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PIK

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Complex Network Approach for Recurrence Analysis of Time Series

Leibniz Gemeinschaft

Outline

- Recurrence plots
- Complex networks
- Complex networks from time series
- Applications

Recurrence

- fundamental characteristic of many dynamical systems
- recurrences in real life:

  Milankovich cycles, weather after storm, El Niño phenomenon, heart beat after exertion, Maya calendar etc.

Recurrence

- Anaxagoras, approx. 450 BC:

  perichoresis: chaotic circular movement

Recurrence

- Poincaré, 1890:

  “a system recurs infinitely many times as close as one wishes to its initial state”

Investigating Recurrence

- Poincaré map
- Recurrence time statistics
- First return map
- Recurrence plot

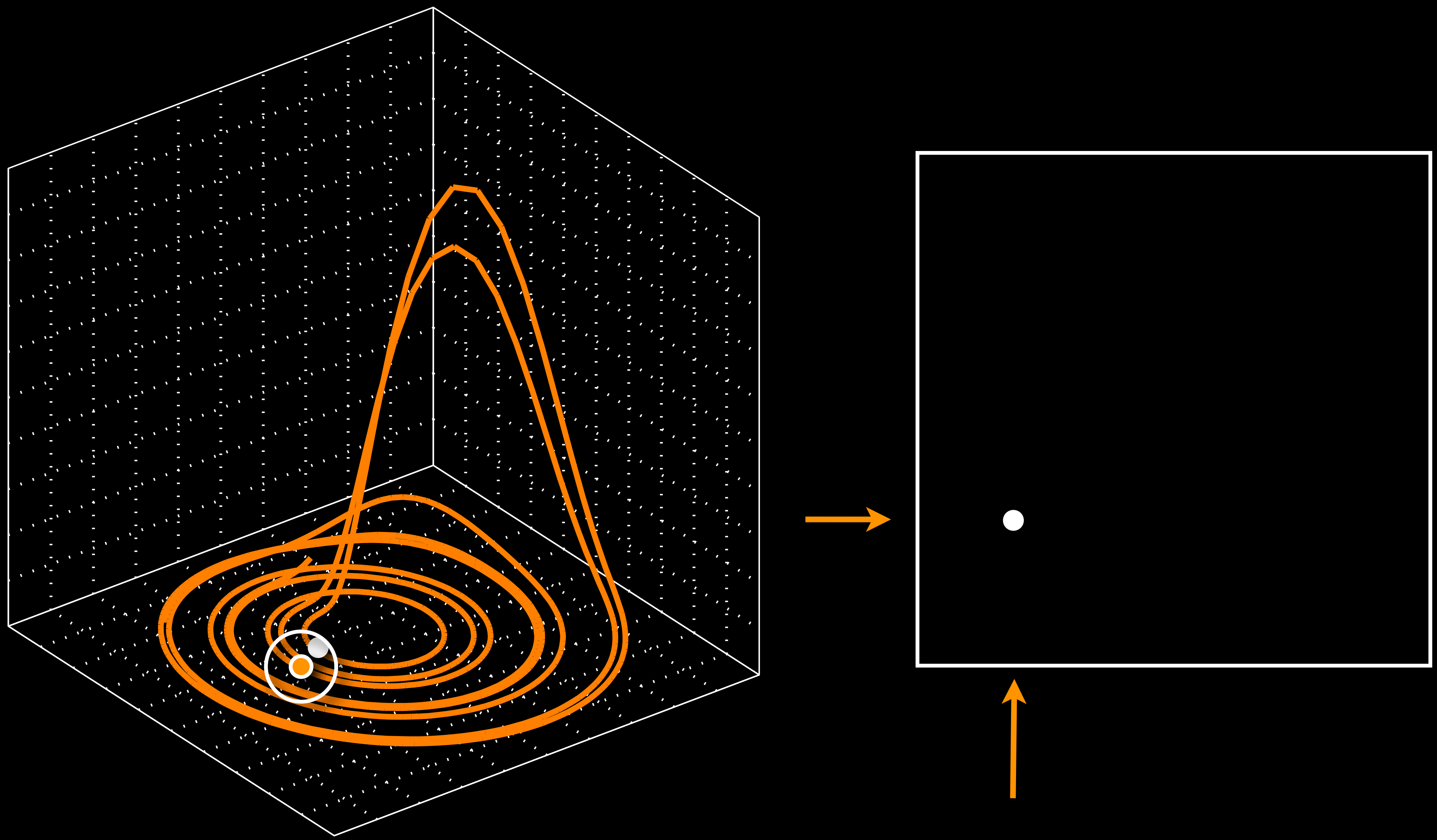
Investigating Recurrence

- Poincaré map
- Recurrence time statistics
- First return map
- Recurrence plot

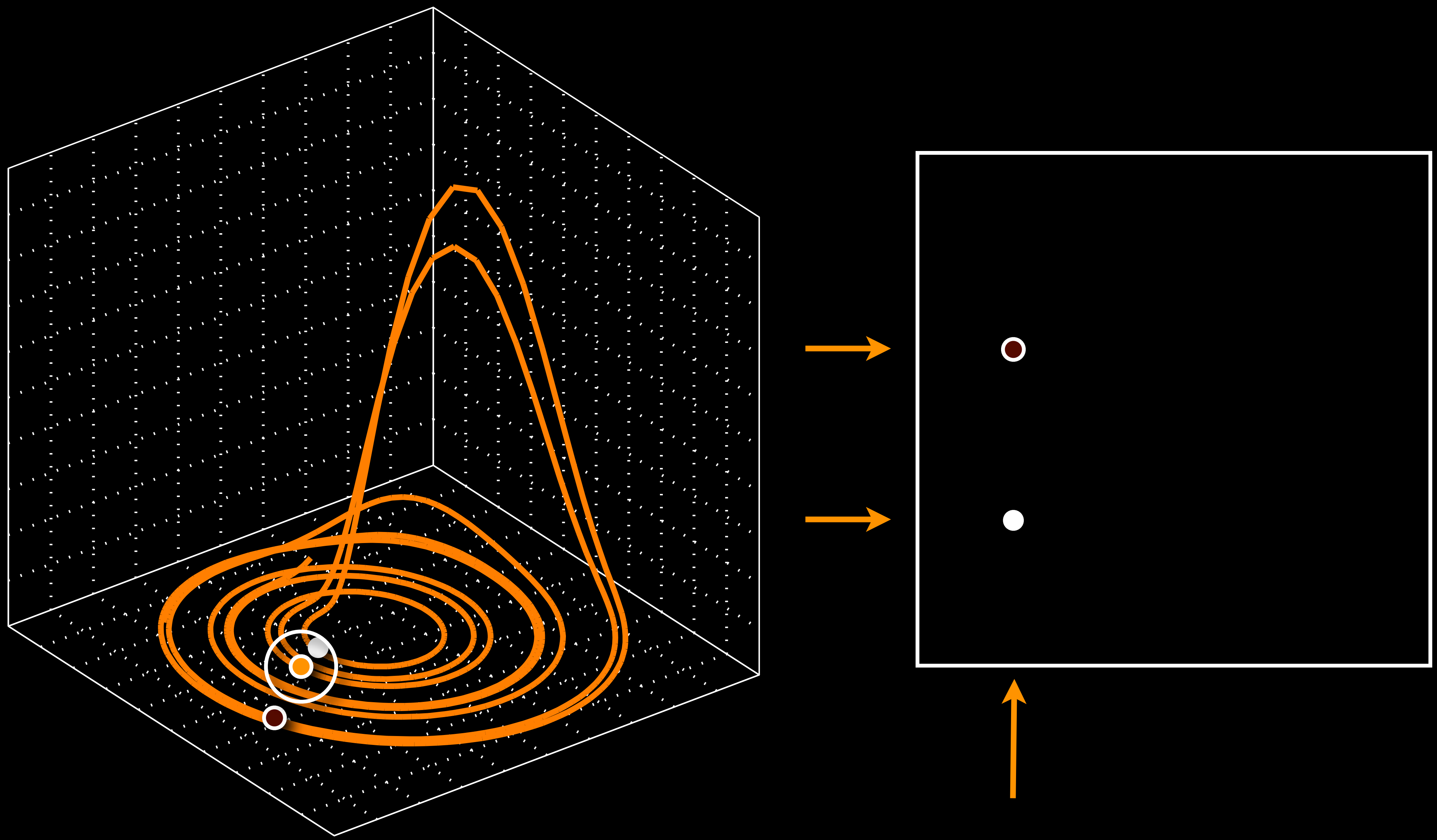
- Recurrence network

Recurrence Plots

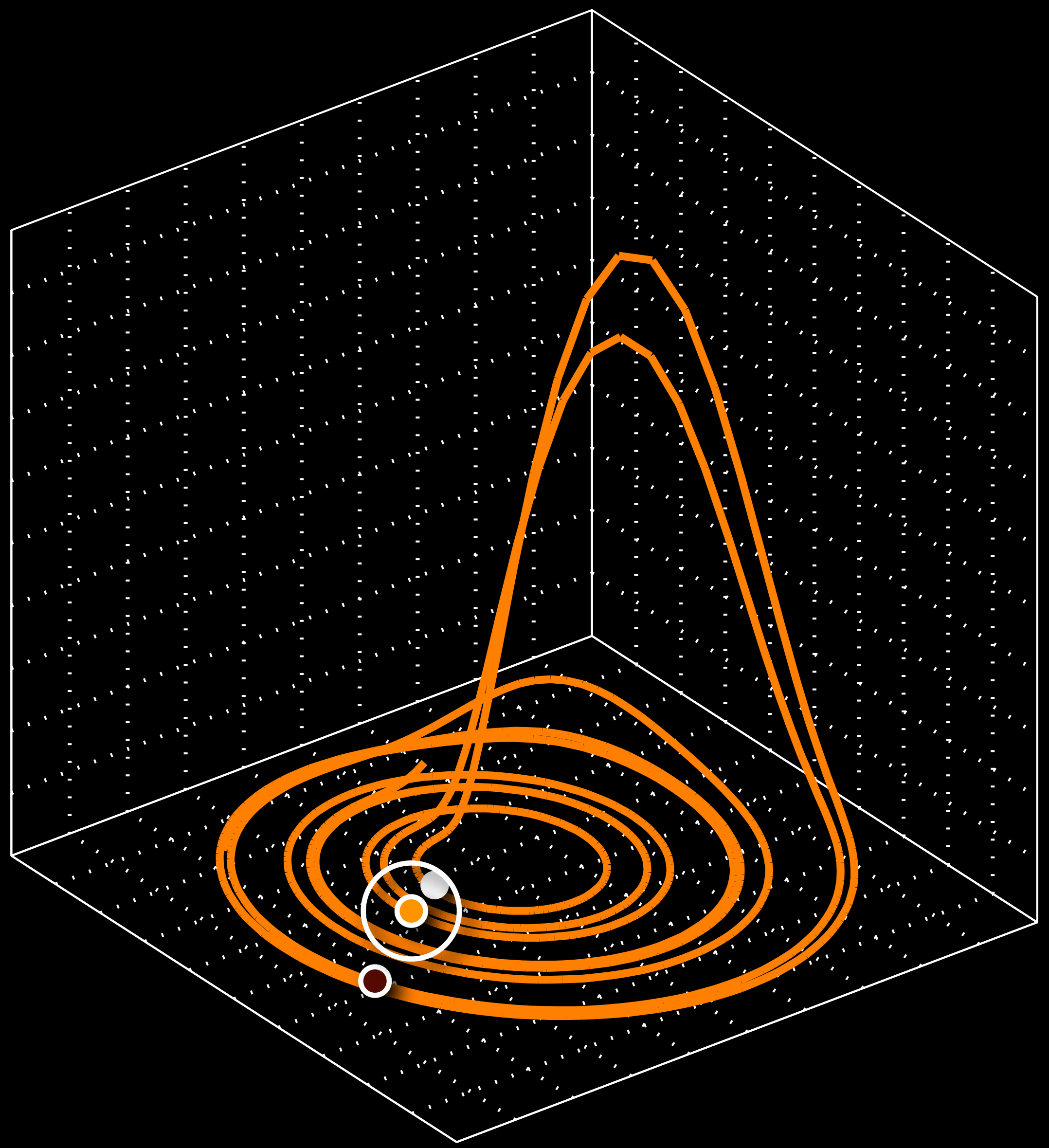
Recurrence Plot

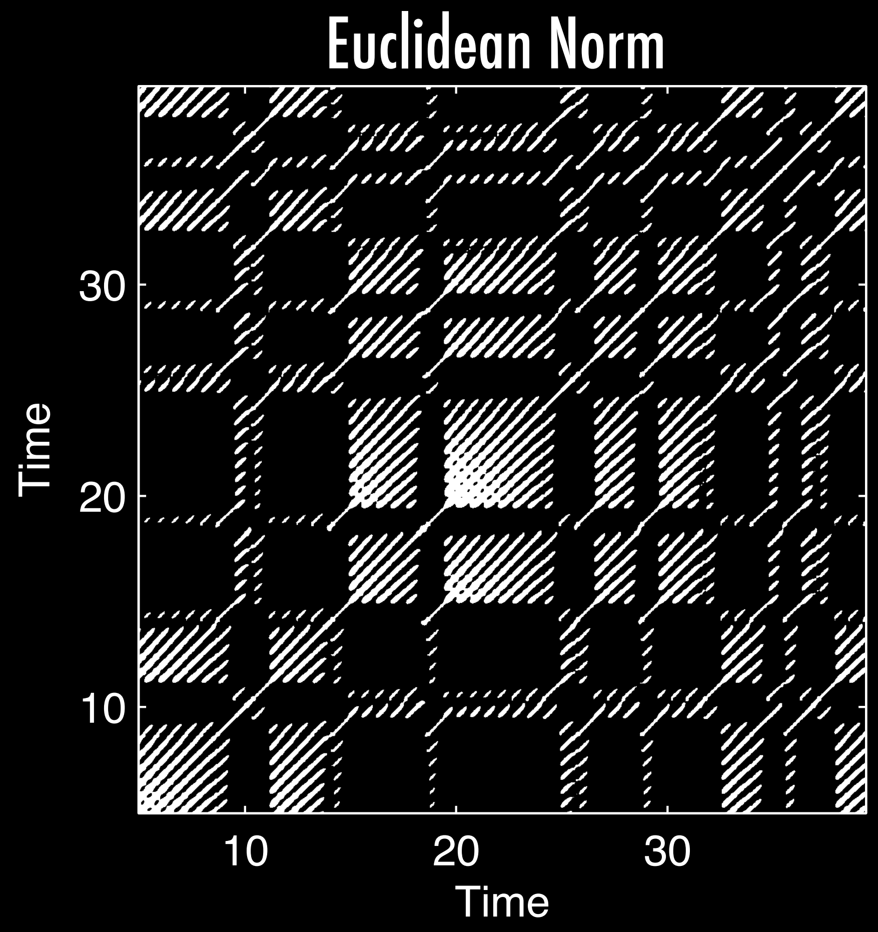


Recurrence Plot



Recurrence Plot





Recurrence Plot

- to visualise the phase space trajectory by its recurrences

$$\mathbf{R}_{i,j} = \begin{cases} 1 : \vec{x}_i \approx \vec{x}_j \\ 0 : \vec{x}_i \not\approx \vec{x}_j \end{cases} \qquad i, j = 1, \ldots, N$$

- formal

$$\mathbf{R}_{i,j}(\varepsilon) = \Theta\left(\varepsilon - \left\|\vec{x}_i - \vec{x}_j\right\|\right), \quad i, j = 1, \ldots, N$$

J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, Europhysics Letters, 5, 1987
N. Marwan et al., Physics Reports, 438, 2007

Recurrence Plot

- to visualise the phase space trajectory by its recurrences

- recurrence matrix:
  - binary
  - symmetric

$R_{i,j} =$

1	1	0	0	1
1	1	1	0	1
0	1	1	0	0
0	0	0	1	1
1	1	0	1	1

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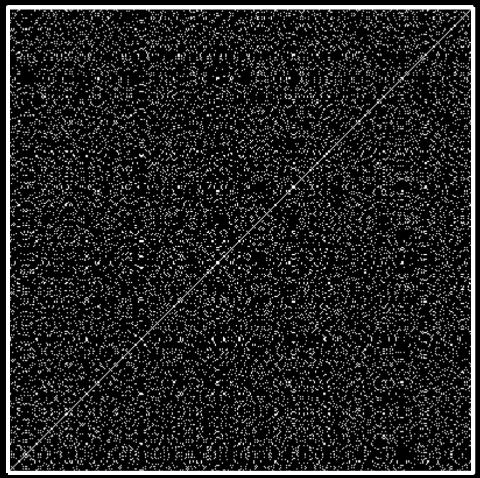
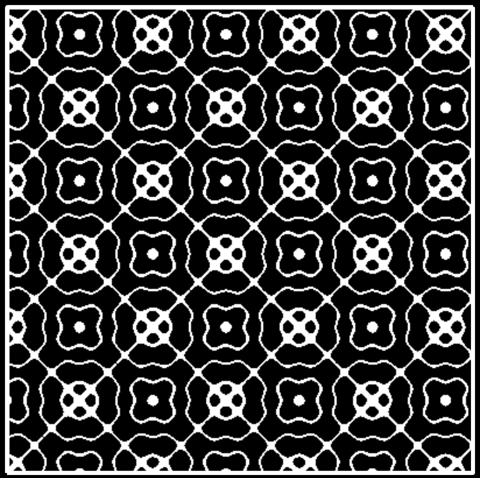
Recurrence Plot Typology

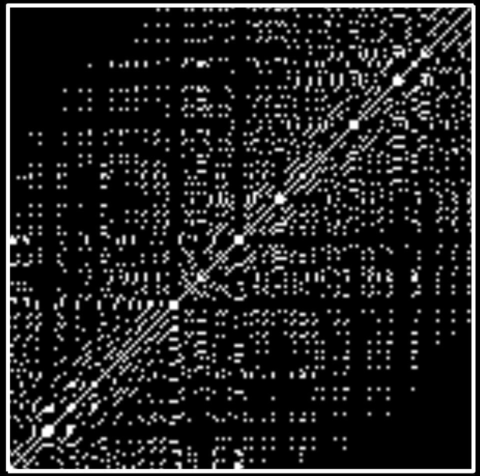
homogeneous



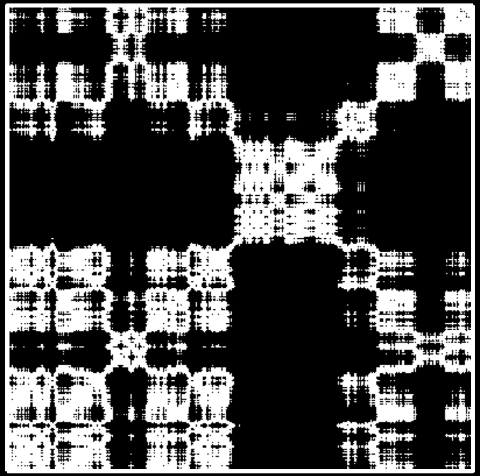
periodic



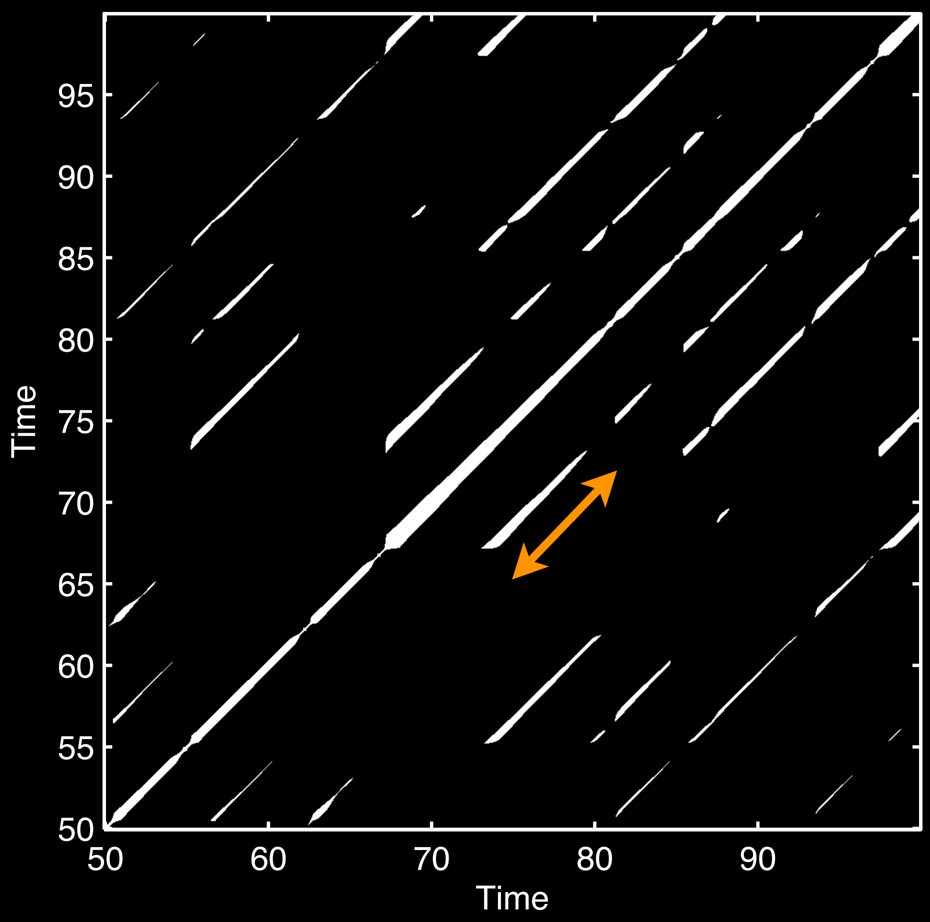
drifty

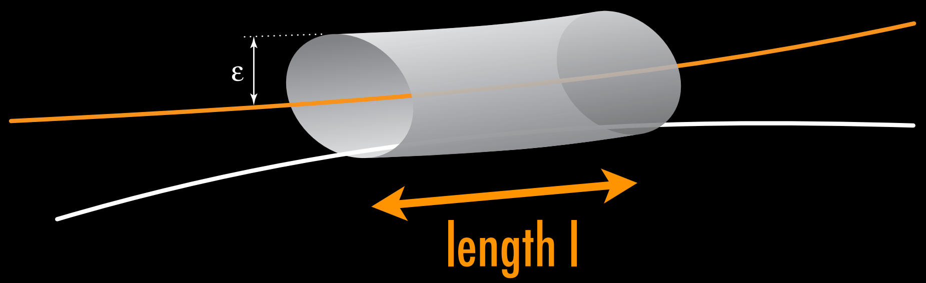


disrupted



Recurrence Plot Quantification



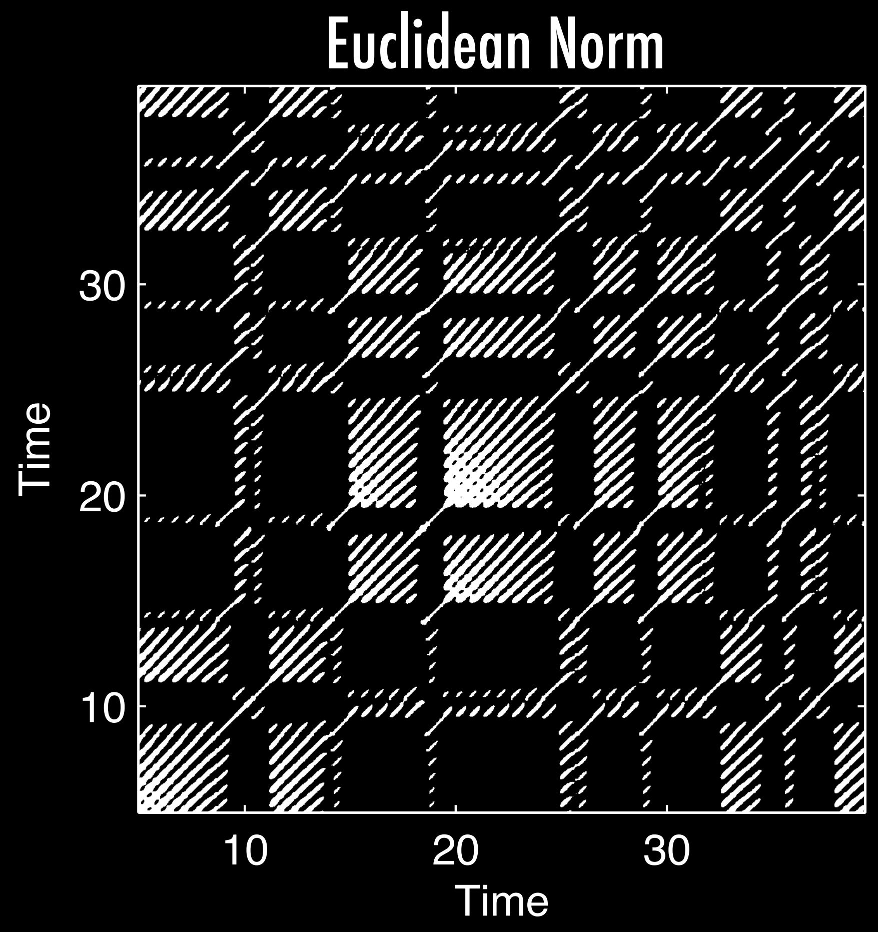


- Line structures related to dynamical properties
- Measures of complexity: quantify line length distribution (recurrence quantification analysis)

J. P. Zbilut & C. L. Webber Jr., Phys. Lett. A 171, 1992
N. Marwan et al., Phys. Rev. E 66, 2002

Recurrence Plot

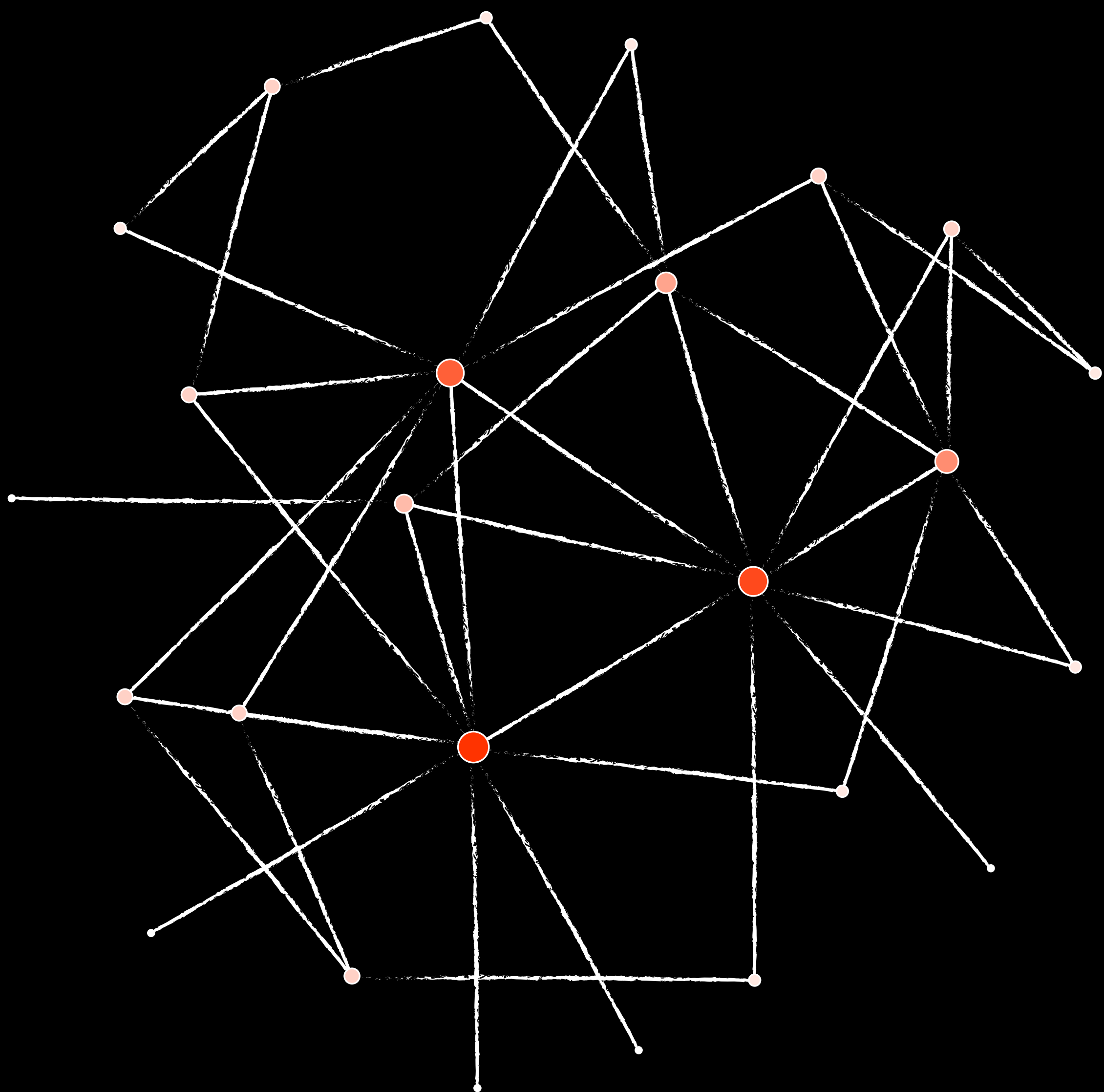
- Transition detection
- Differentiate dynamics
- Finding time scales
- Interrelation detection
- Synchronisation analysis
- Surrogates
- Recurrence time statistics
- etc.



N. Marwan et al., Physics Reports, 438, 2007

Complex Networks

Complex Networks

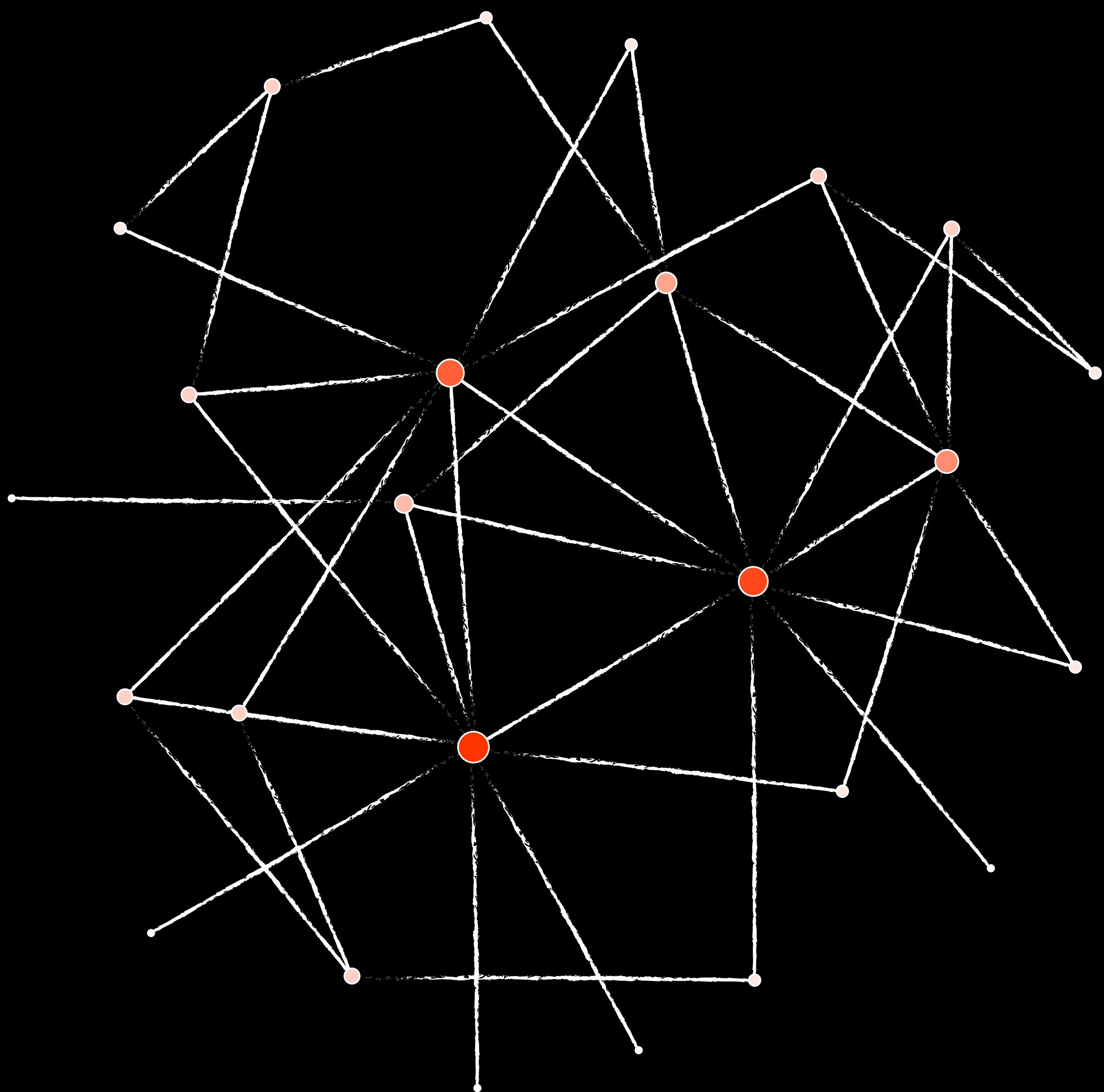


- link matrix (undirected, unweighted network):
  - binary
  - symmetric

$A_{i,j}$ =

0	1	0	0	1
1	0	1	0	1
0	1	0	0	0
0	0	0	0	1
1	1	0	1	0

Complex Networks



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0	1	0	0	1
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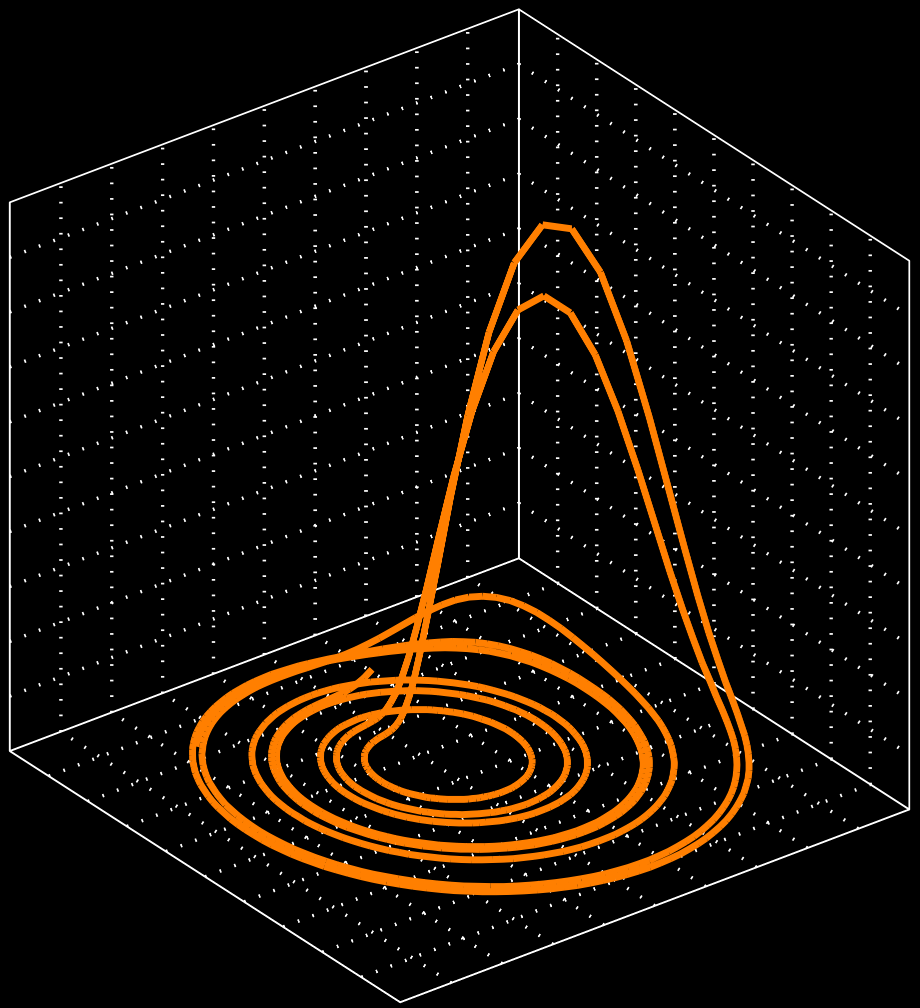
0	1	0	0	0
0	0	0	0	1
1	1	0	1	0

- link matrix: similar to recurrence plot

Time Series Analysis using Complex Networks

- Link matrix = recurrence matrix of time series
- Nodes: states in phase space
- Links: local neighbours of states (i.e. recurrence)
- Path: connected neighbourhoods



Marwan et al., Phys. Lett. A 373, 2009

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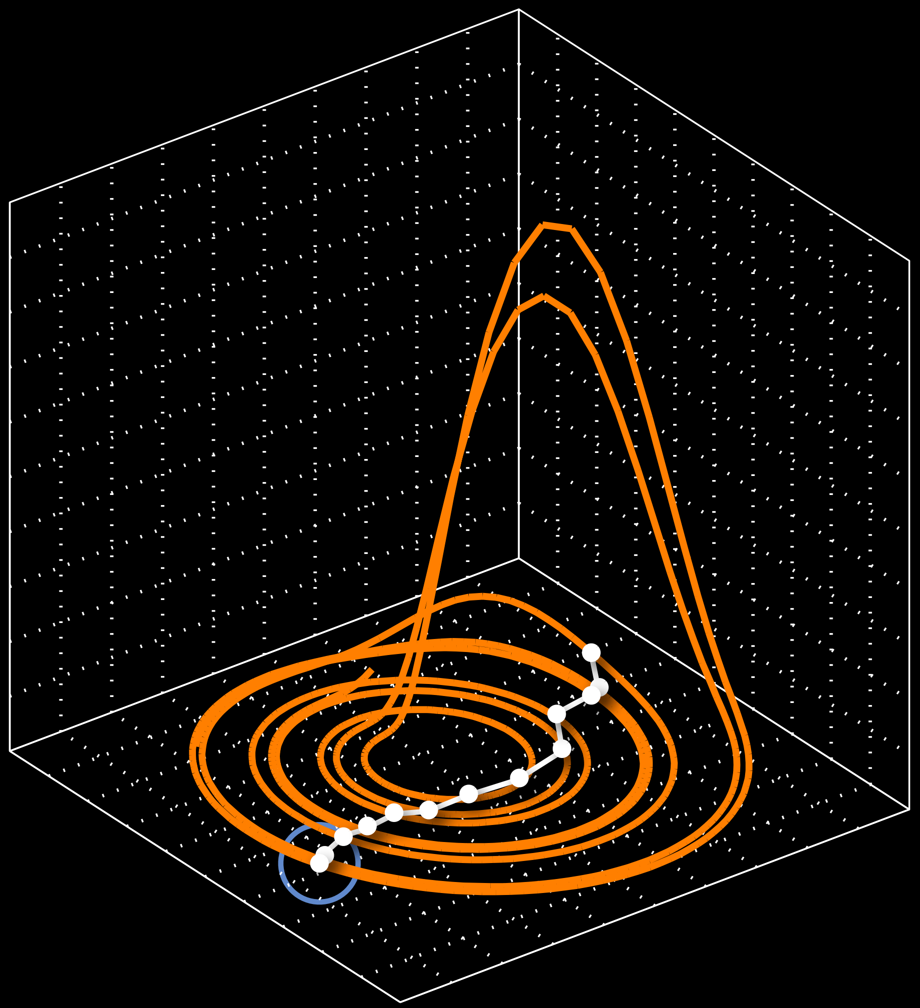

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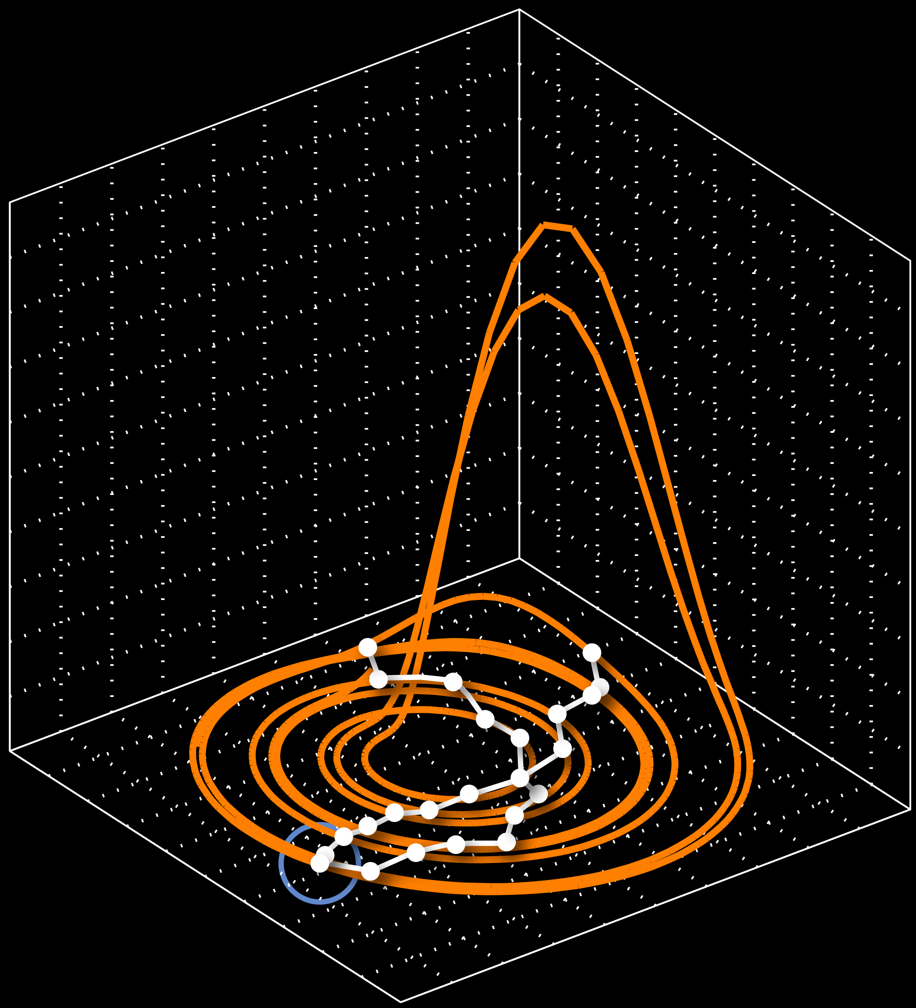
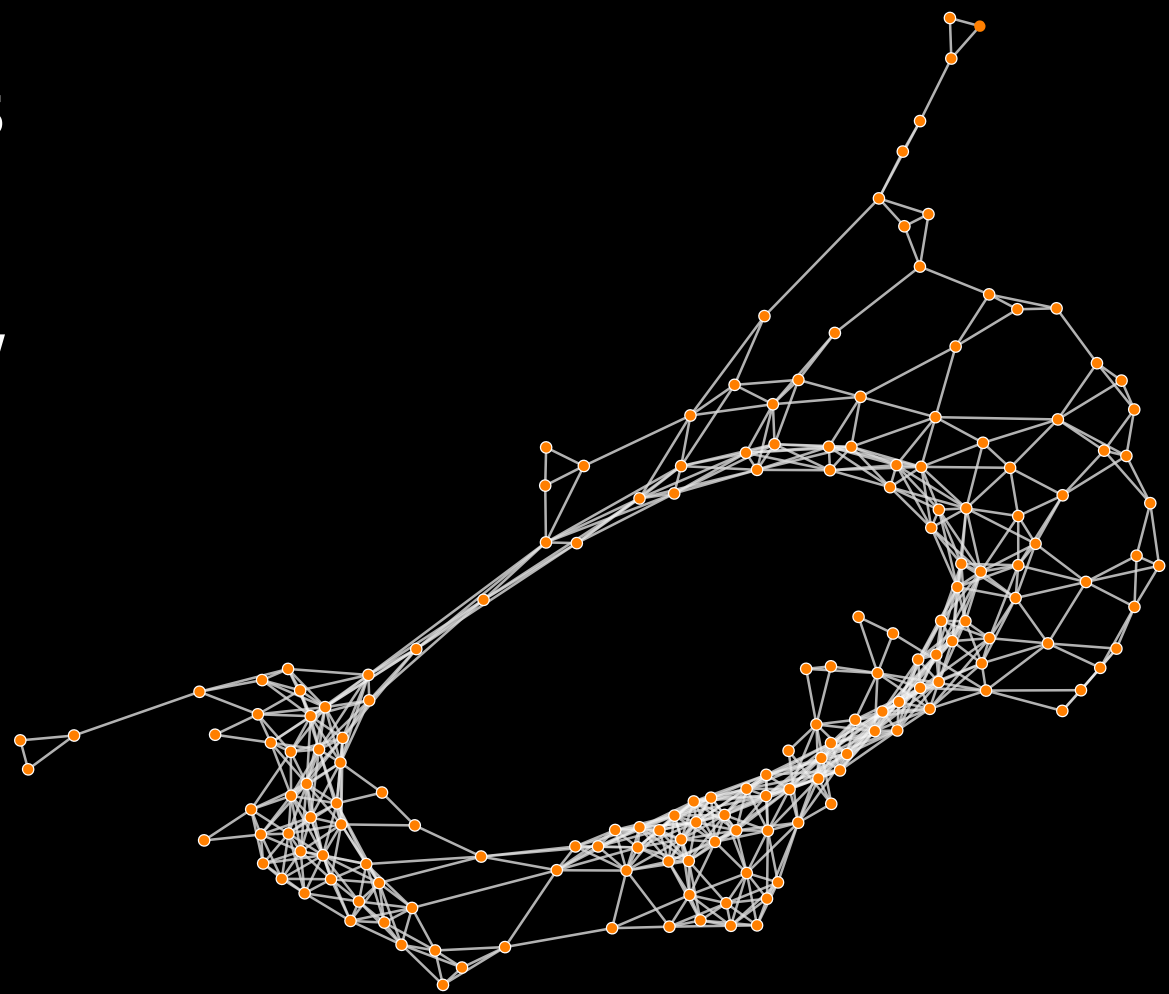


Marwan et al., Phys. Lett. A 373, 2009

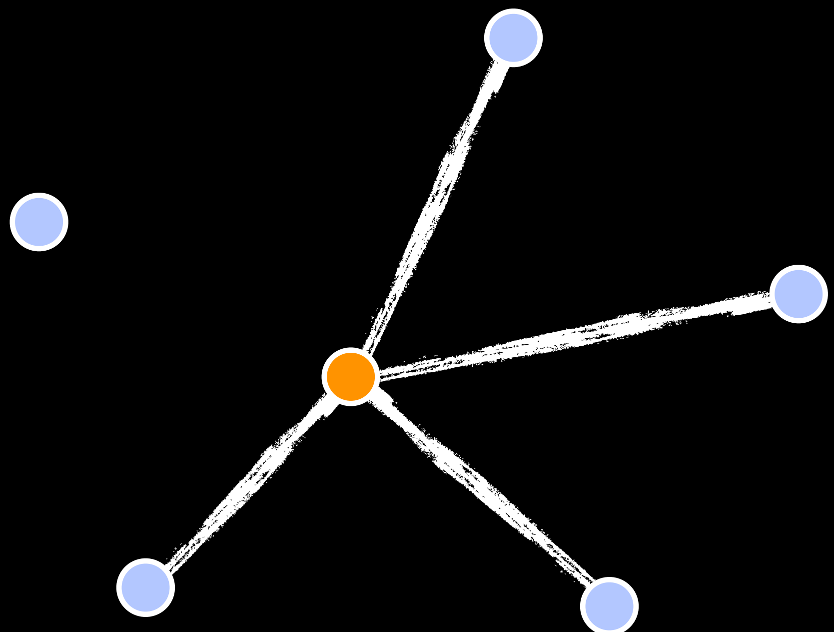
Time Series Analysis using Complex Networks

- Complex network measures applied to recurrence plot
  - measures of complexity explaining dynamical properties complex systems
- „recurrence network“

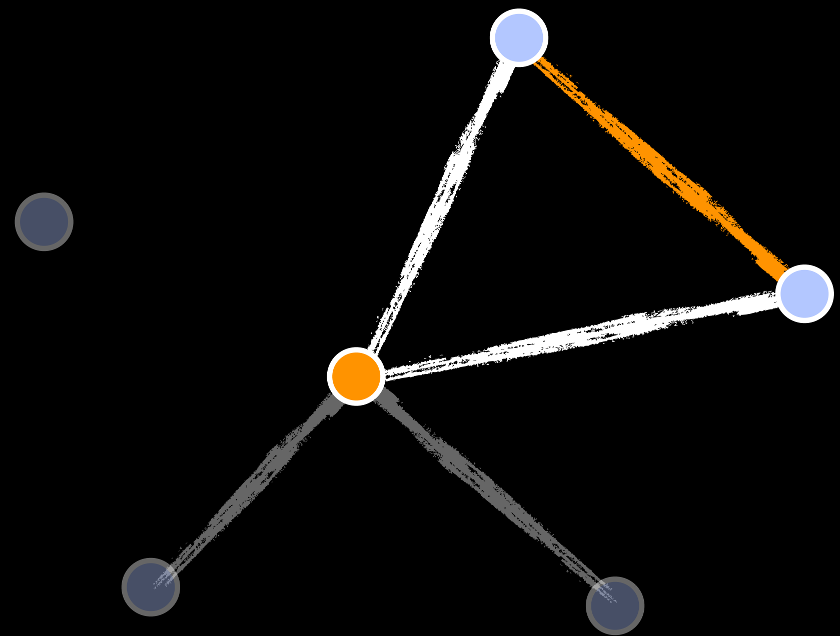


Marwan et al., Phys. Lett. A 373, 2009

Clustering Coefficient

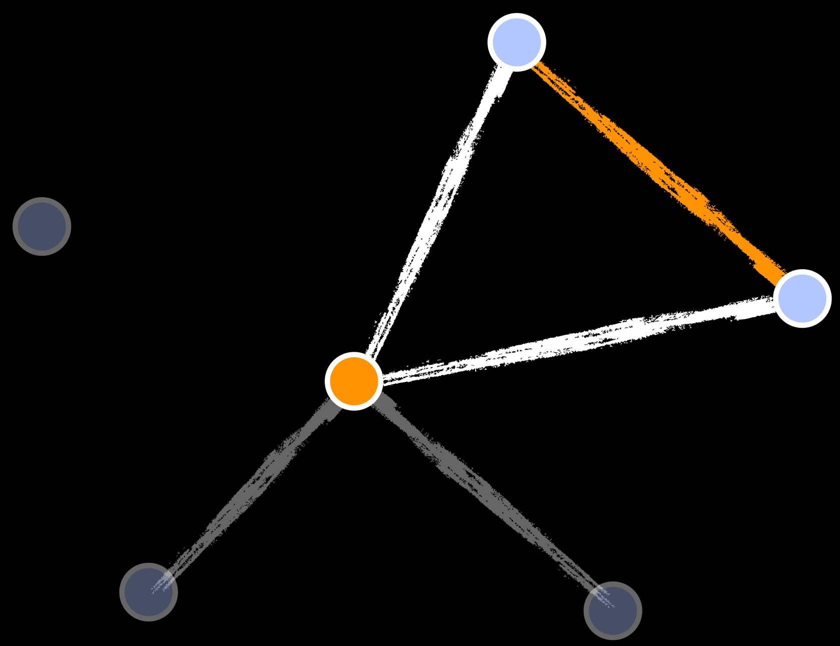


Clustering Coefficient



- probability that neighbours of a node are also connected

Clustering Coefficient

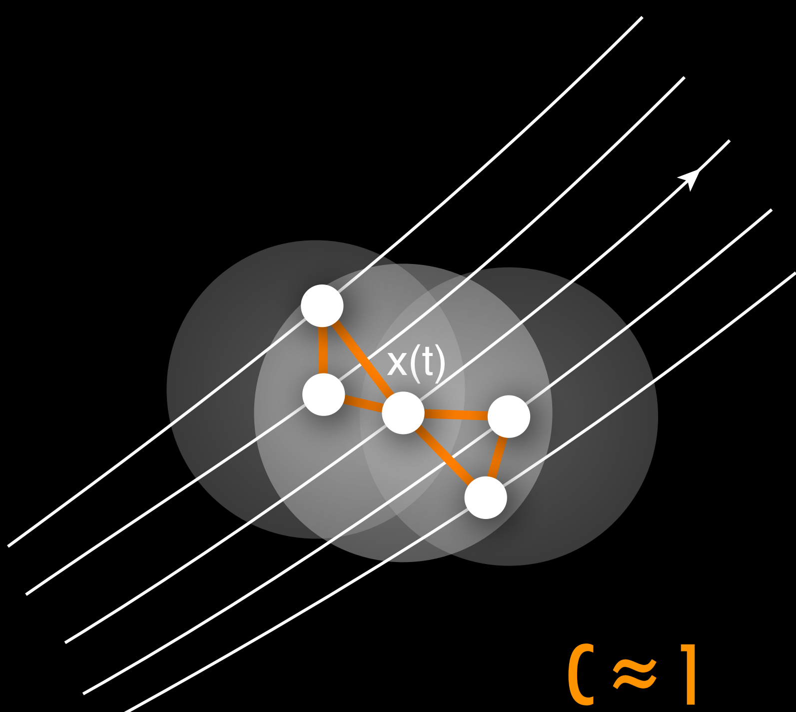


$$C_v = \frac{\sum_{i,j} A_{v,i} A_{i,j} A_{j,v}}{k_v(k_v - 1)}$$

- probability that neighbours of a node are also connected

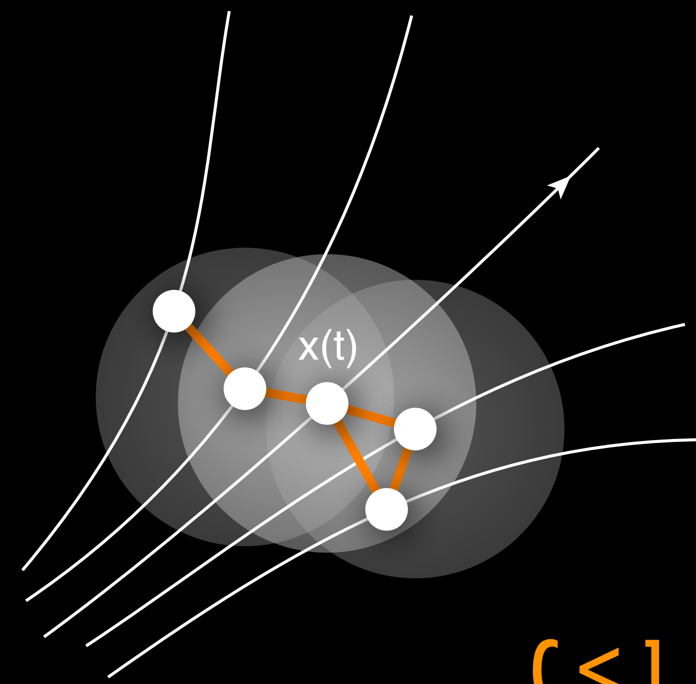
Clustering Coefficient in Phase Space

Regular/ periodic



$C \approx 1$

Diverging/ chaotic



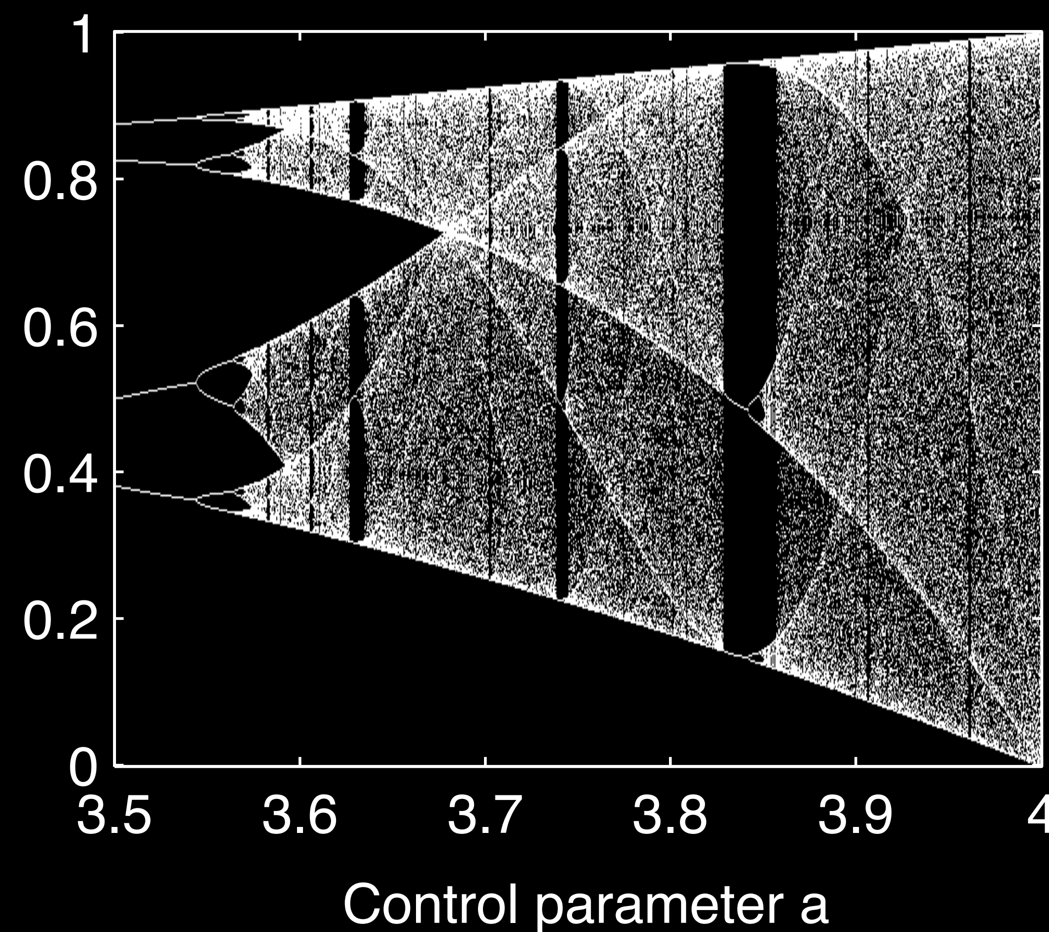
$C < 1$

- clustering coefficient: regularity of dynamics

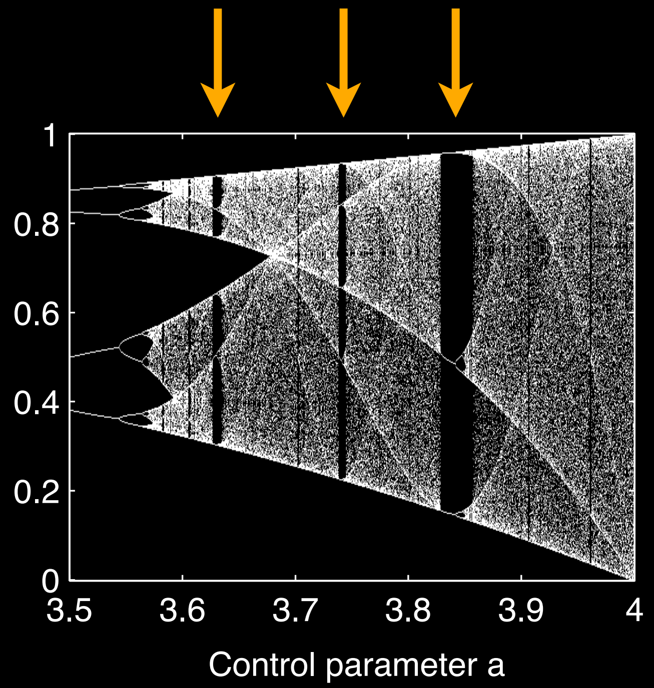
Example: Logistic Map

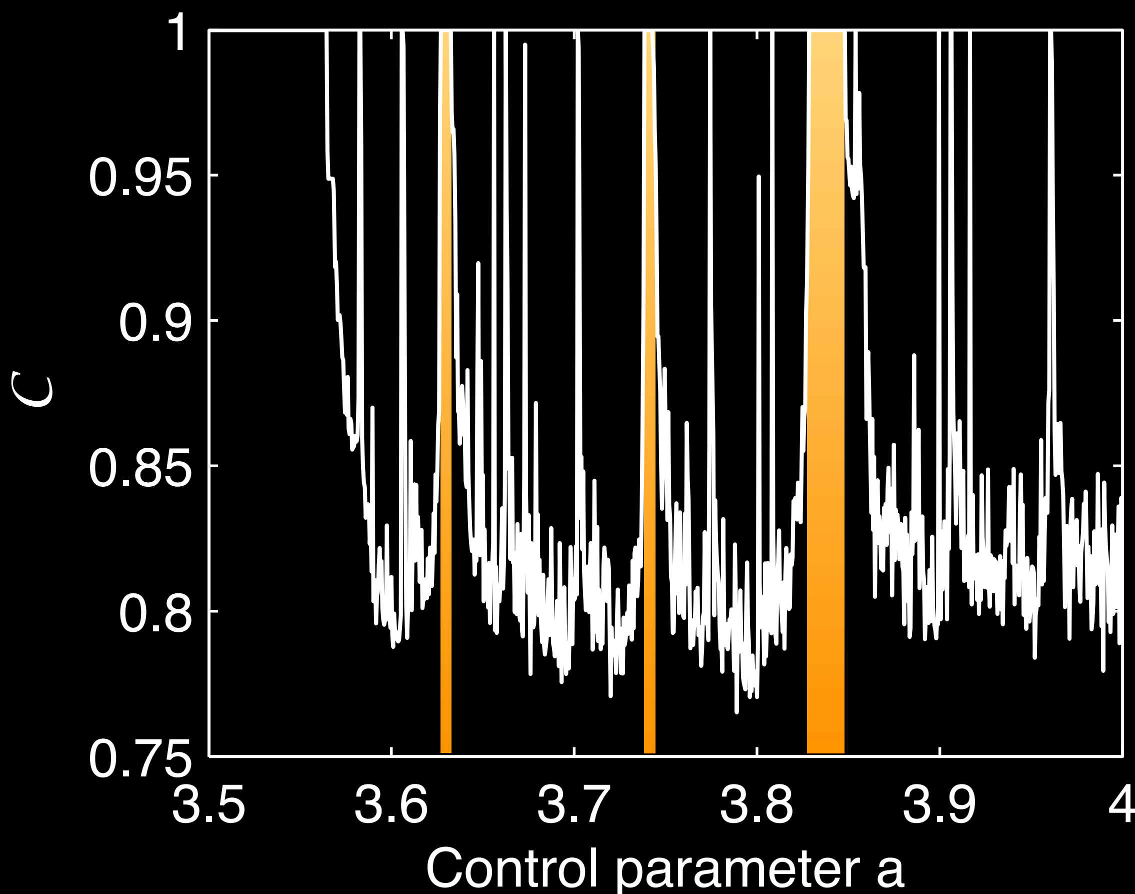
- Logistic map:

$$x_{i+1} = a\, x_i(1 - x_i)$$



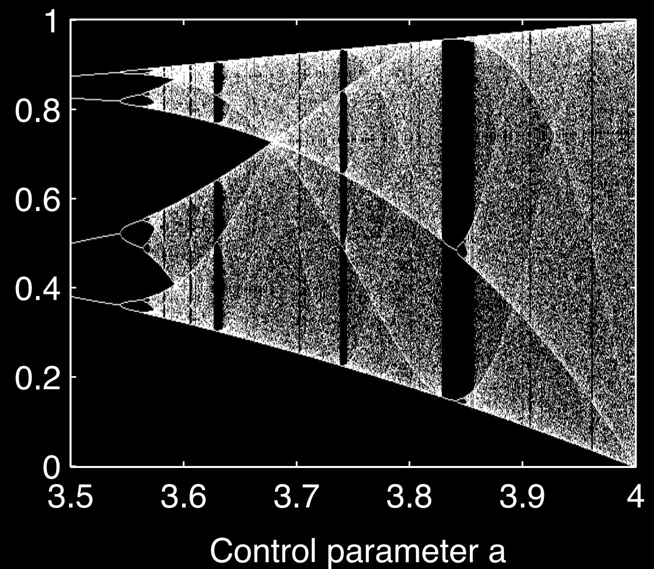
Example: Logistic Map

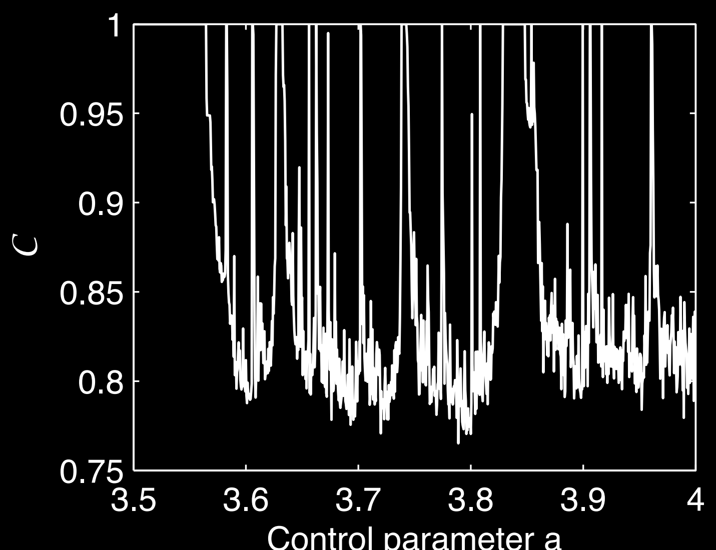


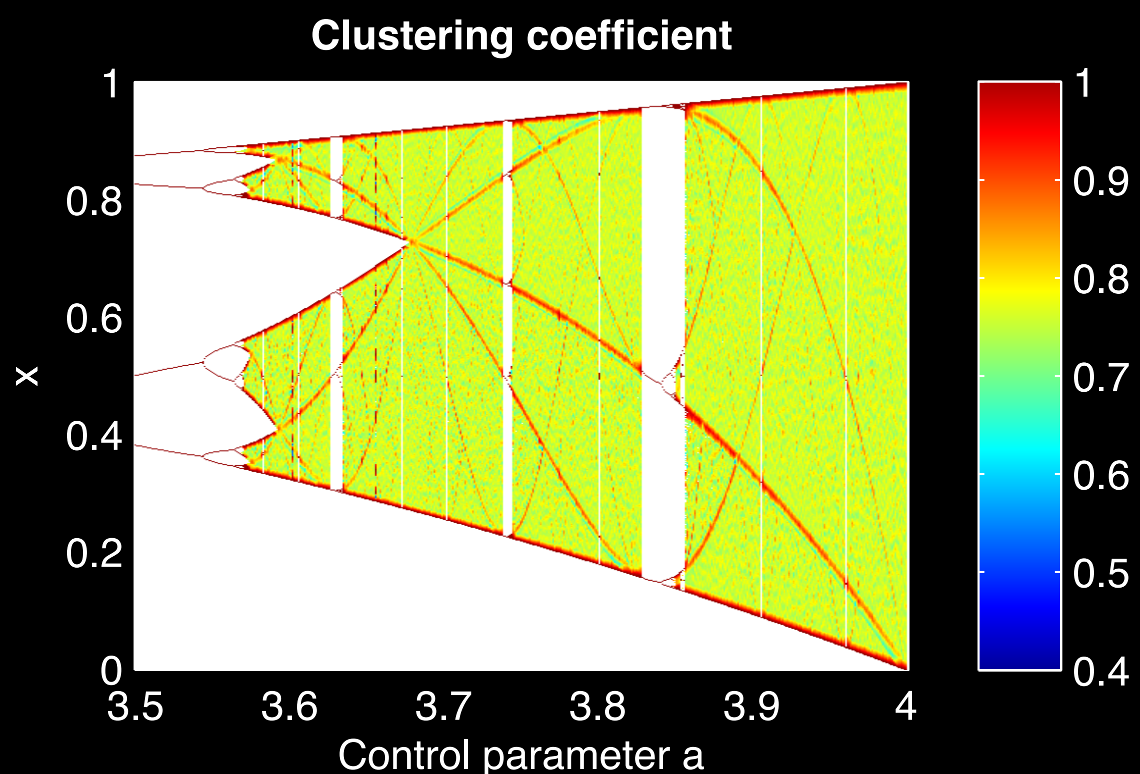


- periodic windows

Example: Logistic Map







- intermittent phases

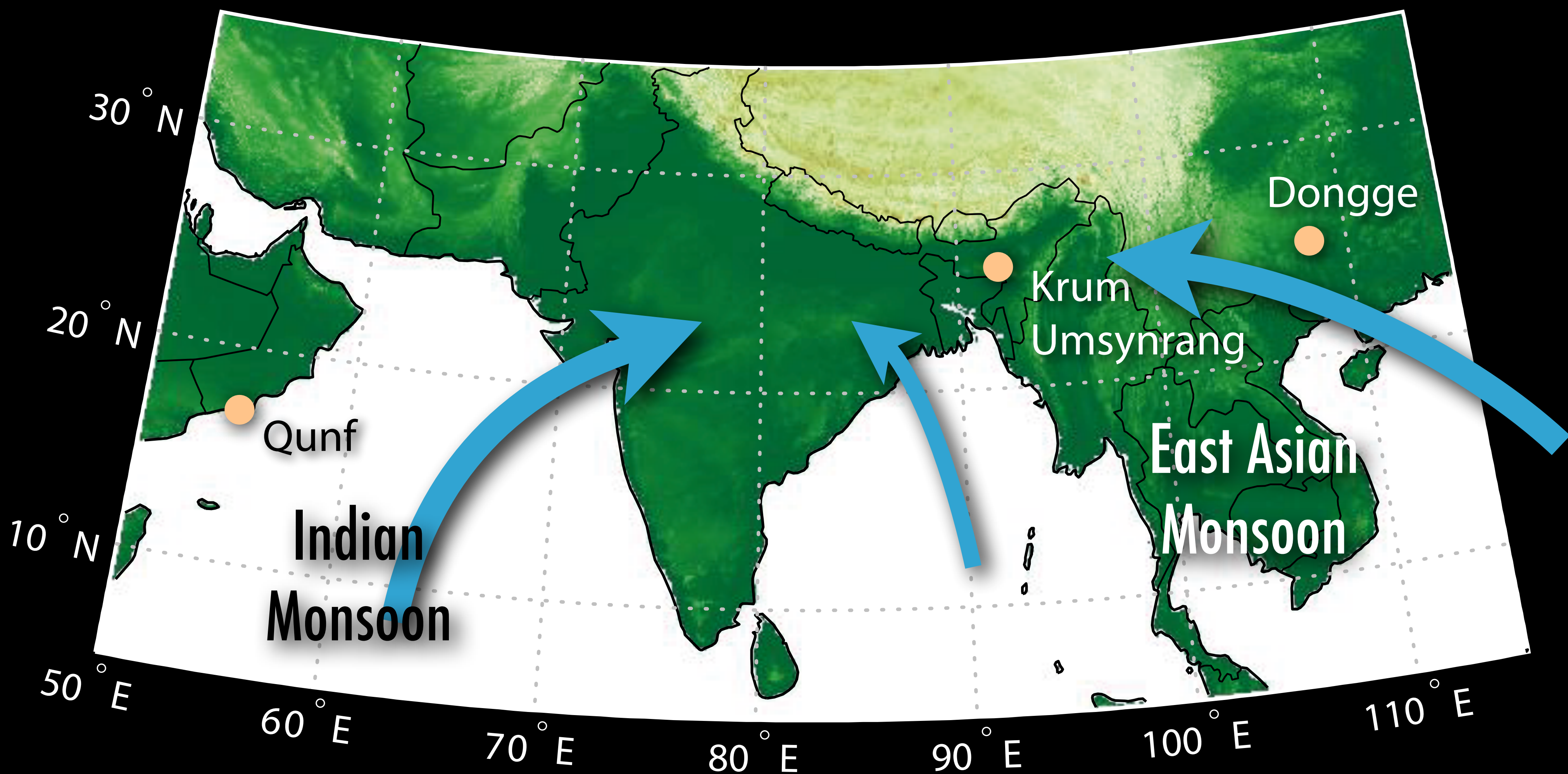
Time Series Analysis using Complex Networks

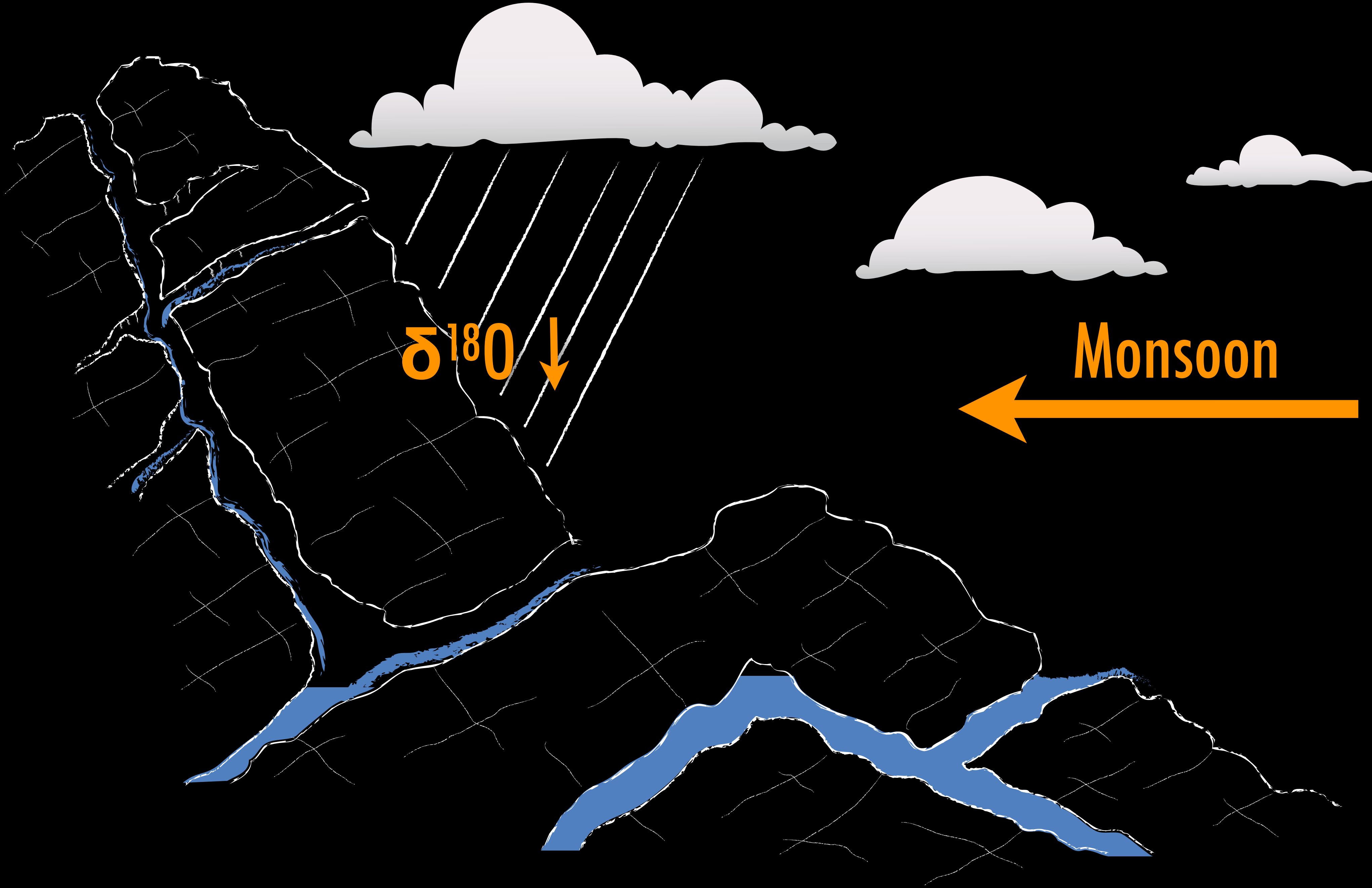
- Degree centrality (recurrence probability)
- Clustering coefficient (regularity)
- Betweenness centrality (attractor fractionation)
- Average shortest path length (mean phase space separation)
- Matching index („twinness“ of states)
- etc.

Donner et al., New J. Phys 12, 2010

Applications

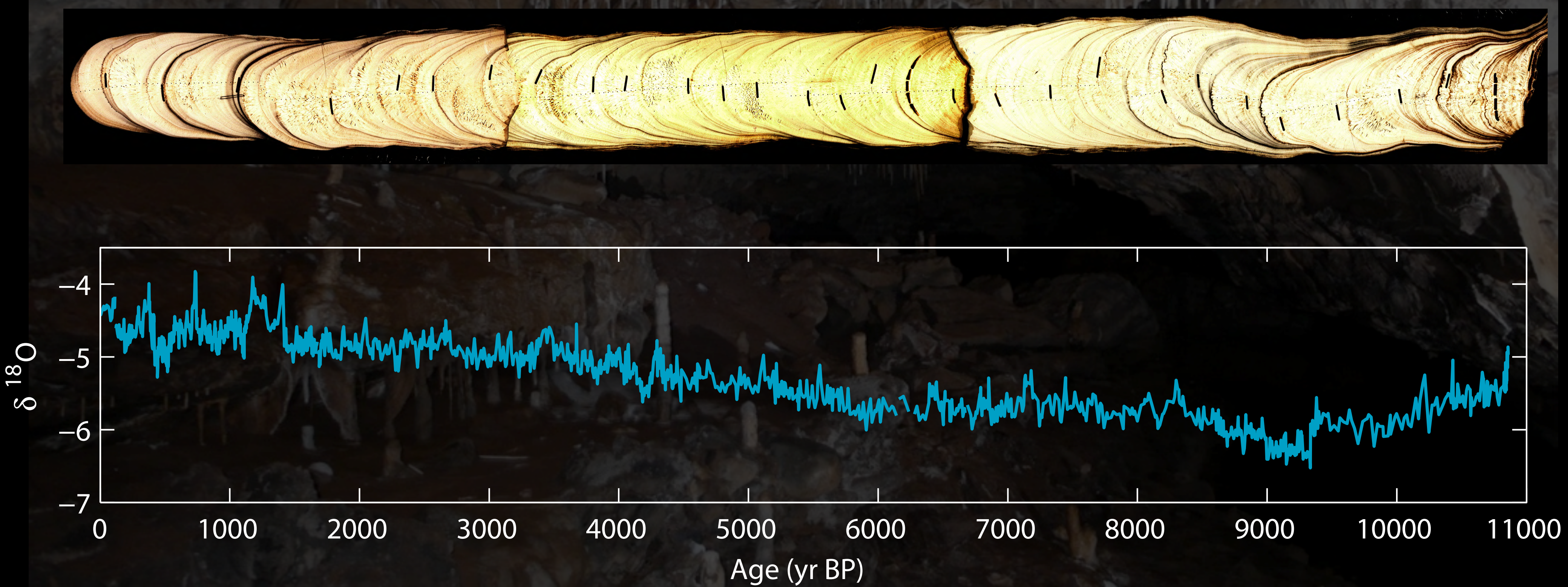
Asian Monsoon



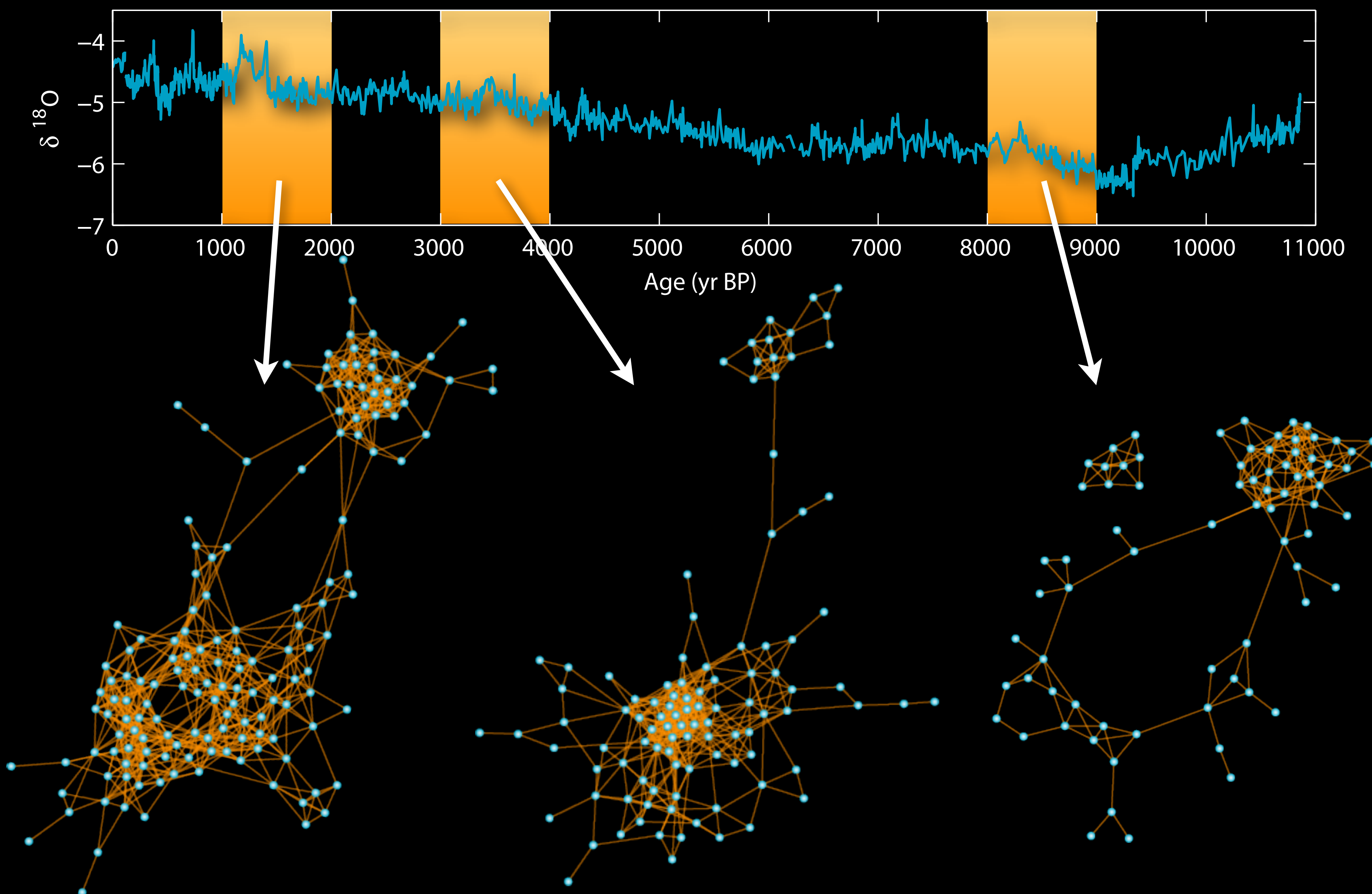

δ18O ↓
Monsoon

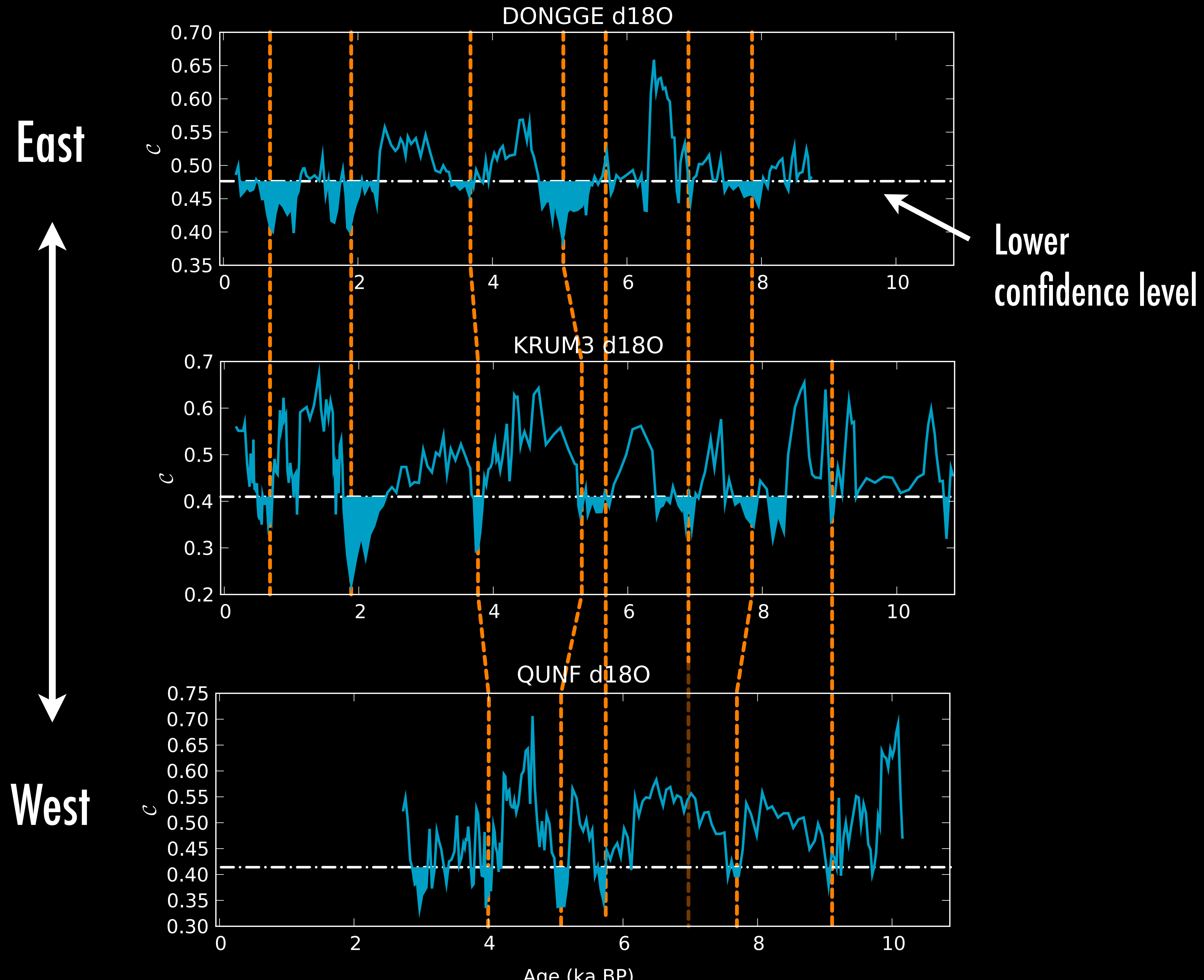
Asian Monsoon

Asian Monsoon

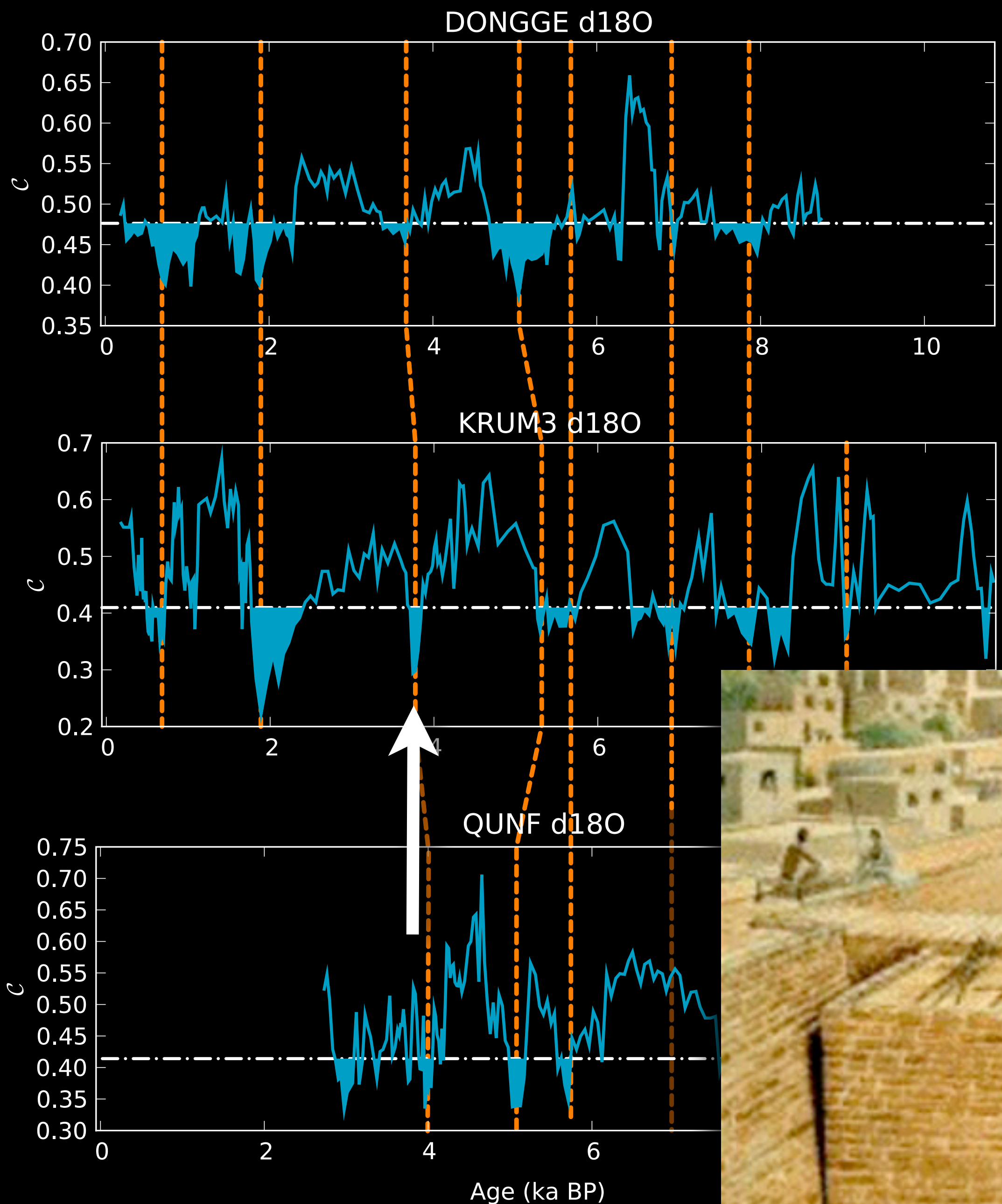


Asian Monsoon





East
West
DONGGE d18O
KRUM3 d18O
QUNF d18O
c
Age (ka BP)
Lower confidence level



3900-3700 yr BP
Harappan culture vanished

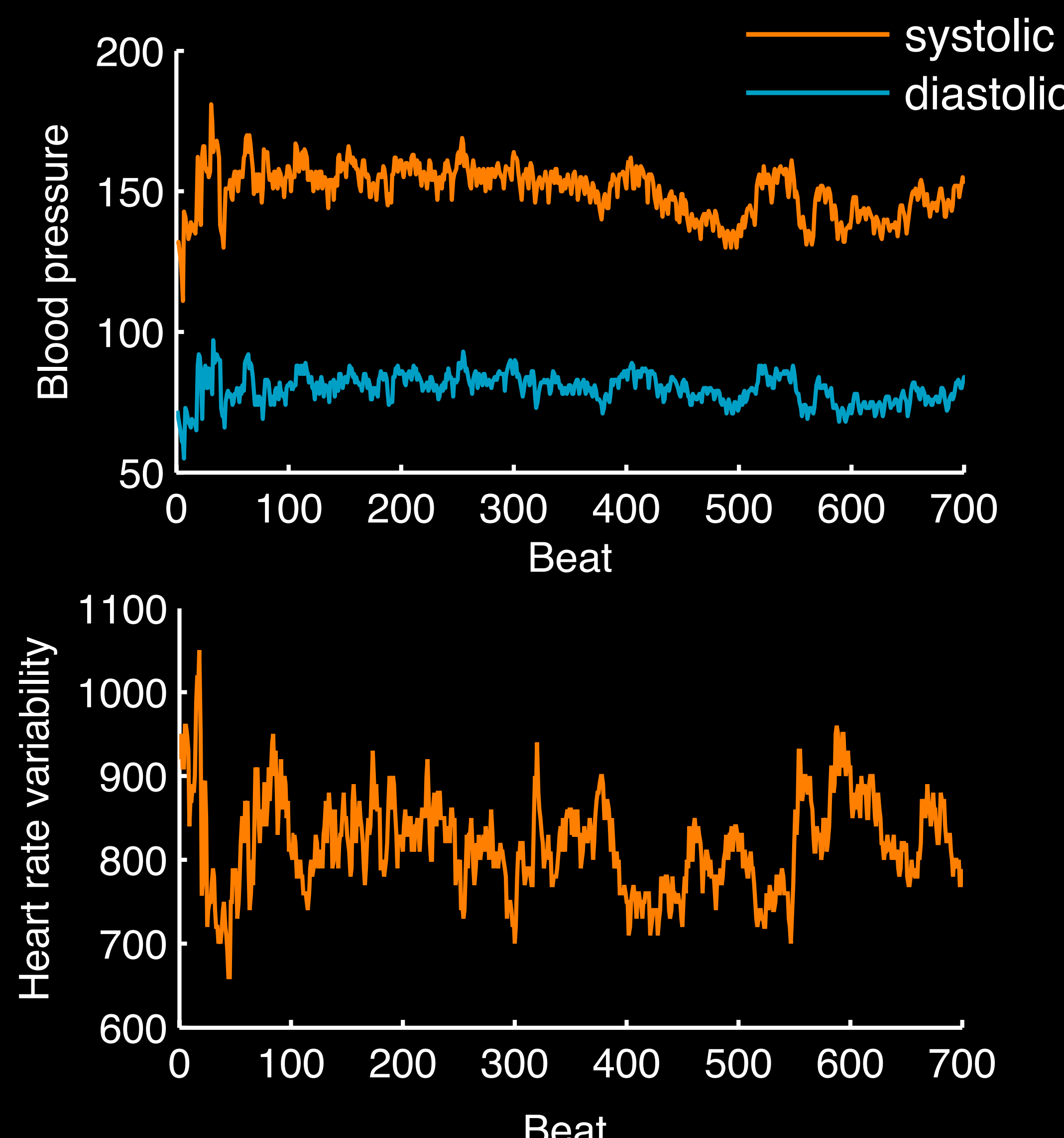
Early Detection of Preeclampsia in Pregnancy

- Life-threatening cramps for mother and fetus
- Under-supply of the fetus
- Growth retardation

‣ positive predictive value appr. 20-30%

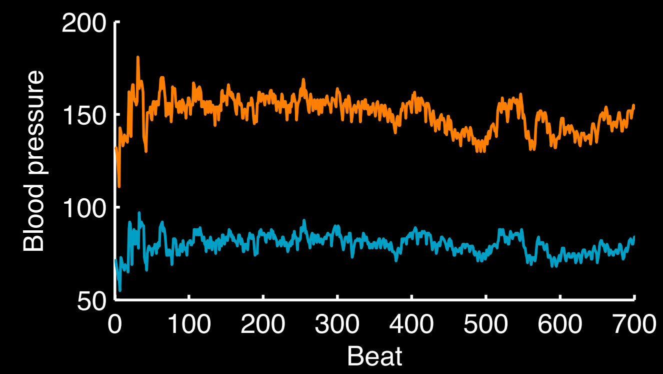
Early Detection of Preeclampsia in Pregnancy

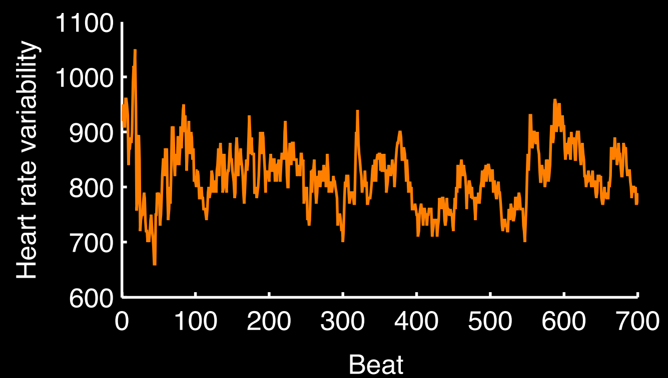
- 20th week of gestation
- Systolic and diastolic blood pressure (S, D)
- Heart rate variability (H)



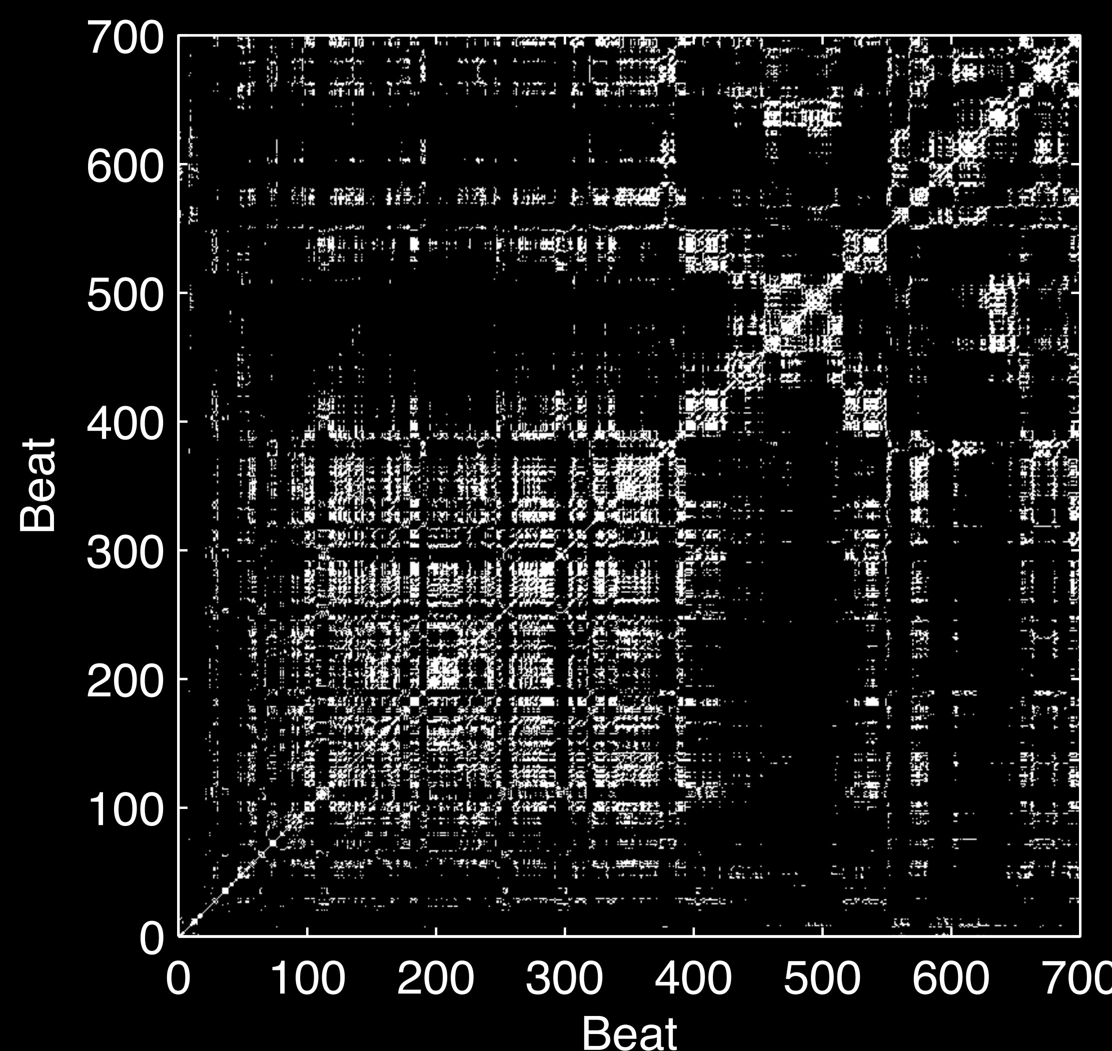
Walther et al., J Hypertens 24, 2006
Malberg et al., Chaos 17, 2007

Early Detection of Preeclampsia in Pregnancy

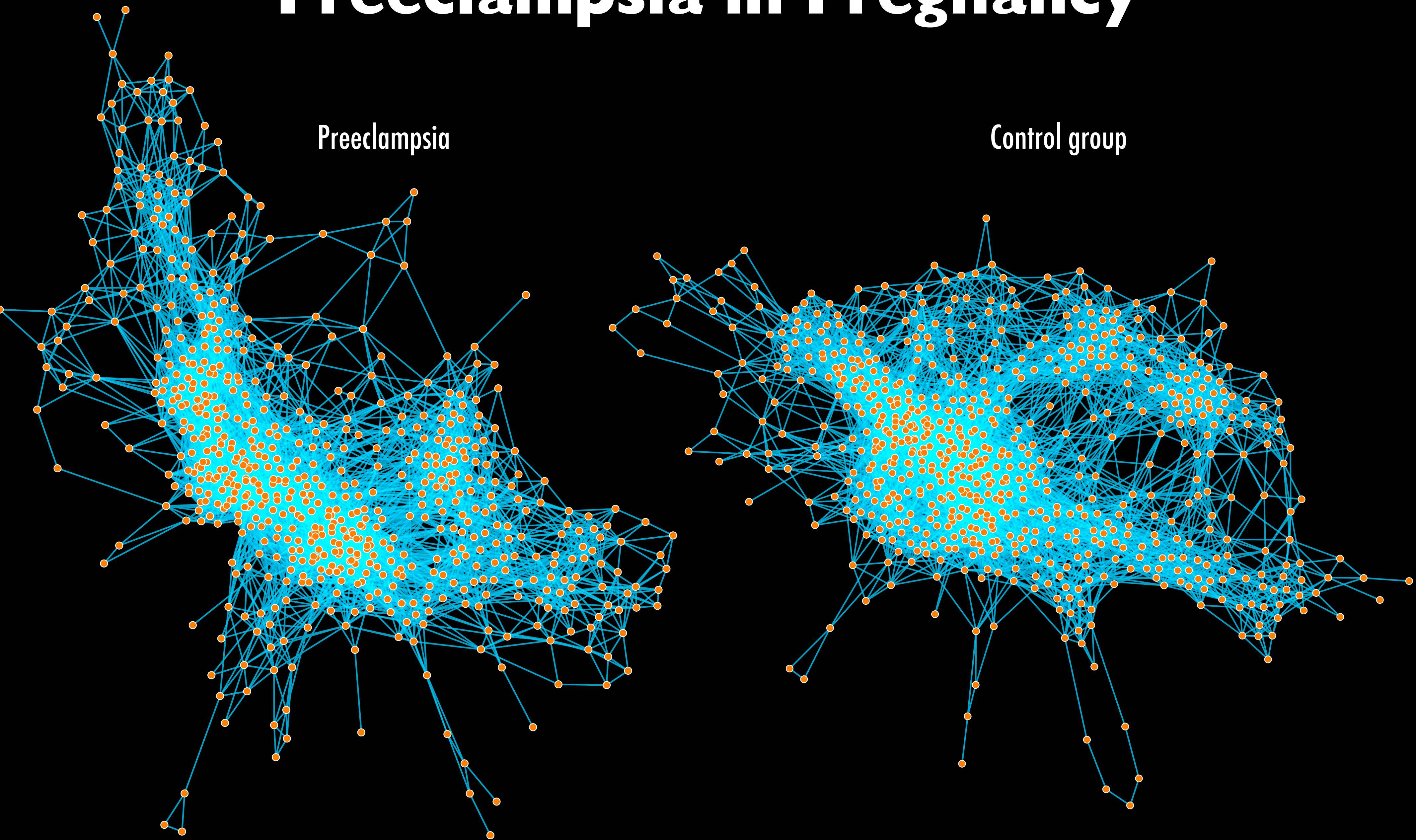




$$\vec{x} = \begin{pmatrix} H \\ D \\ S \end{pmatrix}$$



Early Detection of Preeclampsia in Pregnancy



Early Detection of Preeclampsia in Pregnancy

	Preeclampsia	Control	p
H (ms)	734.5 (±110.8)	760.5 (±111.7)	n.s.
S (mmHg)	123.0 (±15.4)	123.5 (±20.0)	n.s.
D (mmHg)	75.5 (±10.4)	66.6 (±13.9)	n.s.
recurrence rate	0.14 (±0.04)	0.16 (±0.05)	0.0024
laminarity	0.80 (±0.10)	0.83 (±0.08)	n.s.
clustering	0.60 (±0.03)	0.62 (±0.04)	0.0015

positive accuracy value: 60% negative accuracy value: 80%

Summary

- Complex networks from time series
- Recurrence analysis using complex network statistics
- Complementary analysis to traditional recurrence measures

Publications

- N. Marwan, M. C. Romano, M. Thiel, J. Kurths: Recurrence Plots for the Analysis of Complex Systems, Physics Reports, 438(5–6), 237–329 (2007)
- N. Marwan, J. Donges, Y. Zou, R. Donner, J. Kurths: Complex network approach for recurrence analysis of time series, Phys. Lett. A 373, 4246–4254 (2009)
- R. V. Donner, Y. Zou, J. F. Donges, N. Marwan, J. Kurths: Recurrence networks – A novel paradigm for nonlinear time series analysis, New Journal of Physics, 12(3), 033025 (2010)