Recurrence Plots
in Earth Sciences

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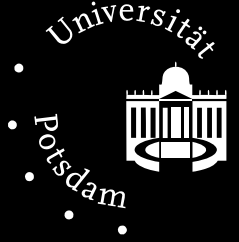


Recurrence

- fundamental characteristic of many dynamical systems
- Poincaré, 1890:

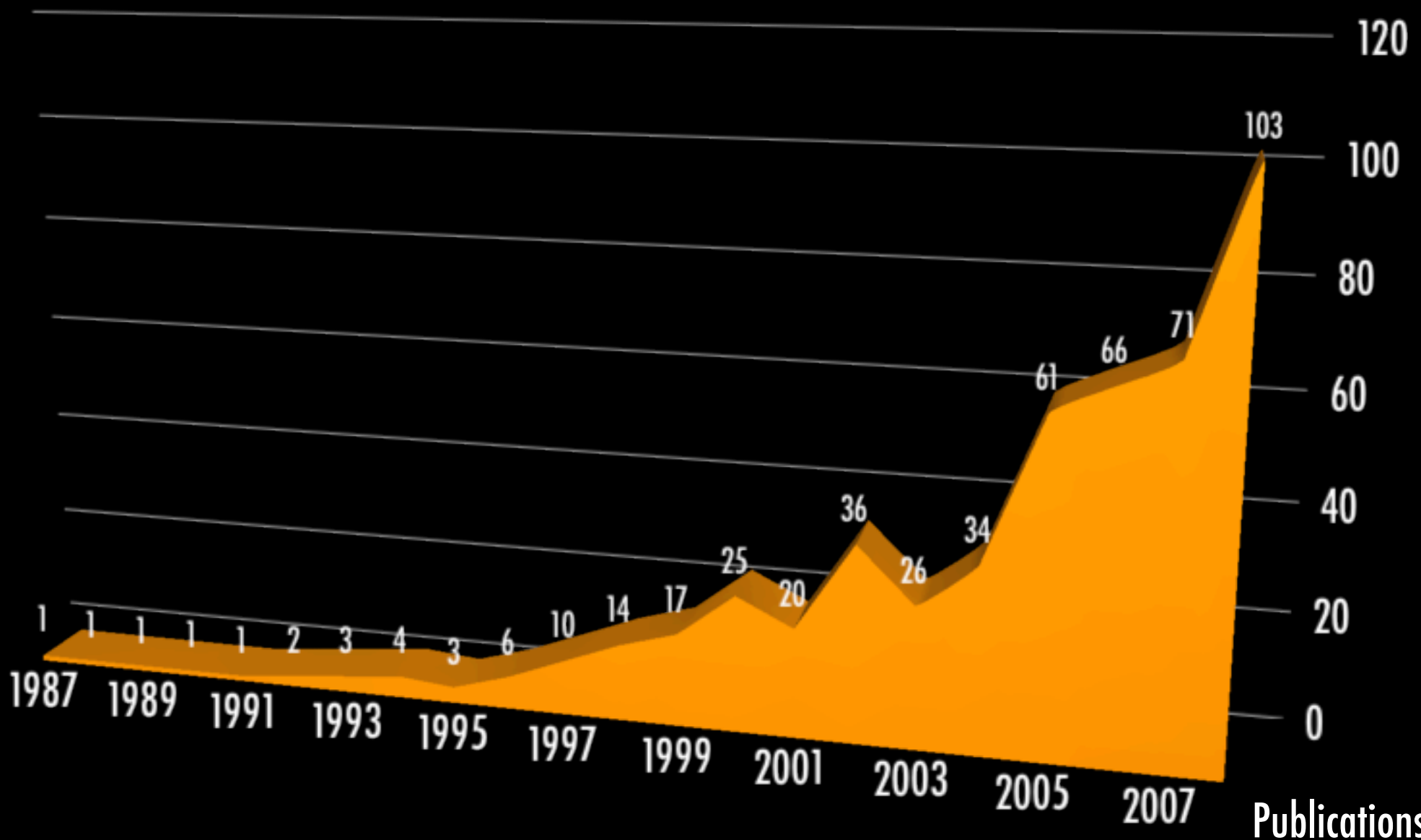
  "a system recurs infinitely many times as close as one wishes to its initial state"
- recurrences in real life:

  Milankovich cycles, weather after storm, El Niño phenomenon, heart beat after exertion, Maya calendar etc.

Investigating System's Dynamics

- Poincaré map
- Recurrence time statistics
- First return map
- Recurrence plot

Historical Review




Historical Review
Introduction
Quantification
Time-dependent Quantification
Cross-RP
Joint-RP
Synchronisation
Dynamical invariants
Surrogates
Coupling direction
Significance tests
1
1
1
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71
103
1987
1989
1991
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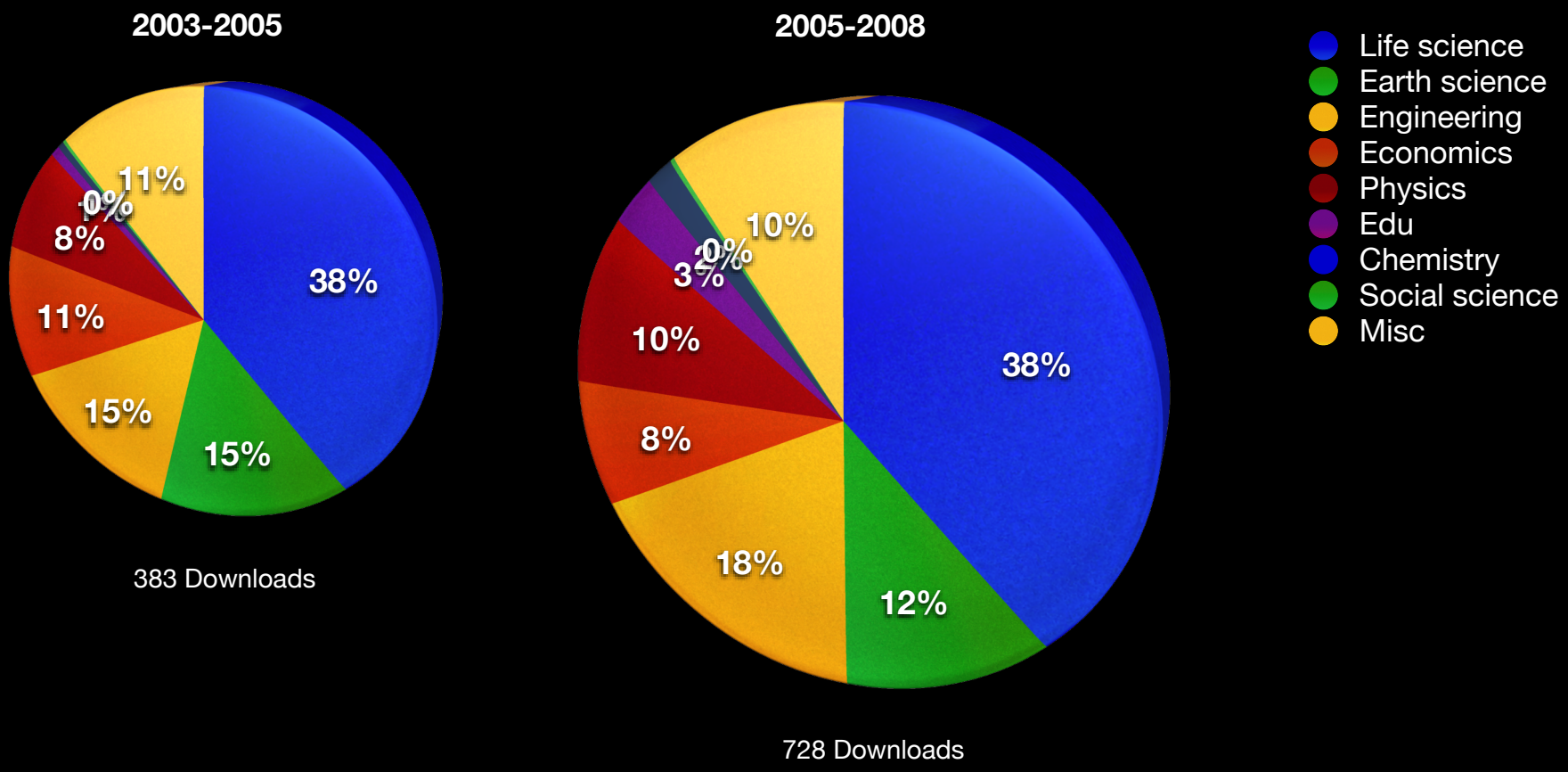
2007
120
100
80
60
40
20
0
Publications

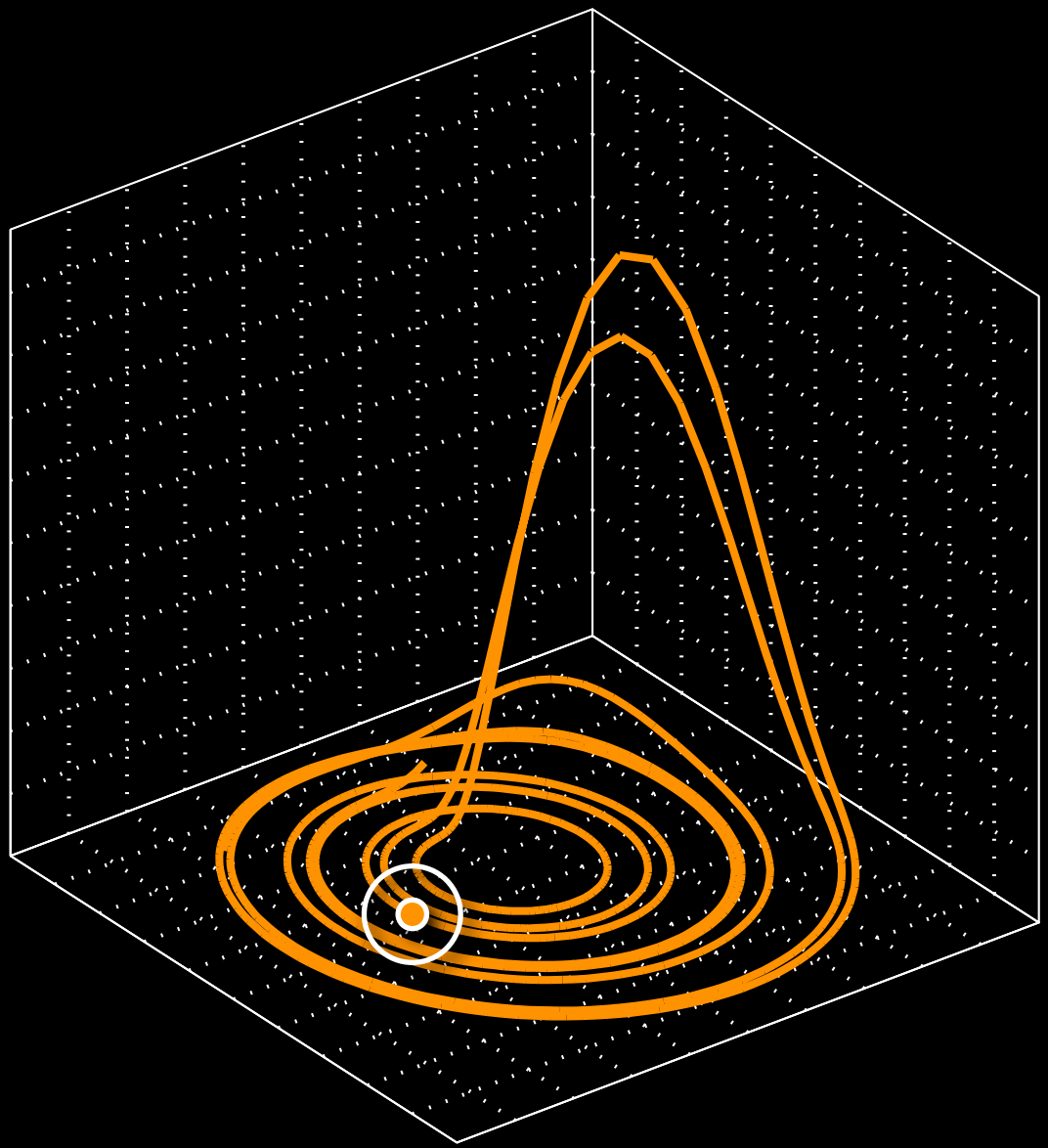
Application Diversity

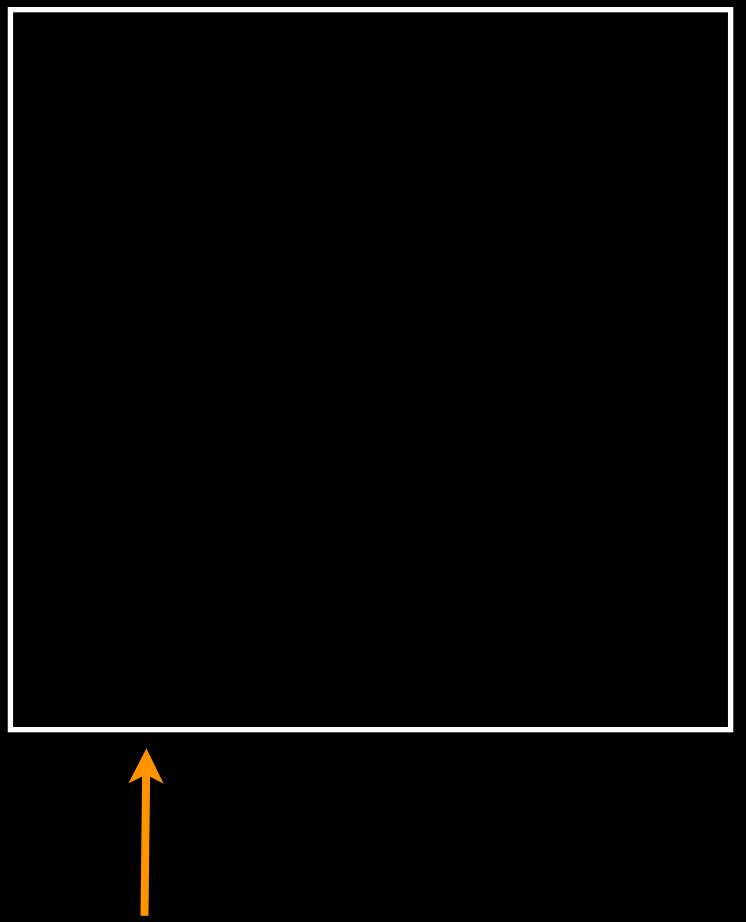


383 Downloads

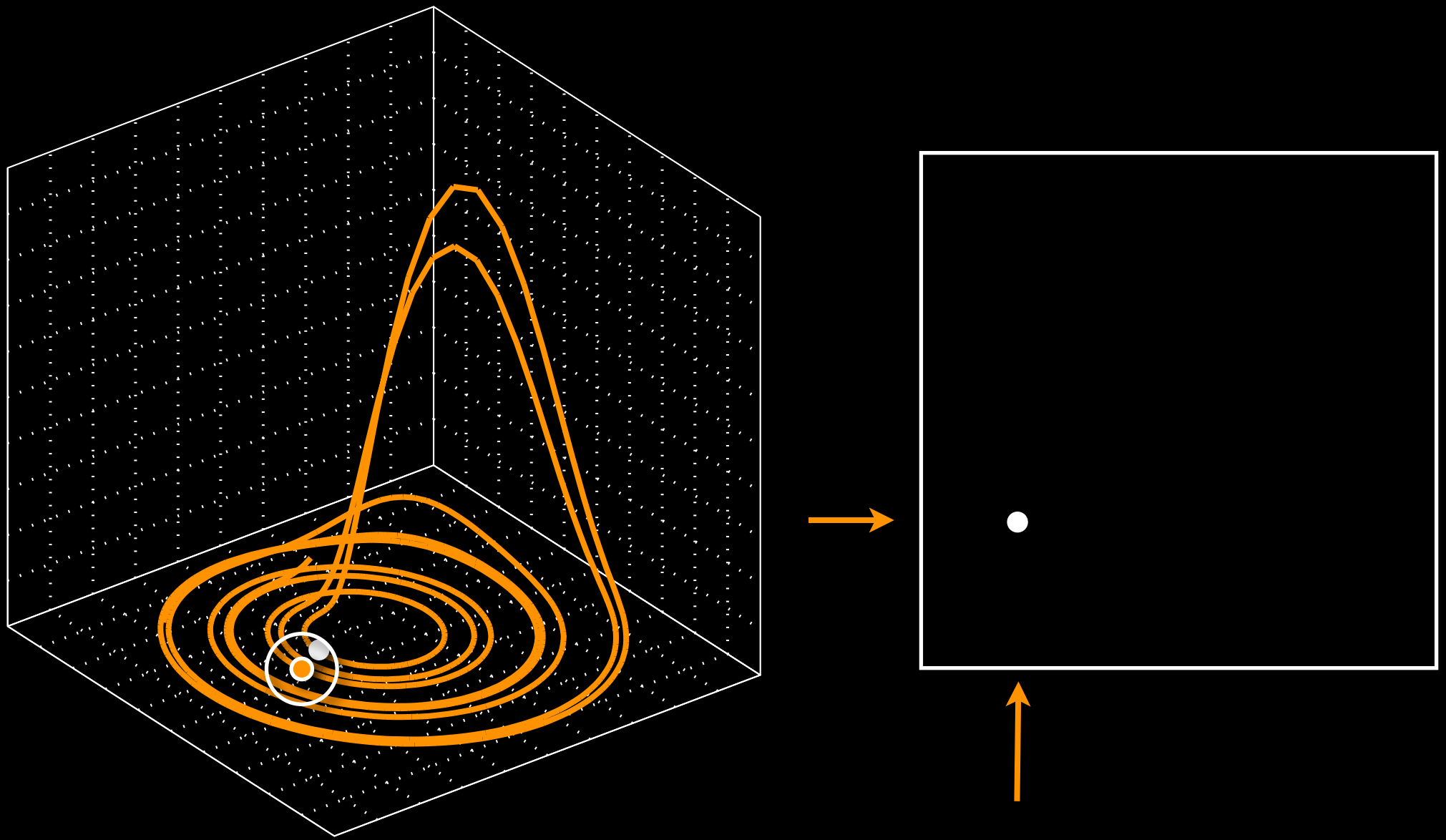
728 Downloads

Recurrence Plot

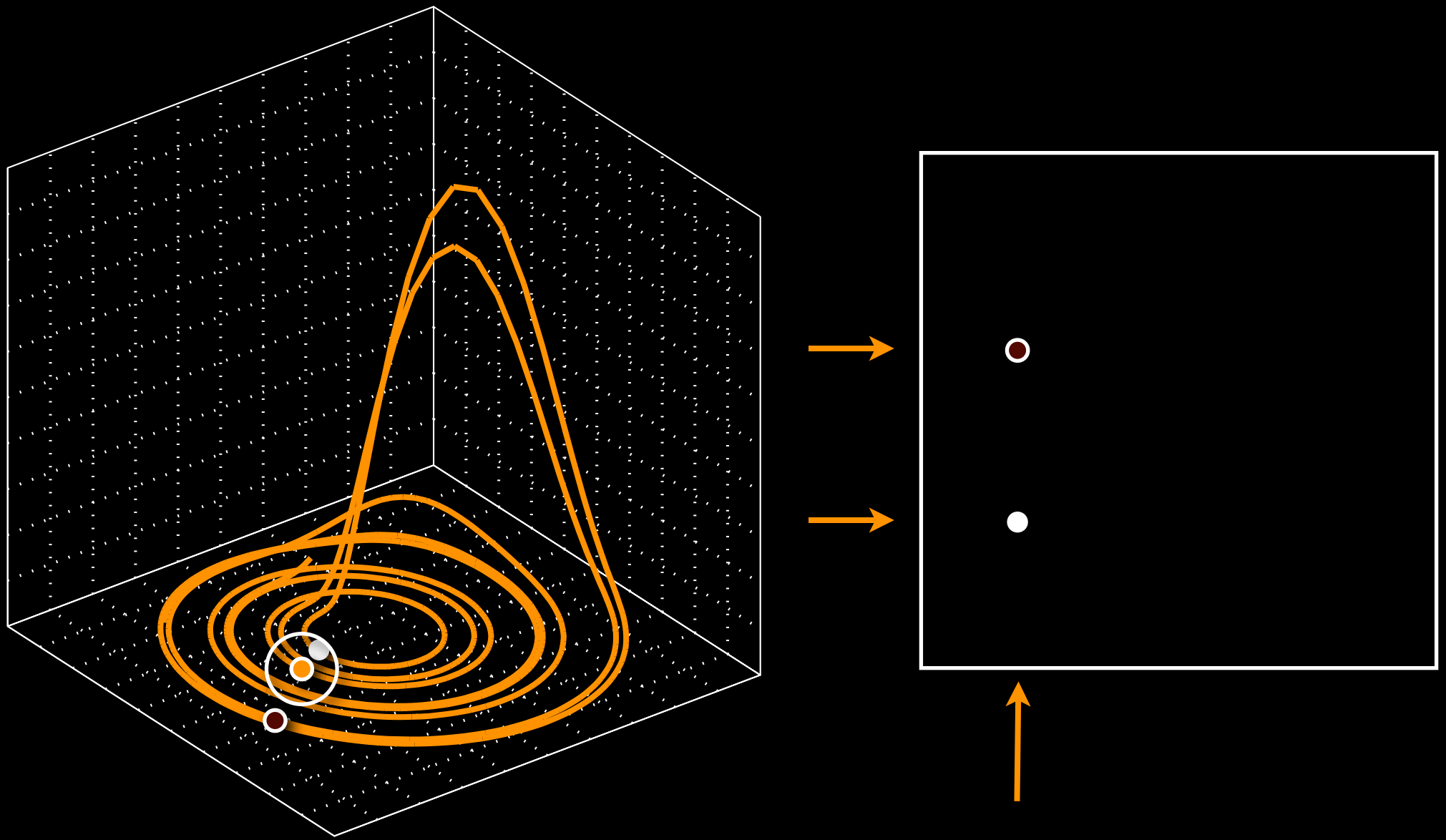




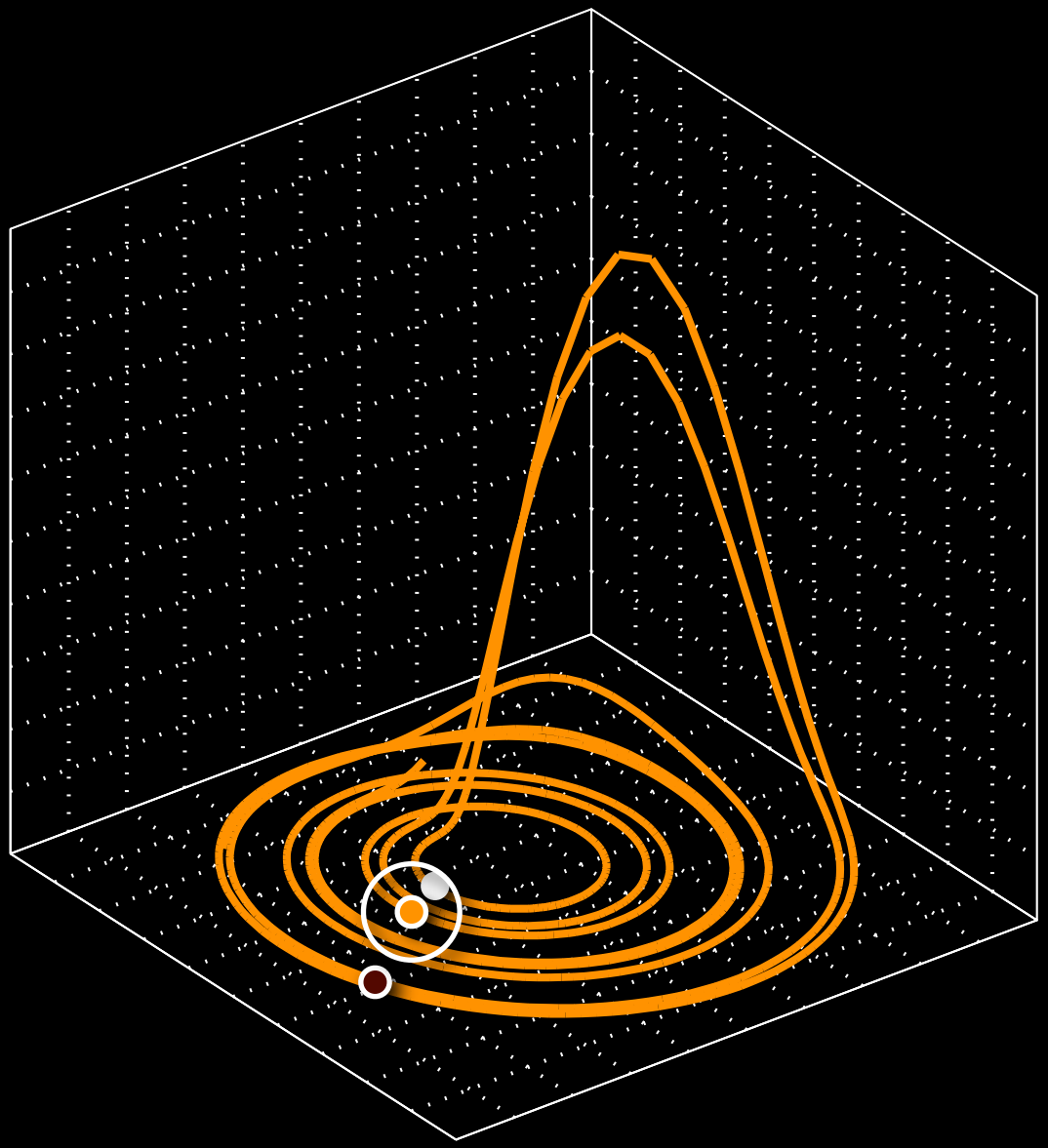
Recurrence Plot

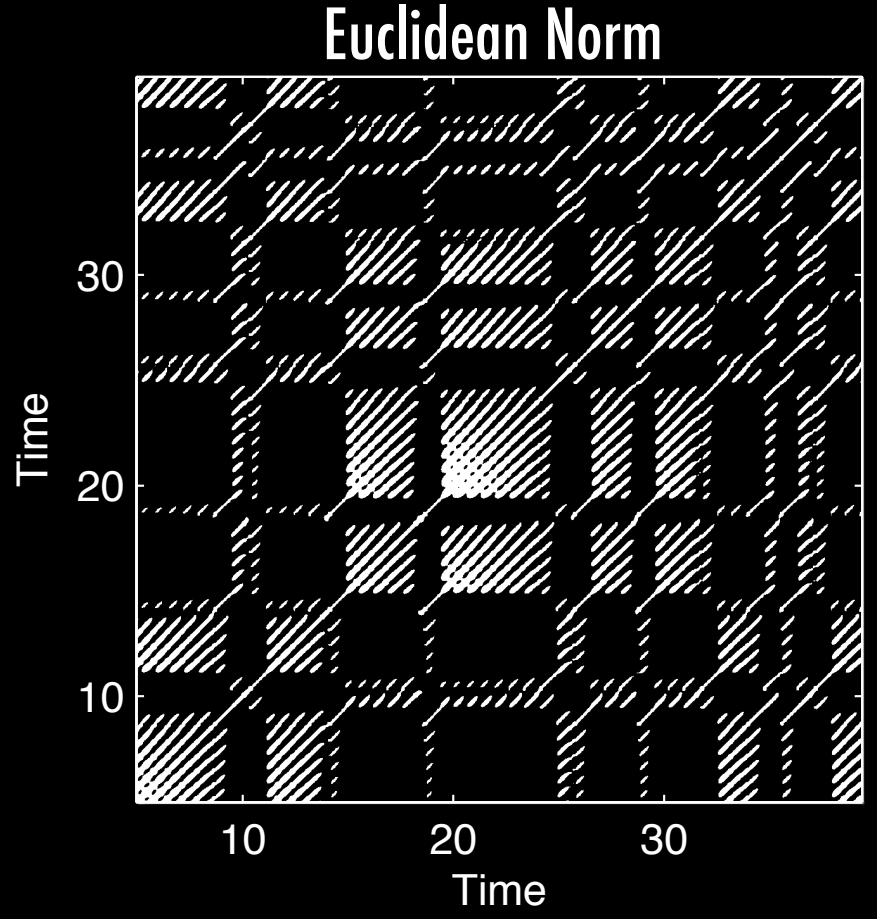


Recurrence Plot



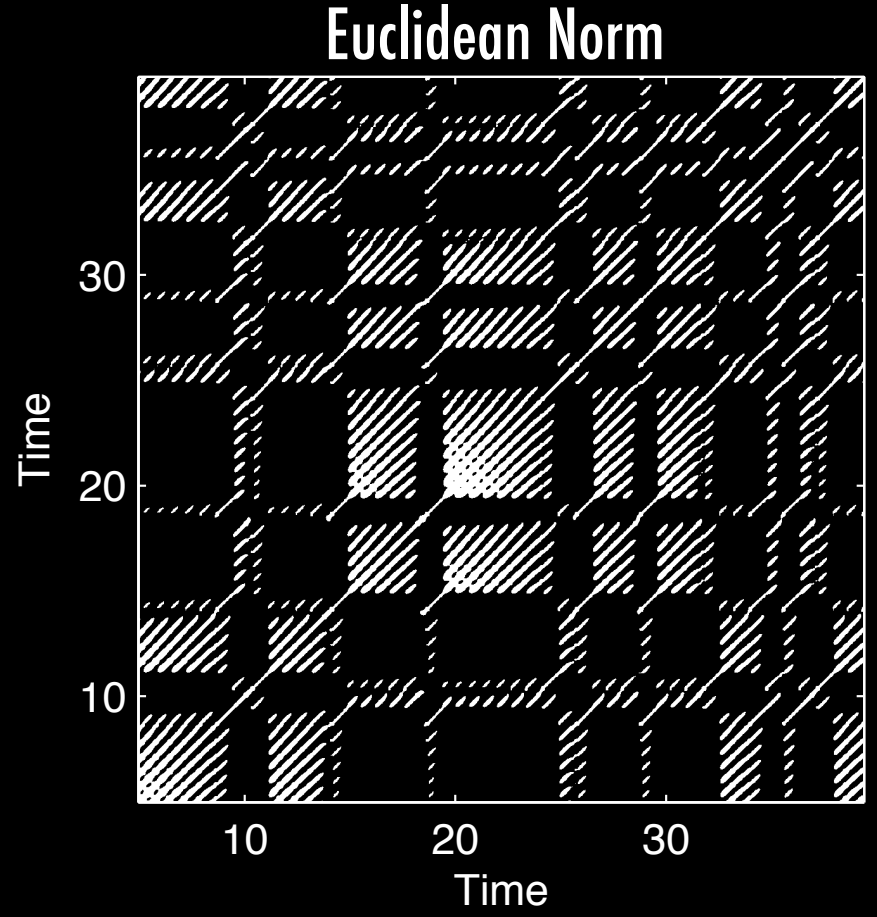
Recurrence Plot





Recurrence Plot

- Transition detection
- Differentiate dynamics
- Finding time scales
- Interrelation detection
- Synchronisation analysis
- Surrogates



nonstationary and short data!

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Recurrence plots for the analysis of complex systems

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Abstract

Recurrence is a fundamental property of dynamical systems, which can be exploited to characterise the system's behaviour in phase space. A powerful tool for their visualisation and analysis called *recurrence plot* was introduced in the late 1980's. This report is a comprehensive overview covering recurrence based methods and their applications with an emphasis on recent developments. After a brief outline of the theory of recurrences, the basic idea of the recurrence plot with its variations is presented. This includes the quantification of recurrence plots, like the recurrence quantification analysis, which is highly effective to detect, e. g., transitions in the dynamics of systems from time series. A main point is how to link recurrences to dynamical invariants and unstable periodic orbits. This and further evidence suggest that recurrences contain all relevant information about a system's behaviour. As the respective phase spaces of two systems change due to coupling, recurrence plots allow studying and quantifying their interaction. This fact also provides us with a sensitive tool for the study of synchronisation of complex systems. In the last part of the report several applications of recurrence plots in economy, physiology, neuroscience, earth sciences, astrophysics and engineering are shown. The

Recurrence Plot

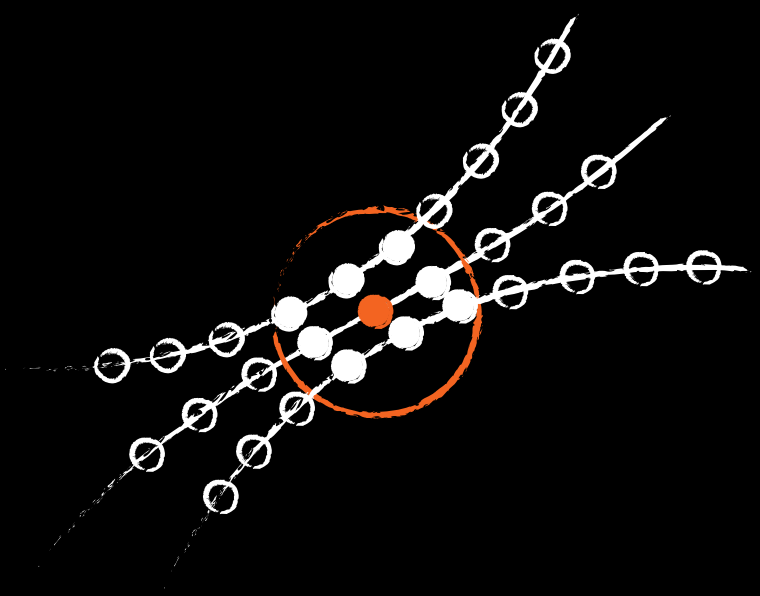
$$\mathbf{R}_{i,j} = \begin{cases} 1 : \vec{x}_i \approx \vec{x}_j \\ 0 : \vec{x}_i \not\approx \vec{x}_j \end{cases} \qquad i, j = 1, \ldots, N$$

Eckmann, Kamphorst, Ruelle, Europhysics Letters, 5, 1987

Recurrence Plot

$$\mathbf{R}_{t_1,t_2} = \Theta\left(\varepsilon - \|\vec{x}(t_1) - \vec{x}(t_2)\|\right)$$

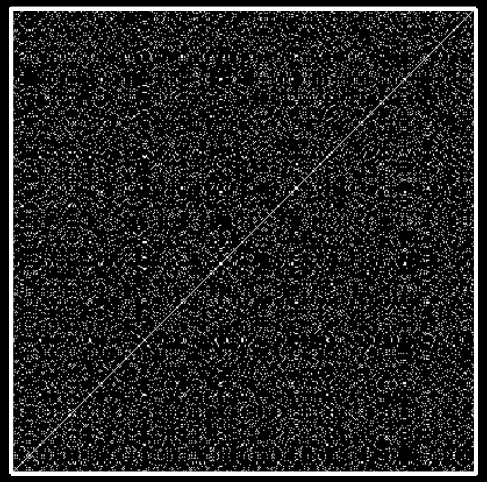
- fixed threshold
  - maximum norm
  - Euclidean norm
  - minimum norm
- fixed number of neighbours
- local rank order

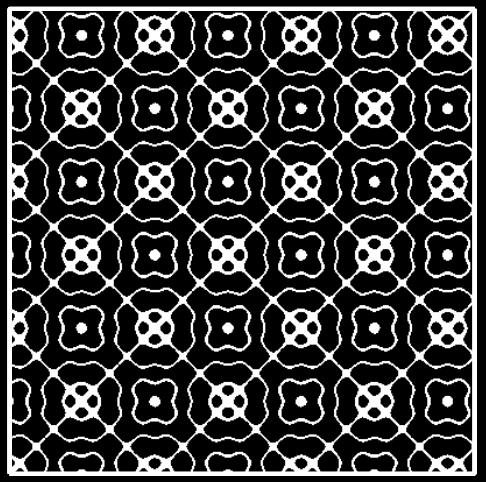


Eckmann, Kamphorst, Ruelle, Europhysics Letters, 5, 1987

Recurrence Plot Typology

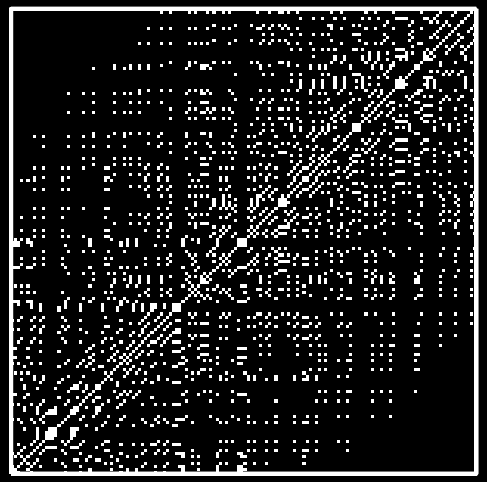
homogeneous

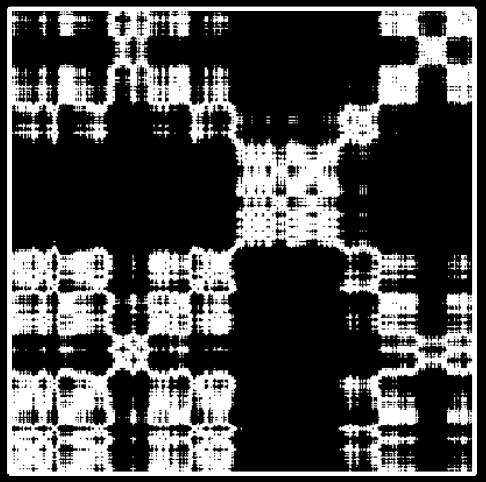




periodic

drifty

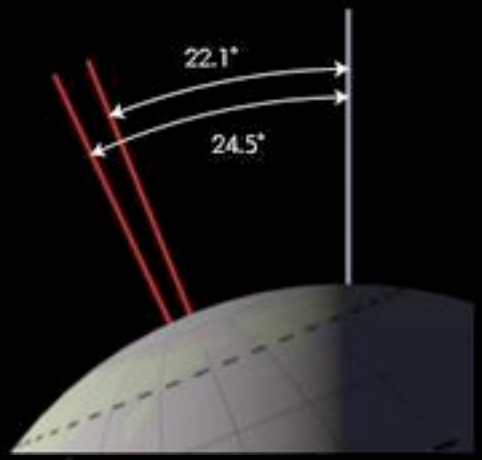




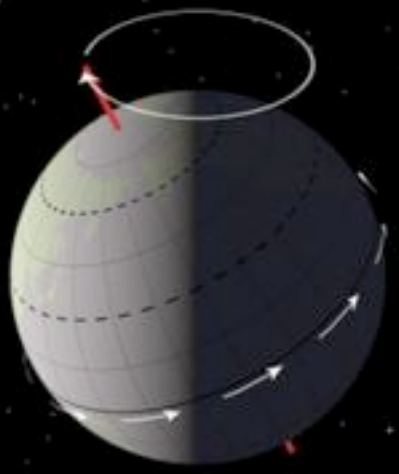
disrupted

Solar Insolation

Eccentricity (413 kyr)



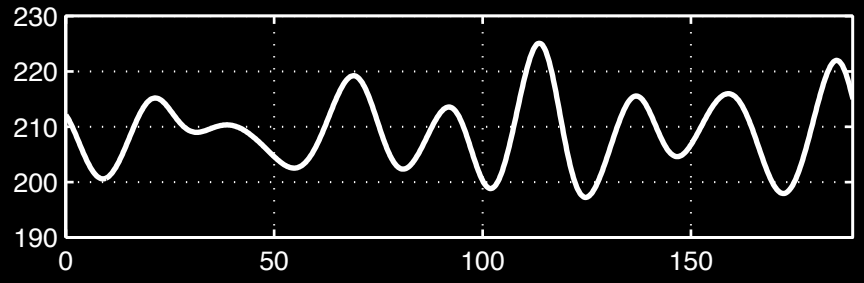
Axial tilt (41 kyr)

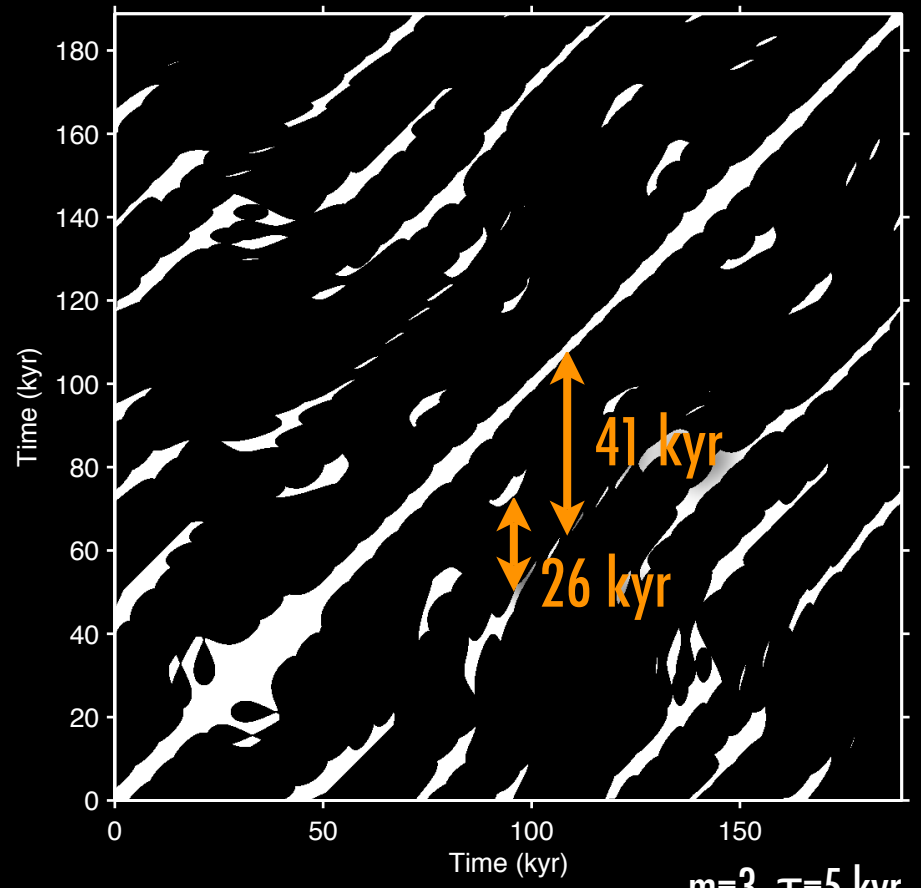


Precession (26 kyr)

Milankovich cycles

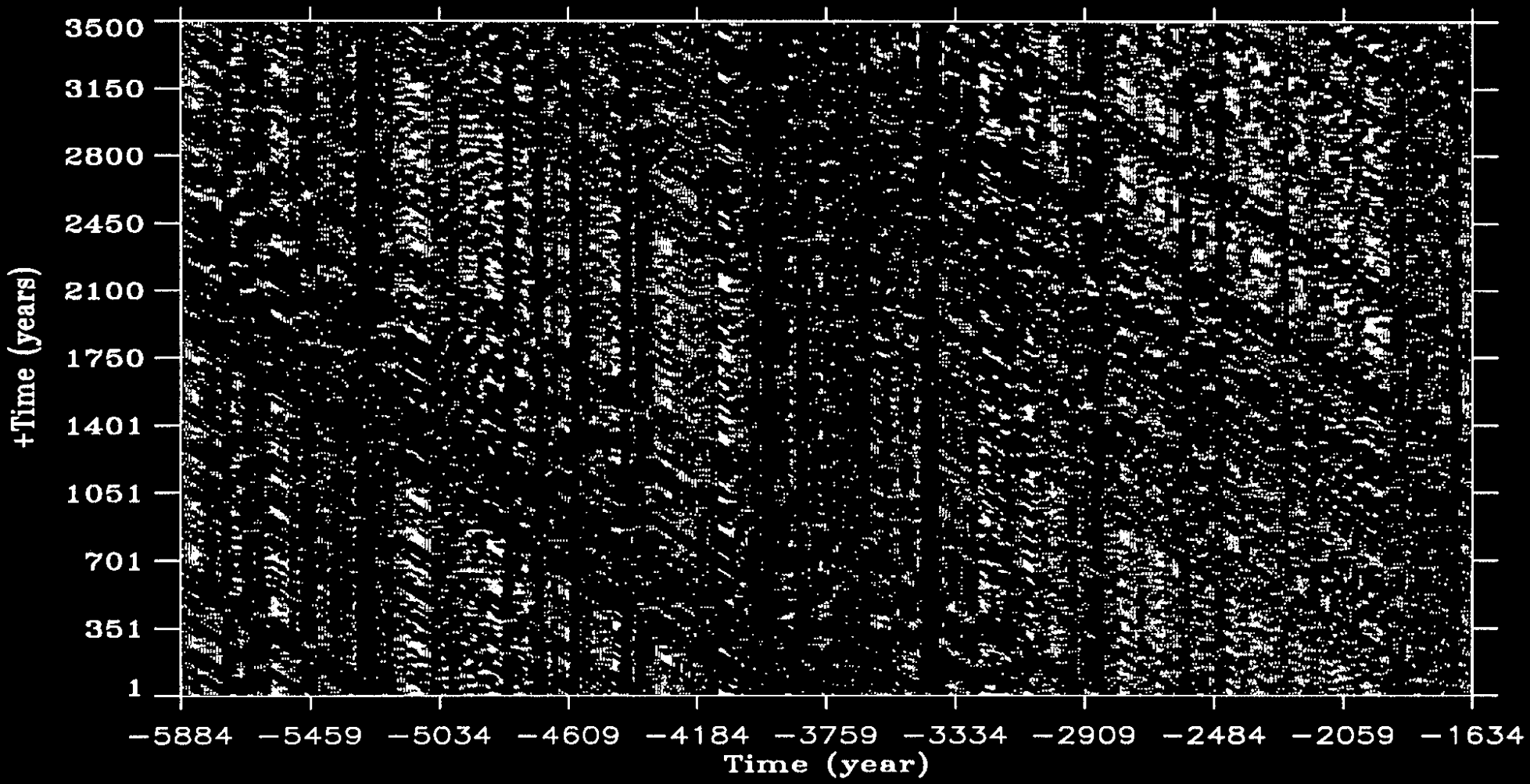
January insolation (44°N)





m=3, τ=5 kyr

Nonlinear variability in solar activity derived from radiocarbon records

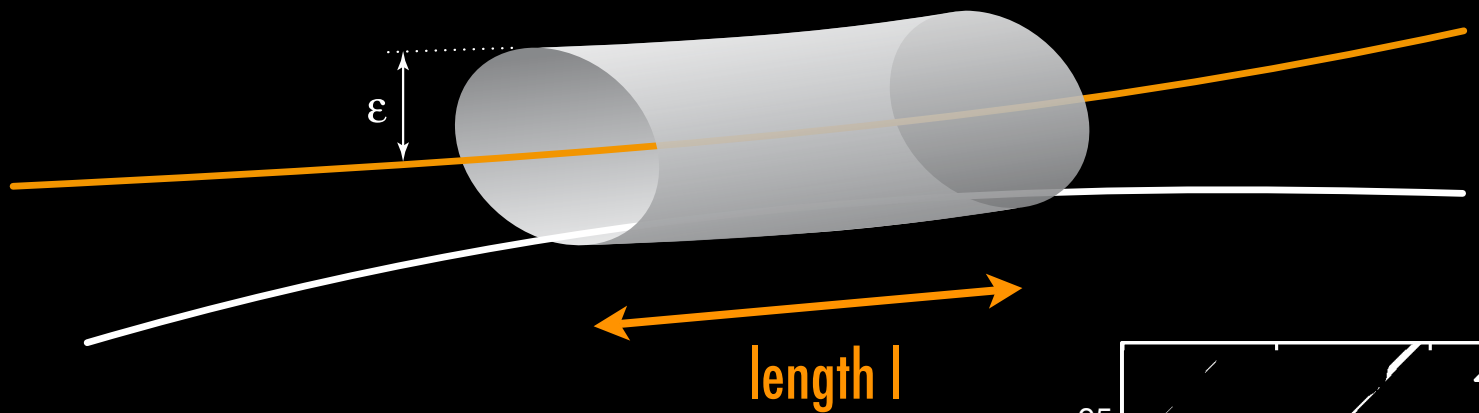


Kurths, Schwarz, Sonett, Parlitz, Nonlinear Processes in Geophysics, 1, 1994

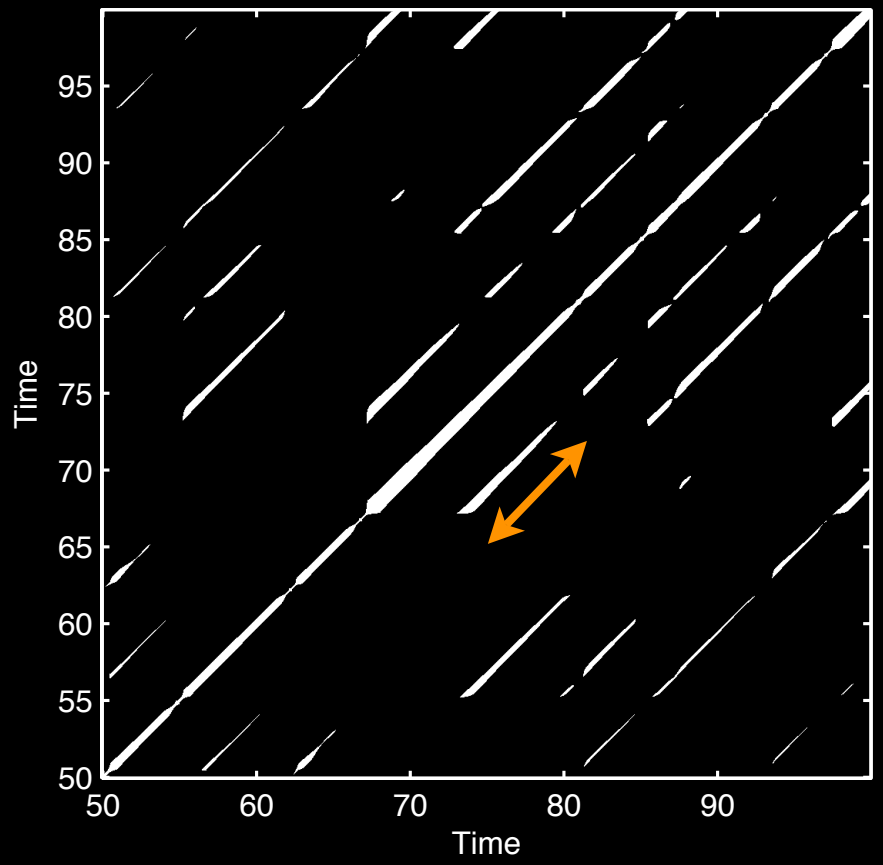
Recurrence Quantification

- quantitative description of RPs
- based on
  - recurrence point density
  - diagonal lines
  - vertical lines

Recurrence Quantification



- number of lines of exactly length l
  - histogram P(l)



Line Based Measures

- Determinism DET

$$DET = \frac{\sum_{l=l_{\min}}^{N} l\, P(l)}{\sum_{l=1}^{N} l\, P(l)}$$

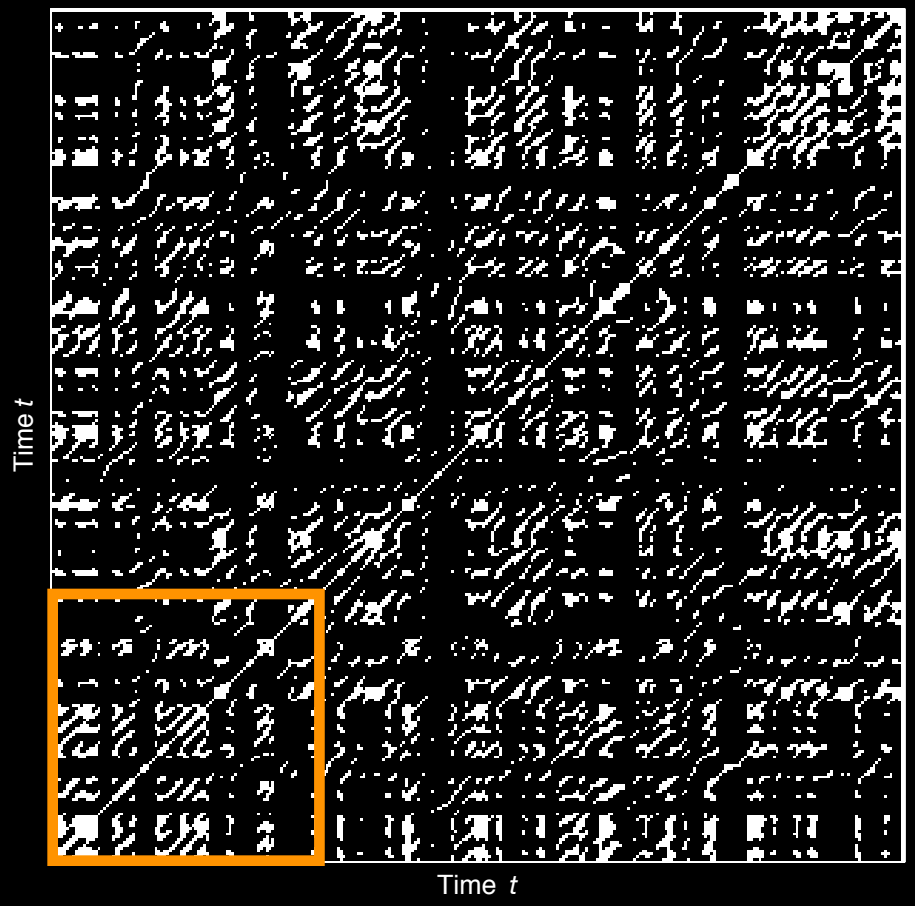
Fraction of points forming diagonal lines

- Mean diagonal line length L

$$L = \frac{\sum_{l=l_{\min}}^{N} l\, P(l)}{\sum_{l=l_{\min}}^{N} P(l)}$$

Recurrence Quantification

- Time dependent analysis:
  - sliding windows over RP
- Detection of transitions



Recurrence Quantification

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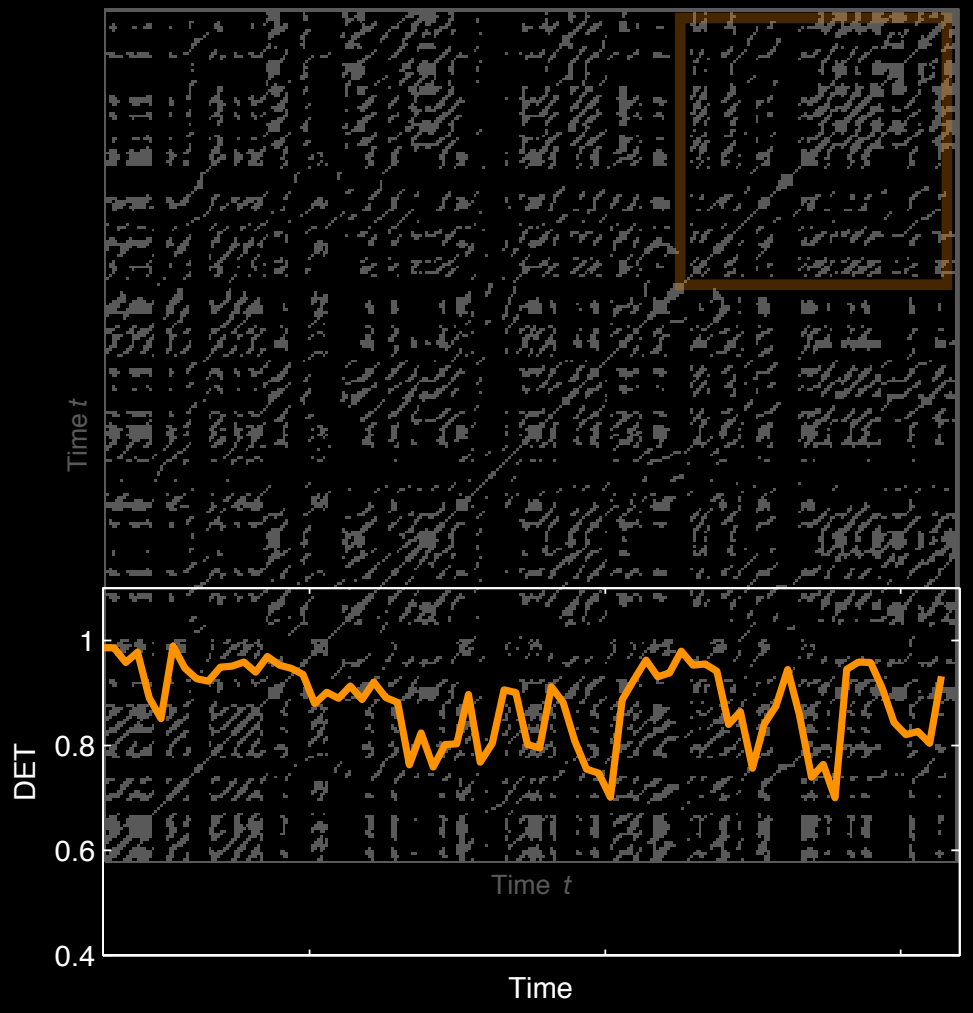
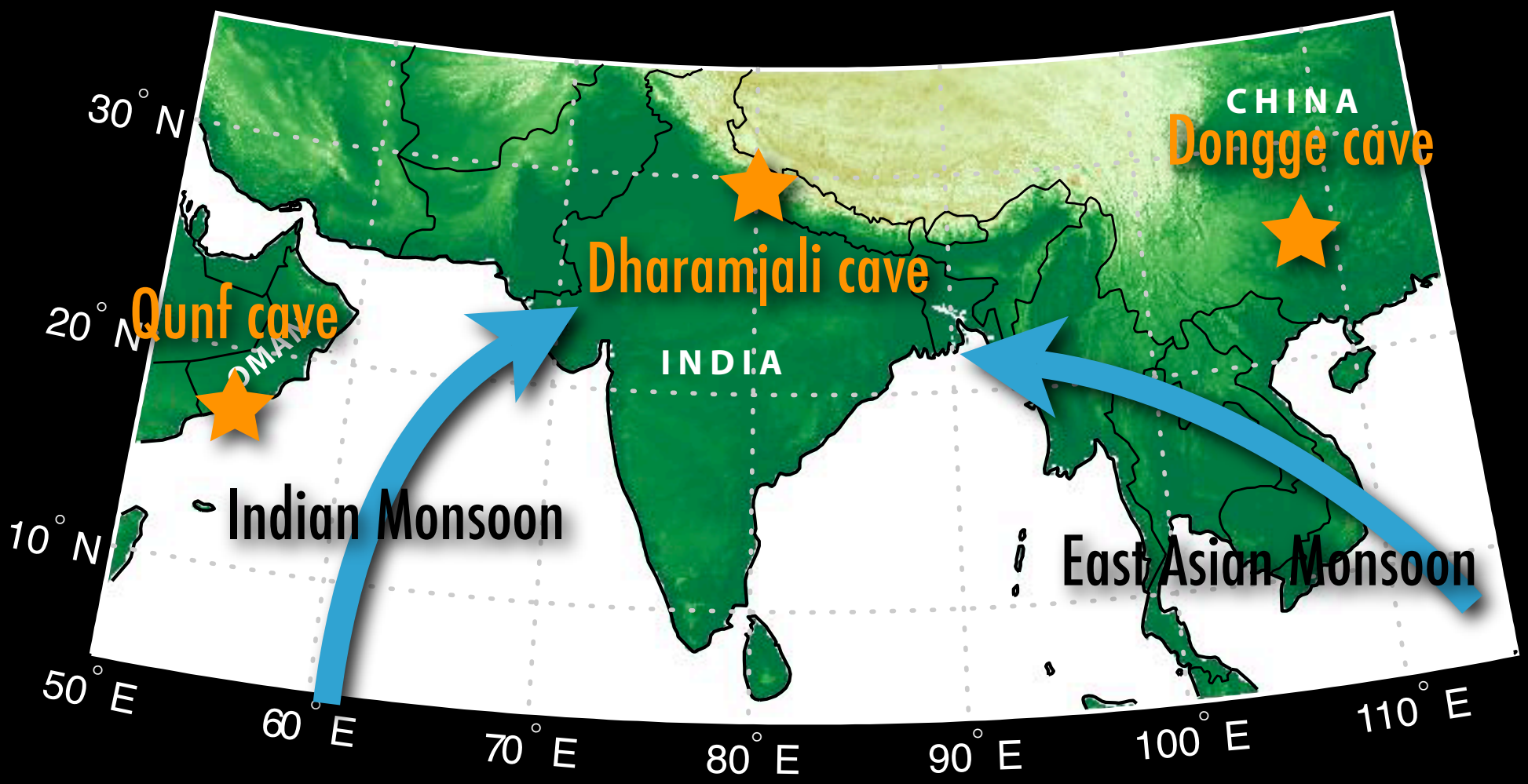


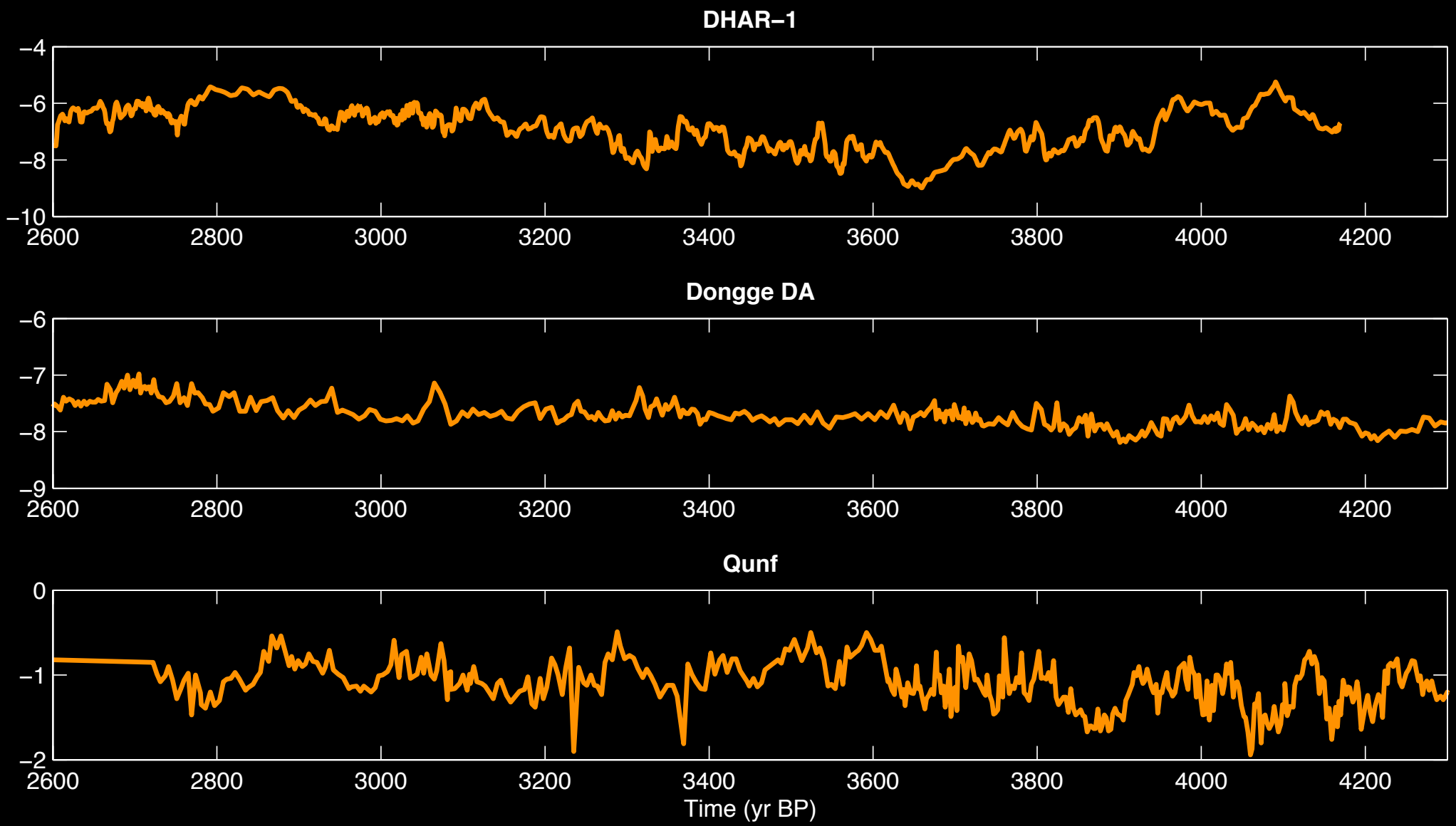
Asian Monsoon – Isotope Records



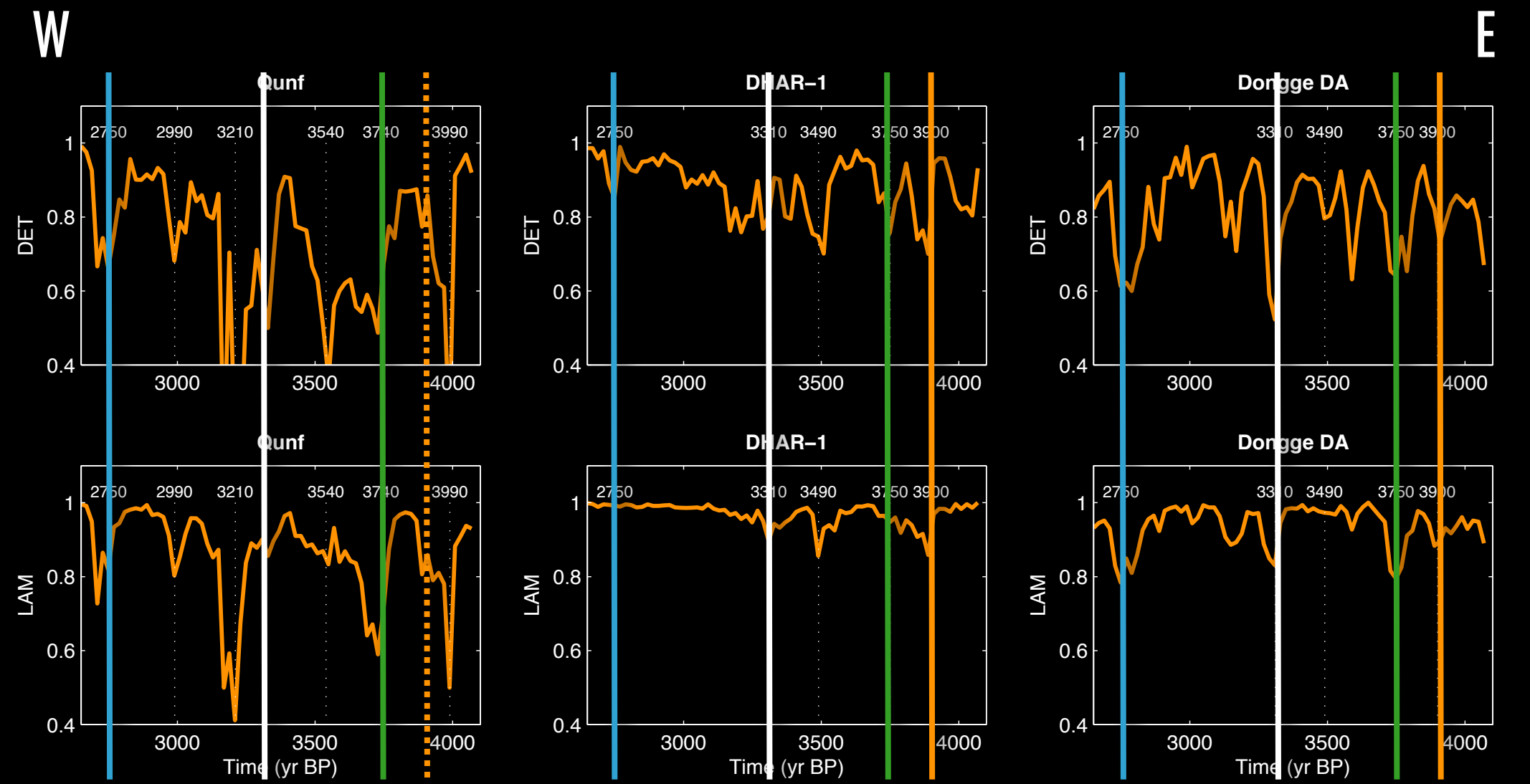
Isotope Records

Isotope Records

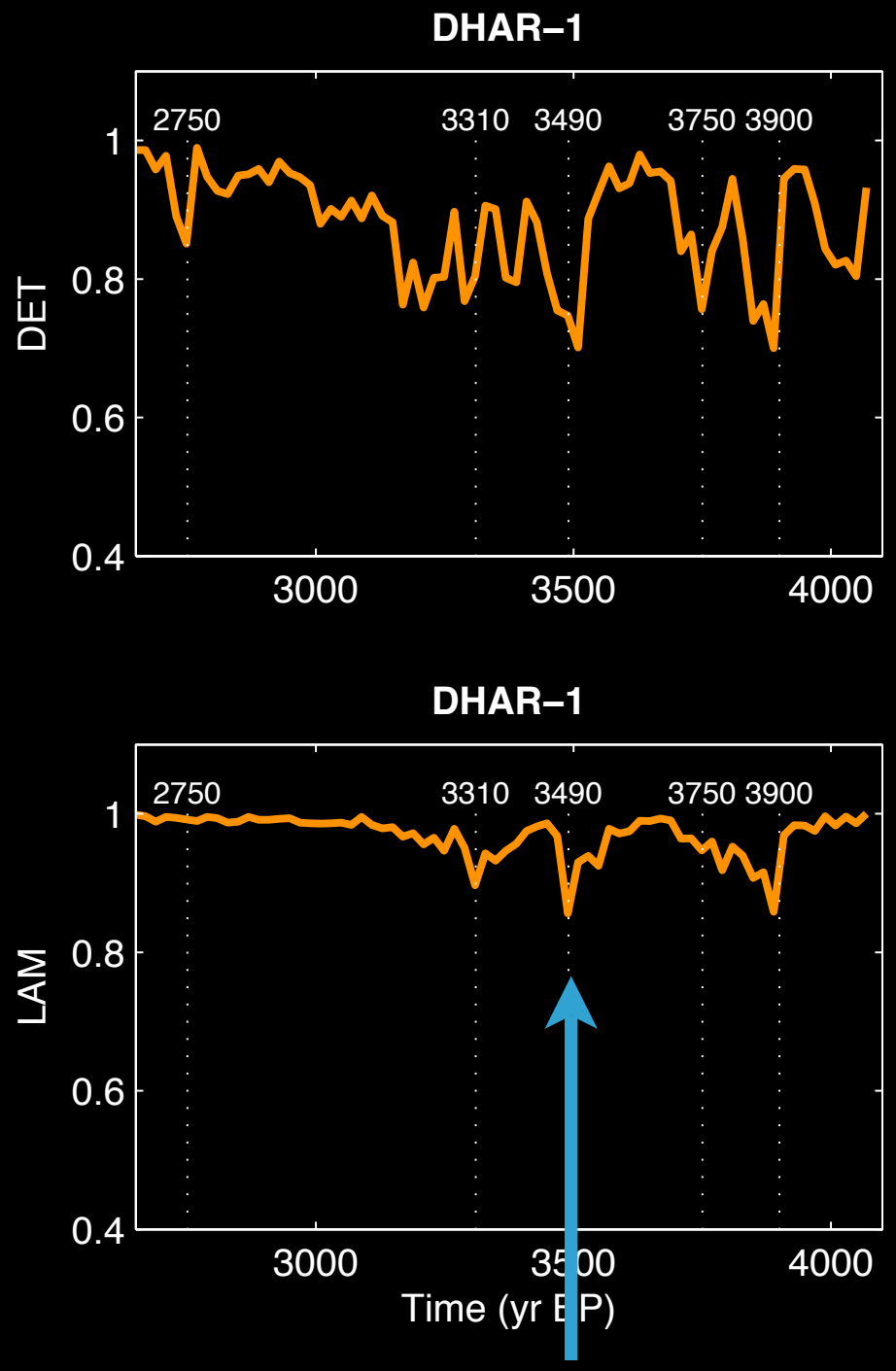
Isotope Records



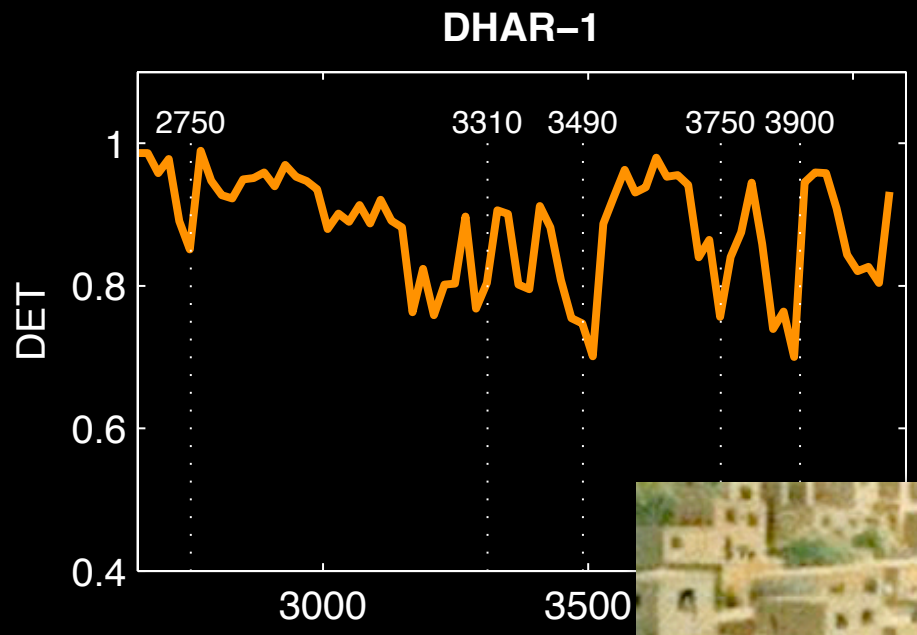
RQA Isotope Records

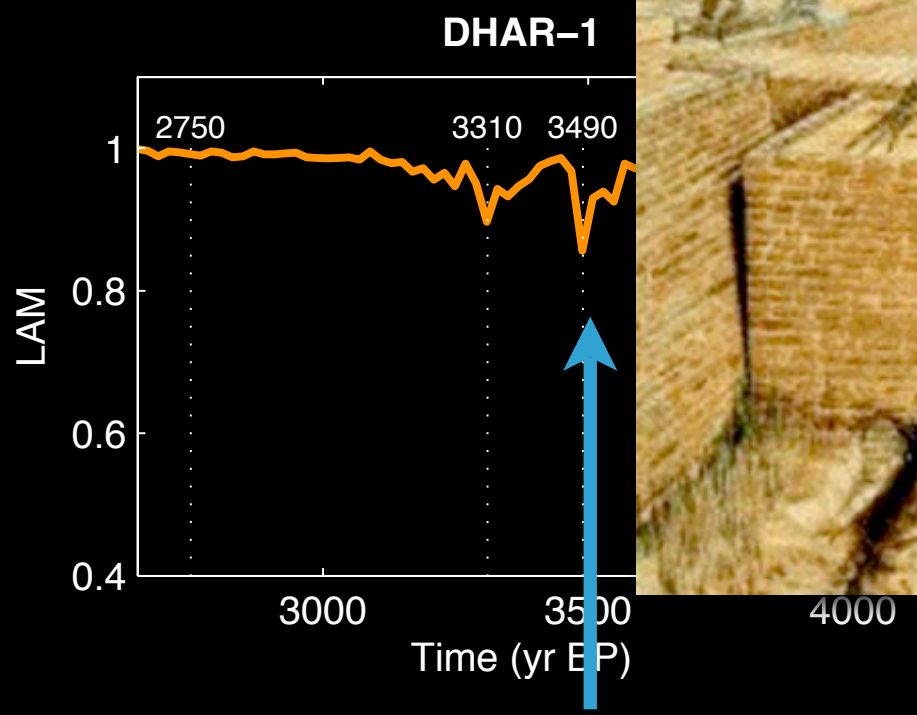


2750 yr BP 3310 yr BP 3740-50 yr BP 3900 yr BP


DHAR–1
2750
3310
3490
3750
3900
DET
1
0.8
0.6
0.4
3000
3500
4000
DHAR–1
2750
3310
3490
3750
3900
LAM
1
0.8
0.6
0.4
3000
3500
4000
Time (yr BP)

1540 B.C.

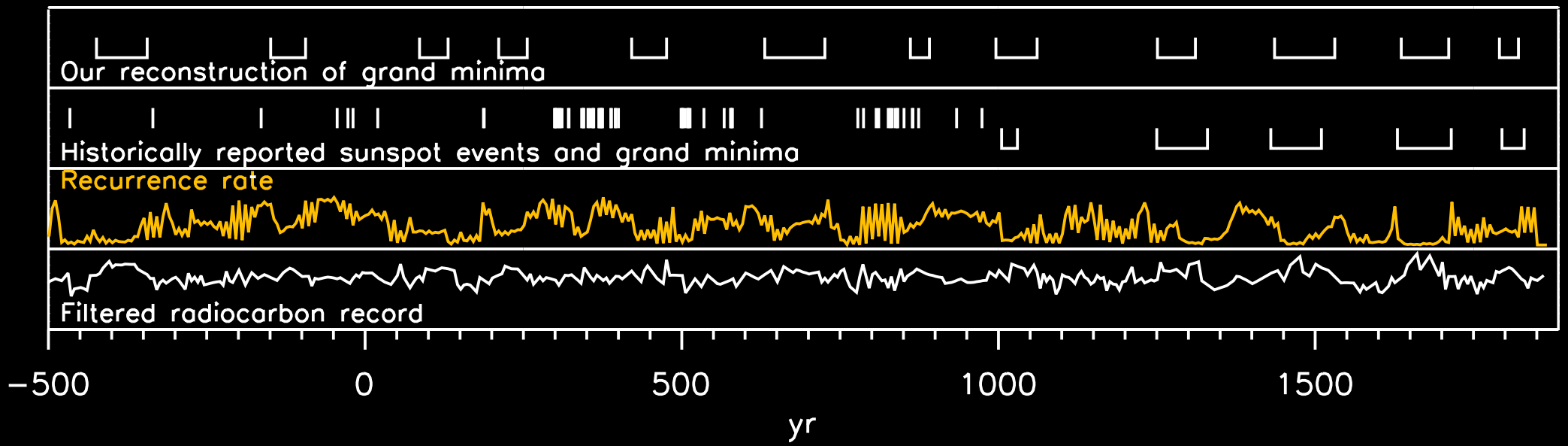




Harappan culture vanished

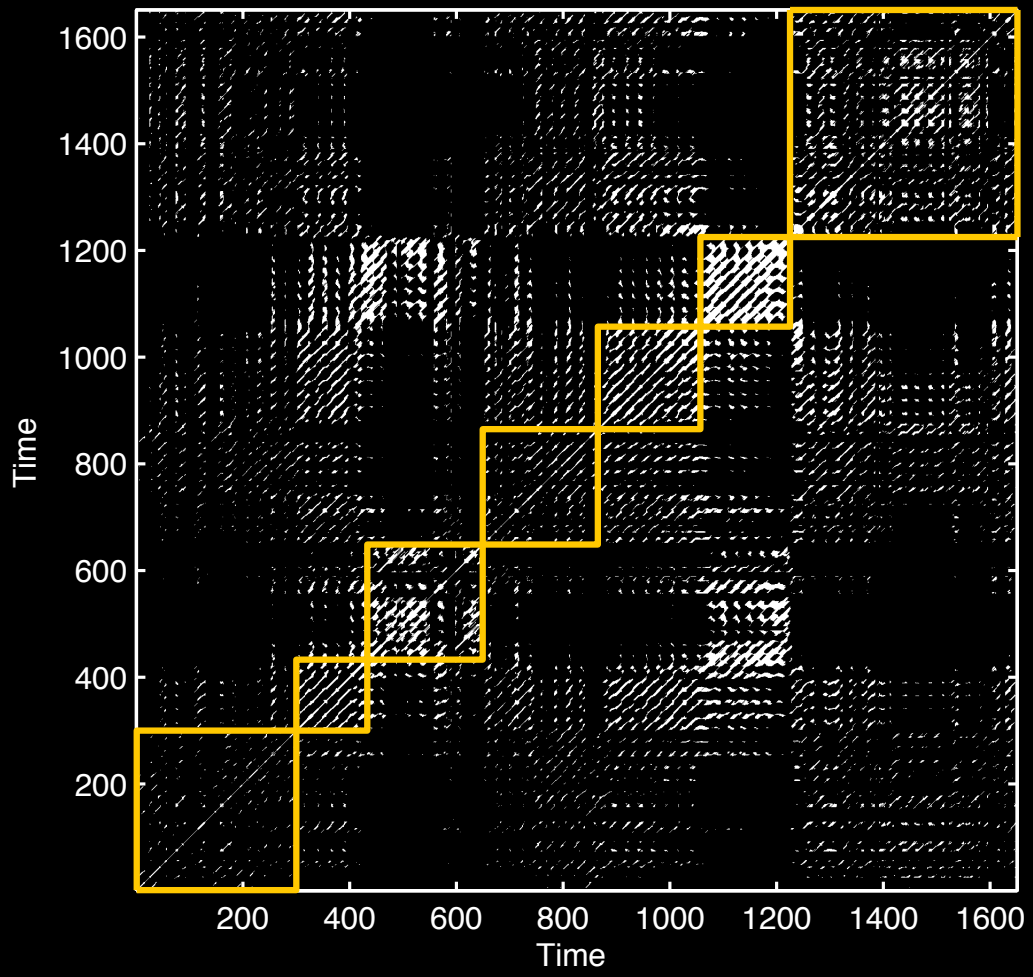
1540 B.C.

Reconstruction of solar activity from radiocarbon records



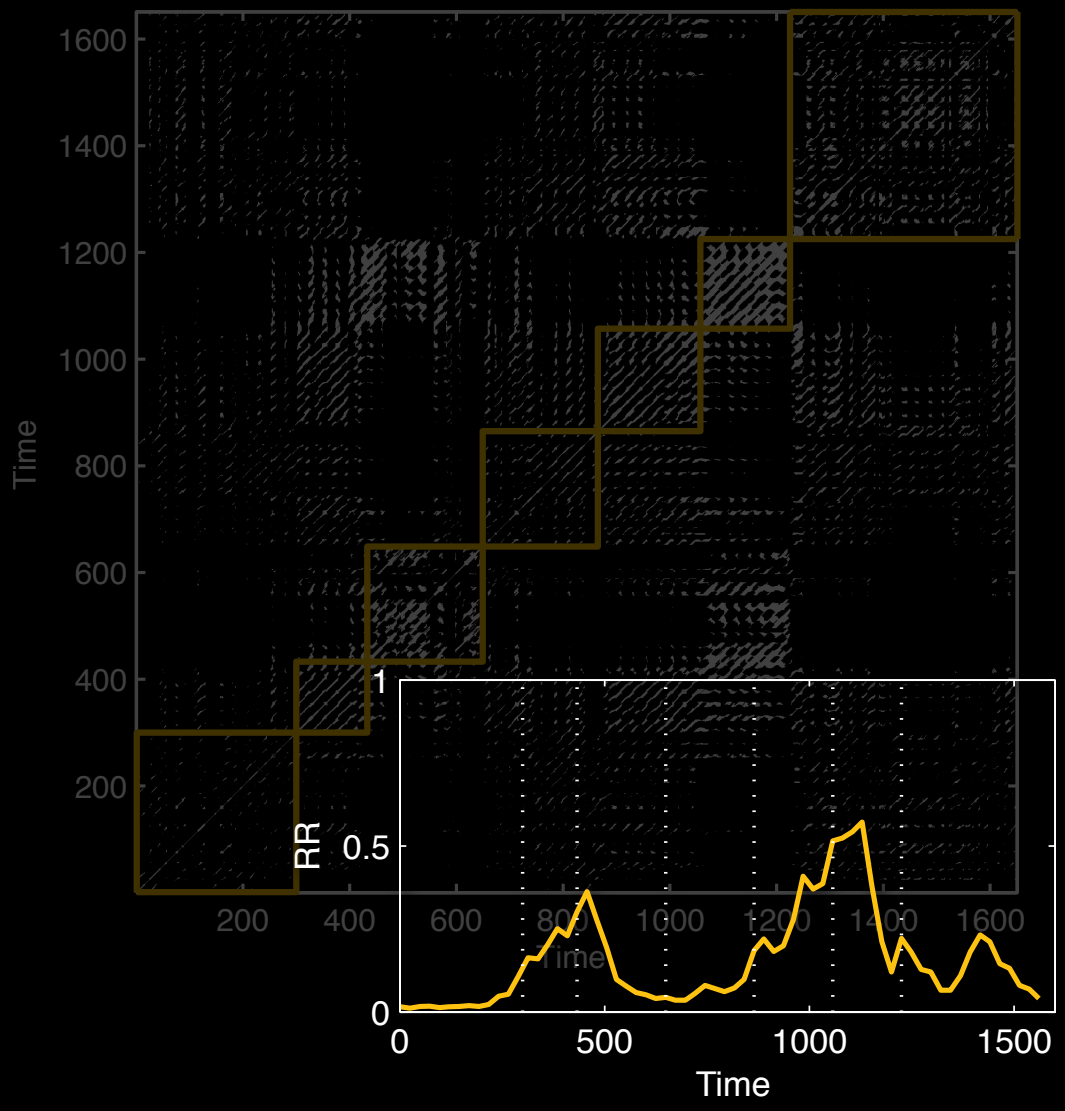
Voss, Kurths, Schwarz, Journal of Geophyscial Research, 101, 1996

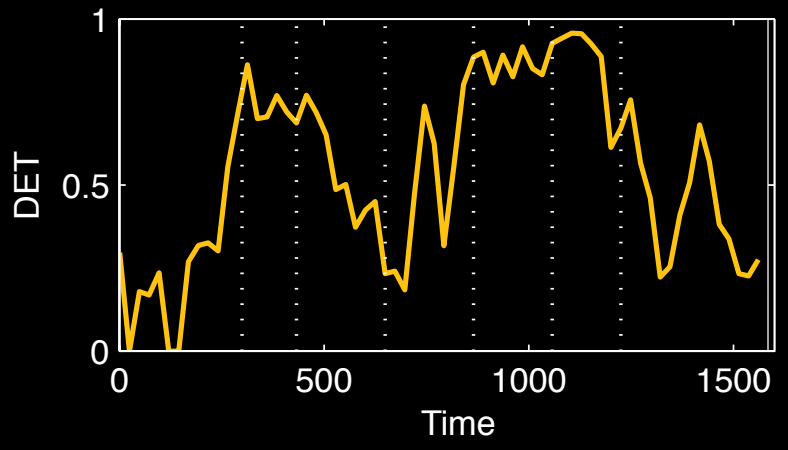
Dynamics of Oxygen Crises in Lakes



Facchini, Mocenni, Marwan, Vicino, Tiezzo, Ecological Modelling, 203, 2007

Dynamics of Oxygen Crises in Lakes



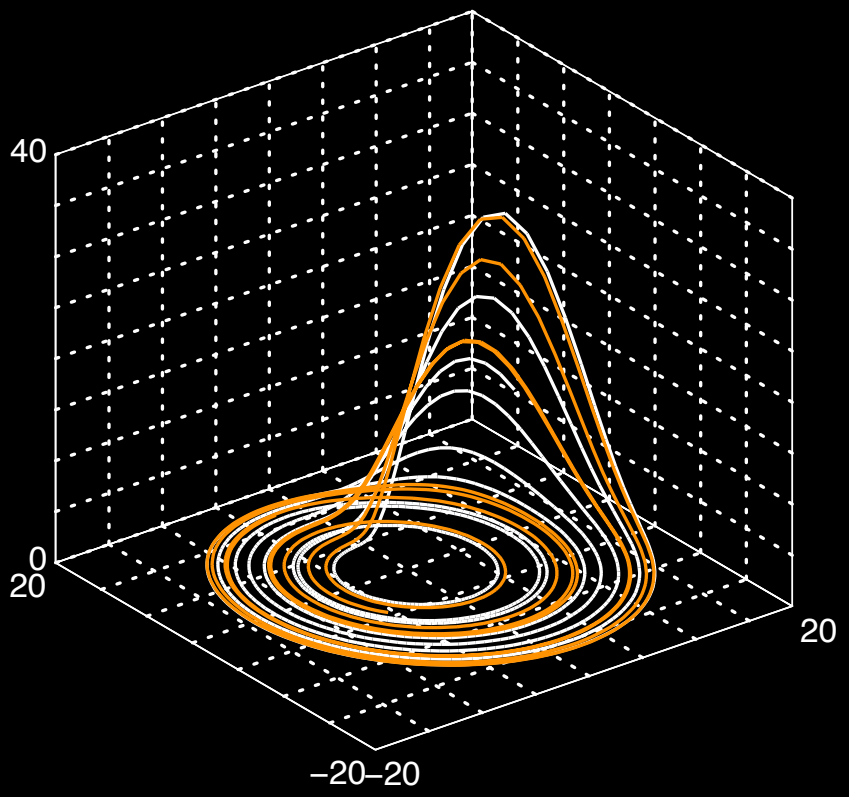


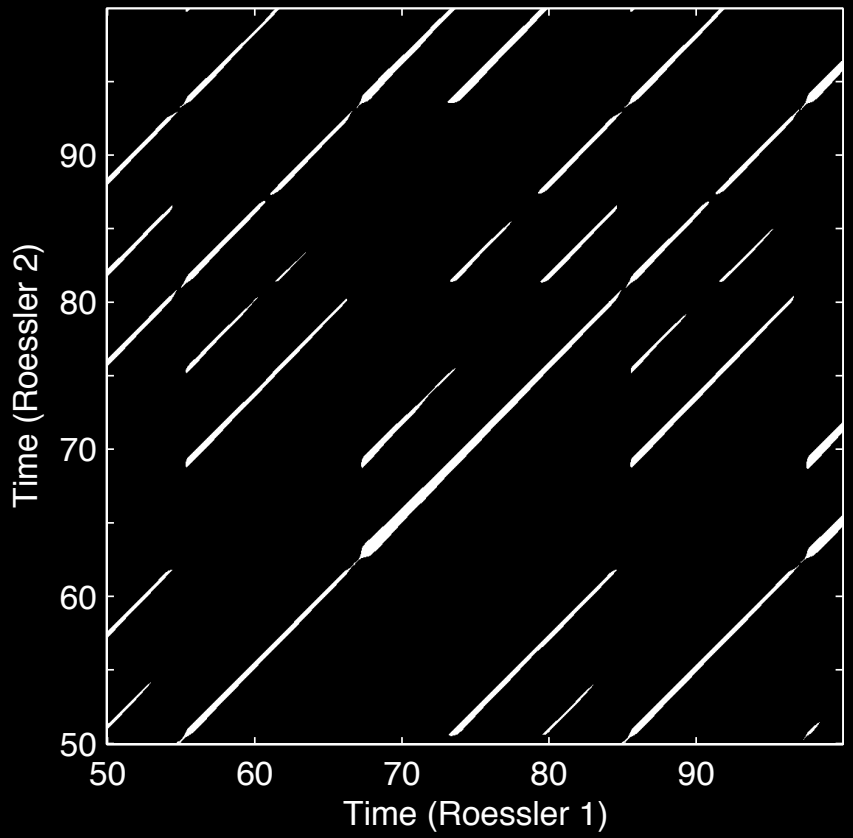
Facchini, Mocenni, Marwan, Vicino, Tiezzo, Ecological Modelling, 203, 2007

Studies of Weak External Forcing in Simulated Shear Displacements

Chelidze, Matcharashvili, Gogiashvili, Lursmanashvili, Devidze, Nonlinear Processes in Geophysics, 12, 2005

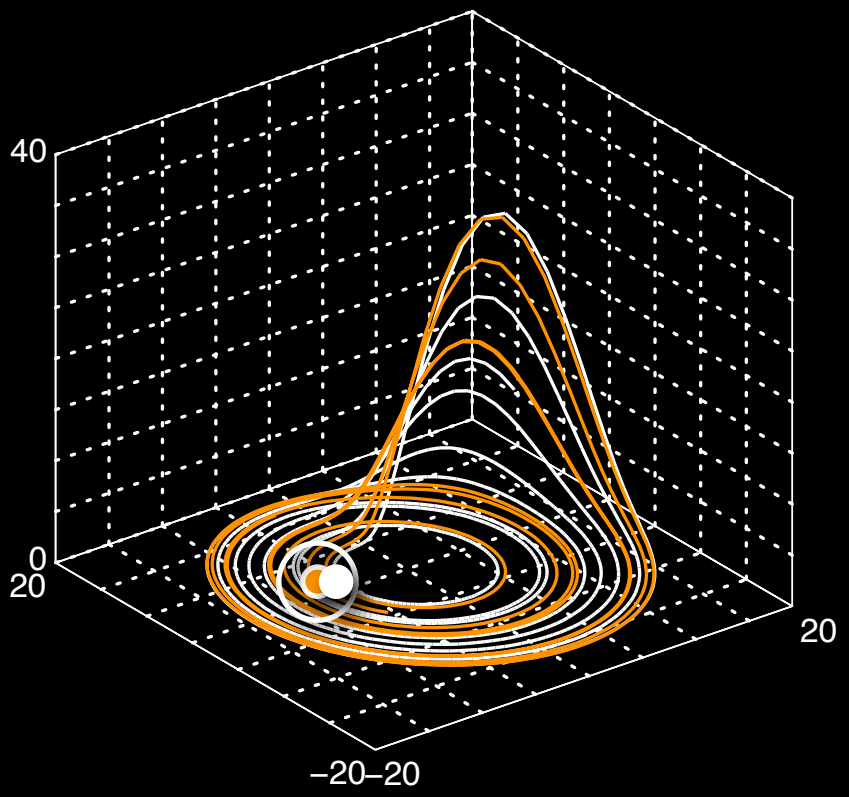
Cross Recurrence Plot

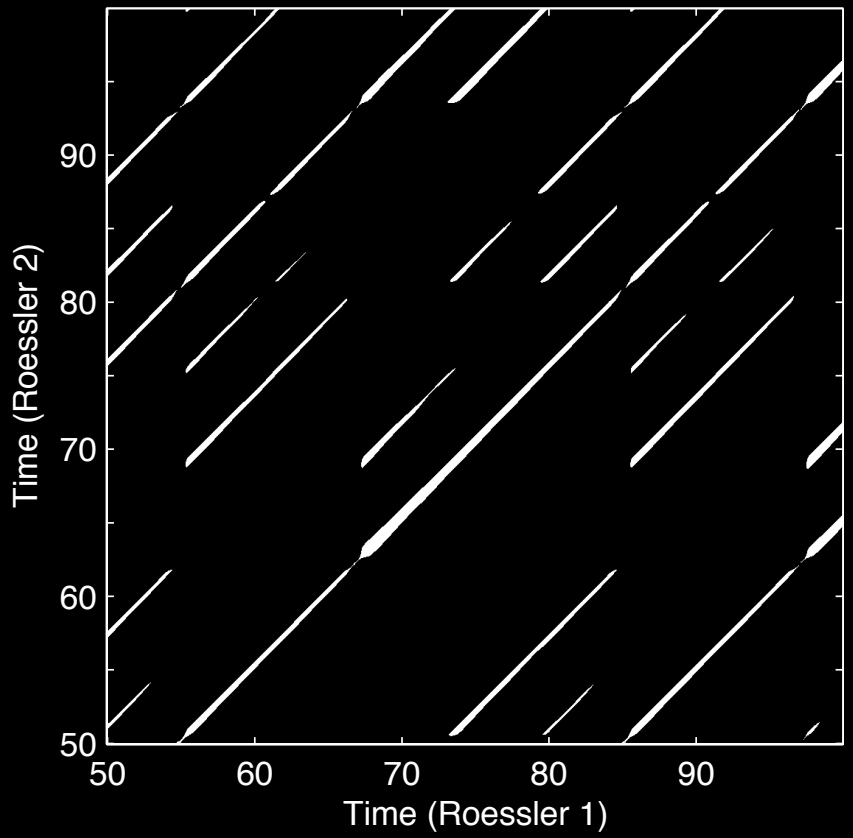




Marwan, Thiel, Nowaczyk, Nonlinear Processes in Geophysics, 9, 2002

Cross Recurrence Plot





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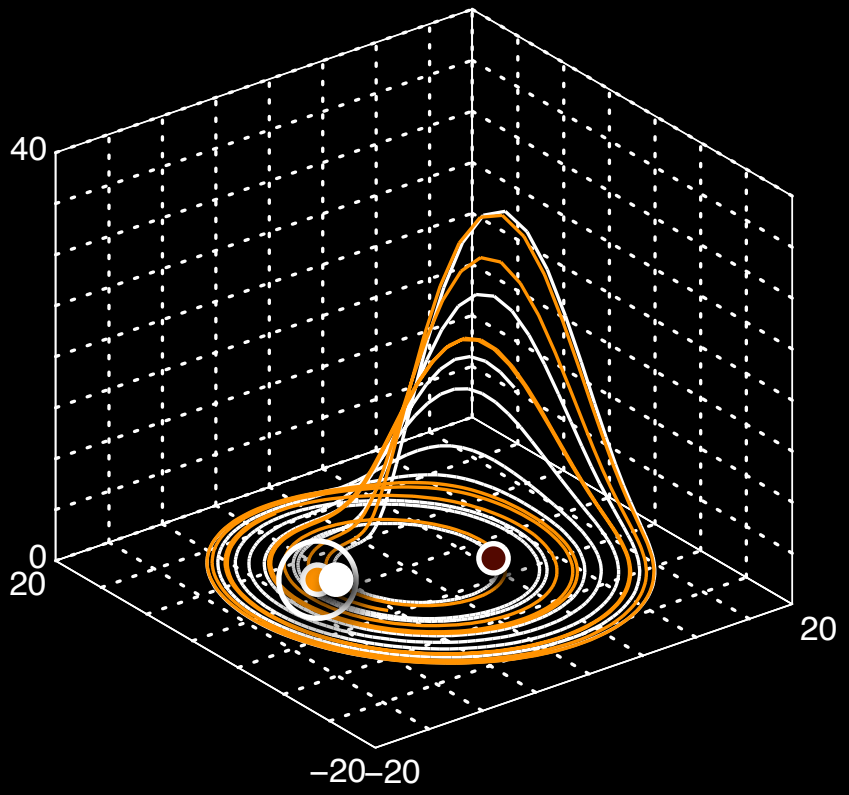

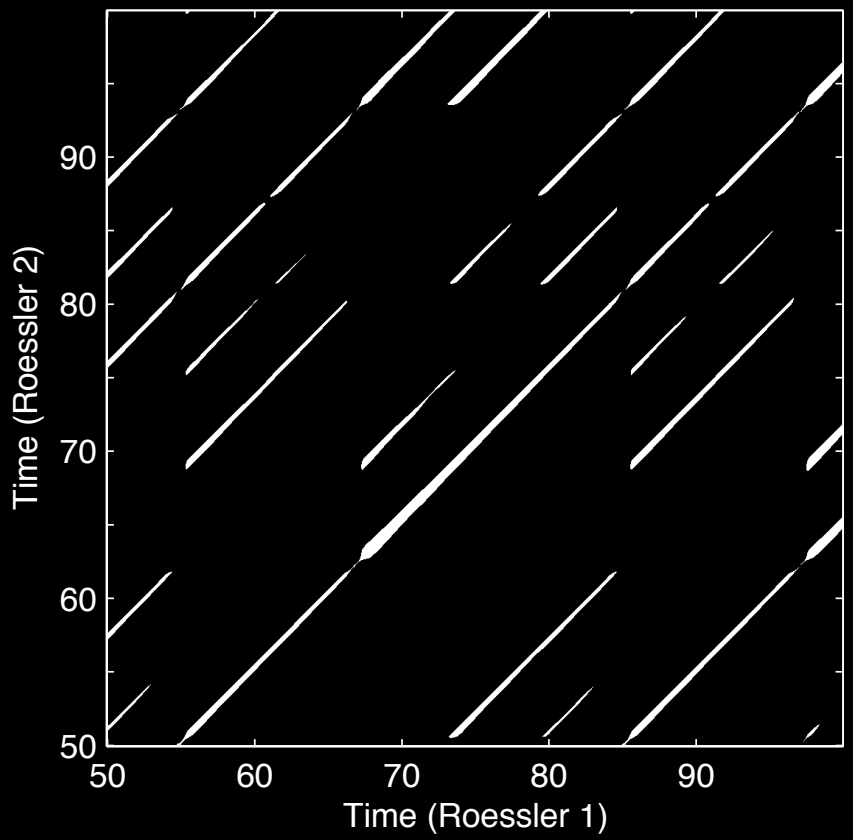

Marwan, Thiel, Nowaczyk, Nonlinear Processes in Geophysics, 9, 2002

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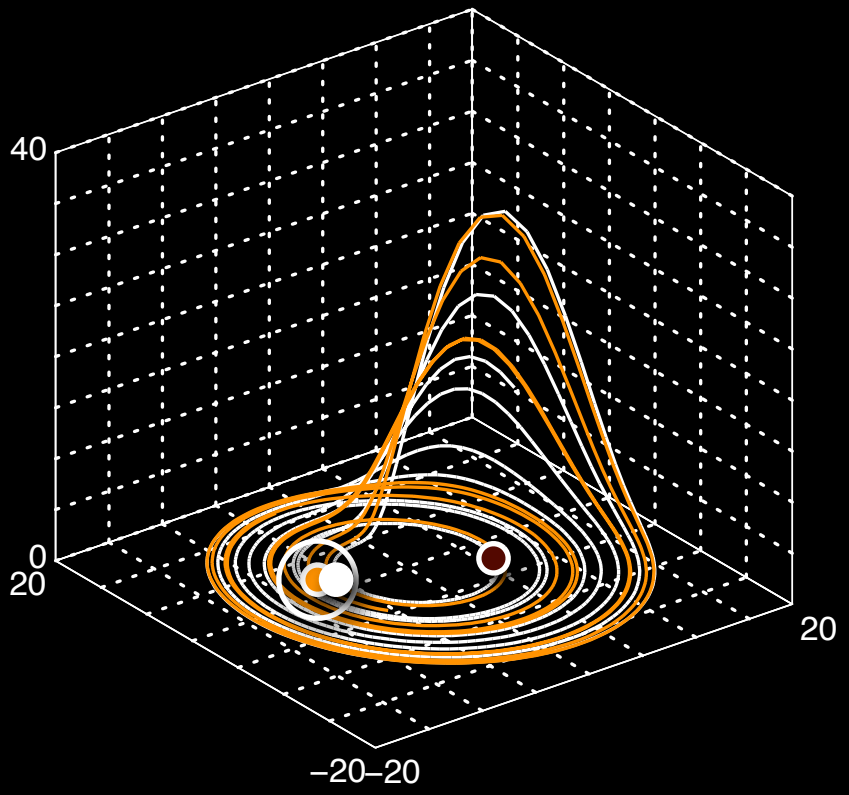
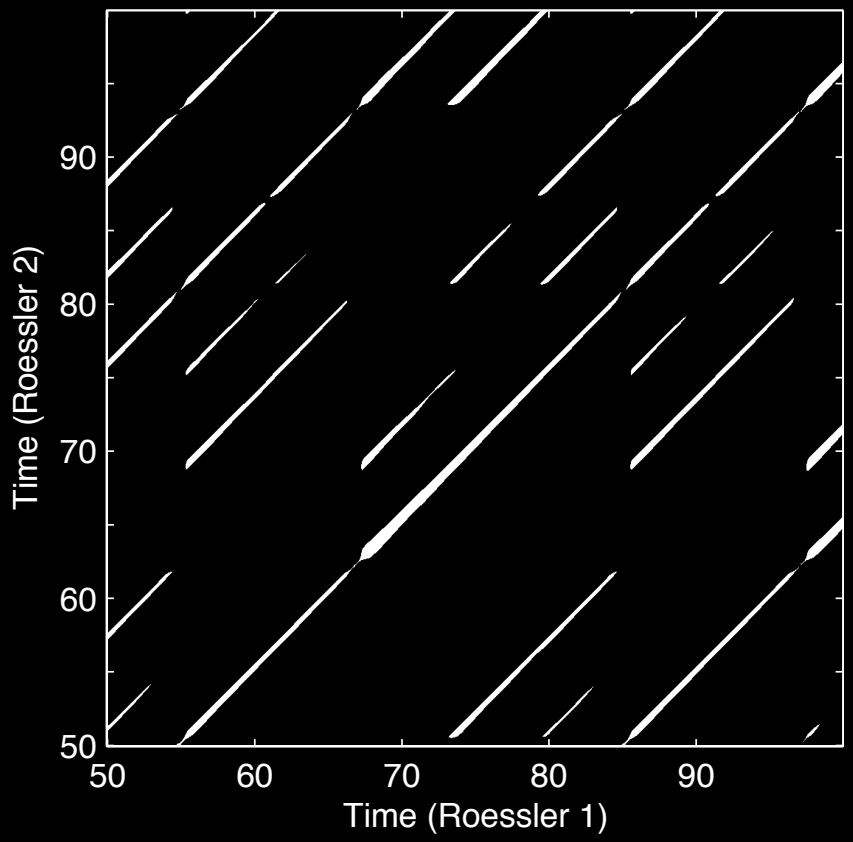
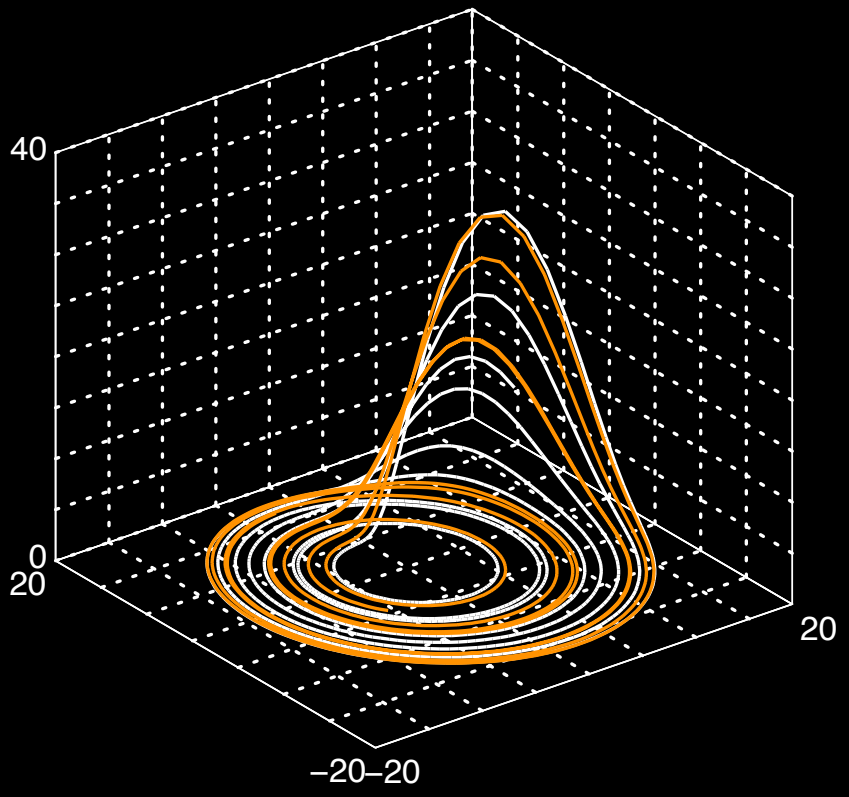


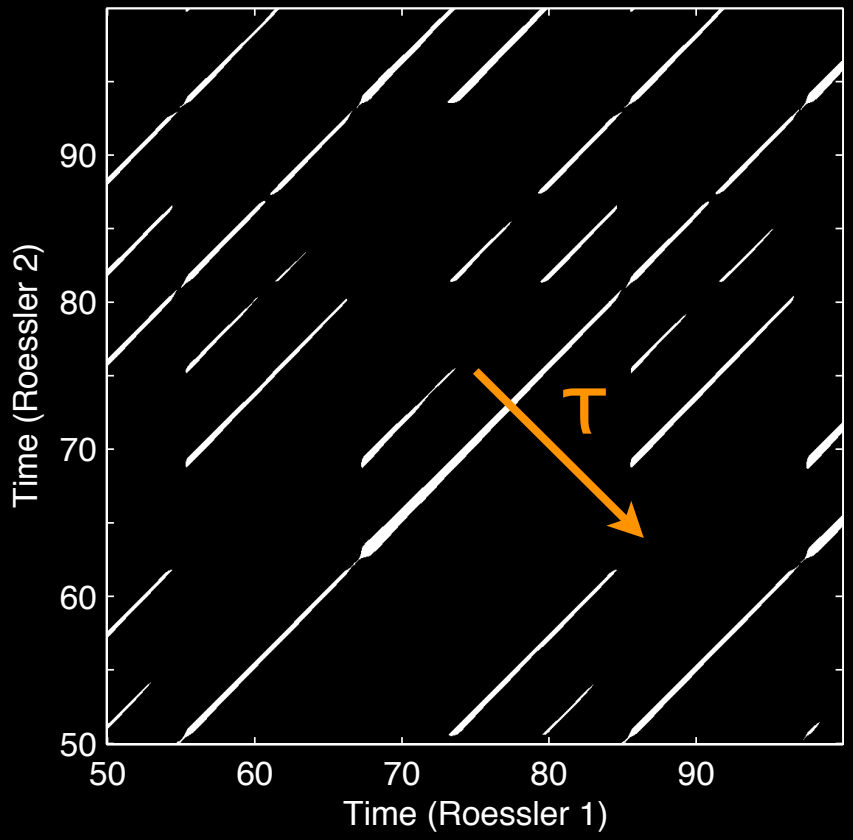


$$\mathbf{CR}^{+}_{i,j} = \Theta(\varepsilon - \|\vec{x}_i - \vec{y}_j\|)$$

Marwan, Thiel, Nowaczyk, Nonlinear Processes in Geophysics, 9, 2002

Cross Recurrence Plot





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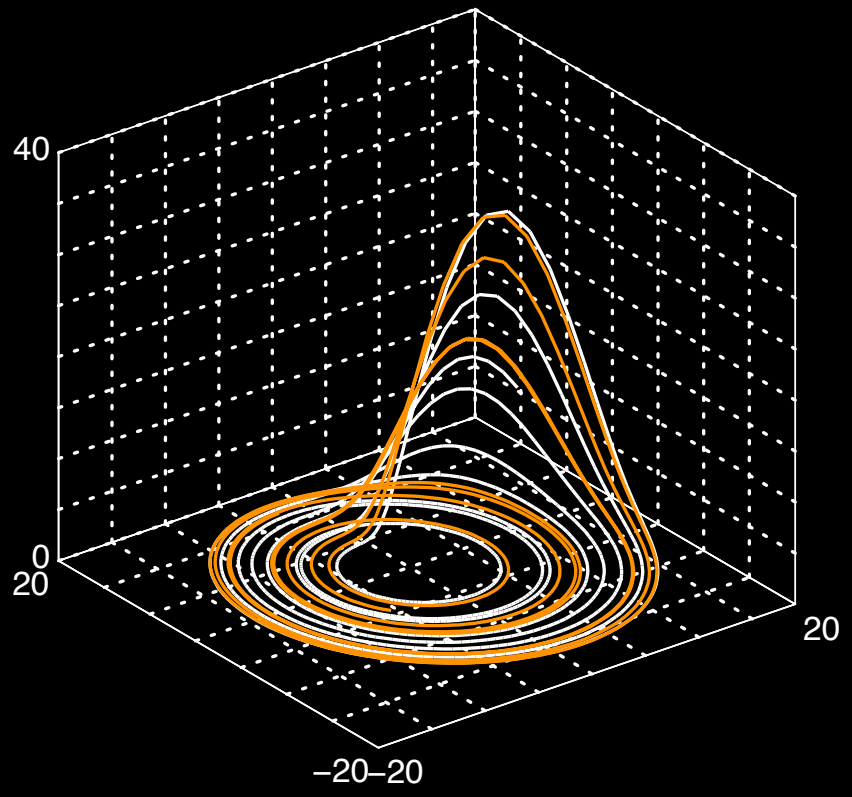
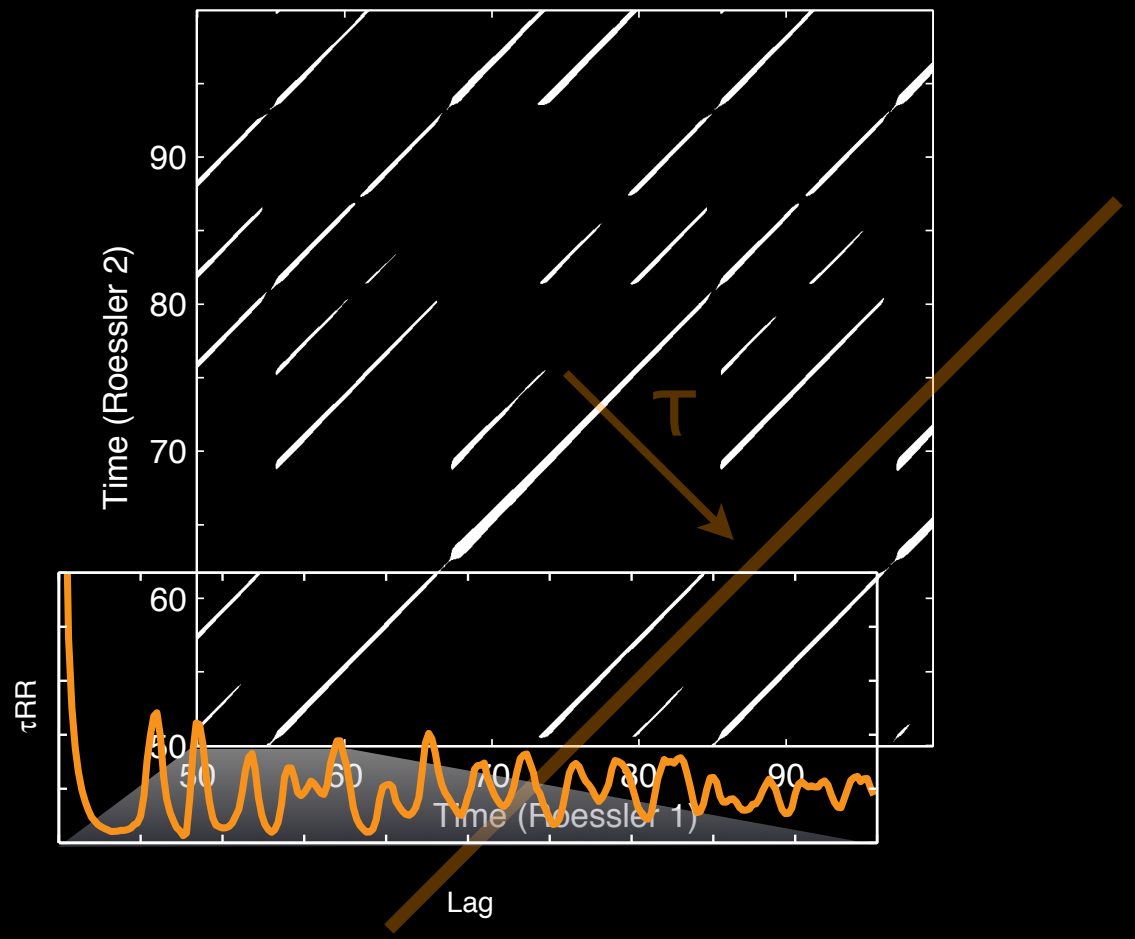




- Detection of interrelations between systems

Cross Recurrence Quantification

- delay dependent RQA measures
  - Recurrence Rate

$$RR(i) = \frac{1}{N-i} \sum_{j=1}^{N-i} \left(\mathbf{CR}^{+}_{j,\,j+i} - \mathbf{CR}^{-}_{j,\,j+i} \right)$$

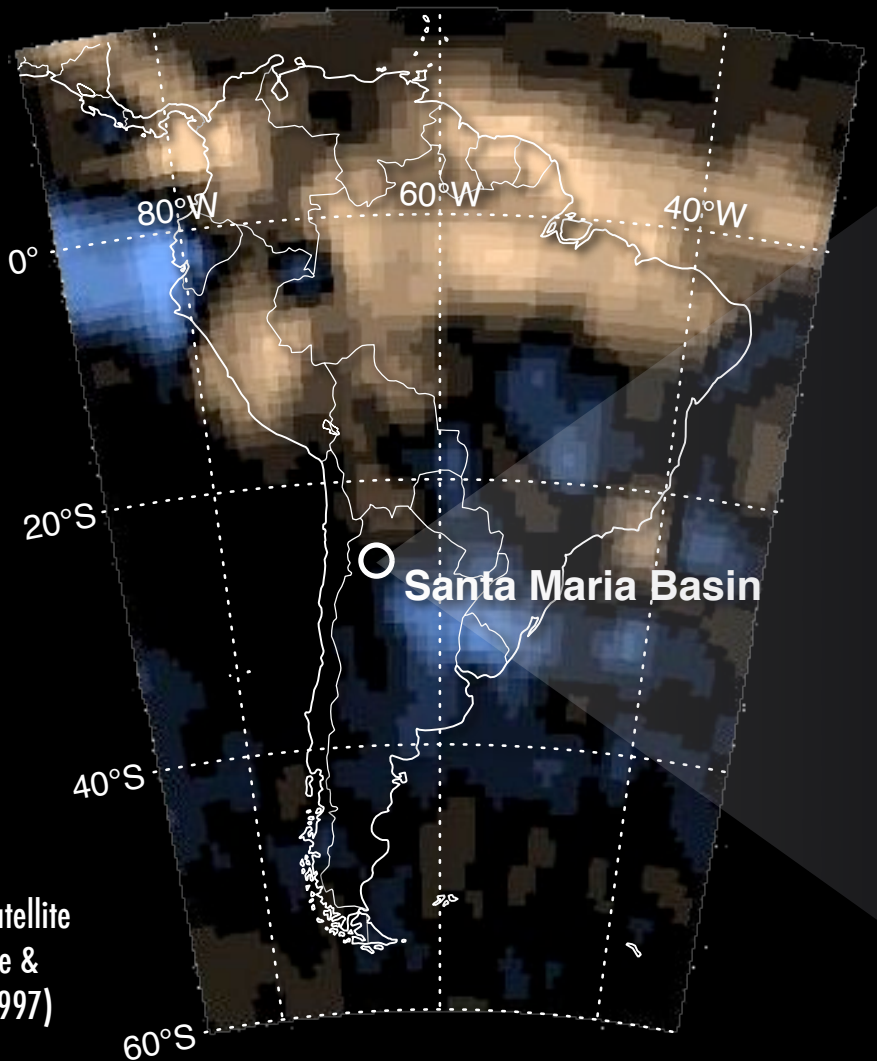
Probability that similar states occur after given delay

  - Mean Diagonal Line Length

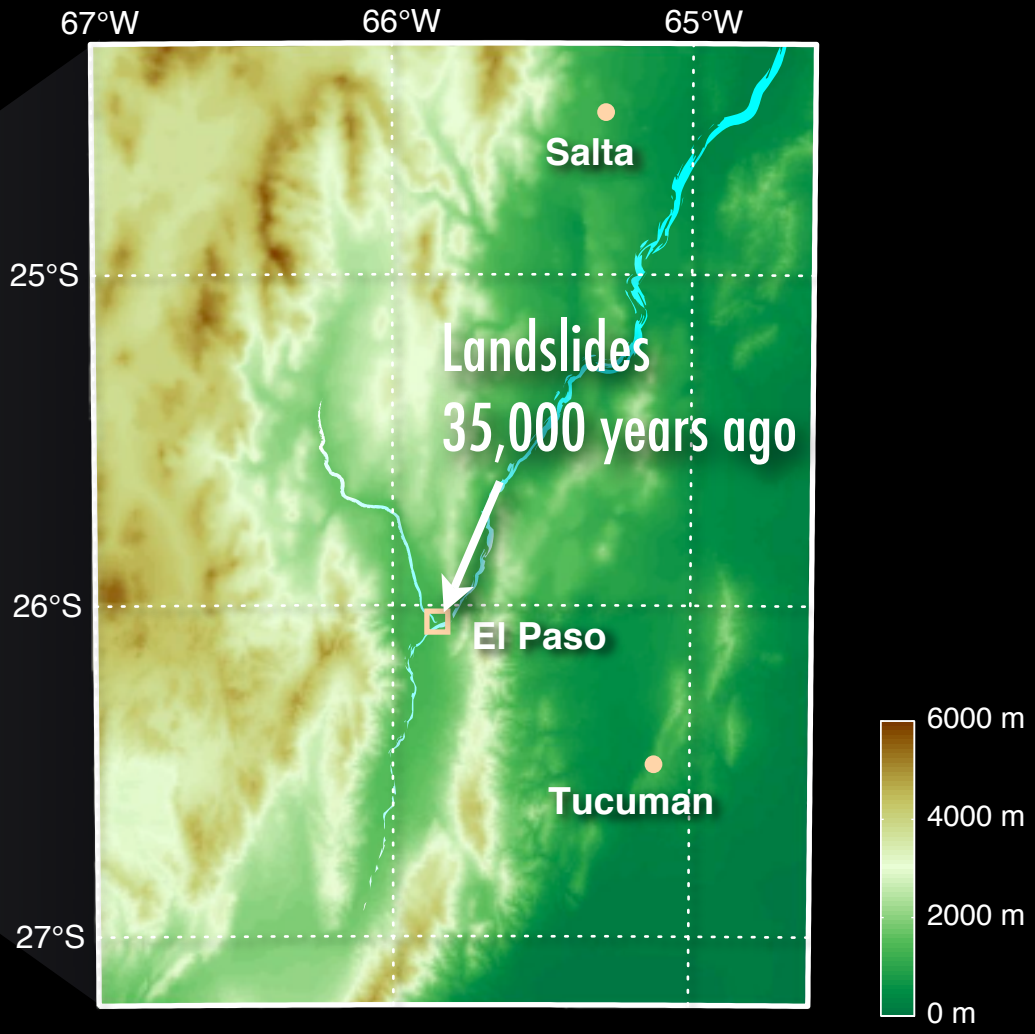
$$L(i) = \frac{\sum_{l=l_{min}}^{N-i} l \, \left[P^{+}(l,\,i) - P^{-}(l,\,i) \right]}{\sum_{l=l_{min}}^{N-i} \left[P^{+}(l,\,i) - P^{-}(l,\,i) \right]}$$

El Niño Southern Oscillation

Marwan, Trauth, Vuille, Kurths, Climate Dynamics, 21, 2003

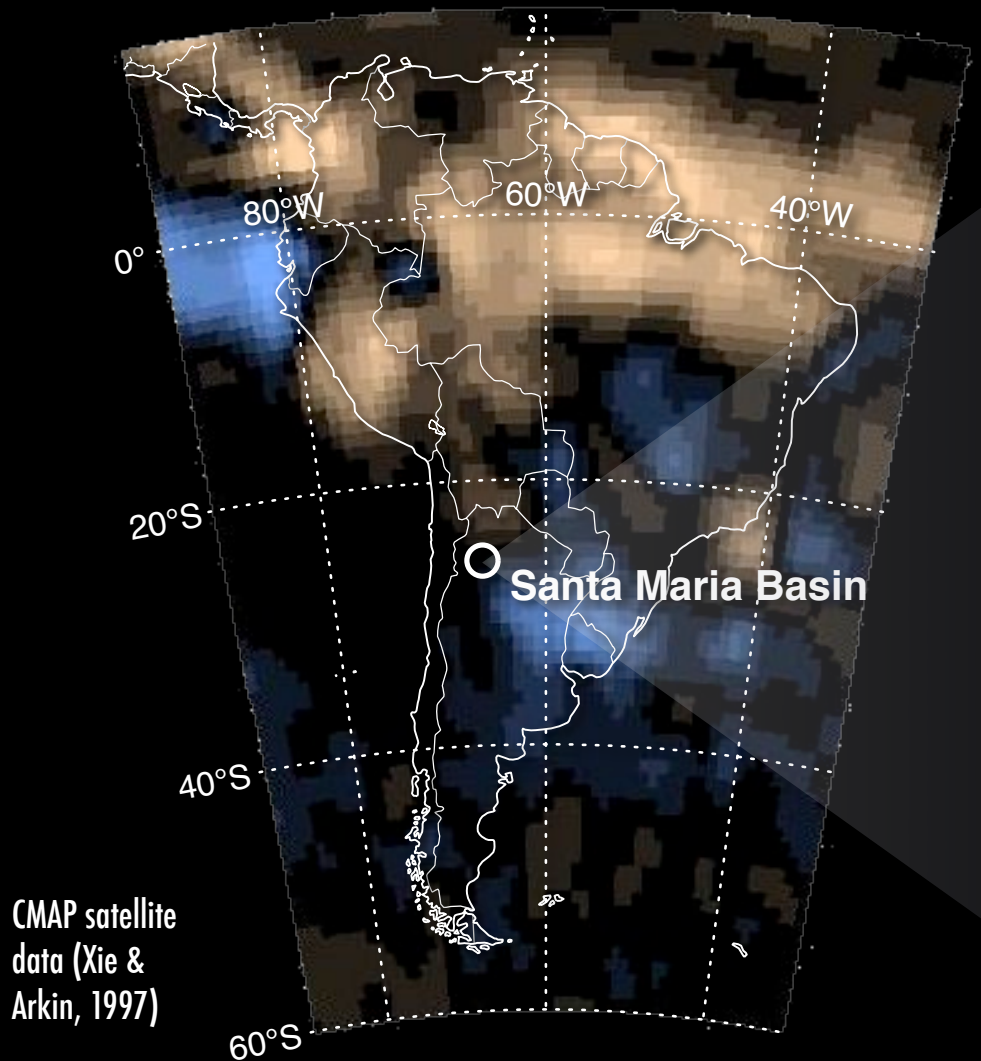


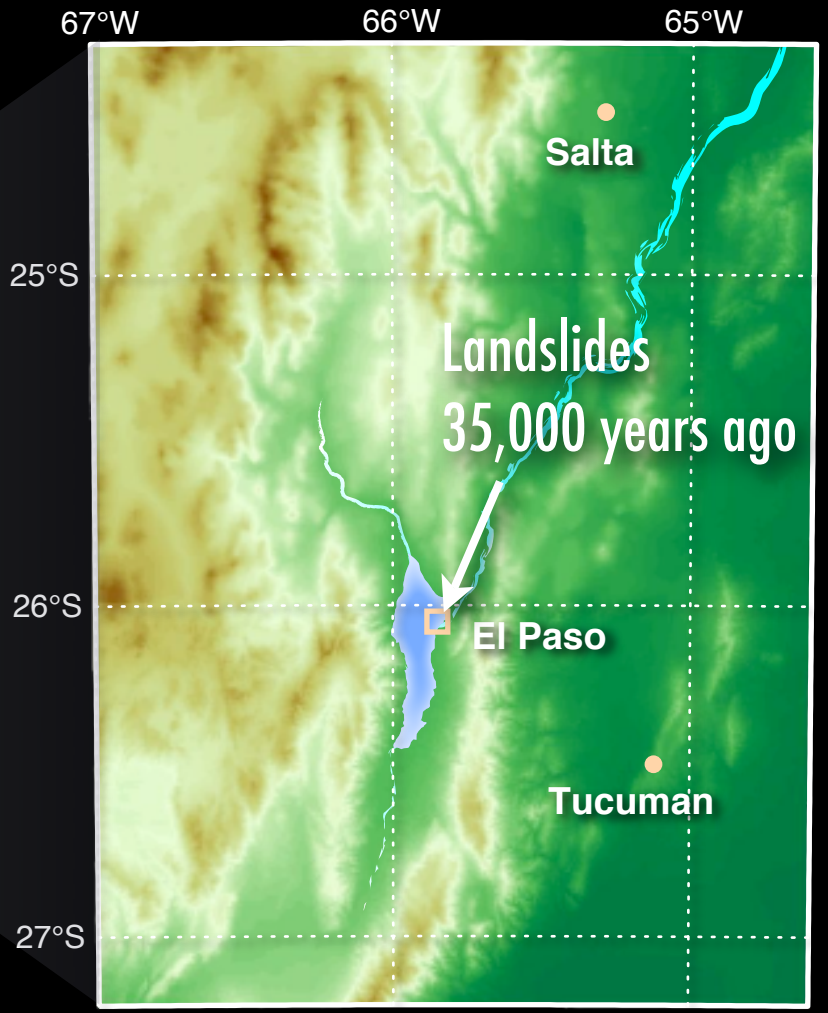
CMAP satellite data (Xie & Arkin, 1997)



Santa Maria Basin

El Niño Southern Oscillation



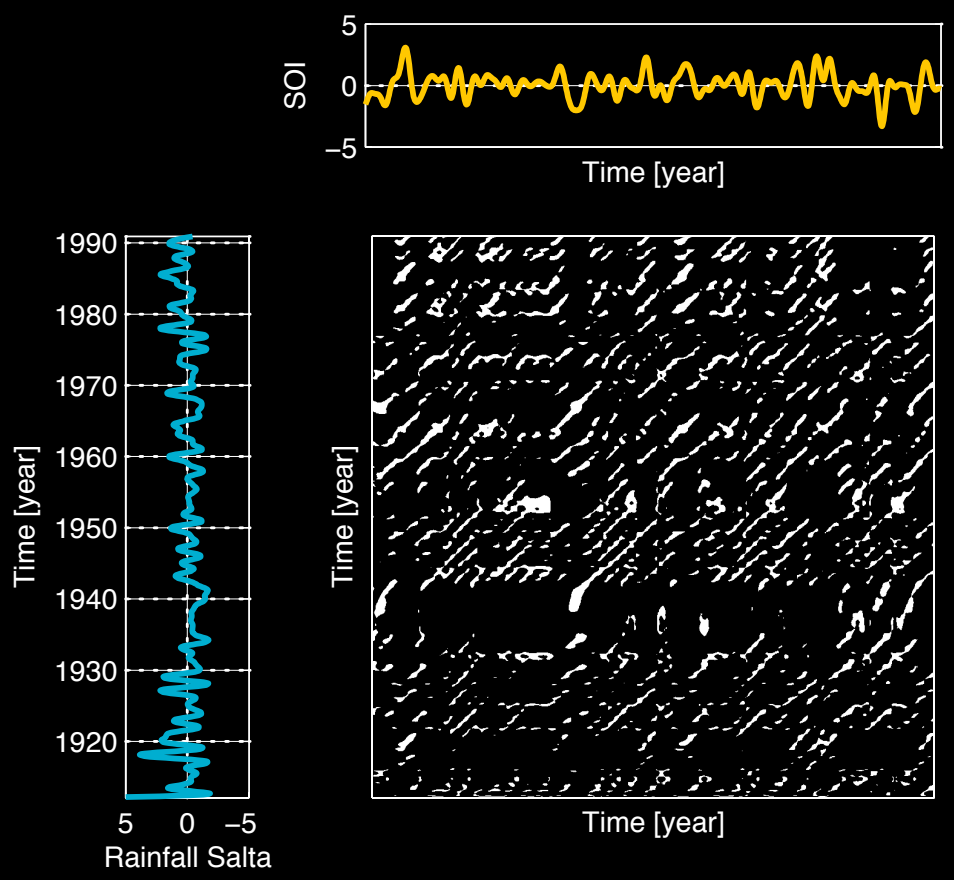


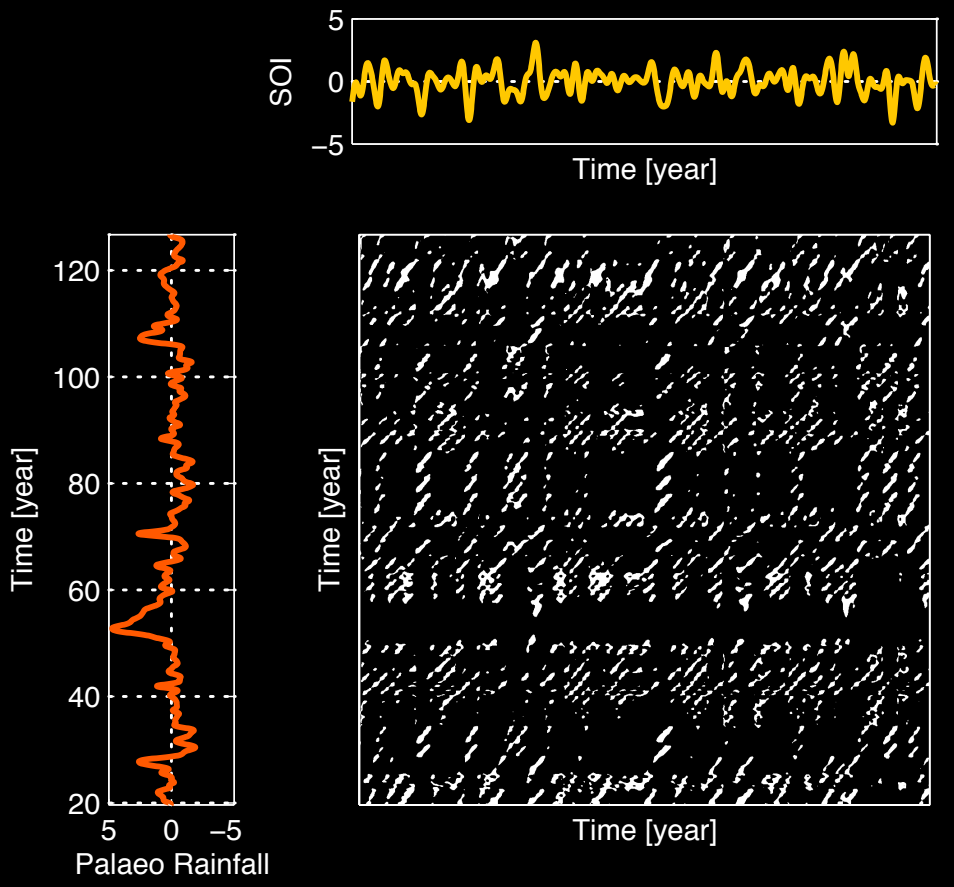
Santa Maria Basin

Landslides and Lake Sediments

Varved Lake Sediments

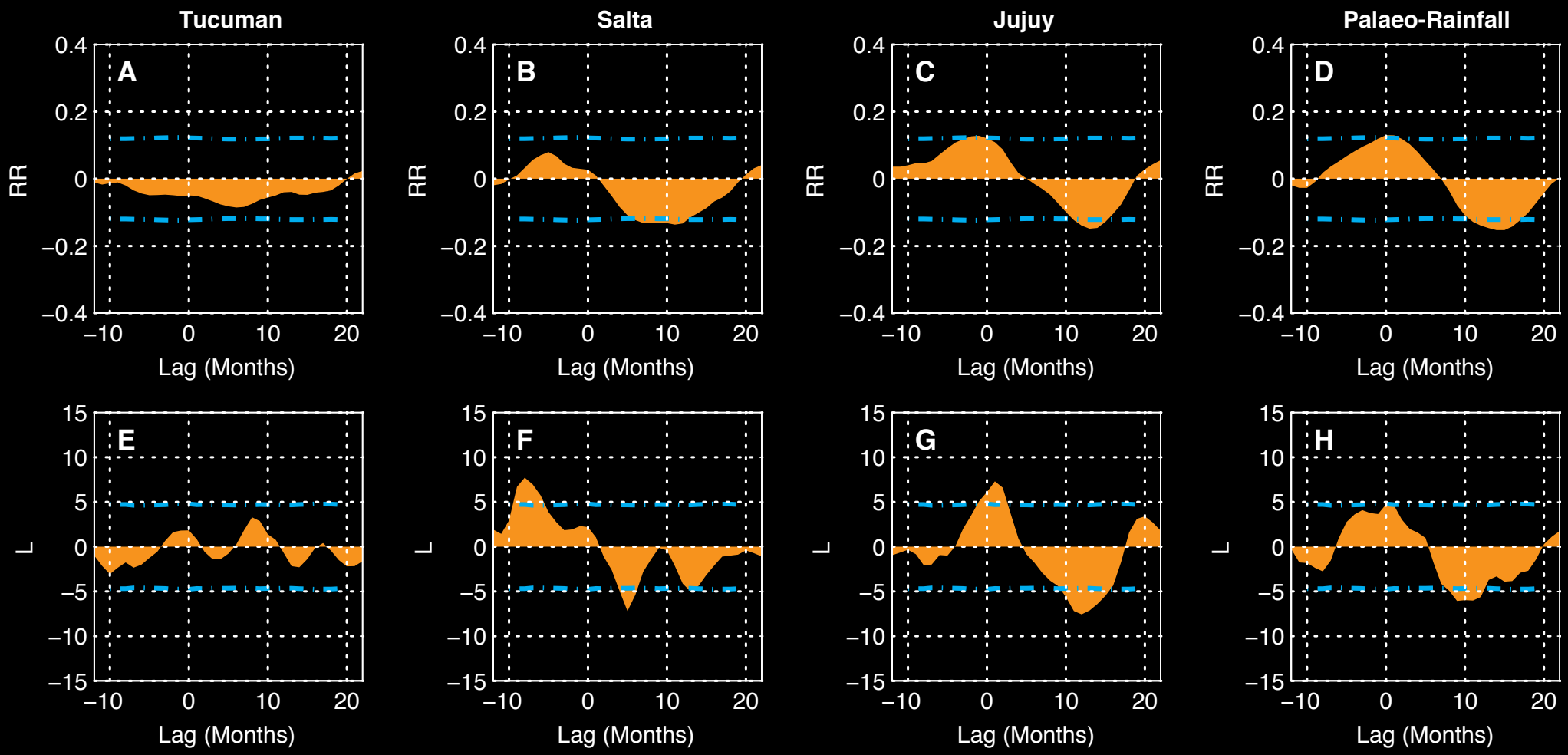
Cross Recurrence Plot





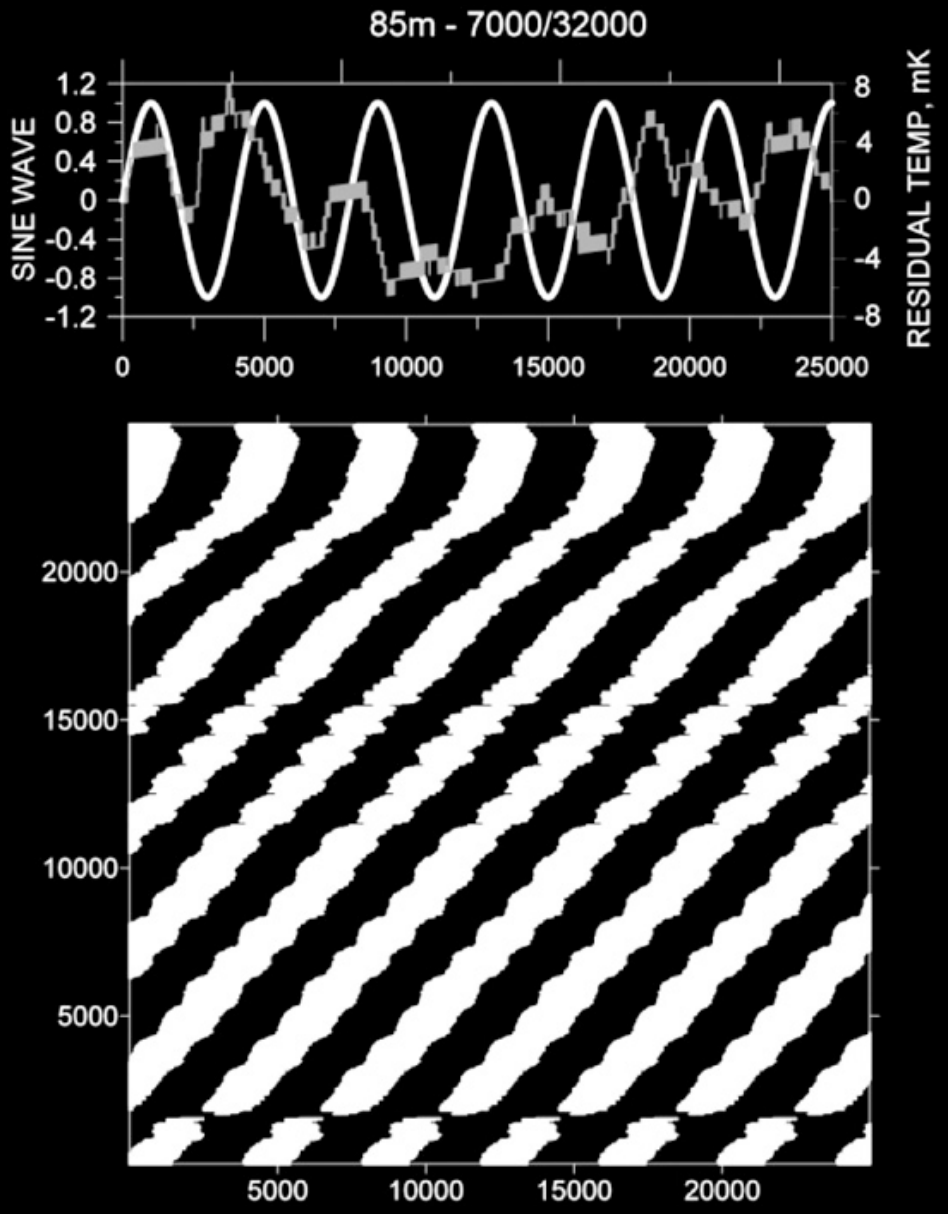
Cross Recurrence Quantification

South ⟷ North



Similar patterns of interrelation

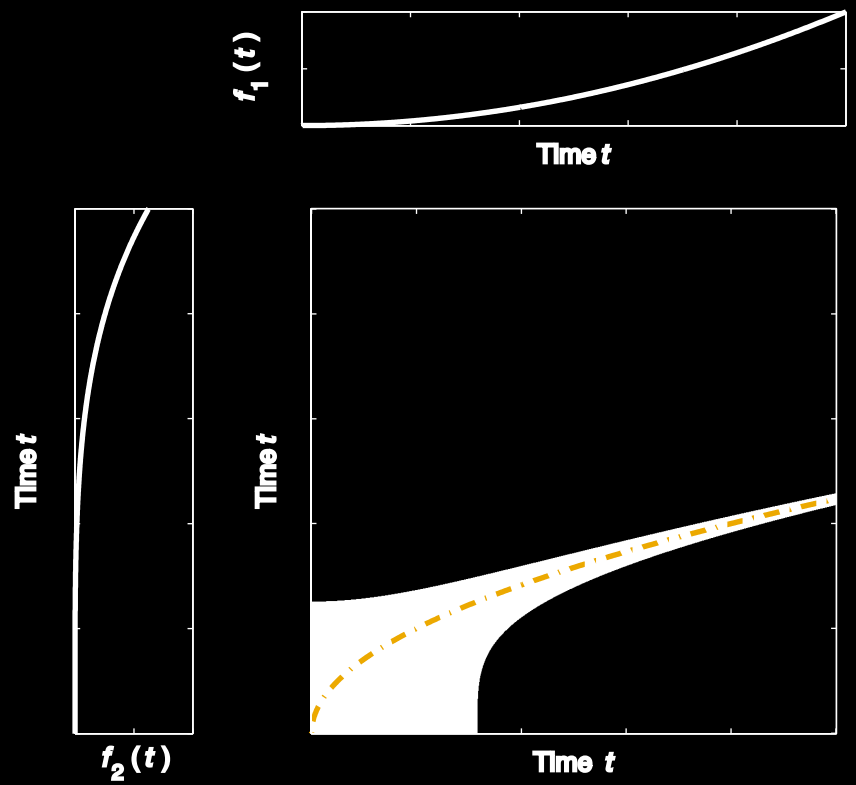
Intra-Borehole Convection Dynamics



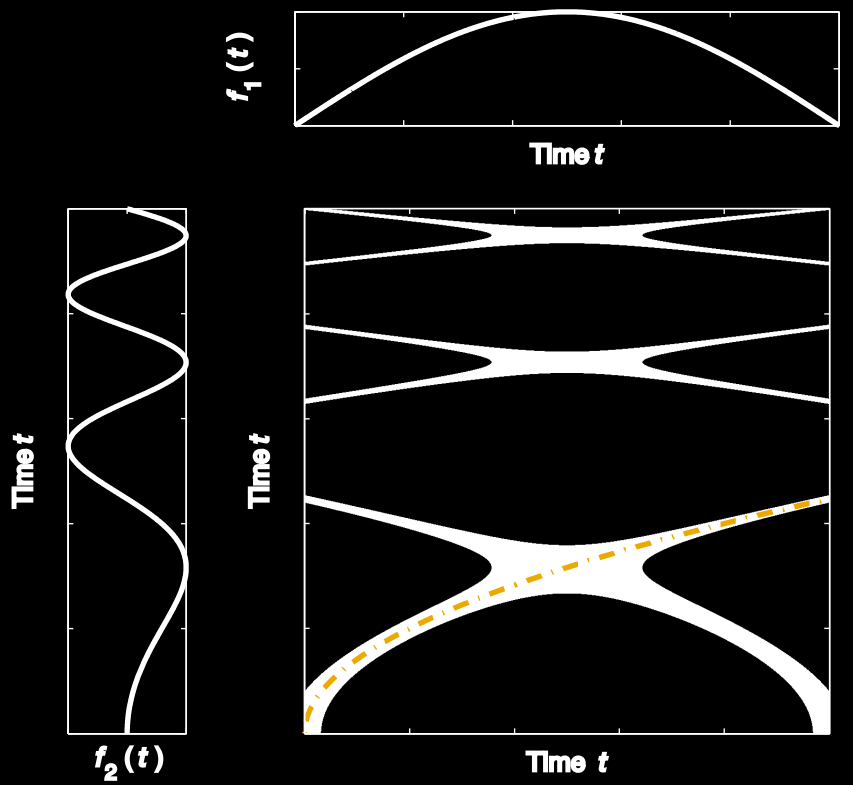
Čermák, Šafandra, Krešl, Studia Geophysica et Geodaetica, 52, 2008

Time Scale Alignment

$f(t) = t^2$



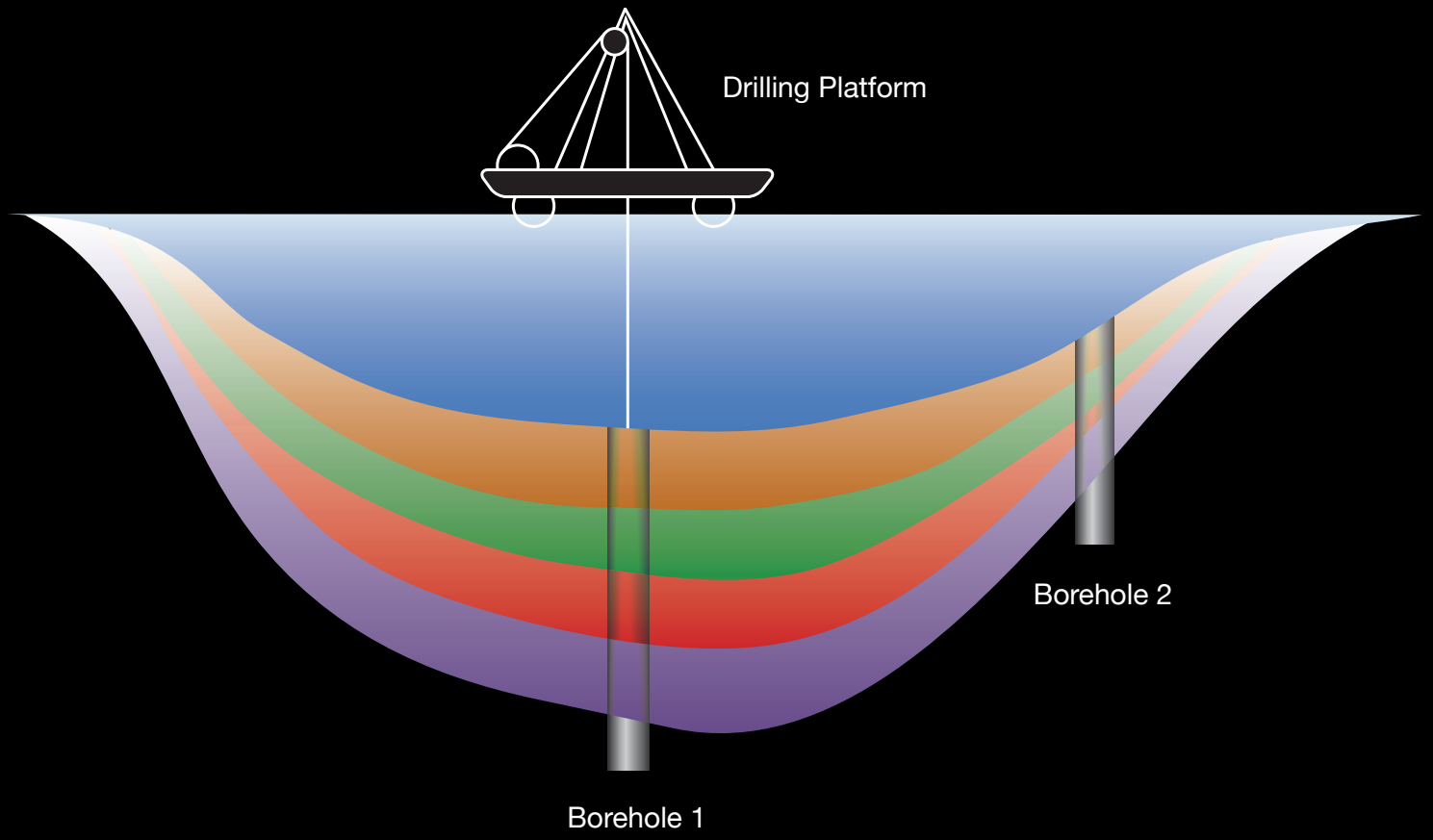
$f(t) = \sin(\pi t)$

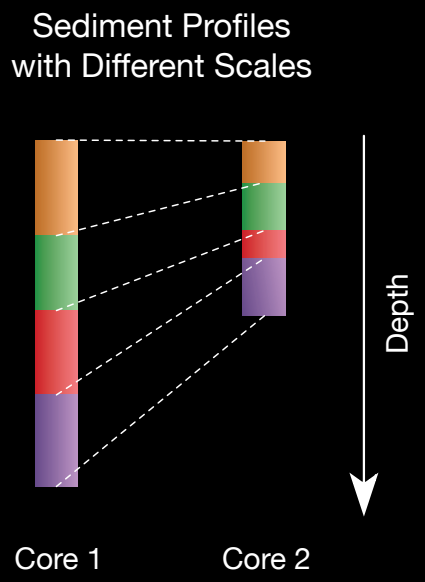


$T_1 = t \qquad T_2 = 5t^2$

Marwan, Kurths, Physics Letters A, 336, 2005

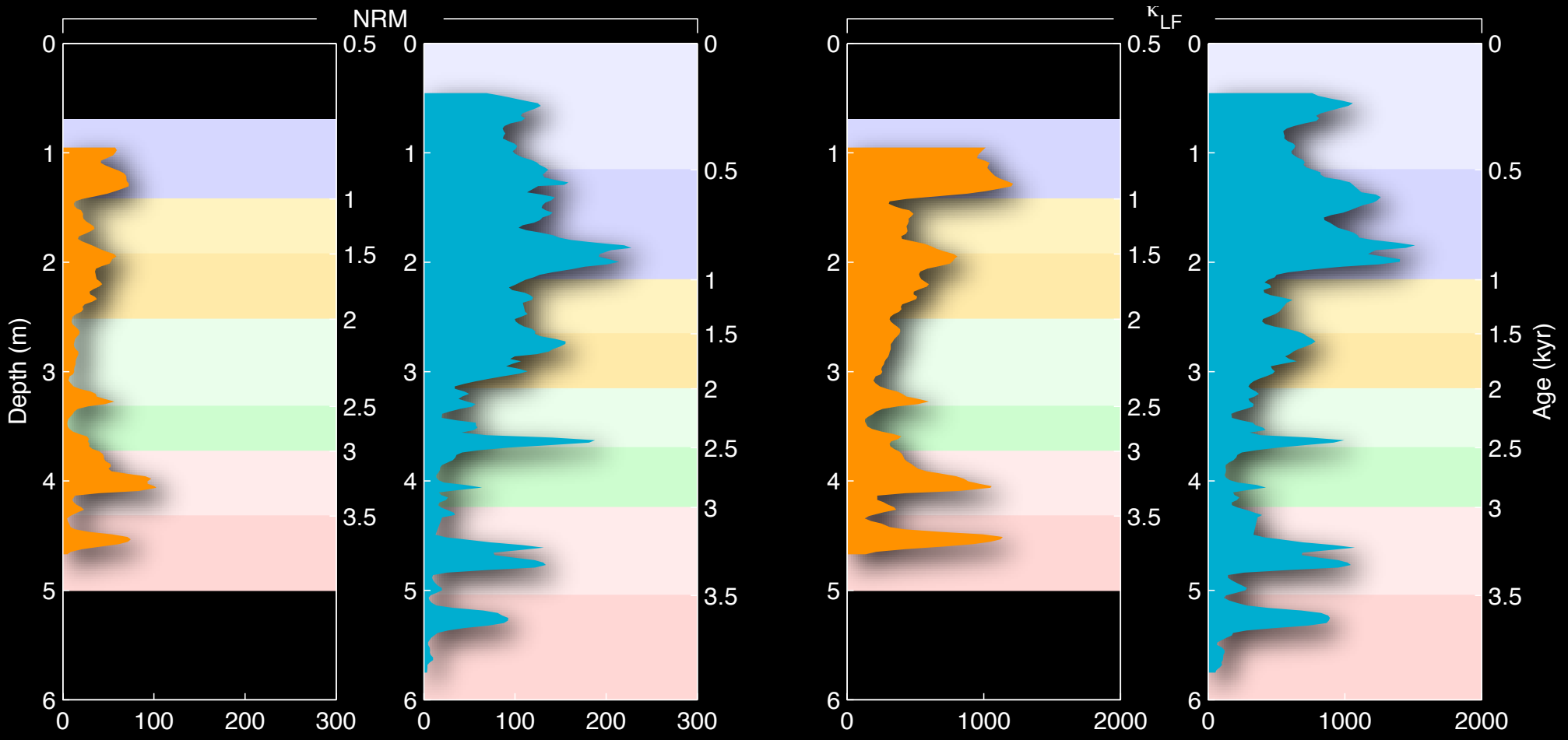
Time Scale Alignment



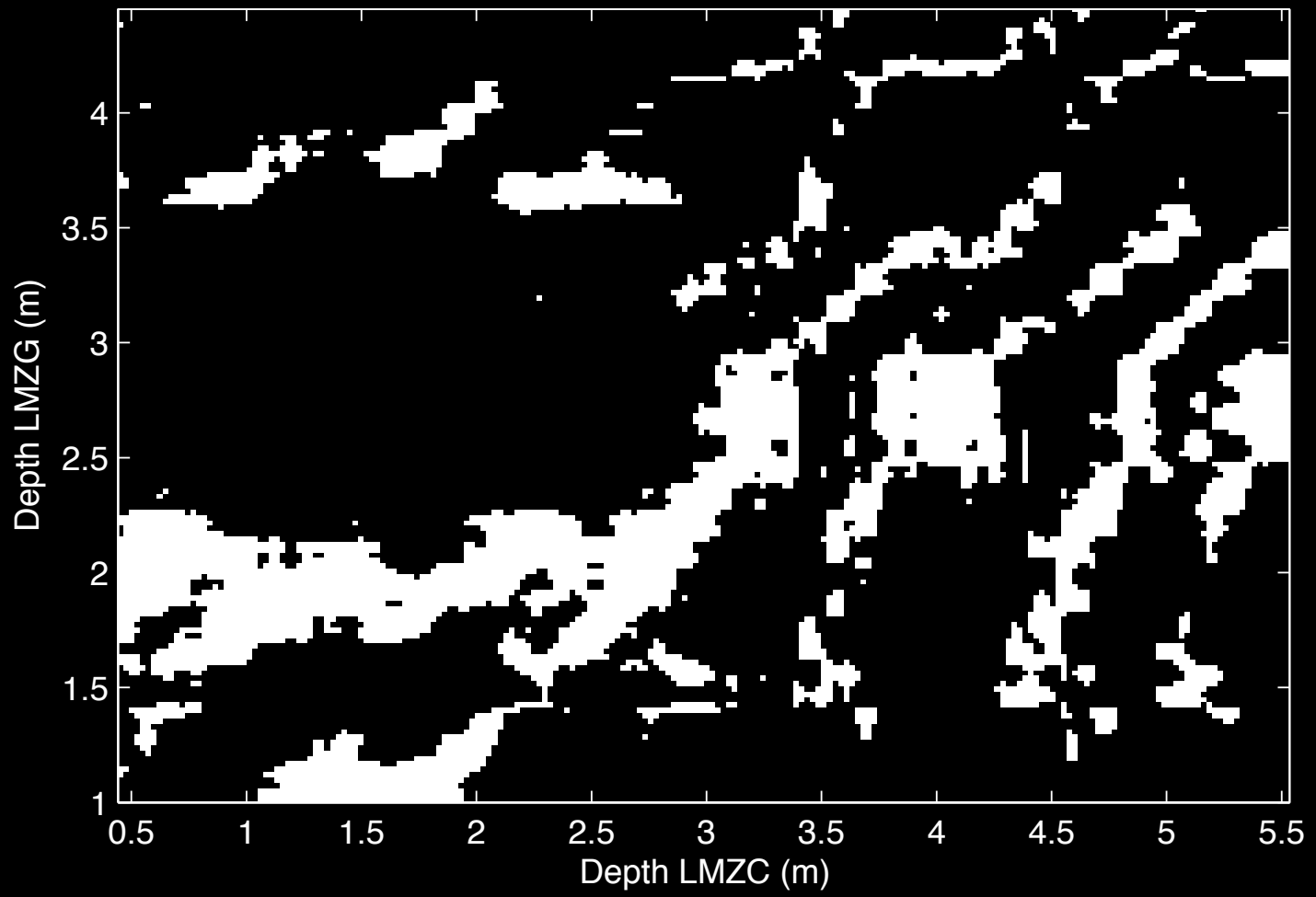


Marwan, Thiel, Nowaczyk, Nonlinear Processes in Geophysics, 9, 2002

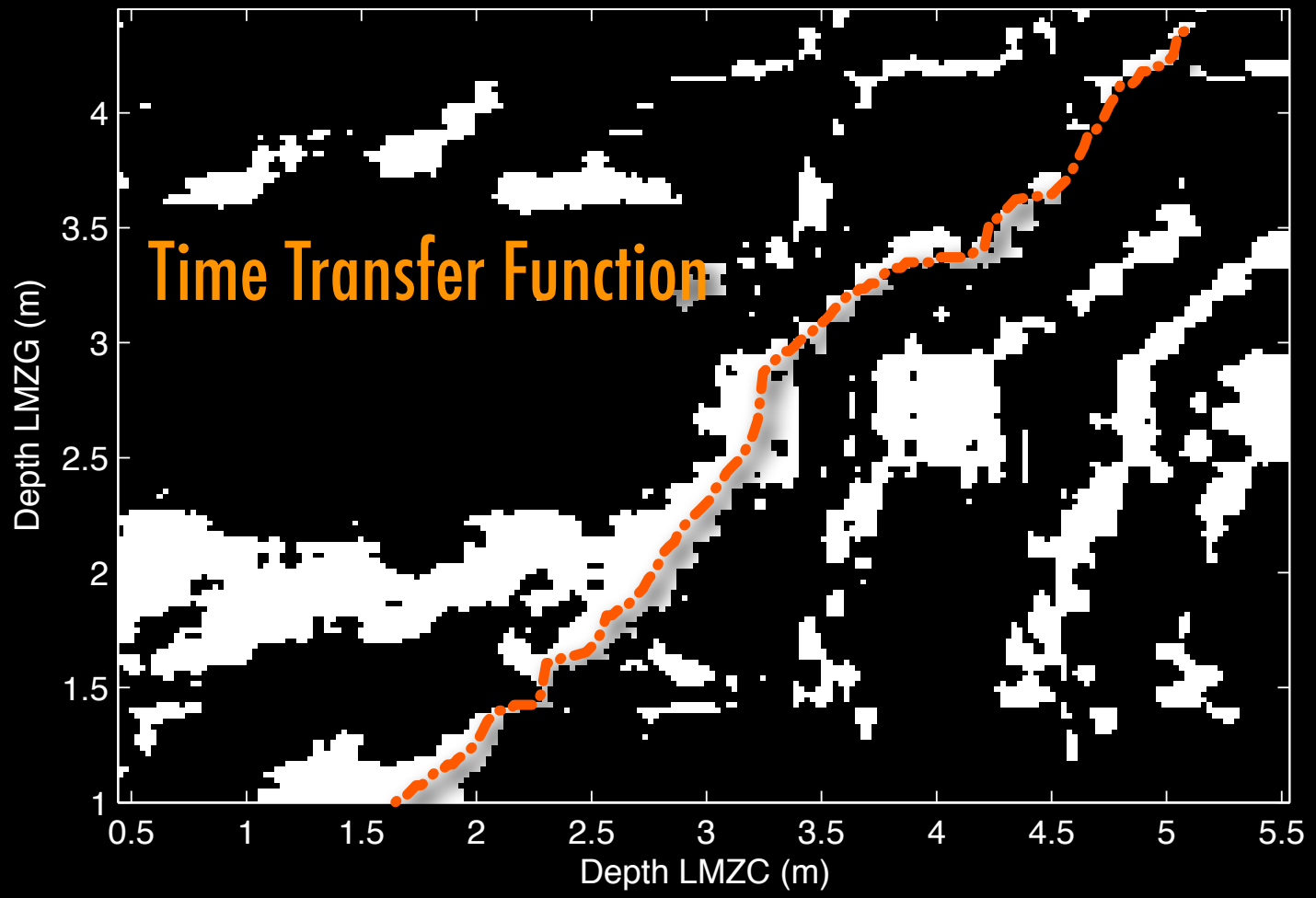
Rock-Magnetic Measurements



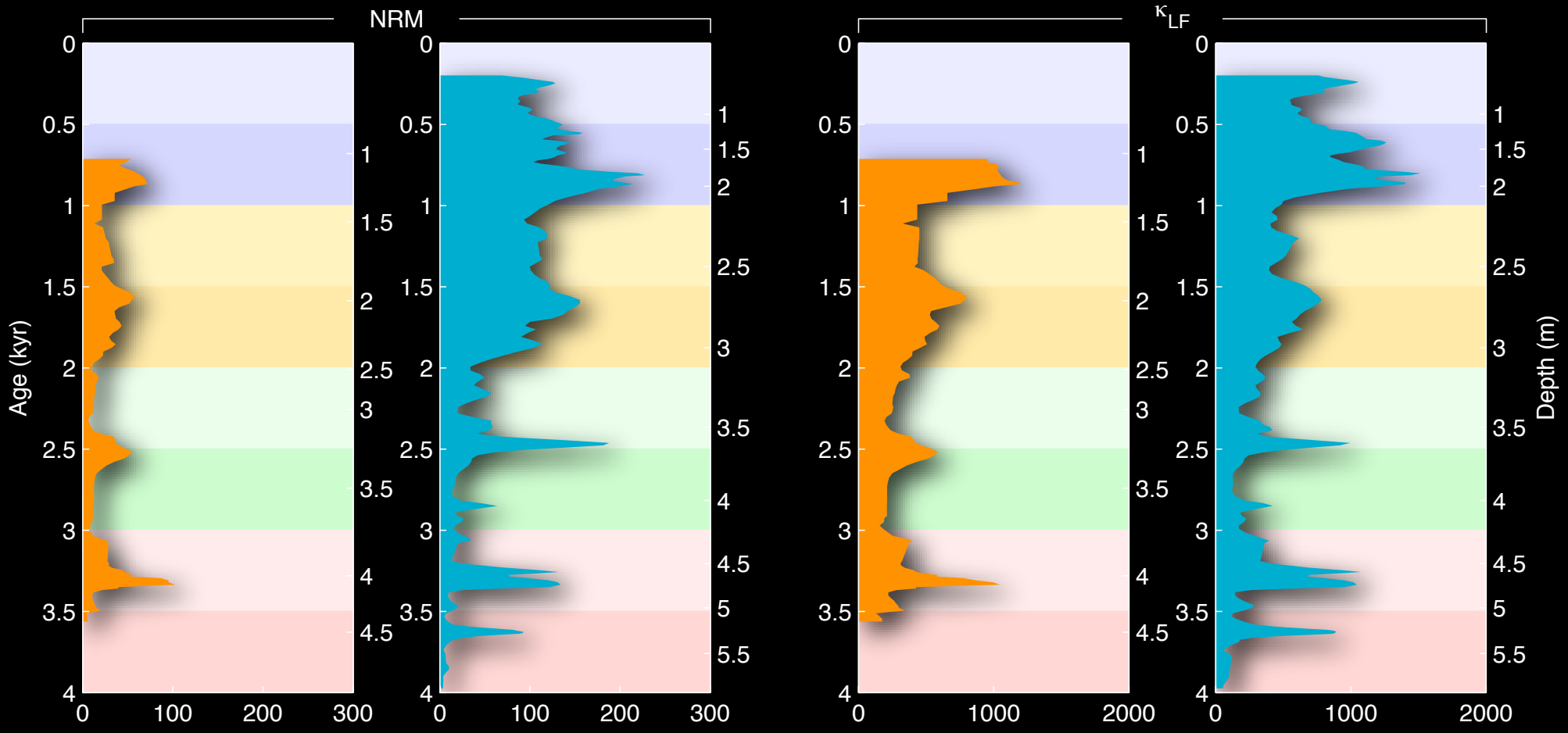
Cross Recurrence Plot



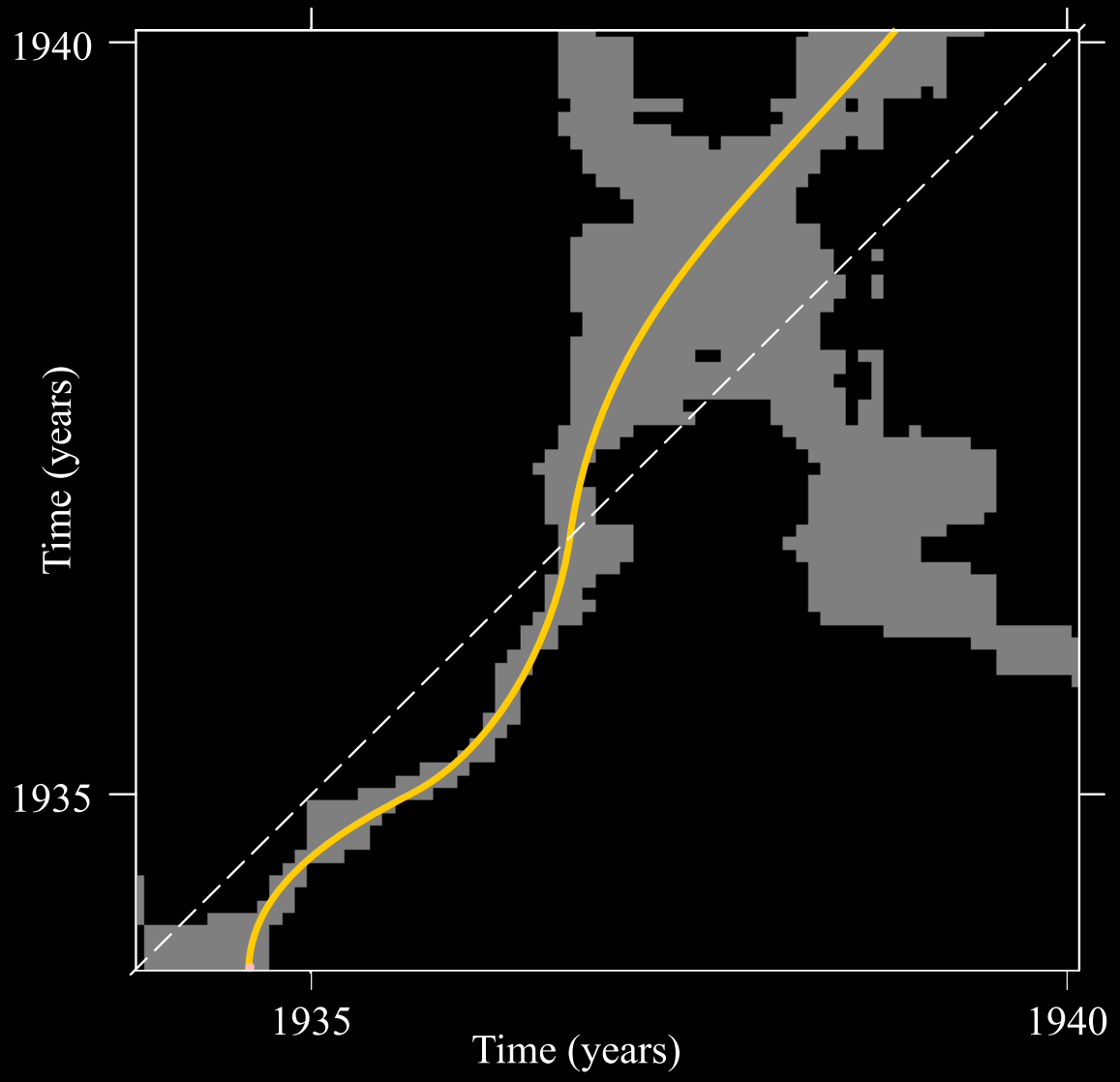
Cross Recurrence Plot



Time Scale Adjustment

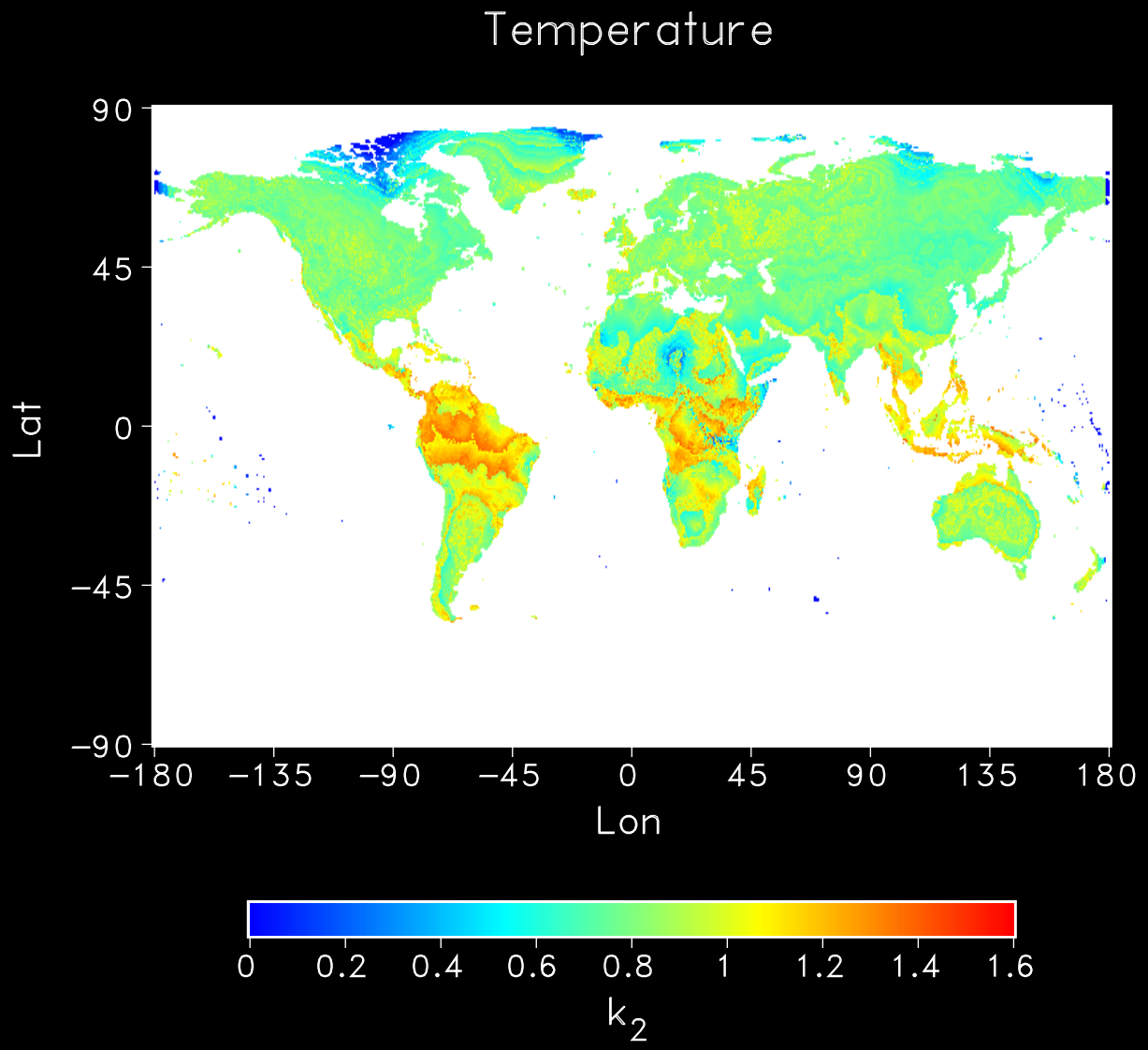


Hemispherical Asymmetry in Solar Activity



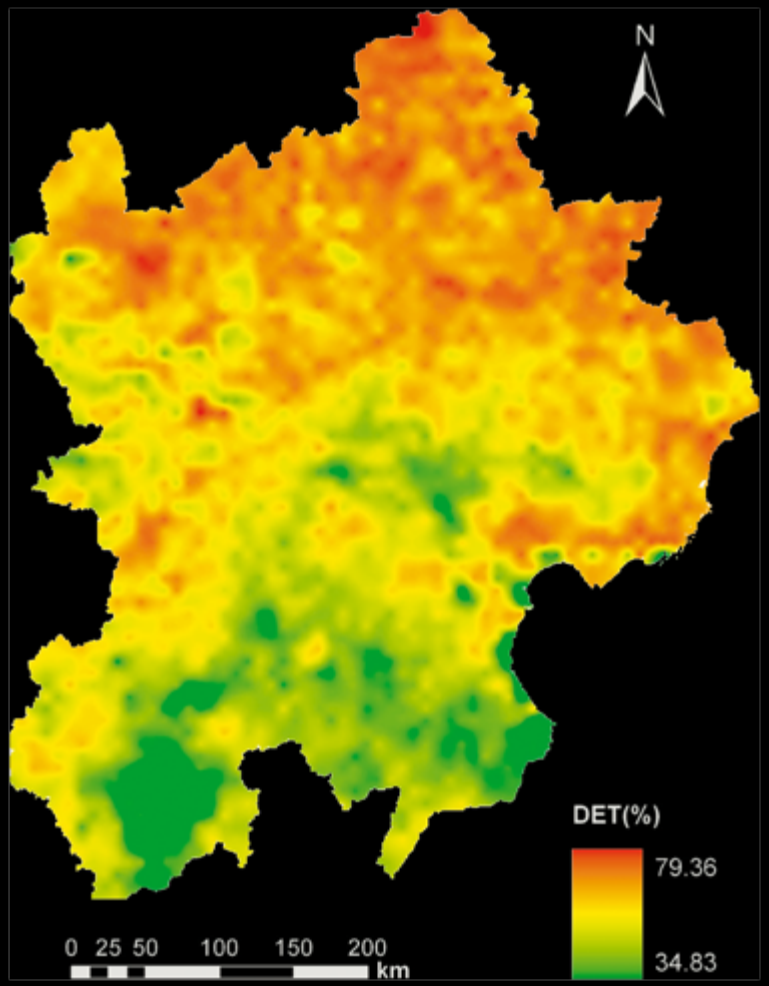
Zolotova, Ponyavin, Astronomy & Astrophysics, 449, 2006

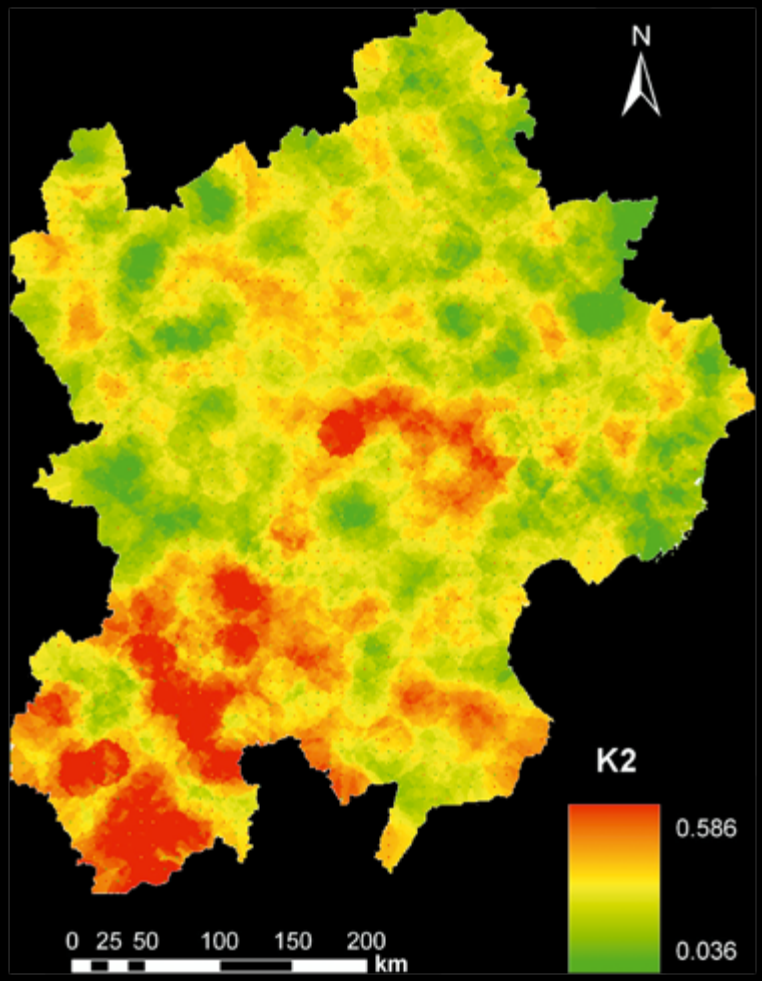
Long-term Predictability of Global Temperature



von Bloh, Romano, Thiel, Nonlinear Processes in Geophysics, 12, 2005

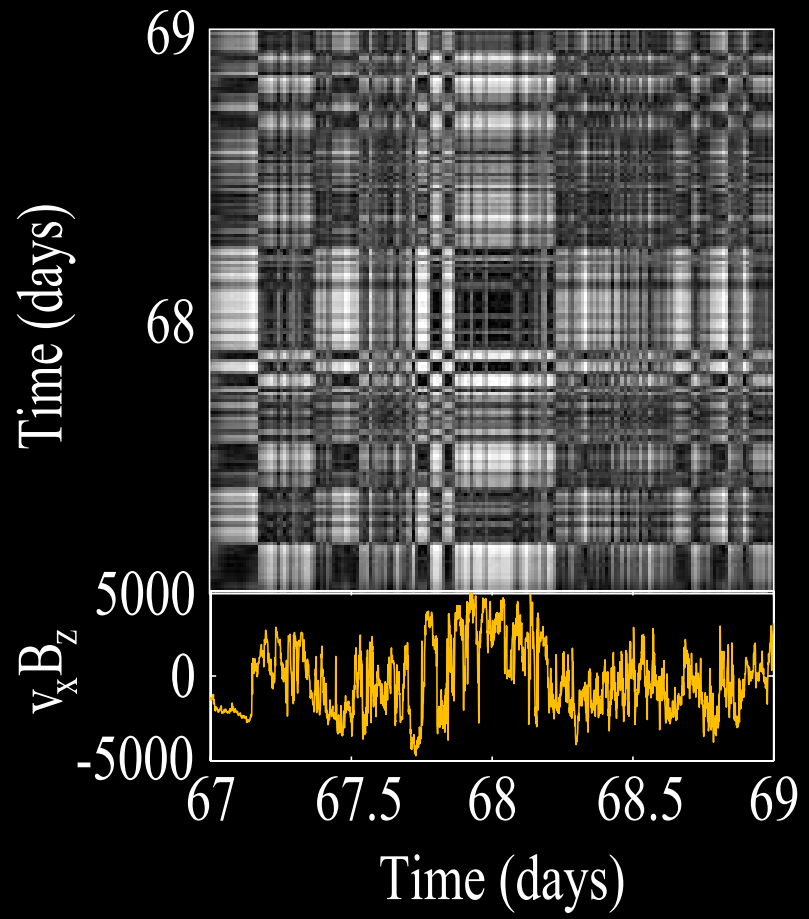
Remote Sensing Analysis (Vegetation Index)

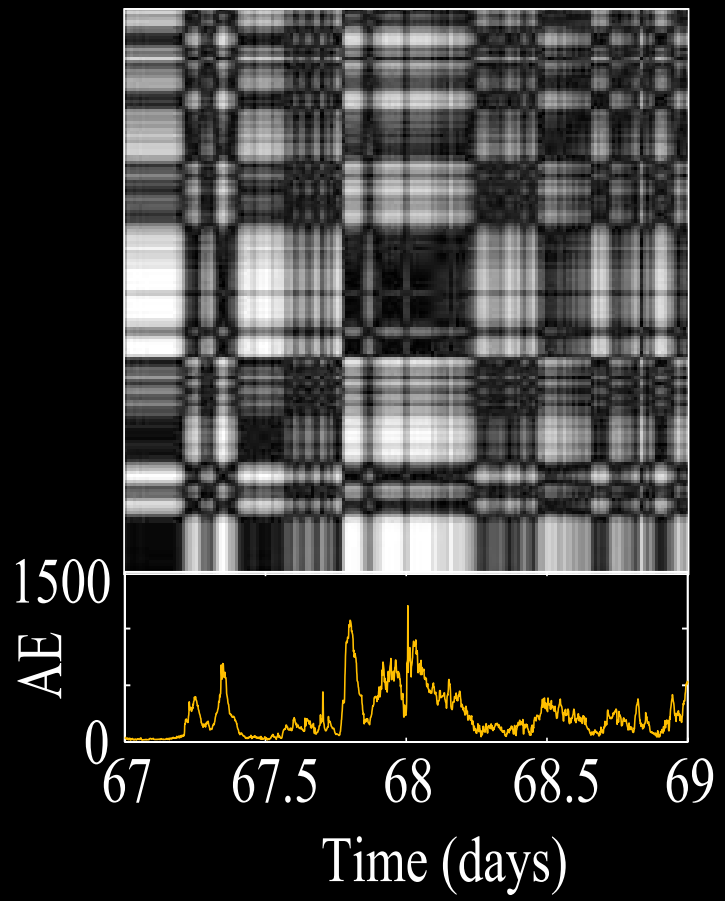




Li, Zhao, Liu, European Physical Journal ST, 164, 2008

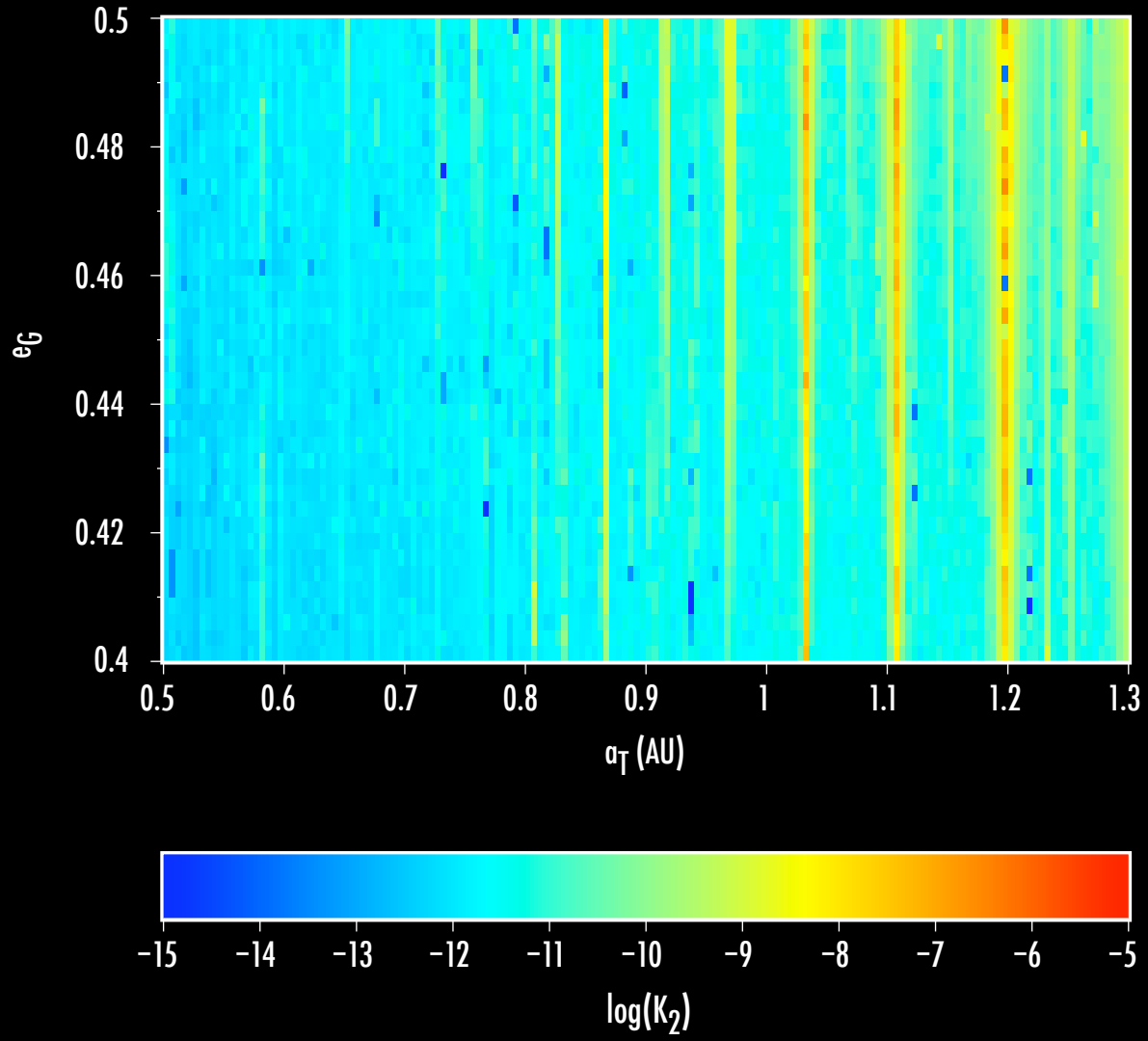
Impact of Solar Wind on Earth – Dynamics of Magnetosphere





March, Chapman, Dendy, Geophysical Research Letters, 32, 2005
Unnikrishnan, Annales Geophysicae, 26, 2008

Search for Habitable Extra-solar Planets



Asghari et al., Astronomy & Astrophysics, 426, 2004

Summary

- powerful techniques
- wide applicability in Earth sciences

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- further information:
  - recurrence plot symposia (Montreal 8/2009)
  - recurrence plot review report
  - recurrence plot website