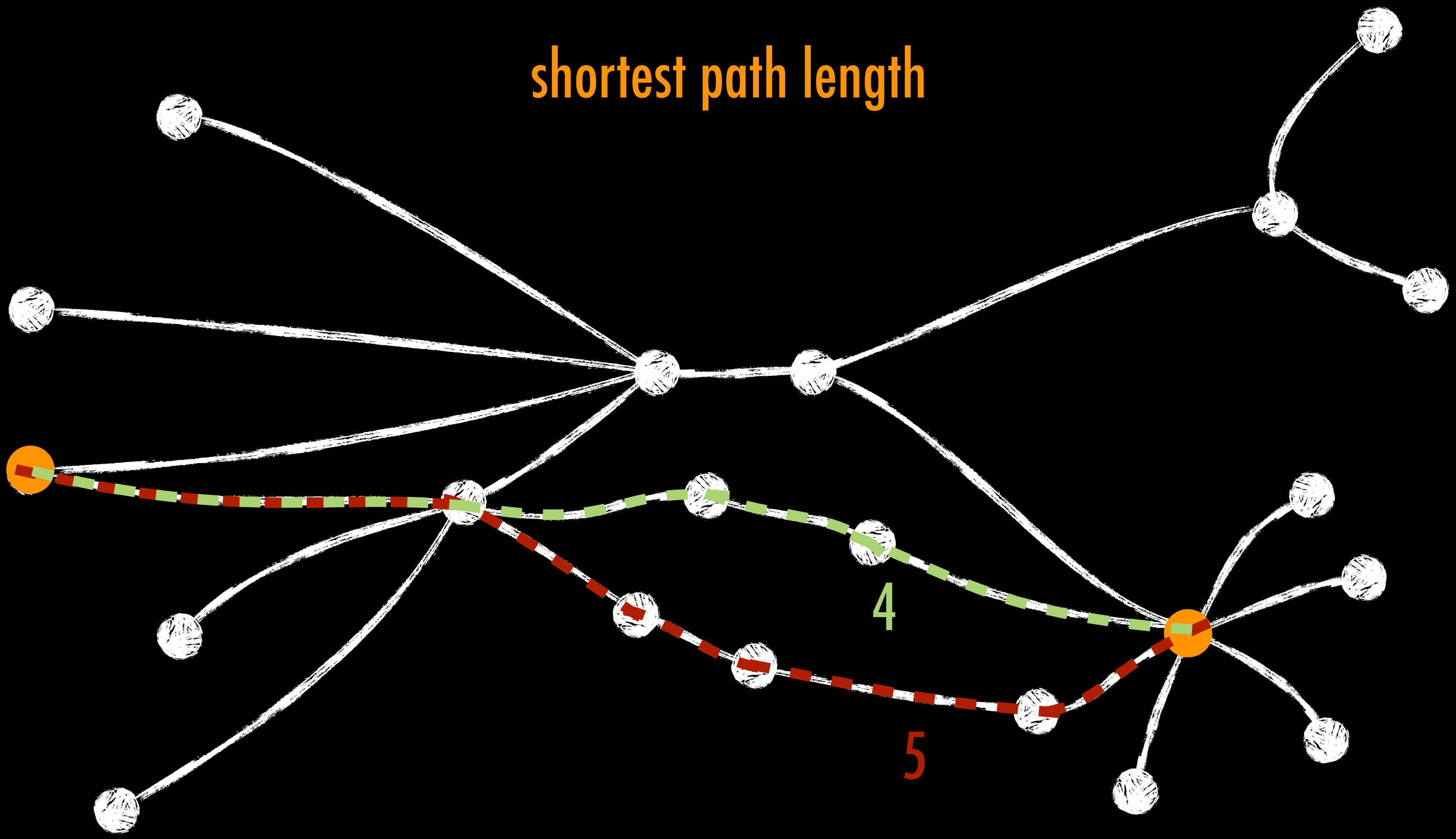
# Network Properties

shortest path length



# Network Properties

shortest path length



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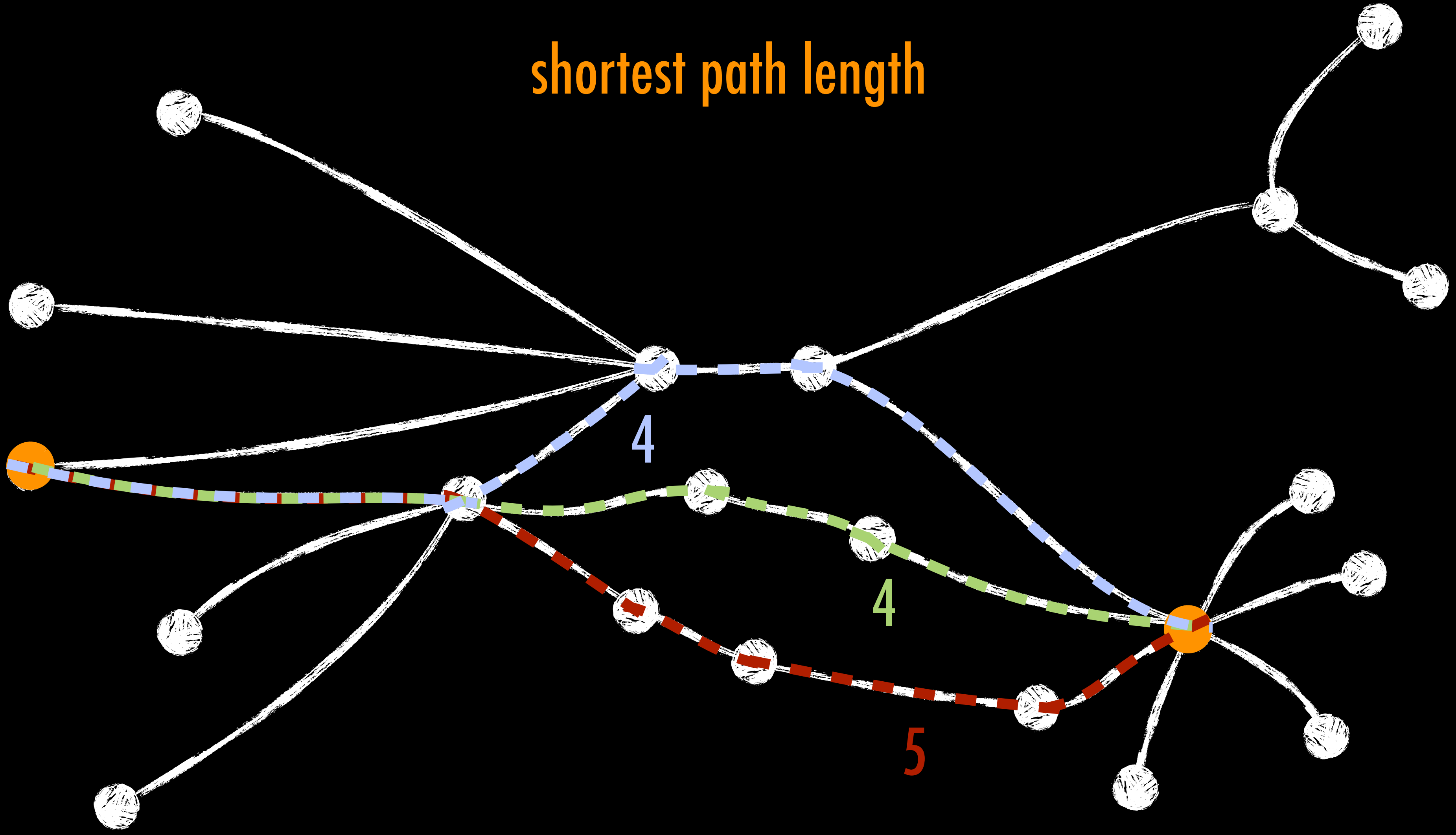
shortest path length



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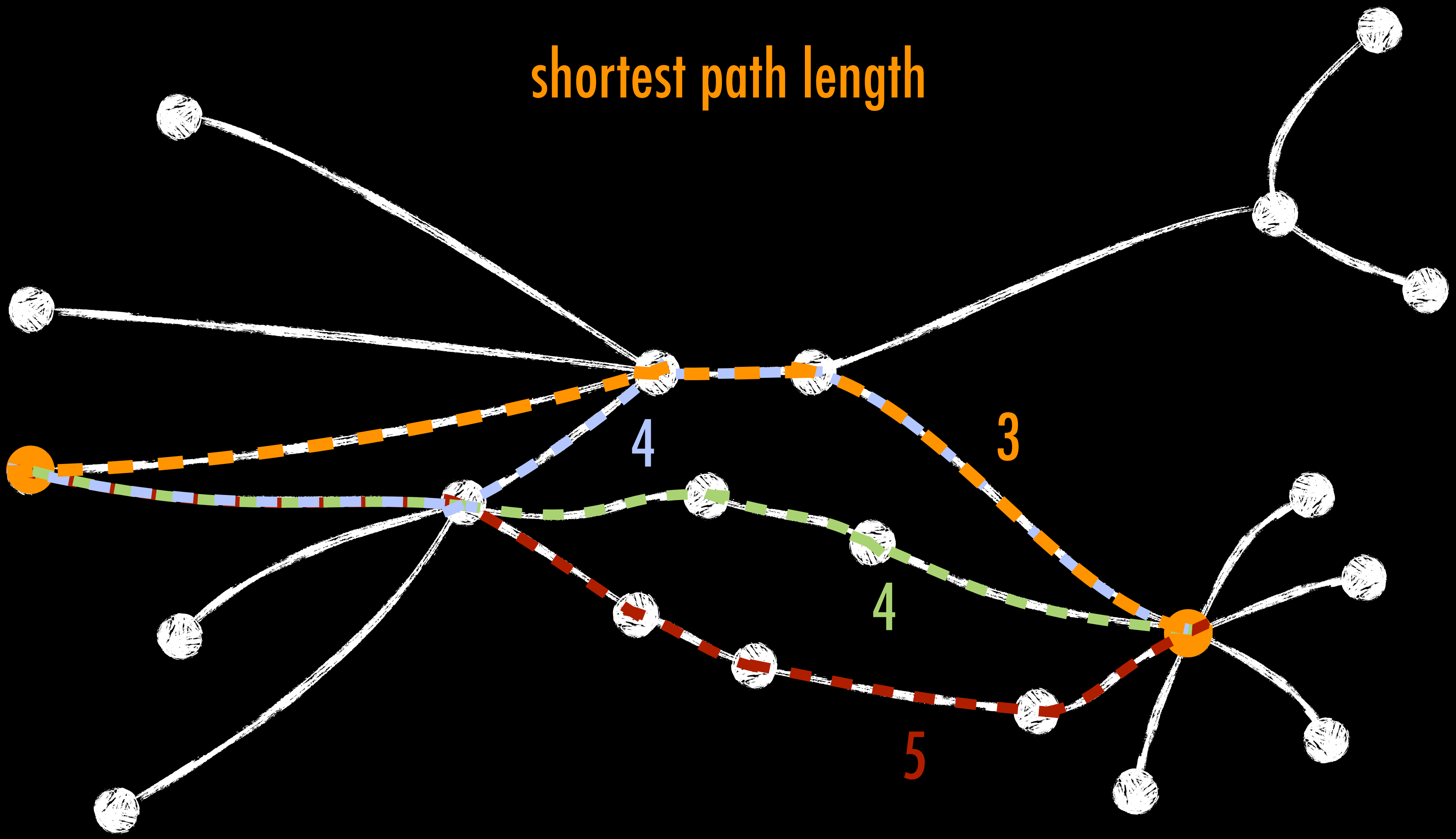

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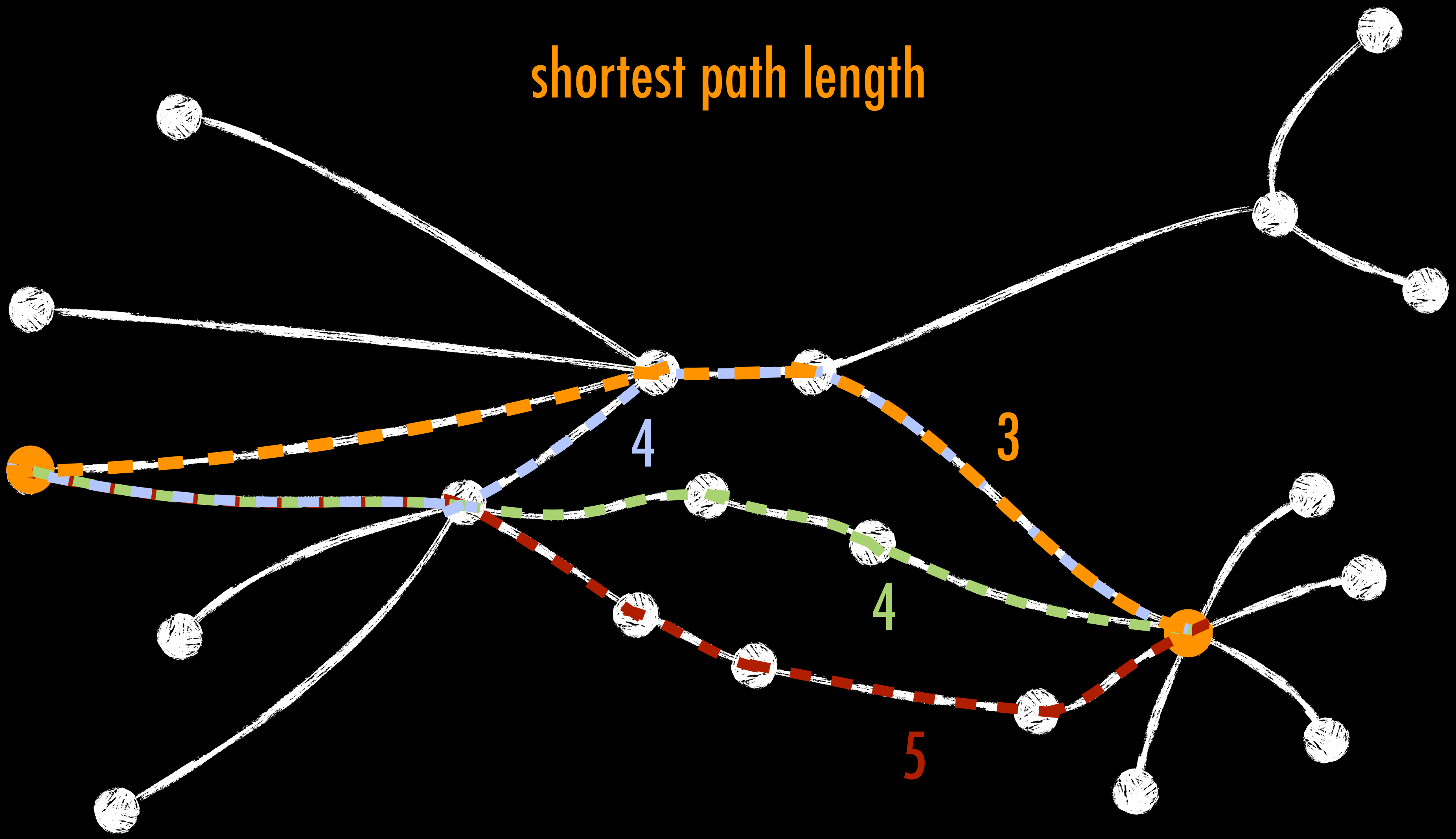
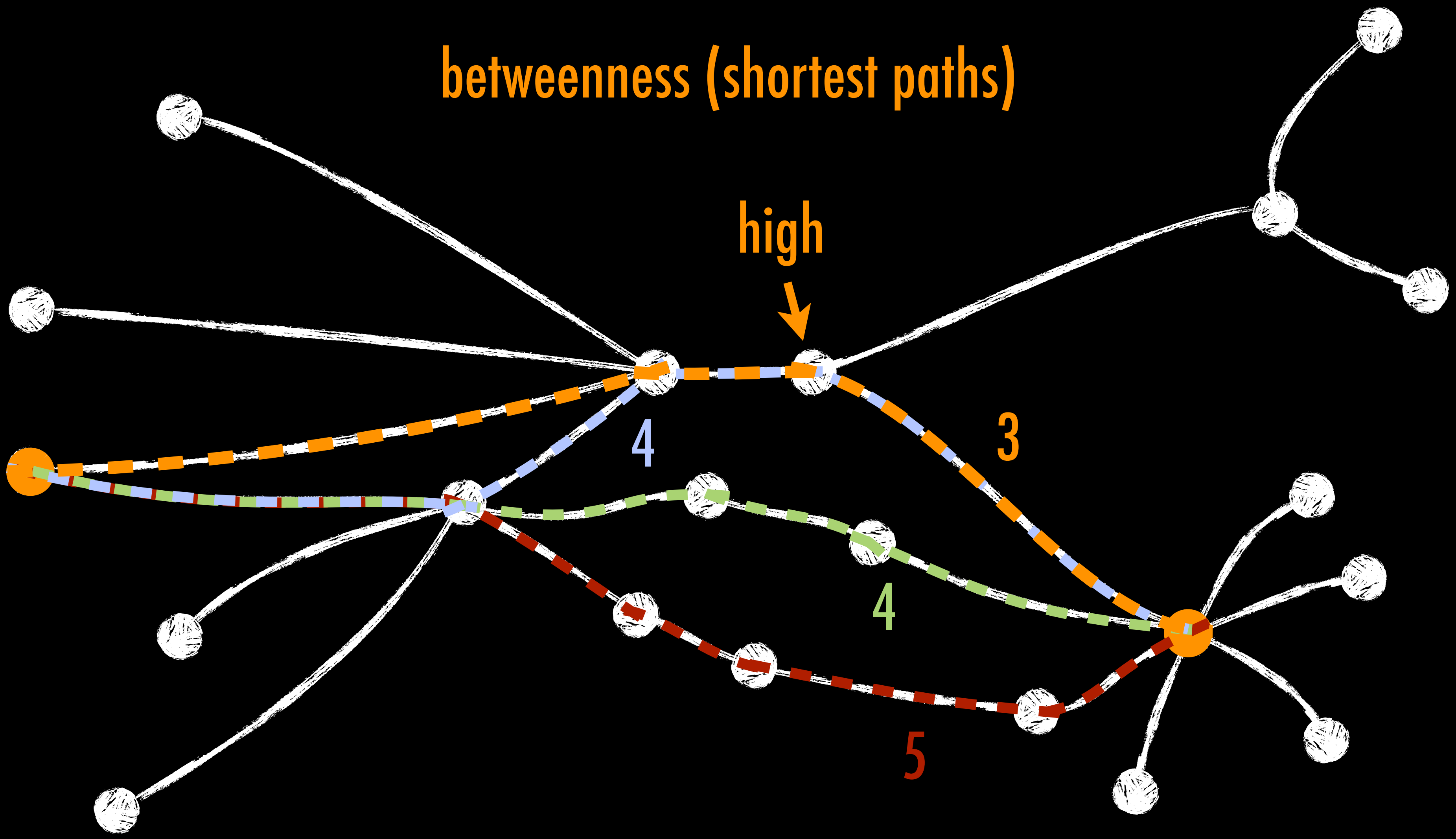


- topology of the network (small-world)

# Network Properties



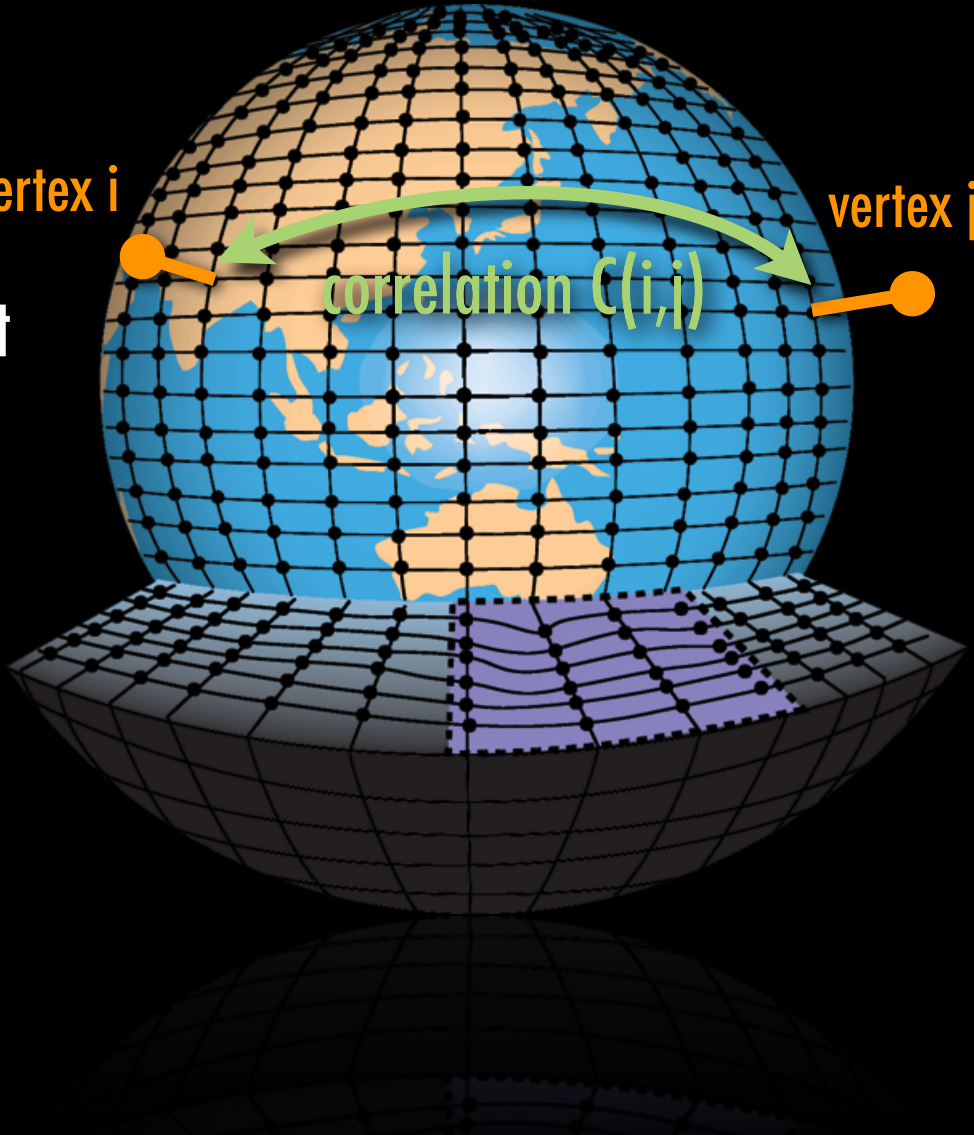
- impact of vertex for information, energy, or matter flow

# Betweenness Centrality

- number of shortest paths through a given vertex
- physics: energy transport on shortest ways
- changes on a vertex of high betweenness: impact on large parts of the network

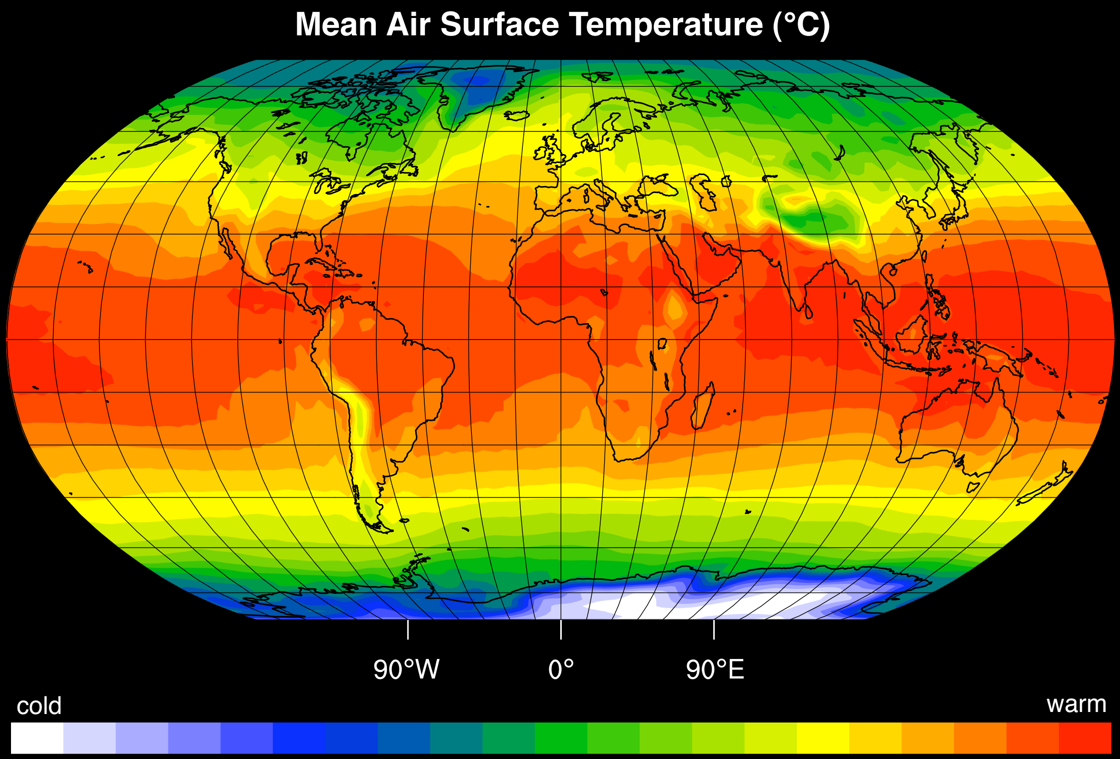
# Network Construction

- Edges: (non)linear correlations between vertices
  - Pearson correlation coefficient
  - mutual information
  - lag correlation
- statistically significant links
- unweighted, undirected network



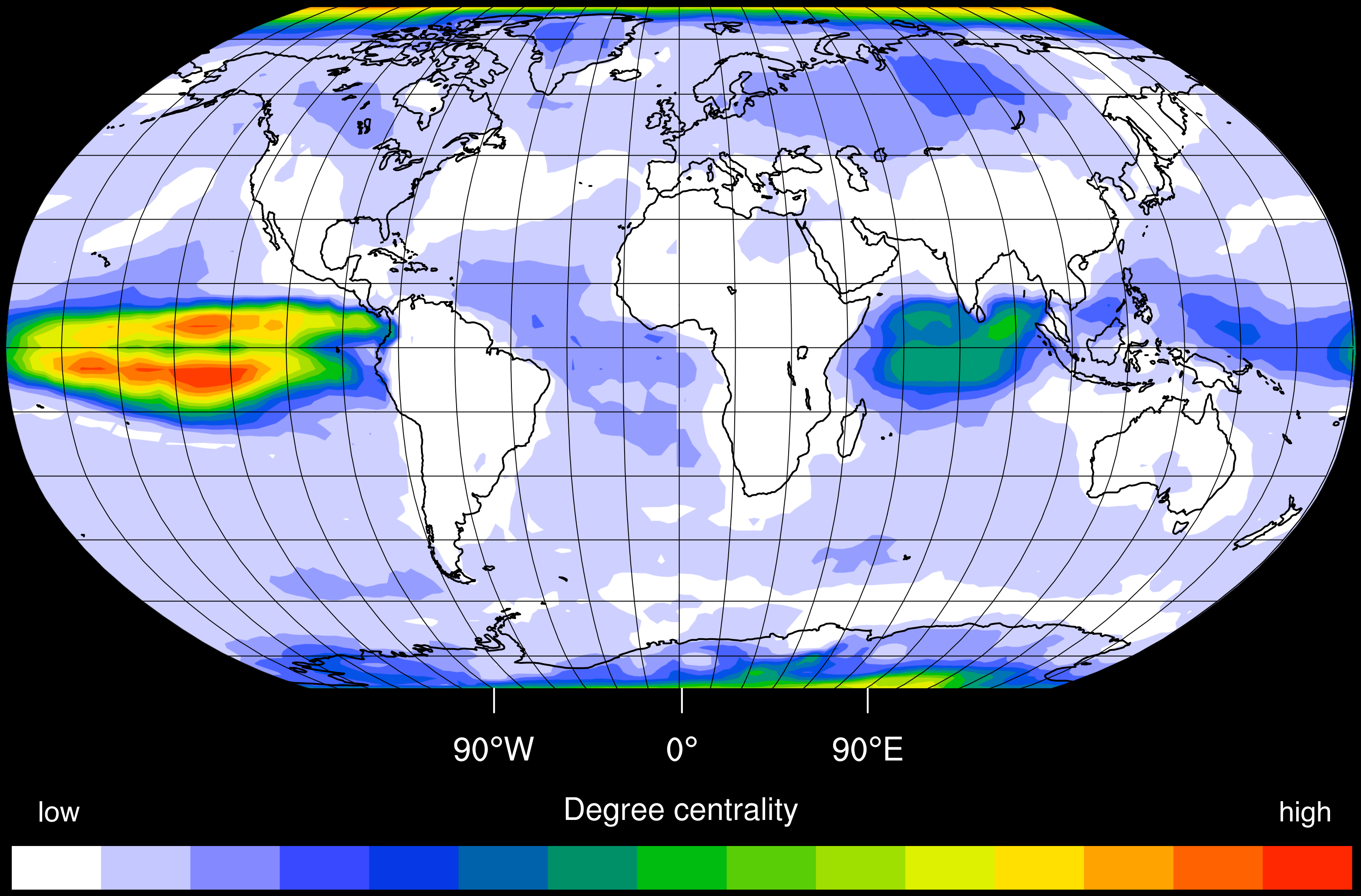
# Data

- NCEP/NCAR reanalysis data
- air temperature (17 levels)
- 1/1948 – 12/2007 (720 samples)
- grid resolution 2.5° × 2.5° (10,224 vertices)



# Surface Level

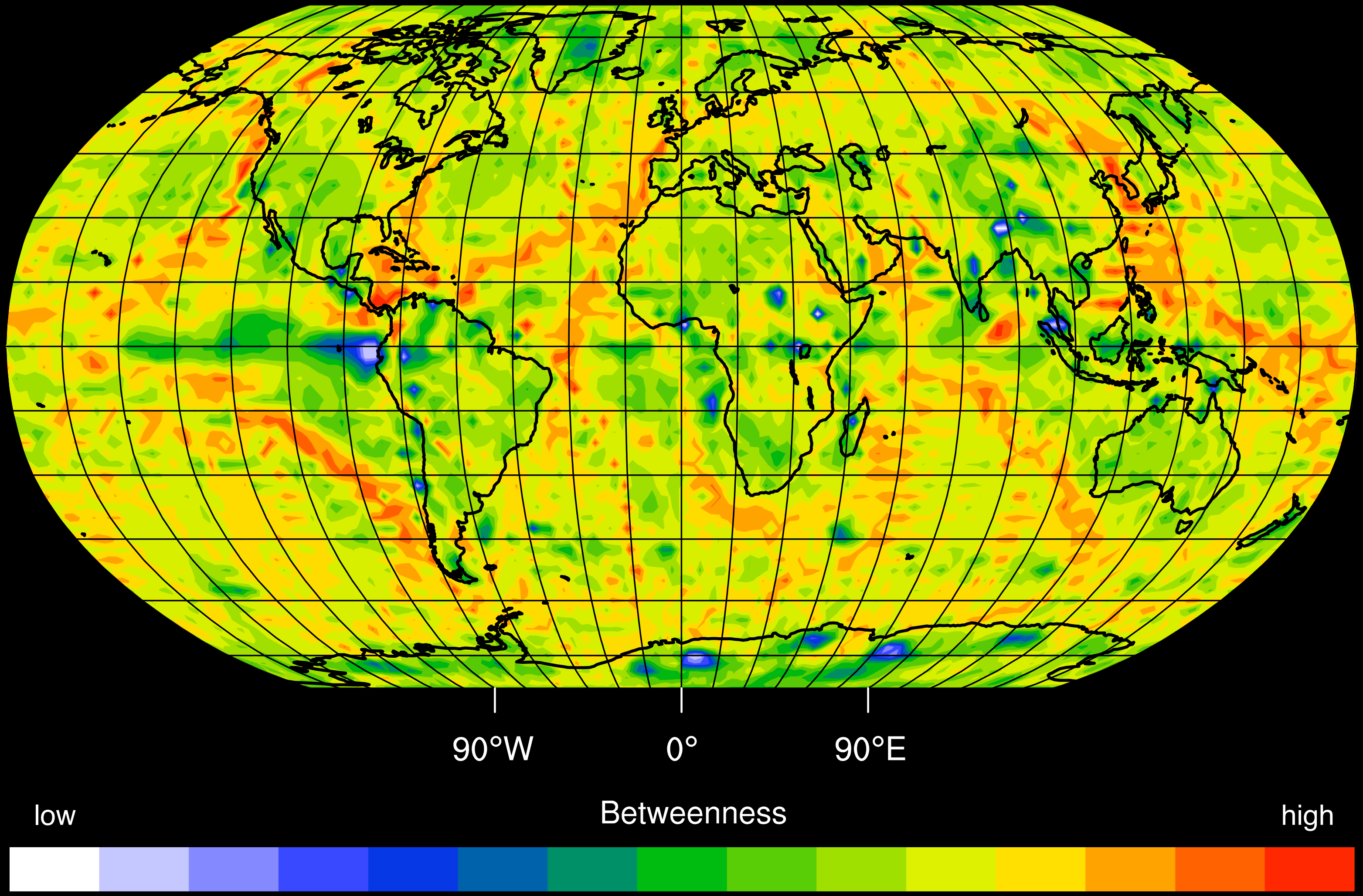
- highly linked regions (hubs)



Donges et al., Europhys. Lett. 87, 48007 (2009)

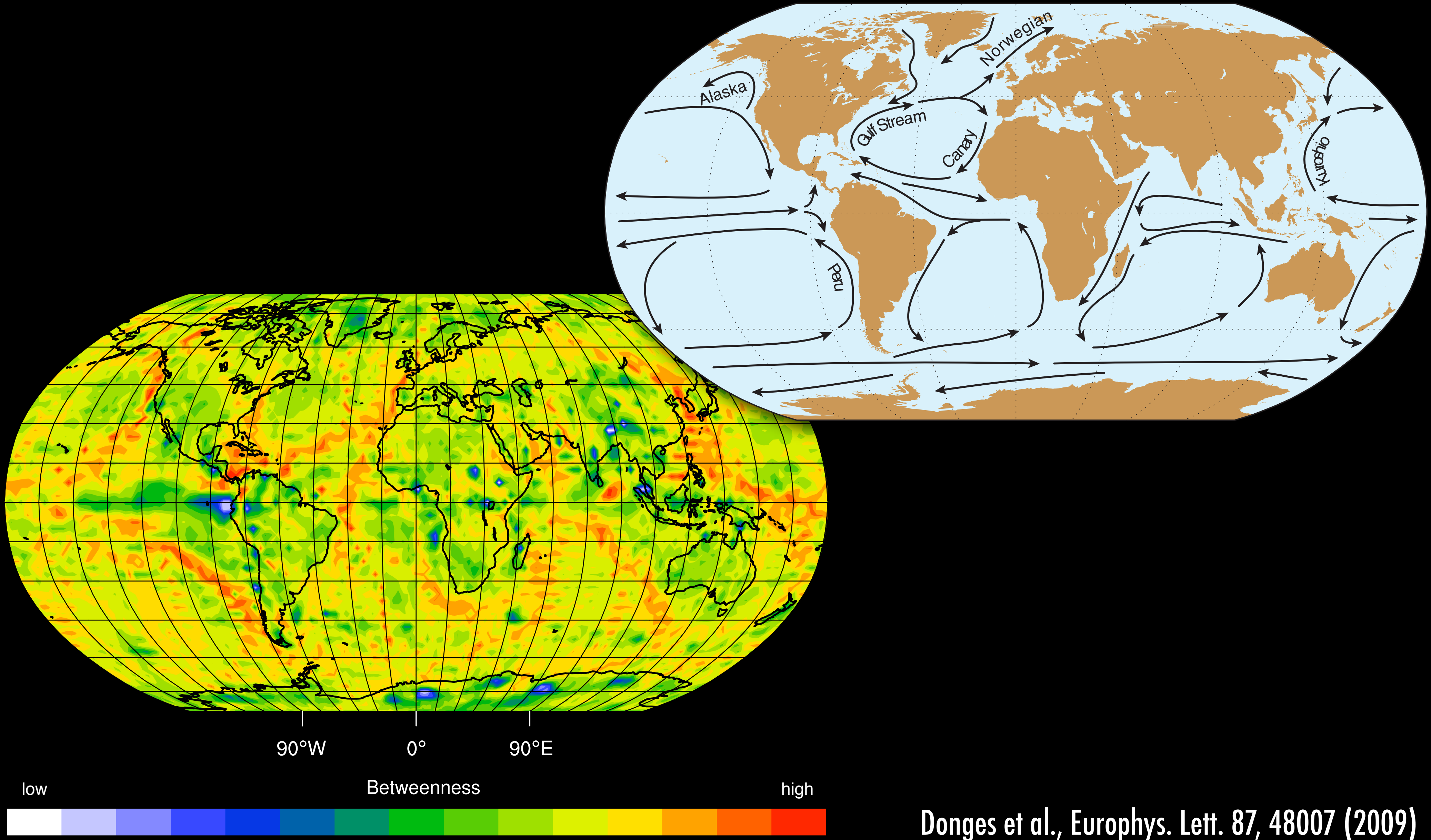
# Surface Level

- energy (heat) transport ways

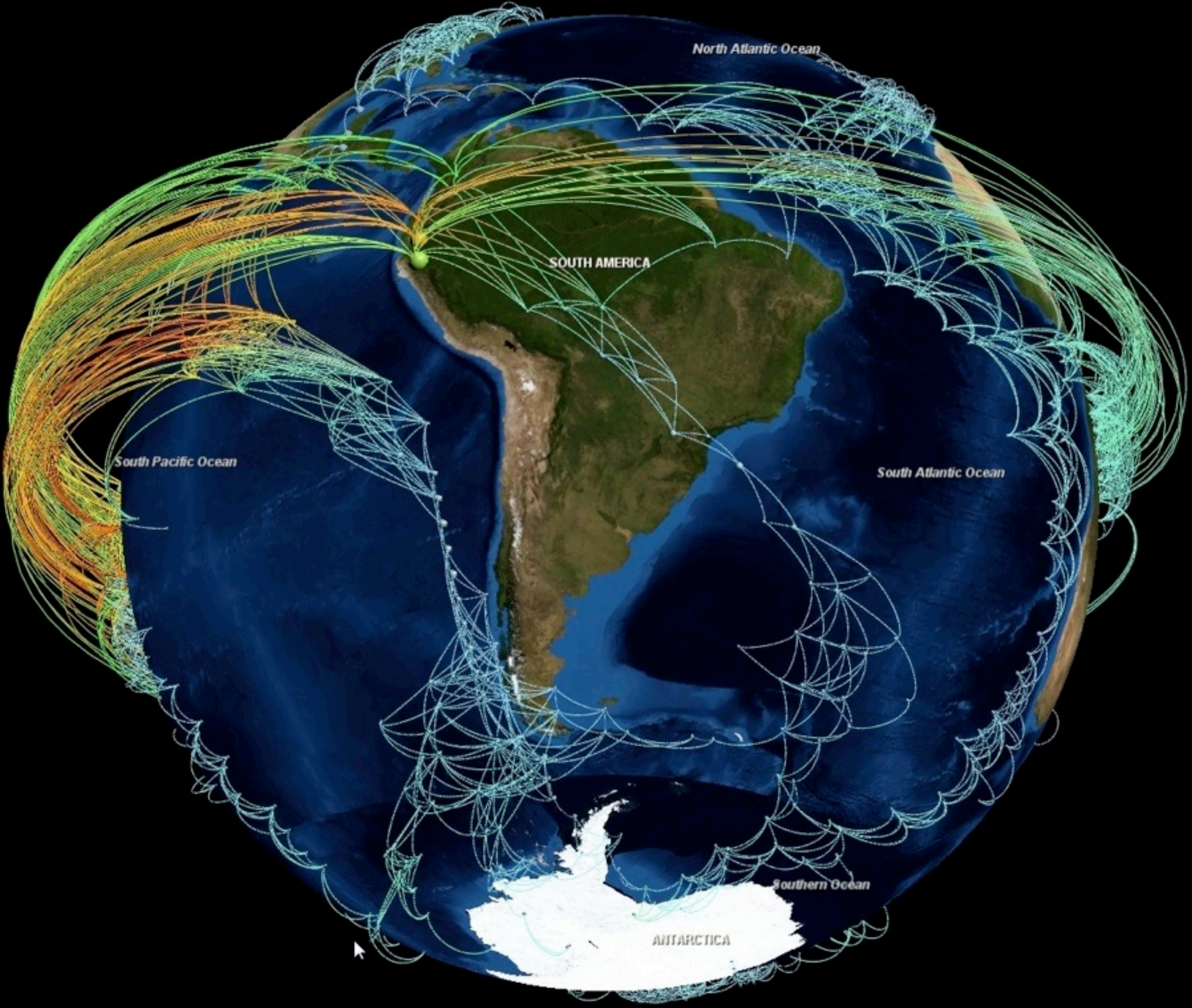


Donges et al., Europhys. Lett. 87, 48007 (2009)

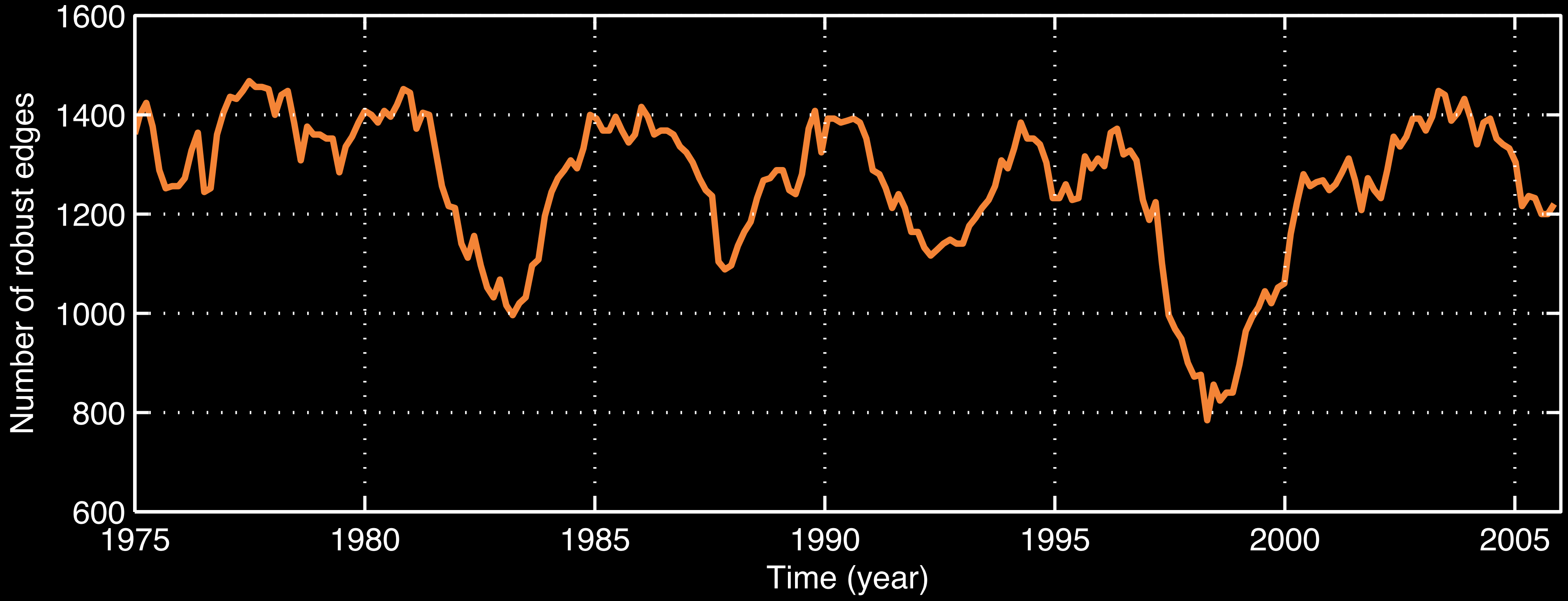
# Surface Level



Donges et al., Europhys. Lett. 87, 48007 (2009)


North Atlantic Ocean
SOUTH AMERICA
South Pacific Ocean
South Atlantic Ocean
Southern Ocean
ANTARCTICA

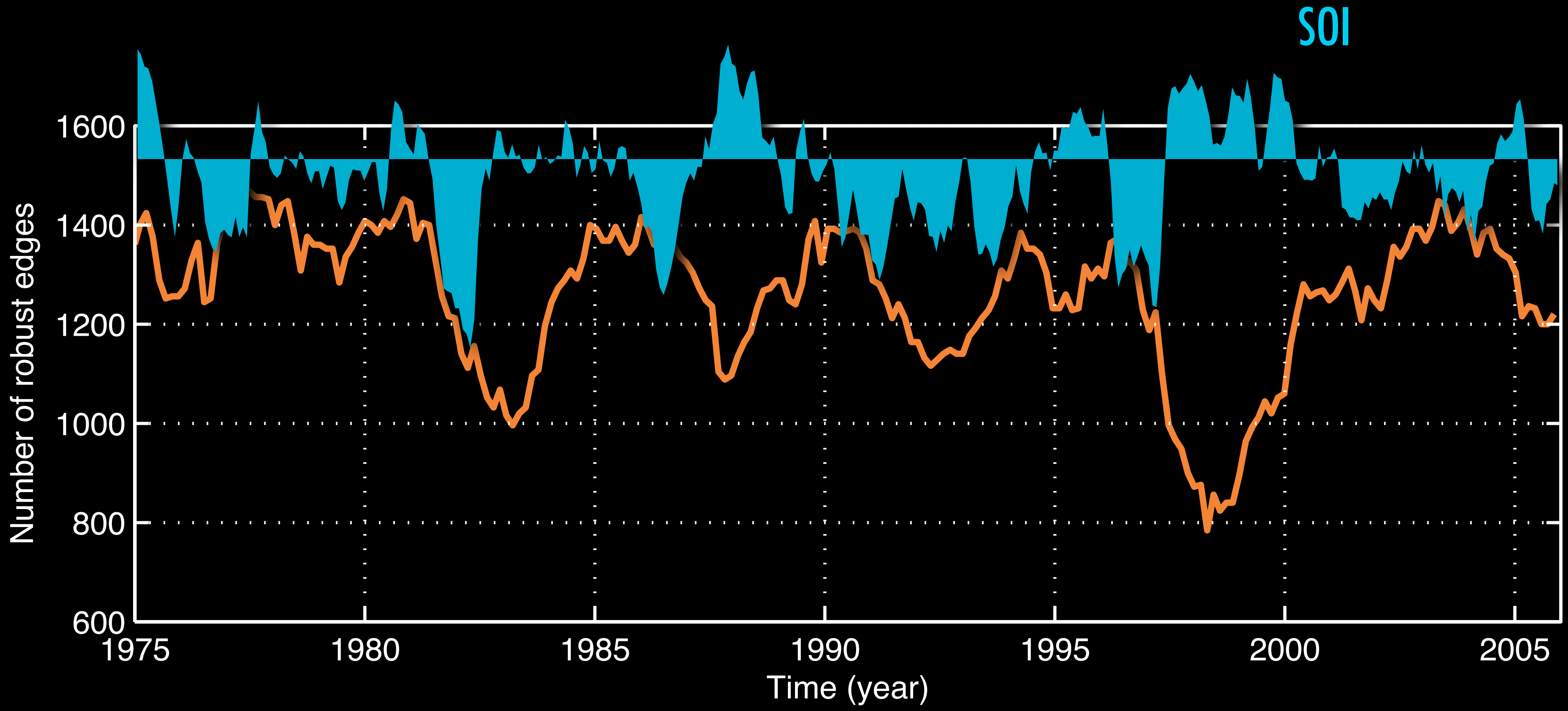
# Evolving Networks



J. Runge, A. Radebach,
J. Zscheischler

- stability of the climate network during El Niño

# Evolving Networks



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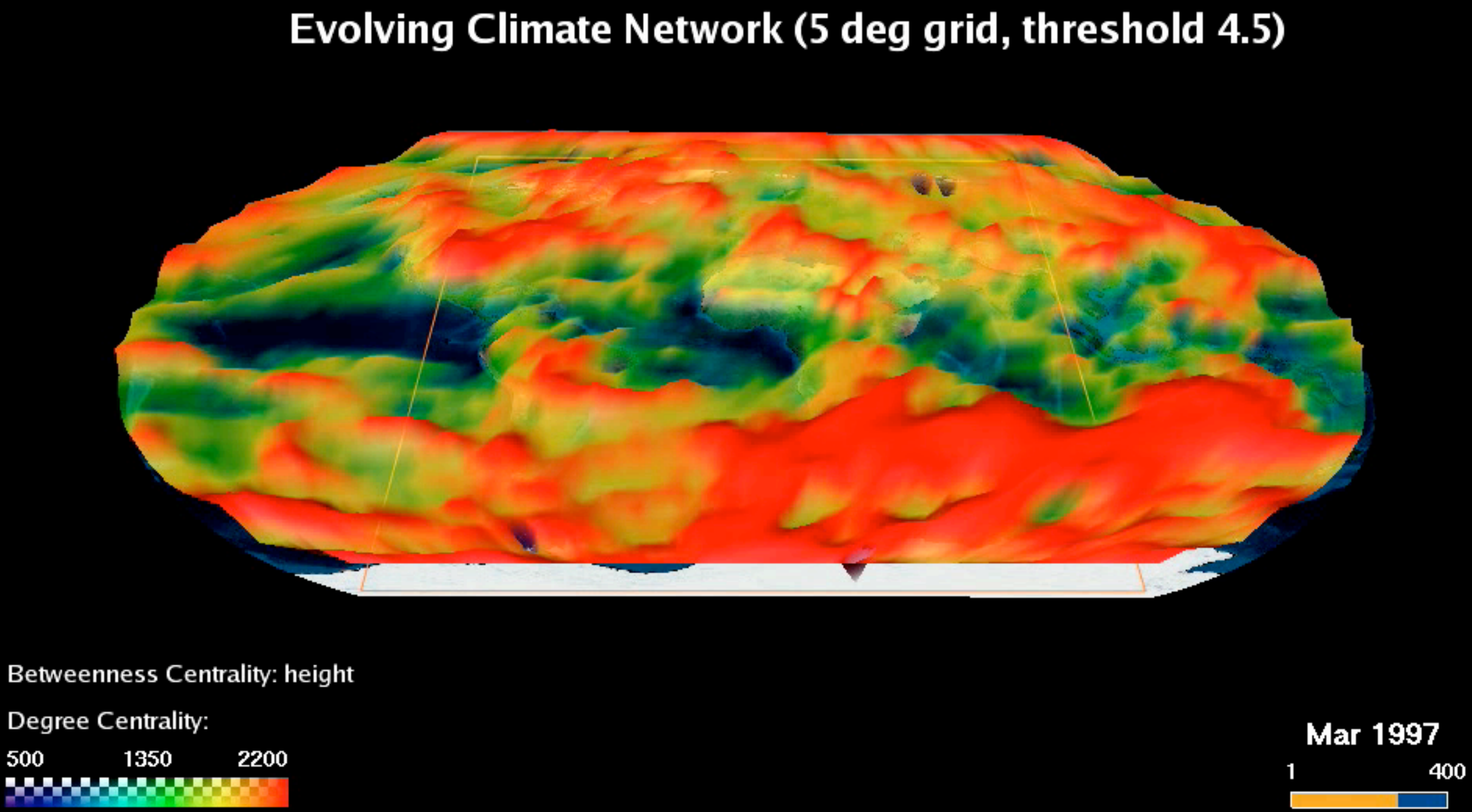
- stability of the climate network during El Niño

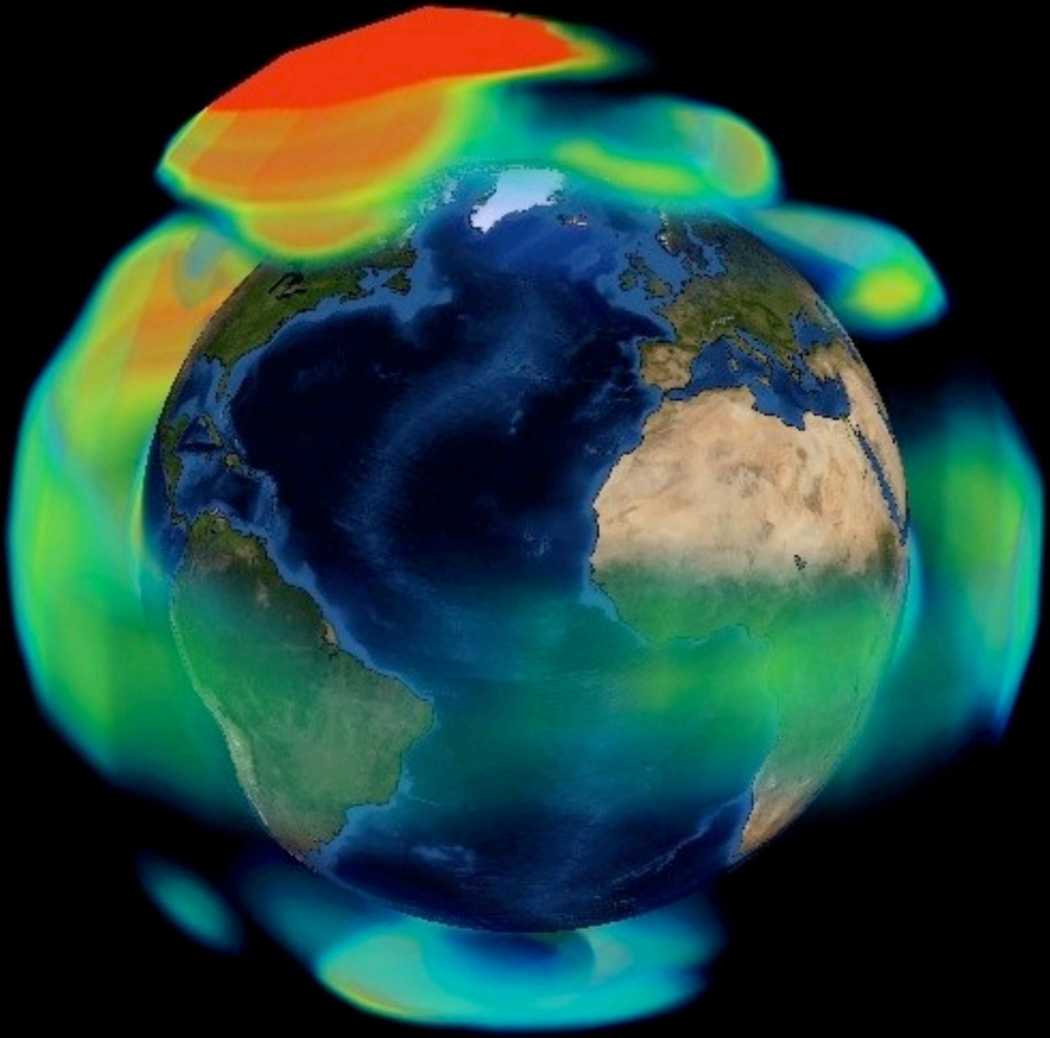
# Evolving Networks



- stability of the climate network during El Niño

# Software for Complex Networks

- iGraph (interfaces to C, R, Python)
  http://igraph.sourceforge.net/
- Network Workbench
  http://nwb.slis.indiana.edu/
- pyRecurrence
  http://www.pik-potsdam.de/members/donges/software



# Modern Nonlinear Approaches for Data Analysis

# Typical Behaviour of Nonlinear Systems

- fractal dimension
- self-similarity
- exponential error growth, non-predictable
- recurrence